ATLAS
OF
FINITE GROUPS

Maximal Subgroups and Ordinary Characters for Simple Groups

BY

J. H. CONWAY
R. T. CURTIS
S. P. NORTON
R. A. PARKER
R. A. WILSON

with computational assistance from

J. G. THACKRAY

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THE GROUPS

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$L_2(27)$	18	He	104
$L_2(29)$	20	$O_7(3)$	106 (108)
$L_2(31)$	21	$S_6(3)$	110 (112)
$A_8 \cong L_4(2)$	22	$G_2(5)$	114
$L_3(4)$	23	$U_6(2)$	115
$U_4(2) \cong S_4(3)$	26	$R(27)$	122 (123)
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INTRODUCTION

1

A survey of the finite simple groups

1. Prelimaries

The theory of groups is the theory of the different possible kinds of symmetry, and as such finds applications throughout mathematics and the sciences whenever symmetrical objects or theories are being discussed. We originally conceived this ATLAS as a work whose aim would be to convey every interesting fact about every interesting finite group. This, of course, was a tall order, only partly fulfilled by the present volume!

The most interesting groups are the simple groups, and perhaps the most impressive result of finite group theory is the recent complete classification of finite simple groups. This volume tabulates, for various finite simple groups G, the following information:

(i) the order of G, and the orders of its Schur multiplier and outer automorphism group;

(ii) various 'constructions' for G, or for concepts closely associated with G;

(iii) information about the subgroups of G and of its automorphism groups (often the complete list of maximal subgroups);

(iv) a 'compound character table' from which it is easy to read off the ordinary character table of G and those of various closely related groups.

Some of the terminology used here will be defined in later sections of this introduction. In general, we expect the reader to be familiar with the basic concepts of group theory and representation theory. The main aim of our definitions is to introduce the notation used later in the ATLAS, which is sometimes idiosyncratic.

According to the classification theorem, the finite simple groups fall into certain infinite families, with 26 extremely interesting exceptions called the *sporadic groups*. Many of the early terms in the infinite families display intriguing special behaviour. Within each family, we have continued until the groups became either too big or too boring, and have tried hard to include any group that had a property or properties not explained by its membership within that family. In doubtful cases, our rule was to *think how far the reasonable person would go, and then go a step further.* For the smaller families, this has had the effect that the last group included gives a very good idea of the structure of the typical element of the family. We have included all of the sporadic groups.

We have chosen to print the particular information described above partly for its obvious utility, and partly because we could manage to obtain a reasonably uniform coverage. Our first priority was to print the ordinary character table, which is beyond doubt the most compendious way of conveying information about a group to the skilled reader. Moreover, the computation of character tables is almost routine for groups of moderate size, while for several notorious groups of immoderate size it has been achieved after heroic efforts.

The complete list of maximal subgroups is very useful in internal investigations of the group, mainly because it reduces the difficult problem of deciding what group certain elements generate to the mere enumeration of cases. It is considerably harder to find all maximal subgroups than to compute the character table, but the answers are now known for a surprisingly large number of groups, and useful partial information is often available for other cases.

The Schur multipliers and outer automorphism groups of all finite simple groups are now known. (It should be noted that the computation of multipliers is delicate, and there have been several amendments to previously published lists.) For most of the groups treated in the ATLAS we also give the character tables for the corresponding covering groups and extensions by automorphisms. The individual tables are easily read off from the handy 'compound character table' into which we have combined them.

The typical expert in this subject knows a vast number of isolated facts about particular groups, but finds it very hard to convey this rather formless information to the earnest student. We hope that such students will find that our 'constructions' answer some of their questions. Usually the object constructed is the simple group itself or a close relative, but we allow it to be *any* concept connected with the group. Most of the interesting facts about our groups are fairly naturally described in these constructions, and we have not hesitated to stretch the 'constructions' format so as to swallow the odd counter-example.

We plead indulgence for the many defects in these rather hasty descriptions. The 'constructions' format arose almost by oversight at a fairly late stage in the preparation of our material, and we have not really had time to organize the information properly. However, we feel that they are already useful enough to be presented to the reader now, rather than postponed to a later volume.

In the rest of this introduction, we shall describe the finite simple groups and their classification, and then explain how to read the tables and text in the main body of the ATLAS.

2. The finite simple groups

Since finite simple groups are the main topic of this ATLAS, we had better describe them!

Combining the results of a large number of authors, we have:

The classification theorem for finite simple groups
The finite simple groups are to be found among:
- the cyclic groups of prime order
- the alternating groups of degree at least 5
- the Chevalley and twisted Chevalley groups, and the Tits group
- the 26 sporadic simple groups.

We briefly describe these classes. More detailed descriptions appear in the subsequent sections of this introduction, and in the ATLAS entries for the individual groups.

The cyclic groups of prime order are the only abelian (commutative) simple groups. There are so many ways in which these groups behave differently to the non-abelian simple groups that the term 'simple group' is often tacitly understood to mean 'non-abelian simple group'.

The fact that the alternating groups of degree at least 5 are non-abelian simple groups has been known since Galois, and implies that generic algebraic equations of degree at least 5 are insoluble by radicals. In fact the true origin of finite group theory is Galois theory.

The Chevalley and twisted Chevalley groups generalize the familiar 'classical' groups (the linear, unitary, symplectic, and orthogonal groups), which are particular cases. There is a sense in which the vast majority of finite simple groups belong to this class. The Tits group, which might have been considered as a sporadic group, is a simple subgroup of index 2 in the twisted group ${}^2F_4(2)$. We shall describe all the Chevalley groups in later sections.

3. The sporadic groups

The sporadic simple groups may be roughly sorted as the *Mathieu groups*, the *Leech lattice groups*, Fischer's *3-transposition groups*, the further *Monster centralizers*, and the half-dozen *oddments*.

For about a hundred years, the only sporadic simple groups were the Mathieu groups M_{11}, M_{12}, M_{22}, M_{23}, M_{24}. These were described by Emil Mathieu in 1861 and 1873 as highly transitive permutation groups, the subscripts indicating the numbers of letters permuted. (It is interesting to note that Mathieu, in the early language of Galois theory, spoke of *highly transitive functions* of several letters, rather than *highly transitive groups*.) One of the first great exercises in representation theory was the computation by Frobenius of the character table of M_{24}. Frobenius also showed that all the Mathieu groups were subgroups of M_{24} (the containment of M_{12} in M_{24} was not known to Mathieu).

The group M_{24} is one of the most remarkable of all finite groups. Many properties of the larger sporadic groups reduce on examination to properties of M_{24}. This centenarian group can still startle us with its youthful acrobatics.

The automorphism group of the Leech lattice, modulo a centre of order 2, is the Conway group Co_1, and by stabilizing sublattices of dimensions 1 and 2 we obtain the other Conway groups Co_2, Co_3, the McLaughlin group M^cL, and the Higman–Sims group HS. The sporadic Suzuki group Suz, and the Hall–Janko group $HJ = J_2$, can also be obtained from the Leech lattice by enlarging the ring of definition. The Leech lattice is a 24-dimensional Euclidean lattice which is easily defined in terms of the Mathieu group M_{24}.

The Fischer groups Fi'_{24}, Fi_{23}, Fi_{22}, were discovered by B. Fischer in the course of his enumeration of *3-transposition groups* (groups generated by a conjugacy class of involutions whose pairwise products have orders at most 3). Fi_{22} and Fi_{23} are 3-transposition groups, and Fi'_{24} has index 2 in the 3-transposition group Fi_{24}. There is a set S of 24 transpositions in Fi_{24} such that the elements of Fi_{24} that fix S as a whole realize exactly the permutations of M_{24} on S.

The Monster group, or Friendly Giant, which was independently discovered by B. Fischer and R. Griess, is the largest of the sporadic groups, with order

$$8080\ 17424\ 79451\ 28758\ 86459\ 90496\ 17107\ 57005\ 75436\ 80000\ 00000$$
$$= 2^{46}.3^{20}.5^{9}.7^{6}.11^{2}.13^{3}.17.19.23.29.31.41.47.59.71.$$

It involves all the sporadic groups so far mentioned. Many of them can be described as the non-abelian composition factors in the centralizers of various elements of the Monster. The remaining sporadic groups that can be so obtained are

$M = F_1$: the Monster itself
$B = F_{2+}$: Fischer's 'Baby Monster'
$Th = F_{3|3}$: the Thompson group
$HN = F_{5+}$: the Harada–Norton group, and
$He = F_{7+}$: the Held group.

The subscript on our F symbol specifies the relevant Monster element. In the same notation, we have $Co_1 = F_{2-}$, $Fi'_{24} = F_{3+}$. It hardly needs to be said that the Mathieu group M_{24} plays a vital role in the structure of the Monster.

The groups we referred to as the oddments are the remaining groups J_1, J_3, J_4 discovered by Janko, and those groups Ru, $O'N$, Ly discovered by Rudvalis, O'Nan, and Lyons. These were originally found in a variety of ways, but are probably now best constructed via matrix groups over finite fields. We

Table 1. The sporadic groups

Group	Order	Investigators	M	A
M_{11}	$2^4.3^2.5.11$	Mathieu	1	1
M_{12}	$2^6.3^3.5.11$	Mathieu	2	2
M_{22}	$2^7.3^2.5.7.11$	Mathieu	12	2
M_{23}	$2^7.3^2.5.7.11.23$	Mathieu	1	1
M_{24}	$2^{10}.3^3.5.7.11.23$	Mathieu	1	1
J_2	$2^7.3^3.5^2.7$	Hall, Janko	2	2
Suz	$2^{13}.3^7.5^2.7.11.13$	Suzuki	6	2
HS	$2^9.3^2.5^3.7.11$	Higman, Sims	2	2
McL	$2^7.3^6.5^3.7.11$	McLaughlin	3	2
Co_3	$2^{10}.3^7.5^3.7.11.23$	Conway	1	1
Co_2	$2^{18}.3^6.5^3.7.11.23$	Conway	1	1
Co_1	$2^{21}.3^9.5^4.7^2.11.13.23$	Conway, Leech	2	1
He	$2^{10}.3^3.5^2.7^3.17$	Held/Higman, McKay	1	2
Fi_{22}	$2^{17}.3^9.5^2.7.11.13$	Fischer	6	2
Fi_{23}	$2^{18}.3^{13}.5^2.7.11.13.17.23$	Fischer	1	1
Fi'_{24}	$2^{21}.3^{16}.5^2.7^3.11.13.17.23.29$	Fischer	3	2
HN	$2^{14}.3^6.5^6.7.11.19$	Harada, Norton/Smith	1	2
Th	$2^{15}.3^{10}.5^3.7^2.13.19.31$	Thompson/Smith	1	1
B	$2^{41}.3^{13}.5^6.7^2.11.13.17.19.23.31.47$	Fischer/Sims, Leon	2	1
M	$2^{46}.3^{20}.5^9.7^6.11^2.13^3.17.19.23.29.31.41.47.59.71$	Fischer, Griess	1	1
J_1	$2^3.3.5.7.11.19$	Janko	1	1
$O'N$	$2^9.3^4.7^3.5.11.19.31$	O'Nan/Sims	3	2
J_3	$2^7.3^5.5.17.19$	Janko/Higman, McKay	3	2
Ly	$2^8.3^7.5^6.7.11.31.37.67$	Lyons/Sims	1	1
Ru	$2^{14}.3^3.5^3.7.13.29$	Rudvalis/Conway, Wales	2	1
J_4	$2^{21}.3^3.5.7.11^3.23.29.31.37.43$	Janko/Norton, Parker, Benson, Conway, Thackray	1	1

give a brief table of the appropriate dimensions and fields:

group:	J_1	J_3	J_4	Ru	$O'N$	Ly
dimension:	7	9	112	28	45	111
field:	$\mathbb{F}_{11}$	$\mathbb{F}_4$	$\mathbb{F}_2$	$\mathbb{F}_2$	$\mathbb{F}_7$	$\mathbb{F}_5$

The 112-dimensional matrices for J_4 are rather complicated, but explicit matrices for the other five cases can be found in the appropriate 'constructions' sections of this ATLAS. The construction for J_4 makes heavy use of the Mathieu group M_{24}, while those for the O'Nan and Lyons groups involve M_{11}. The indicated representation of the Rudvalis group lifts to a complex representation of its double cover.

It has often happened that a sporadic group was predicted to exist some time before its construction, in sufficient detail to give its order, various local subgroups, and sometimes the character table. In such cases we have separated names by a slash in Table 1. Those who were mainly concerned with the prediction appear before the slash; those mainly concerned with the construction after it. Elsewhere in the ATLAS, we usually name the group after its predictor(s) only, in order to avoid a multiplicity of names, and we recommend this practice. The columns headed 'M' and 'A' give the orders of the Schur multiplier and outer automorphism group, which are both cyclic for each sporadic group. For the simple alternating groups ($n \geqslant 5$) we have $M=2$ (see page 236) except that A_6 and A_7 have $M=6$, and $A=2$ except that A_6 has $A=2^2$.

4. Finite fields

The description of the classical groups, and of the Chevalley and twisted Chevalley groups that generalize them, involves heavy use of the properties of finite fields. We summarize the main facts here.

The order of any finite field is a prime power, and for each prime power q there is up to isomorphism just one such field, which we call $\mathbb{F}_q$. The existence of these fields was established by Galois, and their uniqueness for each possible order by Moore. These fields are often referred to as Galois fields, and Dickson's notation $GF(q)$ is often used for $\mathbb{F}_q$.

If p is prime, the field $\mathbb{F}_p$ is just $\mathbb{Z}/p\mathbb{Z}$, the integers taken modulo p, while for $q=p^f$, the field $\mathbb{F}_q$ may be constructed from $\mathbb{F}_p$ by adjoining a root of any irreducible equation of degree f over $\mathbb{F}_p$. It may also be defined non-constructively as the set of solutions of $x^q=x$ in the algebraic closure of $\mathbb{F}_p$.

Since $\mathbb{F}_q$ is a vector space of dimension f over $\mathbb{F}_p$, its additive group is the direct sum of f cyclic groups of order p. The multiplicative group is cyclic of order $q-1$, and a generator for this group is called a *primitive root* for $\mathbb{F}_q$. There are exactly $\phi(q-1)$ primitive roots, where $\phi(n)$ is Euler's totient function. Although primitive roots are easily found by inspection in any particular case, there is no simple formula which gives a primitive root in $\mathbb{F}_q$ for an arbitrary given q.

The automorphism group of $\mathbb{F}_q$ is cyclic of order f, and consists of the maps $x \to x^r$ $(r=1, p, p^2, \ldots, p^{f-1})$, which are called the *Frobenius maps*. The subfields of $\mathbb{F}_q$ are those $\mathbb{F}_r$ for which q is a power of r, and each subfield is the fixed field of the corresponding Frobenius map. If $q=r^s$, then the characteristic polynomial over $\mathbb{F}_r$ of an element x of $\mathbb{F}_q$ is

$$(t-x)(t-x^r)\ldots(t-x^{r^{s-1}}),$$

The coefficients of this polynomial lie in $\mathbb{F}_r$. In particular, the *trace* and *norm* functions from $\mathbb{F}_q$ to $\mathbb{F}_r$ are defined by

$$Tr_{\mathbb{F}_q \to \mathbb{F}_r}(x) = x + x^r + x^{r^2} + \ldots + x^{r^{s-1}}$$
$$N_{\mathbb{F}_q \to \mathbb{F}_r}(x) = x \,.\, x^r \,.\, x^{r^2} \,.\ldots.\, x^{r^{s-1}} = x^{(r^s-1)/(r-1)}.$$

Any linear function from $\mathbb{F}_q$ to $\mathbb{F}_r$ can be written as $Tr_{\mathbb{F}_q \to \mathbb{F}_r}(kx)$ for a unique k in $\mathbb{F}_q$.

2

The classical groups

The *Linear, Unitary, Symplectic,* and *Orthogonal* groups have been collectively known as 'The classical groups' since the publication of Hermann Weyl's famous book of that name, which discussed them over the real and complex fields. Most of their theory has been generalized to the other Chevalley and twisted Chevalley groups. However, the classical definitions require little technical knowledge, lead readily to invariant treatments of the groups, and provide many techniques for easy calculations inside them. In this ATLAS we take a severely classical viewpoint, for the most part. Later in this introduction, however, we shall quickly describe the larger class of groups, and the present section contains some forward references.

1. The groups $GL_n(q)$, $SL_n(q)$, $PGL_n(q)$, and $PSL_n(q) = L_n(q)$

The *general linear group* $GL_n(q)$ consists of all the $n \times n$ matrices with entries in $\mathbb{F}_q$ that have non-zero determinant. Equivalently it is the group of all linear automorphisms of an n-dimensional vector space over $\mathbb{F}_q$. The *special linear group* $SL_n(q)$ is the subgroup of all matrices of determinant 1. The *projective general linear group* $PGL_n(q)$ and *projective special linear group* $PSL_n(q)$ are the groups obtained from $GL_n(q)$ and $SL_n(q)$ on factoring by the scalar matrices contained in those groups.

For $n \geqslant 2$ the group $PSL_n(q)$ is simple except for $PSL_2(2) = S_3$ and $PSL_2(3) = A_4$, and we therefore also call it $L_n(q)$, in conformity with Artin's convention in which single-letter names are used for groups that are 'generally' simple.

The orders of the above groups are given by the formulae

$$|GL_n(q)| = (q-1)N, \qquad |SL_n(q)| = |PGL_n(q)| = N,$$

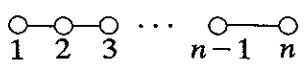
$$|PSL_n(q)| = |L_n(q)| = \frac{N}{d},$$

where

$$N = q^{\frac{1}{2}n(n-1)}(q^n-1)(q^{n-1}-1)\ldots(q^2-1),$$

and $d = (q-1, n)$.

$L_{n+1}(q)$ is the adjoint Chevalley group $A_n(q)$, with Dynkin diagram



(n nodes).

The maximal parabolic subgroup correlated with the node labelled k in the diagram corresponds to the stabilizer of a k-dimensional vector subspace.

2. The groups $GU_n(q)$, $SU_n(q)$, $PGU_n(q)$, and $PSU_n(q) = U_n(q)$

Let V be a vector space over $\mathbb{F}_{q^2}$. Then a function $f(x, y)$ which is defined for all x, y in V and takes values in $\mathbb{F}_{q^2}$ is called a *conjugate-symmetric sesquilinear form* if it satisfies

$$f(\lambda_1 x_1 + \lambda_2 x_2, y) = \lambda_1 f(x_1, y) + \lambda_2 f(x_2, y)$$

(linearity in x), and

$$f(y, x) = \overline{f(x, y)}$$

(conjugate-symmetry), where $x \to \bar{x} = x^q$ is the automorphism of $\mathbb{F}_{q^2}$ whose fixed field is $\mathbb{F}_q$. Such a form is necessarily *semilinear in* y, that is

$$f(x, \lambda_1 y_1 + \lambda_2 y_2) = \overline{\lambda_1} f(x, y_1) + \overline{\lambda_2} f(x, y_2).$$

It is called *singular* if there is some $x_0 \neq 0$ such that $f(x_0, y) = 0$ for all y. The *kernel* is the set of all such x_0. The *nullity* and *rank* are the dimension and codimension of the kernel.

A *Hermitian form* $F(x)$ is any function of the shape $f(x, x)$, where $f(x, y)$ is a conjugate-symmetric sesquilinear form. Since either of the forms F and f determines the other uniquely, it is customary to transfer the application of adjectives freely from one to the other. Thus $F(x) = f(x, x)$ is termed non-singular if and only if $f(x, y)$ is non-singular. Coordinates can always be chosen so that a given non-singular Hermitian form becomes

$$x_1\overline{x_1} + x_2\overline{x_2} + \ldots + x_n\overline{x_n}.$$

The *general unitary group* $GU_n(q)$ is the subgroup of all elements of $GL(q^2)$ that fix a given non-singular Hermitian form, or, equivalently, that fix the corresponding non-singular conjugate-symmetric sesquilinear form. If the forms are chosen to be the canonical one above, then a matrix U belongs to $GU_n(q)$ (is *unitary*) just if $U^{-1} = \bar{U}^t$, the matrix obtained by replacing the entries of U^t by their qth powers.

The determinant of a unitary matrix is necessarily a $(q+1)$st root of unity. The *special unitary group* $SU_n(q)$ is the subgroup of unitary matrices of determinant 1. The *projective general unitary group* $PGU_n(q)$ and *projective special unitary group* $PSU_n(q)$ are the groups obtained from $GU_n(q)$ and $SU_n(q)$ on factoring these groups by the scalar matrices they contain.

For $n \geqslant 2$, the group $PSU_n(q)$ is simple with the exceptions

$$PSU_2(2) = S_3, \qquad PSU_2(3) = A_4, \qquad PSU_3(2) = 3^2 : Q_8,$$

and so we also give it the simpler name $U_n(q)$. We have $U_2(q) = L_2(q)$.

The orders of the above groups are given by

$$|GU_n(q)| = (q+1)N, \qquad |SU_n(q)| = |PGU_n(q)| = N,$$

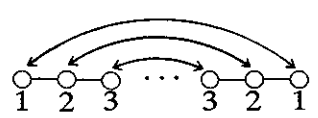
$$|PSU_n(q)| = |U_n(q)| = \frac{N}{d},$$

where

$$N = q^{\frac{1}{2}n(n-1)}(q^n - (-1)^n)(q^{n-1} - (-1)^{n-1})\ldots(q^3+1)(q^2-1),$$

and $d = (q+1, n)$.

$U_{n+1}(q)$ is the twisted Chevalley group ${}^2A_n(q)$, with the Dynkin diagram and twisting automorphism indicated:



(n nodes).

The maximal parabolic subgroup correlated with the orbit of nodes labelled k in the diagram corresponds to the stabilizer of a k-dimensional totally isotropic subspace (i.e. a space on which $F(x)$ or equivalently $f(x, y)$ is identically zero).

3. The groups $Sp_n(q)$ and $PSp_n(q) = S_n(q)$

An *alternating bilinear form* (or *symplectic form*) on a vector space V over $\mathbb{F}_q$ is a function $f(x, y)$ defined for all x, y in V and taking values in $\mathbb{F}_q$, which satisfies

$$f(\lambda_1 x_1 + \lambda_2 x_2, y) = \lambda_1 f(x_1, y) + \lambda_2 f(x_2, y)$$

(linearity in x), and also

$$f(y, x) = -f(x, y) \quad \text{and} \quad f(x, x) = 0$$

(skew-symmetry and alternation). It is automatically linear in y also (and so *bilinear*).

The *kernel* of such a form is the subspace of x such that $f(x, y) = 0$ for all y, and the *nullity* and *rank* of f are the dimension and codimension of its kernel. A form is called non-singular if its nullity is zero. The rank of a symplectic form is necessarily an even number, say $2m$, and coordinates can be chosen so that the form has the shape

$$x_1y_{m+1} + x_2y_{m+2} + \ldots + x_my_{2m} - x_{m+1}y_1 - x_{m+2}y_2 - \ldots - x_{2m}y_m.$$

For an even number $n = 2m$, the *symplectic group* $Sp_n(q)$ is defined as the group of all elements of $GL_n(q)$ that preserve a given non-singular symplectic form $f(x, y)$. Any such matrix necessarily has determinant 1, so that the 'general' and 'special' symplectic groups coincide. The *projective symplectic group* $PSp_n(q)$ is obtained from $Sp_n(q)$ on factoring it by the subgroup of scalar matrices it contains (which has order at most 2). For $2m \geqslant 2$, $PSp_{2m}(q)$ is simple with the exceptions

$$PSp_2(2) = S_3, \qquad PSp_2(3) = A_4, \qquad PSp_4(2) = S_6$$

and so we also call it $S_{2m}(q)$. We have $S_2(q) = L_2(q)$.

If A, B, C, D are $m \times m$ matrices, then $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ belongs to the symplectic group for the canonical symplectic form above just if

$$A^tC - C^tA = 0, \qquad A^tD - C^tB = I, \qquad B^tD - D^tB = 0,$$

where M^t denotes a transposed matrix.

The orders of the above groups are given by

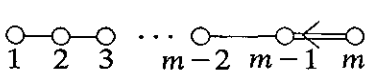
$$|Sp_{2m}(q)| = N, \qquad |PSp_{2m}(q)| = |S_{2m}(q)| = \frac{N}{d},$$

where

$$N = q^{m^2}(q^{2m} - 1)(q^{2m-2} - 1) \ldots (q^2 - 1)$$

and $d = (q - 1, 2)$.

$S_{2m}(q)$ is the adjoint Chevalley group $C_m(q)$ with Dynkin diagram



(m nodes).

The maximal parabolic group correlated with the node labelled k corresponds to the stabilizer of a k-dimensional totally isotropic subspace (that is, a space on which $f(x, y)$ is identically zero).

4. The groups $GO_n(q)$, $SO_n(q)$, $PGO_n(q)$, $PSO_n(q)$, and $O_n(q)$

A *symmetric bilinear form* on a space V over $\mathbb{F}_q$ is a function $f(x, y)$ defined for all x, y in V and taking values in $\mathbb{F}_q$ which satisfies

$$f(\lambda_1x_1 + \lambda_2x_2, y) = \lambda_1 f(x_1, y) + \lambda_2 f(x_2, y)$$

(linearity in x), and also

$$f(y, x) = f(x, y)$$

(symmetry). It is then automatically linear in y. A *quadratic form* on V is a function $F(x)$ defined for x in V and taking values in $\mathbb{F}_q$, for which we have

$$F(\lambda x + \mu y) = \lambda^2 F(x) + \lambda\mu f(x, y) + \mu^2 F(y)$$

for some symmetric bilinear form $f(x, y)$.

The *kernel of* f is the subspace of all x such that $f(x, y) = 0$ for all y, and the *kernel of* F is the set of all x in the kernel of f for which also $F(x) = 0$.

When the characteristic is not 2, F and f uniquely determine each other, so that the two kernels coincide. The literature contains a bewildering variety of terminology adapted to describe the more complicated situations that can hold in characteristic 2. This can be greatly simplified by using only a few standard terms (rank, nullity, non-singular, isotropic), but always being careful to state to which of f and F they apply.

Thus we define the *nullity* and *rank* of either f or F to be the dimension and codimension of its kernel, and say that f or F is *non-singular* just when its nullity is zero. A subspace is said to be (*totally*) *isotropic for* f if $f(x, y)$ vanishes for all x, y in that subspace, and (*totally*) *isotropic for* F if $F(x)$ vanishes for all x in the subspace. When the characteristic is not 2 our adjectives can be freely transferred between f and F.

The *Witt index* of a quadratic form F is the greatest dimension of any totally isotropic subspace for F. It turns out that if two non-singular quadratic forms on the same space over $\mathbb{F}_q$ have the same Witt index, then they are equivalent to scalar multiples of each other. The *Witt defect* is obtained by subtracting the Witt index from its largest possible value, $[\frac{1}{2}n]$. For a non-singular form over a finite field the Witt defect is 0 or 1.

The *general orthogonal group* $GO_n(q, F)$ is the subgroup of all elements of $GL_n(q)$ that fix the particular non-singular quadratic form F. The determinant of such an element is necessarily ± 1, and the *special orthogonal group* $SO_n(q, F)$ is the subgroup of all elements with determinant 1. The *projective general orthogonal group* $PGO_n(q, F)$ and *projective special orthogonal group* $PSO_n(q, F)$ are the groups obtained from $GO_n(q, F)$ and $SO_n(q, F)$ on factoring them by the groups of scalar matrices they contain.

In general $PSO_n(q, F)$ is *not* simple. However, it has a certain subgroup, specified precisely later, that is simple with finitely many exceptions when $n \geqslant 5$. This subgroup, which is always of index at most 2 in $PSO_n(q, F)$, we call $O_n(q, F)$.

When $n = 2m + 1$ is odd, all non-singular quadratic forms on a space of dimension n over $\mathbb{F}_q$ have Witt index m and are equivalent up to scalar factors. When $n = 2m$ is even, there are up to equivalence just two types of quadratic form, the *plus type*, with Witt index m, and the *minus type*, with Witt index $m - 1$. (These statements make use of the finiteness of $\mathbb{F}_q$.) Accordingly, we obtain only the following distinct families of groups:

When n is odd $GO_n(q)$, $SO_n(q)$, $PGO_n(q)$, $PSO_n(q)$, $O_n(q)$, being the values of $GO_n(q, F)$ (etc.) for any non-singular F.

When n is even $GO_n^\varepsilon(q)$, $SO_n^\varepsilon(q)$, $PGO_n^\varepsilon(q)$, $PSO_n^\varepsilon(q)$, $O_n^\varepsilon(q)$ for either sign $\varepsilon = +$ or $-$, being the values of $GO_n(q, F)$ (etc.) for a form F of plus type or minus type respectively.

We now turn to the problem of determining the generally simple group $O_n(q)$ or $O_n^\varepsilon(q)$. This can be defined in terms of the invariant called the *spinor norm*, when q is odd, or in terms of the *quasideterminant*, when q is even. We define these below, supposing $n \geqslant 3$ (the groups are boring for $n \leqslant 2$).

A vector r in V for which $F(r) \neq 0$ gives rise to certain elements of $GO_n(q, F)$ called *reflections*, defined by the formula

$$x \to x - \frac{f(x, r)}{F(r)} \,.\, r.$$

We shall now define a group $\Omega_n^\varepsilon(q)$ of index 1 or 2 in $SO_n^\varepsilon(q)$. The image $P\Omega_n^\varepsilon(q)$ of this group in $PSO_n^\varepsilon(q)$ is the group we call $O_n^\varepsilon(q)$, which is usually simple. The Ω, $P\Omega$ notation was introduced by Dieudonné, who defined $\Omega_n^\varepsilon(q)$ to be the commutator subgroup of $SO_n^\varepsilon(q)$, but we have changed the definition so as to obtain the 'correct' groups (in the Chevalley sense) for small n. For $n \geqslant 5$ our groups agree with Dieudonné's.

When q is odd, $\Omega_n^\varepsilon(q)$ is defined to be the set of all those g in $SO_n^\varepsilon(q)$ for which, when g is expressed in any way as the product of reflections in vectors $r_1, r_2, \ldots, r_t$, we have

$$F(r_1) \,.\, F(r_2) \,.\, \ldots \,.\, F(r_t) \quad \text{a square in } \mathbb{F}_q.$$

The function just defined is called the *spinor norm* of g, and takes values in $\mathbb{F}_q^*/(\mathbb{F}_q^*)^2$, where $\mathbb{F}_q^*$ is the multiplicative group of $\mathbb{F}_q$. Then $\Omega_n^\varepsilon(q)$ has index 2 in $SO_n^\varepsilon(q)$. It contains the scalar matrix -1 just when $n = 2m$ is even and $(4, q^m - \varepsilon) = 4$.

When q is even, then if $n = 2m + 1$ is odd, $SO_n(q) = GO_n(q)$ is isomorphic to the (usually simple) symplectic group $Sp_{2m}(q)$, and we define $\Omega_n(q) = SO_n(q)$. To obtain the isomorphism, observe that the associated symmetric bilinear form f has a one-dimensional kernel, and yields a non-singular symplectic form on $V/ker(f)$.

For arbitrary q, and $n=2m$ even, we define the *quasideterminant* of an element to be $(-1)^k$, where k is the dimension of its fixed space. Then for q odd this homomorphism agrees with the determinant, and for q even we define Ω_n^ε to be its kernel. The quasideterminant can also be written as $(-1)^D$, where D is a polynomial invariant called the Dickson invariant, taking values in $\mathbb{F}_2$.

An alternative definition of the quasideterminant is available. When $\varepsilon=+$ there are two families of maximal isotropic subspaces for F, two spaces being in the same family just if the codimension of their intersection in either of them is even. Then the quasideterminant of an element is 1 or -1 according as it preserves each family or interchanges the two families. When $\varepsilon=-$, the maximal isotropic spaces defined over $\mathbb{F}_q$ have dimension $m-1$, but if we extend the field to $\mathbb{F}_{q^2}$, we obtain two families of m-dimensional isotropic spaces and can use the same definition.

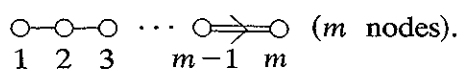
When $n=2m+1$ is odd, the groups have orders

$$|GO_n(q)|=dN, \qquad |SO_n(q)|=|PGO_n(q)|=|PSO_n(q)|=N,$$
$$|\Omega_n(q)|=|P\Omega_n(q)|=|O_n(q)|=N/d,$$

where

$$N=q^{m^2}(q^{2m}-1)(q^{2m-2}-1)\dots(q^2-1)$$

and $d=(2,q-1)$. $O_{2m+1}(q)$ is the adjoint Chevalley group $B_m(q)$, with Dynkin diagram



(m nodes).

The maximal parabolic subgroup correlated with the node labelled k corresponds to the stabilizer of an isotropic k-space for F.

When $n=2m$ is even, the groups have orders

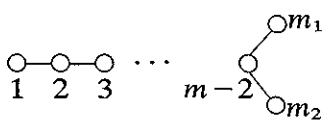
$$|GO_n^\varepsilon(q)|=2N, \qquad |SO_n^\varepsilon(q)|=|PGO_n^\varepsilon(q)|=2N/e,$$
$$|PSO_n^\varepsilon(q)|=2N/e^2, \qquad |\Omega_n^\varepsilon(q)|=N/e,$$
$$|P\Omega_n^\varepsilon(q)|=|O_n(q)|=N/d,$$

where

$$N=q^{m(m-1)}(q^m-\varepsilon)(q^{2m-2}-1)(q^{2m-4}-1)\dots(q^2-1)$$

and $d=(4,q^m-\varepsilon)$, $e=(2,q^m-\varepsilon)$.

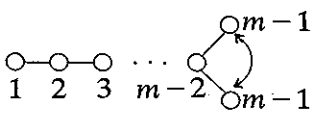
$O_{2m}^+(q)$ is the adjoint Chevalley group $D_m(q)$, with Dynkin diagram



(m nodes).

The maximal parabolic subgroup correlated with the node labelled $k\leqslant m-2$ corresponds to the stabilizer of an isotropic k-space for F. Those correlated with the nodes labelled m_1 and m_2 correspond to stabilizers of members of the two families of isotropic m-spaces for F.

$O_{2m}^-(q)$ is the twisted Chevalley group ${}^2D_m(q)$, with the Dynkin diagram and twisting automorphism



(m nodes).

The maximal parabolic subgroup correlated with the orbit of nodes labelled $k\leqslant m-1$ corresponds to the stabilizer of an isotropic k-space for F.

For $n\leqslant 6$, the orthogonal groups are isomorphic to other classical groups, as follows:

$$O_3(q)=L_2(q), \qquad O_4^+(q)=L_2(q)\times L_2(q), \qquad O_4^-(q)=L_2(q^2),$$
$$O_5(q)=S_4(q), \qquad O_6^+(q)=L_4(q), \qquad O_6^-(q)=U_4(q).$$

The group $O_n(q)$ is simple for $n\geqslant 5$, with the single exception that $O_5(2)=S_4(2)$ is isomorphic to the symmetric group S_6.

5. Classification of points and hyperplanes in orthogonal spaces

Let V be a space equipped with a non-singular quadratic form F. Then for fields of odd characteristic many authors classify the vectors of V into three classes according as

$F(v)=0$,
$F(v)$ a non-zero square,
$F(v)$ a non-square,

since these correspond exactly to the three orbits of projective points under the orthogonal group of F.

In this ATLAS we prefer a different way of making these distinctions, which is independent of the choice of any particular scalar multiple of the quadratic form F, and which does the correct thing in characteristic 2. We say that a subspace H of even dimension on which F is non-singular is of *plus type* or *minus type* according to the type of F when restricted to H, and if H is the hyperplane perpendicular to a vector v, we apply the same adjectives to v. Thus our version of the above classification is

v *isotropic* (or *null*),
v of *plus type*,
v of *minus type*.

Table 2. Structures of classical groups

d	$GL_n(q)$	$PGL_n(q)$	$SL_n(q)$	$PSL_n(q)=L_n(q)$
$(n,q-1)$	$d.\left(\frac{q-1}{d}\times G\right).d$	$G.d$	$d.G$	G

d	$GU_n(q)$	$PGU_n(q)$	$SU_n(q)$	$PSU_n(q)=U_n(q)$
$(n,q+1)$	$d.\left(\frac{q+1}{d}\times G\right).d$	$G.d$	$d.G$	G

d	$Sp_n(q)$	$PSp_n(q)=S_n(q)$
$(2,q-1)=2$	$2.G$	G
$(2,q-1)=1$	G	G

d	$GO_n^\varepsilon(q)$	$PGO_n^\varepsilon(q)$	$SO_n^\varepsilon(q)$	$PSO_n^\varepsilon(q)$	$\Omega_n^\varepsilon(q)$	$P\Omega_n^\varepsilon(q)=O_n^\varepsilon(q)$	
$(4,q^m-\varepsilon)=4$	$2.G.2^2$	$G.2^2$	$2.G.2$	$G.2$	$2.G$	G	$n=2m$
$(4,q^m-\varepsilon)=2$	$2\times G.2$	$G.2$	$2\times G$	G	G	G	$n=2m$
$(4,q^m-\varepsilon)=1$	$G.2$	$G.2$	$G.2$	$G.2$	G	G	$n=2m$
$(2,q-1)=2$	$2\times G.2$	$G.2$	$G.2$	$G.2$	G	G	$n=2m+1$
$(2,q-1)=1$	G	G	G	G	G	G	$n=2m+1$

6. The Clifford algebra and the spin group

The *Clifford algebra* of F is the associative algebra generated by the vectors of V with the relations $x^2 = F(x)$, which imply $xy + yx = f(x, y)$. If V has basis $e_1, \ldots, e_n$, then the Clifford algebra is 2^n-dimensional, with basis consisting of the formal products

$$e_{i_1} e_{i_2} \ldots e_{i_k} \quad (i_1 < i_2 < i_3 < \ldots < i_k,\ k \leq n).$$

The vectors r with $F(r) \neq 0$ generate a subgroup of the Clifford algebra which is a central extension of the orthogonal group, the vector r in the Clifford algebra mapping to the negative of the reflection in r. When the ground field has characteristic $\neq 2$, this remark can be used to construct a proper double cover of the orthogonal group, called the *spin group*.

7. Structure tables for the classical groups

Table 2 describes the structure of all the groups mentioned, in terms of the usually simple group G, which is the appropriate one of $L_n(q)$, $U_n(q)$, $S_n(q)$, $O_n(q)$.

8. Other notations for the simple groups

There are many minor variations such as $L_n(\mathbb{F}_q)$ or $L(n, q)$ for $L_n(q)$ which should give little trouble. However, the reader should be aware that although the 'smallest field' convention which we employ in this ATLAS is rapidly gaining ground amongst group theorists, there are still many people who write $U_n(q^2)$ or $U(n, q^2)$ for what we call $U_n(q)$. Artin's 'single letter for simple group' convention is not universally adopted, so that many authors would use $U_n(q)$ and $O_n(q)$ for what we call $GU_n(q)$ and $GO_n(q)$. The notations $E_2(q)$ and $E_4(q)$ have sometimes been used for $G_2(q)$ and $F_4(q)$.

Dickson's work has had a profound influence on group theory, and his notations still have some currency, but are rapidly becoming obsolete. Here is a brief dictionary:

$$\begin{aligned}
&\text{(Linear fractional) } LF(n, q) = L_n(q)\\
&\text{(Hyperorthogonal) } HO(n, q^2) = U_n(q)\\
&\text{(Abelian linear) } A(2m, q) = S_{2m}(q)\\
&\text{(First orthogonal)} \begin{cases} FO(2m+1, q) = O_{2m+1}(q) \\ FO(2m, q) = O^{\varepsilon}_{2m}(q) \end{cases} \text{where } \varepsilon = \pm 1,\\
&\text{(Second orthogonal) } SO(2m, q) = O^{-\varepsilon}_{2m}(q) \quad q^m \equiv \varepsilon \text{ modulo } 4.\\
&\text{(First hypoabelian) } FH(2m, q) = O^{+}_{2m}(q)\\
&\text{(Second hypoabelian) } SH(2m, q) = O^{-}_{2m}(q)
\end{aligned}$$

Dickson's first orthogonal group is that associated with the quadratic form $x_1^2 + x_2^2 + \ldots + x_n^2$. His orthogonal groups are defined only for odd q, and his hypoabelian groups only for even q. He uses the notation $GLH(n, q)$ (General Linear Homogeneous group) for $GL_n(q)$.

3

The Chevalley and twisted Chevalley groups

1. The untwisted groups

The Chevalley and twisted Chevalley groups include and neatly generalize the classical families of linear, unitary, symplectic, and orthogonal groups.

In the entries for individual groups in the ATLAS, we have preferred to avoid the Chevalley notation, since it requires considerable technical knowledge, and since most of the groups we discuss also have classical definitions. The classical descriptions, when available, permit easy calculation, and lead readily to the desired facts about subgroups, etc.

However, for a full understanding of the entire set of finite simple groups, the Chevalley theory is unsurpassed. In particular the isomorphisms such as $L_4(q) = O_6^+(q)$ between different classical groups of the same characteristic become evident. The full Chevalley theory is beyond the scope of this ATLAS, but in the next few pages we give a brief description for those already acquainted with some of the terminology of Lie groups and Lie algebras. We reject any reproach for the incompleteness of this treatment. It is intended merely to get us to the point where we can list all the groups, and the isomorphisms among them, and also ~~to~~ specify their Schur multipliers and outer automorphism groups.

In 1955 Chevalley discovered a uniform way to define bases for the complex simple Lie algebras in which all their structure constants were rational integers. It follows that analogues of these Lie algebras and the corresponding Lie groups can be defined over arbitrary fields. The resulting groups are now known as the *adjoint Chevalley groups*. Over finite fields, these groups are finite groups which are simple in almost all cases. The definition also yields certain covering groups, which are termed the *universal Chevalley groups*. If a given finite simple group can be expressed as an adjoint Chevalley group, then in all but finitely many cases its abstract universal cover is the corresponding universal Chevalley group.

In the standard notation, the complex Lie algebras are

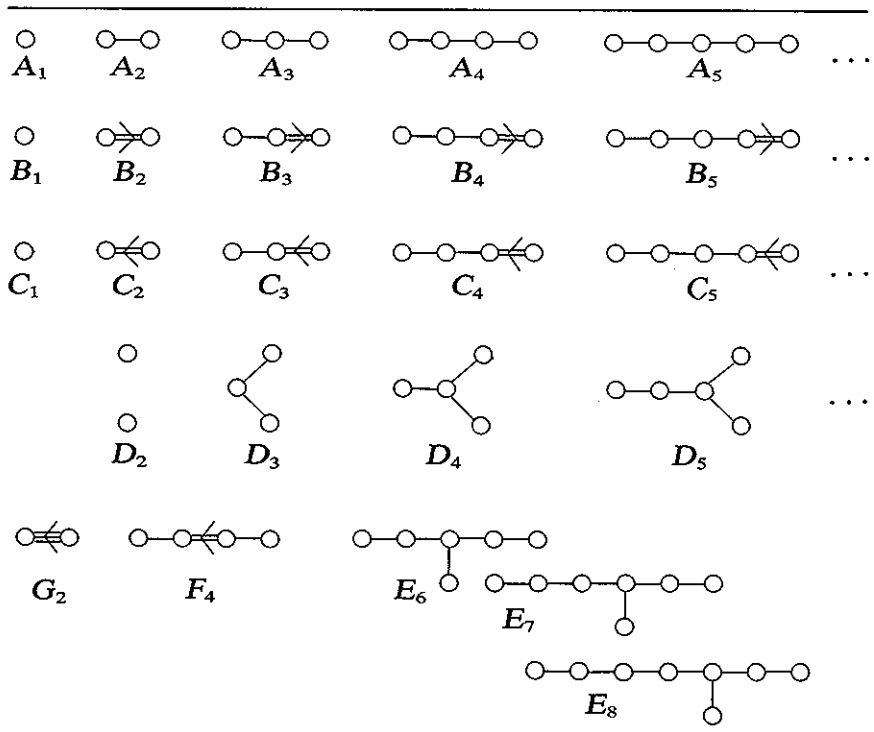
$$A_n \quad B_n \quad C_n \quad D_n \quad G_2 \quad F_4 \quad E_6 \quad E_7 \quad E_8$$

where to avoid repetitions we may demand that $n \geq 1, 2, 3, 4$ for A_n, B_n, C_n, D_n respectively. The corresponding adjoint Chevalley groups are denoted by

$$A_n(q),\ B_n(q),\ C_n(q),\ D_n(q),\ G_2(q),\ F_4(q),\ E_6(q),\ E_7(q),\ E_8(q).$$

The corresponding *Dynkin diagrams*, which specify the structure of the fundamental roots, appear in Table 3. In the cases when a p-fold branch appears ($p = 2$ or 3), the arrowhead points from long roots to short ones, the ratio of lengths being $\sqrt{p}$. We have included the non-simple case $D_2 = A_1 \oplus A_1$, and the repetitions $A_1 = B_1 = C_1$, $B_2 = C_2$, $A_3 = D_3$, since these help in the understanding of the relations between various classical groups.

Table 3. Dynkin diagrams



2. The twisted groups

Steinberg showed that a modification of Chevalley's procedure could be made to yield still more finite groups, and in particular, the unitary groups.

Any symmetry of the Dynkin diagram (preserving the direction of the arrowhead, if any) yields an automorphism of the Lie group or its Chevalley analogues, called an *ordinary graph automorphism*. Let us suppose that α is such an automorphism, of order t, and call it the *twisting automorphism*. We now define the *twisted Chevalley group* ${}^tX_n(q, q^t)$ to be set of elements of $X_n(q^t)$ that are fixed by the quotient of the twisting automorphism and the field automorphism induced by the Frobenius map $x \to x^q$ of F_{q^t}.

The particular cases are

$$\begin{aligned} &{}^2A_n(q, q^2) = U_{n+1}(q), \quad (n \geq 2) \\ &{}^2D_n(q, q^2) = O_{2n}^-(q), \quad (n \geq 3) \\ &{}^3D_4(q, q^3), \\ &{}^2E_6(q, q^2), \end{aligned}$$

the last two families being discovered by Steinberg. We usually abbreviate ${}^tX_n(q, q^t)$ to ${}^tX_n(q)$.

A further modification yields the infinite families of simple groups discovered by Suzuki and Ree. If the Dynkin diagram of X_n has a p-fold edge ($p = 2, 3$), then over fields of characteristic p the Chevalley group is independent of the direction of the arrowhead on that edge. In other words, there is an isomorphism between $X_n(p^f)$ and $Y_n(p^f)$, where Y_n is the diagram obtained from X_n by reversing the direction of the arrowhead. Thus for example, $B_n(2^f) = C_n(2^f)$, or in classical notation $O_{2n+1}(2^f) = S_{2n}(2^f)$, as we have already seen.

In the three cases $B_2 = C_2$, G_2, F_4, the diagram has an automorphism reversing the direction of the p-fold edge, and so over fields of the appropriate characteristic p, the resulting Chevalley groups have a new type of graph automorphism, which we call an *extraordinary graph automorphism*, whose square is the field automorphism induced by the Frobenius map $x \to x^p$.

Now when $q = p^{2m+1}$ is an odd power of this p, the field automorphism induced by $x \to x^{p^{m+1}}$ has the same square, and the elements of $X_n(p^{2m+1})$ fixed by the quotient of these two automorphisms form a new type of twisted Chevalley group, called ${}^2X_n(*, p^{2m+1})$, and usually abbreviated to ${}^2X_n(p^{2m+1})$. The particular cases are

$${}^2B_2(*, 2^{2m+1}) = {}^2C_2(*, 2^{2m+1}), \text{ a } \textit{Suzuki group},$$
$${}^2G_2(*, 3^{2m+1}), \text{ a } \textit{Ree group} \text{ of characteristic 3},$$
$${}^2F_4(*, 2^{2m+1}), \text{ a } \textit{Ree group} \text{ of characteristic 2}.$$

These groups are often written

$$Sz(q), R_1(q), R_2(q),$$

where $q = p^{2m+1}$, but the subscripts 1 and 2 for the two types of Ree group can be omitted without risk of confusion.

It turns out that the first group in each of these families is not simple, but all the later ones are simple. The cases are

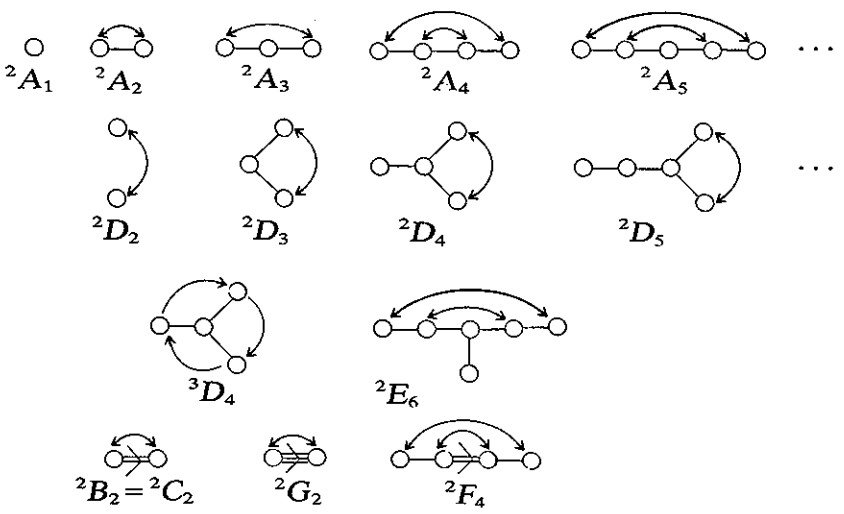
${}^2B_2(2) = 5:4$, the Frobenius group of order 20,

${}^2G_2(3) = L_2(8):3$, the extension of the simple group $L_2(8)$ of order 504 by its field automorphism,

${}^2F_4(2) = T.2$, where T is a simple group not appearing elsewhere in the classification of simple groups, called the *Tits group*.

Table 4 gives the Dynkin diagrams and twisting automorphisms for all the twisted groups. We remark that what we have called simply the twisted Chevalley groups are more fully called

Table 4. Diagrams and twisting automorphisms for twisted groups



the *adjoint* twisted Chevalley groups, and that, like the untwisted groups, they have certain multiple covers called the *universal twisted Chevalley groups*. We have included the untwisted group ${}^2A_1 = A_1$ in the table, and also the case ${}^2D_2$ obtained by twisting a disconnected diagram, and the repetition ${}^2A_3 = {}^2D_3$. The reason is again that these special cases illuminate relations between some classical groups.

3. Multipliers and automorphisms of Chevalley groups

The Schur multiplier has order de, and the outer automorphism group has order dfg, where the order of the base field is $q = p^f$ (p prime), and the numbers d, f, g are tabulated in Table 5. (An entry '2 if ...' means 1 if *not*.)

The Schur multiplier is the direct product of groups of orders d (the *diagonal multiplier*) and e (the *exceptional multiplier*). The diagonal multiplier extends the adjoint group to the corresponding universal Chevalley group. The exceptional multiplier is always a p-group (for the above p), and is trivial except in finitely many cases.

The outer automorphism group is a semidirect product (in this order) of groups of orders d (*diagonal automorphisms*), f (*field automorphisms*), and g (*graph automorphisms modulo field automorphisms*), except that for

$$B_2(2^f), G_2(3^f), F_4(2^f)$$

the (extraordinary) graph automorphism squares to the generating field automorphism. The groups of orders d, e, f, g are cyclic except that 3! indicates the symmetric group of degree 3, and orders written as powers indicate the corresponding direct powers of cyclic groups.

4. Orders of the Chevalley groups

In Table 6, the parameters have been chosen so as to avoid the 'generic' isomorphisms.
N is the order of the universal Chevalley group.
N/d is the order of the adjoint Chevalley group.

5. The simple groups enumerated

The exact list of finite simple groups is obtained from the union of

the list of Chevalley groups (Table 5)
the alternating groups A_n, for $n \geq 5$
the cyclic groups of prime order
the 26 sporadic groups, and finally
the Tits simple group $T = {}^2F_4(2)'$

by taking into account the exceptional isomorphisms below. Each of these isomorphisms is either between two of the above groups, or between one such group and a non-simple group. For the reader's convenience, we give both the Chevalley and classical notations.

The exceptional isomorphisms:

$A_1(2) = L_2(2) \cong S_3$	$B_2(2) = S_4(2) \cong S_6$
$A_1(3) = L_2(3) \cong A_4$	$G_2(2) \cong {}^2A_2(3).2 = U_3(3).2$
$A_1(4) = L_2(4) \cong A_5$	${}^2A_2(2) = U_3(2) \cong 3^2.Q_8$
$A_1(5) = L_2(5) \cong A_5$	${}^2A_3(2) = U_4(2) \cong B_2(3) = S_4(3)$
$A_1(7) = L_2(7) \cong A_2(2) = L_3(2)$	${}^2B_2(2) = Sz(2) \cong 5:4$
$A_1(9) = L_2(9) \cong A_6$	${}^2G_2(3) = R(3) \cong A_1(8).3 = L_2(8).3$
$A_3(2) = L_4(2) \cong A_8$	${}^2F_4(2) = R(2) \cong T.2.$

6. Parabolic subgroups

With every proper subset of the nodes of the Dynkin diagram there is associated a *parabolic subgroup*, which is in structure a p-group extended by the Chevalley group determined by the subdiagram on those nodes. (Here p, as always, denotes the characteristic of the ground field $\mathbb{F}_q$.) The *maximal parabolic subgroups* are those associated to the sets containing all but one of the nodes—we shall say that such a subgroup is *correlated* to the remaining node. According to the *Borel–Tits theorem* the maximal p-local subgroups of a Chevalley group are to be found among its maximal parabolic subgroups. These statements hold true for the twisted groups, provided we replace 'node' by 'orbit of nodes under the twisting automorphism'.

7. The fundamental representations

The ordinary representation theory of Chevalley groups, as recently developed by Deligne and Lusztig, is very complex, and the irreducible modular representations are not yet completely described. But certain important representations (not always irreducible) can be obtained from the representation theory of Lie groups.

The irreducible representations of a Lie group are completely classified, and can be written as 'polynomials' in certain *fundamental representations*, one for each node of the Dynkin diagram. The degrees of the representations for the various Lie groups X_n are given in Table 7. By change of field, we obtain the so-called fundamental representations of the universal Chevalley group $X_n(q)$, which are representations over the field $\mathbb{F}_q$ having the given degrees. In the classical cases these are fairly easily described geometrically—for example in $A_n(q) = L_{n+1}(q)$ the fundamental representation corresponding to the rth node is that of $SL_{n+1}(q)$ on the rth exterior power of the original vector space V.

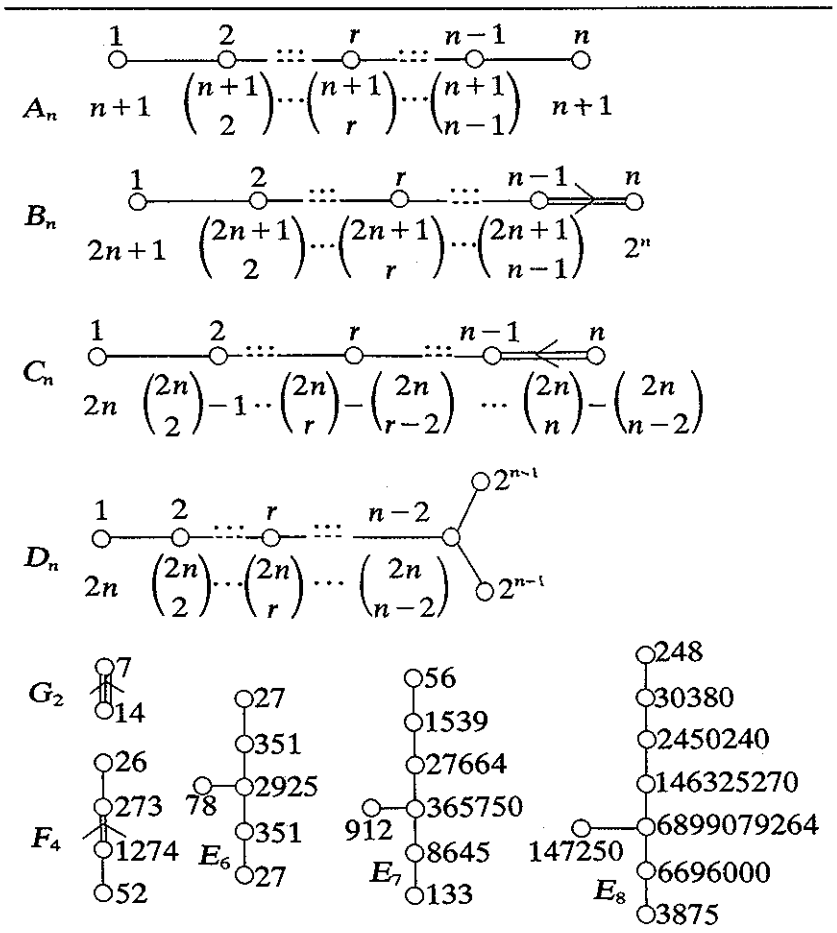
Table 5. Automorphisms and multipliers of the Chevalley groups

Condition	Group	Definitions of d	f	g	Cases when $e \neq 1$
	$A_1(q)$	$(2, q-1)$	$q=p^f$	1	$A_1(4) \to 2$, $A_1(9) \to 3$
$n \geqslant 2$	$A_n(q)$	$(n+1, q-1)$	$q=p^f$	2	$A_2(2) \to 2$, $A_2(4) \to 4^2$, $A_3(2) \to 2$
$n \geqslant 2$	${}^2A_n(q)$	$(n+1, q+1)$	$q^2=p^f$	1	${}^2A_3(2) \to 2$, ${}^2A_3(3) \to 3^2$, ${}^2A_5(2) \to 2^2$
	$B_2(q)$	$(2, q-1)$	$q=p^f$	2 if $p=2$	$B_2(2) \to 2$
f odd	${}^2B_2(q)$	1	$q=2^f$	1	${}^2B_2(8) \to 2^2$
$n \geqslant 3$	$B_n(q)$	$(2, q-1)$	$q=p^f$	1	$B_3(2) \to 2$, $B_3(3) \to 3$
$n \geqslant 3$	$C_n(q)$	$(2, q-1)$	$q=p^f$	1	$C_3(2) \to 2$
	$D_4(q)$	$(2, q-1)^2$	$q=p^f$	3!	$D_4(2) \to 2^2$
	${}^3D_4(q)$	1	$q^3=p^f$	1	none
$n>4$, even	$D_n(q)$	$(2, q-1)^2$	$q=p^f$	2	none
$n>4$, odd	$D_n(q)$	$(4, q^n-1)$	$q=p^f$	2	none
$n \geqslant 4$	${}^2D_n(q)$	$(4, q^n+1)$	$q^2=p^f$	1	none
	$G_2(q)$	1	$q=p^f$	2 if $p=3$	$G_2(3) \to 3$, $G_2(4) \to 2$
f odd	${}^2G_2(q)$	1	$q=3^f$	1	none
	$F_4(q)$	1	$q=p^f$	2 if $p=2$	$F_4(2) \to 2$
f odd	${}^2F_4(q)$	1	$q=2^f$	1	none
	$E_6(q)$	$(3, q-1)$	$q=p^f$	2	none
	${}^2E_6(q)$	$(3, q+1)$	$q^2=p^f$	1	${}^2E_6(2) \to 2^2$
	$E_7(q)$	$(2, q-1)$	$q=p^f$	1	none
	$E_8(q)$	1	$q=p^f$	1	none

Table 6. Orders of Chevalley groups

G	N	d
$A_n(q), n \geqslant 1$	$q^{n(n+1)/2} \prod_{i=1}^{n} (q^{i+1}-1)$	$(n+1, q-1)$
$B_n(q), n \geqslant 2$	$q^{n^2} \prod_{i=1}^{n} (q^{2i}-1)$	$(2, q-1)$
$C_n(q), n \geqslant 3$	$q^{n^2} \prod_{i=1}^{n} (q^{2i}-1)$	$(2, q-1)$
$D_n(q), n \geqslant 4$	$q^{n(n-1)}(q^n-1) \prod_{i=1}^{n-1} (q^{2i}-1)$	$(4, q^n-1)$
$G_2(q)$	$q^6(q^6-1)(q^2-1)$	1
$F_4(q)$	$q^{24}(q^{12}-1)(q^8-1)(q^6-1)(q^2-1)$	1
$E_6(q)$	$q^{36}(q^{12}-1)(q^9-1)(q^8-1)(q^6-1)(q^5-1)(q^2-1)$	$(3, q-1)$
$E_7(q)$	$q^{63}(q^{18}-1)(q^{14}-1)(q^{12}-1)(q^{10}-1)(q^8-1)(q^6-1)(q^2-1)$	$(2, q-1)$
$E_8(q)$	$q^{120}(q^{30}-1)(q^{24}-1)(q^{20}-1)(q^{18}-1)$ $(q^{14}-1)(q^{12}-1)(q^8-1)(q^2-1)$	1
${}^2A_n(q), n \geqslant 2$	$q^{n(n+1)/2} \prod_{i=1}^{n} (q^{i+1}-(-1)^{i+1})$	$(n+1, q+1)$
${}^2B_2(q), q=2^{2m+1}$	$q^2(q^2+1)(q-1)$	1
${}^2D_n(q), n \geqslant 4$	$q^{n(n-1)}(q^n+1) \prod_{i=1}^{n-1} (q^{2i}-1)$	$(4, q^n+1)$
${}^3D_4(q)$	$q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$	1
${}^2G_2(q), q=3^{2m+1}$	$q^3(q^3+1)(q-1)$	1
${}^2F_4(q), q=2^{2m+1}$	$q^{12}(q^6+1)(q^4-1)(q^3+1)(q-1)$	1
${}^2E_6(q)$	$q^{36}(q^{12}-1)(q^9+1)(q^8-1)(q^6-1)(q^5+1)(q^2-1)$	$(3, q+1)$

Table 7. Degrees of the fundamental representations of Lie and Chevalley groups. (The degrees are the numbers below or to the right of the nodes.)



4

How to read this ATLAS: the 'constructions' sections

The typical ATLAS entry has four parts, giving:

1. the order of G, and its Schur multiplier and outer automorphism group;
2. various constructions for G, or related groups or concepts;
3. the maximal subgroups of G and its automorphic extensions;
4. the compound character table, from which the ordinary character tables for various extensions of G can be read off.

The parts are usually in this order, but the exigencies of pagination have occasionally forced us to put the character table first. For some large groups, we can only give partial information. Below, we discuss the type of information contained.

1. Order, multiplier, and outer automorphism group

Every reader of this ATLAS should be familiar with the notion of the *order* of a finite group! The set of all *automorphisms* (isomorphisms from G onto itself) of a given group G forms a group $Aut(G)$ called the *automorphism group* of G. Any element g of G yields a particular automorphism $x \to g^{-1}xg$ called *transformation* or *conjugation* by g, or the inner automorphism determined by g, and the set of all such automorphisms is a normal subgroup $Inn(G)$ of $Aut(G)$, the *inner automorphism group*. It is abstractly isomorphic to the quotient of G by its centre. The quotient group $Aut(G)/Inn(G) = Out(G)$ is the *outer automorphism group* of G.

A *central extension*, or *covering group*, of G is a group H with a central subgroup C whose quotient is G. It is called a *proper covering group* if C is contained in the commutator subgroup of H. The groups C that arise for proper covers of G are each of them quotients of a largest one, called the *Schur multiplier* (or *multiplicator*), Mult(G), of G. The corresponding central extension is only determined up to *isoclinism*, see Chapter 6, Section 7. However, for a perfect group G (and in particular for a simple G), we *do* have a unique *universal covering group*, of which any proper cover of G is a quotient, and for which C is the Schur multiplier of G.

The multiplier may be computed in the following way: Let G be defined by the relations

$$R_1(a_1, a_2, \ldots) = 1, \ldots, R_j(a_1, a_2, \ldots) = 1, \ldots$$

in generators $a_1, \ldots, a_i, \ldots$. Then the modified relations

$$R_j(a_1, \ldots) = z_j, \qquad a_i z_j = z_j a_i$$

define a group H which is a central extension of G by the subgroup

$$C = \langle z_1, z_2, \ldots \rangle$$

of H. Then the Schur multiplier of G is the intersection of C with the commutator subgroup of H. Moreover, if we factor H by a subgroup of C that is maximal subject to its not intersecting the commutator subgroup, we obtain one of the proper covers of G by its multiplier. The multiplier plays an important role in representation theory—see later sections.

2. Constructions

Under this heading, we give brief descriptions of various ways of constructing G, various closely related groups, or objects of which these groups are the automorphism groups. The style is telegraphic, and the reader must expect to fill in many details for himself. However, we have tried wherever possible to make the definitions so explicit that a sufficiently determined reader can completely recover the group or object from our description of it.

For the classical definitions, say of the unitary groups, we have been content to give merely the definition itself, and the structure of the various groups in terms of the simple group G, for example

$$GU_3(5) = 2 \times 3 \,.\, G \,.\, 3, \qquad PGU_3(5) = G \,.\, 3,$$
$$SU_3(5) = 3 \,.\, G, \qquad PSU_3(5) = U_3(5) = G.$$

The most interesting constructions, however, are those that relate the group to some special combinatorial object whose existence is not immediate from the classical definition; for example in the case of $U_3(5)$ the Hoffman–Singleton graph of 50 vertices, each joined to 7 others, whose automorphism group is $G \,.\, 2 = P\sum U_3(5)$. There is no reasonable analogue of this for the other unitary groups. In such cases, the thing we construct may be the combinatorial object, its automorphism group, or preferably both.

The combinatorial object is usually a set of some kind, and often a set of vectors in some space. In the simplest cases we have been able to give explicit names for all the members of this set, and also for all their automorphisms, but more usually we have only been able to specify a generating set of automorphisms. We have usually tried to make this generating set as large as we can, rather than as small as we can, since computations are then easier. Readers who want minimal systems of generators must look elsewhere!

In some large cases we have only been able to give a description of the relevant object rather than an explicit construction. We are pleased to report that in several cases where our description was originally of this kind we have managed, sometimes at the proof stage, to insert a more explicit construction. (The publishers of the ATLAS were not quite so pleased!)

In very complicated cases the work of the construction has been spread over several sections, each individually called a 'construction' because it constructs some portion of the final object. In this way we have even managed to fit a complete and explicit construction for the Monster onto a few ATLAS pages!

In the remainder of this chapter, we briefly describe some notations and ideas that appear repeatedly in the 'constructions' sections.

3. Notation for types of point and hyperplane for orthogonal groups

We attach the terms 'plus' and 'minus' to various notions connected with orthogonal groups. This usage is explained in Chapter 2, Section 5.

4. Projective dimensions and projective counting

Since the group stabilizing a vector also stabilizes its multiples, it is often sensible to use projective terminology, in which the $(k+1)$-dimensional subspaces of an $(n+1)$-dimensional vector

space are called the k-dimensional subspaces of an n-dimensional *projective space*. To avoid confusion, we usually use the terms *point, line, plane,* ... to indicate a space of *projective* dimension $0, 1, 2, \ldots$.

In projective counting, a vector is identified with its non-zero scalar multiples. Thus the 8 vectors from the centre of a cube to its vertices arise in 4 pairs $\{v, -v\}$, and so projectively speaking they yield just 4 distinct points. In such a situation we might write '... 2×4 vectors...' or '..., projectively, 4 vectors...' or, when we wish to indicate just which scalars are to be neglected '... 4 ($\pm$)-vectors...'. Formally, a ($\pm$)-vector is a pair $\{v, -v\}$, and an ω-vector (where $\omega^3 = 1$) is a triple $\{v, \omega v, \omega^2 v\}$.

5. Coordinates for vectors

We adopt various compact notations for vectors. For example $(2^8 0^{16})$ indicates a vector, or set of vectors, having some eight coordinates 2 and the remaining sixteen coordinates 0.

On occasion we indicate that we intend to apply all permutations of the appropriate Symmetric, Alternating, Cyclic, or Dihedral group by the corresponding superscript letter. Thus $(x, y, z, t)^A$ represents the twelve vectors obtained by all even permutations of the coordinates.

6. Quaternionic spaces

A fair number of our constructions involve quaternionic spaces, and as some readers will not be familiar with the necessary distinctions between the roles of the two kinds of multiplication, we state our conventions here. (Care is needed in comparisons with other sources.)

We use *left multiplication* for scalar products, and *right multiplication* for group actions. Thus if $x = (x_1, \ldots, x_n)$ is a vector, λ a scalar, and $\delta = diag(\delta_1, \ldots, \delta_n)$ a diagonal group element, then the scalar product of x by λ is

$$\lambda x = (\lambda x_1, \ldots, \lambda x_n),$$

while the image of x under δ is

$$x\delta = (x_1\delta_1, \ldots, x_n\delta_n).$$

The reader is cautioned that the map $x \to \lambda x$ is *not* a linear operation. Again, in a lattice L over some quaternionic ring R, we might say that $x \equiv y$ (modulo r) to mean that $x - y \in rL$. This does *not* usually imply that $\lambda x \equiv \lambda y$ (modulo r), but if δ is a group operation of some group acting on L, it *will* imply that $x\delta \equiv y\delta$ (modulo r).

7. Presentations

We write presentations in the format

$$\langle \text{generators} \mid \text{relations} \rangle$$

thus for example

$$\langle a, b \mid a^2 = b^3 = (ab)^5 = 1 \rangle$$

is a presentation for A_5. Relations placed in square brackets are redundant.

Coxeter-type presentations

We represent certain relations by diagrams, in which each node refers to a generator g, and indicates the relation $g^2 = 1$. For any two such generators g and h, we have the further relation

$gh = hg$, or $(gh)^2 = 1$, if the corresponding nodes are not joined,
$(gh)^3 = 1$, if they are joined by an unmarked edge,
$(gh)^n = 1$, if they are joined by an edge marked n.

This notation is often combined with the '$\langle \ldots \mid \ldots \rangle$' format, and additional relations may be explicitly adjoined. On occasions we specify certain relations, and then write '+ more relations(?)'—this means that the group certainly satisfies the specified relations, but we do not know whether they suffice for a presentation.

Many interesting presentations for particular groups can be deduced from the information on pages 232–3.

Notations for particular presentations (after Coxeter and Moser)

(l, m, n)	$R^l = S^m = T^n = RST = 1$, or $R^l = S^m = (RS)^n = 1$
$\langle l, m, n \rangle$	$R^l = S^m = T^n = RST$
$(l, m \mid n, k)$	$R^l = S^m = (RS)^n = (R^{-1}S)^k = 1$
$(l, m, n; q)$	$R^l = S^m = T^n = RST = (TSR)^q = 1$, or
	$R^l = S^m = (RS)^n = (R^{-1}S^{-1}RS)^q = 1$
$G^{p,q,r}$	$A^p = B^q = C^r = (AB)^2 = (BC)^2 = (CA)^2$
	$= (ABC)^2 = 1$, or
	$X^2 = Y^2 = (XY)^2 = 1$, and
	$1 = T^2 = (XT)^p = (YT)^q = (XYT)^r$
$G^{p,q,r,s,t,u,v}$	$X^2 = Y^2 = Z^2 = (XY)^2 = (YZ)^2 = (ZX)^2$
	$= 1$, and
	$1 = T^2 = (XT)^p = (YT)^q = (XYT)^r =$
	$(ZT)^s = (XZT)^t = (YZT)^u = (XYZT)^v$

5

How to read this ATLAS: information about subgroups and their structure

1. The maximal subgroups sections

A *maximal subgroup* H of G is a proper subgroup of G that is contained in no strictly larger proper subgroup of G. Whenever we have been able to do so, under the heading 'maximal subgroups' we have listed all the maximal subgroups of G up to conjugacy. On other occasions, we have modified this heading appropriately, or made some explicit qualification.

For each of the listed subgroups H of G, we usually give, in successive columns:

- its *order*
- its *index* in G
- its *structure* in the notation described at the end of this section
- for each automorphic extension $G.A$ of G, the *fusion behaviour* of H under A, and the associated maximal subgroups of $G.A$
- the *permutation character* of G on the cosets of H, when this is sufficiently simple,
- and finally various ways to *specify* a subgroup of type H in G.

In the column for an automorphic extension $G.A$,

- a *splitting symbol* (:) indicates that some conjugate of H is invariant under automorphisms of G that suffice to generate $G.A$ modulo G. (It then gives rise to an 'ordinary' maximal subgroup $H.A$ of $G.A$.)
- a *fusion join* (|) is used to show that several isomorphic groups H_i that are not conjugate in G become conjugate in $G.A$.

These symbols may be followed by structure-symbols for the 'ordinary' and 'novel' maximal subgroups of $G.A$. The maximal subgroups K of $G.A$ are of three types:

- *Trivialities*: these are the groups $G.B$, where B is a maximal subgroup of A. They are ignored in our tables.
- *Ordinaries*: if H is invariant under some automorphisms of G that generate $G.A$ modulo G, it extends to a maximal subgroup $K=H.A$ of $G.A$. In this case, we write the structure of K immediately after the corresponding splitting symbol. The letter H, when used, refers to the corresponding maximal subgroup of G.
- *Novelties*. It can happen that the intersection of K with G is a non-maximal proper subgroup of G. In such cases K is called a *novel* subgroup of $G.A$, and we have written its structure in the row corresponding to one of the maximal subgroups H of G that contains $K \cap G$.

Any maximal subgroup of $G.A$ that occurs after a fusion marker is a novelty; but novelties can also occur after a splitting symbol, in which case they are written *after* the corresponding 'ordinary' maximal subgroup $H.A$.

The *permutation character* of G associated with H is the character of the permutation representation of G acting by right multiplication on the right cosets of H in G. The irreducible constituents of this representation are indicated by their degrees followed by lower case letters a, b, c, . . . , which indicate the successive irreducible representations of G of that degree, in the order in which they appear in the ATLAS character table. A sequence of small letters (not necessarily distinct) after a single number indicates a sum of irreducible constituents all of the same degree. Thus

$$1a+45ab+216a+342aac$$

would be used to abbreviate

$$1a+45a+45b+216a+342a+342a+342c.$$

The *specifications* of subgroups H give information which might help to locate a copy of H inside G. An *abstract specification* expresses H as the normalizer in G of some element or subgroup, or alternatively as the centralizer in G of an outer automorphism. We indicate these (respectively) by writing $N(\ldots)$ or $C(\ldots)$, where the parentheses contain information about the conjugacy classes of the groups being normalized or centralized. The form $C(\ldots)$ is *only* used for the centralizers of *outer* automorphisms.

The class of a cyclic subgroup is indicated by its order, followed by the tag letters of its generators (see 'Class names', Chapter 7, Section 5): subscripts are used to count cyclic subgroups in a larger group, and superscripts indicate direct powers of groups. Thus the symbols below would be used for the normalizers in G of elements or groups of the indicated forms:

$N(2A)$:	an involution in G, of class $2A$.
$C(2B)$:	an outer automorphism of G, of class $2B$.
$C(2)$:	an outer automorphism of order 2 and unspecified class
$N(3A)$:	a group of order 3, with each generator in class $3A$.
$N(5AB)$:	a group of order 5, containing both classes $5A$ and $5B$.
$N(2A^2)$:	a fourgroup, whose involutions are in class $2A$.
$N(3^3)=N(3AB_4C_3D_6)$:	an elementary abelian group of order 27, whose 13 cyclic subgroups number 4 containing both classes $3A$ and $3B$, 3 containing $3C$ only, 6 containing $3D$ only (neglecting the identity element).
$N(2^6)$:	an elementary abelian group of order 2^6, unspecified classes.
$N(2A, 3B, 5CD)$:	an A_5, containing elements of classes $2A$, $3B$, $5C$, $5D$.
$N(2A, 3B, 5CD)^2$:	the direct product of two such A_5 groups.
$N(2A, 2C, 3A, 3B, \ldots)$:	a group containing elements of the indicated classes, among others.

The type of the group being normalized, when it is not obvious, can usually be deduced from the structure information about H. In the case of a non-abelian simple group S that is treated elsewhere in this ATLAS, the conjugacy classes in G of the non-trivial cyclic subgroups of S are listed in the order in which the classes appear in the ATLAS character table for S.

The remaining *specifications* describe subgroups, where possible, as the stabilizers of objects that arise naturally in the various *constructions* for G and related groups. The appropriate column is headed by a keyword which hints at the construction(s) referred to. The entry against a group H hints at an object whose stabilizer is a copy of H. These stabilizers should always be understood in the widest possible sense, so that for instance the stabilizer of a 'vector' would be the set of all group elements that fix that vector up to scalar multiplication (its *projective*

stabilizer). For this reason, we have a slight bias towards projective terminology in these columns.

There are many generic involvements of small Chevalley groups S in greater ones G. Thus $X_n(q^2)$ involves $X_n(q)$, $A_{mn-1}(q)$ involves $A_{n-1}(q)$ wr S_m, and $B_3(q)$ involves $G_2(q)$, which in turn involves both $A_2(q)$ and ${}^2A_2(q)$. We indicate these by writing just the name of S in the appropriate column of the maximal subgroup table for G, even though the actual maximal subgroup of G is often not just the group S, but a more complicated group involving S in its composition series. The structure column should supply the extra information.

2. Notations for group structures

There are many different ways in which structures of groups are indicated in the literature. We recommend the convenient ones below, and have used many of them in our tables of maximal subgroups and elsewhere in this ATLAS.

The notations C_m (*cyclic*), D_m (*dihedral*), Q_m (*quaternionic*, or *dicyclic*), E_m (*elementary abelian*), and X_m or X_m^+ or X_m^- (*extraspecial*), are often used for groups of order m with the indicated structures. (The reader should note that some authors use D_m and Q_m for the dihedral and quaternionic groups of order $2m$ rather than m.) The cyclic, dihedral, and quaternionic groups are defined by the presentations

$$\begin{array}{llll} C_m: & a^m=1 & & \\ D_m: & a^{\frac{1}{2}m}=1, & b^2=1, & b^{-1}ab=a^{-1} \quad (m \text{ even}) \\ Q_m: & a^{\frac{1}{2}m}=1, & b^2=a^{\frac{1}{4}m}, & b^{-1}ab=a^{-1} \quad (4 \text{ divides } m), \end{array}$$

and the elementary abelian and extraspecial groups are described later. The particular group Q_8 is the *quaternion group*, presented by $ij=k$, $jk=i$, $ki=j$, or by $i^2=j^2=k^2=ijk$, in which the common value is -1, a central element of order 2.

There are various ways to combine groups:

$A \times B$ is the *direct product*, or *Cartesian product*, of A and B. It may be defined as the set of ordered pairs (a, b) $(a \in A, b \in B)$, with $(a, b)(a', b') = (aa', bb')$.

$A.B$ or AB denotes any group having a normal subgroup of structure A, for which the corresponding quotient group has structure B. This is called an *upward extension* of A by B, or a *downward extension* of B by A.

$A:B$ or $A:^{\phi}B$ indicates a case of $A.B$ which is a *split extension*, or *semi-direct product*. The structure can be completely described by giving the homomorphism $\phi: B \to Aut(A)$ which shows how B acts by conjugation on A. It may be defined to consist of the ordered pairs (b, a) $(b \in B, a \in A)$, with $(b, a)(b', a') = (bb', a^{\phi(b')}a')$.

$A \cdot B$ indicates any case of $A.B$ which is *not* a split extension.

$A \circ B$ or $A \circ_C B$ indicates the *central product* of A and B over their common central subgroup C. Formally, we have injections $\alpha: C \to A$ and $\beta: C \to B$ of the abstract group C into the centres of A and B, and $A \circ B$ is the quotient of $A \times B$ by the set of ordered pairs of the form $(\alpha(c), \beta(c))$ $(c \in C)$. When it is omitted from the notation, C is usually understood to take the largest possible value.

$A \triangle B$ or $A \triangle_D B$ indicates a *diagonal product* of A and B over their common homomorphic image D. Formally, we have homomorphisms $\gamma: A \to D$ and $\delta: B \to D$ from A and B onto D, and $A \triangle B$ is the subgroup of $A \times B$ consisting of the (a, b) for which $\gamma(a) = \delta(b)$. When D is omitted from the notation, it is usually understood to take the largest possible value.

Central and diagonal products may have more than two factors. The notation $\frac{1}{2}(A \times B)$ denotes a diagonal product in which D has order 2.

$A \triangle\!\!\!\circ B$ or $A \circ_C \triangle_D B$ can be used for a *central-diagonal* product. For suitable groups, and with notations as above, this consists of the ordered pairs (a, b) for which $\gamma(a) = \delta(b)$ in D, taken modulo the pairs of form $(\alpha(c), \beta(c))$ $(c \in C)$.

$A \wr B$, or A wr B, the *wreath product* of A and B, is defined when B is a permutation group on n letters. It is the split extension $A^n:B$, where A^n is the direct product of n copies of A, on which B acts by permuting the factors.

In our structure symbols for complicated groups, we use the abbreviations:

$[m]$ for m an integer, denoting an arbitrary group of order m

m denoting a cyclic group of order m

A^n for the direct product of n groups of structure A. In particular

p^n where p is prime, indicates the elementary abelian group of that order.

Thus the two types of group of order four become

4 (the cyclic group), and 2^2 (the fourgroup).

p^{m+n} indicates a case of $p^m . p^n$, and so on.

Thus 2^{3+2+6} would indicate a 2-group having an elementary abelian normal subgroup of order 8 whose quotient contains a normal fourgroup modulo which becomes elementary abelian of order 2^6.

p^{1+2n} or p_+^{1+2n} or p_-^{1+2n} is used for the particular case of an extraspecial group. For each prime number p and positive n, there are just two types of extraspecial group, which are central products of n non-abelian groups of order p^3. For odd p, the subscript is + or − according as the group has exponent p or p^2. For $p=2$, it is + or − according as the central product has an even or odd number of quaternionic factors.

Products of three or more groups are left-associated, and should only be used when the indicated decomposition is invariant. Thus $A \cdot B:C$ means $(A \cdot B):C$, and implies the existence of a normal subgroup A.

Note to Chapters 6 and 7

The more complicated parts of these chapters are intended to be read in conjunction with one another. In these chapters, G usually denotes a simple group, with Schur multiplier M and outer automorphism group A. If ω is a root of unity, we shall sometimes say that a representation *represents a certain group element by* ω when we mean that it represents that element by ω times an identity matrix. The same phrase is applied to the character of that representation. Two characters of $M.G$ are said to belong to the same *cohort* if they represent the elements of M by the same roots of unity.

The unqualified word 'class' usually refers to a conjugacy class in a group G or $G.a$, while 'character' usually refers to a character of a group G or $m.G$.

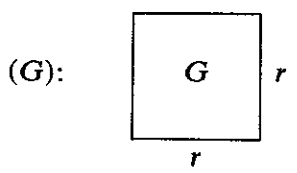
6

How to read this ATLAS: The map of an ATLAS character table

With each character table printed in this ATLAS we give a small map that explains to which groups the various portions of the table refer. The shape of this map depends on the Schur multiplier M and outer automorphism group A. We describe some of the simplest and most common cases first.

1. The case G (that is, trivial M and A)

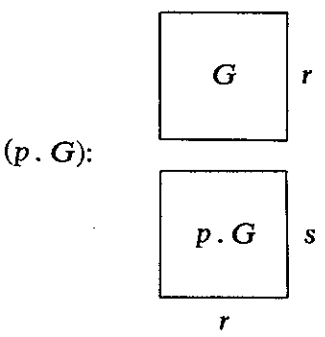
The map is a square



where the right-hand number r is the number of characters of G, and the lower one the number of classes. Of course these two numbers are equal, and usually called the *class number* of G.

2. The case $p.G$ (that is, M cyclic of prime order, A trivial)

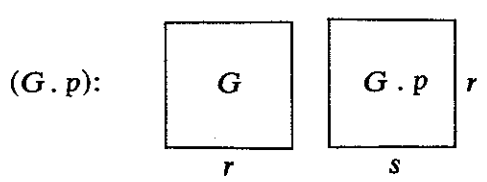
The map is a 'tower' of two squares



The upper square gives the same information as before. The lower one refers to one cohort of characters faithful on the central element, and tells us that there are s characters in that cohort. Since there are $p-1$ such cohorts, the class number of $p.G$ is $r+(p-1)s$. The orders of the elements in these classes are visible in the lifting order rows for $p.G$.

3. The case $G.p$ (M trivial, A cyclic of prime order)

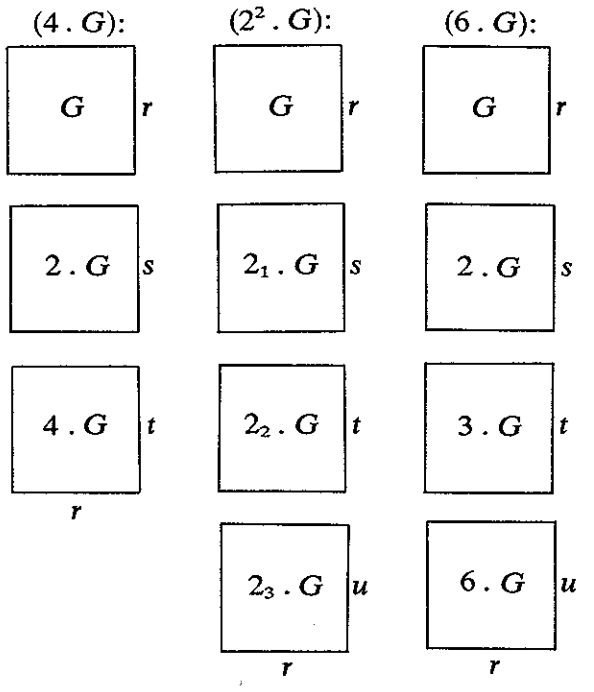
The map consists of two squares side by side:



This tells us that G has class number r, and that each other coset of G in $G.p$ contains s classes of $G.p$. Since $r-s$ characters of G fuse in sets of p, the class number of $G.p$ is $ps+(r-s)/p$, but is easiest found from the indicator column for $G.p$, which reveals the number of characters for that group.

4. The case $M.G$ (arbitrary multiplier, trivial automorphism group)

We illustrate the cases $4.G$, $2^2.G$, and $6.G$:



In the first case $(4.G)$, there is a cohort of r representations in which the central element is mapped to $+1$, and which we regard as representations of G, another cohort of s representations mapping that element to -1, which we regard as representations of $2.G$, and two cohorts of t representations each, mapping the central element to i or $-i$, which are faithful representations of $4.G$. The class numbers of G, $2.G$, and $4.G$ are r, $r+s$, and $r+s+2t$.

In the second case $(2^2.G)$, *no* irreducible representation can be faithful, since the central fourgroup must be represented by a group of roots of unity, which is necessarily cyclic. The map is a tower of four squares, the topmost one corresponding to the representations that are trivial on the central fourgroup, and so can be regarded as representations of G, and the other three corresponding to the three cyclic quotient groups, labelled 2_1, 2_2, 2_3, of the central fourgroup. The class numbers of G, $2_1.G$, $2_2.G$, $2_3.G$, and $2^2.G$ are r, $r+s$, $r+t$, $r+u$, and $r+s+t+u$.

In the third case $(6.G)$, the central subgroup has four cyclic quotients, of orders 1, 2, 3, and 6, and so we again have a tower of four squares, corresponding to groups G, $2.G$, $3.G$, and $6.G$. The map tells us that a central element of order 6 is represented by

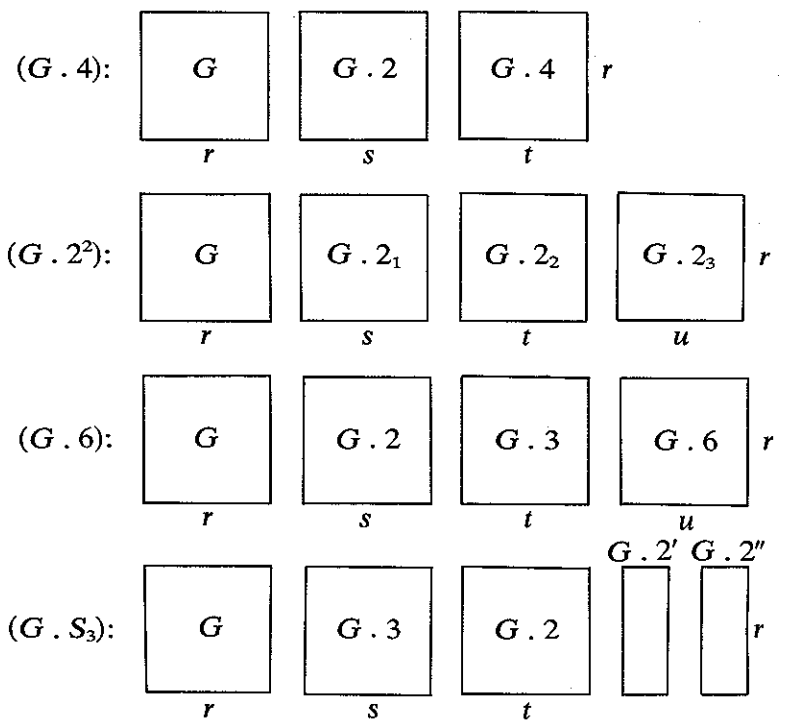
$$\begin{aligned} 1 &\text{ for } r \text{ characters} \\ -1 &\text{ for } s \text{ characters} \\ \omega \text{ or } \omega^2 &\text{ for } t \text{ characters each,} \\ -\omega \text{ or } -\omega^2 &\text{ for } u \text{ characters each,} \\ &\text{where } \omega = \exp(2\pi i/3). \end{aligned}$$

The class numbers of G, $2.G$, $3.G$, and $6.G$ are r, $r+s$, $r+2t$, $r+s+2t+2u$. The cohorts of characters representing the given central element by ω^2 and $-\omega^2$ are not printed.

In the general case $M.G$, there is a tower of squares, one for each cyclic quotient group (of order m, say) of the multiplier. The number to the right of a square is the number of characters in any of the cohorts which realize exactly that quotient group—there are $\phi(m)$ such cohorts, only one of which is printed.

5. The case $G.A$ (trivial multiplier, arbitrary outer automorphism group)

We illustrate the cases $G.4$, $G.2^2$, $G.6$, and $G.S_3$:

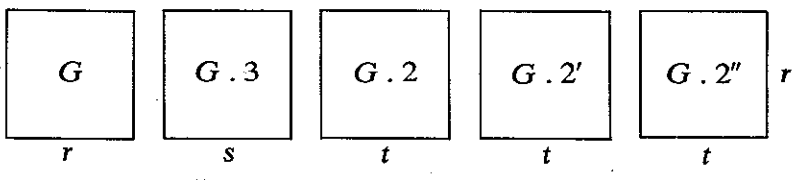


The 'dual' to the statement that any irreducible representation of $M.G$ realizes only a cyclic quotient of M is the much more obvious statement that any conjugacy class of $G.A$ is represented in some $G.a$, where a is a cyclic subgroup of A. Usually therefore an ATLAS table contains a detachment of columns for each cyclic subgroup of the outer automorphism group A. Each 'square' of the map corresponds to a *bicyclic* extension $m.G.a$ (m and a cyclic groups). The experienced user can often deduce information about non-bicyclic extensions, but we emphasize that 'officially'

ATLAS *tables give information only about bicyclic extensions.*

The first three of the above cases are the 'transposes' of the 'dual' cases $4.G$, $2^2.G$, and $6.G$, and we shall say little more about them. In the first case we have r classes of G, s classes of $G.2$ outside G, and $2t$ classes of $G.4$ outside $G.2$, but the exact class numbers of $G.2$ and $G.4$ are best obtained from the indicator columns for those groups. In the second case, we have r classes of G, and s, t, or u classes in the three extensions $G.2$ outside G. The class numbers of these can be found from the appropriate indicator columns, but in general there is no way to read the class number of $G.2^2$ from our table (it is not a bicyclic extension). The trouble is that a character of G that splits in (that is, extends to) each of the three extensions $G.2_1$, $G.2_2$, $G.2_3$ may either *split* to give four characters of $G.2^2$ or *fuse* to give just one, and there is no universal way to discover this from the character values.† This sort of problem does not arise in the third case, since the extending group is cyclic: a character which splits for $G.2$ and $G.3$ necessarily splits to give six characters for $G.6$.

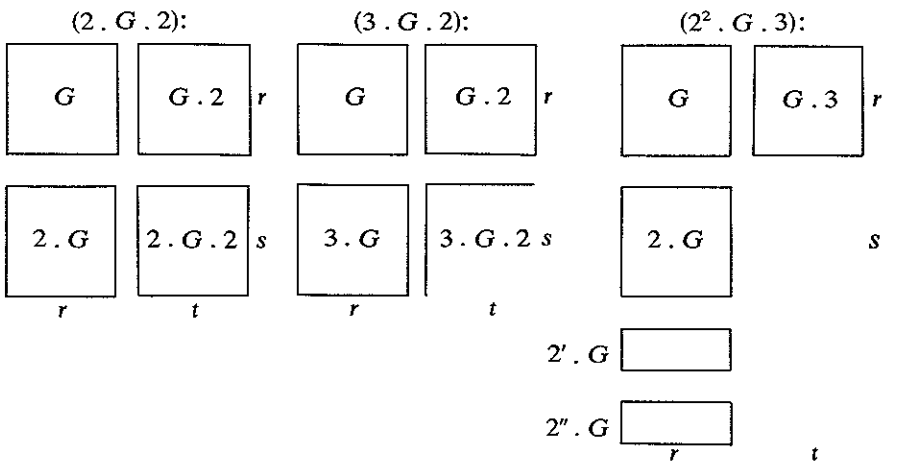
The fourth case, $G.S_3$ introduces new phenomena since the extending group is non-abelian. There are five cyclic subgroups of S_3, one of each order 1 and 3, and three of order 2, and so the full ATLAS table would have five detachments of columns, and a map:



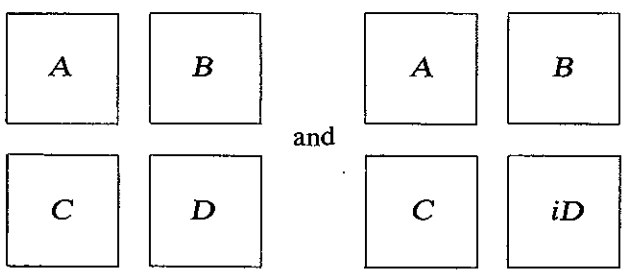
However, the last three blocks are images of each other under the automorphism group, and so contain much the same information. We therefore only print one of them in full (the *leader*), and just the indicator and fusion columns for the other two (its *followers*). In our maps, the 'squares' corresponding to follower cosets that have been suppressed in this way are 'squashed' in the way shown when we introduced this example.

6. The general case $M.G.A$

We illustrate three particular cases:



The first is the very common case $2.G.2$. The conventions about it are essentially the union of those for cases of form $2.G$ and $G.2$, and need little further explanation. However, it can be important to realize that any group $2.G.2$ has an isoclinic variant, and of course the group $2.G.2$ you are investigating may not be the one that we print. For the definitions, see 'Isoclinism' (Section 7) at the end of this chapter. The ATLAS tables for a group $2.G.2$ and its isoclinic variant $(2.G.2)^-$ have the forms

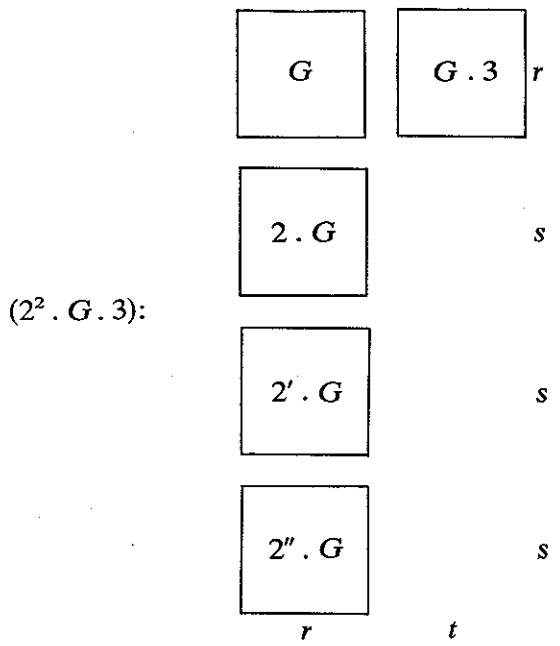


The orders of elements in the variant can be deduced from the statement that if g is an element of $2.G.2$ outside $2.G$, with $g^2 = h$, then in $(2.G.2)^-$ the corresponding element squares to $-h$, where -1 is the central element of order 2. If χ, χ^+, χ^- are corresponding characters of $2.G$, $2.G.2$, and $(2.G.2)^-$ respectively, faithful on the central element of order 2, then $\mathrm{ind}(\chi) = \mathrm{ind}(\chi^+) + \mathrm{ind}(\chi^-)$.

Our second example is for a group $3.G.2$, in which a central element z of order 3 in $3.G$ is inverted by the outer automorphisms. In this case each character of $3.G$ in the printed cohort that represents z by $\omega = \exp(2\pi i/3)$ fuses in $3.G.2$ with one from the unprinted cohort representing z by ω^2. It follows that all entries in the bottom right-hand portion of the table are zero. We therefore leave this portion blank, except for its fusion and indicator columns and lifting order rows, and represent this situation in our map by drawing only the leading two sides of the corresponding square. In such cases, the fusion column for $3.G.2$ will contain symbols $*$, which indicate fusion of the unprinted characters with either their own proxies or the proxies of other unprinted characters, as indicated by the joins (see Chapter 7, Section 18).

† When a character *does* extend to a group $G.A$ (A non-cyclic) the character values we print for the various cyclic extensions should all be part of a single character for $G.A$. In the case $G.2^2$ there is a simple rule which usually settles the question: if a *real-valued* (indicator + or −) character for G extends to characters α, β, γ for each of $G.2_1$, $G.2_2$, $G.2_3$ then it extends to $G.2^2$ if and only if an odd number of α, β, γ are real-valued characters.

The third example is for a group $2^2 . G . 3$, in which the multiplier has three cyclic quotients of order 2, permuted by the automorphism group. The full ATLAS table would contain a detachment of rows for each of the three corresponding groups $2 . G$, $2' . G$, $2'' . G$, and would have map:



We do not draw blocks $2 . G . 3$, $2' . G . 3$, $2'' . G . 3$ since the corresponding groups do not exist. However, the lower three blocks in this full table are images of each other under the automorphism group, and so contain much the same information. We therefore print only one of them in full (the *leader* cohort), and just the lifting order rows for the other two cohorts (its *followers*). (The lifting order rows for triple or higher order multiple covers that have been suppressed in this way are printed in a compressed format—see Chapter 7, Section 9.) In our maps, the 'squares' corresponding to the follower cosets that have been suppressed in this way are 'squashed' in the way shown when we introduced this example.

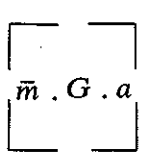
In a completely general case, the Full ATLAS Table (FAT) would contain a detachment of columns for each cyclic subgroup a of the outer automorphism group, and a detachment of rows for each cyclic quotient group m of the multiplier. If a transforms m into a different quotient group, there is no square on the map corresponding to $m . G . a$. If a acts non-trivially upon m, we print only two sides of the corresponding square, as in the case $3 . G . 2$, since the corresponding portion of the character table has all character values zero. There can only be non-zero character values when a centralizes m, and we then draw a full square in our map for the portion $m . G . a$.

In complicated cases, the table we print is usually smaller. The outer automorphism group will have various orbits on the cyclic subgroups of A or quotient groups of M, and from each orbit of the corresponding detachments of columns or rows we print just one in full, but retain only the indicator and fusion columns, or lifting order rows, for its followers. The portions of our maps corresponding to the suppressed objects are 'squashed'.

Of course in a complicated table we might have an orbit of $r > 1$ cosets, and also an orbit of $s > 1$ cohorts! In the cases we print, it happens that of the corresponding rs portions of the full ATLAS table all the ones that contain non-zero character values are equivalent under the automorphisms, and so we need only print one such portion in full.

For an example of a case when these rs portions include several non-zero orbits under the automorphisms, see the parts labelled x and y in our map for $O_8^+(3)$.

There are occasions when cyclic upward and downward extensions $G . a$ and $m . G$ exist, and a centralizes m, but there is no corresponding bicyclic extension $m . G . a$. They are indicated in our maps by a 'broken box'



which contains the name of a rather larger extension that does exist. We defer further explanations until we have explained the notion of isoclinism.

7. Isoclinism

The *commutator* $[x, y] = x^{-1}y^{-1}xy$ of two elements x, y of a group G is unaffected when we multiply x or y by an element of the centre $Z(G)$ of G, and of course its values lie in the *derived group* or *commutator subgroup* $G' = [G, G]$ of G (since this is defined as the group generated by all such commutators). We can therefore regard the commutator function as a map from $G/Z(G) \times G/Z(G)$ to G'. Two groups G and H are said to be *isoclinic* if they have 'the same' commutator map in this sense; in other words if there are isomorphisms $G/Z(G) \to H/Z(H)$ and $G' \to H'$ which identify the two commutator maps.

An equivalent and more concrete definition is to say that G and H are isoclinic if we can enlarge their centres so as to get isomorphic groups. This means that each of G and H can be embedded in a larger group K in such a way that K is generated by $Z(K)$ and G, and also by $Z(K)$ and H.

Philip Hall introduced isoclinism in connection with the enumeration of p-groups. It is also of vital importance in the theory of projective representations and the Schur multiplier. Indeed, any irreducible representation of G can be converted to one of H of the same degree, for which the representing matrices are scalar multiples of those that represent G. To see this, restrict the representation to $Z(G)$, when we obtain a set of scalar matrices to which we can adjoin others which extend this representation to one of $Z(K)$, and hence extend the original representation of G to one of K. The desired representation of H is now obtained by restriction.

Thus the dihedral and quaternionic groups of order 8 are isoclinic: the matrices $\pm\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, $\pm\begin{pmatrix}i & 0\\0 & -i\end{pmatrix}$, $\pm\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$, $\pm\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$ represent the former, and by multiplying the last two pairs of matrices by the scalar matrix $\begin{pmatrix}i & 0\\0 & i\end{pmatrix}$ we obtain a representation of the latter. A similar phenomenon occurs with any faithful representation of a group $H = 2 . G . 2$. The representing matrices will be various matrices $\pm A$ (for elements of $2 . G$), and $\pm B$ (for the remaining elements of H), and there is an isoclinic group K, (also of structure $2 . G . 2$) represented by the matrices $\pm A$ and $\pm iB$. A group of structure $3 . G . 3$ is one of three isoclinic variants, in which the elements in the three cosets of the subgroup $3 . G$ are represented by matrices

$$\omega^n A,\ \omega^n B,\ \omega^n C$$
$$\omega^n A,\ \varepsilon\omega^n B,\ \varepsilon^2\omega^n C$$
$$\omega^n A,\ \varepsilon^2\omega^n B,\ \varepsilon\omega^n C,$$

where $\omega = \exp(2\pi i/3)$, $\varepsilon = \exp(2\pi i/9)$.

A *perfect* group (one for which $G = G'$) has a unique central extension $M . G$ realizing any quotient M of its Schur multiplier. Thus for example there is a unique proper cover $2 . A_n$ of the alternating group A_n for $n \geqslant 5$. For more general groups G, the appropriate groups $M . G$ are only unique up to isoclinism. Thus for example, the symmetric groups S_n $(n \geqslant 5)$ have two double covers, 2^+S_n and 2^-S_n, which can be converted into each other by multiplying the elements that lie above odd permutations by a new central element i, whose square is the central element of order 2 in either group. The one we tabulate is 2^+S_n, in which the transpositions of S_n lift to involutions.

There is a sense in which the different portions of an ATLAS character table (or the squares of its map) correspond to isoclinism classes of bicyclic extensions, rather than to the individual extensions whose characters we choose to print. Changing to a different group in the isoclinism class merely multiplies the associated portions of a table by various roots of unity (and affects their headings).

It can happen that upward and downward cyclic extensions

$G.a$ and $m.G$ exist, and the automorphisms of a centralize m, but no corresponding bicyclic extension $m.G.a$ exists. The relevant isoclinism class of extensions is well-defined, but happens to contain no group of the 'expected' order $|m|.|G|.|a|$. We illustrate with the case $G = A_6$, which has three cyclic automorphic extensions $G.2_1 = S_6$, $G.2_2 = PGL_2(9)$, and $G.2_3 =$ the 'Mathieu' group M_{10}.

The Schur double cover $2.G$, is categorically associated with G, and so any automorphism of G can be lifted to an automorphism of $2.G$. Now it happens that $G.2_1$ and $G.2_2$ are split extensions, from which it follows that there *do* exist groups $2.G.2_1$ and $2.G.2_2$ of the required type. To see this, take an involution x extending G to the appropriate $G.2_i$. Then the automorphism $g \to x^{-1}gx$ of G lifts to an automorphism θ_x, say, of $2.G$, and the split extension $(2.G):2$ of $2.G$ by this automorphism is the desired group $2.G.2_i$ $(i = 1, 2)$.

However, the group $G.2_3$ is a non-split extension of G. We can generate it by G together with an element x of order 4 with $x^2 = y$ in G. It turns out that the automorphism θ_x of $2.G$ obtained by lifting the automorphism $g \to x^{-1}gx$ *interchanges the two preimages of* y *in* $2.G$. It is therefore impossible to achieve this by an element of order 4 (in a putative group $2.G.2_3$) which squares to either preimage of y.

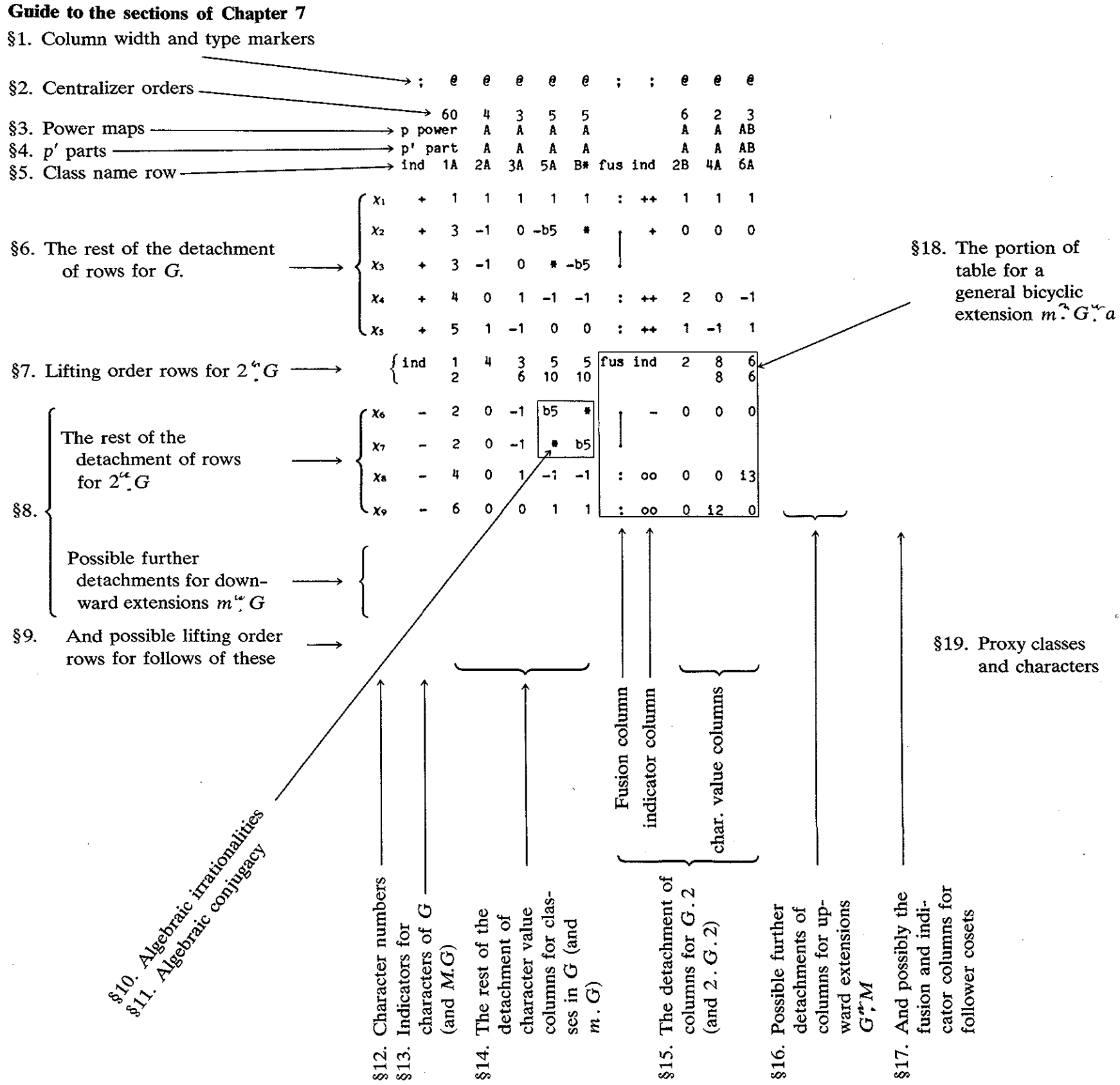
When such a case arises in this ATLAS, we draw a *broken box*, for example:

$$\boxed{4.G.2}$$

in the appropriate place on the map, containing the name of a larger extension that does exist. In our example, this is a group $4.G.2$ which can be obtained as the split extension of $2.G$ by the automorphism θ_x of order 4 just described. (There is also an extension of structure $2^2.G.2$ in this isoclinism class, but we prefer the bicyclic one.)

It is slightly easier in this example to see that no enveloping group of structure $2.G.2^2$ can exist. This is because the outer automorphism of S_6 interchanges the two isoclinism classes of groups $2S_6$. (It interchanges elements of cycle-types $2^1 1^4$ and 2^3, and just one of these lifts to an involution in either $2S_6$, and so neither of these groups can possess an automorphism of the required type.) We remark that the calculations needed to verify our assertions about this example can be performed in $\Gamma L_2(9)$.

Guide to the sections of Chapter 7



7

How to read this ATLAS: character tables

The Guide describes the parts of a typical ATLAS character table. We shall first describe it row-by-row, and then column-by-column. Much of the complexity of this description arises from the need to consider groups with arbitrary Schur multiplier and outer automorphism group—the reader who is not interested in complicated cases should find that the first few sections suffice.

1. Column width and type markers

The first row contains the symbol @ at the head of a column that later contains character values of some conjugacy class, and the symbol ; for one that contains fusion or indicator information.

If the table is read mechanically, this row can be used to determine the widths of the columns, which may be necessary since occasionally the entries in adjacent columns abut. The information in any column is right-justified.

2. Centralizer orders

The second and third rows give the order of the centralizer in G of the typical element of each class. Since the numbers are sometimes too large to fit in the appropriate portion of a single row, two rows are provided, the top one containing the leading digits of large numbers.

The numbers given are the orders of centralizers in the base group G, even for columns that refer to classes in a larger group $G.a$.

3. Power maps

The kth powers of the elements of a given class form another class. The resulting *power maps* between the classes can be found by repeated use of the *prime power maps*, the particular cases when k is prime.

The fourth row gives the tag letters (see Section 5, 'Class names') of the classes that contain the powers $g^p, g^q, g^r, \ldots$ of g, where $p<q<r<\ldots$ are the distinct prime divisors of the order of g. Thus if g has order 60, an entry ABC means that

$$g^2 \text{ is in class } 30A,\ g^3 \text{ in class } 20B,\ g^5 \text{ in class } 12C.$$

If k is a number prime to the order of g, the class of g^k can be found by applying the algebraic conjugacy operator $*k$ to the class of g.

It can happen that the pth power of an element of an automorphic extension $G.a$ does not lie in any of our chosen generator cosets (see Section 16). In such cases the tag letter given is that of the proxy class. We do not give complete power map information for groups $m.G$ and $m.G.a$. A good deal can be deduced from the orders of elements in those groups, and the power maps for the corresponding groups G and $G.a$.

4. p′-parts

If π is any set of primes, we can uniquely write

$$g = g(\pi).g(\pi') = g(\pi').g(\pi),$$

where π contains all the prime divisors of the order of $g(\pi)$, but none of those of the order of $g(\pi')$. The elements $g(\pi)$ and $g(\pi')$, which are certain powers of g, are respectively called the *π-part* and *π'-part* of g. When π contains a single prime p, we write $g(p)$ and $g(p')$ for these, and call them the *p-part* and *p'-part* of g.

The π-parts for general sets π can be found by repeated use of the p'-parts. The fifth row of the table gives the tag letters for $g(p'), g(q'), g(r'), \ldots$, where $p<q<r<\ldots$ are the distinct prime divisors of the order of g. We do not give complete information on the p' parts for multiple covers $m.G$.

5. Class names

The conjugacy classes that contain elements of order n are named $nA, nB, nC, \ldots$. The alphabet used here is potentially infinite, and reads

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R,
S, T, U, V, W, X, Y, Z, A1, B1, . . . , A2, B2, ...

The *class name row* contains the following information:

The headings 'ind' and 'fus' for indicator and fusion columns;
'Master' class name entries, of form nX, meaning just that the column refers to a conjugacy class nX;
'Slave' class name entries, of form $Y*k$ (or $Y**k$, $Y**$, $Y*$), which refer to a class nY, and also give the information that nY can be obtained from the most recent 'master' class nX by applying the algebraic conjugacy operator $*k$ (or $**k$, $**$, $*$).

A 'master' class and the immediately subsequent 'slave' classes complete an *algebraically conjugate family* of classes. The algebraic conjugacy operators are defined on classes as follows:

$(nX)^{*k}$ contans the kth powers of elements of nX;
$(nX)^{**k}$ contains the $(-k)$th powers of elements of nX;
$(nX)^{**}$ contains the inverses of elements of nX; and
$(nX)^{*}$ is the class other than nX containing elements of order n that are powers of elements of nX, when this class is unique.

It is to be understood that k is prime to n.

The values of characters on these classes are the images of their values on nX under the appropriate algebraic conjugacies.

6. Character values, for characters of G

The typical row of the character table consists mostly of character values, together with indicator and fusion information that is described later. Usually the character values are ordinary integers, but certain algebraic irrationalities can also arise. In such cases we *either* print the ATLAS name for the desired irrationality in full, *or* just an algebraic conjugacy operator by which it can be obtained from a nearby entry in the same row. To be precise, this nearby entry can be any entry for a class in the same algebraically conjugate family as the desired one that *is* printed in full.

The complete ATLAS system of names for algebraic irrationalities is given later (Section 10). The following should

suffice for simple cases:

$$rN=\sqrt{N} \qquad zN=\exp(2\pi i/N)$$

$$iN=i\sqrt{N} \qquad yN=2\cos(2\pi/N)$$

$$bN=\begin{cases}\frac{1}{2}(-1+\sqrt{N}) & (N\equiv 1, \bmod 4)\\ \frac{1}{2}(-1+i\sqrt{N}) & (N\equiv -1, \bmod 4)\end{cases} \qquad mN=2i\sin(2\pi/N),$$

where of course i denotes $\sqrt{-1}$.

Any of these algebraic numbers can be expressed as a linear combination of nth roots of unity for some n. If k is prime to n, the algebraic conjugacy operator $*k$ acts on such numbers by replacing these roots of unity by their kth powers. The operator $**k$ is an abbreviation for $*(-k)$, and $**$ is the complex conjugation operator, which is $*(-1)$. The operator $*$ is defined for a quadratic number field, and replaces $r+s\sqrt{N}$ by $r-s\sqrt{N}$ (r, s rational, $\sqrt{N}$ irrational).

7. Lifting order rows for a group $m\,.\,G$

If G has a (cyclic) multiple cover $m\,.\,G$, then g will be the image of m distinct elements $g_0, g_1, \ldots, g_{m-1}$ of $m\,.\,G$. There will be some divisor d of m such that g_i will be conjugate (in $m\,.\,G$) to g_j just when i and j are congruent modulo d, so that $g_0, g_1, \ldots, g_{d-1}$ will represent the distinct conjugacy classes lying above the class of g.

For any such extension $m\,.\,G$ for which characters are printed, we give m *lifting order rows* which contain the orders of the elements $g_0, g_1, \ldots, g_{d-1}$ (and by implication, the number d). There is a slightly different convention for cases when the characters are not printed (Section 9).

8. Character values for a group $m\,.\,G$

We first describe the case $m=2$. Let Ω be the central element of order 2. Any irreducible representation of $2\,.\,G$ must represent Ω by I or $-I$, where I is the appropriate identity matrix, and in the former case we regard the representation as being one of G. In the latter case, the corresponding row contains the indicator of χ, followed by its particular values $\chi(g_0)$, the remaining values being deducible since $\chi(g_1)=-\chi(g_0)$ for these characters.

Thus the character table of $2A_5$ is:

class	$1A_0$	$1A_1$	$2A_0$	$3A_0$	$3A_1$	$5A_0$	$5A_1$	$5B_0$	$5B_1$
order	1	2	4	3	6	5	10	5	10
+	1	1	1	1	1	1	1	1	1
+	3	3	−1	−0	0	$-b_5$	$-b_5$	$-b_5^*$	$-b_5^*$
+	3	3	−1	0	0	$-b_5^*$	$-b_5^*$	$-b_5$	$-b_5$
+	4	4	0	1	1	−1	−1	−1	−1
+	5	5	1	−1	−1	0	0	0	0
−	2	−2	0	−1	1	b_5	$-b_5$	b_5^*	$-b_5^*$
−	2	−2	0	−1	1	b_5^*	$-b_5^*$	b_5	$-b_5$
−	4	−4	0	1	−1	−1	1	−1	1
−	6	−6	0	0	0	1	−1	1	−1

In general, an irreducible character χ of a central extension $M\,.\,G$ of G restricts to M to give a scalar multiple of some linear (that is, degree 1) character χ' of M. The set of χ yielding a particular χ' on M will be called a *cohort* of characters. The values of χ' on M will be all the roots of unity whose order divides some m, and so the characters of the cohort can be regarded as characters of some *cyclic* extension $m\,.\,G$, a quotient of $M\,.\,G$.

For each such cyclic extension $m\,.\,G$, we choose a particular cohort of characters χ for which $\chi'(\Omega)=\omega I$, where Ω generates the central cyclic subgroup of $m\,.\,G$, and ω is a particular primitive mth root of unity. The chosen cohort will be called the *generator cohort* for $m\,.\,G$. A character χ of $m\,.\,G$ from a cohort with $\chi'(\Omega)=\omega_1 I$, where ω_1 is another primitive mth root of unity, is algebraically conjugate to a character from the generator cohort, which we shall call its *proxy*. (The precise rules relating characters to their proxies will be defined in Section 19.)

The detachment of rows for a generator cohort for which we print character values is headed by m lifting order rows as described in the previous section. Each subsequent row really represents $\phi(m)$ algebraically conjugate characters, the printed one being that from the generator cohort, which acts as proxy for the others.

Whenever a row of an ATLAS table is used to represent $M\geqslant 2$ algebraically conjugate characters, the entry in its indicator column is $+M$, $\circ M$, $-M$ according as the indicators (see Section 13) of these characters are +, ○, − (they are necessarily the same). So for $m\geqslant 3$, the characters we print for the generator cohort of a group $m\,.\,G$ all represent $\phi(m)=M$ characters, and are headed by the entry $\circ M$ in the indicator column (their values include multiples of the non-real mth roots of unity).

We illustrate all this by describing the generator cohort of characters for $3A_6$. The appropriate portion of the ATLAS table reads:

ind	1	2	3	3	4	5	5	
	3	6			12	15	15	
	3	6			12	15	15	
χ_{14}	o2	3	−1	0	0	1	−b5	*
χ_{15}	o2	3	−1	0	0	1	*	−b5
χ_{16}	o2	6	2	0	0	0	1	1
χ_{17}	o2	9	1	0	0	1	−1	−1
χ_{18}	o2	15	−1	0	0	−1	0	0

The entries in the lifting order rows give us the number of conjugacy classes (17) of $3A_6$ and the orders of the elements they contain. The characters of this cohort, if written out at length, would look like:

class of g	$1A_0$	$1A_1$	$1A_2$	$2A_0$	$2A_1$	$2A_2$	$3A_0$	$3B_0$
order of g	1	3	3	2	6	6	3	3
values of χ_{14}:	3	3ω	$3\bar{\omega}$	−1	$-\omega$	$-\bar{\omega}$	0	0
χ_{15}:	3	3ω	$3\bar{\omega}$	−1	$-\omega$	$-\bar{\omega}$	0	0
χ_{16}:	6	6ω	$6\bar{\omega}$	2	2ω	$2\bar{\omega}$	0	0
χ_{17}:	9	9ω	$9\bar{\omega}$	1	ω	$\bar{\omega}$	0	0
χ_{18}:	15	15ω	$15\bar{\omega}$	−1	$-\omega$	$-\bar{\omega}$	0	0

$4A_0$	$4A_1$	$4A_2$	$5A_0$	$5A_1$	$5A_2$	$5B_0$	$5B_1$	$5B_2$
4	12	12	5	15	15	5	15	15
1	ω	$\bar{\omega}$	$-b_5$	$-b_5\omega$	$-b_5\bar{\omega}$	$-b_5^*$	$-b_5^*\omega$	$-b_5^*\bar{\omega}$
1	ω	$\bar{\omega}$	$-b_5^*$	$-b_5^*\omega$	$-b_5^*\bar{\omega}$	$-b_5$	$-b_5\omega$	$-b_5\bar{\omega}$
0	0	0	1	ω	$\bar{\omega}$	1	ω	$\bar{\omega}$
1	ω	$\bar{\omega}$	−1	$-\omega$	$-\bar{\omega}$	−1	$-\omega$	$-\bar{\omega}$
−1	$-\omega$	$-\bar{\omega}$	0	0	0	0	0	0

where $\omega=\exp(2\pi i/3)$ is a primitive cube root of unity. The alternative cohort can be obtained by applying any algebraic conjugacy operation that takes ω to $\bar{\omega}$, and the complex conjugacy operator $**$ is the nicest choice. The trivial cohort consisting of the 7 characters for G itself, supplemented by these two cohorts of 5 characters each for $3\,.\,G$, gives the entire set of 17 characters for $3\,.\,G$.

9. Follower cohorts

A rather larger table than the one we print is what we call the Full Atlas Table (FAT), which would contain a full detachment of rows corresponding to the generator cohort of each downward cyclic extension $m\,.\,G$ of G. But for complicated tables the FAT table would be *too* fat, since there can be several cyclic covers $m\,.\,G$ that are equivalent under the action of $Aut(G)$. We thin it out by selecting only one (*leader*) extension $m\,.\,G$ for each family equivalent under $Aut(G)$. The detachment of rows for the leader extension is printed in full, and followed by only the lifting order rows for the remaining (*follower*) extensions equivalent to it.

To save space, the lifting order rows are printed in a slightly different format, needing only two rows. The top row, headed

'and' gives the order of the canonical inverse images g_0 of elements g in G. (We always choose g_0 to have the smallest order of any inverse image of g.) The bottom row, headed 'no'. gives the number of distinct conjugacy classes of $m.G$ represented by the inverse images of g (when this number exceeds 1).

10. The ATLAS notation for algebraic numbers

Note. For greater legibility in this Introduction, we use subscripts and superscripts as in ordinary mathematical practice. However, our notation for algebraic numbers has been designed so that it can all be written on one line without loss of information. Thus what we here write as $y_{24}^{\prime\prime *5}$ appears elsewhere in our tables and text as y''24*5.

As usual, we let i denote a fixed square root of -1. Then for suitable integers N we define:

$$z_N = \exp(2\pi i/N) = z, \text{ say}$$

$$b_N = \tfrac{1}{2}\sum_{t=1}^{N-1} z^{t^2} \quad (N \equiv 1 \bmod 2) \qquad y_N = z + z^n = 2\cos(2\pi/N) \qquad (n = n_2 = -1)$$

$$c_N = \tfrac{1}{3}\sum_{t=1}^{N-1} z^{t^3} \quad (N \equiv 1 \bmod 3) \qquad x_N = z + z^n + z^{n^2} \qquad (n = n_3)$$

$$d_N = \tfrac{1}{4}\sum_{t=1}^{N-1} z^{t^4} \quad (N \equiv 1 \bmod 4) \qquad w_N = z + z^n + z^{n^2} + z^{n^3} \qquad (n = n_4)$$

$$e_N = \tfrac{1}{5}\sum_{t=1}^{N-1} z^{t^5} \quad (N \equiv 1 \bmod 5) \qquad v_N = z + z^n + z^{n^2} + z^{n^3} + z^{n^4} \qquad (n = n_5)$$

$$f_N = \tfrac{1}{6}\sum_{t=1}^{N-1} z^{t^6} \quad (N \equiv 1 \bmod 6) \qquad u_N = z + z^n + z^{n^2} + \ldots + z^{n^5} \qquad (n = n_6)$$

$$g_N = \tfrac{1}{7}\sum_{t=1}^{N-1} z^{t^7} \quad (N \equiv 1 \bmod 7) \qquad t_N = z + z^n + z^{n^2} + \ldots + z^{n^6} \qquad (n = n_7)$$

$$h_N = \tfrac{1}{8}\sum_{t=1}^{N-1} z^{t^8} \quad (N \equiv 1 \bmod 8) \qquad s_N = z + z^n + z^{n^2} + \ldots + z^{n^7} \qquad (n = n_8)$$

$$i_N = i\sqrt{N} \qquad r_N = \sqrt{N}$$

$$m_N = z - z^n = 2(\sin 2\pi/N)i \qquad (n = n_2 = -1)$$

$$l_N = z - z^n + z^{n^2} - z^{n^3} \qquad (n = n_4)$$

$$k_N = z - z^n + \ldots - z^{n^5} \qquad (n = n_6)$$

$$j_N = z - z^n + \ldots - z^{n^7} \qquad (n = n_8).$$

It follows from a famous theorem of Gauss that

$$b_N = \begin{cases} \tfrac{1}{2}(-1+\sqrt{N}) & \text{if } N \equiv 1, \bmod 4 \\ \tfrac{1}{2}(-1+i\sqrt{N}) & \text{if } N \equiv -1, \bmod 4. \end{cases}$$

The numbers $n_2, n_3, \ldots, n_8$ are defined as follows: n_k is the smallest integer in absolute value that has multiplicative order exactly k modulo N, the positive value being chosen when n_k and $-n_k$ have the same order. (This definition ensures that $n_2 = -1$.)

Further algebraic numbers

$$y'_N, y''_N, y'''_N, \ldots, x'_N, x''_N, \ldots, j'_N, j''_N, \ldots,$$

are obtained on replacing n_k in the above definitions by $n'_k, n''_k, \ldots$, where

$$n_k, n_k', n_k'', n_k''', \ldots$$

are *all* the numbers of multiplicative order exactly k modulo N, chosen in the order of preference

$$1, -1, 2, -2, 3, -3, 4, -4, \ldots.$$

Thus the numbers of exact order 2 modulo 24 are, in this order,

$$-1, 5, -5, 7, -7, 11, \ldots,$$

and so we have

$$y'_{24} = z_{24} + z_{24}^{5}, \quad y''_{24} = z_{24} + z_{24}^{-5}, \quad y'''_{24} = z_{24} + z_{24}^{7}, \ldots.$$

The irrational numbers that appear in ATLAS character tables are named in terms of the *atomic* irrationalities we have just defined. However, when several neighbouring numbers in a row are algebraically conjugate, we often print only some of them in full, and for the others indicate only the algebraic conjugacy by which they can be obtained from their neighbours.

Thus the ATLAS table for A_5 contains the 2×2 block of entries

$$\begin{array}{ccc} & 5A & 5B \\ \chi_2 & -b5 & * \\ \chi_3 & * & -b5 \end{array}, \text{ which abbreviates } \begin{array}{ccc} & 5A & 5B \\ \chi_2 & -b_5 & -b_5^*, \\ \chi_3 & -b_5^* & -b_5 \end{array}$$

or, in full,

$$\begin{array}{ccc} & 5A & 5B \\ \chi_2 & -\tfrac{1}{2}(-1+\sqrt{5}) & -\tfrac{1}{2}(-1-\sqrt{5}) \\ \chi_3 & -\tfrac{1}{2}(-1-\sqrt{5}) & -\tfrac{1}{2}(-1+\sqrt{5}), \end{array}$$

because in this instance $*$ indicates the algebraic conjugacy that changes the sign of $\sqrt{5}$, and we have $b_5 = \tfrac{1}{2}(-1+\sqrt{5})$ by Gauss's theorem.

In general, when only the name ($*k$, $**k$, $**$, or $*$) of an algebraic conjugacy appears as a character value entry, it is to be applied to an entry in the same row whose value is given in full, and which refers to a class in the same algebraically conjugate family as the desired one. Such families can be detected by examining the class name row (Section 5). The conjugacy $*k$ replaces nth roots of unity by their kth powers, whenever k is prime to n, and we have

$$**k = *(-k)$$
$$** = *(-1),$$

while $*$ is only applied to a quadratic number field, and takes $r + s\sqrt{N}$ to $r - s\sqrt{N}$ (r, s rational, $\sqrt{N}$ irrational).

The ampersand convention

On rare occasions, we have used another convention to abbreviate some very complicated irrationalities. If q_N denotes any of our atomic irrationalities (e.g. y_{24}''), then linear combinations of algebraic conjugates of q_N are abbreviated as in the following examples:

2qN+3&5-4&7+&9	means $2q_N + 3q_N^{*5} - 4q_N^{*7} + q_N^{*9}$
4qN&3&5&7-3&11	means $4(q_N + q_N^{*3} + q_N^{*5} + q_N^{*7}) - 3q_N^{*11}$
4qN*3&5+&7	means $4(q_N^{*3} + q_N^{*5}) + q_N^{*7}$.

(Only much simpler examples actually occur!) To explain the 'ampersand' syntax in general we remark that '&k' is interpreted as q_N^{*k}, where q_N is the most recently named atomic irrationality, and that the scope of any premultiplying coefficient is broken by a + or − sign, but not by & or $*k$. The algebraic conjugations indicated by the ampersands apply directly to the *atomic* irrationality q_N, even when, as in our last example, q_N first appears with another conjugacy $*k$.

11. A note on algebraic conjugacy

Our algebraic conjugacy operators $*k$ operate on a wide variety of concepts, always defined with respect to some number n prime to k:

Algebraic numbers in the field of nth roots of unity—this is the primitive notion: the action was defined in Section 10.

Elements g of order n: $*k$ takes g to g^k.

The corresponding conjugacy classes: $*k$ takes the class of g to that of g^k.

Cosets C of G in an upward extension of G, where $C^n = G$: $*k$ takes C to C^k.

Characters defined in the field of nth roots of unity: $*k$ acts on their values

Cohorts C of characters faithful exactly on a cyclic quotient of order n of the multiplier: $*k$ takes the cohort representing a given central element by ω to that representing it by ω^k.

We now turn to the column by column description of an ATLAS table.

12. The character number column(s)

In this column the rows containing characters are numbered $\chi_1, \chi_2, \chi_3, \ldots$ for easy reference. In wide tables this column is repeated at the left and right edges of the separate pages.

The reader is warned that in complicated situations a single row may represent several characters.

13. The indicator column for G, and for $m.G$

The Frobenius–Schur *indicator* function for a character χ of a group H may be defined as

$$\operatorname{ind}\chi = \frac{1}{|H|}\sum_{h\in H}\chi(h^2).$$

For an irreducible character χ we have

ind $\chi = 0$ if χ takes any non-real value
ind $\chi = +1$ if χ is afforded by a representation written over the reals
ind $\chi = -1$ for other real-value characters.

The *indicator column* uses the symbols ○, +, − for these three cases.

The rows corresponding to a triple or higher order multiple cover $m.G$ of G each represent $\phi(m)$ distinct characters related by algebraic conjugation. The character values are printed only for one of these characters, which serves as *proxy* for the others. A symbol ○2 or ○4 or ... indicates that the row in being used in this way to represent 2 or 4 or ... characters, all having indicator ○.

14. Columns for classes of G

The first detachment of columns has a column for each conjugacy class of G. Against the typical character χ_n of G it gives the value $\chi_n(g)$ for g in the given class. The notation for algebraic irrationalities has been explained. Thus from the $3A$ column of our example A_5, we read the five character values

$$\left.\begin{array}{l}\chi_1(g)=1\\ \chi_2(g)=0\\ \chi_3(g)=0\\ \chi_4(g)=1\\ \chi_5(g)=-1\end{array}\right\}\text{ giving the values of } A_5 \text{ characters at an element of order 3.}$$

If G has a multiple cover $m.G$, then g will be the image of m distinct elements $g_0, g_1, \ldots, g_{m-1}$ of $m.G$. The rows of the table corresponding to characters faithful on $m.G$ give character values only at g_0, since for these characters we have $\chi_n(g_i) = \omega^i\chi_n(g_0)$, where ω is a primitive mth root of unity. (It follows that $\chi_n(g_i)=0$ unless there are m distinct conjugacy classes g_i above g.) Thus in our example

$$\text{we read}\left\{\begin{array}{l}\chi_6(g_0)=-1\\ \chi_7(g_0)=1\\ \chi_8(g_0)=0\end{array}\right\}\text{ and deduce that}\left\{\begin{array}{l}\chi_6(g_1)=1\\ \chi_7(g_1)=-1\\ \chi_8(g_1)=0\end{array}\right\},$$

for the two inverse images g_0 and g_1 of an element g of class $3A$.

15. The detachment of columns for a group $G.2$

There are simple relations between the characters of G and $G.2$.

The splitting case
The first possibility is that a character χ of G may extend to $G.2$. It then necessarily does so in two ways, giving two characters χ^0 and χ^1 of $G.2$, whose values on elements of $G.2$ outside G are negatives of each other. In this case, we put the *splitting symbol* (:) in the fusion column, the indicators (+, −, or ○) of both χ^0 and χ^1 in the indicator column, and then the values of χ^0 (only) in the columns for classes of $G.2$ outside G.

Thus the row

	ind	1A	2A	3A	5A	5B	fus	ind	2B	4A	6A
χ_4	+	4	0	1	−1	−1	:++		2	0	−1

of our example means that χ_4 splits to yield two characters χ_4^0 and χ_4^1 of $G.2 = S_5$, with the same values as χ_4 on elements of A_5, and otherwise

$$\chi_4^0(2B)=2,\quad \chi_4^0(4A)=0,\quad \chi_4^0(6A)=-1,$$
$$\chi_4^1(2B)=-2,\quad \chi_4^1(4A)=0,\quad \chi_4^1(6A)=1.$$

Both χ_4^0 and χ_4^1 have indicator +.

The fusion case
The other possibility is that two characters χ_m and χ_n of G may fuse to give a single character $\chi_{m,n}$ of $G.2$, with values

$$\chi_{m,n}(g) = \chi_m(g) + \chi_n(g), \text{ for elements of } G$$
$$\chi_{m,n}(g) = 0, \text{ for elements of } G.2 \text{ outside } G.$$

We indicate this case by drawing a *fusion join* between dots against χ_m and χ_n in the fusion column, and then continuing only the first of these two rows into the $G.2$ detachment, with the indicator of $\chi_{m,n}$ in the indicator column, and the values (all 0) of $\chi_{m,n}$ on elements of $G.2$ outside G in the remaining columns of the $G.2$ detachment. Any class of G on which χ_m and χ_n take distinct values fuses with the class on which those values are taken in the other order to give a single class of $G.2$.

Thus from our example we read

	ind	1A	2A	3A	5A	5B	fus	ind	2B	4A	6A
χ_2	+	3	−1	0	$-b5$	*		+	0	0	0
χ_3	+	3	−1	0	*	$-b5$	|				

and deduce that these two characters fuse to give a single character of S_5 with values

	ind	1A	2A	3A	5AB = 5A ∪ 5B	2B	4A	6A
$\chi_{2,3}$	+	6	−2	0	$1 = -b_5 - b_5^*$	0	0	0

The complete character table for S_5 can now be read from our table. It is

	ind	1A	2A	3A	5AB	2B	4A	6A
χ_1^0	+	1	1	1	1	1	1	1
χ_1^1	+	1	1	1	1	−1	−1	−1
$\chi_{2,3}$	+	6	−2	0	1	0	0	0
χ_4^0	+	4	0	1	−1	2	0	−1
χ_4^1	+	4	0	1	−1	−2	0	1
χ_5^0	+	5	1	−1	0	1	−1	1
χ_5^1	+	5	1	−1	0	−1	1	−1

We remark that certain other symbols can appear in the fusion and indicator columns for a group $m.G.a$. They are explained in Section 18.

16. The detachment of columns for a group $G.a$, $a \geqslant 3$

For an automorphic extension $G.a$, where $a \geqslant 3$, we choose in $G.a$ one particular coset of G that generates $G.a$, and call this *the generator coset* for $G.a$. We do not print character values for the classes of $G.a$ lying in cosets that generate $G.a$ but are not the chosen generator coset, since each such class is an algebraic conjugate of a corresponding class in the generator coset, which we call its *proxy* class. The exact rules relating classes to their proxies will be described later. Classes lying in cosets that do not generate $G.a$ are regarded as pertaining to smaller extensions $G.a'$ $(a'<a)$.

Our conventions are the natural extensions of those for groups $G.2$, but more things can happen.

The splitting case
If a character χ of G extends to $G.a$, it does so in a ways, giving a characters $\chi^0, \chi^1, \ldots, \chi^{a-1}$ of $G.a$, whose values at elements of the generator coset are related by

$$\chi^i(g) = \omega^i\chi^0(g),$$

for a primitive ath root of unity ω. Such cases are indicated by the splitting symbol (:) in the fusion column, followed by the indicators for $\chi^0, \chi^1, \ldots, \chi^{a-1}$ (in that order) in the indicator

column, and then the values of χ^0 (only) in the remaining columns of the $G.a$ detachment.

The fusion case

It can happen that a characters $\chi_m, \chi_n, \ldots, \chi_p$ of G may fuse to give a single character $\chi_{m,n,\ldots,p}$ of $G.a$, with values $\chi_m + \chi_n + \ldots + \chi_p$ on G and 0 elsewhere. In this case, the fusion column contains a line joining dots against those characters, and only the earliest of the corresponding rows is continued further into the $G.a$ detachment of columns, by the indicator of $\chi_{m,n,\ldots,p}$ and its values (all 0) on the classes of the generator coset for $G.a$.

Mixed cases

If $a = a_1 a_2$ is composite, it can happen that each of a_2 particular characters of G may extend, as in the splitting case, to give a_1 characters of $G.a_1$, and that the resulting $a_1 a_2$ characters fuse in sets of a_2 to give a_1 characters of $G.a = (G.a_1).a_2$.

In this case our fusion column would contain a_2 joined copies of the splitting symbol (:), the earliest of which would be followed by the indicators of the a_1 resulting characters and their values (all 0) on the ensuing classes. In the case $a = 6$, $a_1 = 3$, $a_2 = 2$ this would look like:

```
   (G)        (G.2)          (G.3)        (G.6)
+ ~~~~~~ | + 0  0...0  : +∞ ~~~~~~ | +∞   0...0
+ ~~~~~~ |             : +∞ ~~~~~~ |
```

where ~~~~~~ indicates a line of character entries. We can regard this *either* as one character of $G.2$ splitting to become three of $G.6$, *or* as three pairs of characters of $G.3$, with each pair fusing to give a single character of $G.6$. Note that the fusion column always refers back to G, and that : means that there has been splitting somewhere on the way from G to $G.a$, while the joins similarly indicate that there has also been fusion somewhere along the way.

In the case $a = 4$, $a_1 = a_2 = 2$, the table would look like:

```
   (G)        (G.2)      (G.4)
+ ~~~~~~ : ++ ~~~~~~ | ++ 0...0
+ ~~~~~~ : ++ ~~~~~~ |
```

and can be regarded as two pairs of characters of $G.2$, each pair fusing to give a single character of $G.4$.

17. Follower cosets

We have already remarked that for any (cyclic) automorphic extension $G.a$, the corresponding detachment of columns of the ATLAS table refers only to the classes in its chosen generator coset, since the values for the other classes of $G.a$ can be deduced from these. If we printed one such detachment for each cyclic subgroup of the automorphism group of G, we should obtain what we call the Full Atlas Table (FAT) for G.

Although it is useful as a theoretical standard, the FAT table is usually too fat. We have thinned it out by selecting only one (*leader*) extension $G.a$ from each family of extensions equivalent under $Aut(G)$. The detachment of columns corresponding to the leader extension is printed in full, and followed by only the indicator and fusion columns for the remaining (*follower*) extensions equivalent to it.

18. The portion of an ATLAS table for a bicyclic extension $m.G.a$

For a bicyclic extension $m.G.a$ in which the central cyclic subgroup m of $m.G$ remains central, our conventions can best be described as the union of those for $m.G$ and $G.a$. The reader should be able to read off the characters of the group $2.G.2$ appropriate to our example $G = A_5$:

class:		$1A_0$	$1A_1$	$2A_0$	$3A_0$	$3A_1$	$5AB_0$	$5AB_1$	$2B_0$	$4A_0$	$4A_1$	$6B_0$	$6B_1$	
order:		1	2	4	3	6	5	10	2	8	8	6	6	
χ_1^0	+	1	1	1	1	1	1	1	1	1	1	1	1	
χ_1^1	+	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	
$\chi_{2,3}$	+	6	6	−2	0	0	1	1	0	0	0	0	0	
χ_4^0	+	4	4	0	1	1	−1	−1	2	0	0	−1	−1	
χ_4^1	+	4	4	0	1	1	−1	−1	−2	0	0	1	1	
χ_5^0	+	5	5	1	−1	−1	0	0	1	−1	−1	1	1	
χ_5^1	+	5	5	1	−1	−1	0	0	−1	1	1	−1	−1	
$\chi_{6,7}$	−	4	−4	0	−2	1	−1	1	0	0	0	0	0	−
χ_8^0	○	4	−4	0	1	−1	−1	1	0	0	0	i_3	$-i_3$	−
χ_8^1	○	4	−4	0	1	−1	−1	1	0	0	0	$-i_3$	i_3	−
χ_9^0	○	6	−6	0	0	0	1	−1	0	i_2	$-i_2$	0	0	−
χ_9^1	○	6	−6	0	0	0	1	−1	0	$-i_2$	i_2	0	0	−
									4	8	8	12	12	

However, you are just as likely to want the characters of the isoclinic variant, also of structure $2.G.2$. To obtain this, multiply (or divide) all the numbers in the boxed portion by i, and use the new orders of elements given below the box, and the new indicators given to the right of it. For the definition of isoclinism, and the rules for computing these new orders and indicators, see Chapter 6, Section 7.

A character of $m.G$ for which we printed the indicator symbol $\circ N$ is the chosen proxy from a set of N algebraically conjugate characters, as we described in Section 8. It is possible that each of these extends to give a characters of $m.G.a$ (see 'the splitting case' in Section 15 or Section 16).

In such a case, we put the symbol $\circ\circ\ldots\circ N$ (with a symbols $\circ$) in the indicator column for $m.G.a$. In general a string of k indicator symbols +, −, ○ followed by a number N indicates that the printed character yields k characters of the given indicators upon multiplication by linear characters of the appropriate group, and that each of these is proxy for a family of N algebraically conjugate characters which have distinct values when restricted to the central cyclic subgroup m of $m.G$.

When the automorphism of the extending cyclic group a acts non-trivially on m, new symbols can also appear in the fusion column for $m.G.a$. The symbols

$$\Big|_{*k}$$

appearing in this column against characters χ_p and χ_q from a given cohort C mean that χ_p fuses with the character in the algebraically conjugate cohort C^{*k} whose proxy is χ_q. The symbol

$$*k$$

in the fusion column against character χ_p means that χ_p fuses with the character in C^{*k} whose proxy is χ_p itself. The algebraic conjugation symbol $*k$ in this may be replaced by any of $**k$, $**$, or $*$.

19. The precise rules for proxy classes and characters

We have already remarked that we print values for classes in only one of the cosets of G generating a given upward cyclic extension $G.a$, and for characters in only one of the cohorts that realize a given cyclic quotient of the multiplier. For most purposes it is unnecessary to select any particular correspondence between the retained objects and the omitted ones, which are merely their algebraic conjugates. However, a particular correspondence is used to give a precise meaning to our power maps and fusion markers.

Suppose we want to find the classes of a coset C^k for which a given class in the printed coset C serves as proxy, where $C^n = G$. If g is any element of the given class, these classes are those of certain powers $g^{k'}$ of g, where it is only necessary to specify the numbers k' modulo the order N of the element g. In the vast majority of cases in this ATLAS, n divides 12, and we specify k' by the congruences:

$k' \equiv k$ modulo n,
$k' \equiv \pm 1$ modulo powers of 2 and 3 dividing N,
$k' \equiv +1$ modulo any divisor of n prime to 2 and 3.

The only other case that actually appears in this ATLAS is for $n=5$. In this case, we specify k' by

$k' \equiv k$ modulo 5,
k' is of order dividing 4 modulo powers of 5 dividing N,
$k' \equiv +1$ modulo any divisor of n prime to 5.

These conventions are the simplest ones that ensure that the algebraic conjugation $*k'$ has the correct effect on nth roots of unity, while fixing 'irrelevant' irrationalities. Any rule that extends this to all numbers n must be to some extent arbitrary.

The rule for proxy characters is precisely similar. The characters in the cohort C^{*k} for which a given character χ from the cohort C serves as proxy are the algebraically conjugate characters $\chi^{*k'}$, where k' is determined just as above from n, the order of the cyclic quotient of the multiplier realized faithfully by χ, and any number N for which the values of χ lie in the field of Nth roots of unity.

20. Abbreviated character tables

There are some groups whose character tables seemed to deserve compression before we felt able to justify their inclusion. In these cases, indicated by the phase 'Abbreviated character table', we have printed just one representative from each algebraically conjugate family of classes or characters.

An entry of form N/D in the centralizer order row indicates that the corresponding class is the only printed member of an algebraically conjugate family of D classes, each of centralizer order N. Such a family behaves in many ways like a single conjugacy class with centralizer order $\frac{N}{D}$. The entry in the class name row consists of the order n of the elements concerned, followed by the tag letters of the first and last classes in the family. The remaining entries in the appropriate column refer only to the first class in the family.

So in our table for $L_3(8)$ (page 74), the column beginning

```
   @
3528/6
   A
   A
 7AF
```

refers to class $7A$, and tells us that this class is the only printed representative of the 6 classes $7A$, $7B$, $7C$, $7D$, $7E$, $7F$, all having centralizer order 3528.

A row beginning $+M$, $\circ M$, or $-M$ indicates that the printed character is the only printed representative of an algebraically conjugate family of M characters, all having the appropriate indicator + or $\circ$ or $-$. Thus the last row of the table for $L_3(8)$ begins +3, indicating that the printed character is one of an algebraically conjugate family of 3 characters, all having indicator +.

The complete table, except possibly for a small amount of the power map information, can be recovered by applying all possible algebraic conjugations to the printed classes and characters.

8

Concluding remarks

We have given a great deal of thought to the arrangement and printing of character tables, and here pass on our observations to potential table compilers who wish to adopt our conventions.

1. Format

Tables with columns of randomly varying widths have a 'ragged' appearance which is very wearying to the reader. In our tables the only permitted widths are 4, 6, 8, . . . spaces. We have taken great pains, particularly with our irrationality conventions, to ensure that most columns will fit into the standard width of 4 spaces, so that large tracts of table have a regular layout. The even-width requirement entails that a wider column is *seen* to be wider—columns whose widths are nearly but not quite equal produce a slightly disorienting effect.

The rows are usually at double line spacing, but have been compressed to one-and-a-half line spacing for a few large tables. The interline spaces dramatically increase legibility, and produce a pleasingly square appearance.

2. Ordering of classes and characters

Many table compilers arrange the conjugacy classes in an order which to some extent reflects the power maps. Thus a class whose elements have prime order p may be followed by classes containing elements of orders $2p$, $3p$, etc., having powers in the original class. The merits of such arrangements are usually more evident to the compiler than the user, who is seldom properly informed about the principles (if any) of the arrangement, and so cannot use the implied power map information. It is better to choose a simpler arrangement, and explicitly indicate the power maps.

In this ATLAS, the conjugacy classes within a given coset are arranged

firstly, by increasing succession of n, the order of their elements;

secondly, for elements with the same n, by decreasing succession of N, their centralizer order;

thirdly, for elements with the same n and N, by increasing succession of d, the degree of the algebraic number field generated by their character values (so that rational elements come first); and

fourthly, for elements with the same n, N, and d, in a manner which seems best compatible with the p' parts, so that, other things being equal, we prefer to arrange elements of order 10 in the same succession as the elements of order 5 that are their odd parts.

Again, the characters within a given cohort are arranged

firstly, in increasing succession of n, their character degree;

secondly, for characters with the same n, in increasing succession of d, the degree of the number field generated by their values;

thirdly, for characters with the same n and d, so that characters that fuse in some automorphic extension group are close together; and

fourthly, so that the resulting blocks of algebraic irrationalities have a readily visible structure.

Of course *any* system of this kind has the merit that if two investigators use the same system for the same group, their character tables will be almost identical. [We do *not* think it would be sensible to make the system so precise that it would only permit one arrangement per group.] Our system tends to put the 'important' elements first, and makes it likely that the conjugacy classes in a subgroup will be in roughly the same order as those in the containing group.

Similar remarks hold for characters. The ordering by degrees is useful in discussions of the possible decompositions of some compound character, and often makes it easier to identify the restriction of a character to a subgroup.

3. Some other 'virtuous' properties

When tabulating characters of a group $M.G$ (or $M.G.A$) with a large multiplier, we have always arranged that for any given g, the displayed pre-images of g in all the cyclic extensions $m.G$ are all images of a 'universal' pre-image of g in $M.G$. 'Dually', we have arranged *whenever possible* that the prolongations of a given character to all the cyclic extensions $G.a$ are restrictions of a 'universal' character defined on $G.A$.

There are still some choices that are more virtuous than others. For example in a group of structure $2^2.G.3$, in which the three involutions of the normal fourgroup are permuted by the automorphism group, only one of the four pre-images of a given element can be invariant under an outer automorphism; this should be the printed 'universal' pre-image. 'Dually', in a group of structure $G.2^2.3$ in which the outer automorphism group is A_4, a character of G which extends to $G.2^2$ does so in four ways, only one of which can be invariant under the additional automorphisms; this character should be the printed one. In groups with complicated multiplier and automorphism group such desiderata can conflict, so we do not regard them as obligatory (and the reader cannot rely on them). But a table which satisfies them is more useful than one that does not, since there is a higher probability that the element the reader wants is exactly that which is tabulated.

4. The reliability of ATLAS information

> With regard to errors in general, whether falling under the denomination of mental, typographical, or accidental, we are conscious of being able to point to a greater number than any critic whatever. Men who are acquainted with the innumerable difficulties attending the execution of a work of such an extensive nature will make proper allowances. To these we appeal, and shall rest satisfied with the judgement they pronounce.
>
> *Preface to the first edition of the Encyclopaedia Brittanica, 1771*

Our fairly wide experience has taught us that published character tables are as likely as not to contain errors (which are usually minor). We have been able to correct errors in our own tables up to a very late date before publication, but the rate of their discovery suggests that the above quotation will become apposite a few months after the ATLAS appears in print.

We first briefly comment on the information *not* contained in the character tables. Experts will know that one of the most common errors in group-theoretical papers is for the structure of

a group to be slightly mis-stated. We hope and believe that our own error rate in this respect is rather smaller than the average, but since the ATLAS makes very many more assertions about group structures than any other work, we are certain that they will not all be exactly correct. We can only counsel the reader who feels he may have discovered an error to check the consistency of the statement he doubts with similar statements about subgroups or with analogous cases.

Although we intended at first to subject all our tables to a large battery of computer tests, we must confess that in the event the only such test that has been consistently applied to every table is the orthogonality check. (There are many other mechanical tests that could be easily applied to ordinary tables, but for which the complexity of applying them to our compound tables has defeated us.) We are confident that the overall error rate in our character tables is very low indeed, but feel that the reader should be aware of the possibility that there are occasional errors, and of their most likely form. To put the problem in perspective, we emphasize again the very large amount of information that this ATLAS contains. If, for example, we were to misprint one in every thousand digits, then the MONSTER character table alone would contain about 100 errors. Our actual error rate is a few orders of magnitude smaller!

If you suspect an error, perhaps the first thing to do is to check that you have not misunderstood the ATLAS conventions, which are sometimes subtler than you may think. Then for:

centralizer orders: check that the given number is the sum of the squares of the moduli of the entries in its column;

power maps and p′ parts: check the mutual consistency of these, for instance using $(x^2)^3=(x^3)^2$, and check the congruence $\chi(g)\equiv\chi(g')$ modulo p, whenever g' is the p' part of g, and χ takes rational values on both. (For irrational values, the modulus is $1-z_p$.)

lifting orders: check consistency with power maps—also the above congruences;

indicators: the indicator should be 0 just if some character value is non-real; also, check the formulae

$$\mathrm{ind}(\chi)=\frac{1}{|G|}\sum_g \chi(g^2) \quad \text{and} \quad R(g)=\frac{1}{|G|}\sum_\chi \mathrm{ind}(\chi)\,.\,\chi(g)$$

where $R(g)$ is the number of square roots of g;

fusion: check that $G\,.\,a$ has the same number of classes as characters, also check compatibility with indicators.

We have discussed these first since our experience suggests that it is this 'peripheral' information that is most likely to be at risk. The character values themselves are much safer.† Any error that remains in them probably concerns an irrational entry, or is an incompatibility between the tables for G and $G\,.\,a$.

An extremely valuable test which we recommend when the above simple rules of thumb have failed is to compute the skew squares

$$\chi^{2-}(g)=\frac{(\chi(g))^2-\chi(g^2)}{2}$$

of one or more characters χ, and check their inner products with selected irreducibles. One should also check the consistency of restriction maps from groups to subgroups whose tables are also known.

Of course it can be a good idea to check a doubtful point against another published table, which might well be the one from which ours was originally derived. A disagreement *probably* means that we believed we had found and corrected an error in the source table, but of course we could be wrong, or could have inadvertently introduced some error ourselves.

If you do discover and confirm an error, please write to us about it!

† Any complacency we might have had in this regard was rudely shattered when the pre-publication version of the table for the outer automorphism group of the Held group was found to contain an error affecting 22 entries (but obeying the orthogonality conditions)! A similar, but much smaller, error was discovered in the table for the outer automorphism group of J_3.

5. Acknowledgements

This is perhaps the place for a comment about references and the bibliography. Our entries for the individual groups contain no explicit references to sources. This is because the few lines we allot to any given construction often contain material culled from many places (sometimes barely remembered) which might have been transformed beyond recognition by our own investigations or transliterations. We intend no disrespect, and beg forgiveness for any offence our policy has caused. We gratefully acknowledge here that we have used information from many published and unpublished sources that are not explicitly referenced in this volume.

The bibliography on pp. 243–51 is restricted to the 26 sporadic groups, and to a number of books and papers which give information about most or all of the infinite families of simple groups. The works by Gorenstein and Davis in the latter portion are very much more extensive bibliographies.

It is a pleasure to record our indebtedness to many colleagues throughout the world who have freely sent us information. We mention particularly our major sources of character tables:

M. Hall, Jr	[the Tits group, $6A_6$, $6A_7$, . . .]
Zia-ud-Din	[S_{12}, S_{13}]
A. O. Morris	[$2S_n$]
J. A. Todd	[the Mathieu groups, $U_4(3)$, . . .]
D. Enomoto	[$G_2(q)$]
H. N. Ward	[${}^2G_2(q)$]
B. Srinivasan	[$S_4(q)$]
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D. Gabrysch	[$F_4(2)$, ${}^2E_6(2)$, . . .]
D. C. Hunt	[many, including the Baby Monster]
J. S. Frame, A. Rudvalis, and collaborators	[many, including $O_n(2)$]
B. Fischer, D. Livingstone, M. Thorne	[the Monster],

and finally the Aachen 'CAS' team led by J. Neubüser and H. Pahlings both for many original tables and for improvements, extensions, and corrections to many others.

Among our group-theoretical colleagues at Cambridge who have used the ATLAS and contributed tables, corrections, improvements, or criticism are David Benson, Patrick Brooke, Mike Guy, David Jackson, Gordon James, Peter Kleidman, Martin Liebeck, Nick Patterson, Larissa Queen, Alex Ryba, Jan Saxl, Peter Smith, and finally John Thompson, who has acted as our friend and mentor throughout.

We owe a particular debt to Jonathan Thackray, who designed and wrote the computer system in which our compound character tables are held. Many errors were discovered and eliminated as a result of his work. We thank Leonard Soicher for his valuable work which produced the Coxeter-type presentations displayed for many groups in this ATLAS.

The authors' names appear on the cover of this ATLAS in what is both the alphabetical order and the chronological order of their involvement with the ATLAS project. Here are some comments about their individual roles:

Robert Curtis and I started the work by designing the ATLAS format and converting many pre-existing tables into it. Curtis also made many investigations into the subgroup structure and primitive permutation representations of various groups, not all of which work has been used in this volume.

Simon Norton constructed the tables for a large number of extensions, including some particularly complicated ones. He has throughout acted as 'troubleshooter'—any difficult problem was automatically referred to him in the confident expectation that it would speedily be solved.

Richard Parker was responsible for the initial 'mechanization' of the ATLAS project, and also did a great part of the more tedious job of entering pre-existing tables into the computer. He has also computed a large number of modular character tables, intended for a later ATLAS publication.

Robert Wilson has investigated, and often completely enumerated, the maximal subgroups of a large number of ATLAS groups. He has greatly increased the usefulness of this ATLAS by adding this and other information, and over the last few years has cheerfully shouldered the enormous task of gathering and transforming our untidy heaps of material into a form fit for publication.

My own function was to initiate and control the entire project, to collaborate with each of the above, and (eventually) to write this *Introduction.* *John Conway*

November 1984
Department of Pure Mathematics
and Mathematical Statistics,
University of Cambridge

THE GROUPS

$A_5 \cong L_2(4) \cong L_2(5)$

Alternating group A_5; Linear group $L_2(4) \cong A_1(4) \cong U_2(4) \cong S_2(4) \cong O_3(4) \cong O_4^-(2)$;

Linear group $L_2(5) \cong A_1(5) \cong U_2(5) \cong S_2(5) \cong O_3(5)$

Order = $60 = 2^2.3.5$ Mult = 2 Out = 2

Constructions

Alternating $S_5 \cong G.2$: all permutations of 5 letters;

$A_5 \cong G$: all even permutations; 2.G and 2.G.2 : the Schur double covers

Linear (4) $GL_2(4) \cong 3 \times G$: all non-singular 2 x 2 matrices over $\mathbf{F}_4$;

$SL_2(4) \cong PGL_2(4) \cong PSL_2(4) \cong G$; $\Gamma L_2(4) \cong (3 \times G).2$; $P\Gamma L_2(4) \cong \Sigma L_2(4) \cong P\Sigma L_2(4) \cong G.2$

Unitary (4) $GU_2(4) \cong 5 \times G$: all 2 x 2 matrices over $\mathbf{F}_{16}$ preserving a non-singular Hermitian form;

$PGU_2(4) \cong SU_2(4) \cong PSU_2(4) \cong G$

Orthogonal (4) $GO_3(4) \cong PGO_3(4) \cong SO_3(4) \cong PSO_3(4) \cong O_3(4) \cong G$: all 2 x 2 matrices over $\mathbf{F}_4$ preserving a non-singular quadratic form; $\Gamma O_3(4) \cong P\Gamma O_3(4) \cong \Sigma O_3(4) \cong P\Sigma O_3(4) \cong G.2$

Orthogonal (2) $GO_4^-(2) \cong PGO_4^-(2) \cong SO_4^-(2) \cong PSO_4^-(2) \cong G.2$: all 4 x 4 matrices over $\mathbf{F}_2$ preserving a quadratic form of Witt defect 1, for example $x_1^2 + x_1x_2 + x_2^2 + x_3x_4$; $O_4^-(2) \cong G$

Linear (5) $GL_2(5) \cong 2.(G \times 2).2$: all non-singular 2 x 2 matrices over $\mathbf{F}_5$;

$PGL_2(5) \cong G.2$; $SL_2(5) \cong 2.G$; $PSL_2(5) \cong G$

Unitary (5) $GU_2(5) \cong 3 \times 2.G.2$: all 2 x 2 matrices over $\mathbf{F}_{25}$ preserving a non-singular Hermitian form;

$PGU_2(5) \cong G.2$; $SU_2(5) \cong 2.G$; $PSU_2(5) \cong G$

Orthogonal (5) $GO_3(5) \cong 2 \times G.2$: the 3 x 3 matrices over $\mathbf{F}_5$ preserving a non-singular quadratic form;

$PGO_3(5) \cong SO_3(5) \cong PSO_3(5) \cong G.2$; $O_3(5) \cong G$

Quaternionic 2.G is the group of those quaternions q for which the coordinates of 2q are :

$(\pm 2,0,0,0)^A$, $(\pm 1,\pm 1,\pm 1,\pm 1)$, $(0,\pm 1,\pm b5,\pm b5^*)^A$; these generate the icosian ring

Icosahedral G x 2 : symmetries of the vectors $(0,\pm 1,\pm b5^*)^C$ (vertices of icosahedron), or of $(\pm 1,\pm 1,\pm 1)$, $(0,\pm b5^*,\pm b5)^C$ (dodecahedron), or of $(\pm 2,0,0)^C$, $(\pm 1,\pm b5,\pm b5^*)^C$ (icosidodecahedron: the reflections in these generate G x 2). These vectors are obtained by deleting the real parts of the above quaternions. G is the group of symmetries of determinant 1.

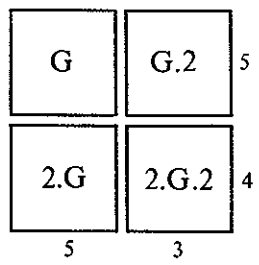
Presentations $2.G \cong \langle 2,3,5\rangle$; $G \cong (2,3,5) \cong G^{3,5,5} \cong \langle x_1,x_2,x_3 \mid x_i^3=(x_ix_j)^2=1\rangle$;

$G.2 \cong (2,4,5;3) \cong (2,5,6;2)$; $G \times 2 \cong$ ——5—— $\cong G^{3,5,10}$; $G.2 \times 2 \cong G^{4,5,6}$

Maximal subgroups					Specifications			
Order	Index	Structure	G.2	Character	Abstract	Alternating	Linear (4)	Orthogonal (4)
12	5	A_4	: S_4	1a+4a	$N(2A^2)$	point	point	isotropic point
10	6	D_{10}	: 5:4	1a+5a	N(5AB)		$O_2^-(4)$, $L_1(16)$	minus line
6	10	S_3	: 2 x S_3	1a+4a+5a	N(3A)	duad	$O_2^+(4)$, base	plus line

Orthogonal (2)	Linear (5)	Orthogonal (5)	Icosahedral
isotropic point		base	base
$O_2^-(4)$	point	isotropic point	pentad axis
non-isotropic point	$O_2^-(5)$, $L_1(25)$	minus point	triad axis

	;	@	@	@	@	@	;	;	@	@	@
		60	4	3	5	5			6	2	3
	p power		A	A	A	A			A	A	AB
	p' part		A	A	A	A			A	A	AB
	ind	1A	2A	3A	5A	B*	fus	ind	2B	4A	6A
χ_1	+	1	1	1	1	1	:	++	1	1	1
χ_2	+	3	-1	0	-b5	*		+	0	0	0
χ_3	+	3	-1	0	*	-b5					
χ_4	+	4	0	1	-1	-1	:	++	2	0	-1
χ_5	+	5	1	-1	0	0	:	++	1	-1	1
	ind	1	4	3	5	5	fus	ind	2	8	6
		2		6	10	10				8	6
χ_6	-	2	0	-1	b5	*		-	0	0	0
χ_7	-	2	0	-1	*	b5					
χ_8	-	4	0	1	-1	-1	:	oo	0	0	i3
χ_9	-	6	0	0	1	1	:	oo	0	i2	0



$L_3(2) \cong L_2(7)$

Linear group $L_3(2) \cong A_2(2)$; Linear group $L_2(7) \cong A_1(7) \cong U_2(7) \cong S_2(7) \cong O_3(7)$

Order = 168 = $2^3.3.7$ Mult = 2 Out = 2

Constructions

Linear (2) $GL_3(2) \cong PGL_3(2) \cong SL_3(2) \cong PSL_3(2) \cong G$: all non-singular 3 x 3 matrices over $\mathbf{F}_2$;

the points and lines of the corresponding projective plane can be numbered (mod 7) so that $L_i = \{P_{i+1}, P_{i+2}, P_{i+4}\}$;

G.2 is obtained by adjoining the duality (graph) automorphism

Linear (7) $GL_2(7) \cong 3 \times 2.G.2$: all non-singular 2 x 2 matrices over $\mathbf{F}_7$;

$PGL_2(7) \cong G.2$; $SL_2(7) \cong 2.G$; $PSL_2(7) \cong G$

Unitary $GU_2(7) \cong 2.(G \times 4).2$: all 2 x 2 matrices over $\mathbf{F}_{49}$ preserving a non-singular Hermitian form;

$PGU_2(7) \cong G.2$; $SU_2(7) \cong 2.G$; $PSU_2(7) \cong G$

Orthogonal $GO_3(7) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbf{F}_7$ preserving a non-singular quadratic form;

$PGO_3(7) \cong SO_3(7) \cong PSO_3(7) \cong G.2$; $O_3(7) \cong G$

Reflection $G \times 2$: symmetries of the lattice Λ_3^{b7} whose minimal (root) vectors are : $(\pm2,0,0)^S$, $(0,\pm b7,\pm b7)^S$, $(\pm1,\pm1,\pm b7^{**})^S$;

the group is generated by reflections in these vectors;

G : the symmetries of determinant 1

Vectors 2G : the symmetries of the set of 8 (±)vectors (where b=b7, p=i7) :

$v_\infty=(p;0,0,0)$ $v_0=(-1;b,b,b)$ $v_1=(-1;b,0,b^2)$ $v_2=(-1;0,b^2,b)$ $v_3=(-1;2,b,0)$ $v_4=(-1;b^2,b,0)$ $v_5=(-1;0,2,b)$ $v_6=(-1;b,0,2)$;

these generate the laminated lattice $\Lambda_{4,b7}$, whose full automorphism group is $2A_7$

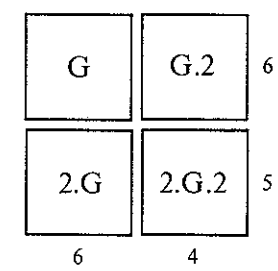
Presentations $G \cong (3,3|4,4) \cong (2,3,7;4) \cong \langle S,T | S^7=(S^4T)^4=1,(ST)^3=T^2[=1]\rangle$;

$G.2 \cong G^{3,7,8} \cong (2,3,8;4)$; $2 \times G.2 \cong G^{3,8,8}$

Maximal subgroups / Specifications

Order	Index	Structure	G.2	Character	Abstract	Linear (2)	Linear (7)	Orthogonal (7)	Reflection
24	7	S_4	D_{12}, D_{16}	1a+6a	$N(2A^2)$	point		base	b7-congruence base
24	7	S_4		1a+6a	$N(2A^2)$	line		base	b7**-congruence base
21	8	7:3	: 7:6	1a+7a	N(7AB)	$L_1(8)$	point	isotropic point	

	;	@	@	@	@	@	@	;	;	@	@	@	@
		168	8	3	4	7	7			6	3	4	4
		p power	A	A	A	A	A			A	AB	A	A
		p' part	A	A	A	A	A			A	AB	A	A
		ind	1A	2A	3A	4A	7A	B**	fus ind	2B	6A	8A	B*
χ_1	+	1	1	1	1	1	1	:	++	1	1	1	1
χ_2	o	3	-1	0	1	b7	**		+	0	0	0	0
χ_3	o	3	-1	0	1	**	b7						
χ_4	+	6	2	0	0	-1	-1	:	++	0	0	r2	-r2
χ_5	+	7	-1	1	-1	0	0	:	++	1	1	-1	-1
χ_6	+	8	0	-1	0	1	1	:	++	2	-1	0	0
	ind	1	4	3	8	7	7	fus ind		4	12	16	16
		2		6	8	14	14				12	16	16
χ_7	o	4	0	1	0	-b7	**		-	0	0	0	0
χ_8	o	4	0	1	0	**	-b7						
χ_9	-	6	0	0	r2	-1	-1	:	--	0	0	y16	*3
χ_{10}	-	6	0	0	-r2	-1	-1	:	--	0	0	*5	y16
χ_{11}	-	8	0	-1	0	1	1	:	--	0	r3	0	0



$A_6 \cong L_2(9) \cong S_4(2)'$

Alternating group A_6; Linear group $L_2(9) \cong A_1(9) \cong U_2(9) \cong S_2(9) \cong O_3(9) \cong O_4^-(3)$;

Derived symplectic group $S_4(2)' \cong C_2(2)' \cong O_5(2)'$

Order = 360 = $2^3.3^2.5$ Mult = 6 Out = 2^2

Constructions

Alternating $S_6 \cong G.2_1$: all permutations on 6 letters;

$A_6 \cong G$: the even permutations; 2.G and $2.G.2_1$: the Schur double covers

The outer automorphism of S_6 interchanges the 15 duads ab (pairs) with the 15 synthemes ab.cd.ef (trisections), so interchanges the 6 points a (which could be specified by 5 duads ab, ac, ad, ae, af) with the 6 totals (sets of 5 synthemes containing all duads)

Linear $GL_2(9) \cong 2.(G \times 4).2_2$: all 2 x 2 non-singular matrices over $\mathbf{F}_9$;

$PGL_2(9) \cong G.2$; $SL_2(9) \cong 2.G$; $PSL_2(9) \cong G$;

$\Gamma L_2(9) \cong 2.(G \times 4).2^2$; $P\Gamma L_2(9) \cong G.2^2$; $\Sigma L_2(9) \cong 2.G.2_1$; $P\Sigma L_2(9) \cong G.2_1$

Unitary $GU_2(9) \cong 5 \times 2.G.2_2$: all matrices over $\mathbf{F}_{81}$ preserving a non-singular Hermitian form;

$PGU_2(9) \cong G.2_2$; $SU_2(9) \cong 2.G$; $PSU_2(9) \cong G$

Orthogonal (9) $GO_3(9) \cong 2 \times G.2_2$: all 3 x 3 matrices over $\mathbf{F}_9$ preserving a non-singular quadratic form;

$PGO_3(9) \cong SO_3(9) \cong PSO_3(9) \cong G.2_2$; $O_3(9) \cong G$; $\Gamma O_3(9) \cong 2 \times G.2^2$; $P\Gamma O_3(9) \cong \Sigma O_3(9) \cong P\Sigma O_3(9) \cong G.2^2$

Orthogonal (3) $GO_4^-(3) \cong 2 \times G.2_1$: all 4 x 4 matrices over $\mathbf{F}_3$ preserving a quadratic form of Witt defect 1, for example $x_1^2 + x_2^2 + x_3^2 - x_4^2$; $PGO_4^-(3) \cong G.2_1$; $SO_4^-(3) \cong 2 \times G$; $PSO_4^-(3) \cong O_4^-(3) \cong G$;

by taking the 4-space of the vectors (x_i) in $\mathbf{F}_3^6$ with $\Sigma x_i=0$, modulo (111111), we see the isomorphism with $2 \times S_6$

Orthogonal (2) $GO_5(2) \cong PGO_5(2) \cong SO_5(2) \cong PSO_5(2) \cong G.2_1$: all 5 x 5 matrices over $\mathbf{F}_2$ preserving a non-singular quadratic form; $O_5(2) \cong G.2_1$

Symplectic $S_4(2) \cong G.2_1$: all 4 x 4 matrices over $\mathbf{F}_2$ preserving a non-singular symplectic form;

Mathieu $M_{10} \cong G.2_3$: the stabilizer of a point in M_{11}, or two points in M_{12};

in M_{24} the subgroups of $M_{10}.2 \cong G.2^2$ have orbits as follows:

G (1,1,10;6,6), $G.2_1$ (2,10;6,6), $G.2_2$ (2,10;12), $G.2_3 \cong M_{10}$ (1,1,10;12)

Hexacode 3.G : the monomial symmetries of the code over $\mathbf{F}_4 = \{0,1,w,\bar{w}\}$ whose 64 = 36+12+9+6+1 words are obtained from (0a 0a bc) (36), (bc bc bc) (12), (00 aa aa) (9), (aa bb cc) (6), (00 00 00) (1) (in which a,b,c is any cyclic permutation of $1,w,\bar{w}$) by bodily permuting the 3 couples, and/or reversing any 2 of them;

the words of weight 6 (total words) are permuted (modulo scalar factors) like the 6 totals of 5 disjoint synthemes

Reflection 2 x 3.G : the symmetries of the $(\pm w)$vectors $(\pm 2,0,0)^S$, $(\pm 1,\pm 1,\pm \bar{h})^S$, $(0,\pm h,\pm h)^S$, $(\pm 1,\pm wc,\pm \bar{w}b)^S$ (where w = z3, b = b5, c = b5*, h = z3 - b5); the group is generated by the 45 reflections in these vectors

Quaternionic 2.G : the symmetries of the (i,w)vectors $(\theta,0)$, $((i+1)w^a,w^b)$, $(w^a,(i-1)w^b)$ where θ and i are quaternions with $\theta^2 = -3$, $i^2 = -1$, $\theta i = -i\theta$, and $2w = -1+\theta$; the group is generated by the 20 quaternionic reflections in these vectors - these reflections have order 3

Presentations $G \cong (3,4,5;2) \cong (5,5|2,4) \cong \langle x_1,x_2,x_3,x_4 | x_i^3=(x_ix_j)^2=1\rangle$;

$G.2_1 \cong$ ——•——•——•—— ; $2 \times G.2_2 \cong G^{4,5,8}$

Maximal subgroups

Order	Index	Structure	$G.2_1$	$G.2_2$	$G.2_3$	$G.2^2$	Character	Specifications Abstract	Alternating
60	6	A_5	: S_5	D_{20}	5:4	10:4	1a+5a	N(2A,3A,5A)	point
60	6	A_5	: S_5				1a+5b	N(2A,3B,5A)	total
36	10	3^2:4	: 3^2:D_8	: 3^2:8	: 3^2:Q_8	: 3^2:[2^4]	1a+9a	N(3^2) = N($3A_2B_2$)	bisection
24	15	S_4	: S_4 x 2	D_{16}	8:2	[2^5]	1a+5a+9a	N($2A^2$), C(2B)	duad
24	15	S_4	: S_4 x 2				1a+5b+9a	N($2A^2$), C(2C)	syntheme

Linear (9)	Orthogonal (9)	Orthogonal (3)	Orthogonal (2)	Symplectic	Mathieu	Hexacode
	icosahedral		minus plane	$O_4^-(2)$, $S_2(4)$	total	coordinate
	icosahedral		minus plane	$O_4^-(2)$, $S_2(4)$	total	weight 6 word
point	isotropic point	isotropic point	plus plane	$O_4^+(2)$	point	
$L_2(3)$	$O_3(3)$, base	non-isotropic point	isotropic point	point	tetrad	weight 4 word
$L_2(3)$	$O_3(3)$, base	non-isotropic point	isotropic line	isotropic line	tetrad	2 weight 6 words

$A_6 \cong L_2(9) \cong S_4(2)'$

	;	@	@	@	@	@	@	@	;	;	@	@	@	@	@	;	;	@	@	@	@	@	;	;	@	@	@
		360	8	9	9	4	5	5			24	24	4	3	3			10	4	4	5	5			2	4	4
	p power		A	A	A	A	A	A			A	A	A	AB	BC			A	A	A	BD	AD			A	A	A
	p' part		A	A	A	A	A	A			A	A	A	AB	BC			A	A	A	AD	BD			A	A	A
	ind	1A	2A	3A	3B	4A	5A	B*	fus	ind	2B	2C	4B	6A	6B	fus	ind	2D	8A	B*	10A	B*	fus	ind	4C	8C	D**
χ_1	+	1	1	1	1	1	1	1	:	++	1	1	1	1	1	:	++	1	1	1	1	1	:	++	1	1	1
χ_2	+	5	1	2	-1	-1	0	0	:	++	3	-1	1	0	-1		+	0	0	0	0	0		+	0	0	0
χ_3	+	5	1	-1	2	-1	0	0	:	++	-1	3	1	-1	0												
χ_4	+	8	0	-1	-1	0	-b5	*		+	0	0	0	0	0	:	++	2	0	0	b5	*		+	0	0	0
χ_5	+	8	0	-1	-1	0	*	-b5								:	++	2	0	0	*	b5					
χ_6	+	9	1	0	0	1	-1	-1	:	++	3	3	-1	0	0	:	++	-1	1	1	-1	-1	:	++	1	-1	-1
χ_7	+	10	-2	1	1	0	0	0	:	++	2	-2	0	-1	1	:	++	0	r2	-r2	0	0	:	oo	0	i2	-i2
	ind	1	4	3	3	8	5	5	fus	ind	2	4	8	6	12	fus	ind	4	16	16	20	20	fus	ind	4	16	16
		2		6	6	8	10	10						6	12				16	16	20	20					
χ_8	-	4	0	-2	1	0	-1	-1	:	--	0	0	0	0	r3		-	0	0	0	0	0		o2			
χ_9	-	4	0	1	-2	0	-1	-1	:	oo	0	0	0	i3	0								*				
χ_{10}	-	8	0	-1	-1	0	-b5	*		-	0	0	0	0	0	:	--	0	0	0	y20	*3		o2			
χ_{11}	-	8	0	-1	-1	0	*	-b5								:	--	0	0	0	*7	y20	*				
χ_{12}	-	10	0	1	1	r2	0	0		-	0	0	0	0	0	:	--	0	y16	*5	0	0		o2			
χ_{13}	-	10	0	1	1	-r2	0	0								:	--	0	*13	y16	0	0	*				
	ind	1	2	3	3	4	5	5	fus	ind	2	2	4	6	6	fus	ind	2	8	8	10	10	fus	ind	4	8	8
		3	6			12	15	15																	12	24	24
		3	6			12	15	15																	12	24	24
χ_{14}	o2	3	-1	0	0	1	-b5	*		o2						*	+							o2	0	0	0
χ_{15}	o2	3	-1	0	0	1	*	-b5	*							*	+										
χ_{16}	o2	6	2	0	0	0	1	1	*	+						*	+						:	oo2	0	i2	-i2
χ_{17}	o2	9	1	0	0	1	-1	-1	*	+						*	+						:	oo2	1	-1	-1
χ_{18}	o2	15	-1	0	0	-1	0	0	*	+						*	+						:	oo2	1	1	1
	ind	1	4	3	3	8	5	5	fus	ind	2	4	8	6	12	fus	ind	4	16	16	20	20	fus	ind	4	16	16
		6	12	6	6	24	30	30						6	12				16	16	20	20			12	48	48
		3	12			24	15	15																	12	48	48
		2				8	10	10																			
		3				24	15	15																			
		6				24	30	30																			
χ_{19}	o2	6	0	0	0	r2	1	1		o2						*	-							o4			
χ_{20}	o2	6	0	0	0	-r2	1	1	*							*	-						*7				
χ_{21}	o2	12	0	0	0	0	b5	*		o2						*	-							o4			
χ_{22}	o2	12	0	0	0	0	*	b5	*							*	-						*7				

G	$G.2_1$	$G.2_2$	$G.2_3$	7
2.G	$2.G.2_1$	$2.G.2_2$	$4.G.2_3$ (*)	6
3.G	$3.G.2_1$	$3.G.2_2$	$3.G.2_3$	5
6.G	$6.G.2_1$	$6.G.2_2$	$12.G.2_3$ (*)	4
7	5	5	3	

(*) There is no group $2.G.2_3$ or $6.G.2_3$.
See the Introduction: 'Isoclinism'

$L_2(8) \cong R(3)'$

Linear group $L_2(8) \cong A_1(8) \cong U_2(8) \cong S_2(8) \cong O_3(8)$; Derived Ree group $R(3)' \cong {}^2G_2(3)'$

Order = $504 = 2^3.3^2.7$ Mult = 1 Out = 3

Constructions

Linear $GL_2(8) \cong 7 \times G$: all non-singular 2×2 matrices over $\mathbb{F}_8$;

$PGL_2(8) \cong SL_2(8) \cong PSL_2(8) \cong G$; $\Gamma L_2(8) \cong (7 \times G).3$; $P\Gamma L_2(8) \cong \Sigma L_2(8) \cong P\Sigma L_2(8) \cong G.3$

Unitary $GU_2(8) \cong 9 \times G$: all 2×2 matrices over $\mathbb{F}_{64}$ preserving a non-singular Hermitian form;

$PGU_2(8) \cong SU_2(8) \cong PSU_2(8) \cong G$

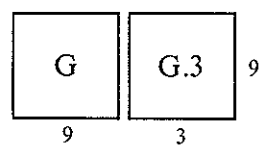
Orthogonal $GO_3(8) \cong PGO_3(8) \cong SO_3(8) \cong PSO_3(8) \cong O_3(8) \cong G$: all 3×3 matrices over $\mathbb{F}_8$ preserving a non-singular quadratic form; $\Gamma O_3(8) \cong P\Gamma O_3(8) \cong \Sigma O_3(8) \cong P\Sigma O_3(8) \cong G.3$

Ree $R(3) \cong {}^2G_2(3) \cong G.3$: the centralizer in $G_2(3)$ of an outer automorphism of order 2; the group acts doubly transitively on $3^3+1 = 28$ points

Presentations $G \cong G^{3,7,9} \cong \langle P,Q \mid P^7=(P^2Q)^3=(P^3Q)^2=(PQ^5)^2=1\rangle$

Maximal subgroups					Specifications			
Order	Index	Structure	G.3	Character	Abstract	Linear	Orthogonal	Ree
56	9	$2^3{:}7$	: $2^3{:}7{:}3$	1a+8a	$N(2A^3)$	point	isotropic point	
18	28	D_{18}	: 9:6	1a+9abc	N(3A)	$O_2^-(8)$, $L_1(64)$	minus point	point
14	36	D_{14}	: 7:6	1a+8a+9abc	N(7ABC)	$O_2^+(8)$, base	plus point	

		;	@	@	@	@	@	@	@	@	@	;	;	@	@	@
			504	8	9	7	7	7	9	9	9			6	2	3
		p power		A	A	A	A	A	A	A	A			A	BA	A
		p' part		A	A	A	A	A	A	A	A			A	BA	A
		ind	1A	2A	3A	7A	B*2	C*4	9A	B*2	C*4	fus	ind	3B	6A	9D
χ_1	+		1	1	1	1	1	1	1	1	1	:	+oo	1	1	1
χ_2	+		7	-1	-2	0	0	0	1	1	1	:	+oo	1	-1	1
χ_3	+		7	-1	1	0	0	0	-y9	*2	*4		+	0	0	0
χ_4	+		7	-1	1	0	0	0	*4	-y9	*2					
χ_5	+		7	-1	1	0	0	0	*2	*4	-y9					
χ_6	+		8	0	-1	1	1	1	-1	-1	-1	:	+oo	2	0	-1
χ_7	+		9	1	0	y7	*2	*4	0	0	0		+	0	0	0
χ_8	+		9	1	0	*4	y7	*2	0	0	0					
χ_9	+		9	1	0	*2	*4	y7	0	0	0					



$L_2(11)$

Linear group $L_2(11) \cong A_1(11) \cong U_2(11) \cong S_2(11) \cong O_3(11)$

Order = 660 = $2^2.3.5.11$ Mult = 2 Out = 2

Constructions

Linear $GL_2(11) \cong 5 \times 2.G.2$: all non-singular 2 x 2 matrices over $\mathbf{F}_{11}$;

$PGL_2(11) \cong G.2$; $SL_2(11) \cong 2.G$; $PSL_2(11) \cong G$

Unitary $GU_2(11) \cong 3 \times 2.(G \times 2).2$: all 2 x 2 matrices over $\mathbf{F}_{121}$ preserving a non-singular Hermitian form;

$PGU_2(11) \cong G.2$; $SU_2(11) \cong 2.G$; $PSU_2(11) \cong G$

Orthogonal $GO_3(11) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbf{F}_{11}$ preserving a non-singular quadratic form;

$PGO_3(11) \cong SO_3(11) \cong PSO_3(11) \cong G.2$; $O_3(11) \cong G$

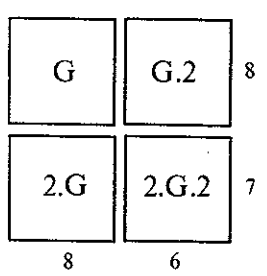
Biplane G : the automorphism group of a system of 11 "points" and 11 "lines" which can be numbered (mod 11) so that $L_i = \{P_{i+1}, P_{i+3}, P_{i+9}, P_{i+5}, P_{i+4}\}$; any pair of points determines <u>two</u> lines; any pair of lines determines <u>two</u> points; G.2 is obtained by adjoining duality

Mathieu G : the stabilizer of a point in the 12-point representation of M_{11}; i.e. the stabilizer of a point and a total in M_{12}; becomes G.2 by adjoining an automorphism interchanging points and totals

Presentations $G \cong G^{5,5,5} \cong \langle \underset{\bullet}{a} \underset{\bullet}{b} \underset{\bullet}{5} \underset{\bullet}{c} \underset{\bullet}{d} \mid (abc)^5=1[=(bcd)^5] \rangle \cong \langle S,T \mid S^{11}=(S^4TS^6T)^2=1,(ST)^3=T^2[=1] \rangle$

Maximal subgroups					Specifications			
Order	Index	Structure	G.2	Character	Abstract	Linear	Orthogonal	Mathieu
60	11	A_5	S_4, D_{20}	1a+10b	N(2A,3A,5AB)		icosahedral	point
60	11	A_5		1a+10b	N(2A,3A,5AB)		icosahedral	total
55	12	11:5	: 11:10	1a+11a	N(11AB)	point	isotropic point	
12	55	D_{12}	: D_{24}	1a+5ab+10bb+12ab	N(2A), N(3A)	$O_2^-(11)$, $L_1(121)$	minus point	duad

		;	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@
			660	12	6	5	5	6	11	11			10	6	5	5	6	6
		p power		A	A	A	A	AA	A	A			A	A	BB	AB	AA	AA
		p' part		A	A	A	A	AA	A	A			A	A	AB	BB	AA	AA
		ind	1A	2A	3A	5A	B*	6A	11A	B**	fus	ind	2B	4A	10A	B*	12A	B*
χ_1	+		1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1
χ_2	o		5	1	-1	0	0	1	b11	**		+	0	0	0	0	0	0
χ_3	o		5	1	-1	0	0	1	**	b11								
χ_4	+		10	-2	1	0	0	1	-1	-1	:	++	0	2	0	0	-1	-1
χ_5	+		10	2	1	0	0	-1	-1	-1	:	++	0	0	0	0	r3	-r3
χ_6	+		11	-1	-1	1	1	-1	0	0	:	++	1	-1	1	1	-1	-1
χ_7	+		12	0	0	b5	*	0	1	1	:	++	2	0	b5	*	0	0
χ_8	+		12	0	0	*	b5	0	1	1	:	++	2	0	*	b5	0	0
	ind		1	4	3	5	5	12	11	11	fus	ind	4	8	20	20	24	24
			2		6	10	10	12	22	22				8	20	20	24	24
χ_9	o		6	0	0	1	1	0	-b11	**		-	0	0	0	0	0	0
χ_{10}	o		6	0	0	1	1	0	**	-b11								
χ_{11}	-		10	0	-2	0	0	0	-1	-1	:	--	0	r2	0	0	r2	r2
χ_{12}	-		10	0	1	0	0	r3	-1	-1	:	--	0	r2	0	0	-y24	*7
χ_{13}	-		10	0	1	0	0	-r3	-1	-1	:	--	0	r2	0	0	*7	-y24
χ_{14}	-		12	0	0	b5	*	0	1	1	:	--	0	0	y20	*3	0	0
χ_{15}	-		12	0	0	*	b5	0	1	1	:	--	0	0	*7	y20	0	0



$L_2(13)$

Linear group $L_2(13) \cong A_1(13) \cong U_2(13) \cong S_2(13) \cong O_3(13)$

Order = 1,092 = $2^2.3.7.13$ Mult = 2 Out = 2

Constructions

Linear $GL_2(13) \cong 3 \times 2.(G \times 2).2$: all non-singular 2×2 matrices over $\mathbb{F}_{13}$;

$PGL_2(13) \cong G.2$; $SL_2(13) \cong 2.G$; $PSL_2(13) \cong G$

Unitary $GU_2(13) \cong 7 \times 2.G.2$: all 2×2 matrices over $\mathbb{F}_{169}$ preserving a non-singular Hermitian form;

$PGU_2(13) \cong G.2$; $SU_2(13) \cong 2.G$; $PSU_2(13) \cong G$

Orthogonal $GO_3(13) \cong 2 \times G.2$: all 3×3 matrices over $\mathbb{F}_{13}$ preserving a non-singular quadratic form;

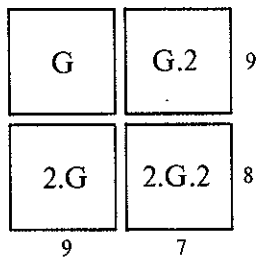
$PGO_3(13) \cong SO_3(13) \cong PSO_3(13) \cong G.2$; $O_3(13) \cong G$

Presentations $G \cong G^{3,7,13} \cong (2,3,7;6) \cong (2,3,7;7) \cong \langle S,T \mid S^7=T^2=(ST)^6=(S^2T)^3=1\rangle \cong \langle S,T \mid S^{13}=(S^4TS^7T)^2=1,(ST)^3=T^2[=1]\rangle$;

$G.2 \cong G^{3,7,12} \cong G^{3,7,14} \cong (2,4,7;3)$; $2 \times G.2 \cong G^{4,6,7}$

Maximal subgroups					Specifications		
Order	Index	Structure	G.2	Character	Abstract	Linear	Orthogonal
78	14	13:6	: 13:12	1a+13a	N(13AB)	point	isotropic point
14	78	D_{14}	: D_{28}	1a+12abc+13a+14aa	N(7ABC), C(2B)	$O_2^-(13)$, $L_1(169)$	minus point
12	91	D_{12}	: D_{24}	1a+12abc+13aa+14aa	N(2A), N(3A)	$O_2^+(13)$, base	plus point
12	91	A_4	: S_4	1a+7ab+12abc+13aa+14a	N(2A^2)		base

	;	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@
		1092	12	6	6	7	7	7	13	13			14	6	6	6	7	7	7
	p power		A	A	AA	A	A	A	A	A			A	A	AA	AA	BB	CB	AB
	p' part		A	A	AA	A	A	A	A	A			A	A	AA	AA	AB	BB	CB
	ind	1A	2A	3A	6A	7A	B*2	C*4	13A	B*	fus	ind	2B	4A	12A	B*	14A	B*5	C*3
χ_1	+	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1
χ_2	+	7	-1	1	-1	0	0	0	-b13	*		+	0	0	0	0	0	0	0
χ_3	+	7	-1	1	-1	0	0	0	*	-b13									
χ_4	+	12	0	0	0	-y7	*2	*4	-1	-1	:	++	2	0	0	0	y7	*2	*4
χ_5	+	12	0	0	0	*4	-y7	*2	-1	-1	:	++	2	0	0	0	*4	y7	*2
χ_6	+	12	0	0	0	*2	*4	-y7	-1	-1	:	++	2	0	0	0	*2	*4	y7
χ_7	+	13	1	1	1	-1	-1	-1	0	0	:	++	1	-1	-1	-1	1	1	1
χ_8	+	14	2	-1	-1	0	0	0	1	1	:	++	0	2	-1	-1	0	0	0
χ_9	+	14	-2	-1	1	0	0	0	1	1	:	++	0	0	r3	-r3	0	0	0
	ind	1 2	4	3 6	12 12	7 14	7 14	7 14	13 26	13 26	fus	ind	4	8 8	24 24	24 24	28 28	28 28	28 28
χ_{10}	-	6	0	0	0	-1	-1	-1	b13	*		-	0	0	0	0	0	0	0
χ_{11}	-	6	0	0	0	-1	-1	-1	*	b13									
χ_{12}	-	12	0	0	0	-y7	*2	*4	-1	-1	:	--	0	0	0	0	*3	y28	*9
χ_{13}	-	12	0	0	0	*4	-y7	*2	-1	-1	:	--	0	0	0	0	*9	*3	y28
χ_{14}	-	12	0	0	0	*2	*4	-y7	-1	-1	:	--	0	0	0	0	y28	*9	*3
χ_{15}	-	14	0	2	0	0	0	0	1	1	:	--	0	r2	r2	r2	0	0	0
χ_{16}	-	14	0	-1	r3	0	0	0	1	1	:	--	0	r2	-y24	*7	0	0	0
χ_{17}	-	14	0	-1	-r3	0	0	0	1	1	:	--	0	r2	*7	-y24	0	0	0



$L_2(17)$

Linear group $L_2(17) \cong A_1(17) \cong U_2(17) \cong S_2(17) \cong O_3(17)$

Order = 2,448 = $2^4.3^2.17$ Mult = 2 Out = 2

Constructions

Linear $GL_2(17) \cong 2.(G \times 8).2$: all non-singular 2 x 2 matrices over $\mathbf{F}_{17}$;

$PGL_2(17) \cong G.2$; $SL_2(17) \cong 2.G$; $PSL_2(17) \cong G$

Unitary $GU_2(17) \cong 9 \times 2.G.2$: all 2 x 2 matrices over $\mathbf{F}_{289}$ preserving a non-singular Hermitian form;

$PGU_2(17) \cong G.2$; $SU_2(17) \cong 2.G$; $PSU_2(17) \cong G$

Orthogonal $GO_3(17) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbf{F}_{17}$ preserving a non-singular quadratic form;

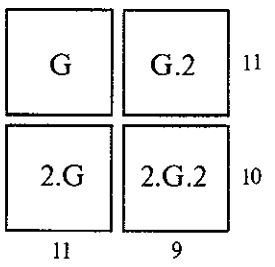
$PGO_3(17) \cong SO_3(17) \cong PSO_3(17) \cong G.2$; $O_3(17) \cong G$

Presentations $G \cong \langle S,T \mid S^9=T^2=(ST)^4=(S^2T)^3=1\rangle \cong \langle S,T \mid S^{17}=(S^4TS^9T)^2=1,(ST)^3=T^2[=1]\rangle$

Maximal subgroups — Specifications

Order	Index	Structure	G.2	Character	Abstract	Linear	Orthogonal
136	18	17:8	: 17:16	1a+17a	N(17AB)	point	isotropic point
24	102	S_4		1a+9ab+16bcd+17a+18a	$N(2A^2)$		base
24	102	S_4		1a+9ab+16bcd+17a+18a	$N(2A^2)$		base
18	136	D_{18}	: D_{36}	1a+9ab+16abcd+17a+18aa	N(3A), C(2B)	$O_2^-(17)$, $L_1(289)$	minus point
16	153	D_{16}	: D_{32}	1a+9ab+16abcd+17aa+18aa	N(2A)	$O_2^+(17)$, base	plus point

	;	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@
		2448	16	9	8	8	8	9	9	9	17	17			18	9	8	8	8	8	9	9	9
	p	power	A	A	A	A	A	A	A	A	A	A			A	AB	A	B	A	B	BA	CA	AA
	p'	part	A	A	A	A	A	A	A	A	A	A			A	AB	A	A	A	A	AB	BB	CB
	ind	1A	2A	3A	4A	8A	B*	9A	B*2	C*4	17A	B*	fus	ind	2B	6A	16A	B*3	C*7	D*5	18A	B*7	C*5
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1
χ_2	+	9	1	0	1	-1	-1	0	0	0	-b17	*		+	0	0	0	0	0	0	0	0	0
χ_3	+	9	1	0	1	-1	-1	0	0	0	*	-b17											
χ_4	+	16	0	-2	0	0	0	1	1	1	-1	-1	:	++	2	2	0	0	0	0	-1	-1	-1
χ_5	+	16	0	1	0	0	0	-y9	*2	*4	-1	-1	:	++	2	-1	0	0	0	0	y9	*2	*4
χ_6	+	16	0	1	0	0	0	*4	-y9	*2	-1	-1	:	++	2	-1	0	0	0	0	*4	y9	*2
χ_7	+	16	0	1	0	0	0	*2	*4	-y9	-1	-1	:	++	2	-1	0	0	0	0	*2	*4	y9
χ_8	+	17	1	-1	1	1	1	-1	-1	-1	0	0	:	++	1	1	-1	-1	-1	-1	1	1	1
χ_9	+	18	2	0	-2	0	0	0	0	0	1	1	:	++	0	0	r2	-r2	r2	-r2	0	0	0
χ_{10}	+	18	-2	0	0	r2	-r2	0	0	0	1	1	:	++	0	0	y16	*3	*7	*5	0	0	0
χ_{11}	+	18	-2	0	0	-r2	r2	0	0	0	1	1	:	++	0	0	*5	y16	*3	*7	0	0	0
	ind	1	4	3	8	16	16	9	9	9	17	17	fus	ind	4	12	32	32	32	32	36	36	36
		2		6	8	16	16	18	18	18	34	34				12	32	32	32	32	36	36	36
χ_{12}	-	8	0	-1	0	0	0	-1	-1	-1	b17	*		-	0	0	0	0	0	0	0	0	0
χ_{13}	-	8	0	-1	0	0	0	-1	-1	-1	*	b17											
χ_{14}	-	16	0	-2	0	0	0	1	1	1	-1	-1	:	--	0	0	0	0	0	0	r3	r3	r3
χ_{15}	-	16	0	1	0	0	0	-y9	*2	*4	-1	-1	:	--	0	r3	0	0	0	0	*13	y36	*11
χ_{16}	-	16	0	1	0	0	0	*4	-y9	*2	-1	-1	:	--	0	r3	0	0	0	0	*11	*13	y36
χ_{17}	-	16	0	1	0	0	0	*2	*4	-y9	-1	-1	:	--	0	r3	0	0	0	0	y36	*11	*13
χ_{18}	-	18	0	0	r2	y16	*3	0	0	0	1	1	:	--	0	0	y32	*13	*9	*5	0	0	0
χ_{19}	-	18	0	0	-r2	*5	y16	0	0	0	1	1	:	--	0	0	*5	y32	*13	*7	0	0	0
χ_{20}	-	18	0	0	r2	-y16	*3	0	0	0	1	1	:	--	0	0	*7	*5	y32	*3	0	0	0
χ_{21}	-	18	0	0	-r2	*5	-y16	0	0	0	1	1	:	--	0	0	*13	*9	*11	y32	0	0	0



A_7

Alternating group A_7

Order = 2,520 = $2^3.3^2.5.7$ Mult = 6 Out = 2

Constructions

Alternating $S_7 \cong G.2$: all permutations of 7 letters;

$A_7 \cong G$: the even permutations; 2.G and 2.G.2 : the Schur double covers

Lattice $2A_7 \cong 2.G$: the symmetries of the lattice $\Lambda_{4,b7}$ whose minimal vectors are obtained from ±(i7;0,0,0) ±(0;i7,0,0) ±(1;x,y,z) ±(-x;1,y,-z) ±(-xy;1,1,1) ±(1;xy,1,-1) ±(xy;0,z,1) ±(-x;yz,0,-1) ±(-1;0,xy,z) ±(0;1,-x,yz) by replacing each of x,y,z by one of b7 or b7**, and cyclically permuting the last 3 coordinates

Vectors $3A_7$: symmetries of the 21 (w)vectors obtained from (20 00 00), (00 11 11), (01 01 $w\bar{w}$) by bodily permuting the 3 couples, and reversing any 2 couples (see A_6 (hexacode));

the lattice these generate has automorphism group $6U_4(3).2$ (see $U_4(3)$)

Unitary $3A_7$ has a 3-dimensional unitary representation over $\mathbb{F}_{25}$ (see $U_3(5)$)

Presentations $G \cong \langle A,B \mid A^4=B^5=(AB)^3=(A^{-1}BA^2B^2)^2=1\rangle \cong \langle x_1,\ldots,x_5 \mid x_i^3=(x_ix_j)^2=1\rangle$;

$G.2 \cong$ ———————— $\cong \langle A,B \mid A^2=[A,B^2]^2=[A,B^3]^2=1;B^7=(AB)^6\rangle$

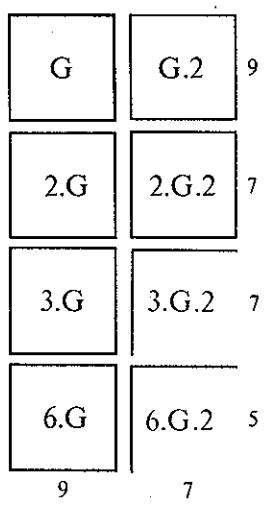
Maximal subgroups

Order	Index	Structure	G.2	Character
360	7	A_6	: S_6	1a+6a
168	15	$L_2(7)$	7:6	1a+14b
168	15	$L_2(7)$		1a+14b
120	21	S_5	: S_5 x 2	1a+6a+14a
72	35	$(A_4 \times 3):2$	: $S_4 \times S_3$	1a+6a+14ab

Specifications

Abstract	Alternating
N(2A,3A,3B,4A,5A)	point
N(2A,3B,4A,7AB)	S(2,3,7)
N(2A,3B,4A,7AB)	S(2,3,7)
N(2A,3A,5A), C(2B)	duad
N(3A), N($2A^2$)	triad

	;	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@
		2520	24	36	9	4	5	12	7	7			120	24	12	6	3	5	6
	p	power	A	A	A	A	A	AA	A	A			A	A	A	AB	BC	AB	AB
	p'	part	A	A	A	A	A	AA	A	A			A	A	A	AB	BC	AB	AB
	ind		1A	2A	3A	3B	4A	5A	6A	7A	B**	fus ind		2B	2C	4B	6B	6C	10A	12A
χ_1	+		1	1	1	1	1	1	1	1	1	: ++		1	1	1	1	1	1	1
χ_2	+		6	2	3	0	0	1	-1	-1	-1	: ++		4	0	2	1	0	-1	-1
χ_3	o		10	-2	1	1	0	0	1	b7	**	+		0	0	0	0	0	0	0
χ_4	o		10	-2	1	1	0	0	1	**	b7									
χ_5	+		14	2	2	-1	0	-1	2	0	0	: ++		6	2	0	0	-1	1	0
χ_6	+		14	2	-1	2	0	-1	-1	0	0	: ++		4	0	-2	1	0	-1	1
χ_7	+		15	-1	3	0	-1	0	-1	1	1	: ++		5	-3	1	-1	0	0	1
χ_8	+		21	1	-3	0	-1	1	1	0	0	: ++		1	-3	-1	1	0	1	-1
χ_9	+		35	-1	-1	-1	1	0	-1	0	0	: ++		5	1	-1	-1	1	0	-1
	ind		1	4	3	3	8	5	12	7	7	fus ind		2	4	8	6	12	10	24
			2		6	6	8	10		14	14							12	10	24
χ_{10}	o		4	0	-2	1	0	-1	0	-b7	**	-		0	0	0	0	0	0	0
χ_{11}	o		4	0	-2	1	0	-1	0	**	-b7									
χ_{12}	-		14	0	2	-1	r2	-1	0	0	0	-		0	0	0	0	0	0	0
χ_{13}	-		14	0	2	-1	-r2	-1	0	0	0									
χ_{14}	-		20	0	-4	-1	0	0	0	-1	-1	: --		0	0	0	0	r3	0	0
χ_{15}	-		20	0	2	2	0	0	0	-1	-1	: --		0	0	0	0	0	0	r6
χ_{16}	-		36	0	0	0	0	1	0	1	1	: --		0	0	0	0	0	r5	0
	ind		1	2	3	3	4	5	6	7	7	fus ind		2	2	4	6	6	10	12
			3	6			12	15	6	21	21									
			3	6			12	15	6	21	21									
χ_{17}	o2		6	2	0	0	0	1	2	-1	-1	* +								
χ_{18}	o2		15	-1	0	0	-1	0	2	1	1	* +								
χ_{19}	o2		15	3	0	0	1	0	0	1	1	* +								
χ_{20}	o2		21	1	0	0	-1	1	-2	0	0	* +								
χ_{21}	o2		21	-3	0	0	1	1	0	0	0	* +								
χ_{22}	o2		24	0	0	0	0	-1	0	b7	**	+2								
χ_{23}	o2		24	0	0	0	0	-1	0	**	b7	*								
	ind		1	4	3	3	8	5	12	7	7	fus ind		2	4	8	6	12	10	24
			6	12	6	6	24	30	12	42	42							12	10	24
			3	12			24	15	12	21	21									
			2				8	10		14	14									
			3				24	15		21	21									
			6				24	30		42	42									
χ_{24}	o2		6	0	0	0	r2	1	0	-1	-1	o2								
χ_{25}	o2		6	0	0	0	-r2	1	0	-1	-1	*								
χ_{26}	o2		24	0	0	0	0	-1	0	b7	**	-2								
χ_{27}	o2		24	0	0	0	0	-1	0	**	b7	*								
χ_{28}	o2		36	0	0	0	0	1	0	1	1	* -								



$L_2(19)$

Linear group $L_2(19) \cong A_1(19) \cong U_2(19) \cong S_2(19) \cong O_3(19)$

Order = 3,420 = $2^2.3^2.5.19$ Mult = 2 Out = 2

Constructions

Linear $GL_2(19) \cong 9 \times 2.G.2$: all non-singular 2 x 2 matrices over $\mathbf{F}_{19}$;

$PGL_2(19) \cong G.2$; $SL_2(19) \cong 2.G$; $PSL_2(19) \cong G$

Unitary $GU_2(19) \cong 5 \times 2.(G \times 2).2$: all 2 x 2 matrices over $\mathbf{F}_{361}$ preserving a non-singular Hermitian form;

$PGU_2(19) \cong G.2$; $SU_2(19) \cong 2.G$; $PSU_2(19) \cong G$

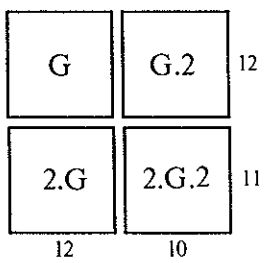
Orthogonal $GO_3(19) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbf{F}_{19}$ preserving a non-singular quadratic form;

$PGO_3(19) \cong SO_3(19) \cong PSO_3(19) \cong G.2$; $O_3(19) \cong G$

Presentations $G \cong G^{3,9,9} \cong (2,5,9;2) \cong \langle \underline{a\ 5\ b\quad c\ 5\ d} \mid (abc)^5=(bcd)^5=1 \rangle \cong \langle S,T \mid S^{19}=(S^4TS^{10}T)^2=1,(ST)^3=T^2[=1]\rangle$;

$G.2 \cong G^{4,5,9}$; $S_3 \times G \cong G^{3,9,10}$

Maximal subgroups					Specifications		
Order	Index	Structure	G.2	Character	Abstract	Linear	Orthogonal
171	20	19:9	: 19:18	1a+19a	N(19AB)	point	isotropic point
60	57	A_5	S_4	1a+18cd+20a	N(2A,3A,5AB)		icosahedral
60	57	A_5		1a+18cd+20a	N(2A,3A,5AB)		icosahedral
20	171	D_{20}	: D_{40}	1a+9ab+18ccdd+20abcd	N(2A), N(5AB)	$O_2^-(19)$, $L_1(361)$	minus point
18	190	D_{18}	: D_{36}	1a+9ab+18ccdd+19a+20abcd	N(3A), C(2B)	$O_2^+(19)$, base	plus point



	;	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@
		3420	20	9	10	10	9	9	9	10	10	19	19			18	10	9	9	9	9	10	10	10	10
	p power		A	A	A	A	A	A	A	BA	AA	A	A			A	A	AB	BA	CA	AA	BA	AA	BA	AA
	p' part		A	A	A	A	A	A	A	AA	BA	A	A			A	A	AB	AB	BB	CB	AA	BA	AA	BA
	ind	1A	2A	3A	5A	B*	9A	B*2	C*4	10A	B*	19A	B**	fus	ind	2B	4A	6A	18A	B*7	C*5	20A	B*3	C*9	D*7
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1
χ_2	o	9	1	0	-1	-1	0	0	0	1	1	b19	**		+	0	0	0	0	0	0	0	0	0	0
χ_3	o	9	1	0	-1	-1	0	0	0	1	1	**	b19												
χ_4	+	18	-2	0	-b5	*	0	0	0	-b5	*	-1	-1	:	++	0	2	0	0	0	0	b5	*	b5	*
χ_5	+	18	-2	0	*	-b5	0	0	0	*	-b5	-1	-1	:	++	0	2	0	0	0	0	*	b5	*	b5
χ_6	+	18	2	0	-b5	*	0	0	0	b5	*	-1	-1	:	++	0	0	0	0	0	0	y20	*3	*9	*7
χ_7	+	18	2	0	*	-b5	0	0	0	*	b5	-1	-1	:	++	0	0	0	0	0	0	*7	y20	*3	*9
χ_8	+	19	-1	1	-1	-1	1	1	1	-1	-1	0	0	:	++	1	-1	1	1	1	1	-1	-1	-1	-1
χ_9	+	20	0	2	0	0	-1	-1	-1	0	0	1	1	:	++	2	0	2	-1	-1	-1	0	0	0	0
χ_{10}	+	20	0	-1	0	0	y9	*2	*4	0	0	1	1	:	++	2	0	-1	y9	*2	*4	0	0	0	0
χ_{11}	+	20	0	-1	0	0	*4	y9	*2	0	0	1	1	:	++	2	0	-1	*4	y9	*2	0	0	0	0
χ_{12}	+	20	0	-1	0	0	*2	*4	y9	0	0	1	1	:	++	2	0	-1	*2	*4	y9	0	0	0	0
	ind	1	4	3	5	5	9	9	9	20	20	19	19	fus	ind	4	8	12	36	36	36	40	40	40	40
		2		6	10	10	18	18	18	20	20	38	38				8	12	36	36	36	40	40	40	40
χ_{13}	o	10	0	1	0	0	1	1	1	0	0	-b19	**		-	0	0	0	0	0	0	0	0	0	0
χ_{14}	o	10	0	1	0	0	1	1	1	0	0	**	-b19												
χ_{15}	-	18	0	0	-2	-2	0	0	0	0	0	-1	-1	:	--	0	r2	0	0	0	0	r2	r2	r2	r2
χ_{16}	-	18	0	0	-b5	*	0	0	0	y20	*3	-1	-1	:	--	0	r2	0	0	0	0	*17	*9	*7	-y40
χ_{17}	-	18	0	0	*	-b5	0	0	0	*7	y20	-1	-1	:	--	0	r2	0	0	0	0	-y40	*17	*9	*7
χ_{18}	-	18	0	0	-b5	*	0	0	0	-y20	*3	-1	-1	:	--	0	r2	0	0	0	0	*7	-y40	*17	*9
χ_{19}	-	18	0	0	*	-b5	0	0	0	*7	-y20	-1	-1	:	--	0	r2	0	0	0	0	*9	*7	-y40	*17
χ_{20}	-	20	0	2	0	0	-1	-1	-1	0	0	1	1	:	--	0	0	0	r3	r3	r3	0	0	0	0
χ_{21}	-	20	0	-1	0	0	y9	*2	*4	0	0	1	1	:	--	0	0	r3	*13	y36	*11	0	0	0	0
χ_{22}	-	20	0	-1	0	0	*4	y9	*2	0	0	1	1	:	--	0	0	r3	*11	*13	y36	0	0	0	0
χ_{23}	-	20	0	-1	0	0	*2	*4	y9	0	0	1	1	:	--	0	0	r3	y36	*11	*13	0	0	0	0

$L_2(16)$

Linear group $L_2(16) \cong A_1(16) \cong U_2(16) \cong S_2(16) \cong O_3(16) \cong O_4^-(4)$

Order = 4,080 = $2^4.3.5.17$ Mult = 1 Out = 4

Constructions

Linear $GL_2(16) \cong 15 \times G$: all non-singular 2 x 2 matrices over $\mathbb{F}_{16}$;

$PGL_2(16) \cong SL_2(16) \cong PSL_2(16) \cong G$; $\Gamma L_2(16) \cong (15 \times G).4$; $P\Gamma L_2(16) \cong \Sigma L_2(16) \cong P\Sigma L_2(16) \cong G.4$

Unitary $GU_2(16) \cong 17 \times G$: all 2 x 2 matrices over $\mathbb{F}_{256}$ preserving a non-singular Hermitian form;

$PGU_2(16) \cong SU_2(16) \cong PSU_2(16) \cong G$

Orthogonal (16) $GO_3(16) \cong PGO_3(16) \cong SO_3(16) \cong PSO_3(16) \cong O_3(16) \cong G$: all 3 x 3 matrices over $\mathbb{F}_{16}$ preserving a non-singular quadratic form; $\Gamma O_3(16) \cong P\Gamma O_3(16) \cong \Sigma O_3(16) \cong P\Sigma O_3(16) \cong G.4$

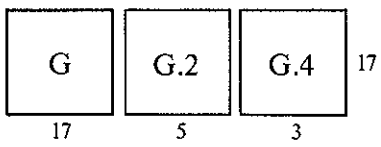
Orthogonal (4) $GO_4^-(4) \cong PGO_4^-(4) \cong SO_4^-(4) \cong PSO_4^-(4) \cong G.2$: all 4 x 4 matrices over $\mathbb{F}_4$ preserving a non-singular quadratic form of Witt defect 1, for example $wx_1^2+x_1x_2+wx_2^2+x_3x_4$; $O_4^-(4) \cong G$

Presentation$ $G \cong \langle P,Q \mid P^{15}=(P^2Q)^3=(P^3Q)^2=(PQ^9)^2=(P^8Q^2)^2=1\rangle$

Maximal subgroups / Specifications

Order	Index	Structure	G.2	G.4	Character	Abstract	Linear
240	17	$2^4:15$	: $2^4:(3xD_{10})$	: $2^4:15:4$	1a+16a	$N(2A^4)$	point
60	68	A_5	: A_5 x 2	: $(A_5x2)\cdot 2$	1a+16a+17abc	N(2A,3A,5AB), C(2B)	$L_2(4)$
34	120	D_{34}	: 17:4	: 17:8	1a+17abcdefg	N(17A-H)	$O_2^-(16)$, $L_1(256)$
30	136	D_{30}	: D_{10} x S_3	: S_3 x 5:4	1a+16a+17abcdefg	N(3A), N(5AB)	$O_2^+(16)$, base

Orthogonal (16)	Orthogonal (4)
isotropic point	isotropic point
$O_3(4)$	non-isotropic point
minus line	$O_2^-(16)$
plus line	pair of isotropic points



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	;	;	@	@	@
	4080	16	15	15	15	15	15	15	15	17	17	17	17	17	17	17	17				60	4	3	5	5			6	2	3
	p power	A	A	A	A	BA	BA	AA	AA	A	A	A	A	A	A	A	A				A	A	AB	BB	AB			B	A	AB
	p' part	A	A	A	A	AA	AA	BA	BA	A	A	A	A	A	A	A	A				A	A	AB	AB	BB			A	A	AB
	ind	1A	2A	3A	5A	B*	15A	B*4	C*2	D*8	17A	B*4	C*2	D*8	E*6	F*7	G*5	H*3	fus ind		2B	4A	6A	10A	B*	fus ind		4B	8A	12A
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	:+o+o		1	1	1
χ_2	+	15	-1	0	0	0	0	0	0	0	-y17	*4	*2	*8	*6	*7	*5	*3		+	0	0	0	0	0		+	0	0	0
χ_3	+	15	-1	0	0	0	0	0	0	0	*4	-y17	*8	*2	*7	*6	*3	*5												
χ_4	+	15	-1	0	0	0	0	0	0	0	*8	*2	-y17	*4	*3	*5	*6	*7		+	0	0	0	0	0					
χ_5	+	15	-1	0	0	0	0	0	0	0	*2	*8	*4	-y17	*5	*3	*7	*6												
χ_6	+	15	-1	0	0	0	0	0	0	0	*3	*5	*6	*7	-y17	*4	*2	*8		+	0	0	0	0	0		+	0	0	0
χ_7	+	15	-1	0	0	0	0	0	0	0	*5	*3	*7	*6	*4	-y17	*8	*2												
χ_8	+	15	-1	0	0	0	0	0	0	0	*7	*6	*3	*5	*8	*2	-y17	*4		+	0	0	0	0	0					
χ_9	+	15	-1	0	0	0	0	0	0	0	*6	*7	*5	*3	*2	*8	*4	-y17												
χ_{10}	+	16	0	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	:	++	4	0	1	-1	-1	:+o+o		2	0	-1
χ_{11}	+	17	1	-1	2	2	-1	-1	-1	-1	0	0	0	0	0	0	0	0	:	++	5	1	-1	0	0	:+o+o		1	-1	1
χ_{12}	+	17	1	2	b5	*	b5	b5	*	*	0	0	0	0	0	0	0	0	:	++	3	-1	0	-b5	*		++	0	0	0
χ_{13}	+	17	1	2	*	b5	*	*	b5	b5	0	0	0	0	0	0	0	0	:	++	3	-1	0	*	-b5					
χ_{14}	+	17	1	-1	*	b5	y15	*4	*2	*8	0	0	0	0	0	0	0	0		+	0	0	0	0	0		+	0	0	0
χ_{15}	+	17	1	-1	*	b5	*4	y15	*8	*2	0	0	0	0	0	0	0	0												
χ_{16}	+	17	1	-1	b5	*	*8	*2	y15	*4	0	0	0	0	0	0	0	0		+	0	0	0	0	0					
χ_{17}	+	17	1	-1	b5	*	*2	*8	*4	y15	0	0	0	0	0	0	0	0												

$L_3(3)$

Linear group $L_3(3) \cong A_2(3)$

Order = 5,616 = $2^4.3^3.13$ Mult = 1 Out = 2

Constructions

Linear $GL_3(3) \cong 2 \times G$: all 3 x 3 non-singular matrices over $\mathbf{F}_3$;

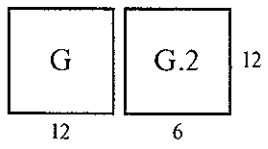
$SL_3(3) \cong PGL_3(3) \cong PSL_3(3) \cong G$; the points and lines of the corresponding projective plane can be numbered (mod 13) so that $L_i = \{P_i, P_{i+1}, P_{i+3}, P_{i+9}\}$;

G.2 is obtained by adjoining the duality (graph) automorphism

Presentation: $G \cong \langle S,T \mid S^6=T^3=(ST)^4=(S^2T)^4=(S^3T)^3=[S^2,(TS^2T)^2]=1\rangle$

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Linear
432	13	$3^2:2S_4$	$3_+^{1+2}:D_8$, $2S_4:2$	1a+12a	$N(3A^2)$	point
432	13	$3^2:2S_4$		1a+12a	$N(3A^2)$	line
39	144	13:3	: 13:6	1a+13a+16abcd+27a+39a	N(13ABCD)	$L_1(27)$
24	234	S_4	: $S_4 \times 2$	1a+12aa+16abcd+26aa+27aa+39a	$N(2A^2)$, C(2B)	$O_3(3)$, base



	;	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@
		5616	48	54	9	8	6	8	8	13	13	13	13			24	24	3	4	6	6
	p power		A	A	A	A	AA	A	A	A	A	A	A			A	A	BB	A	AB	AB
	p' part		A	A	A	A	AA	A	A	A	A	A	A			A	A	BB	A	AB	AB
	ind	1A	2A	3A	3B	4A	6A	8A	B**	13A	B**	C*5	D*8	fus	ind	2B	4B	6B	8C	12A	B*
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1
χ_2	+	12	4	3	0	0	1	0	0	-1	-1	-1	-1	:	++	0	0	0	0	r3	-r3
χ_3	+	13	-3	4	1	1	0	-1	-1	0	0	0	0	:	++	1	-3	1	-1	0	0
χ_4	o	16	0	-2	1	0	0	0	0	d13	**	*5	*8		+	0	0	0	0	0	0
χ_5	o	16	0	-2	1	0	0	0	0	**	d13	*8	*5								
χ_6	o	16	0	-2	1	0	0	0	0	*8	*5	d13	**		+	0	0	0	0	0	0
χ_7	o	16	0	-2	1	0	0	0	0	*5	*8	**	d13								
χ_8	+	26	2	-1	-1	2	-1	0	0	0	0	0	0	:	++	2	-2	-1	0	1	1
χ_9	o	26	-2	-1	-1	0	1	i2	-i2	0	0	0	0		+	0	0	0	0	0	0
χ_{10}	o	26	-2	-1	-1	0	1	-i2	i2	0	0	0	0								
χ_{11}	+	27	3	0	0	-1	0	-1	-1	1	1	1	1	:	++	3	3	0	-1	0	0
χ_{12}	+	39	-1	3	0	-1	-1	1	1	0	0	0	0	:	++	3	-1	0	1	-1	-1

$U_3(3) \cong G_2(2)'$

Unitary group $U_3(3) \cong {}^2A_2(3)$; Derived Chevalley group $G_2(2)'$

Order = 6,048 = $2^5.3^3.7$ Mult = 1 Out = 2

Constructions

Unitary $GU_3(3) \cong 4 \times G$: all 3×3 matrices preserving a non-singular Hermitian form over $\mathbf{F}_9$;

$PGU_3(3) \cong SU_3(3) \cong PSU_3(3) \cong G$

Chevalley $G_2(2) \cong G.2$: adjoint Chevalley group of type G_2 over $\mathbf{F}_2$;

G.2 : the automorphism group of a generalized hexagon of order (2,2) consisting of 63 vertices and 63 edges, each object being incident with 3 of the other type.

Cayley (2) G.2 : the automorphism group of the $\mathbf{F}_2$ Cayley algebra, whose elements are integral Cayley numbers modulo doubles of integral Cayley numbers (see below). A pair $\{x,y\} \neq \{0,1\}$ of elements with $x+y = 1$ is either <u>isotropic</u> with $x^2=y^2=0$ (36 cases), <u>non-isotropic</u> with $x^3=y^3=1$ (28), or <u>mixed</u> with $x^2=0$, $y^2=1$, or vice versa (63).

G.2 : the fixed points of $O_8^+(2)$ under its triality automorphism

Cayley ($\mathbb{Z}$) G.2 : the automorphism group of the integral Cayley algebra in which $i_\infty=1$, $i_{n+1}=i$, $i_{n+2}=j$, $i_{n+4}=k$ define a quaternion subalgebra for each n (subscripts modulo 7); $\sum x_n i_n$ is <u>integral</u> provided that all $2x_n$ are integral and the set of n for which x_n is integral is one of

{0124}, {0235}, {0346}, {0156}, {∞013}, {∞026}, {∞045}, {∞0123456} or their complements.

Alternatively, they can be generated by the even sums of X_∞, X_0, ... X_6 together with $1 = \sum X_n$, with multiplication defined by the equations $2X_n^2 = X_n-1$, $2X_0X_\infty = 1-X_1-X_2-X_4$, $2X_\infty X_0 = 1-X_3-X_5-X_6$ and their images under the coordinate permutation group $L_2(7)$.

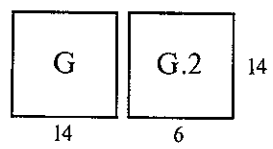
Quaternionic $2 \times G$: the group generated by the 63=3+12+48 quaternionic reflections in the vectors $(2,0,0)^S$ (3), $(0,a,a)^S$ (12), and $(\bar{a},1,1)^S$ (48) and their images under $\mathrm{diag}(\pm1,\pm i,\pm i)^S$, where $2a = -1-i-\sqrt{5}j+k$

Presentation $G.2 \cong \langle \overset{a}{\bullet}\!\!-\!\!\overset{b}{\bullet}\!\!-\!\!\overset{c}{\bullet}\overset{8}{}\overset{d}{\bullet}\!\!-\!\!\overset{e}{\bullet} \mid a=(cd)^4,(bcde)^8=1 \rangle$

Maximal subgroups / Specifications

Order	Index	Structure	G.2	Character	Abstract	Unitary	Cayley (2)	Cayley ($\mathbb{Z}$)
216	28	$3_+^{1+2}:8$	$: 3_+^{1+2}:8:2$	1a+27a	N(3A)	isotropic point	non-isotropic pair	$\langle x\rangle : x^3=1$
168	36	$L_2(7)$	$: L_2(7):2$	1a+7bc+21a	N(2A,3B,4C,7AB)	$\wedge_3^{b7}$	isotropic pair	$\{X_\infty,\ldots,X_6\}$
96	63	$4\cdot S_4$	$: \cancel{4\cdot S_4:2}\ 2_+^{1+4}.S_3$	1a+14a+21a+27a	N(2A)	non-isotropic point	isotropic space	
96	63	$4^2:S_3$	$: 4^2:D_{12}$	1a+7bc+21a+27a	N($2A^2$)	base	mixed pair	$\langle x\rangle : x^2=-1$

Chevalley	Quaternionic
${}^2A_2(2)$	
$A_2(2)$	$\wedge_3^{b7}$
vertex	root vector
edge	base



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@
		6048	96	108	9	96	96	16	12	7	7	8	8	12	12			24	24	3	4	6	6
	p power		A	A	A	A	A	A	AA	A	A	A	B	AB	AA			A	A	BB	C	AD	AD
	p' part		A	A	A	A	A	A	AA	A	A	A	A	AA	AB			A	A	BB	A	AD	AD
	ind	1A	2A	3A	3B	4A	B**	4C	6A	7A	B**	8A	B**	12A	B**	fus	ind	2B	4D	6B	8C	12C	D**
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1
χ_2	−	6	−2	−3	0	−2	−2	2	1	−1	−1	0	0	1	1	:	oo	0	0	0	0	i3	−i3
χ_3	+	7	−1	−2	1	3	3	−1	2	0	0	−1	−1	0	0	:	++	1	−3	1	−1	0	0
χ_4	o	7	3	−2	1	−2i−1	2i−1	1	0	0	0	i	−i	i−1	−i−1		+	0	0	0	0	0	0
χ_5	o	7	3	−2	1	2i−1	−2i−1	1	0	0	0	−i	i	−i−1	i−1								
χ_6	+	14	−2	5	−1	2	2	2	1	0	0	0	0	−1	−1	:	++	2	−2	−1	0	1	1
χ_7	+	21	5	3	0	1	1	1	−1	0	0	−1	−1	1	1	:	++	3	−1	0	1	−1	−1
χ_8	o	21	1	3	0	2i−3	−2i−3	−1	1	0	0	i	−i	−i	i		+	0	0	0	0	0	0
χ_9	o	21	1	3	0	−2i−3	2i−3	−1	1	0	0	−i	i	i	−i								
χ_{10}	+	27	3	0	0	3	3	−1	0	−1	−1	1	1	0	0	:	++	3	3	0	−1	0	0
χ_{11}	o	28	−4	1	1	4i	−4i	0	−1	0	0	0	0	i	−i		+	0	0	0	0	0	0
χ_{12}	o	28	−4	1	1	−4i	4i	0	−1	0	0	0	0	−i	i								
χ_{13}	o	32	0	−4	−1	0	0	0	0	−b7	**	0	0	0	0		+	0	0	0	0	0	0
χ_{14}	o	32	0	−4	−1	0	0	0	0	**	−b7	0	0	0	0								

$L_2(23)$

Linear group $L_2(23) \cong A_1(23) \cong U_2(23) \cong S_2(23) \cong O_3(23)$

Order = 6,072 = $2^3.3.11.23$ Mult = 2 Out = 2

Constructions

Linear $GL_2(23) \cong 11 \times 2.G.2$: all non-singular 2 x 2 matrices over $\mathbb{F}_{23}$;

$PGL_2(23) \cong G.2$; $SL_2(23) \cong 2.G$; $PSL_2(23) \cong G$

Unitary $GU_2(23) \cong 3 \times 2.(G \times 4).2$: all 2 x 2 matrices over $\mathbb{F}_{529}$ preserving a non-singular Hermitian form;

$PGU_2(23) \cong G.2$; $SU_2(23) \cong 2.G$; $PSU_2(23) \cong G$

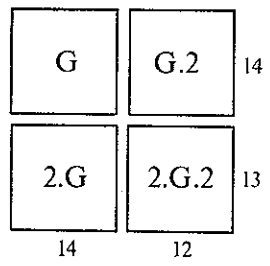
Orthogonal $GO_3(23) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbb{F}_{23}$ preserving a non-singular quadratic form;

$PGO_3(23) \cong SO_3(23) \cong PSO_3(23) \cong G.2$; $O_3(23) \cong G$

Presentations $G \cong (2,3,11;4) \cong \langle S,T \mid S^{23}=(S^4TS^{12}T)^2=1; (ST)^3=T^2[=1]\rangle$; $G.2 \cong G^{3,8,11}$

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications: Abstract	Linear	Orthogonal
253	24	23:11	: 23:22	1a+23a	N(23AB)	point	isotropic point
24	253	S_4		1a+22bcddee+24abcde	$N(2A^2)$		base
24	253	S_4			$N(2A^2)$		base
24	253	D_{24}	: D_{48}	1a+22bbddee+24abcde	N(2A), N(3A)	$O_2^-(23)$, $L_1(529)$	minus point
22	276	D_{22}	: D_{44}	1a+22bbddee+23a+24abcde	N(11ABCDE), C(2B)	$O_2^+(23)$, base	plus point



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@
		6072	24	12	12	12	11	11	11	11	11	12	12	23	23			22	12	12	11	11	11	11	11	12	12	12	12
	p power		A	A	A	AA	A	A	A	A	A	AA	AA	A	A			A	A	A	CB	DB	EB	AB	BB	BB	AB	BA	AA
	p' part		A	A	A	AA	A	A	A	A	A	AA	AA	A	A			A	A	A	AB	BB	CB	DB	EB	AA	AA	AB	AB
	ind	1A	2A	3A	4A	6A	11A	B*3	C*2	D*5	E*4	12A	B*	23A	B**	fus ind		2B	8A	B*	22A	B*3	C*9	D*5	E*7	24A	B*7	C*11	D*5
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	o	11	-1	-1	1	-1	0	0	0	0	0	1	1	b23	**		+	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	o	11	-1	-1	1	-1	0	0	0	0	0	1	1	**	b23														
χ_4	+	22	-2	1	-2	1	0	0	0	0	0	1	1	-1	-1	:	++	0	2	2	0	0	0	0	0	-1	-1	-1	-1
χ_5	+	22	2	-2	0	2	0	0	0	0	0	0	0	-1	-1	:	++	0	r2	-r2	0	0	0	0	0	r2	r2	-r2	-r2
χ_6	+	22	-2	1	2	1	0	0	0	0	0	-1	-1	-1	-1	:	++	0	0	0	0	0	0	0	0	r3	-r3	r3	-r3
χ_7	+	22	2	1	0	-1	0	0	0	0	0	r3	-r3	-1	-1	:	++	0	-r2	r2	0	0	0	0	0	y24	*7	*11	*5
χ_8	+	22	2	1	0	-1	0	0	0	0	0	-r3	r3	-1	-1	:	++	0	-r2	r2	0	0	0	0	0	*7	y24	*5	*11
χ_9	+	23	-1	-1	-1	-1	1	1	1	1	1	-1	-1	0	0	:	++	1	-1	-1	1	1	1	1	1	-1	-1	-1	-1
χ_{10}	+	24	0	0	0	0	y11	*3	*2	*5	*4	0	0	1	1	:	++	2	0	0	y11	*3	*2	*5	*4	0	0	0	0
χ_{11}	+	24	0	0	0	0	*4	y11	*3	*2	*5	0	0	1	1	:	++	2	0	0	*4	y11	*3	*2	*5	0	0	0	0
χ_{12}	+	24	0	0	0	0	*5	*4	y11	*3	*2	0	0	1	1	:	++	2	0	0	*5	*4	y11	*3	*2	0	0	0	0
χ_{13}	+	24	0	0	0	0	*2	*5	*4	y11	*3	0	0	1	1	:	++	2	0	0	*2	*5	*4	y11	*3	0	0	0	0
χ_{14}	+	24	0	0	0	0	*3	*2	*5	*4	y11	0	0	1	1	:	++	2	0	0	*3	*2	*5	*4	y11	0	0	0	0
	ind	1	4	3	8	12	11	11	11	11	11	24	24	23	23	fus ind		4	16	16	44	44	44	44	44	48	48	48	48
		2		6	8	12	22	22	22	22	22	24	24	46	46				16	16	44	44	44	44	44	48	48	48	48
χ_{15}	o	12	0	0	0	0	1	1	1	1	1	0	0	-b23	**		-	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	o	12	0	0	0	0	1	1	1	1	1	0	0	**	-b23														
χ_{17}	-	22	0	-2	r2	0	0	0	0	0	0	r2	r2	-1	-1	:	--	0	y16	*3	0	0	0	0	0	y16	y16	*3	*3
χ_{18}	-	22	0	-2	-r2	0	0	0	0	0	0	-r2	-r2	-1	-1	:	--	0	*5	y16	0	0	0	0	0	*5	*5	y16	y16
χ_{19}	-	22	0	1	-r2	-r3	0	0	0	0	0	y24	*7	-1	-1	:	--	0	*5	y16	0	0	0	0	0	*17	y48	*19	*13
χ_{20}	-	22	0	1	-r2	r3	0	0	0	0	0	*7	y24	-1	-1	:	--	0	*5	y16	0	0	0	0	0	y48	*17	*13	*19
χ_{21}	-	22	0	1	r2	-r3	0	0	0	0	0	-y24	*7	-1	-1	:	--	0	-y16	*3	0	0	0	0	0	*5	*11	*17	y48
χ_{22}	-	22	0	1	r2	r3	0	0	0	0	0	*7	-y24	-1	-1	:	--	0	-y16	*3	0	0	0	0	0	*11	*5	y48	*17
χ_{23}	-	24	0	0	0	0	y11	*3	*2	*5	*4	0	0	1	1	:	--	0	0	0	*7	y44	*19	*9	*5	0	0	0	0
χ_{24}	-	24	0	0	0	0	*4	y11	*3	*2	*5	0	0	1	1	:	--	0	0	0	*5	*7	y44	*19	*9	0	0	0	0
χ_{25}	-	24	0	0	0	0	*5	*4	y11	*3	*2	0	0	1	1	:	--	0	0	0	*9	*5	*7	y44	*19	0	0	0	0
χ_{26}	-	24	0	0	0	0	*2	*5	*4	y11	*3	0	0	1	1	:	--	0	0	0	*19	*9	*5	*7	y44	0	0	0	0
χ_{27}	-	24	0	0	0	0	*3	*2	*5	*4	y11	0	0	1	1	:	--	0	0	0	y44	*19	*9	*5	*7	0	0	0	0

$L_2(25)$

Linear group $L_2(25) \cong A_1(25) \cong U_2(25) \cong S_2(25) \cong O_3(25) \cong O_4^-(5)$

Order = 7,800 = $2^3.3.5^2.13$ Mult = 2 Out = 2^2

Constructions

Linear $GL_2(25) \cong 3 \times 2.(G \times 4).2_1$: all non-singular 2 x 2 matrices over $\mathbb{F}_{25}$;

$PGL_2(25) \cong G.2_1$; $SL_2(25) \cong 2.G$; $PSL_2(25) \cong G$

$\Gamma L_2(25) \cong (3 \times 2.(G \times 4).2_1).2_2$; $P\Gamma L_2(25) \cong G.2^2$; $\Sigma L_2(25) \cong 2.G.2_2$; $P\Sigma L_2(25) \cong G.2_2$

Unitary $GU_2(25) \cong 13 \times 2.G.2_1$: all 2 x 2 matrices over $\mathbb{F}_{625}$ preserving a non-singular Hermitian form;

$PGU_2(25) \cong G.2_1$; $SU_2(25) \cong 2.G$; $PSU_2(25) \cong G$

Orthogonal (25) $GO_3(25) \cong 2 \times G.2_1$: all 3 x 3 matrices over $\mathbb{F}_{25}$ preserving a non-singular quadratic form;

$PGO_3(25) \cong SO_3(25) \cong PSO_3(25) \cong G.2_1$; $O_3(25) \cong G$; $\Gamma O_3(25) \cong 2 \times G.2^2$; $P\Gamma O_3(25) \cong \Sigma O_3(25) \cong P\Sigma O_3(25) \cong G.2^2$

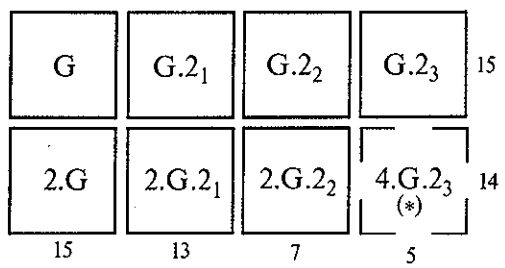
Orthogonal (5) $GO_4^-(5) \cong 2 \times G.2_2$: all 4 x 4 matrices over $\mathbb{F}_5$ preserving a quadratic form of Witt defect 1;

$PGO_4^-(5) \cong G.2_2$; $SO_4^-(5) \cong 2 \times G$; $PSO_4^-(5) \cong O_4^-(5) \cong G$;

Presentation $G \cong \langle a,b,c \mid a^5=b^{12}=c^2=(bc)^2=(ac)^3=(b^4abac)^3=1, b^{-2}ab^2=b^{-1}a^2ba=ab^{-1}a^2b\rangle$

Maximal subgroups

Order	Index	Structure	$G.2_1$	$G.2_2$	$G.2_3$	$G.2^2$	Character	Specifications: Abstract	Linear	Orthogonal (25)	Orthogonal (5)
300	26	$5^2:12$	: $5^2:24$	: $5^2:(4 \times S_3)$	: H.2	: $5^2:24:2$	1a+25a	$N(5A^2)$	point	isotropic point	isotropic point
120	65	S_5		: $S_5 \times 2$			1a+13a+25a+26a	C(2C)	$L_2(5)$	$O_3(5)$	non-isotropic point
120	65	S_5		: $S_5 \times 2$			1a+13b+25a+26a	C(2D)	$L_2(5)$	$O_3(5)$	non-isotropic point
26	300	D_{26}	: D_{52}	: 13:4	: 13:4	: 26:4	1a+13ab+24abcdef+25a+26aacc	N(13A–F), C(2B)	$O_2^-(25)$, $L_1(625)$	minus point	
24	325	D_{24}	: D_{48}	: $D_8 \times S_3$	: $Q_8 \times S_3$	: $H.2^2$	1a+13ab+24abcdef+25aa+26aacc	N(2A), N(3A)	$O_2^+(25)$, base	plus point	



(*) There is no group $2.G.2_3$.
See the Introduction: 'Isoclinism'

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	;	;	@	@	@	@	@	
		7800	24	12	12	25	25	12	12	12	13	13	13	13	13	13			26	12	12	12	12	12	12	13	13	13	13	13	13			120	120	4	6	6	5	5			6	4	4	6	6	
	p power		A	A	A	A	A	AA	AA	AA	A	A	A	A	A	A			A	A	A	BB	AA	BA	AB	FB	EB	BB	AB	DB	CB			A	A	A	AC	AD	AC	BD			A	A	A	AC	AC	
	p' part		A	A	A	A	A	AA	AA	AA	A	A	A	A	A	A			A	A	A	AA	AB	AB	AA	AB	BB	CB	DB	EB	FB			A	A	A	AC	AD	AC	BD			A	A	A	AC	AC	
	ind	1A	2A	3A	4A	5A	5B	6A	12A	B*	13A	B*5	C*4	D*6	E*3	F*2	fus	ind	2B	8A	B*	24A	B*5	C*11	D*7	26A	B*5	C*9	D*7	E*3	F*11	fus	ind	2C	2D	4B	6B	6C	10A	10B	fus	ind	4C	8C	D**	12C	D**	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	:	++	1	1	1	1	1	χ_1
χ_2	+	13	1	1	-1	3	-2	1	-1	-1	0	0	0	0	0	0	┌	+	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	1	5	-1	1	-1	1	0	┌	+	0	0	0	0	0	χ_2
χ_3	+	13	1	1	-1	-2	3	1	-1	-1	0	0	0	0	0	0	└															:	++	5	1	-1	-1	1	0	1	└							χ_3
χ_4	+	24	0	0	0	-1	-1	0	0	0	-y13	*5	*4	*6	*3	*2	:	++	2	0	0	0	0	0	0	y13	*5	*4	*6	*3	*2	┌	+	0	0	0	0	0	0	0	┌	+	0	0	0	0	0	χ_4
χ_5	+	24	0	0	0	-1	-1	0	0	0	*5	-y13	*6	*4	*2	*3	:	++	2	0	0	0	0	0	0	*5	y13	*6	*4	*2	*3	└									└							χ_5
χ_6	+	24	0	0	0	-1	-1	0	0	0	*3	*2	-y13	*5	*4	*6	:	++	2	0	0	0	0	0	0	*3	*2	y13	*5	*4	*6	┌	+	0	0	0	0	0	0	0	┌	+	0	0	0	0	0	χ_6
χ_7	+	24	0	0	0	-1	-1	0	0	0	*2	*3	*5	-y13	*6	*4	:	++	2	0	0	0	0	0	0	*2	*3	*5	y13	*6	*4	└									└							χ_7
χ_8	+	24	0	0	0	-1	-1	0	0	0	*4	*6	*3	*2	-y13	*5	:	++	2	0	0	0	0	0	0	*4	*6	*3	*2	y13	*5	┌	+	0	0	0	0	0	0	0	┌	+	0	0	0	0	0	χ_8
χ_9	+	24	0	0	0	-1	-1	0	0	0	*6	*4	*2	*3	*5	-y13	:	++	2	0	0	0	0	0	0	*6	*4	*2	*3	*5	y13	└									└							χ_9
χ_{10}	+	25	1	1	1	0	0	1	1	1	-1	-1	-1	-1	-1	-1	:	++	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	:	++	5	5	1	-1	-1	0	0	:	++	-1	1	1	-1	-1	χ_{10}
χ_{11}	+	26	2	-1	2	1	1	-1	-1	-1	0	0	0	0	0	0	:	++	0	2	2	-1	-1	-1	-1	0	0	0	0	0	0	:	++	4	4	0	1	1	-1	-1	:	++	-2	0	0	1	1	χ_{11}
χ_{12}	+	26	-2	2	0	1	1	-2	0	0	0	0	0	0	0	0	:	++	0	r2	-r2	r2	-r2	-r2	r2	0	0	0	0	0	0	:	++	6	-6	0	0	0	1	-1	:	oo	0	i2	-i2	0	0	χ_{12}
χ_{13}	+	26	2	-1	-2	1	1	-1	1	1	0	0	0	0	0	0	:	++	0	0	0	r3	-r3	r3	-r3	0	0	0	0	0	0	:	++	4	-4	0	1	-1	-1	1	:	oo	0	0	0	i3	-i3	χ_{13}
χ_{14}	+	26	-2	-1	0	1	1	1	r3	-r3	0	0	0	0	0	0	:	++	0	-r2	r2	y24	*5	*11	*7	0	0	0	0	0	0	┌	+	0	0	0	0	0	0	0	┌	+	0	0	0	0	0	χ_{14}
χ_{15}	+	26	-2	-1	0	1	1	1	-r3	r3	0	0	0	0	0	0	:	++	0	r2	-r2	*5	y24	*7	*11	0	0	0	0	0	0	└									└							χ_{15}
	ind	1 2	4	3 6	8 8	5 10	5 10	12 12	24 24	24 24	13 26	13 26	13 26	13 26	13 26	13 26	fus	ind	4	16 16	16 16	48 48	48 48	48 48	48 48	52 52	52 52	52 52	52 52	52 52	52 52	fus	ind	2	4	8	6	12	10 10	20 20	fus	ind	8	16	16	24	24	
χ_{16}	-	12	0	0	0	-3	2	0	0	0	-1	-1	-1	-1	-1	-1	┌	-	0	0	0	0	0	0	0	0	0	0	0	0	0	:	--	0	0	0	0	0	r5	0	┌	o2						χ_{16}
χ_{17}	-	12	0	0	0	2	-3	0	0	0	-1	-1	-1	-1	-1	-1	└															:	oo	0	0	0	0	0	0	i5	└*							χ_{17}
χ_{18}	-	24	0	0	0	-1	-1	0	0	0	-y13	*5	*4	*6	*3	*2	:	--	0	0	0	0	0	0	0	*9	*7	*23	*11	y52	*5	┌	-	0	0	0	0	0	0	0	┌	o2						χ_{18}
χ_{19}	-	24	0	0	0	-1	-1	0	0	0	*5	-y13	*6	*4	*2	*3	:	--	0	0	0	0	0	0	0	*19	*9	*15	*23	*21	y52	└									└*							χ_{19}
χ_{20}	-	24	0	0	0	-1	-1	0	0	0	*3	*2	-y13	*5	*4	*6	:	--	0	0	0	0	0	0	0	y52	*5	*9	*7	*23	*11	┌	-	0	0	0	0	0	0	0	┌	o2						χ_{20}
χ_{21}	-	24	0	0	0	-1	-1	0	0	0	*2	*3	*5	-y13	*6	*4	:	--	0	0	0	0	0	0	0	*21	y52	*19	*9	*15	*23	└									└*							χ_{21}
χ_{22}	-	24	0	0	0	-1	-1	0	0	0	*4	*6	*3	*2	-y13	*5	:	--	0	0	0	0	0	0	0	*23	*11	y52	*5	*9	*7	┌	-	0	0	0	0	0	0	0	┌	o2						χ_{22}
χ_{23}	-	24	0	0	0	-1	-1	0	0	0	*6	*4	*2	*3	*5	-y13	:	--	0	0	0	0	0	0	0	*15	*23	*21	y52	*19	*9	└									└*							χ_{23}
χ_{24}	-	26	0	2	r2	1	1	0	r2	r2	0	0	0	0	0	0	:	--	0	y16	*5	y16	*5	*5	y16	0	0	0	0	0	0	┌	-	0	0	0	0	0	0	0	┌	o2						χ_{24}
χ_{25}	-	26	0	2	-r2	1	1	0	-r2	-r2	0	0	0	0	0	0	:	--	0	*3	y16	*3	y16	y16	*3	0	0	0	0	0	0	└									└*							χ_{25}
χ_{26}	-	26	0	-1	r2	1	1	-r3	-y24	*7	0	0	0	0	0	0	:	--	0	y16	*5	*19	y48	*17	*13	0	0	0	0	0	0	┌	-	0	0	0	0	0	0	0	┌	o2						χ_{26}
χ_{27}	-	26	0	-1	-r2	1	1	r3	*7	y24	0	0	0	0	0	0	:	--	0	*3	-y16	y48	*5	*11	*17	0	0	0	0	0	0	└									└*							χ_{27}
χ_{28}	-	26	0	-1	-r2	1	1	-r3	y24	*7	0	0	0	0	0	0	:	--	0	*3	-y16	*17	*11	*5	y48	0	0	0	0	0	0	┌	-	0	0	0	0	0	0	0	┌	o2						χ_{28}
χ_{29}	-	26	0	-1	r2	1	1	r3	*7	-y24	0	0	0	0	0	0	:	--	0	y16	*5	*13	*17	y48	*19	0	0	0	0	0	0	└									└*							χ_{29}

M_{11} and $L_2(27)$

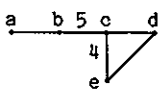
Sporadic Mathieu group M_{11}

Order = 7,920 = $2^4.3^2.5.11$ Mult = 1 Out = 1

Constructions

11-point $G \cong M_{11}$: stabilizer of a point in M_{12};
the automorphism group of the Steiner system S(4,5,11);
automorphisms of the ternary Golay code C_{12} that fix a coordinate

12-point $G \cong M_{11}$: stabilizer of a total in M_{12};
automorphisms of the ternary Golay code C_{12} that fix a weight 12 word (total word);
in particular, the coordinate permutations that fix the set of 22 words obtained from
±(-;++-+++---+-) by cyclic permutations of the last 11 coordinates

Presentations $G \cong \langle A,B,C \mid A^{11}=B^5=C^4=(AC)^3=1, A^B=A^4, B^C=B^2 \rangle \cong \langle$  $\mid (ce)^2=a,\ [(abc)^5=1] \rangle$

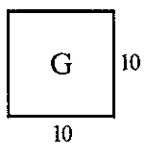
Maximal subgroups

Order	Index	Structure	Character
720	11	$M_{10} \cong A_6{\cdot}2$	1a+10a
660	12	$L_2(11)$	1a+11a
144	55	$M_9{:}2 \cong 3^2{:}Q_8.2$	1a+10a+44a
120	66	S_5	1a+10a+11a+44a
48	165	$M_8{:}S_3 \cong 2{:}S_4$	1a+10a+11a+44aa+55a

Specifications

Abstract	11-point	12-point
	point	total
	total	point
N($3A^2$)	duad	quadrisection
N(2A,3A,5A)	hexad	duad
N(2A)	triad	tetrad

	;	@	@	@	@	@	@	@	@	@	@
		7920	48	18	8	5	6	8	8	11	11
	p	power	A	A	A	A	AA	A	A	A	A
	p'	part	A	A	A	A	AA	A	A	A	A
	ind	1A	2A	3A	4A	5A	6A	8A	B**	11A	B**
χ_1	+	1	1	1	1	1	1	1	1	1	1
χ_2	+	10	2	1	2	0	-1	0	0	-1	-1
χ_3	o	10	-2	1	0	0	1	i2	-i2	-1	-1
χ_4	o	10	-2	1	0	0	1	-i2	i2	-1	-1
χ_5	+	11	3	2	-1	1	0	-1	-1	0	0
χ_6	o	16	0	-2	0	1	0	0	0	b11	**
χ_7	o	16	0	-2	0	1	0	0	0	**	b11
χ_8	+	44	4	-1	0	-1	1	0	0	0	0
χ_9	+	45	-3	0	1	0	0	-1	-1	1	1
χ_{10}	+	55	-1	1	-1	0	-1	1	1	0	0



Linear group $L_2(27) \cong A_1(27) \cong U_2(27) \cong S_2(27) \cong O_3(27)$

Order = 9,828 = $2^2.3^3.7.13$ Mult = 2 Out = 6

Constructions

Linear $GL_2(27) \cong 13 \times 2.G.2$: all non-singular 2 x 2 matrices over $\mathbb{F}_{27}$;
$PGL_2(27) \cong G.2$; $SL_2(27) \cong 2.G$; $PSL_2(27) \cong G$
$\Gamma L_2(27) \cong (13 \times 2.G.2).3$; $P\Gamma L_2(27) \cong G.6$; $\Sigma L_2(27) \cong 2.G.3$; $P\Sigma L_2(27) \cong G.3$

Unitary $GU_2(27) \cong 7 \times 2.(G \times 2).2$: all 2 x 2 matrices over $\mathbb{F}_{729}$ which preserve a non-singular Hermitian form;
$PGU_2(27) \cong G.2$; $SU_2(27) \cong 2.G$; $PSU_2(27) \cong G$

Orthogonal $GO_3(27) \cong 2 \times G.2$: all 3 x 3 matrices preserving a non-singular quadratic form over $\mathbb{F}_{27}$;
$PGO_3(27) \cong SO_3(27) \cong PSO_3(27) \cong G.2$; $O_3(27) \cong G$; $\Gamma O_3(27) \cong 2 \times G.6$; $P\Gamma O_3(27) \cong \Sigma O_3(27) \cong P\Sigma O_3(27) \cong G.6$

Presentation $G \cong \langle a,b,c \mid a^3=b^{13}=c^2=(bc)^2=(ac)^3=1, b^{-3}ab^3=ab^{-1}ab=b^{-1}aba \rangle$

Maximal subgroups

Order	Index	Structure	G.2	G.3	G.6	Character
351	28	$3^3{:}13$	: $3^3{:}26$	: $3^3{:}13{:}3$	: $3^3{:}26{:}3$	1a+27a
28	351	D_{28}	: D_{56}	: 14:6	: 28:6	
26	378	D_{26}	: D_{52}	: 13:6	: 26:6	
12	819	A_4	: S_4	: $A_4 \times 3$	: $S_4 \times 3$	

Specifications

Abstract	Linear	Orthogonal
N($3AB^3$)	point	isotropic point
N(2A), N(7ABC)	$O_2^-(27)$, $L_1(729)$	minus point
N(13A-F), C(2B)	$O_2^+(27)$, base	plus point
N($2A^2$), C(3C)	$L_2(3)$	$O_3(3)$, base

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	;	;	@	@	
		9828	28	27	27	14	14	14	13	13	13	13	13	13	14	14	14			26	14	13	13	13	13	13	13	14	14	14	14	14	14			12	4	3	3			2	2	
	p power		A	A	A	A	A	A	A	A	A	A	A	A	CA	AA	BA			A	A	FB	DB	BB	CB	AB	CA	AA	BA	CA	AA	BA	CA			A	CA	A	B			CB	AA	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	CA			A	A	BB	CB	DB	EB	FB	AA	BA	CA	AA	BA	CA			A	CA	A	A			CB	CA	
	ind	1A	2A	3A	B**	7A	B*3	C*2	13A	B*3	C*4	D*5	E*2	F*6	14A	B*3	C*5	fus	ind	2B	4A	26A	B*3	C*9	D*5	E*11	F*7	28A	B*3	C*9	D*13	E*11	F*5	fus	ind	3C	6A	9A	9B	fus	ind	6B	12A	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	+oo	1	1	1	1	:+oo+oo		1	1	x_1
x_2	o	13	1	b27	**	-1	-1	-1	0	0	0	0	0	0	1	1	1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	ooo	1	1	z3	z3**		+oo	0	0	x_2
x_3	o	13	1	**	b27	-1	-1	-1	0	0	0	0	0	0	1	1	1																	:	ooo	1	1	z3**	z3					x_3
x_4	+	26	-2	-1	-1	-y7	*3	*2	0	0	0	0	0	0	-y7	*3	*2	:	++	0	2	0	0	0	0	0	0	y7	*3	*2	y7	*3	*2		+	0	0	0	0		++	0	0	x_4
x_5	+	26	-2	-1	-1	*2	-y7	*3	0	0	0	0	0	0	*2	-y7	*3	:	++	0	2	0	0	0	0	0	0	*2	y7	*3	*2	y7	*3											x_5
x_6	+	26	-2	-1	-1	*3	*2	-y7	0	0	0	0	0	0	*3	*2	-y7	:	++	0	2	0	0	0	0	0	0	*3	*2	y7	*3	*2	y7											x_6
x_7	+	26	2	-1	-1	-y7	*3	*2	0	0	0	0	0	0	y7	*3	*2	:	++	0	0	0	0	0	0	0	0	*3	*9	y28	*11	*5	*13		+	0	0	0	0		++	0	0	x_7
x_8	+	26	2	-1	-1	*2	-y7	*3	0	0	0	0	0	0	*2	y7	*3	:	++	0	0	0	0	0	0	0	0	y28	*3	*9	*13	*11	*5											x_8
x_9	+	26	2	-1	-1	*3	*2	-y7	0	0	0	0	0	0	*3	*2	y7	:	++	0	0	0	0	0	0	0	0	*9	y28	*3	*5	*13	*11											x_9
x_{10}	+	27	-1	0	0	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	:	++	1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	:	+oo	3	-1	0	0	:+oo+oo		1	-1	x_{10}
x_{11}	+	28	0	1	1	0	0	0	y13	*3	*4	*5	*2	*6	0	0	0	:	++	2	0	y13	*3	*4	*5	*2	*6	0	0	0	0	0	0		+	0	0	0	0		++	0	0	x_{11}
x_{12}	+	28	0	1	1	0	0	0	*4	y13	*3	*6	*5	*2	0	0	0	:	++	2	0	*4	y13	*3	*6	*5	*2	0	0	0	0	0	0											x_{12}
x_{13}	+	28	0	1	1	0	0	0	*3	*4	y13	*2	*6	*5	0	0	0	:	++	2	0	*3	*4	y13	*2	*6	*5	0	0	0	0	0	0											x_{13}
x_{14}	+	28	0	1	1	0	0	0	*5	*2	*6	y13	*3	*4	0	0	0	:	++	2	0	*5	*2	*6	y13	*3	*4	0	0	0	0	0	0		+	0	0	0	0		++	0	0	x_{14}
x_{15}	+	28	0	1	1	0	0	0	*6	*5	*2	*4	y13	*3	0	0	0	:	++	2	0	*6	*5	*2	*4	y13	*3	0	0	0	0	0	0											x_{15}
x_{16}	+	28	0	1	1	0	0	0	*2	*6	*5	*3	*4	y13	0	0	0	:	++	2	0	*2	*6	*5	*3	*4	y13	0	0	0	0	0	0											x_{16}
	ind	1 2	4	3 6	3 6	7 14	7 14	7 14	13 26	13 26	13 26	13 26	13 26	13 26	28 28	28 28	28 28	fus	ind	4	8 8	52 52	52 52	52 52	52 52	52 52	52 52	56 56	56 56	56 56	56 56	56 56	56 56	fus	ind	3 6	12	9 18	9 18	fus	ind	12	24 24	
x_{17}	o	14	0	-b27	**	0	0	0	1	1	1	1	1	1	0	0	0		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	ooo	2	0	-z3	-z3**		-oo	0	0	x_{17}
x_{18}	o	14	0	**	-b27	0	0	0	1	1	1	1	1	1	0	0	0																	:	ooo	2	0	-z3**	-z3					x_{18}
x_{19}	-	26	0	-1	-1	-2	-2	-2	0	0	0	0	0	0	0	0	0	:	--	0	r2	0	0	0	0	0	0	r2	r2	r2	r2	r2	r2	:	-oo	2	0	-1	-1	:-oo-oo		0	r2	x_{19}
x_{20}	-	26	0	-1	-1	-y7	*3	*2	0	0	0	0	0	0	*3	*9	-y28	:	--	0	r2	0	0	0	0	0	0	y56	*25	*9	*15	*17	*23		-	0	0	0	0		--	0	0	x_{20}
x_{21}	-	26	0	-1	-1	*2	-y7	*3	0	0	0	0	0	0	-y28	*3	*9	:	--	0	r2	0	0	0	0	0	0	*9	y56	*25	*23	*15	*17											x_{21}
x_{22}	-	26	0	-1	-1	*3	*2	-y7	0	0	0	0	0	0	*9	-y28	*3	:	--	0	r2	0	0	0	0	0	0	*25	*9	y56	*17	*23	*15											x_{22}
x_{23}	-	26	0	-1	-1	-y7	*3	*2	0	0	0	0	0	0	*3	*9	y28	:	--	0	r2	0	0	0	0	0	0	*15	*17	*23	y56	*25	*9		-	0	0	0	0		--	0	0	x_{23}
x_{24}	-	26	0	-1	-1	*2	-y7	*3	0	0	0	0	0	0	y28	*3	*9	:	--	0	r2	0	0	0	0	0	0	*23	*15	*17	*9	y56	*25											x_{24}
x_{25}	-	26	0	-1	-1	*3	*2	-y7	0	0	0	0	0	0	*9	y28	*3	:	--	0	r2	0	0	0	0	0	0	*17	*23	*15	*25	*9	y56											x_{25}
x_{26}	-	28	0	1	1	0	0	0	y13	*3	*4	*5	*2	*6	0	0	0	:	--	0	0	*9	y52	*23	*7	*5	*11	0	0	0	0	0	0		-	0	0	0	0		--	0	0	x_{26}
x_{27}	-	28	0	1	1	0	0	0	*4	y13	*3	*6	*5	*2	0	0	0	:	--	0	0	*23	*9	y52	*11	*7	*5	0	0	0	0	0	0											x_{27}
x_{28}	-	28	0	1	1	0	0	0	*3	*4	y13	*2	*6	*5	0	0	0	:	--	0	0	y52	*23	*9	*5	*11	*7	0	0	0	0	0	0											x_{28}
x_{29}	-	28	0	1	1	0	0	0	*5	*2	*6	y13	*3	*4	0	0	0	:	--	0	0	*19	*21	*15	*9	y52	*23	0	0	0	0	0	0		-	0	0	0	0		--	0	0	x_{29}
x_{30}	-	28	0	1	1	0	0	0	*6	*5	*2	*4	y13	*3	0	0	0	:	--	0	0	*15	*19	*21	*23	*9	y52	0	0	0	0	0	0											x_{30}
x_{31}	-	28	0	1	1	0	0	0	*2	*6	*5	*3	*4	y13	0	0	0	:	--	0	0	*21	*15	*19	y52	*23	*9	0	0	0	0	0	0											x_{31}

G	G.2.	G.3	G.6
2.G	2.G.2	2.G.3	2.G.6

16 14 4 2

16 15

$L_2(27)$

$L_2(29)$

Linear group $L_2(29) \cong A_1(29) \cong U_2(29) \cong S_2(29) \cong O_3(29)$

Order = 12,180 = $2^2.3.5.7.29$ Mult = 2 Out = 2

Constructions

Linear	$GL_2(29) \cong 7 \times 2.(G \times 2).2$: all non-singular 2 x 2 matrices over $\mathbf{F}_{29}$;
	$PGL_2(29) \cong G.2$; $SL_2(29) \cong 2.G$; $PSL_2(29) \cong G$
Unitary	$GU_2(29) \cong 15 \times 2.G.2$: all 2 x 2 matrices over $\mathbf{F}_{841}$ preserving a non-singular Hermitian form;
	$PGU_2(29) \cong G.2$; $SU_2(29) \cong 2.G$; $PSU_2(29) \cong G$
Orthogonal	$GO_3(29) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbf{F}_{29}$ preserving a non-singular quadratic form;
	$PGO_3(29) \cong SO_3(29) \cong PSO_3(29) \cong G.2$; $O_3(29) \cong G$
Presentations	$G \cong G^{3,7,15} \cong \langle P,Q \mid P^7=(P^2Q)^3=(P^3Q)^2=(PQ^8)^2=1\rangle \cong \langle S,T \mid S^{29}=(S^4TS^{15}T)^2=1,(ST)^3=T^2[=1]\rangle$

G	G.2	17
2.G	2.G.2	16
17	15	

Maximal subgroups / Specifications

Order	Index	Structure	G.2	Character	Abstract	Linear	Orthogonal
406	30	29:14	: 29:28	1a+29a	N(29AB)	point	isotropic point
60	203	A_5	S_4	1a+28defg+30abc	N(2A,3A,5AB)		icosahedral
60	203	A_5		1a+28defg+30abc	N(2A,3A,5AB)		icosahedral
30	406	D_{30}	: D_{60}	1a+28abcdefg+29a+30aabbcc	N(3A), N(5AB), C(2B)	$O_2^-(29)$, $L_1(841)$	minus point
28	435	D_{28}	: D_{56}	1a+28abcdefg+29aa+30aabbcc	N(2A), N(7ABC)	$O_2^+(29)$, base	plus point

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		12180	28	15	15	15	14	14	14	14	14	14	15	15	15	15	29	29			30	14	15	15	15	14	14	14	14	14	14	15	15	15	15	
	p	power	A	A	A	A	A	A	A	BA	CA	AA	BA	AA	BA	AA	A	A			A	A	AB	BB	AB	BA	CA	AA	BA	CA	AA	BBA	CAA	DBA	AAA	
	p'	part	A	A	A	A	A	A	A	AA	BA	CA	AA	BA	AA	BA	A	A			A	A	AB	AB	BB	AA	BA	CA	AA	BA	CA	AAA	BBA	CAA	DBA	
	ind		1A	2A	3A	5A	B*	7A	B*2	C*3	14A	B*5	C*3	15A	B*2	C*4	D*7	29A	B*	fus	ind	2B	4A	6A	10A	B*	28A	B*9	C*3	D*13	E*5	F*11	30AB	*13	C*11	D*7	
χ_1		+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2		+	15	-1	0	0	0	1	1	1	-1	-1	-1	0	0	0	0	-b29	*		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_2
χ_3		+	15	-1	0	0	0	1	1	1	-1	-1	-1	0	0	0	0	*	-b29																		χ_3
χ_4		+	28	0	1	-2	-2	0	0	0	0	0	0	1	1	1	1	-1	-1	:	++	2	0	-1	2	2	0	0	0	0	0	0	-1	-1	-1	-1	χ_4
χ_5		+	28	0	-2	-b5	*	0	0	0	0	0	0	-b5	*	-b5	*	-1	-1	:	++	2	0	2	b5	*	0	0	0	0	0	0	b5	*	b5	*	χ_5
χ_6		+	28	0	-2	*	-b5	0	0	0	0	0	0	*	-b5	*	-b5	-1	-1	:	++	2	0	2	*	b5	0	0	0	0	0	0	*	b5	*	b5	χ_6
χ_7		+	28	0	1	-b5	*	0	0	0	0	0	0	*7	-y15	*2	*4	-1	-1	:	++	2	0	-1	b5	*	0	0	0	0	0	0	*7	y15	*2	*4	χ_7
χ_8		+	28	0	1	*	-b5	0	0	0	0	0	0	*4	*7	-y15	*2	-1	-1	:	++	2	0	-1	*	b5	0	0	0	0	0	0	*4	*7	y15	*2	χ_8
χ_9		+	28	0	1	-b5	*	0	0	0	0	0	0	*2	*4	*7	-y15	-1	-1	:	++	2	0	-1	b5	*	0	0	0	0	0	0	*2	*4	*7	y15	χ_9
χ_{10}		+	28	0	1	*	-b5	0	0	0	0	0	0	-y15	*2	*4	*7	-1	-1	:	++	2	0	-1	*	b5	0	0	0	0	0	0	y15	*2	*4	*7	χ_{10}
χ_{11}		+	29	1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	0	0	:	++	1	-1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	χ_{11}
χ_{12}		+	30	2	0	0	0	y7	*2	*3	y7	*2	*3	0	0	0	0	1	1	:	++	0	2	0	0	0	y7	*2	*3	y7	*2	*3	0	0	0	0	χ_{12}
χ_{13}		+	30	2	0	0	0	*3	y7	*2	*3	y7	*2	0	0	0	0	1	1	:	++	0	2	0	0	0	*3	y7	*2	*3	y7	*2	0	0	0	0	χ_{13}
χ_{14}		+	30	2	0	0	0	*2	*3	y7	*2	*3	y7	0	0	0	0	1	1	:	++	0	2	0	0	0	*2	*3	y7	*2	*3	y7	0	0	0	0	χ_{14}
χ_{15}		+	30	-2	0	0	0	y7	*2	*3	-y7	*2	*3	0	0	0	0	1	1	:	++	0	0	0	0	0	*3	y28	*9	*11	*13	*5	0	0	0	0	χ_{15}
χ_{16}		+	30	-2	0	0	0	*3	y7	*2	*3	-y7	*2	0	0	0	0	1	1	:	++	0	0	0	0	0	*9	*3	y28	*5	*11	*13	0	0	0	0	χ_{16}
χ_{17}		+	30	-2	0	0	0	*2	*3	y7	*2	*3	-y7	0	0	0	0	1	1	:	++	0	0	0	0	0	y28	*9	*3	*13	*5	*11	0	0	0	0	χ_{17}
		ind	1	4	3	5	5	7	7	7	28	28	28	15	15	15	15	29	29	fus	ind	4	8	12	20	20	56	56	56	56	56	56	60	60	60	60	
			2		6	10	10	14	14	14	28	28	28	30	30	30	30	58	58				8	12	20	20	56	56	56	56	56	56	60	60	60	60	
χ_{18}		-	14	0	-1	-1	-1	0	0	0	0	0	0	-1	-1	-1	-1	b29	*		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{18}
χ_{19}		-	14	0	-1	-1	-1	0	0	0	0	0	0	-1	-1	-1	-1	*	b29																		χ_{19}
χ_{20}		-	28	0	1	-2	-2	0	0	0	0	0	0	1	1	1	1	-1	-1	:	--	0	0	r3	0	0	0	0	0	0	0	0	r3	r3	r3	r3	χ_{20}
χ_{21}		-	28	0	-2	-b5	*	0	0	0	0	0	0	-b5	*	-b5	*	-1	-1	:	--	0	0	0	y20	*7	0	0	0	0	0	0	y20	*7	*9	*3	χ_{21}
χ_{22}		-	28	0	-2	*	-b5	0	0	0	0	0	0	*	-b5	*	-b5	-1	-1	:	--	0	0	0	*3	y20	0	0	0	0	0	0	*3	y20	*7	*9	χ_{22}
χ_{23}		-	28	0	1	-b5	*	0	0	0	0	0	0	*7	-y15	*2	*4	-1	-1	:	--	0	0	r3	y20	*7	0	0	0	0	0	0	*13	*11	*23	-y60	χ_{23}
χ_{24}		-	28	0	1	*	-b5	0	0	0	0	0	0	*4	*7	-y15	*2	-1	-1	:	--	0	0	r3	*3	y20	0	0	0	0	0	0	-y60	*13	*11	*23	χ_{24}
χ_{25}		-	28	0	1	-b5	*	0	0	0	0	0	0	*2	*4	*7	-y15	-1	-1	:	--	0	0	r3	-y20	*7	0	0	0	0	0	0	*23	-y60	*13	*11	χ_{25}
χ_{26}		-	28	0	1	*	-b5	0	0	0	0	0	0	-y15	*2	*4	*7	-1	-1	:	--	0	0	r3	*3	-y20	0	0	0	0	0	0	*11	*23	-y60	*13	χ_{26}
χ_{27}		-	30	0	0	0	0	2	2	2	0	0	0	0	0	0	0	1	1	:	--	0	r2	0	0	0	r2	r2	r2	r2	r2	r2	0	0	0	0	χ_{27}
χ_{28}		-	30	0	0	0	0	y7	*2	*3	*3	y28	*9	0	0	0	0	1	1	:	--	0	r2	0	0	0	y56	*9	*25	*15	*23	*17	0	0	0	0	χ_{28}
χ_{29}		-	30	0	0	0	0	*3	y7	*2	*9	*3	y28	0	0	0	0	1	1	:	--	0	r2	0	0	0	*25	y56	*9	*17	*15	*23	0	0	0	0	χ_{29}
χ_{30}		-	30	0	0	0	0	*2	*3	y7	y28	*9	*3	0	0	0	0	1	1	:	--	0	r2	0	0	0	*9	*25	y56	*23	*17	*15	0	0	0	0	χ_{30}
χ_{31}		-	30	0	0	0	0	y7	*2	*3	*3	-y28	*9	0	0	0	0	1	1	:	--	0	r2	0	0	0	*15	*23	*17	y56	*9	*25	0	0	0	0	χ_{31}
χ_{32}		-	30	0	0	0	0	*3	y7	*2	*9	*3	-y28	0	0	0	0	1	1	:	--	0	r2	0	0	0	*17	*15	*23	*25	y56	*9	0	0	0	0	χ_{32}
χ_{33}		-	30	0	0	0	0	*2	*3	y7	-y28	*9	*3	0	0	0	0	1	1	:	--	0	r2	0	0	0	*23	*17	*15	*9	*25	y56	0	0	0	0	χ_{33}

$L_2(31)$

Linear group $L_2(31) \cong A_1(31) \cong U_2(31) \cong S_2(31) \cong O_3(31)$

Order = 14,880 = $2^5.3.5.31$ Mult = 2 Out = 2

Constructions

Linear $GL_2(31) \cong 15 \times 2.G.2$: all non-singular 2 x 2 matrices over $\mathbf{F}_{31}$;

$PGL_2(31) \cong G.2$; $SL_2(31) \cong 2.G$; $PSL_2(31) \cong G$

Unitary $GU_2(31) \cong 2.(G \times 16).2$: all 2 x 2 matrices over $\mathbf{F}_{961}$ preserving a non-singular Hermitian form;

$PGU_2(31) \cong G.2$; $SU_2(31) \cong 2.G$; $PSU_2(31) \cong G$

Orthogonal $GO_3(31) \cong 2 \times G.2$: all 3 x 3 matrices over $\mathbf{F}_{31}$ preserving a non-singular quadratic form;

$PGO_3(31) \cong SO_3(31) \cong PSO_3(31) \cong G.2$; $O_3(11) \cong G$

Presentation: $G \cong \langle S,T \mid S^{31}=(S^4TS^{16}T)^2=1,(ST)^3=T^2[=1]\rangle$

G	G.2	18	
2.G	2.G.2	17	
18	16		

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications: Abstract	Linear	Orthogonal
465	32	31:15	: 31:30	1a+31a	N(31AB)	point	isotropic point
60	248	A_5		1a+31a+30defg+32abc	N(2A,3A,5AB)		icosahedral
60	248	A_5		1a+31a+30defg+32abc	N(2A,3A,5AB)		icosahedral
32	465	D_{32}	: D_{64}	1a+30ddeeffgg+32abcdefg	N(2A)	$O_2^-(961)$, $L_1(31)$	minus point
30	496	D_{30}	: D_{60}	1a+30ddeeffgg+31a+32abcdefg	N(3A), N(5AB), C(2B)	$O_2^+(31)$, base	plus point

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		14880	32	15	16	15	15	16	16	15	15	15	15	16	16	16	16	31	31			30	15	15	15	15	15	15	15	16	16	16	16	16	16	16	16	
	p	power	A	A	A	A	A	A	A	BA	AA	BA	AA	A	B	A	B	A	A			A	AB	BB	AB	BBA	CAA	DBA	AAA	A	B	C	D	A	B	C	D	
	p'	part	A	A	A	A	A	A	A	AA	BA	AA	BA	A	A	A	A	A	A			A	AB	AB	BB	AAA	BBA	CAA	DBA	A	A	A	A	A	A	A	A	
	ind		1A	2A	3A	4A	5A	B*	8A	B*	15A	B*2	C*4	D*7	16A	B*3	C*7	D*5	31A	B**	fus ind	2B	6A	10A	B*	30AB	*13C	*11	D*7	32A	B*3	C*9	D*5E	*15F	*13	G*7H	*11	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	: ++		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	o	15	-1	0	-1	0	0	-1	-1	0	0	0	0	1	1	1	1	b31	**	+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_2
χ_3	o	15	-1	0	-1	0	0	-1	-1	0	0	0	0	1	1	1	1	**	b31																			χ_3
χ_4	+	30	-2	0	-2	0	0	2	2	0	0	0	0	0	0	0	0	-1	-1	: ++		0	0	0	0	0	0	0	0	r2	-r2	r2	-r2	r2	-r2	r2	-r2	χ_4
χ_5	+	30	-2	0	2	0	0	0	0	0	0	0	0	-r2	r2	-r2	r2	-1	-1	: ++		0	0	0	0	0	0	0	0	y16	*3	*7	*5	y16	*3	*7	*5	χ_5
χ_6	+	30	-2	0	2	0	0	0	0	0	0	0	0	r2	-r2	r2	-r2	-1	-1	: ++		0	0	0	0	0	0	0	0	*5	y16	*3	*7	*5	y16	*3	*7	χ_6
χ_7	+	30	2	0	0	0	0	-r2	r2	0	0	0	0	-y16	*3	*7	*5	-1	-1	: ++		0	0	0	0	0	0	0	0	y32	*3	*9	*5	*15	*13	*7	*11	χ_7
χ_8	+	30	2	0	0	0	0	r2	-r2	0	0	0	0	*5	-y16	*3	*7	-1	-1	: ++		0	0	0	0	0	0	0	0	*11	y32	*3	*9	*5	*15	*13	*7	χ_8
χ_9	+	30	2	0	0	0	0	-r2	r2	0	0	0	0	*7	*5	-y16	*3	-1	-1	: ++		0	0	0	0	0	0	0	0	*7	*11	y32	*3	*9	*5	*15	*13	χ_9
χ_{10}	+	30	2	0	0	0	0	r2	-r2	0	0	0	0	*3	*7	*5	-y16	-1	-1	: ++		0	0	0	0	0	0	0	0	*13	*7	*11	y32	*3	*9	*5	*15	χ_{10}
χ_{11}	+	31	-1	1	-1	1	1	-1	-1	1	1	1	1	-1	-1	-1	-1	0	0	: ++		1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	χ_{11}
χ_{12}	+	32	0	-1	0	2	2	0	0	-1	-1	-1	-1	0	0	0	0	1	1	: ++		2	-1	2	2	-1	-1	-1	-1	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	+	32	0	2	0	b5	*	0	0	b5	*	b5	*	0	0	0	0	1	1	: ++		2	2	b5	*	b5	*	b5	*	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}	+	32	0	2	0	*	b5	0	0	*	b5	*	b5	0	0	0	0	1	1	: ++		2	2	*	b5	*	b5	*	b5	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	+	32	0	-1	0	b5	*	0	0	*7	y15	*2	*4	0	0	0	0	1	1	: ++		2	-1	b5	*	*7	y15	*2	*4	0	0	0	0	0	0	0	0	χ_{15}
χ_{16}	+	32	0	-1	0	*	b5	0	0	*4	*7	y15	*2	0	0	0	0	1	1	: ++		2	-1	*	b5	*4	*7	y15	*2	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	+	32	0	-1	0	b5	*	0	0	*2	*4	*7	y15	0	0	0	0	1	1	: ++		2	-1	b5	*	*2	*4	*7	y15	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}	+	32	0	-1	0	*	b5	0	0	y15	*2	*4	*7	0	0	0	0	1	1	: ++		2	-1	*	b5	y15	*2	*4	*7	0	0	0	0	0	0	0	0	χ_{18}
	ind	1	4	3	8	5	5	16	16	15	15	15	15	32	32	32	32	31	31	fus ind		4	12	20	20	60	60	60	60	64	64	64	64	64	64	64	64	
		2		6	8	10	10	16	16	30	30	30	30	32	32	32	32	62	62				12	20	20	60	60	60	60	64	64	64	64	64	64	64	64	
χ_{19}	o	16	0	1	0	1	1	0	0	1	1	1	1	0	0	0	0	-b31	**	-		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{19}
χ_{20}	o	16	0	1	0	1	1	0	0	1	1	1	1	0	0	0	0	**	-b31																			χ_{20}
χ_{21}	-	30	0	0	r2	0	0	y16	*3	0	0	0	0	y32	*3	*9	*5	-1	-1	: --		0	0	0	0	0	0	0	0	y64	*3	*9	*27	*17	*13	*25	*11	χ_{21}
χ_{22}	-	30	0	0	-r2	0	0	*5	y16	0	0	0	0	*11	y32	*3	*9	-1	-1	: --		0	0	0	0	0	0	0	0	*21	y64	*3	*9	*27	*17	*13	*25	χ_{22}
χ_{23}	-	30	0	0	r2	0	0	-y16	*3	0	0	0	0	*7	*11	y32	*3	-1	-1	: --		0	0	0	0	0	0	0	0	*7	*21	y64	*3	*9	*27	*17	*13	χ_{23}
χ_{24}	-	30	0	0	-r2	0	0	*5	-y16	0	0	0	0	*13	*7	*11	y32	-1	-1	: --		0	0	0	0	0	0	0	0	*19	*7	*21	y64	*3	*9	*27	*17	χ_{24}
χ_{25}	-	30	0	0	r2	0	0	y16	*3	0	0	0	0	-y32	*3	*9	*5	-1	-1	: --		0	0	0	0	0	0	0	0	*15	*19	*7	*21	y64	*3	*9	*27	χ_{25}
χ_{26}	-	30	0	0	-r2	0	0	*5	y16	0	0	0	0	*11	-y32	*3	*9	-1	-1	: --		0	0	0	0	0	0	0	0	*5	*15	*19	*7	*21	y64	*3	*9	χ_{26}
χ_{27}	-	30	0	0	r2	0	0	-y16	*3	0	0	0	0	*7	*11	-y32	*3	-1	-1	: --		0	0	0	0	0	0	0	0	*23	*5	*15	*19	*7	*21	y64	*3	χ_{27}
χ_{28}	-	30	0	0	-r2	0	0	*5	-y16	0	0	0	0	*13	*7	*11	-y32	-1	-1	: --		0	0	0	0	0	0	0	0	*29	*23	*5	*15	*19	*7	*21	y64	χ_{28}
χ_{29}	-	32	0	-1	0	2	2	0	0	-1	-1	-1	-1	0	0	0	0	1	1	: --		0	r3	0	0	r3	r3	r3	r3	0	0	0	0	0	0	0	0	χ_{29}
χ_{30}	-	32	0	2	0	b5	*	0	0	b5	*	b5	*	0	0	0	0	1	1	: --		0	0	y20	*7	y20	*7	*9	*3	0	0	0	0	0	0	0	0	χ_{30}
χ_{31}	-	32	0	2	0	*	b5	0	0	*	b5	*	b5	0	0	0	0	1	1	: --		0	0	*3	y20	*3	y20	*7	*9	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}	-	32	0	-1	0	b5	*	0	0	*7	y15	*2	*4	0	0	0	0	1	1	: --		0	r3	y20	*7	*13	*11	*23	-y60	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}	-	32	0	-1	0	*	b5	0	0	*4	*7	y15	*2	0	0	0	0	1	1	: --		0	r3	*3	y20	-y60	*13	*11	*23	0	0	0	0	0	0	0	0	χ_{33}
χ_{34}	-	32	0	-1	0	b5	*	0	0	*2	*4	*7	y15	0	0	0	0	1	1	: --		0	r3	-y20	*7	*23	-y60	*13	*11	0	0	0	0	0	0	0	0	χ_{34}
χ_{35}	-	32	0	-1	0	*	b5	0	0	y15	*2	*4	*7	0	0	0	0	1	1	: --		0	r3	*3	-y20	*11	*23	-y60	*13	0	0	0	0	0	0	0	0	χ_{35}

$A_8 \cong L_4(2)$

Alternating group A_8; Linear group $L_4(2) \cong A_3(2) \cong O_6^+(2)$

Order = 20,160 = $2^6.3^2.5.7$ Mult = 2 Out = 2

Constructions

Alternating $S_8 \cong G.2$: all permutations on 8 letters;

$A_8 \cong G$: all even permutations; 2.G and 2.G.2 : the Schur double covers

Linear $GL_4(2) \cong PGL_4(2) \cong SL_4(2) \cong PSL_4(2) \cong G$: all non-singular 4 x 4 matrices over $\mathbf{F}_2$;

the points P_i and planes Q_i of the corresponding projective 3-space can be numbered (mod 15) so that

$Q_i = \{P_i, P_{i+5}, P_{i+10}, P_{i+1}, P_{i+2}, P_{i+4}, P_{i+8}\}$;

G.2 is obtained by adjoining the duality (graph) automorphism, interchanging points and planes

Orthogonal $GO_6^+(2) \cong PGO_6^+(2) \cong SO_6^+(2) \cong PSO_6^+(2) \cong G.2$: all 6 x 6 matrices over $\mathbf{F}_2$ fixing a quadratic form of Witt defect 0, for example $x_1x_2 + x_3x_4 + x_5x_6$; $O_6^+(2) \cong G$;

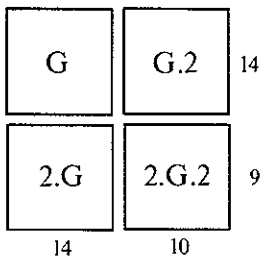
by taking the 6-space to be the even weight vectors in $\mathbf{F}_2^8$ modulo (11111111), with form equal to half the weight, we see the isomorphism with S_8

Mathieu G : the stabilizer in M_{24} of an octad and a point outside it; the group acts as A_8 on the octad, and $L_4(2)$ on its complement

Presentations G.2 ≅ ——————————— ; $G \cong \langle x_1,\ldots,x_6 \mid x_i^3=(x_ix_j)^2=1\rangle$

Maximal subgroups / Specifications

Order	Index	Structure	G.2	Character	Abstract	Alternating	Linear	Orthogonal
2520	8	A_7	: S_7	1a+7a		point		
1344	15	$2^3:L_3(2)$	$2^4:S_4$,	1a+14a	$N(2A^3)$	S(3,4,8)	point	isotropic plane
1344	15	$2^3:L_3(2)$	$L_3(2):2$	1a+14a	$N(2A^3)$	S(3,4,8)	plane	isotropic plane
720	28	S_6	: S_6 x 2	1a+7a+20a	C(2C)	duad	$S_4(2)$	non-isotropic point
576	35	$2^4:(S_3xS_3)$	: $(S_4xS_4):2$	1a+14a+20a	$N(2^4) = N(2A_9B_6)$	bisection	line	isotropic point
360	56	$(A_5x3):2$	: S_5 x S_3	1a+7a+20a+28a	N(3A), N(2B,3A,5A)	triad	$L_2(4)$	$O_3(4)$



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@
		20160	192	96	180	18	16	8	15	12	6	7	7	15	15			720	48	48	16	18	18	6	4	5	6
		p power	A	A	A	A	A	B	A	AB	BA	A	A	AA	AA			A	A	B	B	AC	BC	BD	A	AC	AC
		p' part	A	A	A	A	A	A	A	AB	BA	A	A	AA	AA			A	A	A	A	AC	BC	BD	A	AC	AC
		ind	1A	2A	2B	3A	3B	4A	4B	5A	6A	6B	7A	B**	15A	B**	fus ind	2C	2D	4C	4D	6C	6D	6E	8A	10A	12A
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	: ++	1	1	1	1	1	1	1	1	1	1
χ_2	+	7	-1	3	4	1	-1	1	2	0	-1	0	0	-1	-1		: ++	5	1	3	-1	2	-1	1	-1	0	0
χ_3	+	14	6	2	-1	2	2	0	-1	-1	0	0	0	-1	-1		: ++	4	0	-2	2	1	-2	0	0	-1	1
χ_4	+	20	4	4	5	-1	0	0	0	1	1	-1	-1	0	0		: ++	10	2	2	2	1	1	-1	0	0	-1
χ_5	+	21	-3	1	6	0	1	-1	1	-2	0	0	0	1	1		: ++	9	-3	3	-1	0	0	0	1	-1	0
χ_6	o	21	-3	1	-3	0	1	-1	1	1	0	0	0	b15	**		+	0	0	0	0	0	0	0	0	0	0
χ_7	o	21	-3	1	-3	0	1	-1	1	1	0	0	0	**	b15												
χ_8	+	28	-4	4	1	1	0	0	-2	1	-1	0	0	1	1		: ++	10	2	-2	-2	1	1	-1	0	0	1
χ_9	+	35	3	-5	5	2	-1	-1	0	1	0	0	0	0	0		: ++	5	-3	1	1	-1	2	0	-1	0	1
χ_{10}	o	45	-3	-3	0	0	1	1	0	0	0	b7	**	0	0		+	0	0	0	0	0	0	0	0	0	0
χ_{11}	o	45	-3	-3	0	0	1	1	0	0	0	**	b7	0	0												
χ_{12}	+	56	8	0	-4	-1	0	0	1	0	-1	0	0	1	1		: ++	4	4	0	0	-2	1	1	0	-1	0
χ_{13}	+	64	0	0	4	-2	0	0	-1	0	0	1	1	-1	-1		: ++	16	0	0	0	-2	-2	0	0	1	0
χ_{14}	+	70	-2	2	-5	1	-2	0	0	-1	1	0	0	0	0		: ++	10	-2	-4	0	1	1	1	0	0	-1
	ind	1	2	4	3	3	4	8	5	12	6	7	7	15	15	fus ind		2	4	8	8	6	6	12	8	10	24
		2			6	6			10		6	14	14	30	30										8	10	24
χ_{15}	+	8	0	0	-4	2	0	0	-2	0	0	1	1	1	1		: oo	0	0	0	0	0	0	0	2i	0	0
χ_{16}	o	24	0	0	-6	0	0	0	-1	0	0	b7	**	-1	-1		-	0	0	0	0	0	0	0	0	0	0
χ_{17}	o	24	0	0	-6	0	0	0	-1	0	0	**	b7	-1	-1												
χ_{18}	-	48	0	0	6	0	0	0	-2	0	0	-1	-1	1	1		: --	0	0	0	0	0	0	0	0	0	r6
χ_{19}	o	56	0	0	-4	-1	0	0	1	0	i3	0	0	1	1		-	0	0	0	0	0	0	0	0	0	0
χ_{20}	o	56	0	0	-4	-1	0	0	1	0	-i3	0	0	1	1												
χ_{21}	o	56	0	0	2	2	0	0	1	0	0	0	0	b15	**		-	0	0	0	0	0	0	0	0	0	0
χ_{22}	o	56	0	0	2	2	0	0	1	0	0	0	0	**	b15												
χ_{23}	-	64	0	0	4	-2	0	0	-1	0	0	1	1	-1	-1		: --	0	0	0	0	0	0	0	0	r5	0

$L_3(4)$

Linear group $L_3(4) \cong A_2(4)$

Order = 20,160 = $2^6.3^2.5.7$ **Mult** = 4 x 4 x 3 **Out** = 2 x S_3

Constructions

Linear $GL_3(4) \cong 3.G.3$: all non-singular 3 x 3 matrices over $\mathbb{F}_4$;

$PGL_3(4) \cong G.3$; $SL_3(4) \cong 3.G$; $PSL_3(4) \cong G$;

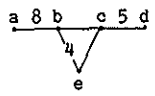
$G.2_3$ etc. are obtained by adjoining the duality (graph) automorphism;

$\Gamma L_3(4) \cong 3.G.3.2_2$; $P\Gamma L_3(4) \cong G.3.2_2$; $\Sigma L_3(4) \cong 3.G.2_2$; $P\Sigma L_3(4) \cong G.2_2$

Mathieu $M_{21}.S_3 \cong G.3.2_2$: the set-stabilizer of a triad in M_{24}; $M_{21} \cong G$: the pointwise stabilizer;

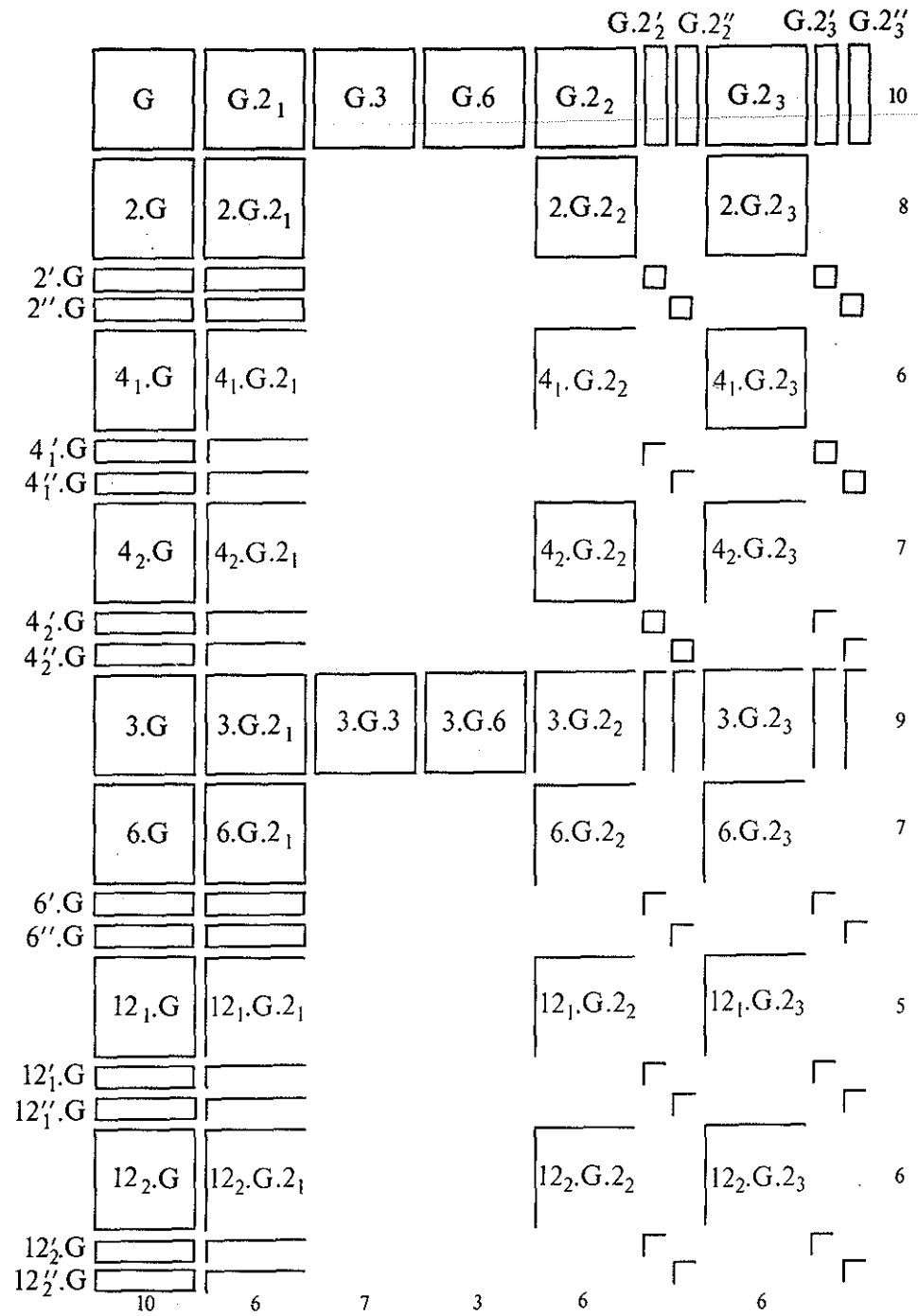
the remaining 21 points form the projective plane whose lines are the 21 pentads which complete the triad to an octad

Lattice $6.G.2_1$ stabilizes the complex lattice Λ_6^{z3}, whose full automorphism group is $6U_4(3).2$ (see $U_4(3)$)

Presentation $12_2.G.2_3 \cong \langle$  $\mid (be)^2=(cde)^5, d=(ab)^4, 1=[e,(cbaba)^5] \rangle$

The multiplier M is generated by elements a,b,c,d satisfying $abc = a^4 = b^4 = c^4 = d^3 = 1$. The table shows how these are transformed by elements in the printed cosets and represented by characters in the printed cohorts. There is an automorphism acting as (abc)(d) on M, and as (XX'X") on cosets and cohorts.

coset						cohort							
1	2_1	3	6	2_2	2_3	1	2	4_1	4_2	3	6	12_1	12_2
a	a^3	b	b^3	a	a^3	1	1	1	-1	1	1	1	-1
b	b^3	c	c^3	c	c^3	1	-1	i	i	1	-1	i	i
c	c^3	a	a^3	b	b^3	1	-1	-i	i	1	-1	-i	i
d	d	d	d	d^2	d^2	1	1	1	1	z3	z3	z3	z3



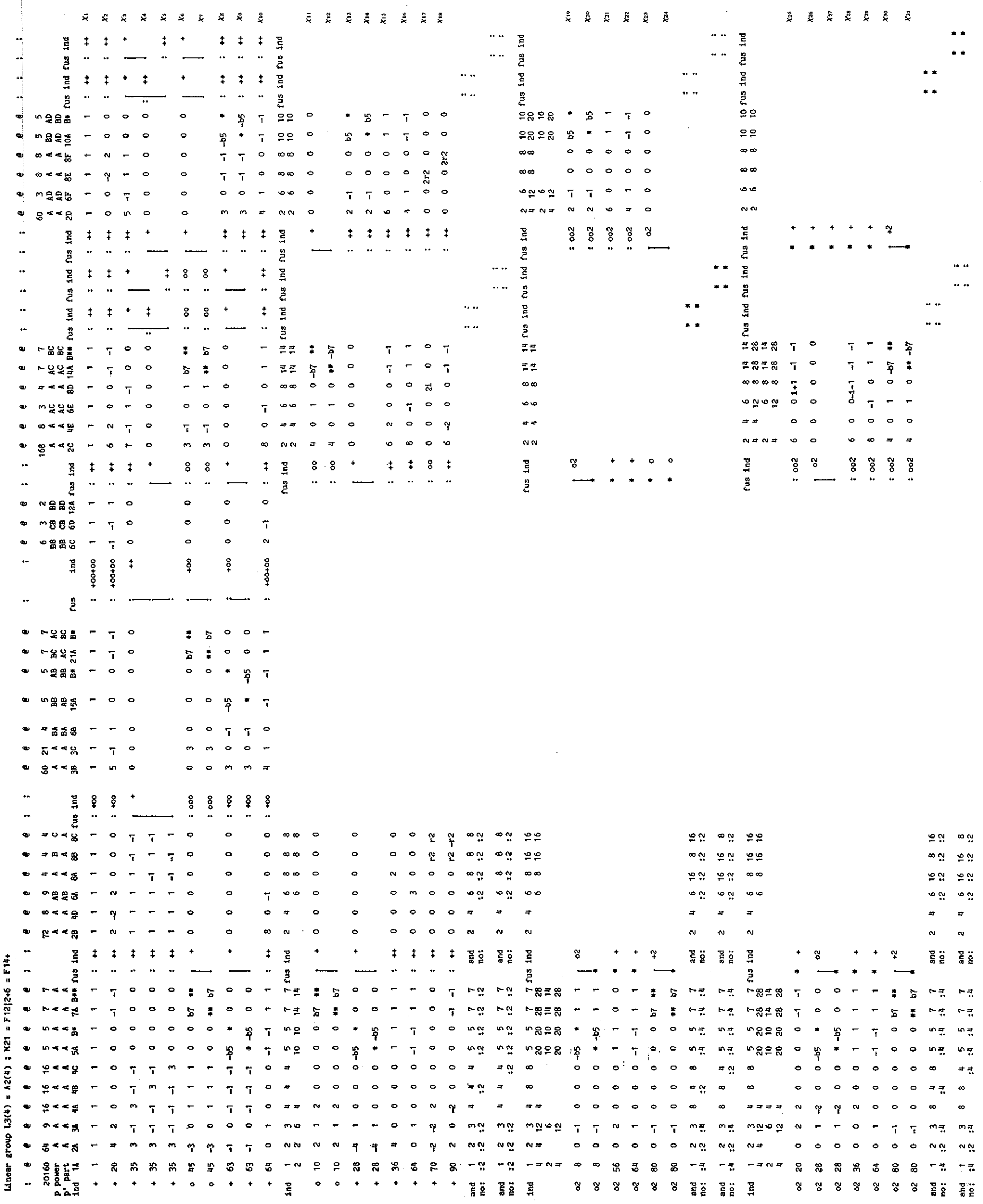
Maximal subgroups

Order	Index	Structure	$G.2_1$	$G.3$	$G.6$	$G.2_2$	$G.3.2_2$	$G.2_3$	$G.3.2_3$	$G.2^2$	$G.D_{12}$
960	21	$2^4:A_5$	$2^{2+4}.3.2$,	: $2^4:(3xA_5)$	$2^{2+4}.3.2$,	: 2^4S_5	: $2^4(3xA_5)2$	$2^{2+4}.3.2$,	$2^{2+4}.3^2.2$,	$2^{2+4}.3.2^2$,	$2^{2+4}.3^2.2^2$,
960	21	$2^4:A_5$	S_5	: $2^4:(3xA_5)$	$3xS_5$	: 2^4S_5	: $2^4(3xA_5)2$	$2 \times A_5$	$S_3 \times A_5$	$2 \times S_5$	$S_3 \times S_5$
360	56	A_6	: M_{10}			: S_6		: $PGL_2(9)$		: $A_6 \cdot 2^2$	
360	56	A_6	: M_{10}								
360	56	A_6	: M_{10}								
168	120	$L_2(7)$	: $L_2(7):2$	7:3 x 3	7:6 x 3	: $L_2(7) \times 2$	7:3 x S_3	: $L_2(7):2$	(7:3x3):2	: $L_2(7):2 \times 2$	7:6 x S_3
168	120	$L_2(7)$	: $L_2(7):2$								
168	120	$L_2(7)$	: $L_2(7):2$								
72	280	$3^2:Q_8$	: $3^2:Q_8 \times 2$	: $3^2:2A_4$	: $3^2:2A_4 \times 2$	: $3^2:Q_8.2$	: $3^2:2S_4$	: $3^2:Q_8.2$	: $3^2:2S_4$	: $3^2:Q_8.2 \times 2$	: $3^2:2S_4 \times 2$

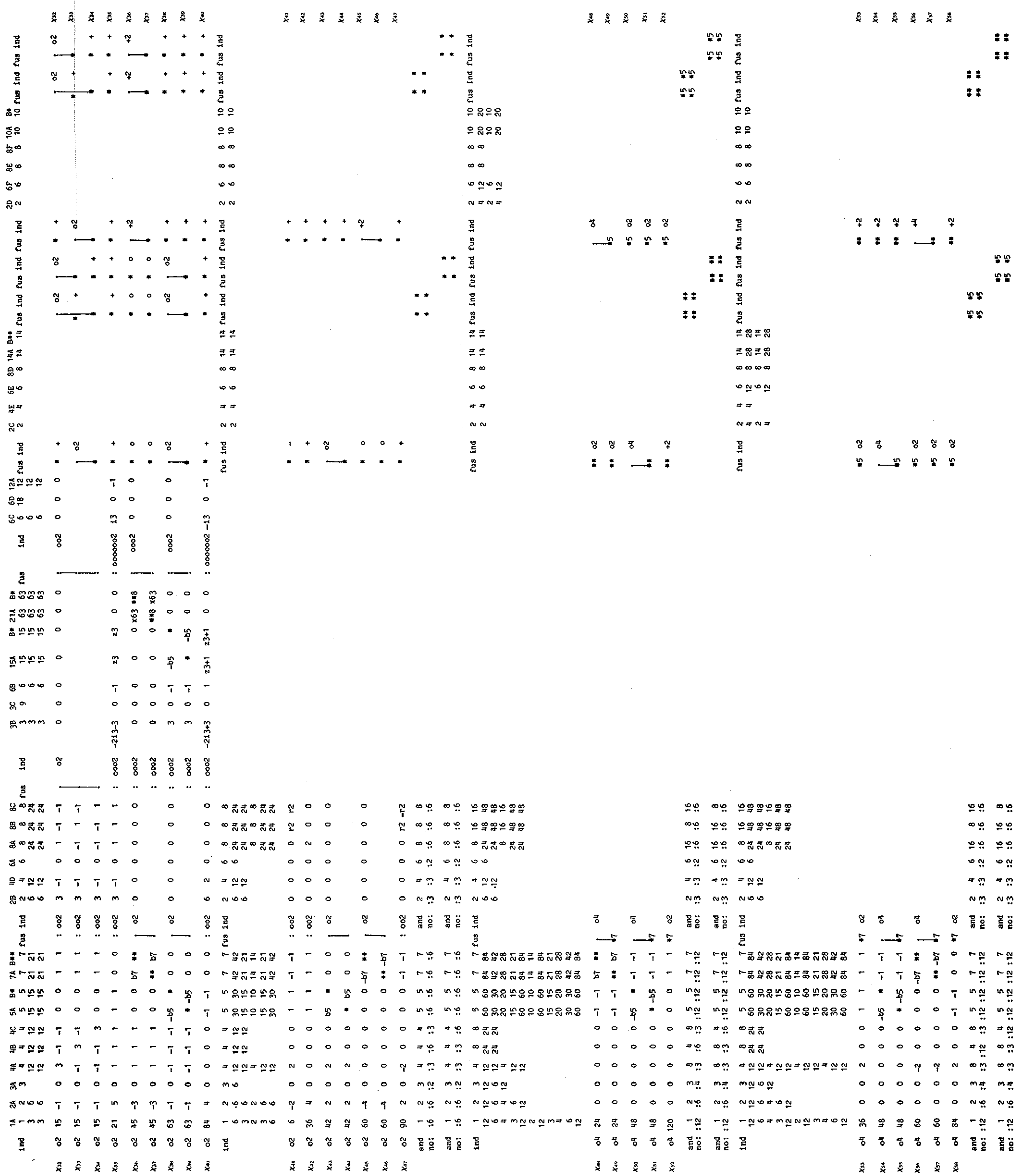
Specifications

Character	Abstract	Linear	Mathieu
1a+20a	$N(2A^4)$	point	point
1a+20a	$N(2A^4)$	line	pentad
1a+20a+35a	N(2A,3A,3A,4A,5AB)	oval	hexad
1a+20a+35b	N(2A,3A,3A,4B,5AB)	oval	hexad
1a+20a+35c	N(2A,3A,3A,4C,5AB)	oval	hexad
1a+20a+35a+64a	N(2A,3A,4A,7AB)	$L_3(2)$	heptad
1a+20a+35b+64a	N(2A,3A,4B,7AB)	$L_3(2)$	heptad
1a+20a+35c+64a	N(2A,3A,4C,7AB)	$L_3(2)$	heptad
1a+20a+35abc+45ab+64a	$N(3A^2)$		ennead

$L_3(4)$



$L_3(4)$



$U_4(2) \cong S_4(3)$

Unitary group $U_4(2) \cong {}^2A_3(2) \cong O_6^-(2)$; Symplectic group $S_4(3) \cong C_2(3) \cong O_5(3)$

Order = 25,920 = $2^6.3^4.5$ Mult = 2 Out = 2

Constructions

Unitary $GU_4(2) \cong 3 \times G$: all 4 x 4 matrices over $\mathbb{F}_4$ preserving a non-singular Hermitian form;

$PGU_4(2) \cong SU_4(2) \cong PSU_4(2) \cong G$

Orthogonal (2) $GO_6^-(2) \cong PGO_6^-(2) \cong SO_6^-(2) \cong PSO_6^-(2) \cong G.2$: all 6 x 6 matrices over $\mathbb{F}_2$ preserving a non-singular quadratic form of Witt defect 1, for example $x_1^2+x_1x_2+x_2^2+x_3x_4+x_5x_6$; $O_6^-(2) \cong G$

Orthogonal (3) $GO_5(3) \cong 2 \times G.2$: all 5 x 5 matrices over $\mathbb{F}_3$ preserving a non-singular quadratic form;

$PGO_5(3) \cong SO_5(3) \cong PSO_5(3) \cong G.2$; $O_5(3) \cong G$

G : the automorphism group of a generalized 4-gon of order (3,3) consisting of 40 vertices and 40 edges, each object being incident with 4 of the other type

Symplectic $Sp_4(3) \cong 2.G$: all 4 x 4 matrices over $\mathbb{F}_3$ preserving a non-singular symplectic form;

$PSp_4(3) \cong S_4(3) \cong G$;

Weyl G.2 : the Weyl group of E_6, generated by the 36 reflections in the minimal (root) vectors of the E_6 lattice; these may be taken as

$(0,0,0,0,0,0;1,-1)$ (1), $(1,-1,0,0,0,0;0,0)$ (15), $(\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2};\frac{1}{2},-\frac{1}{2})$ (20) (first base)

or $(\pm1,\pm1,0,0,0;0,0,0)$ (20), $\pm(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2};\frac{1}{2},\frac{1}{2},\frac{1}{2})$ (even sum) (16) (second base)

where the coordinates before the semicolon may be freely permuted.

The coordinates are obtained from the E_8 root lattice (see $O_8^+(2)$) by imposing the conditions

$x_1+x_2+x_3+x_4+x_5+x_6=0=x_7+x_8$ (first base) or $x_6=x_7=x_8$ (second base).

The Weyl group permutes the 27 vectors (which belong to one coset of the lattice in its dual E_6^*)

$a_i = (\frac{5}{6},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6};\frac{1}{2},-\frac{1}{2})$ (6), $b_i = (\frac{5}{6},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6},-\frac{1}{6};-\frac{1}{2},\frac{1}{2})$ (6), $c_{ij} = (-\frac{2}{3},-\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3};0,0)$ (15);

the full automorphism group of the lattice is 2 x G.2 and is obtained by adjoining negation.

Reducing modulo 2 shows the isomorphism with $O_6^-(2)$.

Biflection 2 x G : the group generated by the 45 complex reflections of order 2 in the vectors

$(\pm2,0,0,0,0)^C$ (5), $(0,\pm w,\pm\bar{w},\pm\bar{w},\pm w)^C$ (40), where w = z3.

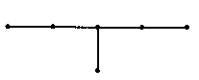
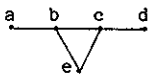
Reducing mod θ = i3 shows the isomorphism with $O_5(3)$.

6 x G : the full automorphism group of the lattice these generate.

Triflection 3 x 2.G : the group generated by the 40 complex reflections of order 3 (triflections) in the ($\pm$w)vectors $(\theta;0,0,0)$ $(0;\theta,0,0)$ $(0;w^a,w^b,w^c)$ $(w^a;0,w^b,-w^c)$, where the last 3 coordinates may be cyclically permuted; the triflection in r takes v to $v - (1-w)(v.r)r/(r.r)$, where $x.y = \Sigma x_i\bar{y}_i$. Reducing mod 2 shows the isomorphism with $U_4(2)$; reducing mod θ shows the isomorphism with $S_4(3)$.

Schläfli G.2 : the group of the incidence-preserving permutations of the 27 lines on the general cubic surface in projective 3-space : a_i (6), b_i (6), c_{ij} (15) (i,j = 1, ... 6, i≠j). These lie in threes in 45 tritangent planes (a_i,b_j,c_{ij}) (30), (c_{ij},c_{kl},c_{mn}) (15) (i,j,k,l,m,n, distinct), and two lines intersect if and only if they lie in some tritangent plane. There are 36 "double sixes" such as $\{a_i|b_j\}$, in which each line of either "six" meets all but one line of the other "six".

Hesse G.2 : the stabilizer of a bitangent in Hesse's group (see $S_6(2)$) of symmetries of the 28 bitangents to the general quartic curve in the complex projective plane; the bitangents may be labelled with unordered pairs from {1,2,3,4,5,6,a,b} and the 27 bitangents other than ab then correspond naturally with Schläfli's 27 lines: ai $\longleftrightarrow a_i$, bj $\longleftrightarrow b_j$, ij $\longleftrightarrow c_{ij}$. Generating permutations may easily be obtained from Hesse's bifid maps (see $S_6(2)$) e.g. (a123|b456) and (ab12|3456)(a 1)(b 2). The 45 quartets {ab,bj,ji,ia} (30) and {ab,ij,kl,mn} (15) containing ab yield the 45 tritangent planes.

Presentations G.2 ≅  ; G x 2 ≅ < 

| $(bcbe)^3=1$ >

Maximal subgroups

Order	Index	Structure	G.2	Character
960	27	$2^4:A_5$	$: 2^4:S_5$	1a+6a+20a
720	36	S_6	$: S_6 \times 2$	1a+15b+20a
648	40	$3_+^{1+2}:2A_4$	$: 3_+^{1+2}:2S_4$	1a+15a+24a
648	40	$3^3:S_4$	$: 3^3:(S_4 \times 2)$	1a+15b+24a
576	45	$2^{\cdot}(A_4 \times A_4).2$	: H.2	1a+20a+24a

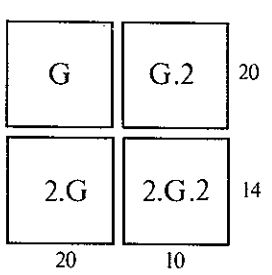
Specifications

Abstract	Unitary	Orthogonal (2)
$N(2^4) = N(2A_5B_{10})$	isotropic line	isotropic point
N(2B,3C,3D,4B,5A), C(2C)	$S_4(2)$	non-isotropic point
N(3AB)	non-isotropic point	$U_3(2)$
$N(3^3) = N(3AB_4C_3D_6)$	base	$O_2^-(2) wr S_3$
N(2A)	isotropic point	isotropic line

Orthogonal (3)	Symplectic	Weyl	Biflection	Triflection	Schläfli
base		minimal vector of E_6^*			line
minus point		root vector of E_6			double six
isotropic line	point			reflection	
isotropic point	isotropic line				trisection
plus point	$S_2(3)wr2$		reflection		tritangent plane

$U_4(2) \cong S_4(3)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	
		25920	576	96	648	648	108	54	48	8	5	72	72	36	36	18	12	9	9	12	12				720	48	48	16	18	18	6	4	5	6	
	p	power	A	A	A	A	A	A	A	B	A	BA	AA	CA	CA	DA	CB	A	B	BA	AA				A	A	B	B	CC	DC	DD	A	AC	FC	
	p'	part	A	A	A	A	A	A	A	A	A	AA	BA	CA	CA	DA	CB	A	A	AA	BA				A	A	A	A	CC	DC	DD	A	AC	CC	
	ind	1A	2A	2B	3A	B**	3C	3D	4A	4B	5A	6A	B**	6C	D**	6E	6F	9A	B**	12A	B**	fus	ind		2C	2D	4C	4D	6G	6H	6I	8A	10A	12C	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++		1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	o	5	-3	1	**	-b27	-1	2	1	-1	0	**	z3-1	i3	-i3	0	1	-z3	**	z3	**		+		0	0	0	0	0	0	0	0	0	0	χ_2
χ_3	o	5	-3	1	-b27	**	-1	2	1	-1	0	z3-1	**	-i3	i3	0	1	**	-z3	**	z3														χ_3
χ_4	+	6	-2	2	-3	-3	3	0	2	0	1	1	1	1	1	-2	-1	0	0	-1	-1	:	++		4	0	-2	2	1	-2	0	0	-1	1	χ_4
χ_5	o	10	2	-2	3z3-2	**	1	1	2	0	0	**	-b27	-1	-1	-1	1	**	z3	-z3	**		+		0	0	0	0	0	0	0	0	0	0	χ_5
χ_6	o	10	2	-2	**	3z3-2	1	1	2	0	0	-b27	**	-1	-1	-1	1	z3	**	**	-z3														χ_6
χ_7	+	15	-1	-1	6	6	3	0	3	-1	0	2	2	-1	-1	2	-1	0	0	0	0	:	++		5	-3	1	1	-1	2	0	-1	0	1	χ_7
χ_8	+	15	7	3	-3	-3	0	3	-1	1	0	1	1	-2	-2	1	0	0	0	-1	-1	:	++		5	1	3	-1	2	-1	1	-1	0	0	χ_8
χ_9	+	20	4	4	2	2	5	-1	0	0	0	-2	-2	1	1	1	1	-1	-1	0	0	:	++		10	2	2	2	1	1	-1	0	0	-1	χ_9
χ_{10}	+	24	8	0	6	6	0	3	0	0	-1	2	2	2	2	-1	0	0	0	0	0	:	++		4	4	0	0	-2	1	1	0	-1	0	χ_{10}
χ_{11}	+	30	-10	2	3	3	3	3	-2	0	0	-1	-1	-1	-1	-1	-1	0	0	1	1	:	++		10	-2	-4	0	1	1	1	0	0	-1	χ_{11}
χ_{12}	o	30	6	2	3b27	**	-3	0	2	0	0	z3-1	**	-i3	i3	0	-1	0	0	**	-z3		+		0	0	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	o	30	6	2	**	3b27	-3	0	2	0	0	**	z3-1	i3	-i3	0	-1	0	0	-z3	**														χ_{13}
χ_{14}	o	40	-8	0	-3i3-5	3i3-5	-2	1	0	0	0	-2z3	**	-2z3	**	1	0	**	z3	0	0		+		0	0	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	o	40	-8	0	3i3-5	-3i3-5	-2	1	0	0	0	**	-2z3	**	-2z3	1	0	z3	**	0	0														χ_{15}
χ_{16}	o	45	-3	-3	-9z3	**	0	0	1	1	0	3z3	**	0	0	0	0	0	0	z3	**		+		0	0	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	o	45	-3	-3	**	-9z3	0	0	1	1	0	**	3z3	0	0	0	0	0	0	**	z3														χ_{17}
χ_{18}	+	60	-4	4	6	6	-3	-3	0	0	0	2	2	-1	-1	-1	1	0	0	0	0	:	++		10	2	-2	-2	1	1	-1	0	0	1	χ_{18}
χ_{19}	+	64	0	0	-8	-8	4	-2	0	0	-1	0	0	0	0	0	0	1	1	0	0	:	++		16	0	0	0	-2	-2	0	0	1	0	χ_{19}
χ_{20}	+	81	9	-3	0	0	0	0	-3	-1	1	0	0	0	0	0	0	0	0	0	0	:	++		9	-3	3	-1	0	0	0	1	-1	0	χ_{20}
	ind	1	2	4	3	3	3	3	4	8	5	6	6	6	6	6	12	9	9	12	12	fus	ind		4	2	8	8	12	12	6	8	20	24	
		2			6	6	6	6	4		10	6	6			6		18	18	12	12						8						8	20	24	
χ_{21}	o	4	0	0	b27	**	-2	1	2	0	-1	z3-1	**	0	0	i3	0	**	z3	**	-z3		+		0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}	o	4	0	0	**	b27	-2	1	2	0	-1	**	z3-1	0	0	-i3	0	z3	**	-z3	**														χ_{22}
χ_{23}	-	20	0	0	-7	-7	2	2	2	0	0	3	3	0	0	0	0	-1	-1	-1	-1	:	oo		0	0	2i2	0	0	0	0	i2	0	-i2	χ_{23}
χ_{24}	o	20	0	0	3z3+8	**	2	2	2	0	0	-3z3	**	0	0	0	0	-z3	**	-z3	**		+		0	0	0	0	0	0	0	0	0	0	χ_{24}
χ_{25}	o	20	0	0	**	3z3+8	2	2	2	0	0	**	-3z3	0	0	0	0	**	-z3	**	-z3														χ_{25}
χ_{26}	o	20	0	0	3i3+2	-3i3+2	-4	-1	2	0	0	-i3	i3	0	0	i3	0	**	-z3	-1	-1		+		0	0	0	0	0	0	0	0	0	0	χ_{26}
χ_{27}	o	20	0	0	-3i3+2	3i3+2	-4	-1	2	0	0	i3	-i3	0	0	-i3	0	-z3	**	-1	-1														χ_{27}
χ_{28}	o	36	0	0	9z3	**	0	0	2	0	1	3z3	**	0	0	0	0	0	0	-z3	**		+		0	0	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	o	36	0	0	**	9z3	0	0	2	0	1	**	3z3	0	0	0	0	0	0	**	-z3														χ_{29}
χ_{30}	-	60	0	0	-3	-3	-6	0	-2	0	0	3	3	0	0	0	0	0	0	1	1	:	oo		0	0	2i2	0	0	0	0	-i2	0	-i2	χ_{30}
χ_{31}	o	60	0	0	**	-3b27	0	3	-2	0	0	**	z3-1	0	0	-i3	0	0	0	z3	**		+		0	0	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}	o	60	0	0	-3b27	**	0	3	-2	0	0	z3-1	**	0	0	i3	0	0	0	**	z3														χ_{32}
χ_{33}	-	64	0	0	-8	-8	4	-2	0	0	-1	0	0	0	0	0	0	1	1	0	0	:	oo		0	0	0	0	0	0	0	0	i5	0	χ_{33}
χ_{34}	-	80	0	0	8	8	2	-4	0	0	0	0	0	0	0	0	0	-1	-1	0	0	:	oo		0	0	4i2	0	0	0	0	0	0	i2	χ_{34}



Sz(8)

Suzuki group $Sz(8) \cong {}^2B_2(8)$

Order = 29,120 = $2^6.5.7.13$ Mult = 2^2 Out = 3

Constructions

Suzuki $Sz(8) \cong G$: the centralizer in $S_4(8)$ of an outer (graph) automorphism of order 2;

G : all 4 x 4 matrices over $\mathbf{F}_8$ preserving the set of vectors (t,x,y,z) for which $xy+(x^{s+2}+y^s+z)t=0$, where $s : x \to x^4$ is the automorphism of $\mathbf{F}_8$ with $s^2=2$; projectively this defines an oval of $8^2+1=65$ points on which G acts doubly transitively

Orthogonal 2.G has an 8-dimensional orthogonal representation over $\mathbf{F}_5$; it may be generated by the maps A and B which take $(x_\infty,x_0,\ldots,x_6)$ (subscripts mod 7) to $(x_4,-x_5,x_2,x_1,-x_6,x_\infty,x_0,x_3)$ and $(x_\infty,x_1,x_2,x_3,x_4,x_5,x_6,x_0)$, respectively, together with the map

$C : x_\infty \to 2x_\infty + x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

$x_t \to x_\infty - x_{-t} - x_{3-t} + x_{4-t} + 3x_{1-t} + 3x_{2-t} + 3x_{6-t}$

Complex G has a 14-dimensional complex representation which we write as a 7-dimensional representation over the quaternion algebra with units 1,j,k,l and complex coefficients a+bi; G is generated by

$A' : (q_0,\ldots q_6) \to (-iq_0,-ikq_1,-ilq_2,lq_3,-ijq_4,kq_5,jq_6)$

$B' : (q_0,\ldots q_6) \to (q_1,q_2,q_3,q_4,q_5,q_6,q_0)$

$C' : -4iq_n \to (i+j+k+l)q_{-n}+(i-l)q_{1-n}+(i-j)q_{2-n}+(l-k)q_{3-n}+(i-k)q_{4-n}+(k-j)q_{5-n}+(j-l)q_{6-n}$

this is connected with the fact that one of the involution centralizers in the Rudvalis group is $2^2 \times G$

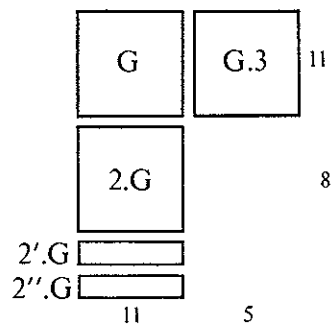
Maximal subgroups

Order	Index	Structure	G.3	Character
448	65	$2^{3+3}:7$	: $2^{3+3}:7:3$	1a+64a
52	560	13:4	: 13:12	1a+35abc+64a+65aabbcc
20	1456	5:4	: 5:4 x 3	
14	2080	D_{14}	: 7:6	

Specifications

Abstract	Suzuki	Orthogonal
$N(2A^3)$	parabolic	base
N(13ABC)		
N(5A), C(3A)	Sz(2)	
N(7ABC)	$L_1(8)$	

	;	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@
		29120	64	16	16	5	7	7	7	13	13	13			20	4	4	4	5
	p power		A	A	A	A	A	A	A	A	A	A			A	AA	AB	AA	AA
	p' part		A	A	A	A	A	A	A	A	A	A			A	AA	AA	AB	AA
	ind	1A	2A	4A	B**	5A	7A	B*2	C*4	13A	B*3	C*9	fus	ind	3A	6A	12A	B*	15A
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	:	+oo	1	1	1	1	1
χ_2	o	14	-2	2i	-2i	-1	0	0	0	1	1	1	:	ooo	-1	1	-i	i	-1
χ_3	o	14	-2	-2i	2i	-1	0	0	0	1	1	1	:	ooo	-1	1	i	-i	-1
χ_4	+	35	3	-1	-1	0	0	0	0	-c13	*3	*9		+	0	0	0	0	0
χ_5	+	35	3	-1	-1	0	0	0	0	*9	-c13	*3							
χ_6	+	35	3	-1	-1	0	0	0	0	*3	*9	-c13							
χ_7	+	64	0	0	0	-1	1	1	1	-1	-1	-1	:	+oo	4	0	0	0	-1
χ_8	+	65	1	1	1	0	y7	*2	*4	0	0	0		+	0	0	0	0	0
χ_9	+	65	1	1	1	0	*4	y7	*2	0	0	0							
χ_{10}	+	65	1	1	1	0	*2	*4	y7	0	0	0							
χ_{11}	+	91	-5	-1	-1	1	0	0	0	0	0	0	:	+oo	1	1	-1	-1	1
	ind	1	2	4	4	5	7	7	7	13	13	13							
		2				10	14	14	14	26	26	26							
χ_{12}	+	40	0	0	0	0	-y7	*2	*4	1	1	1							
χ_{13}	+	40	0	0	0	0	*4	-y7	*2	1	1	1							
χ_{14}	+	40	0	0	0	0	*2	*4	-y7	1	1	1							
χ_{15}	+	56	0	0	0	1	0	0	0	c13	*3	*9							
χ_{16}	+	56	0	0	0	1	0	0	0	*9	c13	*3							
χ_{17}	+	56	0	0	0	1	0	0	0	*3	*9	c13							
χ_{18}	+	64	0	0	0	-1	1	1	1	-1	-1	-1							
χ_{19}	+	104	0	0	0	-1	-1	-1	-1	0	0	0							
	and	1	2	4	4	5	7	7	7	13	13	13							
	no:	:2				:2	:2	:2	:2	:2	:2	:2							
	and	1	2	4	4	5	7	7	7	13	13	13							
	no:	:2				:2	:2	:2	:2	:2	:2	:2							



$L_2(32)$

Linear group $L_2(32) \cong A_1(32) \cong U_2(32) \cong S_2(32) \cong O_3(32)$

Order = 32,736 = $2^5.3.11.31$ Mult = 1 Out = 5

Constructions

Linear $GL_2(32) \cong 31 \times G$: all non-singular 2 x 2 matrices over $\mathbb{F}_{32}$;

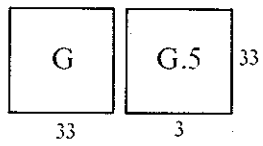
$PGL_2(32) \cong SL_2(32) \cong PSL_2(32) \cong G$; $\Gamma L_2(32) \cong (31 \times G).5$; $P\Gamma L_2(32) \cong \Sigma L_2(32) \cong P\Sigma L_2(32) \cong G.5$

Unitary $GU_2(32) \cong 33 \times G$: all 2 x 2 matrices over $\mathbb{F}_{1024}$ preserving a non-singular Hermitian form;

$PGU_2(32) \cong SU_2(32) \cong PSU_2(32) \cong G$

Orthogonal $GO_3(32) \cong PGO_3(32) \cong SO_3(32) \cong PSO_3(32) \cong O_3(32) \cong G$: all 3 x 3 matrices over $\mathbb{F}_{32}$ preserving a non-singular quadratic form; $\Gamma O_3(32) \cong P\Gamma O_3(32) \cong \Sigma O_3(32) \cong P\Sigma O_3(32) \cong G.5$

Presentations $G \cong \langle P,Q \mid P^{31}=(P^2Q)^3=(P^3Q)^2=(PQ^6)^2=(P^2Q^4P^4Q^5)^2=(P^2Q^5P^4Q^4)^2=1\rangle$



Maximal subgroups					Specifications		
Order	Index	Structure	G.5	Character	Abstract	Linear	Orthogonal
992	33	2^5:31	: 2^5:31:5	1a+32a	$N(2A^5)$	point	isotropic point
66	496	D_{66}	: 33:10	1a+33a-o	N(3A), N(11ABCDEF)	$O_2^-(32)$, $L_1(1024)$	minus line
62	528	D_{62}	: 31:10	1a+32a+33a-o	N(31A-O)	$O_2^+(32)$, base	plus line

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	
		32736	32	33	33	33	33	33	33	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	33	33	33	33	33	33	33	33	33	33			6	2	3	
	p	power	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	DA	EA	AA	BA	CA	DA	EA	AA	BA	CA			A	AA	AA	
	p'	part	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	CA	DA	EA	AA	BA	CA	DA	EA			A	AA	AA	
	ind	1A	2A	3A	11A	B*2	C*4	D*3	E*5	31A	B*2	C*4	D*8	E*15	F*5	G*10	H*11	I*9	J*13	K*6	L*12	M*7	N*14	O*3	33A	B*2	C*4	D*8	E*16	F*10	G*13	H*7	I*14	J*5	fus	ind	5A	10A	15A	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	: +0000	1	1	1	1	χ_1
χ_2	+	31	-1	1	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	: +0000	1	-1	1		χ_2
χ_3	+	31	-1	-2	-y11	*2	*4	*3	*5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-y11	*2	*4	*3	*5	-y11	*2	*4	*3	*5		+	0	0	0	χ_3
χ_4	+	31	-1	-2	*5	-y11	*2	*4	*3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*5	-y11	*2	*4	*3	*5	-y11	*2	*4	*3						χ_4
χ_5	+	31	-1	-2	*3	*5	-y11	*2	*4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*3	*5	-y11	*2	*4	*3	*5	-y11	*2	*4						χ_5
χ_6	+	31	-1	-2	*4	*3	*5	-y11	*2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*4	*3	*5	-y11	*2	*4	*3	*5	-y11	*2						χ_6
χ_7	+	31	-1	-2	*2	*4	*3	*5	-y11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*2	*4	*3	*5	-y11	*2	*4	*3	*5	-y11						χ_7
χ_8	+	31	-1	1	-y11	*2	*4	*3	*5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*8	*16	-y33	*2	*4	*14	*5	*10	*13	*7		+	0	0	0	χ_8
χ_9	+	31	-1	1	*5	-y11	*2	*4	*3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*4	*8	*16	-y33	*2	*7	*14	*5	*10	*13						χ_9
χ_{10}	+	31	-1	1	*3	*5	-y11	*2	*4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*2	*4	*8	*16	-y33	*13	*7	*14	*5	*10						χ_{10}
χ_{11}	+	31	-1	1	*4	*3	*5	-y11	*2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-y33	*2	*4	*8	*16	*10	*13	*7	*14	*5						χ_{11}
χ_{12}	+	31	-1	1	*2	*4	*3	*5	-y11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*16	-y33	*2	*4	*8	*5	*10	*13	*7	*14						χ_{12}
χ_{13}	+	31	-1	1	-y11	*2	*4	*3	*5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*14	*5	*10	*13	*7	*8	*16	-y33	*2	*4		+	0	0	0	χ_{13}
χ_{14}	+	31	-1	1	*5	-y11	*2	*4	*3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*7	*14	*5	*10	*13	*4	*8	*16	-y33	*2						χ_{14}
χ_{15}	+	31	-1	1	*3	*5	-y11	*2	*4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*13	*7	*14	*5	*10	*2	*4	*8	*16	-y33						χ_{15}
χ_{16}	+	31	-1	1	*4	*3	*5	-y11	*2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*10	*13	*7	*14	*5	-y33	*2	*4	*8	*16						χ_{16}
χ_{17}	+	31	-1	1	*2	*4	*3	*5	-y11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*5	*10	*13	*7	*14	*16	-y33	*2	*4	*8						χ_{17}
χ_{18}	+	32	0	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	: +0000	2	0	-1		χ_{18}
χ_{19}	+	33	1	0	0	0	0	0	0	y31	*2	*4	*8	*15	*5	*10	*11	*9	*13	*6	*12	*7	*14	*3	0	0	0	0	0	0	0	0	0	0		+	0	0	0	χ_{19}
χ_{20}	+	33	1	0	0	0	0	0	0	*15	y31	*2	*4	*8	*13	*5	*10	*11	*9	*3	*6	*12	*7	*14	0	0	0	0	0	0	0	0	0	0						χ_{20}
χ_{21}	+	33	1	0	0	0	0	0	0	*8	*15	y31	*2	*4	*9	*13	*5	*10	*11	*14	*3	*6	*12	*7	0	0	0	0	0	0	0	0	0	0						χ_{21}
χ_{22}	+	33	1	0	0	0	0	0	0	*4	*8	*15	y31	*2	*11	*9	*13	*5	*10	*7	*14	*3	*6	*12	0	0	0	0	0	0	0	0	0	0						χ_{22}
χ_{23}	+	33	1	0	0	0	0	0	0	*2	*4	*8	*15	y31	*10	*11	*9	*13	*5	*12	*7	*14	*3	*6	0	0	0	0	0	0	0	0	0	0						χ_{23}
χ_{24}	+	33	1	0	0	0	0	0	0	*6	*12	*7	*14	*3	y31	*2	*4	*8	*16	*5	*10	*11	*9	*13	0	0	0	0	0	0	0	0	0	0		+	0	0	0	χ_{24}
χ_{25}	+	33	1	0	0	0	0	0	0	*3	*6	*12	*7	*14	*15	y31	*2	*4	*8	*13	*5	*10	*11	*9	0	0	0	0	0	0	0	0	0	0						χ_{25}
χ_{26}	+	33	1	0	0	0	0	0	0	*14	*3	*6	*12	*7	*8	*15	y31	*2	*4	*9	*13	*5	*10	*11	0	0	0	0	0	0	0	0	0	0						χ_{26}
χ_{27}	+	33	1	0	0	0	0	0	0	*7	*14	*3	*6	*12	*4	*8	*15	y31	*2	*11	*9	*13	*5	*10	0	0	0	0	0	0	0	0	0	0						χ_{27}
χ_{28}	+	33	1	0	0	0	0	0	0	*12	*7	*14	*3	*6	*2	*4	*8	*15	y31	*10	*11	*9	*13	*5	0	0	0	0	0	0	0	0	0	0						χ_{28}
χ_{29}	+	33	1	0	0	0	0	0	0	*5	*10	*11	*9	*13	*6	*12	*7	*14	*3	y31	*2	*4	*8	*15	0	0	0	0	0	0	0	0	0	0		+	0	0	0	χ_{29}
χ_{30}	+	33	1	0	0	0	0	0	0	*13	*5	*10	*11	*9	*3	*6	*12	*7	*14	*15	y31	*2	*4	*8	0	0	0	0	0	0	0	0	0	0						χ_{30}
χ_{31}	+	33	1	0	0	0	0	0	0	*9	*13	*5	*10	*11	*14	*3	*6	*12	*7	*8	*15	y31	*2	*4	0	0	0	0	0	0	0	0	0	0						χ_{31}
χ_{32}	+	33	1	0	0	0	0	0	0	*11	*9	*13	*5	*10	*7	*14	*3	*6	*12	*4	*8	*15	y31	*2	0	0	0	0	0	0	0	0	0	0						χ_{32}
χ_{33}	+	33	1	0	0	0	0	0	0	*10	*11	*9	*13	*5	*12	*7	*14	*3	*6	*2	*4	*8	*15	y31	0	0	0	0	0	0	0	0	0	0						χ_{33}

$U_3(4)$

Unitary group $U_3(4) \cong {}^2A_2(4)$

Order = 62,400 = $2^6.3.5^2.13$ Mult = 1 Out = 4

Construction

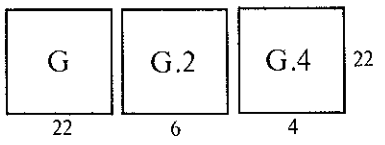
Unitary $GU_3(4) \cong 5 \times G$: all 3 x 3 matrices over $\mathbb{F}_{16}$ preserving a non-singular Hermitian form;

$PGU_3(4) \cong SU_3(4) \cong PSU_3(4) \cong G$;

Design G : the automorphism group of a Steiner system S(2,5,65) whose blocks are the sets of 5 isotropic points corresponding to vectors orthogonal to a non-isotropic vector

Maximal subgroups

Order	Index	Structure	G.2	G.4	Character	Specifications Abstract	Specifications Unitary
960	65	2^{2+4}:15	: 2^{2+4}:(3xD_{10})	: 2^{2+4}:(3xD_{10})·2	1a+64a	N($2A^2$)	isotropic point
300	208	5 x A_5	: D_{10} x A_5	: (D_{10} x A_5)·2	1a+39ab+64a+65a	N(5ABCD)	non-isotropic point
150	416	5^2:S_3	: 5^2:D_{12}	: 5^2:(4 x S_3)	1a+39ab+52abcd+64a+65a	N(5^2)	base
39	1600	13:3	: 13:6	: 13:12		N(13ABCD)	$U_1(64)$



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	;	;	@	@	@	@	
		62400	320	15	16	300	300	300	300	25	25	20	20	20	20	13	13	13	13	15	15	15	15				60	3	8	8	5	5			6	3	4	4	
	p power		A	A	A	A	A	A	A	A	A	CA	DA	BA	AA	A	A	A	A	DA	CA	AA	BA				A	AB	A	A	FB	EB			B	AB	A	A	
	p' part		A	A	A	A	A	A	A	A	A	AA	BA	CA	DA	A	A	A	A	AA	BA	CA	DA				A	AB	A	A	EB	FB			A	AB	A	A	
	ind	1A	2A	3A	4A	5A	B**	C*2	D*3	5E	F*	10A	B**	C*7	D*3	13A	B**	C*5	D*8	15A	B**	C*2	D*8	fus	ind		2B	6A	8A	B**	10A	B*	fus	ind	4B	12A	16A	16B	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++		1	1	1	1	1	1	:+o+o		1	1	1	1	χ_1
χ_2	-	12	-4	0	0	-3	-3	-3	-3	2	2	1	1	1	1	-1	-1	-1	-1	0	0	0	0	:	oo		0	0	2i	-2i	0	0	:oooo		0	0-i+1		i-1	χ_2
χ_3	o	13	-3	1	1	z5-3&2	**	*2	*3	-b5	*	z5&2	**	*2	*3	0	0	0	0	z5	**	*2	*3		+		0	0	0	0	0	0		+	0	0	0	0	χ_3
χ_4	o	13	-3	1	1	**	z5-3&2	*3	*2	-b5	*	**	z5&2	*3	*2	0	0	0	0	**	z5	*3	*2																χ_4
χ_5	o	13	-3	1	1	*3	*2	z5-3&2	**	*	-b5	*3	*2	z5&2	**	0	0	0	0	*3	*2	z5	**		+		0	0	0	0	0	0		+	0	0	0	0	χ_5
χ_6	o	13	-3	1	1	*2	*3	**	z5-3&2	*	-b5	*2	*3	**	z5&2	0	0	0	0	*2	*3	**	z5																χ_6
χ_7	+	39	7	0	-1	-3b5	-3b5	*	*	*	b5+2	b5	b5	*	*	0	0	0	0	0	0	0	0	:	++		3	0	-1	-1	*	-b5	:	++	0	0	0	0	χ_7
χ_8	+	39	7	0	-1	*	*	-3b5	-3b5	b5+2	*	*	*	b5	b5	0	0	0	0	0	0	0	0	:	++		3	0	-1	-1	-b5	*	:						χ_8
χ_9	o	52	4	1	0	4z5+3&2	**	*2	*3	b5	*	*2	*3	**	-z5	0	0	0	0	z5	**	*2	*3		+		0	0	0	0	0	0		+	0	0	0	0	χ_9
χ_{10}	o	52	4	1	0	**	4z5+3&2	*3	*2	b5	*	*3	*2	-z5	**	0	0	0	0	**	z5	*3	*2																χ_{10}
χ_{11}	o	52	4	1	0	*3	*2	4z5+3&2	**	*	b5	-z5	**	*2	*3	0	0	0	0	*3	*2	z5	**		+		0	0	0	0	0	0							χ_{11}
χ_{12}	o	52	4	1	0	*2	*3	**	4z5+3&2	*	b5	**	-z5	*3	*2	0	0	0	0	*2	*3	**	z5																χ_{12}
χ_{13}	+	64	0	1	0	4	4	4	4	-1	-1	0	0	0	0	-1	-1	-1	-1	1	1	1	1	:	++		4	1	0	0	-1	-1	:+o+o		2	-1	0	0	χ_{13}
χ_{14}	+	65	1	-1	1	5	5	5	5	0	0	1	1	1	1	0	0	0	0	-1	-1	-1	-1	:	++		5	-1	1	1	0	0	:+o+o		1	1	-1	-1	χ_{14}
χ_{15}	o	65	1	-1	1	5z5	**	*2	*3	0	0	z5	**	*2	*3	0	0	0	0	-z5	**	*2	*3		+		0	0	0	0	0	0		+	0	0	0	0	χ_{15}
χ_{16}	o	65	1	-1	1	**	5z5	*3	*2	0	0	**	z5	*3	*2	0	0	0	0	**	-z5	*3	*2																χ_{16}
χ_{17}	o	65	1	-1	1	*3	*2	5z5	**	0	0	*3	*2	z5	**	0	0	0	0	*3	*2	-z5	**		+		0	0	0	0	0	0							χ_{17}
χ_{18}	o	65	1	-1	1	*2	*3	**	5z5	0	0	*2	*3	**	z5	0	0	0	0	*2	*3	**	-z5																χ_{18}
χ_{19}	o	75	-5	0	-1	0	0	0	0	0	0	0	0	0	0	-d13	**	*5	*8	0	0	0	0		+		0	0	0	0	0	0		+	0	0	0	0	χ_{19}
χ_{20}	o	75	-5	0	-1	0	0	0	0	0	0	0	0	0	0	**	-d13	*8	*5	0	0	0	0																χ_{20}
χ_{21}	o	75	-5	0	-1	0	0	0	0	0	0	0	0	0	0	*8	*5	-d13	**	0	0	0	0		+		0	0	0	0	0	0							χ_{21}
χ_{22}	o	75	-5	0	-1	0	0	0	0	0	0	0	0	0	0	*5	*8	**	-d13	0	0	0	0																χ_{22}

Sporadic Mathieu group M_{12}

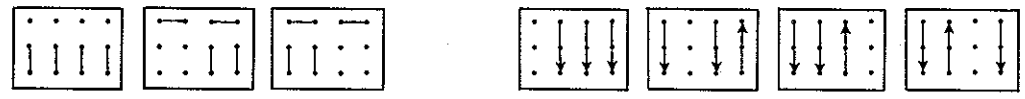
Order = 95,040 = $2^6.3^3.5.11$ Mult = 2 Out = 2

Constructions

Golay (3) code 2G : the automorphism group of the unique dimension 6, length 12 code over $\mathbb{F}_3$ of minimal weight 6 (the ternary Golay code) which is defined in several ways below.

The weight distributions of the code and the cocode (words modulo the code) are $0^1 6^{2x132} 9^{2x220} 12^{2x12}$ and $0^1 1^{2x12} 2^{2x132} 3^{2x220}$, respectively. The minimal weight representatives for the cocode are unique except that words of weight 3 are congruent in sets of 4 (whose supports are linked threes).

Steiner hexads G : the automorphism group of the Steiner system S(5,6,12) of 132 special hexads from a 12-set, such that each pentad is in just one special hexad. The hexads are the supports of the weight 6 words in the ternary Golay code. For a group element of shape $2^4 1^4$ or $3^3 1^3$, the fixed points together with any cycle form a hexad. Below we give, in the MINIMOG array, 3 involutions of a 4-group whose fixed tetrads (linked fours) are disjoint, and 4 elements of a 9-group any 2 of whose fixed triads (linked threes) form a hexad.



MINIMOG The MINIMOG constructs the code in such a way that both signed and unsigned hexads are easily recognized. Each word is written over $\mathbb{F}_3$ in three rows (row_o, row_+, row_-) of four digits each, and belongs to the code just if:

sum of each column = -(sum of row_o)

$row_+ - row_-$ is one of the tetracode words:

oooo o+++ o--- +o+- +-o+ ++-o -o-+ -+o- --+o (or in general $(a_\infty;a_0,a_1,a_2) = (s;a,a+s,a+2s)$.

For example we check -

+	+	+	+		column sums:	-	- - -
o	o	+	-	thus:	row_o sum: +	+	+ + +
+	+	o	-			o	o + -
						+	+ o -
					$row_+ - row_-$:	-	- + o

A hexad is in the Steiner system just if its column distribution is $3^2 0^2$, $3^1 1^3$, $2^3 0^1$, or $2^2 1^2$ and its "odd-men-out" form part of a tetracode word. For example

column distribution:	0 3 0 3	1 1 3 1	2 0 2 2	1 1 2 2
	* * / * * / * *	* * / * * / * *	* * * / * * / *	* * * / * * / *
odd-men-out:	? ? ? ?	+ o ? -	- ? - +	+ o + -

(if a point in a column is the only one in (resp. not in) the hexad, it is the odd man out for that column). We find the corresponding Golay code words using cols and tets:

col-col	col+tet	tet-tet	col+col-tet	a col	a tet
+ - / + - / + -	+ + / + - / + +	+ o + + / - - / -	- + + / - o + / + o	+ / + / +	+ / + / + / +

(Modulo the code we have (every col) ≡ -(every tet), where cols and tets are the obvious words of weight 3 and 4 corresponding to columns and tetracode words.)

Here is a dictionary between numberings of the 12-set and the MINIMOG array:

$-e_0$	e_3	e_∞	$-e_2$
$-e_5$	e_9	e_8	$-e_X$
$-e_4$	e_1	e_6	$-e_7$

modulo 11

6	3	0	9
5	2	7	10
4	1	8	11

shuffle

6	3	0	9
18	2	16	13
4	1	8	12

modulo 23

$-d_0$	d_1	d_3	d_2
d_4	d_6	d_8	d_{10}
$-d_5$	$-d_7$	$-d_9$	$-d_{11}$

lexicographic

Mathieu 24 G : the stabilizer in M_{24} of a weight 12 word (dodecad) in the binary Golay code. The hexads arise from octads of S(5,8,24) meeting the dodecad in 6 points. The outer automorphism interchanges the dodecad with its complement. The standard pair of dodecads is

Q = {0, quadratic residues modulo 23}, N = {∞, non-residues modulo 23}.

If we define n^+ = whichever of n, -n is in Q n^- = whichever of 8/n, -8/n is in N

we convert the modulo 23 numbering of the MOG into shuffle numberings for Q and N, thus:

0	∞	1	11	2	22
19	3	20	4	10	18
15	6	14	16	17	8
5	9	21	13	7	12

=

0^+	0^-	1^+	7^-	2^+	8^-
2^-	3^+	5^-	4^+	10^-	5^+
1^-	6^+	6^-	7^+	9^-	8^+
3^-	9^+	4^-	10^+	11^-	11^+

M_{12}

Modulo 11 The ternary Golay code is a quadratic residue code, modulo 11. Letting Q = {0, quadratic residues} = {0,1,3,4,5,9} and N = {∞,2,6,7,8,X=10 , the words $w_\infty = -\Sigma e_i$ and $w_i = \Sigma e_{q-i} - \Sigma e_{n-i}$ (over $\mathbf{F}_3$) and their negatives are called <u>total words</u>, and span the code. 2G is generated by the maps

$$e_i \to e_{A(i)} \qquad w_i \to w_{A^{-1}(i)}$$
$$e_i \to e_{B(i)} \qquad w_i \to w_{B(i)}$$
$$e_i \to \pm e_{C(i)} \qquad w_i \to \mp w_{C(i)} \qquad \text{(upper sign just on Q)}$$
$$e_i \to e_{D(i)} \qquad w_i \to w_{D(i)}$$

where A(i) = i+1, B(i) = 3i, C(i) = -1/i and D = (2X)(34)(59)(67). The group $\langle A,B,C\rangle \cong SL_2(11)$, so all automorphisms of the projective line are in G. The code is self-dual in the sense that $\Sigma c_i w_i = 0$ just when (c_i) is in the code. There is an outer automorphism E that interchanges the actions on e_i and w_i, and takes $A \to A^{-1}$, $B \to B$, $C \to C^{-1} = -C$, $D \to D$.

Loop G is generated by the multiplication maps $x \to x.a$ in the loop $\{E_\infty, E_0, \ldots E_X\}$ of order 12 defined by $E_t.E_t = E_\infty = 1$, $E_t.E_{t+q} = E_{t+2q}$, $E_t.E_{t+n} = E_{t-n}$ (q in Q, n in N). For each p in $L_2(11)$ there is a permutation p* such that $E_a.E_b = E_c \Rightarrow E_{p(a)}.E_{p(b)} = E_{p^*(c)}$ and G is generated by the p and p*. For the permutations given above, A* = A, B* = B, C* = D. These results extend (with some subtlety) to the loop of order 24 defined by $E_t.E_t = -E_\infty$, $E_t.E_{t+q} = -E_{t+2q}$, $E_t.E_{t+n} = E_{t-n}$, whose right multiplications generate 2G.

Orthogonal (2) G is the intersection of two subgroups A_{12} in $O^-_{10}(2)$, as follows:

the even subsets of {0,1,...11} modulo complementation form an orthogonal 10-space over $\mathbf{F}_2$ with quadratic form $q(X) = \frac{1}{2}|X|$. The reflections $x \to x - (x.r)r$ in the 11 duads r = {0,1},...{10,11} generate one subgroup S_{12}, while those in the 11 hexads with sum 21, namely

r = {1,2,3,4,5,6} {0,1,2,3,7,8} {0,1,2,4,5,9} {0,1,3,4,6,7} {0,1,2,3,5,10} {0,1,2,4,6,8}
{0,2,3,4,5,7} {0,1,2,3,6,9} {0,1,3,4,5,8} {0,1,2,5,6,7} {0,1,2,3,4,11}

generate the other. The outer automorphism of $O^-_{10}(2)$ interchanges the generators in the order given. The above hexads generate the Steiner system by reflection and complementation.

Shuffle G : the case n = 12 of the following construction:

If n|12 we define M_n to be the group of permutations of {0,1,...n-1} generated by the <u>reversal</u> and <u>Mongean shuffle</u> permutations: R : t → n-1-t S : t → min{2t,2n-1-2t}.

If mn = 12, the stabilizer in G of the canonical partition 0.../n.../2n.../... into m sets of n is a subgroup $M_m \times M_n$ acting on one of the standard arrays displayed:

0	1	2	3	4	5	6	7	8	9	10	11

0	1	2	3	4	5
6	7	8	9	10	11

0	1	2	3
7	6	5	4
8	9	10	11

0	1	2
3	4	5
6	7	8
9	10	11

0	1
3	2
4	5
7	6
8	9
11	10

0
1
2
3
4
5
6
7
8
9
10
11

The Steiner system is the same as that given under the "Orthogonal" construction above.

The outer automorphism : R → R, S → S^{-1} conjugates $M_m \times M_n$ to $M_n \times M_m$.

The shuffle numbering is related to the modulo 11 numbering as follows:

Shuffle:	0	1	2	3	4	5	6	7	8	9	10	11
Modulo 11:	∞	1	9	3	4	5	0	8	6	2	X	7
(mnemonic:	∞	1	-2	3	4	5	0	-3	6	-9	-12	-15)

G x 2 : the group of permutations on {0,1,2,...,23} generated by the <u>turnaround</u> permutations

$T_d : qd + r \to qd + d - 1 - r \quad (0 \le r < d)$ for each divisor d of 24

There is an outer automorphism taking T_d to $T_{24/d}.(T_{24})^{d+24/d}$

Hadamard 2G : the automorphism group of any 12 x 12 Hadamard matrix (that is, a matrix with orthogonal rows and entries ±1); the outer automorphism interchanges rows with columns. Particular definitions of Hadamard matrices are

(modulo 11) : $a_{ij} = \pm 1$ according as i is in Q or N

(shuffle) : $a_{ij} = (-1)^{[(ij+i+j-1)/13]}$

An automorphism is a monomial permutation of rows that has the same effect as some monomial permutation of columns.

Lexicographic The code is generated by 6 words, written $(d_0,\ldots d_{11})$, each the lexicographically earliest word that differs in at least 6 places from all linear combinations of its predecessors:

000000111111 000011001122 000101010212 001001012021 010001021201 100001022110

Presentations $G \cong G^{3,5,5,5,5,5,5} \cong \langle A,B,C,D \mid A^{11}=B^5=C^2=D^2=(BC)^2=(BD)^2=(AC)^3=(AD)^3=(DCB)^2=1, A^B=A^3\rangle$

$\cong \langle A,C,D \mid A^{11}=C^2=D^2=(AC)^3=(AD)^3=(CD)^{10}=1, A^2(CD)^2A=(CD)^2\rangle$

M_{12}

Maximal subgroups					Specifications		
Order	Index	Structure	G.2	Character	Abstract	Mathieu 12	Golay (3) code
7920	12	M_{11}	$L_2(11):2$	1a+11a		point	± weight 1 cocode word
7920	12	M_{11}		1a+11b		total	± weight 12 code word
1440	66	$M_{10}:2 \cong A_6 \cdot 2^2$		1a+11a+54a	N(2B,3A,3A,4B,5A)	duad	± weight 2 cocode words
1440	66	$M_{10}:2 \cong A_6 \cdot 2^2$		1a+11b+54a	N(2B,3A,3A,4A,5A)	hexad pair	± weight 6 code words
660	144	$L_2(11)$	: $L_2(11):2$, S_5	1a+16ab+45a+66a	N(2A,3B,5A,6A,11AB)	projective line	
432	220	$M_9:S_3 \cong 3^2:2S_4$	$3_+^{1+2}:D_8$	1a+11a+54a+55a+99a	$N(3A^2)$	triad	± weight 9 code word
432	220	$M_9:S_3 \cong 3^2:2S_4$		1a+11b+54a+55a+99a	$N(3A^2)$	linked threes	± weight 3 cocode word
240	396	$2 \times S_5$	: $(2^2 \times A_5):2$	1a+16ab+45a+54aa+66a+144a	N(2A), N(2B,3B,5A)	2 x 6 array	$M_2 \times M_6$
192	495	$M_8.S_4 \cong 2_+^{1+4}.S_3$	: H.2	1a+11ab+54aa+55a+66a+99a+144a	N(2B)	tetrad	
192	495	$4^2:D_{12}$	: H.2	1a+16ab+45a+54aa+66a+99a+144a	$N(2B^2)$	linked fours	
72	1320	$A_4 \times S_3$	: $S_4 \times S_3$		$N(2A^2)$, N(3B)	4 x 3 array	$M_4 \times M_3$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@
		95040	240	192	54	36	32	32	10	12	6	8	8	10	11	11			120	24	12	6	10	10	6	6	6
	p	power	A	A	A	A	B	B	A	BA	AB	A	B	AA	A	A			A	B	A	BC	AC	AC	AD	BC	BC
	p'	part	A	A	A	A	A	A	A	BA	AB	A	A	AA	A	A			A	A	A	BC	AC	AC	BD	AC	AC
	ind	1A	2A	2B	3A	3B	4A	4B	5A	6A	6B	8A	8B	10A	11A	B**	fus	ind	2C	4C	4D	6C	10B	C*	12A	12B	C*
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1
χ_2	+	11	-1	3	2	-1	-1	3	1	-1	0	-1	1	-1	0	0		+	0	0	0	0	0	0	0	0	0
χ_3	+	11	-1	3	2	-1	3	-1	1	-1	0	1	-1	-1	0	0											
χ_4	o	16	4	0	-2	1	0	0	1	1	0	0	0	-1	b11	**		+	0	0	0	0	0	0	0	0	0
χ_5	o	16	4	0	-2	1	0	0	1	1	0	0	0	-1	**	b11											
χ_6	+	45	5	-3	0	3	1	1	0	-1	0	-1	-1	0	1	1	:	++	5	-3	1	-1	0	0	1	0	0
χ_7	+	54	6	6	0	0	2	2	-1	0	0	0	0	1	-1	-1	:	++	0	0	0	0	r5	-r5	0	0	0
χ_8	+	55	-5	7	1	1	-1	-1	0	1	1	-1	-1	0	0	0	:	++	5	1	-1	-1	0	0	-1	1	1
χ_9	+	55	-5	-1	1	1	3	-1	0	1	-1	-1	1	0	0	0		+	0	0	0	0	0	0	0	0	0
χ_{10}	+	55	-5	-1	1	1	-1	3	0	1	-1	1	-1	0	0	0											
χ_{11}	+	66	6	2	3	0	-2	-2	1	0	-1	0	0	1	0	0	:	++	6	2	0	0	1	1	0	-1	-1
χ_{12}	+	99	-1	3	0	3	-1	-1	-1	-1	0	1	1	-1	0	0	:	++	1	-3	-1	1	1	1	-1	0	0
χ_{13}	+	120	0	-8	3	0	0	0	0	0	1	0	0	0	-1	-1	:	++	0	0	0	0	0	0	0	r3	-r3
χ_{14}	+	144	4	0	0	-3	0	0	-1	1	0	0	0	-1	1	1	:	++	4	0	2	1	-1	-1	-1	0	0
χ_{15}	+	176	-4	0	-4	-1	0	0	1	-1	0	0	0	1	0	0	:	++	4	0	-2	1	-1	-1	1	0	0
	ind	1	4	2	3	3	4	4	5	12	6	8	8	20	11	11	fus	ind	2	4	8	6	10	10	24	12	12
		2		2	6	6			10		6	8	8	20	22	22				4	8				24	12	12
χ_{16}	o	10	0	-2	1	-2	0	0	0	0	1	i2	i2	0	-1	-1	:	oo	0	2	i2	0	0	0	i2	-1	-1
χ_{17}	o	10	0	-2	1	-2	0	0	0	0	1	-i2	-i2	0	-1	-1	:	oo	0	2	-i2	0	0	0	-i2	-1	-1
χ_{18}	+	12	0	4	3	0	0	0	2	0	1	0	0	0	1	1	:	++	0	0	0	0	0	0	0	r3	-r3
χ_{19}	-	32	0	0	-4	2	0	0	2	0	0	0	0	0	-1	-1	:	oo	0	0	2i2	0	0	0	-i2	0	0
χ_{20}	o	44	0	4	-1	2	0	0	-1	0	1	0	0	i5	0	0		+	0	0	0	0	0	0	0	0	0
χ_{21}	o	44	0	4	-1	2	0	0	-1	0	1	0	0	-i5	0	0											
χ_{22}	o	110	0	-6	2	2	0	0	0	0	0	i2	-i2	0	0	0		+	0	0	0	0	0	0	0	0	0
χ_{23}	o	110	0	-6	2	2	0	0	0	0	0	-i2	i2	0	0	0											
χ_{24}	+	120	0	8	3	0	0	0	0	0	-1	0	0	0	-1	-1	:	++	0	4	0	0	0	0	0	1	1
χ_{25}	o	160	0	0	-2	-2	0	0	0	0	0	0	0	0	-b11	**		+	0	0	0	0	0	0	0	0	0
χ_{26}	o	160	0	0	-2	-2	0	0	0	0	0	0	0	0	**	-b11											

G	G.2	15
2.G	2.G.2	11
15	9	

$U_3(5)$

Unitary group $U_3(5) \cong {}^2A_2(5)$

Order = 126,000 = $2^4.3^2.5^3.7$ Mult = 3 Out = S_3

Constructions

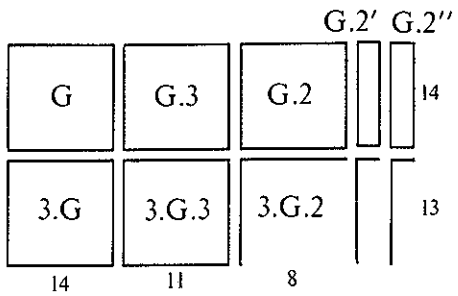
Unitary $2 \times 3.G.3 \cong GU_3(5)$: all 3×3 matrices over $\mathbb{F}_{25}$ preserving a non-singular Hermitian form;

$PGU_3(5) \cong G.3$; $SU_3(5) \cong 3.G$; $PSU_3(5) \cong G$;

Graph G.2 : the automorphism group of the Hoffman-Singleton graph, a rank 3 graph on 50 vertices, in which the vertex stabilizer is A_7, and the suborbits are 1, 7, 42. The vertices may be labelled by ∞ (1), 0,1,...6 (7), and the 42 totals (see A_6) on 6-element subsets of {0,1,...6}. The vertex i joins ∞ and the totals on the set disjoint from {i}; two totals are joined if A_7 contains an element of order 5 fixing them both. There are two systems of 50 sets (co-cliques) of 15 points no two of which are joined.

Leech $2 \times G.S_3$: the set stabilizer in $2Co_1$ of an S-lattice of type $3^{1+3}2^3$ (see Co_1); the pointwise stabilizer is G; hence G appears in Co_2, Co_3, HiS, and McL, which stabilize various sublattices of this S-lattice.

Remark : G.2 is a maximal subgroup of the Rudvalis group.



Maximal subgroups

Order	Index	Structure	G.3	G.2	$G.S_3$	Character	Specifications: Abstract	Unitary	Graph
2520	50	A_7	$6^2:S_3$,	: S_7	$6^2:D_{12}$,	1a+21a+28a	N(...5B,..)		vertex
2520	50	A_7	7:3x3	$L_2(7):2$	(7:3x3):2	1a+21a+28b	N(...5C,..)		co-clique
2520	50	A_7				1a+21a+28c	N(...5D,..)		co-clique
1000	126	$5_+^{1+2}:8$	: $5_+^{1+2}:24$	: $5_+^{1+2}:8:2$	: $5_+^{1+2}:24:2$	1a+125a	N(5A)	isotropic point	
720	175	$M_{10} \cong A_6 \cdot 2_3$	$3^2:2A_4$	: $A_6 \cdot 2^2$	$3^2:2S_4$	1a+21a+28a+125a	N(...5B,..)		edge
720	175	$M_{10} \cong A_6 \cdot 2_3$				1a+21a+28b+125a	N(...5C,..)		
720	175	$M_{10} \cong A_6 \cdot 2_3$				1a+21a+28c+125a	N(...5D,..)		
240	525	$2S_5$	: $3 \times 2S_5$	: $2S_5.2$	: $(3 \times 2S_5).2$		N(2A)	non-isotropic point	Petersen graph

$U_3(5)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	;	;	;	;	
		126000	240	36	8	250	25	25	25	12	7	7	8	8	10			240	21	240	12	8	10	7	7	8	8	10			120	120	6	4	5	6	10	10					
		p power	A	A	A	A	A	A	A	AA	A	A	A	A	AA			A	A	BA	BA	BA	AB	BC	AC	AA	AB	AAB			A	A	AB	A	BB	AB	AB	AB					
		p' part	A	A	A	A	A	A	A	AA	A	A	A	A	AA			A	A	BA	BA	BA	AB	AC	BC	BA	BB	AAB			A	A	AB	A	BB	AB	AB	AB					
		ind 1A	2A	3A	4A	5A	5B	5C	5D	6A	7A	B**	8A	B**	10A	fus	ind	3B	3C	6B	6C	12A	15A	21A	B*	24A	B*	30A	fus	ind	2B	4B	6D	8C	10B	12B	20A	B**	fus	ind	fus	ind	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	+oo	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	:	++	:	++	x_1
x_2	−	20	−4	2	0	−5	0	0	0	2	−1	−1	0	0	1	:	−oo	−4	−1	−4	2	0	1	−1	−1	0	0	1	:	oo	0	0	0	0	0	0	i5	−i5	:	oo	:	oo	x_2
x_3	+	21	5	3	1	−4	1	1	1	−1	0	0	−1	−1	0	:	+oo	−3	0	5	−1	1	2	0	0	−1	−1	0	:	++	1	5	1	−1	1	−1	0	0	:	++	:	++	x_3
x_4	+	28	4	1	0	3	3	−2	−2	1	0	0	0	0	−1		+	0	0	0	0	0	0	0	0	0	0	0	:	++	4	4	1	0	−1	1	−1	−1		+		+	x_4
x_5	+	28	4	1	0	3	−2	3	−2	1	0	0	0	0	−1															+	0	0	0	0	0	0	0	0	:	++			x_5
x_6	+	28	4	1	0	3	−2	−2	3	1	0	0	0	0	−1																										:	++	x_6
x_7	+	84	−4	3	0	9	−1	−1	−1	−1	0	0	0	0	1	:	+oo	0	0	8	2	0	0	0	0	0	0	−2	:	++	4	−4	1	0	−1	−1	1	1	:	++	:	++	x_7
x_8	+	105	1	−3	1	5	0	0	0	1	0	0	−1	−1	1	:	+oo	9	0	1	1	1	−1	0	0	−1	−1	1	:	++	5	1	−1	−1	0	1	1	1	:	++	:	++	x_8
x_9	+	125	5	−1	1	0	0	0	0	−1	−1	−1	1	1	0	:	+oo	5	−1	5	−1	1	0	−1	−1	1	1	0	:	++	5	5	−1	1	0	−1	0	0	:	++	:	++	x_9
x_{10}	+	126	6	0	−2	1	1	1	1	0	0	0	0	0	1	:	+oo	6	0	6	0	−2	1	0	0	0	0	1	:	++	6	−6	0	0	1	0	−1	−1	:	++	:	++	x_{10}
x_{11}	o	126	−6	0	0	1	1	1	1	0	0	0	i2	−i2	−1	:	ooo	6	0	−6	0	0	1	0	0	i2	−i2	−1		+	0	0	0	0	0	0	0	0		+		+	x_{11}
x_{12}	o	126	−6	0	0	1	1	1	1	0	0	0	−i2	i2	−1	:	ooo	6	0	−6	0	0	1	0	0	−i2	i2	−1															x_{12}
x_{13}	o	144	0	0	0	−6	−1	−1	−1	0	−b7	**	0	0	0	:	ooo	0	−3	0	0	0	0	−b7	**	0	0	0		+	0	0	0	0	0	0	0	0		+		+	x_{13}
x_{14}	o	144	0	0	0	−6	−1	−1	−1	0	**	−b7	0	0	0	:	ooo	0	−3	0	0	0	0	**	−b7	0	0	0															x_{14}
	ind	1	2	3	4	5	5	5	5	6	7	7	8	8	10	fus	ind	3	9	6	6	12	15	63	63	24	24	30	fus	ind	2	4	6	8	10	12	20	20	fus	ind	fus	ind	
		3	6		12	15	15	15	15	6	21	21	24	24	30			3		6	6	12	15	63	63	24	24	30															
		3	6		12	15	15	15	15	6	21	21	24	24	30			3		6	6	12	15	63	63	24	24	30															
x_{15}	o2	21	−3	0	1	−4	1	1	1	0	0	0	1	1	2	:ooo2		2i3+3	0	2i3+3	−i3	1	−z3	0	0	1	1	−z3	*	+									*	+	*	+	x_{15}
x_{16}	o2	21	5	0	1	−4	1	1	1	2	0	0	−1	−1	0	:ooo2		2i3+3	0	−2i3−1	−1	1	−z3	0	0	−1	−1	z3+2	*	+									*	+	*	+	x_{16}
x_{17}	o2	48	0	0	0	−2	3	−2	−2	0	−1	−1	0	0	0		o2	0	0	0	0	0	0	0	0	0	0	0	*	+										o2		o2	x_{17}
x_{18}	o2	48	0	0	0	−2	−2	3	−2	0	−1	−1	0	0	0															o2									*	+	*		x_{18}
x_{19}	o2	48	0	0	0	−2	−2	−2	3	0	−1	−1	0	0	0														*										*		*	+	x_{19}
x_{20}	o2	84	−4	0	0	9	−1	−1	−1	2	0	0	0	0	1	:ooo2		−4i3	0	−4	−i3−1	0	i3	0	0	0	0	1	*	+									*	+	*	+	x_{20}
x_{21}	o2	105	9	0	1	5	0	0	0	0	0	0	1	1	−1	:ooo2		−2i3+3	0	−2i3+3	i3	1	z3+1	0	0	1	1	z3+1	*	+									*	+	*	+	x_{21}
x_{22}	o2	105	1	0	1	5	0	0	0	−2	0	0	−1	−1	1	:ooo2		−2i3+3	0	2i3+7	1	1	z3+1	0	0	−1	−1	−z3−1	*	+									*	+	*	+	x_{22}
x_{23}	o2	126	6	0	−2	1	1	1	1	0	0	0	0	0	1	:ooo2		6	0	6	0	−2	1	0	0	0	0	1	*	+									*	+	*	+	x_{23}
x_{24}	o2	126	−6	0	0	1	1	1	1	0	0	0	i2	−i2	−1	:ooo2		6	0	−6	0	0	1	0	0	i2	−i2	−1		+2										+2		+2	x_{24}
x_{25}	o2	126	−6	0	0	1	1	1	1	0	0	0	−i2	i2	−1	:ooo2		6	0	−6	0	0	1	0	0	−i2	i2	−1	*										*		*		x_{25}
x_{26}	o2	144	0	0	0	−6	−1	−1	−1	0	−b7	**	0	0	0	:ooo2		0	0	0	0	0	0	−x'63	*55	0	0	0		+2										+2		+2	x_{26}
x_{27}	o2	144	0	0	0	−6	−1	−1	−1	0	**	−b7	0	0	0	:ooo2		0	0	0	0	0	0	*55	−x'63	0	0	0	*										*		*		x_{27}

J_1

Sporadic Janko group J_1

Order = 175,560 = $2^3.3.5.7.11.19$ Mult = 1 Out = 1

Constructions

Janko G is a subgroup of $G_2(11)$: in the 7-space of pure imaginary Cayley numbers over F_{11}, spanned by i_t (subscripts mod 7), G may be generated by the elements

$P : i_t \to i_{t+1}$; $Q : i_t \to i_{2t}$; $R : i_t \to \pm i_t$ (- for t=0,3,5,6) ;

$S : i_t \to -2i_{-t} + (i_{1-t}+i_{2-t}+i_{4-t}) + 3(i_{6-t}+i_{5-t}+i_{3-t})$.

In the corresponding projective 6-space there is an orbit of 1596 (isotropic) "Whitelaw points" each lying in two out of 266 normal sextic curves of 12 such points.

Graph G : the automorphism group of an 11-valent graph on 266 points in which the point stabilizer is $L_2(11)$ and the suborbits have size 1 (distance 0), 11 (distance 1), 110 (distance 2), 132 (distance 3) and 12 (distance 4, or opposite). There are 11 involutions indicated by superscripts i,j from {0,1,2,...,8,9,X}, the involution 0 being DCDCD below. In these terms the 266 points can be written P (1), P^i (11), $P^{i,j}$ (110), Q_x (12), and $Q_x{}^i$ (132), for x = ∞,0,1,...,9,X; P is joined to P^i and opposite to Q_x. The group is generated by the elements

A : t → t+1, B : t → 3t, on i,j,x

C : (2X)(34)(59)(67) on i,j, and (∞0)(1X)(25)(37)(48)(69) on x

$D : (P^{t,t+q} , Q_{q-t}{}^{5q-t}) (P^{t,t-q} , P^{-t,-t-q}) (Q_t{}^{t+q} , P^{3q-t,6q-t}) (Q_t{}^{t-q} , Q_{-t-2q}{}^{-t-3q})$

$(P , Q_\infty) (P^t , Q_{-t}{}^{-t}) (Q_t , Q_\infty{}^{-t})$, (t = 0,1,...,9,X, q = 1,3,4,5,9)

Orthogonal (2) In the 20-space over F_2 of vectors $v = \Sigma x_t e_t + \Sigma y_t f_t$ (subscripts mod 11) with $\Sigma x_t = \Sigma y_t = 0$ and invariant quadratic form $q(v) = \Sigma x_t y_t$, the above generators act as follows:

$A : e_t \to e_{t+1}$ $f_t \to f_{t+1}$

$B : e_t \to e_{3t}$ $f_t \to f_{3t}$

$C : e_0 \to e_0$ $f_0 \to f_0 + e_0'$, where $e_t' = \Sigma e_i$ $(i \neq t)$

$e_t \to e_{1/t}$ $f_t \to f_{1/t} + e_0 + e_{3/t} + e_{4/t} + e_{8/t}$ (t = 1,3,4,5,9)

$e_t \to e_{9/t}$ $f_t \to f_{9/t} + e_0 + e_{3/t} + e_{4/t} + e_{8/t} + e_{1/t} + e_{5/t}$ (t = 2,6,7,8,10)

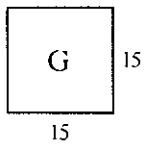
$D : e_t \to f_{-t}$ $f_t \to e_{-t}$

Presentation $G = \langle \underline{a \quad b\ 5\ c \quad d\ 5\ e} \mid (abc)^5=1, (cde)^5=a \rangle$

Remark : G is the centralizer in the O'Nan group of an outer automorphism of order 2.

Maximal subgroups				Specifications			
Order	Index	Structure	Character	Abstract	Graph	Janko	Orthogonal (2)
660	266	$L_2(11)$	1a+56ab+76a+77a		point	sextic curve	10-space : $\{e_t'\}$
168	1045	2^3:7:3		$N(2A^3)$		base : $\{\pm i_t\}$	
120	1463	2 x A_5		N(2A)	edge		point : e_0'
114	1540	19:6		N(19ABC)		non-isotropic point	
110	1596	11:10		N(11A)	opposite points	Whitelaw point	base : $\{e_t',f_t'\}$
60	2926	D_6 x D_{10}		N(3A), N(5AB)	pentagon	non-isotropic point	point
42	4180	7:6		N(7A)		point : (1111111)	

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		175560	120	30	30	30	6	7	10	10	11	15	15	19	19	19
	p power		A	A	A	A	AA	A	BA	AA	A	BA	AA	A	A	A
	p' part		A	A	A	A	AA	A	AA	BA	A	AA	BA	A	A	A
	ind	1A	2A	3A	5A	B*	6A	7A	10A	B*	11A	15A	B*	19A	B*2	C*4
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	+	56	0	2	-2b5	*	0	0	0	0	1	b5	*	-1	-1	-1
χ_3	+	56	0	2	*-2b5		0	0	0	0	1	*	b5	-1	-1	-1
χ_4	+	76	4	1	1	1	1	-1	-1	-1	-1	1	1	0	0	0
χ_5	+	76	-4	1	1	1	-1	-1	1	1	-1	1	1	0	0	0
χ_6	+	77	5	-1	2	2	-1	0	0	0	0	-1	-1	1	1	1
χ_7	+	77	-3	2	b5	*	0	0	b5	*	0	b5	*	1	1	1
χ_8	+	77	-3	2	*	b5	0	0	*	b5	0	*	b5	1	1	1
χ_9	+	120	0	0	0	0	0	1	0	0	-1	0	0	c19	*2	*4
χ_{10}	+	120	0	0	0	0	0	1	0	0	-1	0	0	*4	c19	*2
χ_{11}	+	120	0	0	0	0	0	1	0	0	-1	0	0	*2	*4	c19
χ_{12}	+	133	5	1	-2	-2	-1	0	0	0	1	1	1	0	0	0
χ_{13}	+	133	-3	-2	-b5	*	0	0	b5	*	1	-b5	*	0	0	0
χ_{14}	+	133	-3	-2	*	-b5	0	0	*	b5	1	*	-b5	0	0	0
χ_{15}	+	209	1	-1	-1	-1	1	-1	1	1	0	-1	-1	0	0	0



Alternating group A_9

Order = 181,440 = $2^6.3^4.5.7$ Mult = 2 Out = 2

Constructions

Alternating $S_9 \cong G.2$: all permutations of 9 letters;

$A_9 \cong G$: all even permutations; 2.G and 2.G.2 : the Schur double covers

Orthogonal 2G has an 8-dimensional real orthogonal representation, which may be generated (with subscripts modulo 7) by

$A : e_t \to e_{t+1}$, $B : e_t \to e_{2t}$, $C : e_t \to e_{-1/t}$, $D : e_t \to \pm e_t$ (− for 0,3,5,6) and

$E : 4e_\infty \to -2e_\infty-3e_0-e_3-e_5-e_6$ $\quad 4e_0 \to -2e_0+3e_\infty+e_1+e_2+e_4$

$4e_n \to -2e_n-3e_{3n}+e_\infty-e_{5n}+e_{6n}$ (n = 3,5,6) $\quad 4e_q \to -2e_q+3e_{5q}-e_0-e_{6q}+e_{3q}$ (q = 1,2,4)

which permute $\{\infty,0,\ldots 6,Z\}$ as (0123456), (124)(365), (∞0)(16)(23)(45), (∞0)(23)(15)(46), and (0∞Z) respectively

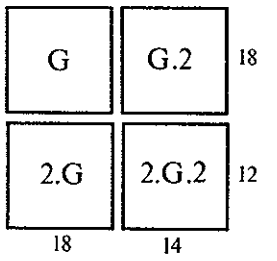
Presentations $G \cong \langle x_1,\ldots x_7 | x_i^3=(x_ix_j)^2=1\rangle$; G.2 ≅

Maximal subgroups

Order	Index	Structure	G.2	Character
20160	9	A_8	: S_8	1a+8a
5040	36	S_7	: S_7 x 2	1a+8a+27a
2160	84	$(A_6 \times 3):2$	: $S_6 \times S_3$	1a+8a+27a+48a
1512	120	$L_2(8):3$		1a+35a+84a
1512	120	$L_2(8):3$		1a+35b+84a
1440	126	$(A_5 \times A_4):2$	: $S_5 \times S_4$	1a+8a+27a+42a+48a
648	280	$3^3:S_4$	: $3^3:(2 \times S_4)$	1a+27a+48a+84a+120a
216	840	$3^2:2A_4$	: $3^2:2S_4$	

Specifications

Abstract	Alternating
	point
C(2C)	duad
N(3A)	triad
N(2B,3B,7A,9A)	projective line
N(2B,3B,7A,9B)	projective line
N(2A^2), N(2A,3A,5A)	tetrad
N(3^3) = N(3A$_3$B$_4$C$_6$)	trisection
N(3B^2)	S(2,3,9)



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;
					1															
		181440	480	192	080	81	54	24	16	60	24	6	7	9	9	20	12	15	15	
	p	power	A	A	A	A	A	A	B	A	AA	CB	A	B	B	AA	AA	AA	AA	
	p'	part	A	A	A	A	A	A	A	A	AA	CB	A	A	A	AA	AA	AA	AA	
	ind		1A	2A	2B	3A	3B	3C	4A	4B	5A	6A	6B	7A	9A	9B	10A	12A	15A	B**	fus ind
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	: ++
χ_2	+	8	4	0	5	−1	2	2	0	3	1	0	1	−1	−1	−1	−1	0	0	: ++
χ_3	o	21	1	−3	−3	3	0	−1	1	1	1	0	0	0	0	1	−1	b15	**	+
χ_4	o	21	1	−3	−3	3	0	−1	1	1	1	0	0	0	0	1	−1	**	b15	
χ_5	+	27	7	3	9	0	0	1	−1	2	1	0	−1	0	0	2	1	−1	−1	: ++
χ_6	+	28	4	−4	10	1	1	0	0	3	−2	−1	0	1	1	−1	0	0	0	: ++
χ_7	+	35	−5	3	5	−1	2	−1	−1	0	1	0	0	2	−1	0	−1	0	0	+
χ_8	+	35	−5	3	5	−1	2	−1	−1	0	1	0	0	−1	2	0	−1	0	0	
χ_9	+	42	6	2	0	−3	3	0	2	−3	0	−1	0	0	0	1	0	0	0	: ++
χ_{10}	+	48	8	0	6	3	0	0	0	−2	2	0	−1	0	0	−2	0	1	1	: ++
χ_{11}	+	56	−4	0	11	2	2	−2	0	1	−1	0	0	−1	−1	1	1	1	1	: ++
χ_{12}	+	84	4	4	−6	3	3	0	0	−1	−2	1	0	0	0	−1	0	−1	−1	: ++
χ_{13}	+	105	5	1	15	−3	−3	−1	1	0	−1	1	0	0	0	0	−1	0	0	: ++
χ_{14}	+	120	0	8	0	3	−3	0	0	0	0	−1	1	0	0	0	0	0	0	: ++
χ_{15}	+	162	6	−6	0	0	0	0	−2	−3	0	0	1	0	0	1	0	0	0	: ++
χ_{16}	+	168	4	0	−15	−3	0	−2	0	3	1	0	0	0	0	−1	1	0	0	: ++
χ_{17}	+	189	−11	−3	9	0	0	1	1	−1	1	0	0	0	0	−1	1	−1	−1	: ++
χ_{18}	+	216	−4	0	−9	0	0	2	0	1	−1	0	−1	0	0	1	−1	1	1	: ++
	ind	1	4	2	3	3	3	8	4	5	12	6	7	9	9	20	24	15	15	fus ind
		2			6	6	6			10		6	14	18	18		24	30	30	
χ_{19}	+	8	0	0	−4	−1	2	0	0	−2	0	0	1	2	−1	0	0	1	1	+
χ_{20}	+	8	0	0	−4	−1	2	0	0	−2	0	0	1	−1	2	0	0	1	1	
χ_{21}	o	48	0	0	6	3	0	0	0	−2	0	0	−1	0	0	0	i6	1	1	−
χ_{22}	o	48	0	0	6	3	0	0	0	−2	0	0	−1	0	0	0	−i6	1	1	
χ_{23}	+	56	0	0	−16	2	2	0	0	−4	0	0	0	−1	−1	0	0	−1	−1	: oo
χ_{24}	+	112	0	0	4	4	4	0	0	2	0	0	0	1	1	0	0	−1	−1	: oo
χ_{25}	o	120	0	0	0	3	−3	0	0	0	0	i3	1	0	0	0	0	0	0	−
χ_{26}	o	120	0	0	0	3	−3	0	0	0	0	−i3	1	0	0	0	0	0	0	
χ_{27}	+	160	0	0	−20	−2	−2	0	0	0	0	0	−1	1	1	0	0	0	0	: oo
χ_{28}	o	168	0	0	12	−3	0	0	0	−2	0	0	0	0	0	0	0	b15	**	−
χ_{29}	o	168	0	0	12	−3	0	0	0	−2	0	0	0	0	0	0	0	**	b15	
χ_{30}	+	224	0	0	−4	−1	2	0	0	4	0	0	0	−1	−1	0	0	1	1	: oo

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;
		5														
		040	144	240	16	72	72	18	18	9	4	10	12	7	10	
		A	A	A	A	AC	AD	CC	CD	BD	B	AC	AC	AC	AC	
		A	A	A	A	AC	AD	CC	CD	BD	A	AC	AC	AC	AC	
	ind	2C	2D	4C	4D	6C	6D	6E	6F	6G	8A	10B	12B	14A	20A	
χ_1	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
χ_2	++	6	2	4	0	3	−1	0	2	−1	0	1	1	−1	−1	
χ_3	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
χ_4																
χ_5	++	15	3	5	1	3	3	0	0	0	−1	0	−1	1	0	
χ_6	++	14	−2	6	−2	2	−2	−1	1	1	0	−1	0	0	1	
χ_7	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
χ_8																
χ_9	++	14	2	−4	0	2	2	−1	−1	−1	0	−1	2	0	1	
χ_{10}	++	20	4	0	0	2	−2	2	−2	1	0	0	0	−1	0	
χ_{11}	++	14	−6	4	0	−1	3	2	0	0	0	−1	1	0	−1	
χ_{12}	++	14	−2	−6	2	2	−2	−1	1	1	0	−1	0	0	−1	
χ_{13}	++	35	−1	5	1	−1	−1	−1	−1	−1	1	0	−1	0	0	
χ_{14}	++	20	4	0	0	−4	4	−1	1	1	0	0	0	−1	0	
χ_{15}	++	36	0	−6	−2	0	0	0	0	0	0	1	0	1	−1	
χ_{16}	++	14	2	−4	0	−1	−1	2	2	−1	0	−1	−1	0	1	
χ_{17}	++	21	−3	1	1	−3	−3	0	0	0	−1	1	1	0	1	
χ_{18}	++	6	−6	−4	0	3	3	0	0	0	0	1	−1	−1	1	
	ind	2	4	8	8	6	12	6	12	12	8	10	24	14	40	
										12	8			14	40	
χ_{19}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
χ_{20}																
χ_{21}	−	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
χ_{22}																
χ_{23}	oo	0	0	0	0	0	0	0	0	0	2i	0	0	0	0	
χ_{24}	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	i10	
χ_{25}	−	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
χ_{26}																
χ_{27}	oo	0	0	0	0	0	0	0	0	0	0	0	0	i7	0	
χ_{28}	−	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
χ_{29}																
χ_{30}	oo	0	0	0	0	0	0	0	0	0	3i	0	0	0	0	

$L_3(5)$

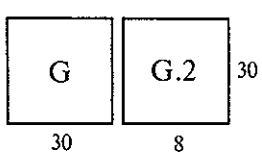
Linear group $L_3(5) \cong A_2(5)$

Order = 372,000 = $2^5.3.5^3.31$ Mult = 1 Out = 2

Constructions

Linear $GL_3(5) \cong 4 \times G$: all non-singular 3 x 3 matrices over $\mathbb{F}_5$;

$PGL_3(5) \cong SL_3(5) \cong PSL_3(5) \cong G$



Maximal subgroups					Specifications	
Order	Index	Structure	G.2	Character	Abstract	Linear
12000	31	5^2:$GL_2(5)$	5_+^{1+2}:$[2^5]$,	1a+30a	$N(5A^2)$	point
12000	31	5^2:$GL_2(5)$	$4S_5.2$	1a+30a	$N(5A^2)$	line
120	3100	S_5	: S_5 x 2		N(2A,3A,5B), C(2B)	$O_3(5)$
96	3875	4^2:S_3	: 4^2:D_{12}		$N(2A^2)$	base
93	4000	31:3	: 31:6		N(31ABCDEFGHIJ)	$L_1(125)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	
		372000	480	24	480	480	16	500	25	24	24	24	20	24	24	20	20	24	24	24	24	31	31	31	31	31	31	31	31	31	31				120	120	6	4	5	6	10	10	
	p	power	A	A	A	A	A	A	A	AA	A	B	AA	AB	AA	AA	AB	AB	BA	AB	BA	A	A	A	A	A	A	A	A	A	A				A	A	AB	C	BB	AD	AD	AD	
	p'	part	A	A	A	A	A	A	A	AA	A	A	AA	AA	AB	AA	AB	AA	AB	AA	AB	A	A	A	A	A	A	A	A	A	A				A	A	AB	A	BB	AD	AD	AD	
	ind	1A	2A	3A	4A	B**	4C	5A	5B	6A	8A	B**	10A	12A	B**	20A	B**	24A	B**	C**7	D*7	31A	B**	C*2D**	2	E*4F**	4	G*8H**	8I*16J*	15	fus	ind	2B	4D	6R	8C	10B	12C	20C	D*	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	χ_1
χ_2	+	30	6	0	6	6	2	5	0	0	0	0	1	0	0	1	1	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	:	++	0	0	0	0	0	0	r5	-r5	χ_2
χ_3	+	31	7	1	-5	-5	-1	6	1	1	-1	-1	2	1	1	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	:	++	1	5	1	-1	1	-1	0	0	χ_3
χ_4	o	31	-5	1	6i-1	-6i-1	1	6	1	1	-i	i	0	-1	-1	i-1	-i-1	-i	i	-i	i	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	χ_4
χ_5	o	31	-5	1	-6i-1	6i-1	1	6	1	1	i	-i	0	-1	-1	-i-1	i-1	i	-i	i	-i	0	0	0	0	0	0	0	0	0											χ_5
χ_6	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	x31	**	*2	**2	*4	**4	*8	**8	*16	*15		+	0	0	0	0	0	0	0	0	χ_6
χ_7	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	**	x31	**2	*2	**4	*4	**8	*8	*15	*16											χ_7
χ_8	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	*16	*15	x31	**	*2	**2	*4	**4	*8	**8		+	0	0	0	0	0	0	0	0	χ_8
χ_9	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	*15	*16	**	x31	**2	*2	**4	*4	**8	*8											χ_9
χ_{10}	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	*8	**8	*16	*15	x31	**	*2	**2	*4	**4		+	0	0	0	0	0	0	0	0	χ_{10}
χ_{11}	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	**8	*8	*15	*16	**	x31	**2	*2	**4	*4											χ_{11}
χ_{12}	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	*4	**4	*8	**8	*16	*15	x31	**	*2	**2		+	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	**4	*4	**8	*8	*15	*16	**	x31	**2	*2											χ_{13}
χ_{14}	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	*2	**2	*4	**4	*8	**8	*16	*15	x31	**		+	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	o	96	0	0	0	0	0	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	**2	*2	**4	*4	**8	*8	*15	*16	**	x31											χ_{15}
χ_{16}	+	124	4	1	4	4	0	-1	-1	1	2	2	-1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	:	++	4	-4	1	0	-1	-1	1	1	χ_{16}
χ_{17}	+	124	4	1	4	4	0	-1	-1	1	-2	-2	-1	1	1	-1	-1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	:	++	4	4	1	0	-1	1	-1	-1	χ_{17}
χ_{18}	o	124	4	1	-4	-4	0	-1	-1	1	2i	-2i	-1	-1	-1	1	1	-i	i	-i	i	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	χ_{18}
χ_{19}	o	124	4	1	-4	-4	0	-1	-1	1	-2i	2i	-1	-1	-1	1	1	i	-i	i	-i	0	0	0	0	0	0	0	0	0	0											χ_{19}
χ_{20}	o	124	-4	-2	4i	-4i	0	-1	-1	2	0	0	1	-2i	2i	-i	i	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	χ_{20}
χ_{21}	o	124	-4	-2	-4i	4i	0	-1	-1	2	0	0	1	2i	-2i	i	-i	0	0	0	0	0	0	0	0	0	0	0	0	0	0											χ_{21}
χ_{22}	o	124	-4	1	4i	-4i	0	-1	-1	-1	0	0	1	i	-i	-i	i	y'24	**	**7	*7	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	χ_{22}
χ_{23}	o	124	-4	1	-4i	4i	0	-1	-1	-1	0	0	1	-i	i	i	-i	**	y'24	*7	**7	0	0	0	0	0	0	0	0	0	0											χ_{23}
χ_{24}	o	124	-4	1	4i	-4i	0	-1	-1	-1	0	0	1	i	-i	-i	i	**7	*7	y'24	**	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	χ_{24}
χ_{25}	o	124	-4	1	-4i	4i	0	-1	-1	-1	0	0	1	-i	i	i	-i	*7	**7	**	y'24	0	0	0	0	0	0	0	0	0	0											χ_{25}
χ_{26}	+	125	5	-1	5	5	1	0	0	-1	-1	-1	0	-1	-1	0	0	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	:	++	5	5	-1	1	0	-1	0	0	χ_{26}
χ_{27}	+	155	11	-1	-1	-1	-1	5	0	-1	1	1	1	-1	-1	-1	-1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	:	++	5	1	-1	-1	0	1	1	1	χ_{27}
χ_{28}	o	155	-1	-1	6i-5	-6i-5	1	5	0	-1	i	-i	-1	1	1	i	-i	i	-i	i	-i	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	o	155	-1	-1	-6i-5	6i-5	1	5	0	-1	-i	i	-1	1	1	-i	i	-i	i	-i	i	0	0	0	0	0	0	0	0	0	0											χ_{29}
χ_{30}	+	186	-6	0	6	6	-2	11	1	0	0	0	-1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	6	-6	0	0	1	0	-1	-1	χ_{30}

Sporadic Mathieu group M_{22}

Order = 443,520 = $2^7.3^2.5.7.11$ Mult = 12 Out = 2

Constructions

Mathieu For almost all purposes, M_{22} is best studied as a subgroup of M_{24} (q.v.). Thus $M_{22}.2 \cong G.2$ is the set stabilizer of a duad in M_{24}, and $M_{22} \cong G$ is the pointwise stabilizer.

$M_{22}.2 \cong G.2$: the automorphism group of the Steiner system S(3,6,22), whose 77 hexads are derived from the octads of S(5,8,24) containing the fixed duad.

Graph G.2 : the automorphism group of a rank 3 graph on 77 points, with suborbits 1, 16, 60, and point stabilizer $2^4{:}S_6$. (Every hexad of S(3,6,22) is disjoint from 16 others and meets the remaining 60 in 2 points each.)

Lattice $2M_{22}.2 \cong 2.G.2$: the automorphism group of a 10-dimensional lattice with coordinates (in $\mathbf{Z}[b7]$) indexed by the triples of 6 points, modulo complementation. The lattice is defined by requiring all coordinates x_{abc} to be congruent modulo b7 and their sum to be divisible by 2b7**, while their sums over sets of the form S(ab.cd.ef) = {ace,acf,ade,adf} and S(ab) = {abc,abd,abe,abf} must be divisible by 2 and $(b7)^2$ respectively. There is a monomial group $2^5{:}S_6$ generated by permutations of {a,b,c,d,e,f} and sign changes on the sets S(ab.cd.ef). With the natural norm $\frac{1}{4}\Sigma|x_{abc}|^2$ the vectors of norms 4 and 5 fall into four orbits 4X, 4Y, 5X, 5Y. The vectors of 4X are the images under the monomial group of

(4,0,0,0,0,0,0,0,0,0)

(i7,1,1,1,1,1,1,1,1,1)

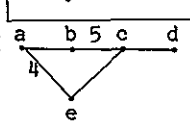
(1,1,1,1,b7**,b7**,b7**,b7**,b7**,b7**) (1 on S(ab))

(b7,b7,b7,b7,0,0,0,0,2,-2) (b7 on S(ab), 0 on S(ab.cd.ef))

and fall into 77 congruence bases modulo b7**.

Unitary In a 6-dimensional vector space of vectors (ab cd ef) over $\mathbf{F}_4$ with unitary norm $\bar{a}b+a\bar{b}+\bar{c}d+c\bar{d}+\bar{e}f+e\bar{f}$, the group $3M_{22} \cong 3G$ is the automorphism group of a configuration of 22 isotropic vector 3-spaces. Each non-zero isotropic vector appears in just two of these spaces. The coordinates (ab cd ef) of a typical vector in each of the 22 3-spaces are given in the MOG array below, where x,y,z are independent elements of $\mathbf{F}_4 = \{0,1,w,\bar{w}\}$, X = y+z, Y = x+z, Z = x+y, S = x+y+z, and A' = wA, B" = $\bar{w}$B :

		x0 y0 z0	SX SY SZ	SX S'Y' S"Z"	S"X" S'Y' SZ
x"X" yY zZ	x'X' yY zZ	x0 0y 0z	SX YS ZS	SX Y'S' Z"S"	S"X" Y'S' ZS
xX y"Y" zZ	xX y'Y' zZ	0x y0 0z	XS SY ZS	XS S'Y' Z"S"	X"S" S'Y' ZS
xX yY z"Z"	xX yY z'Z'	0x 0y z0	XS YS SZ	XS Y'S' S"Z"	X"S" Y'S' SZ

Presentation 3.G ≅ < 

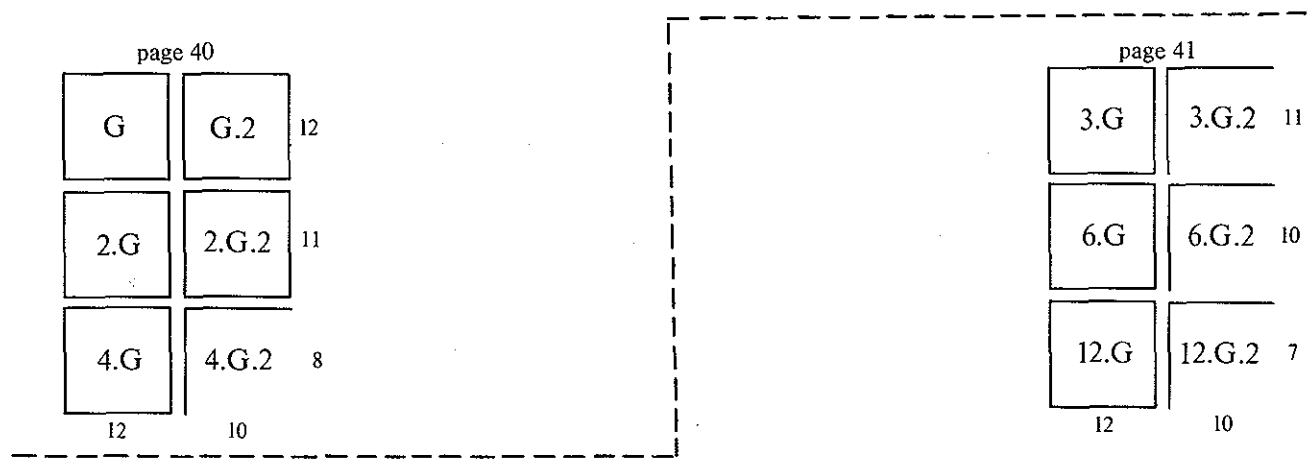
| (eab)^3=(abc)^5=(bce)^5[=(bcd)^5]=(aecd)^4=1 > : adjoin (abce)^8 = 1 for G.

Maximal subgroups

Order	Index	Structure	G.2	Character
20160	22	$L_3(4)$	: $L_3(4){:}2_2$	1a+21a
5760	77	$2^4{:}A_6$	: $2^4{:}S_6$	1a+21a+55a
2520	176	A_7		1a+21a+154a
2520	176	A_7		1a+21a+154a
1920	231	$2^4{:}S_5$	: $2^5{:}S_5$	1a+21a+55a+154a
1344	330	$2^3{:}L_3(2)$	: $2^3{:}L_3(2) \times 2$	1a+21a+55a+99a+154a
720	616	$M_{10} \cong A_6{\cdot}2_3$	: $A_6{\cdot}2^2$	1a+21a+55a+154a+385a
660	672	$L_2(11)$	: $L_2(11){:}2$	1a+21a+55a+154a+210a+231a

Specifications

Abstract	Mathieu	Lattice	Unitary
	point	5X 1-space mod b7	isotropic 3-space
$N(2A^4)$	hexad	4X or 5X 1-space mod b7**	isotropic 3-space
	heptad		
	heptad		
$N(2A^4)$	duad	4Y 1-space mod b7	isotropic 1-space
$N(2A^3)$, C(2B)	octad	4Y 1-space mod b7**	isotropic 3-space
	(do)decad	5Y 1-space mod b7**	pair of isotropic 3-spaces
	endecad		non-isotropic 1-space



M_{22}

	;	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@
																1										
	443520	384	36	32	16	5	12	7	7	8	11	11				344	320	48	32	6	8	5	6	7	7
	p power	A	A	A	A	A	AA	A	A	A	A	A				A	A	A	A	AB	A	AC	AC	AB	BB
	p' part	A	A	A	A	A	AA	A	A	A	A	A				A	A	A	A	AB	A	AC	AC	AB	BB
	ind 1A	2A	3A	4A	4B	5A	6A	7A	B**	8A	11A	B**	fus	ind		2B	2C	4C	4D	6B	8B	10A	12A	14A	B**
χ_1	+ 1	1	1	1	1	1	1	1	1	1	1	1	:	++		1	1	1	1	1	1	1	1	1	1
χ_2	+ 21	5	3	1	1	1	-1	0	0	-1	-1	-1	:	++		7	-1	-1	3	1	1	-1	-1	0	0
χ_3	o 45	-3	0	1	1	0	0	b7	**	-1	1	1	:	oo		3	-5	3	-1	0	1	0	0	b7	**
χ_4	o 45	-3	0	1	1	0	0	**	b7	-1	1	1	:	oo		3	-5	3	-1	0	1	0	0	**	b7
χ_5	+ 55	7	1	3	-1	0	1	-1	-1	1	0	0	:	++		13	5	1	1	1	-1	0	1	-1	-1
χ_6	+ 99	3	0	3	-1	-1	0	1	1	-1	0	0	:	++		15	-1	3	-1	0	-1	-1	0	1	1
χ_7	+ 154	10	1	-2	2	-1	1	0	0	0	0	0	:	++		14	6	2	2	-1	0	1	-1	0	0
χ_8	+ 210	2	3	-2	-2	0	-1	0	0	0	1	1	:	++		14	-10	-2	2	-1	0	0	1	0	0
χ_9	+ 231	7	-3	-1	-1	1	1	0	0	-1	0	0	:	++		7	-9	-1	-1	1	-1	1	-1	0	0
χ_{10}	o 280	-8	1	0	0	0	1	0	0	0	b11	**		+		0	0	0	0	0	0	0	0	0	0
χ_{11}	o 280	-8	1	0	0	0	1	0	0	0	**	b11													
χ_{12}	+ 385	1	-2	1	1	0	-2	0	0	1	0	0	:	++		21	5	-3	-3	0	1	0	0	0	0
	ind 1	2	3	4	4	5	6	7	7	8	11	11	fus	ind		2	2	4	4	6	8	10	12	14	14
	2	2	6	4		10	6	14	14	8	22	22				2		4		6		10	12	14	14
χ_{13}	o 10	2	1	2	0	0	-1	b7	**	0	-1	-1	:	oo		4	0	-2	0	1	0	0	1	-b7	**
χ_{14}	o 10	2	1	2	0	0	-1	**	b7	0	-1	-1	:	oo		4	0	-2	0	1	0	0	1	**	-b7
χ_{15}	+ 56	-8	2	0	0	1	-2	0	0	0	1	1	:	++		0	0	0	0	0	0	r5	0	0	0
χ_{16}	+ 120	-8	3	0	0	0	1	1	1	0	-1	-1	:	++		8	0	4	0	-1	0	0	1	1	1
χ_{17}	o 126	6	0	-2	0	1	0	0	0	0	b11	**		+		0	0	0	0	0	0	0	0	0	0
χ_{18}	o 126	6	0	-2	0	1	0	0	0	0	**	b11													
χ_{19}	o 154	2	1	-2	0	-1	-1	0	0	2i	0	0		+		0	0	0	0	0	0	0	0	0	0
χ_{20}	o 154	2	1	-2	0	-1	-1	0	0	-2i	0	0													
χ_{21}	+ 210	10	3	2	0	0	1	0	0	0	1	1	:	++		28	0	2	0	1	0	0	-1	0	0
χ_{22}	+ 330	2	-3	2	0	0	-1	1	1	0	0	0	:	++		20	0	-2	0	-1	0	0	1	-1	-1
χ_{23}	+ 440	-8	-1	0	0	0	1	-1	-1	0	0	0	:	++		8	0	-4	0	-1	0	0	-1	1	1
	ind 1	2	3	4	8	5	6	7	7	8	11	11	fus	ind		2	4	8	4	6	8	20	24	14	14
	4	4	12	4		20	12	28	28	8	44	44				2		8		6		20	24	14	14
	2		6			10		14	14	8	22	22													
	4		12			20		28	28	8	44	44													
χ_{24}	o2 56	0	2	0	0	1	0	0	0	2z8	1	1		o2											
χ_{25}	o2 56	0	2	0	0	1	0	0	0	-2z8	1	1	*												
χ_{26}	o2 144	0	0	0	0	-1	0	-b7	**	0	1	1	*	o											
χ_{27}	o2 144	0	0	0	0	-1	0	**	-b7	0	1	1	*	o											
χ_{28}	o2 160	0	-2	0	0	0	0	-1	-1	0	-b11	**		-2											
χ_{29}	o2 160	0	-2	0	0	0	0	-1	-1	0	**	-b11	*												
χ_{30}	o2 176	0	-4	0	0	1	0	1	1	0	0	0	*	-											
χ_{31}	o2 560	0	2	0	0	0	0	0	0	0	-1	-1	*	-											

		1A	2A	3A	4A	4B	5A	6A	7A	B**	8A	11A	B**			2B	2C	4C	4D	6B	8B	10A	12A	14A	B**
	ind	1	2	3	4	4	5	6	7	7	8	11	11	fus	ind	2	2	4	4	6	8	10	12	14	14
		3	6		12	12	15	6	21	21	24	33	33												
		3	6		12	12	15	6	21	21	24	33	33												
χ_{32}	o2	21	5	0	1	1	1	2	0	0	-1	-1	-1	*	+										
χ_{33}	o2	45	-3	0	1	1	0	0	b7	**	-1	1	1	*	o										
χ_{34}	o2	45	-3	0	1	1	0	0	**	b7	-1	1	1	*	o										
χ_{35}	o2	99	3	0	3	-1	-1	0	1	1	-1	0	0	*	+										
χ_{36}	o2	105	9	0	1	1	0	0	0	0	1	-b11	**		+2										
χ_{37}	o2	105	9	0	1	1	0	0	0	0	1	**-b11		*											
χ_{38}	o2	210	2	0	-2	-2	0	2	0	0	0	1	1	*	+										
χ_{39}	o2	231	7	0	-1	-1	1	-2	0	0	-1	0	0	*	+										
χ_{40}	o2	231	-9	0	3	-1	1	0	0	0	1	0	0	*	+										
χ_{41}	o2	330	-6	0	-2	2	0	0	1	1	0	0	0	*	+										
χ_{42}	o2	384	0	0	0	0	-1	0	-1	-1	0	-1	-1	*	+										
	ind	1	2	3	4	4	5	6	7	7	8	11	11	fus	ind	2	2	4	4	6	8	10	12	14	14
		6	6	6	12	12	30	6	42	42	24	66	66			2		4		6		10	12	14	14
		3	6		12	12	15	6	21	21	24	33	33												
		2	2		4		10	6	14	14	8	22	22												
		3	6		12		15	6	21	21	24	33	33												
		6	6		12		30	6	42	42	24	66	66												
χ_{43}	o2	66	-6	0	2	0	1	0	b7	**	0	0	0	*	o										
χ_{44}	o2	66	-6	0	2	0	1	0	**	b7	0	0	0	*	o										
χ_{45}	o2	120	-8	0	0	0	0	-2	1	1	0	-1	-1	*	+										
χ_{46}	o2	126	6	0	-2	0	1	0	0	0	0	b11	**		+2										
χ_{47}	o2	126	6	0	-2	0	1	0	0	0	0	**	b11	*											
χ_{48}	o2	210	10	0	2	0	0	-2	0	0	0	1	1	*	+										
χ_{49}	o2	210	-6	0	-2	0	0	0	0	0	2i	1	1		+2										
χ_{50}	o2	210	-6	0	-2	0	0	0	0	0	-2i	1	1	*											
χ_{51}	o2	330	2	0	2	0	0	2	1	1	0	0	0	*	+										
χ_{52}	o2	384	0	0	0	0	-1	0	-1	-1	0	-1	-1	*	+										
	ind	1	2	3	4	8	5	6	7	7	8	11	11	fus	ind	2	4	8	4	6	8	20	24	14	14
		12	12	12	12	24	60	12	84	84	24	132	132			2		8		6		20	24	14	14
		6	6	6	12	24	30	6	42	42	24	66	66												
		4	4	12	4		20	12	28	28	8	44	44												
		3	6		12		15	6	21	21	24	33	33												
		12	12		12		60	12	84	84	24	132	132												
		2					10		14	14	8	22	22												
		12					60		84	84	24	132	132												
		3					15		21	21	24	33	33												
		4					20		28	28	8	44	44												
		6					30		42	42	24	66	66												
		12					60		84	84	24	132	132												
χ_{53}	o4	120	0	0	0	0	0	0	1	1	2z8	-1	-1		o4										
χ_{54}	o4	120	0	0	0	0	0	0	1	1	-2z8	-1	-1	**											
χ_{55}	o4	144	0	0	0	0	-1	0	-b7	**	0	1	1	**	o2										
χ_{56}	o4	144	0	0	0	0	-1	0	**	-b7	0	1	1	**	o2										
χ_{57}	o4	336	0	0	0	0	1	0	0	0	0	-b11	**		-4										
χ_{58}	o4	336	0	0	0	0	1	0	0	0	0	**-b11		**											
χ_{59}	o4	384	0	0	0	0	-1	0	-1	-1	0	-1	-1	**	-2										

$J_2 = HJ = F_{5-}$

Sporadic Hall-Janko group J_2

Order = 604,800 = $2^7.3^3.5^2.7$ Mult = 2 Out = 2

Constructions

Graph $J_2.2 \cong G.2$: the automorphism group of a rank 3 graph on 100 points, with suborbits 1, 36, 63, and point stabilizer $U_3(3).2 \cong G_2(2)$. This graph is one of a series continuing through $G_2(4)$ and Suz.

Quaternionic $2J_2 \cong 2.G$: a group generated by 315 quaternionic reflections. There is a diagonal subgroup 2^{3+4} of right multiplications diag(g_1,g_2,g_3) (g_t in {±1,±i,±j,±k}) with $g_1g_2g_3 = \pm 1$, and the reflecting vectors (roots) are the images under this of $(2,0,0)^S$ (3), $(0,h,h)^S$ (24), $(\bar{h},1,1)^S$ (96), and $(1,wc,b\bar{w})^S$ (192) (where $w=\frac{1}{2}(-1+i+j+k)=z3$, b=b5, c=b5*, h=z3-b5). The ring of left scalar multiplications is the icosian ring, which contains $2A_5$ as a group of units (see A_5). The quaternionic reflections $x \longrightarrow x - 2\langle x,m\rangle m/\langle m,m\rangle$ preserve the quaternionic inner product $\langle x,y\rangle = \Sigma x_t\bar{y}_t$. The Euclidean norm defined by N(x) = a+b when $\langle x,x\rangle$ = a+b√5 (a,b rational), converts the lattice generated by the above vectors into the Leech lattice. Some of the subgroups are definable over proper subrings, often realizable by complex numbers, for example $\langle hj\rangle \cong \mathbb{Z}[b7]$.

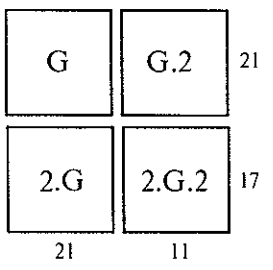
Presentations $G \cong \langle a,b,u,v \mid u=ab, v=ab^{-1}, a^2=b^3=u^{15}=(u^4v^2u^3v^3)^2=(u^3v(u^2v^2)^2)^2=1\rangle$;

$2\cdot(S_3 \times G) \cong \langle$ a 5 b c 8 d e $\mid a = (cd)^4 \rangle$: adjoin $(bcde)^{10} = 1$ for 2 x G, $(abcdecbdc)^7 = 1$ for G.

$G.2 \cong \langle$ a b c 8 d e f $\mid a=(cd)^4, (bcde)^8=1 \rangle$

Maximal subgroups / Specifications

Order	Index	Structure	G.2	Character	Abstract	Graph	Quaternionic
6048	100	$U_3(3)$	: $U_3(3):2$	1a+36a+63a		point	⟨hj,i⟩
2160	280	$3\cdot PGL_2(9)$	: $3\cdot A_6\cdot 2^2$	1a+63a+90a+126a	N(3A)	decad	⟨z3,b5⟩
1920	315	$2_-^{1+4}:A_5$	: $2_-^{1+4}.S_5$	1a+14ab+36a+90a+160a	N(2A)		reflection
1152	525	$2^{2+4}:(3xS_3)$	: H.2	1a+36a+63a+90a+160a+175a	$N(2A^2)$		base
720	840	$A_4 \times A_5$	: $(A_4xA_5):2$	1a+63a+90a+126a+160a+175a+225a	$N(2B^2)$, N(2A,3B,5AB)		icosahedron
600	1008	$A_5 \times D_{10}$	: $(A_5xD_{10})\cdot 2$	1a+14ab+90aa+126a+160a+225a+288a	N(5AB), N(2B,3A,5CD)		pentad axis
336	1800	$L_3(2):2$	: $L_3(2):2 \times 2$		N(2A,3B,4A,7A), C(2C)	edge	⟨hj⟩
300	2016	$5^2:D_{12}$	: $5^2:(4xS_3)$		$N(5^2) = N(5AB_3CD_3)$		
60	10080	A_5	: S_5		N(2B,3B,5CD)		vector



$J_2 = HJ = F_5$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	
			1		1																																
		604800	920	240	080	36	96	300	300	50	50	24	12	7	8	20	20	10	10	12	15	15				336	48	12	6	48	16	6	6	7	12	12	
		p power	A	A	A	A	A	A	A	A	A	AA	BB	A	A	BB	AB	DA	CA	AA	BA	AA				A	A	B	BC	A	A	AB	BC	AC	AB	AB	
		p' part	A	A	A	A	A	A	A	A	A	AA	BB	A	A	AB	BB	CA	DA	AA	AA	BA				A	A	A	BC	A	A	AB	BC	AC	AB	AB	
		ind	1A	2A	2B	3A	3B	4A	5A	B*	5C	D*	6A	6B	7A	8A	10A	B*	10C	D*	12A	15A	B*	fus	ind	2C	4B	4C	6C	8B	8C	12B	12C	14A	24A	B*	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++		1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	14	-2	2	5	-1	2	-3b5	*	b5+2	*	1	-1	0	0	b5	*	-b5	*	-1	0	0		+		0	0	0	0	0	0	0	0	0	0	0	χ_2
χ_3	+	14	-2	2	5	-1	2	*	-3b5	*	b5+2	1	-1	0	0	*	b5	*	-b5	-1	0	0															χ_3
χ_4	+	21	5	-3	3	0	1	b5+4	*	-2b5	*	-1	0	0	-1	b5	*	0	0	1	*	-b5		+		0	0	0	0	0	0	0	0	0	0	0	χ_4
χ_5	+	21	5	-3	3	0	1	*	b5+4	*	-2b5	-1	0	0	-1	*	b5	0	0	1	-b5	*															χ_5
χ_6	+	36	4	0	9	0	4	-4	-4	1	1	1	0	1	0	0	0	-1	-1	1	-1	-1	:	++		6	-2	0	0	2	2	1	0	-1	-1	-1	χ_6
χ_7	+	63	15	-1	0	3	3	3	3	-2	-2	0	-1	0	1	-1	-1	0	0	0	0	0	:	++		7	3	-1	1	-3	1	0	-1	0	0	0	χ_7
χ_8	+	70	-10	-2	7	1	2	-5b5	*	0	0	-1	1	0	0	-b5	*	0	0	-1	b5	*		+		0	0	0	0	0	0	0	0	0	0	0	χ_8
χ_9	+	70	-10	-2	7	1	2	*	-5b5	0	0	-1	1	0	0	*	-b5	0	0	-1	*	b5															χ_9
χ_{10}	+	90	10	6	9	0	-2	5	5	0	0	1	0	-1	0	1	1	0	0	1	-1	-1	:	++		6	2	0	0	4	0	-1	0	-1	1	1	χ_{10}
χ_{11}	+	126	14	6	-9	0	2	1	1	1	1	-1	0	0	0	1	1	-1	-1	-1	1	1	:	++		0	4	0	0	2	-2	1	0	0	-1	-1	χ_{11}
χ_{12}	+	160	0	4	16	1	0	-5	-5	0	0	0	1	-1	0	-1	-1	0	0	0	1	1	:	++		8	0	2	-1	0	0	0	-1	1	0	0	χ_{12}
χ_{13}	+	175	15	-5	-5	1	-1	0	0	0	0	3	1	0	-1	0	0	0	0	-1	0	0	:	++		7	-1	1	1	-1	-1	-1	1	0	-1	-1	χ_{13}
χ_{14}	+	189	-3	-3	0	0	-3	-3b5	*	b5+2	*	0	0	0	1	b5	*	b5	*	0	0	0		+		0	0	0	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	+	189	-3	-3	0	0	-3	*	-3b5	*	b5+2	0	0	0	1	*	b5	*	b5	0	0	0															χ_{15}
χ_{16}	+	224	0	-4	8	-1	0	2r5-1	-2r5-1	2b5	*	0	-1	0	0	1	1	0	0	0	*	-b5		+		0	0	0	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	+	224	0	-4	8	-1	0	-2r5-1	2r5-1	*	2b5	0	-1	0	0	1	1	0	0	0	-b5	*															χ_{17}
χ_{18}	+	225	-15	5	0	3	-3	0	0	0	0	0	-1	1	-1	0	0	0	0	0	0	0	:	++		1	-3	-1	1	3	-1	0	-1	1	0	0	χ_{18}
χ_{19}	+	288	0	4	0	-3	0	3	3	-2	-2	0	1	1	0	-1	-1	0	0	0	0	0	:	++		8	0	-2	-1	0	0	0	1	1	0	0	χ_{19}
χ_{20}	+	300	-20	0	-15	0	4	0	0	0	0	1	0	-1	0	0	0	0	0	1	0	0	:	++		6	-2	0	0	-2	-2	1	0	-1	1	1	χ_{20}
χ_{21}	+	336	16	0	-6	0	0	-4	-4	1	1	-2	0	0	0	0	0	1	1	0	-1	-1	:	++		0	0	0	0	0	0	0	0	0	r6	-r6	χ_{21}
	ind	1	2	4	3	3	4	5	5	5	5	6	12	7	8	20	20	10	10	12	15	15	fus	ind		4	4	8	12	8	8	12	24	28	24	24	
		2	2		6	6	4	10	10	10	10	6		14	8			10	10	12	30	30						8		8		12	24	28	24	24	
χ_{22}	-	6	-2	0	-3	0	2	-2b5	*	*	b5-1	1	0	-1	0	0	0	-b5	*	-1	b5	*		-		0	0	0	0	0	0	0	0	0	0	0	χ_{22}
χ_{23}	-	6	-2	0	-3	0	2	*	-2b5	b5-1	*	1	0	-1	0	0	0	*	-b5	-1	*	b5															χ_{23}
χ_{24}	-	14	6	0	-4	2	2	4	4	-1	-1	0	0	0	0	0	0	1	1	2	1	1	:	--		0	0	r2	0	-2r2	0	0	r2	0	r2	r2	χ_{24}
χ_{25}	o	50	10	0	5	2	2	0	0	0	0	1	0	1	2i	0	0	0	0	-1	0	0		-		0	0	0	0	0	0	0	0	0	0	0	χ_{25}
χ_{26}	o	50	10	0	5	2	2	0	0	0	0	1	0	1	-2i	0	0	0	0	-1	0	0															χ_{26}
χ_{27}	-	56	-8	0	2	2	0	-2b5	*	*	b5-1	-2	0	0	0	0	0	b5	*	0	b5	*		-		0	0	0	0	0	0	0	0	0	0	0	χ_{27}
χ_{28}	-	56	-8	0	2	2	0	*	-2b5	b5-1	*	-2	0	0	0	0	0	*	b5	0	*	b5															χ_{28}
χ_{29}	-	64	0	0	-8	-2	0	2r5+4	-2r5+4	2b5	*	0	0	1	0	0	0	0	0	0	b5	*		-		0	0	0	0	0	0	0	0	0	0	0	χ_{29}
χ_{30}	-	64	0	0	-8	-2	0	-2r5+4	2r5+4	*	2b5	0	0	1	0	0	0	0	0	0	*	b5															χ_{30}
χ_{31}	-	84	4	0	-15	0	4	-6	-6	-1	-1	1	0	0	0	0	0	-1	-1	1	0	0	:	--		0	0	0	0	0	0	r3	0	0	r3	-r3	χ_{31}
χ_{32}	-	126	-10	0	-9	0	2	3r5+1	-3r5+1	-2b5	*	-1	0	0	0	0	0	0	0	-1	1	1		-		0	0	0	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}	-	126	-10	0	-9	0	2	-3r5+1	3r5+1	*	-2b5	-1	0	0	0	0	0	0	0	-1	1	1															χ_{33}
χ_{34}	-	216	24	0	0	0	0	6	6	1	1	0	0	-1	0	0	0	-1	-1	0	0	0	:	--		0	0	0	0	0	0	0	0	r7	0	0	χ_{34}
χ_{35}	-	252	-20	0	9	0	4	2	2	2	2	1	0	0	0	0	0	0	0	1	-1	-1	:	--		0	0	0	0	0	0	r3	0	0	-r3	r3	χ_{35}
χ_{36}	-	336	16	0	-6	0	0	-4	-4	1	1	-2	0	0	0	0	0	1	1	0	-1	-1	:	--		0	0	0	0	4r2	0	0	0	0	r2	r2	χ_{36}
χ_{37}	-	350	-10	0	-10	2	-6	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	:	--		0	0	r2	0	2r2	0	0	r2	0	-r2	-r2	χ_{37}
χ_{38}	-	448	0	0	16	-2	0	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	1	1	:	--		0	0	2r2	0	0	0	0	-r2	0	0	0	χ_{38}

$S_4(4)$

Symplectic group $S_4(4) \cong C_2(4) \cong O_5(4)$

Order = 979,200 = $2^8.3^2.5^2.17$ Mult = 1 Out = 4

Constructions

Symplectic $Sp_4(4) \cong PSp_4(4) \cong S_4(4) \cong G$: all 4 x 4 matrices over $\mathbb{F}_4$ preserving a non-singular symplectic form;

$\Gamma S_4(4) \cong G.2$;

$C_2(4).2 \cong G.2$: the automorphism group of a generalised 4-gon of order (4,4), consisting of 85 vertices and 85 edges, each object being incident with 5 of the other type

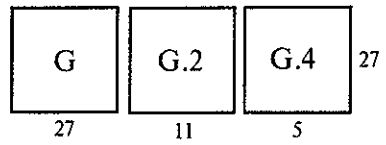
G.4 is obtained by adjoining the duality (graph) automorphism, interchanging vertices and edges

Orthogonal $GO_5(4) \cong PGO_5(4) \cong SO_5(4) \cong PSO_5(4) \cong G$: all 5 x 5 matrices over $\mathbb{F}_4$ preserving a non-singular quadratic form;

$O_5(4) \cong G$;

Presentation $G \times 2 \cong \langle \overset{a}{\bullet}\!\!-\!\!\overset{5}{\bullet}\!\!-\!\!\overset{b}{\bullet}\!\!-\!\!\overset{5}{\bullet}\!\!-\!\!\overset{c}{\bullet}\!\!-\!\!\overset{d}{\bullet}\!\!-\!\!\overset{e}{\bullet} \mid (abc)^4=1 \rangle$

Remark : G.2 is a maximal subgroup of the Held group.



Maximal subgroups / Specifications

Order	Index	Structure	G.2	G.4	Character	Abstract	Symplectic	Orthogonal
11520	85	$2^6:(3xA_5)$	: $2^6:(3xA_5):2$	$(2^2x2^{2+4}:3):12$	1a+34a+50a	$N(2A^2)$, $N(2C^4)$	point	isotropic point
11520	85	$2^6:(3xA_5)$	: $2^6:(3xA_5):2$		1a+34b+50a	$N(2B^2)$, $N(2C^4)$	isotropic line	isotropic line
8160	120	$L_2(16):2$	: $L_2(16):4$	17:16	1a+34a+85a	N(2C,3A,5AB,...)	$O_4^-(4)$	minus hyperplane
8160	120	$L_2(16):2$	: $L_2(16):4$		1a+34b+85b	N(2C,3B,5CD,...)	$S_2(16)$	
7200	136	$(A_5xA_5):2$	: $(A_5xA_5):2^2$	$5^2:[2^5]$	1a+50a+85a	$N(2B,3B,5CD)^2$	$O_4^+(4)$	plus hyperplane
7200	136	$(A_5xA_5):2$	: $(A_5xA_5):2^2$		1a+50a+85b	$N(2A,3A,5AB)^2$	$S_2(4)wr2$	
720	1360	S_6	: $S_6 \times 2$	: $(S_6 \times 2)\cdot 2$		N(2C,3A,3B,4B,5E), C(2D)	$S_4(2)$	$O_5(2)$

$S_4(4)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	
			3	3																																															
	979200	840	840	256	180	180	32	32	300	300	300	300	25	12	12	20	20	20	20	15	15	15	15	17	17	17	17			720	48	48	16	18	18	8	8	5	6	6			20	4	4	4	5	
	p power	A	A	A	A	A	C	C	A	A	A	A	A	AA	BB	BA	AA	DB	CB	BA	AA	DB	CB	A	A	A	A			A	A	B	C	AD	BD	A	A	ED	AC	BD			D	E	A	A	EF	
	p' part	A	A	A	A	A	A	A	A	A	A	A	A	AA	BB	AA	BA	CB	DB	AA	BA	CB	DB	A	A	A	A			A	A	A	A	AD	BD	A	A	ED	AC	BD			A	A	A	A	EF	
	ind	1A	2A	2B	2C	3A	3B	4A	4B	5A	B*	5C	D*	5E	6A	6B	10A	B*	10C	D*	15A	B*	15C	D*	17A	B*2	C*3	D*6	fus ind		2D	4C	4D	4E	6C	6D	8A	B**	10E	12A	12B	fus ind		4F	8C	16A	B*	20A	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	:+o+o		1	1	1	1	1	x_1
x_2	+	18	-6	-6	2	0	0	-2	2	3	3	3	3	-2	0	0	-1	-1	-1	-1	0	0	0	0	1	1	1	1	:	oo	0	0	0	0	0	0	-2i	2i	0	0	0	:oooo-i+1		i-1	0	0-i+1		x_2
x_3	+	34	10	-6	2	1	4	2	-2	4	4	-1	-1	-1	1	0	0	0	-1	-1	1	1	-1	-1	0	0	0	0	:	++	4	4	0	0	1	-2	0	0	-1	1	0	:	++	0	0	0	0	0	x_3
x_4	+	34	-6	10	2	4	1	2	-2	-1	-1	4	4	-1	0	1	-1	-1	0	0	-1	-1	1	1	0	0	0	0	:	++	4	0	4	0	-2	1	0	0	-1	0	1	:							x_4
x_5	+	50	10	10	2	5	5	-2	2	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	-1	:	++	10	2	2	2	1	1	0	0	0	-1	-1	:+o+o		0	0	r2	-r2	0	x_5
x_6	+	51	-13	3	3	3	0	-1	-1	b5+4	*	*	3b5	1	-1	0	b5	*	*	-b5	*	-b5	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	x_6
x_7	+	51	-13	3	3	3	0	-1	-1	*	b5+4	3b5	*	1	-1	0	*	b5	-b5	*	-b5	*	0	0	0	0	0	0																						x_7
x_8	+	51	3	-13	3	0	3	-1	-1	3b5	*	b5+4	*	1	0	-1	-b5	*	b5	*	0	0	*	-b5	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0								x_8
x_9	+	51	3	-13	3	0	3	-1	-1	*	3b5	*	b5+4	1	0	-1	*	-b5	*	b5	0	0	-b5	*	0	0	0	0																						x_9
x_{10}	+	85	21	5	5	4	-5	1	1	5	5	0	0	0	0	-1	1	1	0	0	-1	-1	0	0	0	0	0	0	:	++	5	-3	1	1	2	-1	-1	-1	0	0	1	:	++	0	0	0	0	0	x_{10}
x_{11}	+	85	5	21	5	-5	4	1	1	0	0	5	5	0	-1	0	0	0	1	1	0	0	-1	-1	0	0	0	0	:	++	5	1	-3	1	-1	2	-1	-1	0	1	0								x_{11}
x_{12}	+	153	9	9	-7	0	0	1	1	3	3	3	3	3	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	:	++	9	-3	-3	1	0	0	1	1	-1	0	0	:+o+o		1	1	-1	-1	1	x_{12}
x_{13}	+	204	-4	12	-4	3	0	0	0	4b5+1	*	*	-3b5	-1	-1	0	1	1	*	b5	*	-b5	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	x_{13}
x_{14}	+	204	-4	12	-4	3	0	0	0	*	4b5+1	-3b5	*	-1	-1	0	1	1	b5	*	-b5	*	0	0	0	0	0	0																						x_{14}
x_{15}	+	204	12	-4	-4	0	3	0	0	-3b5	*	4b5+1	*	-1	0	-1	b5	*	1	1	0	0	*	-b5	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0								x_{15}
x_{16}	+	204	12	-4	-4	0	3	0	0	*	-3b5	*	4b5+1	-1	0	-1	*	b5	1	1	0	0	-b5	*	0	0	0	0																						x_{16}
x_{17}	+	225	-15	-15	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d17	*2	*3	*6		+	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	x_{17}
x_{18}	+	225	-15	-15	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*2	d17	*6	*3																						x_{18}
x_{19}	+	225	-15	-15	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*6	*3	d17	*2		+	0	0	0	0	0	0	0	0	0	0	0								x_{19}
x_{20}	+	225	-15	-15	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*3	*6	*2	d17																						x_{20}
x_{21}	+	255	-17	15	-1	-3	0	-1	-1	-5b5	*	0	0	0	1	0	-b5	*	0	0	b5	*	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	x_{21}
x_{22}	+	255	-17	15	-1	-3	0	-1	-1	*	-5b5	0	0	0	1	0	*	-b5	0	0	*	b5	0	0	0	0	0	0																						x_{22}
x_{23}	+	255	15	-17	-1	0	-3	-1	-1	0	0	-5b5	*	0	0	1	0	0	-b5	*	0	0	b5	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0								x_{23}
x_{24}	+	255	15	-17	-1	0	-3	-1	-1	0	0	*	-5b5	0	0	1	0	0	*	-b5	0	0	*	b5	0	0	0	0																						x_{24}
x_{25}	+	256	0	0	0	4	4	0	0	-4	-4	-4	-4	1	0	0	0	0	0	0	-1	-1	-1	-1	1	1	1	1	:	++	16	0	0	0	-2	-2	0	0	1	0	0	:+o+o		4	0	0	0	-1	x_{25}
x_{26}	+	340	4	20	4	1	-5	0	0	-5	-5	0	0	0	1	-1	-1	-1	0	0	1	1	0	0	0	0	0	0	:	++	10	-2	2	-2	1	1	0	0	0	1	-1	:	++	0	0	0	0	0	x_{26}
x_{27}	+	340	20	4	4	-5	1	0	0	0	0	-5	-5	0	-1	1	0	0	-1	-1	0	0	1	1	0	0	0	0	:	++	10	2	-2	-2	1	1	0	0	0	-1	1								x_{27}

$S_6(2)$

Symplectic group $S_6(2) \cong C_3(2)$; Orthogonal group $O_7(2) \cong B_3(2)$

Order = 1,451,520 = $2^9.3^4.5.7$ Mult = 2 Out = 1

Constructions

Symplectic $Sp_6(2) \cong PSp_6(2) \cong S_6(2) \cong G$: all 6 x 6 matrices over $\mathbf{F}_2$ preserving a non-singular symplectic form

Orthogonal $GO_7(2) \cong PGO_7(2) \cong SO_7(2) \cong PSO_7(2) \cong O_7(2) \cong G$: all 7 x 7 matrices over $\mathbf{F}_2$ preserving a non-singular quadratic form;

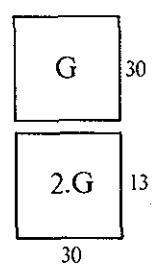
Weyl 2 x G : the Weyl group of E_7; the vectors may be taken as the vectors of the E_8 root lattice (see $O_8^+(2)$) with $x_1+\ldots+x_8 = 0$ (first base) or $x_7 = x_8$ (second base); the minimal (root) vectors are (in the first base) $(1,-1,0,0,0,0,0,0)^S$ (28) and $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})^S$ (35), reflections in which correspond respectively to transpositions (ab) in S_8, and Hesse's bifid maps (abcd|efgh) (see below); the minimal vectors of the dual lattice E_7^* are $\pm(\frac{3}{4},\frac{3}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4})^S$ and correspond to the 28 bitangents ab.
Reducing mod 2 shows the isomorphism with $O_7(2)$.

Hesse G : there are 28 bitangents to the general plane quartic curve, which in Cayley's notation are labelled by the 28 unordered pairs ab, ... gh of 8 objects. The group is generated by Hesse's bifid maps such as (abcd|efgh) = (ab,cd)(ac,bd)(ad,bc)(ef,gh)(eg,fh)(eh,fg). The relations $(abcd|efgh)^2=1$, (abcd|efgh)(abce|dfgh)=(abce|dfgh)(de), (abcd|efgh)(abef|cdgh)=(abgh|cdef)(ab)(cd)(ef)(gh) show that the subgroup S_8 of permutations of {1,2,...8} has index 36, and coset representatives the identity together with the 35 bifid maps. These 35 maps together with the 28 transpositions of S_8 form the conjugacy class of "Steiner transpositions"; the group preserves the 315 quartets such as {ab,cd,ef,gh} (105), {ab,bc,cd,da} (210), whose 8 points of contact lie on a conic

Presentation G x 2 ≅

Maximal subgroups

Order	Index	Structure	Character	Specifications: Abstract	Symplectic	Orthogonal	Weyl	Hesse
51840	28	$U_4(2):2$	1a+27a		$O_6^-(2)$	minus hyperplane	minimal vector of E_7^*	bitangent
40320	36	S_8	1a+35b		$O_6^+(2)$	plus hyperplane		osculating cubic
23040	63	$2^5:S_6$	1a+27a+35b	N(2A)	point	isotropic point	root vector	Steiner transposition
12096	120	$U_3(3):2$	1a+35a+84a		$G_2(2)$			
10752	135	$2^6:L_3(2)$	1a+15a+35b+84a	$N(2B^3)$	isotropic plane	isotropic plane		
4608	315	$2.[2^6]:(S_3 \times S_3)$	1a+27a+35b+84a+168a	N(2B)	isotropic line	isotropic line		conic
4320	336	$S_3 \times S_6$	1a+27a+35b+105b+168a	N(3A), N(2C,3A,3C,4B,5A)	non-isotropic line			
1512	960	$L_2(8):3$	1a+70a+84a+105b+280a+420a	N(2B,3B,7A,9A)	$S_2(8)$			



$S_6(2)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			23	4	1		2																										
	145	1520	040	608	536	384	160	648	108	384	192	192	128	32	30	144	144	72	48	36	36	12	7	16	16	9	10	24	24	12	15		
	p	power	A	A	A	A	A	A	A	B	C	C	B	C	A	AA	AB	BB	AC	CA	CB	CD	A	A	D	B	AA	DB	DC	CA	AA		
	p'	part	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	BB	AC	CA	CB	CD	A	A	A	A	AA	AB	AC	BA	AA		
	ind	1A	2A	2B	2C	2D	3A	3B	3C	4A	4B	4C	4D	4E	5A	6A	6B	6C	6D	6E	6F	6G	7A	8A	8B	9A	10A	12A	12B	12C	15A		
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	7	-5	-1	3	-1	4	-2	1	3	1	-3	-1	1	2	-2	2	2	0	1	-1	-1	0	-1	1	1	0	-2	0	0	-1	χ_2
χ_3	+	15	-5	7	3	-1	0	-3	3	-1	-3	1	3	1	0	-2	-2	1	0	1	1	-1	1	1	-1	0	0	0	-2	-1	0	χ_3
χ_4	+	21	9	-3	1	-3	6	3	0	5	-1	3	1	-1	1	0	0	3	-2	0	0	0	0	-1	1	0	-1	2	0	-1	1	χ_4
χ_5	+	21	-11	5	5	-3	6	3	0	1	-3	-3	1	1	1	-2	2	-1	2	-2	2	0	0	-1	-1	0	-1	0	0	1	1	χ_5
χ_6	+	27	15	3	7	3	9	0	0	3	1	5	-1	1	2	3	3	0	1	0	0	0	-1	1	-1	0	0	1	-1	0	-1	χ_6
χ_7	+	35	-5	3	-5	3	5	-1	2	7	-1	-1	-1	-1	0	1	-3	3	1	-2	0	0	0	1	1	-1	0	-1	-1	1	0	χ_7
χ_8	+	35	15	11	7	3	5	-1	2	-1	5	1	3	1	0	3	-1	-1	1	0	2	0	0	-1	1	-1	0	-1	1	-1	0	χ_8
χ_9	+	56	-24	-8	8	0	11	2	2	0	4	-4	0	0	1	-3	1	-2	-1	0	-2	0	0	0	0	-1	1	1	-1	0	1	χ_9
χ_{10}	+	70	-10	-10	6	-2	-5	7	1	2	2	2	2	-2	0	-1	-1	-1	3	-1	-1	1	0	0	0	1	0	-1	-1	-1	0	χ_{10}
χ_{11}	+	84	4	20	4	4	-6	3	3	4	0	0	4	0	-1	-2	2	-1	-2	1	-1	1	0	0	0	0	-1	0	0	1	-1	χ_{11}
χ_{12}	+	105	-35	1	5	1	15	-3	-3	5	-1	-5	1	-1	0	1	1	1	-1	1	1	1	0	1	-1	0	0	-1	1	-1	0	χ_{12}
χ_{13}	+	105	25	-7	9	1	0	6	3	-3	-3	-3	-3	1	0	4	-4	2	0	1	-1	1	0	-1	-1	0	0	0	0	0	0	χ_{13}
χ_{14}	+	105	5	17	-3	-7	0	6	3	-3	3	-1	1	-1	0	2	2	2	0	-1	-1	-1	0	1	-1	0	0	0	2	0	0	χ_{14}
χ_{15}	+	120	40	-8	8	0	15	-6	0	0	-4	4	0	0	0	1	1	-2	-1	-2	-2	0	1	0	0	0	0	-1	1	0	0	χ_{15}
χ_{16}	+	168	40	8	8	8	6	6	-3	0	0	0	0	0	-2	-2	2	2	2	1	-1	-1	0	0	0	0	0	0	0	0	1	χ_{16}
χ_{17}	+	189	21	-3	-11	-3	9	0	0	9	1	1	1	1	-1	-3	-3	0	1	0	0	0	0	-1	-1	0	1	1	1	0	-1	χ_{17}
χ_{18}	+	189	-51	-3	13	-3	9	0	0	-3	1	1	-3	1	-1	-3	-3	0	1	0	0	0	0	1	1	0	-1	1	1	0	-1	χ_{18}
χ_{19}	+	189	-39	21	1	-3	9	0	0	-3	-5	-1	1	-1	-1	3	3	0	1	0	0	0	0	-1	1	0	1	1	-1	0	-1	χ_{19}
χ_{20}	+	210	50	2	2	-6	15	3	0	-2	2	2	-2	-2	0	-1	-1	-1	-1	2	2	0	0	0	0	0	0	-1	-1	1	0	χ_{20}
χ_{21}	+	210	10	-14	10	2	-15	-6	3	6	-2	-2	-2	-2	0	1	1	-2	1	1	1	-1	0	0	0	0	0	1	1	0	0	χ_{21}
χ_{22}	+	216	-24	24	8	0	-9	0	0	0	-4	4	0	0	1	-3	-3	0	-1	0	0	0	-1	0	0	0	1	-1	1	0	1	χ_{22}
χ_{23}	+	280	-40	-8	-8	8	10	10	1	0	0	0	0	0	0	2	-2	-2	-2	-1	1	-1	0	0	0	1	0	0	0	0	0	χ_{23}
χ_{24}	+	280	40	24	8	0	-5	-8	-2	0	4	-4	0	0	0	1	-3	0	-1	-2	0	0	0	0	0	1	0	1	-1	0	0	χ_{24}
χ_{25}	+	315	-45	-21	3	3	0	-9	0	-5	3	3	3	-1	0	0	0	3	0	0	0	0	0	-1	-1	0	0	0	0	1	0	χ_{25}
χ_{26}	+	336	-16	16	-16	0	6	-6	0	0	0	0	0	0	1	2	-2	-2	2	2	-2	0	0	0	0	0	-1	0	0	0	1	χ_{26}
χ_{27}	+	378	-30	-6	2	-6	-9	0	0	6	2	2	-2	2	-2	3	3	0	-1	0	0	0	0	0	0	0	0	-1	-1	0	1	χ_{27}
χ_{28}	+	405	45	-27	-3	-3	0	0	0	-3	-3	-3	5	1	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	χ_{28}
χ_{29}	+	420	20	4	-12	4	0	-3	3	-4	0	0	-4	0	0	-4	4	1	0	-1	1	1	0	0	0	0	0	0	0	-1	0	χ_{29}
χ_{30}	+	512	0	0	0	0	-16	8	-4	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	-1	χ_{30}
	ind	1 2	4	2	4	2	3 6	3 6	3 6	4 4	8	8	4	8	5 10	12	6	6 6	12	12	6	6	7 14	8	8 8	9 18	20 20	24	24	12 12	15 30		
χ_{31}	+	8	0	0	0	0	-4	-1	2	4	0	0	0	0	-2	0	0	3	0	0	0	0	1	0	2	-1	0	0	0	1	1	χ_{31}
χ_{32}	+	48	0	0	0	0	-12	3	0	8	0	0	0	0	-2	0	0	3	0	0	0	0	-1	0	0	0	0	0	0	-1	-2	χ_{32}
χ_{33}	o	64	0	0	0	0	4	-8	-2	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	1	i5	0	0	0	-1	χ_{33}
χ_{34}	o	64	0	0	0	0	4	-8	-2	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	1	-i5	0	0	0	-1	χ_{34}
χ_{35}	+	112	0	0	0	0	-8	-5	-2	8	0	0	0	0	2	0	0	3	0	0	0	0	0	0	0	1	0	0	0	-1	2	χ_{35}
χ_{36}	+	112	0	0	0	0	4	4	4	8	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	-1	χ_{36}
χ_{37}	+	120	0	0	0	0	0	3	6	-4	0	0	0	0	0	0	0	3	0	0	0	0	1	0	-2	0	0	0	0	-1	0	χ_{37}
χ_{38}	+	168	0	0	0	0	-24	-3	0	4	0	0	0	0	-2	0	0	-3	0	0	0	0	0	0	-2	0	0	0	0	1	1	χ_{38}
χ_{39}	+	280	0	0	0	0	-20	1	4	-4	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	2	1	0	0	0	-1	0	χ_{39}
χ_{40}	+	448	0	0	0	0	16	16	-2	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	χ_{40}
χ_{41}	+	512	0	0	0	0	-16	8	-4	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	-1	χ_{41}
χ_{42}	+	560	0	0	0	0	20	-7	2	8	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	-1	0	0	0	-1	0	χ_{42}
χ_{43}	+	720	0	0	0	0	0	-9	0	-8	0	0	0	0	0	0	0	3	0	0	0	0	-1	0	0	0	0	0	0	1	0	χ_{43}

A_{10}

Alternating group A_{10}

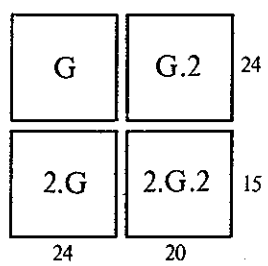
Order = 1,814,400 = $2^7.3^4.5^2.7$ Mult = 2 Out = 2

Constructions

Alternating $S_{10} \cong G.2$: all permutations of 10 letters;

$A_{10} \cong G$: all even permutations; 2.G and 2.G.2 : the Schur double covers

Presentations $G \cong \langle x_1,\ldots x_8 | x_i^3=(x_ix_j)^2=1\rangle$; $G.2 \cong$



Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Alternating
181440	10	A_9	: S_9	1a+9a		point
40320	45	S_8	: S_8 x 2	1a+9a+35a	C(2C)	duad
15120	120	$(A_7x3):2$	: S_7 x S_3	1a+9a+35a+75a	N(3A)	triad
14400	126	$(A_5xA_5):4$	: $(S_5xS_5):2$	1a+35a+90a	$N(2A,3A,5A)^2$	bisection
8640	210	$(A_6xA_4):2$	: S_6 x S_4	1a+9a+35a+75a+90a	$N(2A^2)$	tetrad
1920	945	$2^4:S_5$	: $2^5:S_5$	1a+35a+42a+90a+225a+252a+300a	$N(2^4) = N(2A_{10}B_5)$, C(2D)	quinquisection
720	2520	$M_{10} \cong A_6{\cdot}2_3$	: $A_6{\cdot}2^2$		N(2B,3C,4C,5B)	S(3,4,10)

A_{10}

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			2		7																							40	1		1																	
	1814400		880	384	560	216	81	96	96	32	300	25	72	72	12	21	8	9	9	20	12	12	15	21	21			320	920	576	440	32	32	360	72	72	36	24	9	8	30	5	36	36	7	10	15	
	p power		A	A	A	A	A	A	A	B	A	A	AA	BA	BB	A	C	C	C	AA	AB	BA	AA	AA	AA			A	A	A	A	A	B	AC	BE	AE	BC	BD	CE	C	AC	BD	AD	BD	AC	AD	ABD	
	p' part		A	A	A	A	A	A	A	A	A	A	AA	BA	BB	A	A	A	A	AA	AB	BA	AA	AA	AA			A	A	A	A	A	A	AC	BE	AE	BC	BD	CE	A	AC	BD	AD	BD	AC	AD	ABD	
	ind	1A	2A	2B	3A	3B	3C	4A	4B	4C	5A	5B	6A	6B	6C	7A	8A	9A	9B	10A	12A	12B	15A	21A	B*	fus	ind	2C	2D	2E	4D	4E	4F	6D	6E	6F	6G	6H	6I	8B	10B	10C	12C	12D	14A	20A	30A	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	9	5	1	6	3	0	-1	3	1	4	-1	2	-1	1	2	-1	0	0	0	0	-1	1	-1	-1	:	++	7	-1	3	5	1	-1	4	3	0	1	-1	0	1	2	-1	2	-1	0	0	-1	χ_2
χ_3	+	35	11	3	14	2	-1	3	3	-1	5	0	2	2	0	0	1	-1	-1	1	0	0	-1	0	0	:	++	21	5	5	9	1	1	6	2	2	0	2	-1	-1	1	0	0	0	0	-1	1	χ_3
χ_4	+	36	8	-4	15	3	0	-2	2	0	6	1	-1	-1	-1	1	0	0	0	-2	-1	1	0	1	1	:	++	20	-4	0	10	-2	0	5	3	-3	-1	-1	0	0	0	1	1	1	-1	0	0	χ_4
χ_5	+	42	6	2	0	3	-3	-4	0	2	-3	2	0	3	-1	0	0	0	0	1	0	-1	0	0	0	:	++	14	-10	2	-4	0	-2	2	-1	2	-1	-1	-1	0	-1	0	2	-1	0	1	2	χ_5
χ_6	+	75	15	3	15	0	3	-3	1	-1	0	0	3	0	0	-2	-1	0	0	0	1	0	0	1	1	:	++	35	-5	7	5	1	-1	5	-2	1	2	-2	1	-1	0	0	-1	2	0	0	0	χ_6
χ_7	+	84	0	-4	21	3	3	2	-2	0	4	-1	-3	3	-1	0	0	0	0	0	1	-1	1	0	0	:	++	28	4	-8	10	-2	0	1	1	1	1	1	1	0	-2	-1	1	1	0	0	1	χ_7
χ_8	+	90	14	2	6	3	0	4	0	2	-5	0	2	-1	-1	-1	0	0	0	-1	0	1	1	-1	-1	:	++	34	10	6	-4	0	2	4	-3	0	1	1	0	0	-1	0	2	-1	-1	1	-1	χ_8
χ_9	+	126	-14	6	21	6	0	0	-4	-2	1	1	1	-2	0	0	0	0	0	1	-1	0	1	0	0	:	++	14	6	-6	4	0	-2	-1	0	3	2	0	0	0	-1	1	1	-2	0	-1	-1	χ_9
χ_{10}	+	160	16	0	34	-2	-2	0	0	0	5	0	-2	-2	0	-1	0	1	1	1	0	0	-1	-1	-1	:	++	64	0	0	16	0	0	4	0	0	-2	0	0	0	-1	0	-2	-2	1	1	-1	χ_{10}
χ_{11}	+	210	6	-6	-21	0	3	0	-4	2	5	0	3	0	0	0	0	0	0	1	-1	0	-1	0	0	:	++	14	-10	2	-4	0	-2	-1	2	-1	2	2	-1	0	-1	0	-1	2	0	1	-1	χ_{11}
χ_{12}	+	224	-16	0	14	2	-1	0	0	0	-1	-1	2	2	0	0	0	2	-1	-1	0	0	-1	0	0	│	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	+	224	-16	0	14	2	-1	0	0	0	-1	-1	2	2	0	0	0	-1	2	-1	0	0	-1	0	0	│																						χ_{13}
χ_{14}	+	225	5	9	15	-6	0	3	-1	1	0	0	-1	2	0	1	-1	0	0	0	-1	0	0	1	1	:	++	55	15	3	5	1	-1	-5	0	3	-2	0	0	1	0	0	-1	2	-1	0	0	χ_{14}
χ_{15}	+	252	8	4	-21	3	0	2	-2	0	2	2	-1	-1	1	0	0	0	0	-2	1	-1	-1	0	0	:	++	28	20	0	-10	2	0	1	3	-3	1	-1	0	0	-2	0	-1	-1	0	0	1	χ_{15}
χ_{16}	+	288	16	0	-6	6	0	0	0	0	-7	-2	-2	-2	0	1	0	0	0	1	0	0	-1	1	1	:	++	64	0	0	-16	0	0	4	0	0	-2	0	0	0	-1	0	2	2	1	-1	-1	χ_{16}
χ_{17}	+	300	0	4	-15	3	3	-2	2	0	0	0	-3	3	1	-1	0	0	0	0	-1	1	0	-1	-1	:	++	20	-20	-8	-10	2	0	5	1	1	-1	1	1	0	0	0	-1	-1	-1	0	0	χ_{17}
χ_{18}	+	315	19	-5	21	-3	0	-1	-1	-1	-5	0	1	1	1	0	1	0	0	-1	-1	-1	1	0	0	:	++	91	-5	3	-1	-1	-1	1	-3	-3	1	1	0	1	1	0	-1	-1	0	-1	1	χ_{18}
χ_{19}	+	350	-10	-2	35	-1	-1	-2	-2	2	0	0	-1	-1	1	0	0	-1	-1	0	1	1	0	0	0	:	++	70	-10	-10	10	2	2	-5	-1	-1	1	-1	-1	0	0	0	1	1	0	0	0	χ_{19}
χ_{20}	+	384	0	0	-24	0	-3	0	0	0	4	-1	0	0	0	-1	0	0	0	0	0	0	1	-b21	*	│	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{20}
χ_{21}	+	384	0	0	-24	0	-3	0	0	0	4	-1	0	0	0	-1	0	0	0	0	0	0	1	*	-b21	│																						χ_{21}
χ_{22}	+	450	10	2	-15	-3	0	-2	-2	-2	0	0	1	1	-1	2	0	0	0	0	1	1	0	-1	-1	:	++	70	-10	6	-10	-2	2	-5	3	3	1	-1	0	0	0	0	-1	-1	0	0	0	χ_{22}
χ_{23}	+	525	-15	5	0	-3	3	-1	3	1	0	0	0	-3	-1	0	1	0	0	0	0	-1	0	0	0	:	++	35	-5	7	5	1	-1	-10	1	-2	-1	1	1	-1	0	0	2	-1	0	0	0	χ_{23}
χ_{24}	+	567	-9	-9	0	0	0	3	3	-1	-3	2	0	0	0	0	-1	0	0	1	0	0	0	0	0	:	++	63	15	-9	-9	-1	-1	0	0	0	0	0	0	-1	3	0	0	0	0	1	0	χ_{24}
	ind	1 2	4	2	3 6	3 6	3 6	8	8	4	5 10	5 10	12	12	6	7 14	8 8	9 18	9 18	20	24 24	24 24	15 30	21 42	21 42	fus	ind	2	2	4	8	8	4	6	12	12	6	6	12 12	8	10	10 10	24	24	14 14	40 40	30 30	
χ_{25}	+	16	0	0	-8	4	-2	0	0	0	-4	1	0	0	0	2	0	1	1	0	0	0	2	-1	-1	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r5	0	0	0	0	0	χ_{25}
χ_{26}	o	48	0	0	6	0	3	0	0	0	-2	-2	0	0	0	-1	0	0	0	0	i6	0	1	-1	-1	│	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{26}
χ_{27}	o	48	0	0	6	0	3	0	0	0	-2	-2	0	0	0	-1	0	0	0	0	-i6	0	1	-1	-1	│																						χ_{27}
χ_{28}	+	64	0	0	-20	4	1	0	0	0	-6	-1	0	0	0	1	0	-2	1	0	0	0	0	1	1	│	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	+	64	0	0	-20	4	1	0	0	0	-6	-1	0	0	0	1	0	1	-2	0	0	0	0	1	1	│																						χ_{29}
χ_{30}	+	216	0	0	-36	0	0	0	0	0	-4	1	0	0	0	-1	2	0	0	0	0	0	-1	-1	-1	│	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{30}
χ_{31}	+	216	0	0	-36	0	0	0	0	0	-4	1	0	0	0	-1	-2	0	0	0	0	0	-1	-1	-1	│																						χ_{31}
χ_{32}	+	336	0	0	0	6	3	0	0	0	6	1	0	0	0	0	0	0	0	0	0	r6	0	0	0	│	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}	+	336	0	0	0	6	3	0	0	0	6	1	0	0	0	0	0	0	0	0	0	-r6	0	0	0	│																						χ_{33}
χ_{34}	+	384	0	0	-24	0	-3	0	0	0	4	-1	0	0	0	-1	0	0	0	0	0	0	1	-b21	*	│	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{34}
χ_{35}	+	384	0	0	-24	0	-3	0	0	0	4	-1	0	0	0	-1	0	0	0	0	0	0	1	*	-b21	│																						χ_{35}
χ_{36}	+	400	0	0	-20	-8	4	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i7	0	0	χ_{36}
χ_{37}	+	432	0	0	36	0	0	0	0	0	-8	2	0	0	0	-2	0	0	0	0	0	0	1	1	1	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i15	χ_{37}
χ_{38}	+	448	0	0	28	4	-2	0	0	0	-2	-2	0	0	0	0	0	1	1	0	0	0	-2	0	0	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i10	0	χ_{38}
χ_{39}	+	800	0	0	20	-4	-1	0	0	0	0	0	0	0	0	2	0	-1	-1	0	0	0	0	-1	-1	:	oo	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0	χ_{39}

$L_3(7)$

Linear group $L_3(7) \cong A_2(7)$

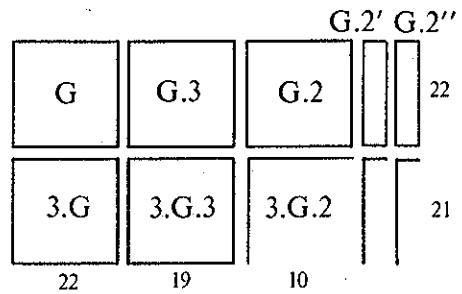
Order = 1,876,896 = $2^5.3^2.7^3.19$ Mult = 3 Out = S_3

Constructions

Linear $GL_3(7) \cong 2 \times 3.G.3$: all non-singular 3×3 matrices over $\mathbb{F}_7$;

$PGL_3(7) \cong G.3$; $SL_3(7) \cong 3.G$; $PSL_3(7) \cong G$

Remark : The O'Nan group contains two classes of maximal subgroups isomorphic to $L_3(7):2$.



Maximal subgroups

Order	Index	Structure	G.3	G.2	$G.S_3$	Character	Specifications: Abstract	Specifications: Linear
32928	57	$7^2:2{\cdot}L_2(7):2$	: $7^2:GL_2(7)$	$7_+^{1+2}:(3 \times D_8)$,	$7_+^{1+2}:6^2:2$,	1a+56a	$N(7A^2)$	point
32928	57	$7^2:2{\cdot}L_2(7):2$	: $7^2:GL_2(7)$	$2{\cdot}(L_2(7)\times 2).2$	$GL_2(7).2$	1a+56a	$N(7A^2)$	line
336	5586	$L_2(7):2$		: $L_2(7):2 \times 2$			N(2A,3A,4A,7B), C(2B)	$O_3(7)$
336	5586	$L_2(7):2$					N(2A,3A,4A,7C)	$O_3(7)$
336	5586	$L_2(7):2$					N(2A,3A,4A,7D)	$O_3(7)$
72	26068	$(3 \times A_4):2$	: $6^2:S_3$	: $S_3 \times S_4$	: $6^2:D_{12}$		N(3A), $N(2A^2)$	base
72	26068	$3^2:Q_8$	: $3^2:2A_4$	: $3^2:Q_8:2$	: $3^2:2S_4$		$N(3A^2)$	
57	32928	19:3	: 57:3	: 19:6	: 57:6		N(19ABCDEF)	$L_1(343)$

$L_3(7)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	;	;	;	;	
		1876896	672	36	16	12	686	49	49	49	16	16	14	16	16	16	16	19	19	19	19	19	19			672	57	672	12	16	14	16	16	14	16	16	16	16	19	19	19	19	19	19			336	336	6	8	6	7	8	8	14	14					
	p power		A	A	A	AA	A	A	A	A	A	A	AA	A	A	B	B	A	A	A	A	A	A			A	A	BA	BA	BA	AB	AB	AA	AAB	AC	AD	BB	BA	DC	CC	FC	EC	BC	AC			A	A	AB	A	AB	BB	A	A	AB	AB					
	p' part		A	A	A	AA	A	A	A	A	A	A	AA	A	A	A	A	A	A	A	A	A	A			A	A	BA	BA	BA	AB	BA	BB	AAB	BA	BB	BC	BD	AC	BC	CC	DC	EC	FC			A	A	AB	A	AB	BB	A	A	AB	AB					
	ind	1A	2A	3A	4A	6A	7A	7B	7C	7D	8A	B*	14A	16A	B**	C*3	D**3	19A	B**	C**2	D*2	E*4	F**4	fus	ind	3B	3C	6B	6C	12A	21A	24A	B*	42A	48A	B*31	C*37	D**5	57A	B*37	C**2	D**5	E*4	F*34	fus	ind	2B	4B	6D	8C	12B	14B	16E	F*	28A	B*	fus	ind	fus	ind	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	+oo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	:	++	:	++	χ_1
χ_2	+	56	8	2	0	2	7	0	0	0	0	0	1	0	0	0	0	-1	-1	-1	-1	-1	-1	:	+oo	8	-1	8	2	0	1	0	0	1	0	0	0	0	-1	-1	-1	-1	-1	-1	:	++	0	0	0	0	0	0	0	0	r7	-r7	:	++	:	++	χ_2
χ_3	+	57	-7	3	1	-1	8	1	1	1	1	1	0	-1	-1	-1	-1	0	0	0	0	0	0	:	+oo	9	0	-7	-1	1	2	1	1	0	-1	-1	-1	-1	0	0	0	0	0	0	:	++	1	-7	1	1	-1	1	-1	-1	0	0	:	++	:	++	χ_3
χ_4	+	152	8	-1	0	-1	5	5	-2	-2	0	0	1	0	0	0	0	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	8	8	-1	0	-1	1	0	0	1	1	|	+	|	+	χ_4
χ_5	+	152	8	-1	0	-1	5	-2	5	-2	0	0	1	0	0	0	0	0	0	0	0	0	0	|																					|	+	0	0	0	0	0	0	0	0	0	0	:	++	|		χ_5
χ_6	+	152	8	-1	0	-1	5	-2	-2	5	0	0	1	0	0	0	0	0	0	0	0	0	0	|																					|												|		:	++	χ_6
χ_7	o	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	x19	**	**2	*2	*4	**4	:	ooo	0	3	0	0	0	0	0	0	0	0	0	0	0	x19	**	**2	*2	*4	**4	|	+	0	0	0	0	0	0	0	0	0	0	|	+	|	+	χ_7
χ_8	o	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	**	x19	*2	**2	**4	*4	:	ooo	0	3	0	0	0	0	0	0	0	0	0	0	0	**	x19	*2	**2	**4	*4	|												|		|		χ_8
χ_9	o	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	*4	**4	x19	**	**2	*2	:	ooo	0	3	0	0	0	0	0	0	0	0	0	0	0	*4	**4	x19	**	**2	*2	|	+	0	0	0	0	0	0	0	0	0	0	|	+	|	+	χ_9
χ_{10}	o	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	**4	*4	**	x19	*2	**2	:	ooo	0	3	0	0	0	0	0	0	0	0	0	0	0	**4	*4	**	x19	*2	**2	|												|		|		χ_{10}
χ_{11}	o	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	**2	*2	*4	**4	x19	**	:	ooo	0	3	0	0	0	0	0	0	0	0	0	0	0	**2	*2	*4	**4	x19	**	|	+	0	0	0	0	0	0	0	0	0	0	|	+	|	+	χ_{11}
χ_{12}	o	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	*2	**2	**4	*4	**	x19	:	ooo	0	3	0	0	0	0	0	0	0	0	0	0	0	*2	**2	**4	*4	**	x19	|												|		|		χ_{12}
χ_{13}	+	342	6	0	-2	0	-1	-1	-1	-1	2	2	-1	0	0	0	0	0	0	0	0	0	0	:	+oo	6	0	6	0	-2	-1	2	2	-1	0	0	0	0	0	0	0	0	0	0	:	++	6	6	0	2	0	-1	0	0	-1	-1	:	++	:	++	χ_{13}
χ_{14}	+	342	6	0	2	0	-1	-1	-1	-1	0	0	-1	r2	r2	-r2	-r2	0	0	0	0	0	0	:	+oo	6	0	6	0	2	-1	0	0	-1	r2	r2	-r2	-r2	0	0	0	0	0	0	:	++	6	-6	0	0	0	-1	r2	-r2	1	1	:	++	:	++	χ_{14}
χ_{15}	+	342	6	0	2	0	-1	-1	-1	-1	0	0	-1	-r2	-r2	r2	r2	0	0	0	0	0	0	:	+oo	6	0	6	0	2	-1	0	0	-1	-r2	-r2	r2	r2	0	0	0	0	0	0	:	++	6	-6	0	0	0	-1	-r2	r2	1	1	:	++	:	++	χ_{15}
χ_{16}	o	342	-6	0	0	0	-1	-1	-1	-1	-r2	r2	1	m16	**	*3	**3	0	0	0	0	0	0	:	ooo	6	0	-6	0	0	-1	-r2	r2	1	m16	**	*3	**3	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	|	+	|	+	χ_{16}
χ_{17}	o	342	-6	0	0	0	-1	-1	-1	-1	-r2	r2	1	**	m16	**3	*3	0	0	0	0	0	0	:	ooo	6	0	-6	0	0	-1	-r2	r2	1	**	m16	**3	*3	0	0	0	0	0	0	|												|		|		χ_{17}
χ_{18}	o	342	-6	0	0	0	-1	-1	-1	-1	r2	-r2	1	**3	*3	m16	**	0	0	0	0	0	0	:	ooo	6	0	-6	0	0	-1	r2	-r2	1	**3	*3	m16	**	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	|	+	|	+	χ_{18}
χ_{19}	o	342	-6	0	0	0	-1	-1	-1	-1	r2	-r2	1	*3	**3	**	m16	0	0	0	0	0	0	:	ooo	6	0	-6	0	0	-1	r2	-r2	1	*3	**3	**	m16	0	0	0	0	0	0	|												|		|		χ_{19}
χ_{20}	+	343	7	1	-1	1	0	0	0	0	-1	-1	0	-1	-1	-1	-1	1	1	1	1	1	1	:	+oo	7	1	7	1	-1	0	-1	-1	0	-1	-1	-1	-1	1	1	1	1	1	1	:	++	7	7	1	-1	1	0	-1	-1	0	0	:	++	:	++	χ_{20}
χ_{21}	+	399	-1	3	-1	-1	7	0	0	0	-1	-1	-1	1	1	1	1	0	0	0	0	0	0	:	+oo	15	0	-1	-1	-1	1	-1	-1	-1	1	1	1	1	0	0	0	0	0	0	:	++	7	-1	1	-1	-1	0	1	1	-1	-1	:	++	:	++	χ_{21}
χ_{22}	+	456	-8	-3	0	1	15	1	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	:	+oo	0	0	16	-2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	:	++	8	-8	-1	0	1	1	0	0	-1	-1	:	++	:	++	χ_{22}
	ind	1	2	3	4	6	7	7	7	7	8	8	14	16	16	16	16	19	19	19	19	19	19	fus	ind	3	9	6	6	12	21	24	24	42	48	48	48	48	171	171	171	171	171	171	fus	ind	2	4	6	8	12	14	16	16	28	28	fus	ind	fus	ind	
		3	6		12	6	21	21	21	21	24	24	42	48	48	48	48	57	57	57	57	57	57			3		6	6	12	21	24	24	42	48	48	48	48	171	171	171	171	171	171																	
		3	6		12	6	21	21	21	21	24	24	42	48	48	48	48	57	57	57	57	57	57			3		6	6	12	21	24	24	42	48	48	48	48	171	171	171	171	171	171																	
χ_{23}	o2	57	9	0	1	0	8	1	1	1	1	1	2	1	1	1	1	0	0	0	0	0	0	:	ooo2	-4i3-3	0	-4i3-3	-i3	1	-z3	1	1	-z3	1	1	1	1	0	0	0	0	0	0	*	+											*	+	*	+	χ_{23}
χ_{24}	o2	57	-7	0	1	2	8	1	1	1	1	1	0	-1	-1	-1	-1	0	0	0	0	0	0	:	ooo2	-4i3-3	0	4i3+5	-1	1	-z3	1	1	z3+2	-1	-1	-1	-1	0	0	0	0	0	0	*	+											*	+	*	+	χ_{24}
χ_{25}	o2	96	0	0	0	0	-2	5	-2	-2	0	0	0	0	0	0	0	1	1	1	1	1	1	|	o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*	+											|	o2	|	o2	χ_{25}
χ_{26}	o2	96	0	0	0	0	-2	-2	5	-2	0	0	0	0	0	0	0	1	1	1	1	1	1	|																					|	o2											*	+	|		χ_{26}
χ_{27}	o2	96	0	0	0	0	-2	-2	-2	5	0	0	0	0	0	0	0	1	1	1	1	1	1	|																					|												|		*	+	χ_{27}
χ_{28}	o2	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	x19	**	**2	*2	*4	**4	:	ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	*4	*10	x171	*37	**2	**5	|	+2											|	+2	|	+2	χ_{28}
χ_{29}	o2	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	**	x19	*2	**2	**4	*4	:	ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	*10	*4	*37	x171	**5	**2	|												|		|		χ_{29}
χ_{30}	o2	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	*4	**4	x19	**	**2	*2	:	ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	**2	**5	*61	*34	x171	*37	|	+2											|	+2	|	+2	χ_{30}
χ_{31}	o2	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	**4	*4	**	x19	*2	**2	:	ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	**5	**2	*34	*61	*37	x171	|												|		|		χ_{31}
χ_{32}	o2	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	**2	*2	*4	**4	x19	**	:	ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	x171	*37	*43	*22	*61	*34	|	+2											|	+2	|	+2	χ_{32}
χ_{33}	o2	288	0	0	0	0	-6	1	1	1	0	0	0	0	0	0	0	*2	**2	**4	*4	**	x19	:	ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	*37	x171	*22	*43	*34	*61	|												|		|		χ_{33}
χ_{34}	o2	342	6	0	-2	0	-1	-1	-1	-1	2	2	-1	0	0	0	0	0	0	0	0	0	0	:	ooo2	6	0	6	0	-2	-1	2	2	-1	0	0	0	0	0	0	0	0	0	0	*	+											*	+	*	+	χ_{34}
χ_{35}	o2	342	6	0	2	0	-1	-1	-1	-1	0	0	-1	r2	r2	-r2	-r2	0	0	0	0	0	0	:	ooo2	6	0	6	0	2	-1	0	0	-1	-r2	-r2	r2	r2	0	0	0	0	0	0	*	+											*	+	*	+	χ_{35}
χ_{36}	o2	342	6	0	2	0	-1	-1	-1	-1	0	0	-1	-r2	-r2	r2	r2	0	0	0	0	0	0	:	ooo2	6	0	6	0	2	-1	0	0	-1	r2	r2	-r2	-r2	0	0	0	0	0	0	*	+											*	+	*	+	χ_{36}
χ_{37}	o2	342	-6	0	0	0	-1	-1	-1	-1	-r2	r2	1	m16	**	*3	**3	0	0	0	0	0	0	:	ooo2	6	0	-6	0	0	-1	-r2	r2	1	m16	**	*3	**3	0	0	0	0	0	0	|	+2											|	+2	|	+2	χ_{37}
χ_{38}	o2	342	-6	0	0	0	-1	-1	-1	-1	-r2	r2	1	**	m16	**3	*3	0	0	0	0	0	0	:	ooo2	6	0	-6	0	0	-1	-r2	r2	1	**	m16	**3	*3	0	0	0	0	0	0	|												|		|		χ_{38}
χ_{39}	o2	342	-6	0	0	0	-1	-1	-1	-1	r2	-r2	1	**3	*3	m16	**	0	0	0	0	0	0	:	ooo2	6	0	-6	0	0	-1	r2	-r2	1	**3	*3	m16	**	0	0	0	0	0	0	|	+2											|	+2	|	+2	χ_{39}
χ_{40}	o2	342	-6	0	0	0	-1	-1	-1	-1	r2	-r2	1	*3	**3	**	m16	0	0	0	0	0	0	:	ooo2	6	0	-6	0	0	-1	r2	-r2	1	*3	**3	**	m16	0	0	0	0	0	0	|												|		|		χ_{40}
χ_{41}	o2	399	15	0	-1	0	7	0	0	0	-1	-1	1	-1	-1	-1	-1	0	0	0	0	0	0	:	ooo2	-4i3+3	0	-4i3+3	-i3	-1	-z3-1	-1	-1	-z3-1	-1	-1	-1	-1	0	0	0	0	0	0	*	+											*	+	*	+	χ_{41}
χ_{42}	o2	399	-1	0	-1	2	7	0	0	0	-1	-1	-1	1	1	1	1	0	0	0	0	0	0	:	ooo2	-4i3+3	0	4i3+11	-1	-1	-z3-1	-1	-1	z3+1	1	1	1	1	0	0	0	0	0	0	*	+											*	+	*	+	χ_{42}
χ_{43}	o2	456	-8	0	0	-2	15	1	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	:	ooo2	-8i3	0	-8	i3+1	0	-i3	0	0	-1	0	0	0	0	0	0	0	0	0	0	*	+											*	+	*	+	χ_{43}

$U_4(3)$

Unitary group $U_4(3) \cong {}^2A_3(3) \cong O_6^-(3)$

Order = 3,265,920 = $2^7.3^6.5.7$ Mult = $3^2 \times 4$ Out = D_8

Constructions

Unitary (3) $GU_4(3) \cong 4.G.4$: all 4 x 4 matrices over $\mathbf{F}_9$ preserving a non-singular Hermitian form;

$PGU_4(3) \cong G.4$; $SU_4(3) \cong 4.G$; $PSU_4(3) \cong G$

Orthogonal (3) $GO_6^-(3) \cong 2.G.(2^2)_{122}$: all 6 x 6 matrices over $\mathbf{F}_3$ preserving a non-singular quadratic form of Witt defect 1, for example $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2$; $PGO_6^-(3) \cong G.(2^2)_{122}$; $SO_6^-(3) \cong SO_6^-(3) \cong 2.G.2_1$; $PSO_6^-(3) \cong G.2_1$; $O_6^-(3) \cong G$

Unitary (2) $3.G.2_2$ has a 6-dimensional unitary representation over $\mathbf{F}_4$ (see $U_6(2)$). This can be obtained from the representation of $6_1.G.2_2$ as a complex reflection group (see below).

Leech $U_4(3).D_8 \cong G.D_8$: the set stabilizer in Co_1 of an S-lattice of type $2^{1+4}3^2$; the pointwise stabilizer is G (see Co_1); hence G is the point stabilizer in the 275-point representation of the McLaughlin group.

Complex Leech $(2x3^2).G.D_8$: the normalizer in $2Co_1$ of an elementary Abelian 3^2-group; $(2x3^2).G$: the centralizer, i.e. the centralizer in 6Suz of an element of order 3. When the Leech lattice vectors are written as quaternionic vectors $(q_1,\ldots,q_6)$ the 3^2-group is generated by left and right multiplication by $w = \frac{1}{2}(-1+i+j+k)$

Reflection $6_1.G.2_2$: the automorphism group of the 6-dimensional lattice over $\mathbf{Z}[w]$ (where w = z3) whose typical vector is $v_1+\theta v_2$ (with θ = i3, v_1 in the E_6 root lattice, v_2 in its dual). The group is generated by the 126 reflections in the 6 x 126 minimal vectors, which are listed (modulo scalars) in four bases below. Each base renders a different maximal subgroup monomial. The standard norm in the n-base is $\frac{1}{n}\Sigma|x_i|^2$, and modulo scalars there are 126, 672, 3402 vectors of norm 2, 3, 4 respectively. Read modulo θ these numbers become 126, 112, 126 and we obtain the "Orthogonal (3)" construction. Read modulo 2 they become 126, 672, 567 and we have the "Unitary (2)" construction.

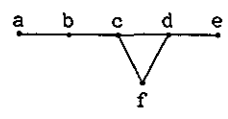
In the 2-base the vectors are just those whose coordinates reduce modulo 2 to give a hexacode word (see A_6). The monomial subgroup (of 6.G.2) is $(2^6 \times 3)A_6$, and the minimal vectors are the images under this group of $(2,0,0,0,0,0)$ (6) and $(0,1,0,1,w,\bar{w})$ (120).

In the 3-base the monomial subgroup is $(2 \times 3^5)S_6$ and the minimal vectors are of form $(w^a\theta,-w^b\theta,0,0,0,0)^S$ (45) and $(w^a,w^b,w^c,w^d,w^e,w^f)^S$ $(a+b+c+d+e+f \equiv 0 \bmod 3)$ (81).

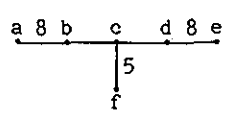
In the 4-base the monomial subgroup is $(3 \times 2^5)S_6$ and the minimal vectors are $(\pm2,\pm2,0,0,0,0)^S$ (30) and $(\pm\theta,\pm1,\pm1,\pm1,\pm1,\pm1)^S$ (even signs) (96).

In the 7-base the monomial group is $6 \times S_7$ and the minimal vectors are $(2+3w,-2-3w,0,0,0,0,0)^S$ (21) and $(2w,-2-w,-2-w,1,1,1,1)^S$ (105).

(Another type of 7-base is obtained by taking the complex conjugates of these numbers.)

Presentations $6_1.G.2_2 \cong$ 

$\langle\ |\ (cdcf)^3=1\ \rangle$;

$3^2.G.(2^2)_{133} \cong$ 

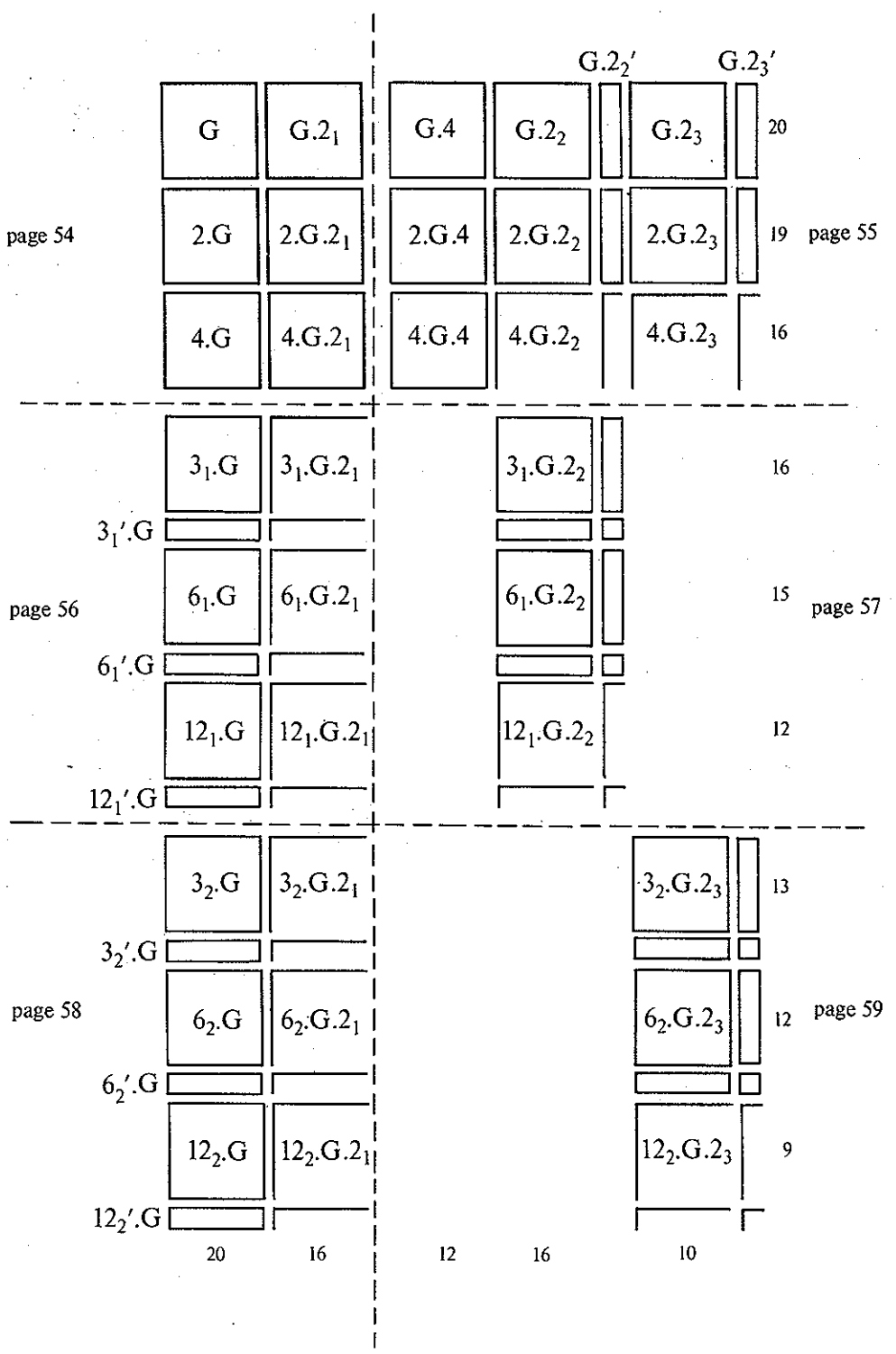
$\langle\ |\ f=(ab)^4=(de)^4,\ 1=(bcdf)^5\ \rangle$; normal 3^2 is generated by $(cbaba)^5$ and $(cdede)^5$

See also page 232.

Maximal subgroups

Order	Index	Structure	$G.2_1$	$G.4$	$G.2_2$	$G.(2^2)_{122}$	$G.2_3$	$G.(2^2)_{133}$	$G.D_8$
29160	112	$3^4:A_6$	: $3^4:(2xA_6)$	: $3^4:(2xA_6)\cdot2$	: $3^4:S_6$	: $3^4:(S_6x2)$	: $3^4:M_{10}$	: $3^4:(2xM_{10})$	: $3^4:(2xA_6\cdot2^2)$
25920	126	$U_4(2)$	: $U_4(2):2$	$(S_6 \times 2)\cdot2$	: $U_4(2) \times 2$	: $U_4(2):2 \times 2$	$A_6\cdot2^2$	$A_6\cdot2^2 \times 2$	$(A_6\cdot2^2 \times 2)\cdot2$
25920	126	$U_4(2)$	: $U_4(2):2$		: $U_4(2):2$	: $U_4(2):2 \times 2$			
20160	162	$L_3(4)$	: $L_3(4):2_2$				: $L_3(4):2_3$	: $L_3(4):2^2$	
20160	162	$L_3(4)$	: $L_3(4):2_2$				: $L_3(4):2_1$	: $L_3(4):2^2$	
11664	280	$3_+^{1+4}.2S_4$	: $3_+^{1+4}.4S_4$	: $3_+^{1+4}.4S_4\cdot2$	: $3_+^{1+4}.2S_4:2$	: $3_+^{1+4}.4S_4:2$	: $3_+^{1+4}.(2S_4x2)$	: $3_+^{1+4}.2_-^{1+4}.S_3$	: $3_+^{1+4}.2_-^{1+4}.D_{12}$
6048	540	$U_3(3)$	: $U_3(3) \times 2$	: $U_3(3) \times 4$	: $U_3(3):2$	: $U_3(3):2 \times 2$	: $U_3(3):2$	: $U_3(3):2 \times 2$	: $(U_3(3) \times 4):2$
5760	567	$2^4:A_6$	: $2^4:S_6$	4^3S_4	: $2^4:S_6$	: $2^5:S_6$	$(4^2x2)S_4$	$(4^2x2)(2xS_4)$	$4^3(2xS_4)$
5760	567	$2^4:A_6$	: $2^4:S_6$		: $2^5:A_6$	: $2^5:S_6$			
2520	1296	A_7			: S_7				
2520	1296	A_7			: S_7				
2520	1296	A_7							
2520	1296	A_7							
1152	2835	$2(A_4xA_4).4$	: $4(A_4xA_4).4$	: $4(S_4xS_4).2$	: $H.2$	: $2\cdot(2^2x(A_4xA_4).4)$	: $H.2$	: $H.2^2$	: $4(S_4xS_4).2^2$
720	4536	$M_{10} \cong A_6\cdot2_3$	: $A_6\cdot2^2$				: $M_{10} \times 2$	: $H.2^2$	
720	4536	$M_{10} \cong A_6\cdot2_3$	: $A_6\cdot2^2$				: $A_6\cdot2^2$	: $H.2^2$	

$U_4(3)$



Specifications

Characters	Abstract	Unitary	Orthogonal	Reflection
1a+21a+90a		isotropic line	isotropic point	3-base
1a+35a+90a	C(2D)	$S_4(3)$	non-isotropic point	norm 2 point
1a+35b+90a		$S_4(3)$	non-isotropic point	norm 4 point mod θ
1a+21a+140a				
1a+21a+140a				
1a+90a+189a	N(3A)	isotropic point	isotropic line	
1a+140a+189a+210a	C(2B)	non-isotropic point	$U_3(3)$	
1a+21a+90a+140a+315a			base	4-base
1a+21a+90a+140a+315b			base	2-base
1a+21a+90a+140a+315a+729a				7-base
1a+21a+90a+140a+315a+729a				7-base
1a+21a+90a+140a+315b+729a				
1a+21a+90a+140a+315b+729a				
1a+35ab+90aa+140a+189a+315ab+729a+896a	N(2A)	$U_2(3)wr2$	$O_2^-(3) \times O_4^+(3)$	
	C(2F)	$O_4^-(3)$		
		$O_4^-(3)$		

$U_4(3)$

	;	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	;	;	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	
			1	5																					6														
		3265920	152	832	972	972	81	96	16	5	72	36	36	7	7	8	27	27	27	27	12				048	720	576	64	108	18	18	9	8	5	72	72	18	18	7	7
		p power	A	A	A	A	A	A	A	A	AA	BA	CA	A	A	A	A	A	A	A	AA				A	A	A	A	AB	BC	CC	DB	A	AC	AC	AC	BC	CC	AB	BB
		p' part	A	A	A	A	A	A	A	A	AA	BA	CA	A	A	A	A	A	A	A	AA				A	A	A	A	AB	BC	CC	DB	A	AC	AC	AC	BC	CC	AB	BB
	ind	1A	2A	3A	3B	3C	3D	4A	4B	5A	6A	6B	6C	7A	B**	8A	9A	B**	9C	D**	12A	fus	ind		2B	2C	4C	4D	6D	6E	6F	6G	8B	10A	12B	C**	12D	12E	14A	B**	
X_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	X_1
X_2	+	21	5	-6	3	3	3	1	1	1	2	-1	-1	0	0	-1	0	0	0	0	-2	:	++		-7	1	5	-3	2	1	1	-1	-1	1	2	2	-1	-1	0	0	X_2
X_3	+	35	3	8	8	-1	-1	3	-1	0	0	0	3	0	0	-1	2	2	-1	-1	0	:	++		7	-5	-1	-1	-2	-2	1	1	-1	0	2	2	2	-1	0	0	X_3
X_4	+	35	3	8	-1	8	-1	3	-1	0	0	3	0	0	0	-1	-1	-1	2	2	0	:	++		7	-5	-1	-1	-2	1	-2	1	-1	0	2	2	-1	2	0	0	X_4
X_5	+	90	10	9	9	9	0	-2	2	0	1	1	1	-1	-1	0	0	0	0	0	1	:	++		6	10	10	2	-3	1	1	0	0	0	1	1	1	1	-1	-1	X_5
X_6	+	140	12	5	-4	-4	5	4	0	0	-3	0	0	0	0	0	-1	-1	-1	-1	1	:	++		28	0	4	4	1	0	0	1	0	0	1	1	-2	-2	0	0	X_6
X_7	+	189	-3	27	0	0	0	5	1	-1	3	0	0	0	0	1	0	0	0	0	-1	:	++		21	9	-3	-3	3	0	0	0	1	-1	3	3	0	0	0	0	X_7
X_8	+	210	2	21	3	3	3	-2	-2	0	5	-1	-1	0	0	0	0	0	0	0	1	:	++		14	-10	10	2	5	-1	-1	-1	0	0	1	1	1	1	0	0	X_8
X_9	o	280	-8	10	10	1	1	0	0	0	-2	-2	1	0	0	0	b27	**	1	1	0		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_9
X_{10}	o	280	-8	10	10	1	1	0	0	0	-2	-2	1	0	0	0	**	b27	1	1	0																				X_{10}
X_{11}	o	280	-8	10	1	10	1	0	0	0	-2	1	-2	0	0	0	1	1	b27	**	0		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{11}
X_{12}	o	280	-8	10	1	10	1	0	0	0	-2	1	-2	0	0	0	1	1	**	b27	0																				X_{12}
X_{13}	+	315	11	-9	18	-9	0	-1	-1	0	-1	2	-1	0	0	1	0	0	0	0	-1	:	++		-21	-5	7	-1	-3	-2	1	0	1	0	1	1	-2	1	0	0	X_{13}
X_{14}	+	315	11	-9	-9	18	0	-1	-1	0	-1	-1	2	0	0	1	0	0	0	0	-1	:	++		-21	-5	7	-1	-3	1	-2	0	1	0	1	1	1	-2	0	0	X_{14}
X_{15}	+	420	4	-39	6	6	-3	4	0	0	1	-2	-2	0	0	0	0	0	0	0	1	:	++		28	0	12	-4	1	0	0	1	0	0	-3	-3	0	0	0	0	X_{15}
X_{16}	-	560	-16	-34	2	2	2	0	0	0	2	2	2	0	0	0	-1	-1	-1	-1	0	:	oo		0	0	0	0	0	0	0	0	0	0	-6i	6i	0	0	0	0	X_{16}
X_{17}	o	640	0	-8	-8	-8	1	0	0	0	0	0	0	b7	**	0	1	1	1	1	0	:	oo		-32	0	0	0	4	0	0	1	0	0	0	0	0	0	b7	**	X_{17}
X_{18}	o	640	0	-8	-8	-8	1	0	0	0	0	0	0	**	b7	0	1	1	1	1	0	:	oo		-32	0	0	0	4	0	0	1	0	0	0	0	0	0	**	b7	X_{18}
X_{19}	+	729	9	0	0	0	0	-3	1	-1	0	0	0	1	1	-1	0	0	0	0	0	:	++		-27	9	9	1	0	0	0	0	-1	-1	0	0	0	0	1	1	X_{19}
X_{20}	+	896	0	32	-4	-4	-4	0	0	1	0	0	0	0	0	0	-1	-1	-1	-1	0	:	++		0	16	0	0	0	-2	-2	0	0	1	0	0	0	0	0	0	X_{20}
	ind	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind		2	4	4	4	6	12	12	6	8	20	12	12	12	12	14	14	
		2	2	6	6	6	6	4		10	6	6	6	14	14	8	18	18	18	18	12				2	4	4		6	12	12	6	8	20	12	12	12	12	14	14	
X_{21}	-	20	4	-7	2	2	2	4	0	0	1	-2	-2	-1	-1	0	-1	-1	-1	-1	1	:	--		-8	0	0	0	1	0	0	-2	0	0	-3	3	0	0	-1	-1	X_{21}
X_{22}	+	56	-8	2	11	2	2	0	0	1	-2	1	-2	0	0	0	2	2	-1	-1	0	:	oo		0	4i	-8i	0	0	i	-2i	0	0	-i	-2i	-2i	i	-2i	0	0	X_{22}
X_{23}	+	56	-8	2	2	11	2	0	0	1	-2	-2	1	0	0	0	-1	-1	2	2	0	:	oo		0	4i	-8i	0	0	-2i	i	0	0	-i	-2i	-2i	-2i	i	0	0	X_{23}
X_{24}	+	70	-2	16	7	7	-2	2	0	0	4	1	1	0	0	0	1	1	1	1	2	:	oo		0	10i	-4i	0	0	i	i	0	-2i	0	2i	2i	-i	-i	0	0	X_{24}
X_{25}	o	70	-2	-11	7	-2	-2	2	0	0	1	1	-2	0	0	0	b27	**	1	1	-1		-		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{25}
X_{26}	o	70	-2	-11	7	-2	-2	2	0	0	1	1	-2	0	0	0	**	b27	1	1	-1																				X_{26}
X_{27}	o	70	-2	-11	-2	7	-2	2	0	0	1	-2	1	0	0	0	1	1	b27	**	-1		-		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{27}
X_{28}	o	70	-2	-11	-2	7	-2	2	0	0	1	-2	1	0	0	0	1	1	**	b27	-1																				X_{28}
X_{29}	+	120	-8	12	-6	-6	3	0	0	0	4	-2	-2	1	1	0	0	0	0	0	0	:	++		8	0	0	0	8	0	0	-1	0	0	0	0	0	0	1	1	X_{29}
X_{30}	o	210	10	21	3	3	3	2	0	0	1	1	1	0	0	2i	0	0	0	0	-1	:	oo		28	10i	4i	0	1	i	i	1	0	0	-2i-3	-2i+3	i	i	0	0	X_{30}
X_{31}	o	210	10	21	3	3	3	2	0	0	1	1	1	0	0	-2i	0	0	0	0	-1	:	oo		28	-10i	-4i	0	1	-i	-i	1	0	0	2i-3	2i+3	-i	-i	0	0	X_{31}
X_{32}	+	504	-8	18	18	-9	0	0	0	-1	-2	-2	1	0	0	0	0	0	0	0	0	:	oo		0	-4i	-8i	0	0	2i	-i	0	0	i	-2i	-2i	-2i	i	0	0	X_{32}
X_{33}	+	504	-8	18	-9	18	0	0	0	-1	-2	1	-2	0	0	0	0	0	0	0	0	:	oo		0	-4i	-8i	0	0	-i	2i	0	0	i	-2i	-2i	i	-2i	0	0	X_{33}
X_{34}	-	540	12	-27	0	0	0	4	0	0	-3	0	0	1	1	0	0	0	0	0	1	:	--		-48	0	0	0	-3	0	0	0	0	0	-3	3	0	0	1	1	X_{34}
X_{35}	+	560	-16	-34	2	2	2	0	0	0	2	2	2	0	0	0	-1	-1	-1	-1	0	:	oo		0	0	-16i	0	0	0	0	0	0	0	2i	2i	2i	2i	0	0	X_{35}
X_{36}	+	630	14	-18	9	9	0	-6	0	0	2	-1	-1	0	0	0	0	0	0	0	0	:	oo		0	10i	-4i	0	0	i	i	0	2i	0	2i	2i	-i	-i	0	0	X_{36}
X_{37}	o	640	0	-8	-8	-8	1	0	0	0	0	0	0	b7	**	0	1	1	1	1	0	:	oo		-32	0	0	0	4	0	0	1	0	0	0	0	0	0	b7	**	X_{37}
X_{38}	o	640	0	-8	-8	-8	1	0	0	0	0	0	0	**	b7	0	1	1	1	1	0	:	oo		-32	0	0	0	4	0	0	1	0	0	0	0	0	0	**	b7	X_{38}
X_{39}	+	896	0	32	-4	-4	-4	0	0	1	0	0	0	0	0	0	-1	-1	-1	-1	0	:	oo		0	16i	0	0	0	-2i	-2i	0	0	i	0	0	0	0	0	0	X_{39}
	ind	1	2	3	3	3	3	4	8	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind		2	8	4	4	6	24	24	6	8	40	12	12	12	12	14	14	
		4	4	12	12	12	12	4		20	12	12	12	28	28	8	36	36	36	36	12				4	8	4		12	24	24	12	8	40	12	12	12	12	28	28	
		2		6	6	6	6	4		10	6			14	14	8	18	18	18	18	12				2		4		6			6	8	40	12	12	12	12	14	14	
		4		12	12	12	12	4		20	12			28	28	8	36	36	36	36	12				4		4		12			12	8	40	12	12	12	12	28	28	
X_{40}	o2	20	0	-7	2	2	2	2	0	0	-3	0	0	-1	-1-i-1		-1	-1	-1	-1	-1	:	oo2		-6	0	2i+2	0	3	0	0	0-i-1		0	2i-1	-i+2	-i-1	-i-1	1	1	X_{40}
X_{41}	o2	120	0	12	-6	-6	3	4	0	0	0	0	0	1	1	0	0	0	0	0	-2	:	oo2		20	0	-4i-4	0	2	0	0	-1	0	0	2i+2	2i+2	-i-1	-i-1	-1	-1	X_{41}
X_{42}	o2	140	0	5	-4	-4	5	-2	0	0	-3	0	0	0	0	i+1	-1	-1	-1	-1	1	:	oo2		14	0	6i+6	0	5	0	0	-1	i+1	0	-3	-3i	0	0	0	0	X_{42}
X_{43}	o2	224	0	8	8	-10	-1	0	0	-1	0	0	0	0	0	0	-b27	**	-1	-1	0		o2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{43}
X_{44}	o2	224	0	8	8	-10	-1	0	0	-1	0	0	0	0	0	0	**	-b27	-1	-1	0																				X_{44}
X_{45}	o2	224	0	8	-10	8	-1	0	0	-1	0	0	0	0	0	0	-1	-1	-b27	**	0		o2		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{45}
X_{46}	o2	224	0	8	-10	8	-1	0	0	-1	0	0	0	0	0	0	-1	-1	**	-b27	0																				X_{46}
X_{47}	o2	280	0	37	10	10	1	4	0	0	-3	0	0	0	0	0	1	1	1	1	1	:	oo2		28	0	4i+4	0	1	0	0	1	0	0	4i+1	i+4	i+1	i+1	0	0	X_{47}
X_{48}	o2	280	0	-17	10	-8	1	4	0	0	3	0	0	0	0	0	-2	-2	1	1	1	:	oo2		28	0	4i+4	0	1	0	0	1	0	0	-2i+1	i-2	i+1	-2i-2	0	0	X_{48}
X_{49}	o2	280	0	-17	-8	10	1	4	0	0	3	0	0	0	0	0	1	1	-2	-2	1	:	oo2		28	0	4i+4	0	1	0	0	1	0	0	-2i+1	i-2	-2i-2	i+1	0	0	X_{49}
X_{50}	o2	420	0	-39	6	6	-3	2	0	0	-3	0	0	0	0	i+1	0	0	0	0	-1	:	oo2		-14	0	2i+2	0	-5	0	0	1	i+1	0	2i-1	-i+2	-i-1	-i-1	0	0	X_{50}
X_{51}	o2	540	0	-27	0	0	0	-2	0	0	-3	0	0	1	1-i-1		0	0	0	0	1	:	oo2		6	0	6i+6	0	-3	0	0	0-i-1		0	-3	-3i	0	0	-1	-1	X_{51}
X_{52}	o2	640	0	-8	-8	-8	1	0	0	0	0	0	0	b7	**	0	1	1	1	1	0	:	oo2		-32	0	0	0	4	0	0	1	0	0	0	0	0	0	b7	**	X_{52}
X_{53}	o2	640	0	-8	-8	-8	1	0	0	0	0	0	0	**	b7	0	1	1	1	1	0	:	oo2		-32	0	0	0	4	0	0	1	0	0	0	0	0	0	**	b7	X_{53}
X_{54}	o2	840	0	3	12	12	3	-4	0	0	3	0	0	0	0	0	0	0	0	0	-1	:	oo2		-28	0	12i+12	0	-1	0	0	-1	0	0	3	3i	0	0	0	0	X_{54}
X_{55}	o2	896	0	32	-4	-4	-4	0	0	1	0	0	0	0	0	0	-1	-1	-1	-1	0	:	oo2		0	0	0	0	0	0	0	0	0	w'40	0	0	0	0	0	0	X_{55}

$U_4(3)$

	fus	ind	6048 B A 4E	96 B A 4F	20 C A 4G	48 C A 8C	16 C A 8D	108 DE AE 12F	12 DF AF 12G	9 GE DE 12H	5 AG AG 20A	6 BC AC 24A	7 AE AE 28A	7 BE BE B*	fus	ind
χ_1	:	+o+o	1	1	1	1	1	1	1	1	1	1	1	1	:	++
χ_2	:	+o+o	7	-1	-1	-3	1	-2	2	1	-1	0	0	0	:	++
χ_3		++	0	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_4															:	++
χ_5	:	+o+o	6	-2	0	4	0	-3	1	0	0	1	-1	-1	:	++
χ_6	:	+o+o	14	-2	0	2	2	5	1	-1	0	-1	0	0	:	++
χ_7	:	+o+o	21	5	-1	1	1	3	-1	0	-1	1	0	0	:	++
χ_8	:	+o+o	28	4	0	2	-2	1	1	1	0	-1	0	0	:	++
χ_9		+	0	0	0	0	0	0	0	0	0	0	0	0	:	oo
χ_{10}															:	oo
χ_{11}																+
χ_{12}																
χ_{13}		++	0	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_{14}															:	++
χ_{15}	:	+o+o	14	-2	0	-2	-2	5	1	-1	0	1	0	0	:	++
χ_{16}	:	oooo	28i+28	-4i-4	0	0	0	i+1	-i-1	i+1	0	0	0	0	:	oo
χ_{17}	:	oooo	32	0	0	0	0	-4	0	-1	0	0	-b7	**		+
χ_{18}	:	oooo	32	0	0	0	0	-4	0	-1	0	0	**	-b7		
χ_{19}	:	+o+o	27	3	1	-3	1	0	0	0	1	0	-1	-1	:	++
χ_{20}	:	+o+o	0	0	4	0	0	0	0	0	-1	0	0	0	:	++
	fus	ind	4 4	4 4	8	8 8	8 8	12 12	12 12	12 12	40 40	24 24	28 28	28 28	fus	ind
χ_{21}	:	-o-o	-6	2	0	2	-2	3	-1	0	0	-1	1	1	:	oo
χ_{22}		oo	0	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_{23}															:	++
χ_{24}	:	oooo	7i+7	3i+3	0	i+1	-i-1	-2i-2	0	i+1	0	i+1	0	0	:	++
χ_{25}		-	0	0	0	0	0	0	0	0	0	0	0	0	:	oo
χ_{26}															:	oo
χ_{27}																+
χ_{28}																
χ_{29}	:	+o+o	20	4	0	0	0	2	-2	-1	0	0	-1	-1	:	++
χ_{30}	:	oooo	-7i+21	-3i+1	0	i+3	-i+1	2i+3	1	-i	0	i	0	0		+
χ_{31}	:	oooo	7i+21	3i+1	0	-i+3	i+1	-2i+3	1	i	0	-i	0	0		
χ_{32}		oo	0	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_{33}															:	++
χ_{34}	:	-o-o	6	-2	0	2	-2	-3	1	0	0	-1	-1	-1	:	oo
χ_{35}	:	oooo	28i+28	-4i-4	0	0	0	i+1	-i-1	i+1	0	0	0	0	:	++
χ_{36}	:	oooo	21i+21	i+1	0	-3i-3	-i-1	3i+3	i+1	0	0	0	0	0	:	++
χ_{37}	:	oooo	32	0	0	0	0	-4	0	-1	0	0	-b7	**		+
χ_{38}	:	oooo	32	0	0	0	0	-4	0	-1	0	0	**	-b7		
χ_{39}	:	oooo	0	0	0	0	0	0	0	0	w'40	0	0	0	:	++
	fus	ind	4 4 4 4	4 4 4 4	16	8 8 8 8	8 8	12 12 12 12	12 12 12 12	12 12 12 12	80 80 80 80	24 24 24 24	28 28 28 28	28 28 28 28	fus	ind
χ_{40}	:	oooo2	i-7	i-3	0	2	0	i+2	i	i-1	0	-1	i	i	*	+
χ_{41}	:	oooo2	6i+14	-2i-2	0	0	0	-3i+5	i+1	-1	0	0	-i	-i	*	+
χ_{42}	:	oooo2	-7i+21	i+1	0	2	0	2i+3	-2i+1	-i	0	-1	0	0	*	+
χ_{43}		o2	0	0	0	0	0	0	0	0	0	0	0	0	*	o
χ_{44}															*	o
χ_{45}																+2
χ_{46}															*	
χ_{47}	:	oooo2	28	-4	0	4	0	1	-1	1	0	1	0	0	*	+
χ_{48}		o2	0	0	0	0	0	0	0	0	0	0	0	0	*	+
χ_{49}	:														*	+
χ_{50}	:	oooo2	21i-7	5i-3	0	-2	0	3i+2	-i	-1	0	1	0	0	*	+
χ_{51}	:	oooo2	-27i+21	-3i+1	0	-2	0	3	1	0	0	1	i	i	*	+
χ_{52}	:	oooo2	32	0	0	0	0	-4	0	-1	0	0	-b7	**		+2
χ_{53}	:	oooo2	32	0	0	0	0	-4	0	-1	0	0	**	-b7	*	
χ_{54}	:	oooo2	28	-4	0	-4	0	1	-1	1	0	-1	0	0	*	+
χ_{55}	:	oooo2	0	0	0	0	0	0	0	0	w'80	0	0	0	*	+

	fus	ind	25 920 A A 2D	576 A A 2E	96 A A 4H	48 A A 4I	648 AD AD 6H	648 AD AD I**	108 CD CD 6J	54 BD BD 6K	36 CE CE 6L	9 DE DE 6M	8 A A 8E	5 AD AD 10B	12 AH AH 12I	6 BI BI 12J	9 BI AD 18A	9 AH BD B**	fus	ind	fus	ind
χ_1			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	:	++
χ_2			9	1	-3	1	0	0	-3	3	1	1	-1	-1	0	1	0	0	:	++	:	++
χ_3			15	-1	-1	3	6	6	3	0	-1	-1	-1	0	2	0	0	0	:	++		+
χ_4			-5	-5	3	-1	4	4	-2	1	-2	1	-1	0	0	-1	1	1	:	++		
χ_5			30	6	2	2	3	3	3	3	3	0	0	0	-1	-1	0	0	:	++	:	++
χ_6			20	4	4	0	-7	-7	2	2	-2	1	0	0	1	0	-1	-1	:	++	:	++
χ_7			9	9	1	-3	9	9	0	0	0	0	1	-1	1	0	0	0	:	++	:	++
χ_8			30	-10	2	-2	3	3	3	3	-1	-1	0	0	-1	1	0	0	:	++	:	++
χ_9			40	-8	0	0-6i3+4	6i3+4		1	-2	1	1	0	0	0	0	**	z3		+		o
χ_{10}			40	-8	0	0	6i3+4-6i3+4		1	-2	1	1	0	0	0	0	z3	**				o
χ_{11}			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	oo		
χ_{12}																			:	oo		
χ_{13}			75	3	-1	3	3	3	-3	0	-3	0	1	0	-1	0	0	0	:	++		+
χ_{14}			15	-9	-5	-1	-3	-3	0	3	0	0	1	0	1	-1	0	0	:	++		
χ_{15}			60	-4	4	0	-3	-3	-6	0	2	-1	0	0	1	0	0	0	:	++	:	++
χ_{16}			0	0	0	0	6i3	-6i3	0	0	0	0	0	0	0	0	-i3	i3	:	oo	:	oo
χ_{17}			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+
χ_{18}																						
χ_{19}			81	9	-3	-3	0	0	0	0	0	0	-1	1	0	0	0	0	:	++	:	++
χ_{20}			64	0	0	0	-8	-8	4	-2	0	0	0	-1	0	0	1	1	:	++	:	++
	fus	ind	2	2	4	4 4	6 6	6 6	6 6	6 6	6 6	6 6	8	10 10	12 12	12 12	18 18	18 18	fus	ind	fus	ind
χ_{21}			0	0	0	0	3i3	-3i3	0	0	0	0	0	0	i3	0	-i3	i3	:	oo	:	oo
χ_{22}			24	0	0	4	6	6	0	3	0	0	0	-1	0	1	0	0	:	++		+
χ_{23}			16	0	0	0	-2	-2	1	4	3	0	0	1	0	0	1	1	:	++		
χ_{24}			20	0	0	2	2	2	5	-1	3	0	0	0	0	-1	-1	-1	:	++	:	++
χ_{25}			20	0	0	2	3i3+2	-3i3+2	-4	-1	0	0	0	0	i3	-1	**	-z3		+		o
χ_{26}			20	0	0	2	-3i3+2	3i3+2	-4	-1	0	0	0	0	-i3	-1	-z3	**				o
χ_{27}			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	oo		
χ_{28}																			:	oo		
χ_{29}			0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	:	++	:	++
χ_{30}			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+
χ_{31}																						
χ_{32}			96	0	0	0	6	6	3	0	-3	0	0	1	0	0	0	0	:	++		+
χ_{33}			24	0	0	-4	6	6	0	3	0	0	0	-1	0	-1	0	0	:	++		
χ_{34}			0	0	0	0	9i3	-9i3	0	0	0	0	0	0	-i3	0	0	0	:	oo	:	oo
χ_{35}			80	0	0	0	-10	-10	-4	2	0	0	0	0	0	0	-1	-1	:	++	:	++
χ_{36}			60	0	0	-2	6	6	-3	-3	3	0	0	0	0	1	0	0	:	++	:	++
χ_{37}			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+
χ_{38}																						
χ_{39}			64	0	0	0	-8	-8	4	-2	0	0	0	-1	0	0	1	1	:	++	:	++
	fus	ind	4 4 4 4	2	4	4 4	12 12	12 12	12 12	12 12	6 6	6 6	8	20 20	12 12	12 12	36 36	36 36	fus	ind	fus	ind
χ_{40}																			*	+	*	+
χ_{41}																			*	+	*	+
χ_{42}																			*	+	*	+
χ_{43}																				+2		o2
χ_{44}																			*			o2
χ_{45}																			*	o	*	
χ_{46}																			*	o		
χ_{47}																			*	+	*	+
χ_{48}																			*	+		o2
χ_{49}																			*	+	*	
χ_{50}																			*	+	*	+
χ_{51}																			*	+	*	+
χ_{52}																				+2		+2
χ_{53}																			*		*	
χ_{54}																			*	+	*	+
χ_{55}																			*	+	*	+

	fus	ind	720 A A 2F	48 A A 4J	9 DF DF 6N	48 A A 8F	16 A A 8G	4 B A 8H	5 AF AF 10C	6 AJ AJ 12J	12 AF AF 24B	12 AF AF C**	fus	ind
χ_1			1	1	1	1	1	1	1	1	1	1	:	++
χ_2			1	-3	1	3	-1	-1	1	0	0	0	:	++
χ_3		+	0	0	0	0	0	0	0	0	0	0		+
χ_4														
χ_5			0	4	0	2	-2	0	0	1	-1	-1	:	++
χ_6			10	2	1	2	2	0	0	-1	-1	-1	:	++
χ_7			9	1	0	1	1	-1	-1	1	1	1	:	++
χ_8			-10	2	-1	4	0	0	0	-1	1	1	:	++
χ_9			o	0	0	0	0	0	0	0	0	0		o
χ_{10}			o	0	0	0	0	0	0	0	0	0		o
χ_{11}														
χ_{12}														
χ_{13}			+	0	0	0	0	0	0	0	0	0		+
χ_{14}														
χ_{15}			++	-10	-2	-1	2	2	0	0	1	-1	-1	: ++
χ_{16}			oo	0	0	0	0	0	0	0	0	16	-16	: oo
χ_{17}			+	0	0	0	0	0	0	0	0	0		+
χ_{18}														
χ_{19}			++	9	-3	0	3	-1	1	-1	0	0	0	: ++
χ_{20}			++	16	0	-2	0	0	0	1	0	0	0	: ++
	fus	ind	2	4	6 6	8 8	8	8 8	10 10	12 12	24 24	24 24	fus	ind
χ_{21}			oo	0	0	0	0	0	0	i3	i3	-i3	:	oo
χ_{22}			+	0	0	0	0	0	0	0	0	0		+
χ_{23}														
χ_{24}			++	0	0	0	2r2	0	r2	0	0	-r2	-r2	: ++
χ_{25}			o	0	0	0	0	0	0	0	0	0		o
χ_{26}			o	0	0	0	0	0	0	0	0	0		o
χ_{27}														
χ_{28}														
χ_{29}			++	0	0	3	0	0	0	0	0	0	:	++
χ_{30}			+	0	0	0	0	0	0	0	0	0		+
χ_{31}														
χ_{32}			+	0	0	0	0	0	0	0	0	0		+
χ_{33}														
χ_{34}			oo	0	0	0	0	0	0	i3	-i3	i3	:	oo
χ_{35}			++	0	0	0	4r2	0	0	0	r2	r2	:	++
χ_{36}			++	0	0	0	2r2	0	-r2	0	0	-r2	-r2	: ++
χ_{37}			+	0	0	0	0	0	0	0	0	0		+
χ_{38}														
χ_{39}			++	0	0	0	0	0	r5	0	0	0	:	++
	fus	ind	2	4	6 6	8 8	8	8 8	10 10	12 12	24 24	24 24	fus	ind
χ_{40}													*	+
χ_{41}													*	+
χ_{42}													*	+
χ_{43}														o2
χ_{44}														o2
χ_{45}													*	
χ_{46}													*	
χ_{47}													*	+
χ_{48}														o2
χ_{49}													*	
χ_{50}													*	+
χ_{51}													*	+
χ_{52}														+2
χ_{53}													*	
χ_{54}													*	+
χ_{55}													*	+

$U_4(3)$

		1A	2A	3A	3B	3C	3D	4A	4B	5A	6A	6B	6C	7A	B**	8A	9A	B**	9C	D**	12A			2B	2C	4C	4D	6D	6E	6F	6G	8B	10A	12B	C**	12D	12E	14A	B**	
	ind	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind	2	2	4	4	6	6	6	6	8	10	12	12	12	12	14	14	
		3	6	3	3			12	12	15	6	6	6	21	21	24	9	9			12																				
		3	6	3	3			12	12	15	6	6	6	21	21	24	9	9			12																				
X_{56}	o2	15	-1	6	3	0	0	3	-1	0	2	-1	2	1	1	1	**	z3-1	0	0	0	*	+																		X_{56}
X_{57}	o2	21	5	3	6	0	0	1	1	1	-1	2	2	0	0	-1	z3-1	**	0	0	1	*	+																		X_{57}
X_{58}	o2	105	9	15	3	0	0	1	1	0	3	3	0	0	0	1	-z3+1	**	0	0	1	*	+																		X_{58}
X_{59}	o2	105	-7	15	3	0	0	5	1	0	-1	-1	2	0	0	-1	-z3+1	**	0	0	-1	*	+																		X_{59}
X_{60}	o2	105	9	-12	12	0	0	1	1	0	0	0	0	0	0	1	i3	-i3	0	0	-2	*	+																		X_{60}
X_{61}	o2	210	2	3	15	0	0	-2	-2	0	-1	-1	2	0	0	0	**	-z3+1	0	0	1	*	+																		X_{61}
X_{62}	o2	315	-5	-36	9	0	0	3	-1	0	4	1	-2	0	0	-1	0	0	0	0	0	*	+																		X_{62}
X_{63}	o2	336	16	-6	6	0	0	0	0	1	-2	-2	-2	0	0	0	-i3	i3	0	0	0	*	+																		X_{63}
X_{64}	o2	360	8	-18	-9	0	0	0	0	0	2	-1	2	b7	**	0	0	0	0	0	0	*	o																		X_{64}
X_{65}	o2	360	8	-18	-9	0	0	0	0	0	2	-1	2	**	b7	0	0	0	0	0	0	*	o																		X_{65}
X_{66}	o2	384	0	24	12	0	0	0	0	-1	0	0	0	-1	-1	0	**	z3-1	0	0	0	*	+																		X_{66}
X_{67}	o2	420	4	33	-6	0	0	4	0	0	1	-2	-2	0	0	0	i3	-i3	0	0	1	*	+																		X_{67}
X_{68}	o2	630	6	9	-9	0	0	2	-2	0	-3	3	0	0	0	0	0	0	0	0	-1	*	+																		X_{68}
X_{69}	o2	729	9	0	0	0	0	-3	1	-1	0	0	0	1	1	-1	0	0	0	0	0	*	+																		X_{69}
X_{70}	o2	756	-12	27	0	0	0	-4	0	1	3	0	0	0	0	0	0	0	0	0	-1	*	+																		X_{70}
X_{71}	o2	945	-15	-27	0	0	0	1	1	0	-3	0	0	0	0	1	0	0	0	0	1	*	+																		X_{71}
	and no:	1 :3	2 :3	3 :3	3	3 :3	3	4 :3	4 :3	5 :3	6 :3	6 :3	6 :3	7 :3	7 :3	8 :3	9	9	9 :3	9 :3	12 :3		and no:	2	2	4	4	6	6	6	6	8	10	12	12	12	12	14	14	
	ind	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind	2	4	4	4	6	12	12	6	8	20	12	12	12	12	14	14	
		6	6	6	6	6	6	12	12	30	6	6	6	42	42	24	18	18	18	18	12			2	4	4		6	12	12	6	8	20	12	12	12	12	14	14	
		3	6	3	3			12	12	15	6	6	6	21	21	24	9	9			12																				
		2	2	6	6			4		10	6	6	6	14	14	8	18	18			12																				
		3	6	3	3			12		15	6	6	6	21	21	24	9	9			12																				
		6	6	6	6			12		30	6	6	6	42	42	24	18	18			12																				
X_{72}	o2	6	-2	-3	3	0	0	2	0	1	1	1	-2	-1	-1	0	i3	-i3	0	0	-1	*	-																		X_{72}
X_{73}	o2	84	4	-15	6	0	0	4	0	-1	1	-2	-2	0	0	0	**	-z3+1	0	0	1	*	-																		X_{73}
X_{74}	o2	120	-8	-6	15	0	0	0	0	0	-2	1	-2	1	1	0	z3-1	**	0	0	0	*	-																		X_{74}
X_{75}	o2	126	-10	18	9	0	0	2	0	1	2	-1	2	0	0	0	0	0	0	0	2	*	-																		X_{75}
X_{76}	o2	210	-6	-24	-3	0	0	6	0	0	0	3	0	0	0	0	-i3	i3	0	0	0	*	-																		X_{76}
X_{77}	o2	270	6	27	0	0	0	2	0	0	3	0	0	-b7	**	0	0	0	0	0	-1	*	o																		X_{77}
X_{78}	o2	270	6	27	0	0	0	2	0	0	3	0	0	**	-b7	0	0	0	0	0	-1	*	o																		X_{78}
X_{79}	o2	336	16	-6	6	0	0	0	0	1	-2	-2	-2	0	0	0	-i3	i3	0	0	0	*	-																		X_{79}
X_{80}	o2	384	0	24	12	0	0	0	0	-1	0	0	0	-1	-1	0	**	z3-1	0	0	0	*	-																		X_{80}
X_{81}	o2	420	-12	-21	12	0	0	-4	0	0	3	0	0	0	0	0	-z3+1	**	0	0	-1	*	-																		X_{81}
X_{82}	o2	630	-18	9	-9	0	0	2	0	0	-3	-3	0	0	0	0	0	0	0	0	-1	*	-																		X_{82}
X_{83}	o2	630	-2	9	-9	0	0	-2	0	0	1	1	-2	0	0	21	0	0	0	0	1	*	o																		X_{83}
X_{84}	o2	630	-2	9	-9	0	0	-2	0	0	1	1	-2	0	0	-21	0	0	0	0	1	*	o																		X_{84}
X_{85}	o2	840	8	12	6	0	0	0	0	0	-4	2	2	0	0	0	**	-z3+1	0	0	0	*	-																		X_{85}
X_{86}	o2	840	8	-42	-3	0	0	0	0	0	2	-1	2	0	0	0	z3-1	**	0	0	0	*	-																		X_{86}
	and no:	1 :6	2 :6	3 :6	3 :2	3 :6	3 :2	4 :6	4 :3	5 :6	6 :6	6 :6	6 :6	7 :6	7 :6	8 :6	9 :2	9 :2	9 :6	9 :6	12 :6		and no:	2 :2	4 :2	4 :2	4	6 :2	12 :2	12 :2	6 :2	8 :2	20 :2	12 :2	12 :2	12 :2	12 :2	14 :2	14 :2	
	ind	1	2	3	3	3	3	4	8	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind	2	8	4	4	6	24	24	6	8	40	12	12	12	12	14	14	
		12	12	12	12	12	12	12	24	60	12	12	12	84	84	24	36	36	36	36	12			4	8	4		12	24	24	12	8	40	12	12	12	12	28	28	
		6	6	6	6	6	6	12	24	30	6	6	6	42	42	24	18	18	18	18	12			2		4		6			6	8	40	12	12	12	12	14	14	
		4	4	12	12	12	12	4		20	12	12	12	28	28	8	36	36	36	36	12			4		4		12			12	8	40	12	12	12	12	28	28	
		3	6	3	3			12		15	6	6	6	21	21	24	9	9			12																				
		12	12	12	12			12		60	12	12	12	84	84	24	36	36			12																				
		2		6	6			4		10	6			14	14	8	18	18			12																				
		12		12	12			12		60	12			84	84	24	36	36			12																				
		3		3	3			12		15	6			21	21	24	9	9			12																				
		4		12	12			4		20	12			28	28	8	36	36			12																				
		6		6	6			12		30	6			42	42	24	18	18			12																				
		12		12	12			12		60	12			24	84	8	36	36			12																				
X_{87}	o4	84	0	-15	6	0	0	2	0	-1	-3	0	0	0	0	-i-1	**	-z3+1	0	0	-1	*5	o2																		X_{87}
X_{88}	o4	120	0	21	6	0	0	4	0	0	-3	0	0	1	1	0	-i3	i3	0	0	1	*5	o2																		X_{88}
X_{89}	o4	216	0	-27	0	0	0	4	0	1	-3	0	0	-1	-1	0	0	0	0	0	1	*5	o2																		X_{89}
X_{90}	o4	384	0	24	12	0	0	0	0	-1	0	0	0	-1	-1	0	**	z3-1	0	0	0	*5	o2																		X_{90}
X_{91}	o4	420	0	-21	12	0	0	2	0	0	3	0	0	0	0	i+1	-z3+1	**	0	0	-1	*5	o2																		X_{91}
X_{92}	o4	420	0	33	-6	0	0	2	0	0	-3	0	0	0	0	i+1	i3	-i3	0	0	-1	*5	o2																		X_{92}
X_{93}	o4	480	0	-24	6	0	0	0	0	0	0	0	0	-b7	**	0	z3-1	**	0	0	0	*5	o2																		X_{93}
X_{94}	o4	480	0	-24	6	0	0	0	0	0	0	0	0	**	-b7	0	z3-1	**	0	0	0	*5	o2																		X_{94}
X_{95}	o4	504	0	-9	-18	0	0	4	0	-1	3	0	0	0	0	0	0	0	0	0	1	*5	o2																		X_{95}
X_{96}	o4	756	0	27	0	0	0	2	0	1	3	0	0	0	0	-i-1	0	0	0	0	-1	*5	o2																		X_{96}
X_{97}	o4	840	0	-15	-12	0	0	-4	0	0	-3	0	0	0	0	0	-i3	i3	0	0	-1	*5	o2																		X_{97}
X_{98}	o4	840	0	12	6	0	0	-4	0	0	0	0	0	0	0	0	**	-z3+1	0	0	2	*5	o2																		X_{98}
	and no:	1 :12	2 :6	3 :12	3 :4	3 :12	3 :4	4 :12	8 :3	5 :12	6 :12	6 :6	6 :6	7 :12	7 :12	8 :12	9 :4	9 :4	9 :12	9 :12	12 :12		and no:	2 :4	8 :2	4 :4	4	6 :4	24 :2	24 :2	6 :4	8 :4	40 :4	12 :4	12 :4	12 :4	12 :4	24 :4	24 :4	

$U_4(3)$

		4E	4F	4G	8C	8D	12F	12G	12H	20A	24A	28A	B*		2D	2E	4H	4I	6H	I**	6J	6K	6L	6M	8E	10B	12I	12J	18A	B**		2F	4J	6N	8F	8G	8H	10C	12J	24B	C**	
														fus ind	2	2	4	4	6	6	6	6	6	6	8	10	12	12	18	18	fus ind											
															6	6	12	12	6	6	6	6			24	30	12	12	18	18												
															6	6	12	12	6	6	6	6			24	30	12	12	18	18												
	χ_{56}													: oo2	5	-3	1	1	-4z3	**	2	-1	0	0	-1	0	-2	1	-z3	**	*	+	χ_{56}										
	χ_{57}													: oo2	11	3	-1	3-2i3-1	2i3-1		2	2	0	0	1	1	-1	0	**	-z3	*	+	χ_{57}										
	χ_{58}													: oo2	25	9	1	1	2i3+1	-2i3+1	4	1	0	0	1	0	1	1	**	z3	*	+	χ_{58}										
	χ_{59}													: oo2	5	-3	1	1	4i3-7	-4i3-7	2	-1	0	0	-1	0	1	1	**	-z3	*	+	χ_{59}										
	χ_{60}													: oo2	35	3	3	3-2i3-4	2i3-4		-4	2	0	0	-1	0	0	0	-1	-1	*	+	χ_{60}										
	χ_{61}													: oo2	50	-6	-2	2-2i3-7	2i3-7		2	-1	0	0	0	0	1	-1	-z3	**	*	+	χ_{61}										
	χ_{62}													: oo2	45	-3	-3	1	0	0	-6	-3	0	0	1	0	0	1	0	0	*	+	χ_{62}										
	χ_{63}													: oo2	64	0	0	0	2i3+4	-2i3+4	-2	4	0	0	0	-1	0	0	1	1	*	+	χ_{63}										
	χ_{64}													o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+2	χ_{64}										
	χ_{65}																															*	χ_{65}										
	χ_{66}													: oo2	64	0	0	0	-8z3	**	4	-2	0	0	0	-1	0	0	z3	**	*	+	χ_{66}										
	χ_{67}													: oo2	20	-12	-4	0	4i3-1	-4i3-1	2	2	0	0	0	0	-1	0	-1	-1	*	+	χ_{67}										
	χ_{68}													: oo2	30	6	2	2	6i3+3	-6i3+3	0	-3	0	0	0	0	-1	-1	0	0	*	+	χ_{68}										
	χ_{69}													: oo2	81	9	-3	-3	0	0	0	0	0	0	-1	1	0	0	0	0	*	+	χ_{69}										
	χ_{70}													: oo2	36	-12	4	0	9	9	0	0	0	0	0	1	1	0	0	0	*	+	χ_{70}										
	χ_{71}													: oo2	45	-3	5	-3	-9	-9	0	0	0	0	1	0	-1	0	0	0	*	+	χ_{71}										
														and	2	2	4	4	6	6	6	6	6	6	8	10	12	12	18	18	:	:										
														no:																		:	:										
														fus ind	2	2	4	4	6	6	6	6	6	6	8	10	12	12	18	18	fus ind											
															6	6	12	12	6	6	6	6	6	6	24	30	12	12	18	18												
															6	6	12	12	6	6	6	6			24	30	12	12	18	18												
															2			4	6	6	6	6				10	12	12	18	18												
															6			12	6	6	6	6				30	12	12	18	18												
															6			12	6	6	6	6				30	12	12	18	18												
	χ_{72}													: oo2	4	0	0	2	-i3-2	i3-2	-2	1	0	0	0	-1	i3	-1	1	1	*	+	χ_{72}										
	χ_{73}													: oo2	16	0	0	0	-i3-8	i3-8	-2	-2	0	0	0	1	i3	0	z3	**	*	+	χ_{73}										
	χ_{74}													: oo2	40	0	0	4	-4i3-2	4i3-2	-2	1	0	0	0	0	0	1	**	z3	*	+	χ_{74}										
	χ_{75}													: oo2	36	0	0	2	0	0	6	3	0	0	0	1	0	-1	0	0	*	+	χ_{75}										
	χ_{76}													: oo2	20	0	0	2	4i3+8	-4i3+8	-4	-1	0	0	0	0	0	-1	-1	-1	*	+	χ_{76}										
	χ_{77}													o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+2	χ_{77}										
	χ_{78}																															*	χ_{78}										
	χ_{79}													: oo2	16	0	0	0	-8z3+6	**	-2	-2	0	0	0	1	0	0	1	1	*	+	χ_{79}										
	χ_{80}													: oo2	64	0	0	0	-8z3	**	4	-2	0	0	0	-1	0	0	z3	**	*	+	χ_{80}										
	χ_{81}													: oo2	80	0	0	0	i3-4	-i3-4	-4	2	0	0	0	0	-i3	0	**	-z3	*	+	χ_{81}										
	χ_{82}													: oo2	60	0	0	-2	3i3+6	-3i3+6	0	3	0	0	0	0	i3	1	0	0	*	+	χ_{82}										
	χ_{83}													o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+2	χ_{83}										
	χ_{84}																															*	χ_{84}										
	χ_{85}													: oo2	80	0	0	0	8z3	**	2	-4	0	0	0	0	0	0	-z3	**	*	+	χ_{85}										
	χ_{86}													: oo2	40	0	0	-4	-4i3-2	4i3-2	-2	1	0	0	0	0	0	-1	**	z3	*	+	χ_{86}										
														and	2	2	4	4	6	6	6	6	6	6	8	10	12	12	18	18	:	:										
														no:	:2			:2	:2	:2	:2	:2	:2	:2		:2	:2	:2	:2	:2	:	:										
														fus ind	4	2	4	4	12	12	12	12	6	6	8	20	12	12	36	36	fus ind											
															12	6	12	12	12	12	12	12	6	6	24	60	12	12	36	36												
															12	6	12	12	12	12	12	12			24	60	12	12	36	36												
															4			4	12	12	12	12				20	12	12	36	36												
															12			12	12	12	12	12				60	12	12	36	36												
															12			12	12	12	12	12				60	12	12	36	36												
	χ_{87}												*7	o2																	**	+2	χ_{87}										
	χ_{88}												*7	o2																	**	+2	χ_{88}										
	χ_{89}												*7	o2																	**	+2	χ_{89}										
	χ_{90}												*7	o2																	**	+2	χ_{90}										
	χ_{91}												*7	o2																	**	+2	χ_{91}										
	χ_{92}												*7	o2																	**	+2	χ_{92}										
	χ_{93}													o4																		+4	χ_{93}										
	χ_{94}												*7																		**		χ_{94}										
	χ_{95}												*7	o2																	**	+2	χ_{95}										
	χ_{96}												*7	o2																	**	+2	χ_{96}										
	χ_{97}												*7	o2																	**	+2	χ_{97}										
	χ_{98}												*7	o2																	**	+2	χ_{98}										
														and	4	2	4	4	12	12	12	12	6	6	8	20	12	12	36	36	*7	*7										
														no:	:2			:2	:2	:2	:2	:2	:2	:2		:2	:2	:2	:2	:2	*7	*7										

$U_4(3)$

		1A	2A	3A	3B	3C	3D	4A	4B	5A	6A	6B	6C	7A	B**	8A	9A	B**	9C	D**	12A			2B	2C	4C	4D	6D	6E	6F	6G	8B	10A	12B	C**	12D	12E	14A	B**	
	ind	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind	2	2	4	4	6	6	6	6	8	10	12	12	12	12	14	14	
		3	6	3				12	12	15	6	6	6	21	21	24					12																			
		3	6	3				12	12	15	6	6	6	21	21	24					12																			
X_{99}	o2	36	4	9	0	0	0	4	0	1	1	-2	-2	1	1	0	0	0	0	0	1	*	+																	X_{99}
X_{100}	o2	45	-3	-9	0	0	0	1	1	0	3	0	0	b7	**	-1	0	0	0	0	1	*	o																	X_{100}
X_{101}	o2	45	-3	-9	0	0	0	1	1	0	3	0	0	**	b7	-1	0	0	0	0	1	*	o																	X_{101}
X_{102}	o2	126	14	-9	0	0	0	2	2	1	-1	2	2	0	0	0	0	0	0	0	-1	*	+																	X_{102}
X_{103}	o2	189	-3	27	0	0	0	5	1	-1	3	0	0	0	0	1	0	0	0	0	-1	*	+																	X_{103}
X_{104}	o2	315	11	18	0	0	0	-1	-1	0	2	2	2	0	0	1	0	0	0	0	2	*	+																	X_{104}
X_{105}	o2	315	-5	18	0	0	0	3	-1	0	-2	-2	4	0	0	-1	0	0	0	0	0	*	+																	X_{105}
X_{106}	o2	315	-5	18	0	0	0	3	-1	0	-2	4	-2	0	0	-1	0	0	0	0	0	*	+																	X_{106}
X_{107}	o2	630	6	-45	0	0	0	2	-2	0	3	0	0	0	0	0	0	0	0	0	-1	*	+																	X_{107}
X_{108}	o2	720	16	18	0	0	0	0	0	0	-2	-2	-2	-1	-1	0	0	0	0	0	0	*	+																	X_{108}
X_{109}	o2	729	9	0	0	0	0	-3	1	-1	0	0	0	1	1	-1	0	0	0	0	0	*	+																	X_{109}
X_{110}	o2	756	-12	27	0	0	0	-4	0	1	3	0	0	0	0	0	0	0	0	0	-1	*	+																	X_{110}
X_{111}	o2	945	-15	-27	0	0	0	1	1	0	-3	0	0	0	0	1	0	0	0	0	1	*	+																	X_{111}
	and	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12		and	2	2	4	4	6	6	6	6	8	10	12	12	12	12	14	14	
	no:	:3	:3	:3				:3	:3	:3	:3	:3	:3	:3	:3	:3					:3		no:																	
	ind	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind	2	4	4	4	6	12	12	6	8	20	12	12	12	12	14	14	
		6	6	6	6	6	6	12	12	30	6	6	6	42	42	24	18	18	18	18	12			2	4	4		6	12	12	6	8	20	12	12	12	12	14	14	
		3	6	3				12	12	15	6	6	6	21	21	24					12																			
		2	2	6				4		10	6	6	6	14	14	8					12																			
		3	6	3				12		15	6	6	6	21	21	24					12																			
		6	6	6				12		30	6	6	6	42	42	24					12																			
X_{112}	o2	90	2	-18	0	0	0	6	0	0	2	2	2	-1	-1	0	0	0	0	0	0	*	-																	X_{112}
X_{113}	o2	126	-10	-9	0	0	0	2	0	1	-1	-4	2	0	0	0	0	0	0	0	-1	*	-																	X_{113}
X_{114}	o2	126	-10	-9	0	0	0	2	0	1	-1	2	-4	0	0	0	0	0	0	0	-1	*	-																	X_{114}
X_{115}	o2	126	6	-9	0	0	0	-2	0	1	3	0	0	0	0	2i	0	0	0	0	1	*	o																	X_{115}
X_{116}	o2	126	6	-9	0	0	0	-2	0	1	3	0	0	0	0	-2i	0	0	0	0	1	*	o																	X_{116}
X_{117}	o2	270	6	27	0	0	0	2	0	0	3	0	0	-b7	**	0	0	0	0	0	-1	*	o																	X_{117}
X_{118}	o2	270	6	27	0	0	0	2	0	0	3	0	0	**	-b7	0	0	0	0	0	-1	*	o																	X_{118}
X_{119}	o2	504	-8	-36	0	0	0	0	0	-1	4	-2	-2	0	0	0	0	0	0	0	0	*	-																	X_{119}
X_{120}	o2	540	12	-27	0	0	0	4	0	0	-3	0	0	1	1	0	0	0	0	0	1	*	-																	X_{120}
X_{121}	o2	630	-18	36	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	2	*	-																	X_{121}
X_{122}	o2	720	16	18	0	0	0	0	0	0	-2	-2	-2	-1	-1	0	0	0	0	0	0	*	-																	X_{122}
X_{123}	o2	1260	-4	-9	0	0	0	-4	0	0	-1	2	2	0	0	0	0	0	0	0	-1	*	-																	X_{123}
	and	1	2	3	3	3	3	4	4	5	6	6	6	7	7	8	9	9	9	9	12		and	2	4	4	4	6	12	12	6	8	20	12	12	12	12	14	14	
	no:	:6	:6	:6	:2	:2	:2	:6	:3	:6	:6	:6	:6	:6	:6	:6	:2	:2	:2	:2	:6		no:	:2	:2	:2		:2	:2	:2	:2	:2	:2	:2	:2	:2	:2	:2	:2	
	ind	1	2	3	3	3	3	4	8	5	6	6	6	7	7	8	9	9	9	9	12	fus	ind	2	8	4	4	6	24	24	6	8	40	12	12	12	12	14	14	
		12	12	12	12	12	12	12	24	60	12	12	12	84	84	24	36	36	36	36	12			4	8	4		12	24	24	12	8	40	12	12	12	12	28	28	
		6	6	6	6	6	6	12	24	30	6	6	6	42	42	24	18	18	18	18	12			2		4		6			6	8	40	12	12	12	12	14	14	
		4	4	12	12	12	12	4		20	12	12	12	28	28	8	36	36	36	36	12			4		4		12			12	8	40	12	12	12	12	28	28	
		3	6	3				12		15	6	6	6	21	21	24					12																			
		12	12	12				12		60	12	12	12	84	84	24					12																			
		2		6				4		10	6			14	14	8					12																			
		12		12				12		60	12			84	84	24					12																			
		3		3				12		15	6			21	21	24					12																			
		4		12				4		20	12			28	28	8					12																			
		6		6				12		30	6			42	42	24					12																			
		12		12				12		60	12			84	84	24					12																			
X_{124}	o4	36	0	9	0	0	0	2	0	1	-3	0	0	1	1	i+1	0	0	0	0	-1	*5	o2																	X_{124}
X_{125}	o4	216	0	-27	0	0	0	4	0	1	-3	0	0	-1	-1	0	0	0	0	0	1	*5	o2																	X_{125}
X_{126}	o4	360	0	9	0	0	0	-4	0	0	-3	0	0	b7	**	0	0	0	0	0	-1	*5	o2																	X_{126}
X_{127}	o4	360	0	9	0	0	0	-4	0	0	-3	0	0	**	b7	0	0	0	0	0	-1	*5	o2																	X_{127}
X_{128}	o4	504	0	-36	0	0	0	4	0	-1	0	0	0	0	0	0	0	0	0	0	-2	*5	o2																	X_{128}
X_{129}	o4	504	0	45	0	0	0	4	0	-1	-3	0	0	0	0	0	0	0	0	0	1	*5	o2																	X_{129}
X_{130}	o4	540	0	-27	0	0	0	-2	0	0	-3	0	0	1	1	-i-1	0	0	0	0	1	*5	o2																	X_{130}
X_{131}	o4	756	0	27	0	0	0	2	0	1	3	0	0	0	0	-i-1	0	0	0	0	-1	*5	o2																	X_{131}
X_{132}	o4	1260	0	-9	0	0	0	-2	0	0	3	0	0	0	0	i+1	0	0	0	0	1	*5	o2																	X_{132}
	and	1	2	3	3	3	3	4	8	5	6	6	6	7	7	8	9	9	9	9	12		and	2	8	4	4	6	24	24	6	8	40	12	12	12	12	14	14	
	no:	:12	:6	:12	:4	:4	:4	:12	:3	:12	:12	:6	:6	:12	:12	:12	:4	:4	:4	:4	:12		no:	:4	:2	:4		:4	:2	:2	:4	:4	:4	:4	:4	:4	:4	:4	:4	

$U_4(3)$

	4E	4F	4G	8C	8D	12F	12G	12H	20A	24A	28A	B*	2D	2E	4H	4I	6H	I**	6J	6K	6L	6M	8E	10B	12I	12J	18A	B**
χ_{99} : oo2																												
χ_{100} o2																												
χ_{101}																												
χ_{102} : oo2																												
χ_{103} : oo2																												
χ_{104} : oo2																												
χ_{105} o2																												
χ_{106}																												
χ_{107} : oo2																												
χ_{108} : oo2																												
χ_{109} : oo2																												
χ_{110} : oo2																												
χ_{111} : oo2																												
and																												
no:																												
fus ind																												
χ_{112} : oo2																												
χ_{113} o2																												
χ_{114}																												
χ_{115} o2																												
χ_{116}																												
χ_{117} o2																												
χ_{118}																												
χ_{119} : oo2																												
χ_{120} : oo2																												
χ_{121} : oo2																												
χ_{122} : oo2																												
χ_{123} : oo2																												
and																												
no:																												
fus ind																												
χ_{112} : oo2																												
χ_{113} o2																												
χ_{114}																												
χ_{115} o2																												
χ_{116}																												
χ_{117} o2																												
χ_{118}																												
χ_{119} : oo2																												
χ_{120} : oo2																												
χ_{121} : oo2																												
χ_{122} : oo2																												
χ_{123} : oo2																												
and																												
no:																												
fus ind																												
χ_{124} *7 o2																												
χ_{125} *7 o2																												
χ_{126} o4																												
χ_{127} *7																												
χ_{128} *7 o2																												
χ_{129} *7 o2																												
χ_{130} *7 o2																												
χ_{131} *7 o2																												
χ_{132} *7 o2																												
and																												
no:																												

		2F	4J	6N	8F	8G	8H	10C	12J	24B	C**		
	fus ind	2	4	6	8	8	8	10	12	24	24	fus ind	
		6	12		24	24	24	30	12	24	24		
		6	12		24	24	24	30	12	24	24		
χ_{99}	: oo2	6	-2	0	2	2	0	1	1	-1	-1	* +	χ_{99}
χ_{100}	o2	0	0	0	0	0	0	0	0	0	0	+2	χ_{100}
χ_{101}												*	χ_{101}
χ_{102}	: oo2	6	2	0	4	0	0	1	-1	1	1	* +	χ_{102}
χ_{103}	: oo2	9	1	0	1	1	-1	-1	1	1	1	* +	χ_{103}
χ_{104}	: oo2	15	3	0	-3	1	1	0	0	0	0	* +	χ_{104}
χ_{105}	o2	0	0	0	0	0	0	0	0	0	0	o2	χ_{105}
χ_{106}												*	χ_{106}
χ_{107}	: oo2	0	4	0	2	-2	0	0	1	-1	-1	* +	χ_{107}
χ_{108}	: oo2	0	0	0	0	0	0	0	0	i6	-i6	* +	χ_{108}
χ_{109}	: oo2	9	-3	0	3	-1	1	-1	0	0	0	* +	χ_{109}
χ_{110}	: oo2	6	-2	0	-2	-2	0	1	1	1	1	* +	χ_{110}
χ_{111}	: oo2	15	-1	0	-1	-1	-1	0	-1	-1	-1	* +	χ_{111}
	and	2	4	6	8	8	8	10	12	24	24	: :	
	no:											: :	
	fus ind	2	4	6	8	8	8	10	12	24	24	fus ind	
		6	12		24	24	24	30	12	24	24		
		6	12		24	24	24	30	12	24	24		
					8		8	10	12	24	24		
					24		24	30	12	24	24		
					24		24	30	12	24	24		
χ_{112}	: oo2	0	0	0	2r2	0	r2	0	0	-r2	-r2	* +	χ_{112}
χ_{113}	o2	0	0	0	0	0	0	0	0	0	0	o2	χ_{113}
χ_{114}												*	χ_{114}
χ_{115}	o2	0	0	0	0	0	0	0	0	0	0	+2	χ_{115}
χ_{116}												*	χ_{116}
χ_{117}	o2	0	0	0	0	0	0	0	0	0	0	+2	χ_{117}
χ_{118}												*	χ_{118}
χ_{119}	: oo2	0	0	0	0	0	0	r5	0	0	0	* +	χ_{119}
χ_{120}	: oo2	0	0	0	0	0	0	0	i3	-i3	i3	* +	χ_{120}
χ_{121}	: oo2	0	0	0	2r2	0	-r2	0	0	-r2	-r2	* +	χ_{121}
χ_{122}	: oo2	0	0	0	4r2	0	0	0	0	r2	r2	* +	χ_{122}
χ_{123}	: oo2	0	0	0	0	0	0	0	i3	i3	-i3	* +	χ_{123}
	and	2	4	6	8	8	8	10	12	24	24	: :	
	no:			:2	:2		:2	:2	:2	:2	:2	: :	
	fus ind	2	4	6	8	8	8	10	12	24	24	fus ind	
		6	12		24	24	24	30	12	24	24		
		6	12		24	24	24	30	12	24	24		
					8		8	10	12	24	24		
					24		24	30	12	24	24		
					24		24	30	12	24	24		
χ_{124}	*7 o2											** +2	χ_{124}
χ_{125}	*7 o2											** +2	χ_{125}
χ_{126}	o4											+4	χ_{126}
χ_{127}	*7											**	χ_{127}
χ_{128}	*7 o2											** +2	χ_{128}
χ_{129}	*7 o2											** +2	χ_{129}
χ_{130}	*7 o2											** +2	χ_{130}
χ_{131}	*7 o2											** +2	χ_{131}
χ_{132}	*7 o2											** +2	χ_{132}
	and	2	4	6	8	8	8	10	12	24	24	*7 *7	
	no:			:2	:2		:2	:2	:2	:2	:2	*7 *7	

$G_2(3)$

Chevalley group $G_2(3)$

Order = 4,245,696 = $2^6.3^6.7.13$ Mult = 3 Out = 2

Constructions

Chevalley G : the adjoint Chevalley group of type G_2 over F_3; G.2 is obtained by adjoining the graph automorphism;

G : the automorphism group of a generalized hexagon of order (3,3) consisting of 364 vertices and 364 edges, each object being incident with 4 of the other type; G.2 is obtained by adjoining an automorphism interchanging vertices and edges

Cayley G : the automorphism group of the mod 3 Cayley algebra of vectors $\Sigma a_n i_n$, with a_n in F_3 (subscripts mod 7), where n runs over $\{\infty,0,1,2,3,4,5,6\}$ and $i_\infty=1$, $i_{n+1}=i$, $i_{n+2}=j$, $i_{n+4}=k$ form a quaternion subalgebra

14-dimensional G.2 acts on a 14-space with base $\{e_\infty,e_0,\ldots e_{12}\}$ and is generated by

$A : e_t \to e_{t+1}$, $B : e_t \to e_{3t}$, $C : (e_2\ e_{12}\ -e_8)\ (e_6\ e_{10}\ -e_{11})\ (e_5\ e_4\ -e_7)$

$D : \sqrt{-3}.e_\infty \to 3e_\infty + \sqrt{3}.\Sigma e_i\ (i \neq \infty)$

$\sqrt{-3}.e_i \to \sqrt{3}e_\infty -3(e_{5i}+e_{5i+2}+e_{5i+5}+e_{5i+4}) +(e_{5i+1}+e_{5i+3}+e_{5i+4}+e_{5i+7}+e_{5i+8}+e_{5i+9}+e_{5i+10}+e_{5i+11}+e_{5i+12})$

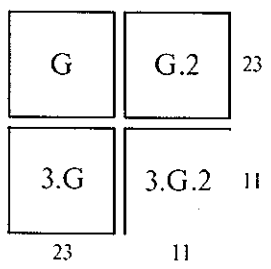
$D^2: e_\infty \to -e_\infty$, $e_i \to -e_{2-i}-e_{4-i}-e_{5-i}-e_{6-i}+e_{7-i}+e_{8-i}-e_{10-i}+e_{11-i}-e_{12-i}$

The subgroup $G_2(3) = \langle A,B,C,C^D\rangle$ is represented by real matrices. There are 2 x 378 images under $G_2(3)$ of each of the vectors $(\sqrt{3}|0^{13})$ and $(3|\sqrt{3}^{13})$. These vectors are fixed by the respective subgroups $\langle A,B,C\rangle$ and $\langle A,B,C^D\rangle$ of type $L_3(3)$, and negated by D^2. The two orbits are interchanged by outer elements of $G_2(3).2$.

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	
				5	5																							1											
	4245696	576	832	832	729	162	162	96	96	72	72	18	18	7	8	8	27	27	27	12	12	13	13				512	24	27	18	18	6	6	7	9	9	9	
	p power	A	A	A	A	A	A	A	A	AA	BA	DA	EA	A	A	B	C	C	C	AA	BB	A	A				A	A	CB	EB	EB	CC	CC	AB	AE	CE	BE	
	p' part	A	A	A	A	A	A	A	A	AA	BA	DA	EA	A	A	A	A	A	A	AA	BB	A	A				A	A	CB	EB	EB	DC	DC	AB	AB	BB	CB	
	ind	1A	2A	3A	3B	3C	3D	3E	4A	4B	6A	6B	6C	6D	7A	8A	8B	9A	9B	C**	12A	12B	13A	B*	fus	ind	2B	4C	6E	6F	G**	12C	D*	14A	18A	18B	C**	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	14	-2	5	5	-4	2	-1	2	2	1	1	-2	1	0	0	0	2	-1	-1	-1	-1	1	1	:	oo	0	0	0	i3	-i3	0	0	0	0	-i3	i3	χ_2
χ_3	o	64	0	-8	-8	1	4	-2	0	0	0	0	0	0	1	0	0	1	b27	**	0	0	-1	-1	:	oo	8	0	-1	2z3	**	0	0	1	-1	**	-z3	χ_3
χ_4	o	64	0	-8	-8	1	4	-2	0	0	0	0	0	0	1	0	0	1	**	b27	0	0	-1	-1	:	oo	8	0	-1	**	2z3	0	0	1	-1	-z3	**	χ_4
χ_5	+	78	-2	-3	-3	-3	-3	6	2	2	1	1	1	-2	1	0	0	0	0	0	-1	-1	0	0	:	++	6	-2	-3	0	0	1	1	-1	0	0	0	χ_5
χ_6	+	91	-5	10	10	10	1	1	3	3	-2	-2	1	1	0	-1	-1	1	1	1	0	0	0	0	:	++	7	-1	-2	1	1	-1	-1	0	1	1	1	χ_6
χ_7	+	91	3	-8	19	1	4	-2	3	-1	0	3	0	0	0	1	-1	-2	1	1	0	-1	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_7
χ_8	+	91	3	19	-8	1	4	-2	-1	3	3	0	0	0	0	-1	1	-2	1	1	-1	0	0	0														χ_8
χ_9	+	104	8	14	14	5	2	-1	0	0	2	2	2	-1	-1	0	0	2	-1	-1	0	0	0	0	:	++	8	0	-1	-1	-1	0	0	1	2	-1	-1	χ_9
χ_{10}	+	168	8	6	6	6	-3	6	0	0	2	2	-1	2	0	0	0	0	0	0	0	0	-1	-1	:	++	0	0	0	0	0	r3	-r3	0	0	0	0	χ_{10}
χ_{11}	+	182	6	20	-7	-7	2	2	2	2	0	-3	0	0	0	0	0	-1	-1	-1	2	-1	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}	+	182	6	-7	20	-7	2	2	2	2	-3	0	0	0	0	0	0	-1	-1	-1	-1	2	0	0														χ_{12}
χ_{13}	+	273	-7	30	3	3	3	3	-3	1	2	-1	-1	-1	0	1	-1	0	0	0	0	1	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}	+	273	-7	3	30	3	3	3	1	-3	-1	2	-1	-1	0	-1	1	0	0	0	1	0	0	0														χ_{14}
χ_{15}	+	448	0	16	16	-11	-2	-2	0	0	0	0	0	0	0	0	0	1	1	1	0	0	b13	*		+	0	0	0	0	0	0	0	0	0	0	0	χ_{15}
χ_{16}	+	448	0	16	16	-11	-2	-2	0	0	0	0	0	0	0	0	0	1	1	1	0	0	*	b13														χ_{16}
χ_{17}	+	546	2	-21	6	6	-3	-3	-2	6	-1	2	-1	-1	0	0	0	0	0	0	1	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}	+	546	2	6	-21	6	-3	-3	6	-2	2	-1	-1	-1	0	0	0	0	0	0	0	1	0	0														χ_{18}
χ_{19}	+	728	-8	26	-28	-1	-1	-1	0	0	-2	4	1	1	0	0	0	-1	-1	-1	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{19}
χ_{20}	+	728	-8	-28	26	-1	-1	-1	0	0	4	-2	1	1	0	0	0	-1	-1	-1	0	0	0	0														χ_{20}
χ_{21}	+	729	9	0	0	0	0	0	-3	-3	0	0	0	0	1	-1	-1	0	0	0	0	0	1	1	:	++	27	3	0	0	0	0	0	-1	0	0	0	χ_{21}
χ_{22}	+	819	3	9	9	9	0	0	-1	-1	-3	-3	0	0	0	1	1	0	0	0	-1	-1	0	0	:	++	21	-3	3	0	0	0	0	0	0	0	0	χ_{22}
χ_{23}	+	832	0	-32	-32	-5	4	4	0	0	0	0	0	0	-1	0	0	1	1	1	0	0	0	0	:	++	8	0	-1	2	2	0	0	1	-1	-1	-1	χ_{23}
	ind	1	2	3	3	3	3	3	4	4	6	6	6	6	7	8	8	9	9	9	12	12	13	13	fus	ind	2	4	6	6	6	12	12	14	18	18	18	
		3	6						12	12					21	24	24				12	12	39	39														
		3	6						12	12					21	24	24				12	12	39	39														
χ_{24}	o2	27	3	0	0	0	0	0	-1	3	0	0	0	0	-1	-1	1	0	0	0	2	0	1	1		o2												χ_{24}
χ_{25}	o2	27	3	0	0	0	0	0	3	-1	0	0	0	0	-1	1	-1	0	0	0	0	2	1	1	*													χ_{25}
χ_{26}	o2	189	-3	0	0	0	0	0	-3	-3	0	0	0	0	0	1	1	0	0	0	0	0	-b13	*		o2												χ_{26}
χ_{27}	o2	189	-3	0	0	0	0	0	-3	-3	0	0	0	0	0	1	1	0	0	0	0	0	*	-b13	*													χ_{27}
χ_{28}	o2	351	15	0	0	0	0	0	3	3	0	0	0	0	1	1	1	0	0	0	0	0	0	0	*	+												χ_{28}
χ_{29}	o2	351	-9	0	0	0	0	0	-1	3	0	0	0	0	1	1	-1	0	0	0	2	0	0	0		o2												χ_{29}
χ_{30}	o2	351	-9	0	0	0	0	0	3	-1	0	0	0	0	1	-1	1	0	0	0	0	2	0	0	*													χ_{30}
χ_{31}	o2	378	-6	0	0	0	0	0	-2	6	0	0	0	0	0	0	0	0	0	0	-2	0	1	1		o2												χ_{31}
χ_{32}	o2	378	-6	0	0	0	0	0	6	-2	0	0	0	0	0	0	0	0	0	0	0	-2	1	1	*													χ_{32}
χ_{33}	o2	729	9	0	0	0	0	0	-3	-3	0	0	0	0	1	-1	-1	0	0	0	0	0	1	1	*	+												χ_{33}
[60] χ_{34}	o2	1728	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	-1	-1	*	+												χ_{34}

$G_2(3)$ and $S_4(5)$

Maximal subgroups					Specifications		
Order	Index	Structure	G.2	Character	Abstract	Chevalley	Cayley
12096	351	$U_3(3):2$		1a+168a+182b		${}^2A_2(3)$	minus point
12096	351	$U_3(3):2$		1a+168a+182a			$G_2(\mathbb{Z})$
11664	364	$(3_+^{1+2}x3^2):2S_4$	$3^2.(3x3_+^{1+2}):D_8$	1a+91b+104a+168a	$N(3B^2)$, $N(3A)$	vertex	isotropic point
11664	364	$(3_+^{1+2}x3^2):2S_4$		1a+91c+104a+168a	$N(3A^2)$, $N(3B)$	edge	null subalgebra
11232	378	$L_3(3):2$		1a+91b+104a+182b		$A_2(3)$	plus point
11232	378	$L_3(3):2$		1a+91c+104a+182a			
1512	2808	$L_2(8):3$	$: L_2(8):3 \times 2$		$N(2A,3C,7A,9A)$, $C(2B)$	${}^2G_2(3)$	Ree
1344	3159	$2^3{\cdot}L_3(2)$	$: 2^3{\cdot}L_3(2):2$		$N(2A^3)$		base
1092	3888	$L_2(13)$	$: L_2(13):2$		$N(2A,3D,6C,7A,13AB)$		
576	7371	$2_+^{1+4}:3^2.2$	$: 2_+^{1+4}.(S_3xS_3)$		$N(2A)$	$D_2(3)$	quaternion subalgebra



Symplectic group $S_4(5) \cong C_2(5) \cong O_5(5)$

Order = 4,680,000 = $2^6.3^2.5^4.13$ Mult = 2 Out = 2

Constructions

Symplectic $Sp_4(5) \cong 2.G$: all 4 x 4 matrices over $\mathbf{F}_5$ preserving a non-singular symplectic form;

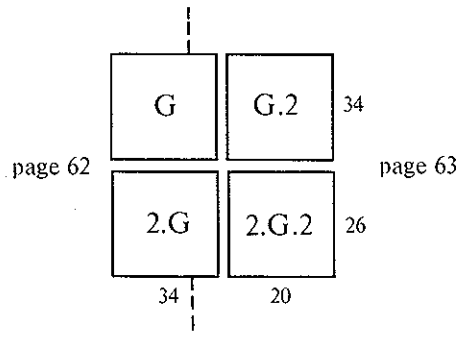
$PSp_4(5) \cong S_4(5) \cong G$;

$C_2(5) \cong G$: the automorphism group of a generalized 4-gon of order (5,5), consisting of 156 vertices and 156 edges, each object being incident with 6 of the other type

Orthogonal $GO_5(5) \cong 2 \times G.2$: all 5 x 5 matrices over $\mathbf{F}_5$ preserving a non-singular quadratic form;

$PGO_5(5) \cong SO_5(5) \cong PSO_5(5) \cong G.2$; $O_5(5) \cong G$

Maximal subgroups					Specifications		
Order	Index	Structure	G.2	Character	Abstract	Symplectic	Orthogonal
30000	156	$5_+^{1+2}:4A_5$	$: 5_+^{1+2}:4S_5$	1a+65b+90a	$N(5AB)$	point	isotropic line
30000	156	$5^3:(2xA_5)^{\cdot}2$	$: 5^3:(4xS_5)$	1a+65a+90a	$N(5^3)$	isotropic line	isotropic point
15600	300	$L_2(25):2_2$	$: L_2(25):2_2 \times 2$	1a+65a+104b+130a	$N(2B,3A,5C,5D,...)$, $C(2C)$	$S_2(25)$	$O_4^-(5)$
14400	325	$2^{\cdot}(A_5xA_5).2$	$: 2^{\cdot}(A_5xA_5).2^2$	1a+90a+104b+130a	$N(2A)$	$S_2(5)wr2$	$O_4^+(5)$
960	4875	$2^4:A_5$	$: 2^4:S_5$		$N(2^4) = N(2A_5B_{10})$		base
720	6500	$S_3 \times S_5$	$: D_{12} \times S_5$		$N(3A)$, $N(2B,3A,5C)$, $C(2D)$	$U_2(5)$	$O_2^-(5) \times O_3(5)$
480	9750	$(2^2 \times A_5):2$	$: D_8 \times S_5$		$N(2B)$, $N(2B,3A,5D)$		$O_2^+(5) \times O_3(5)$
360	13000	A_6	$: S_6$		$N(2B,3A,3B,5EF)$		



$S_4(5)$

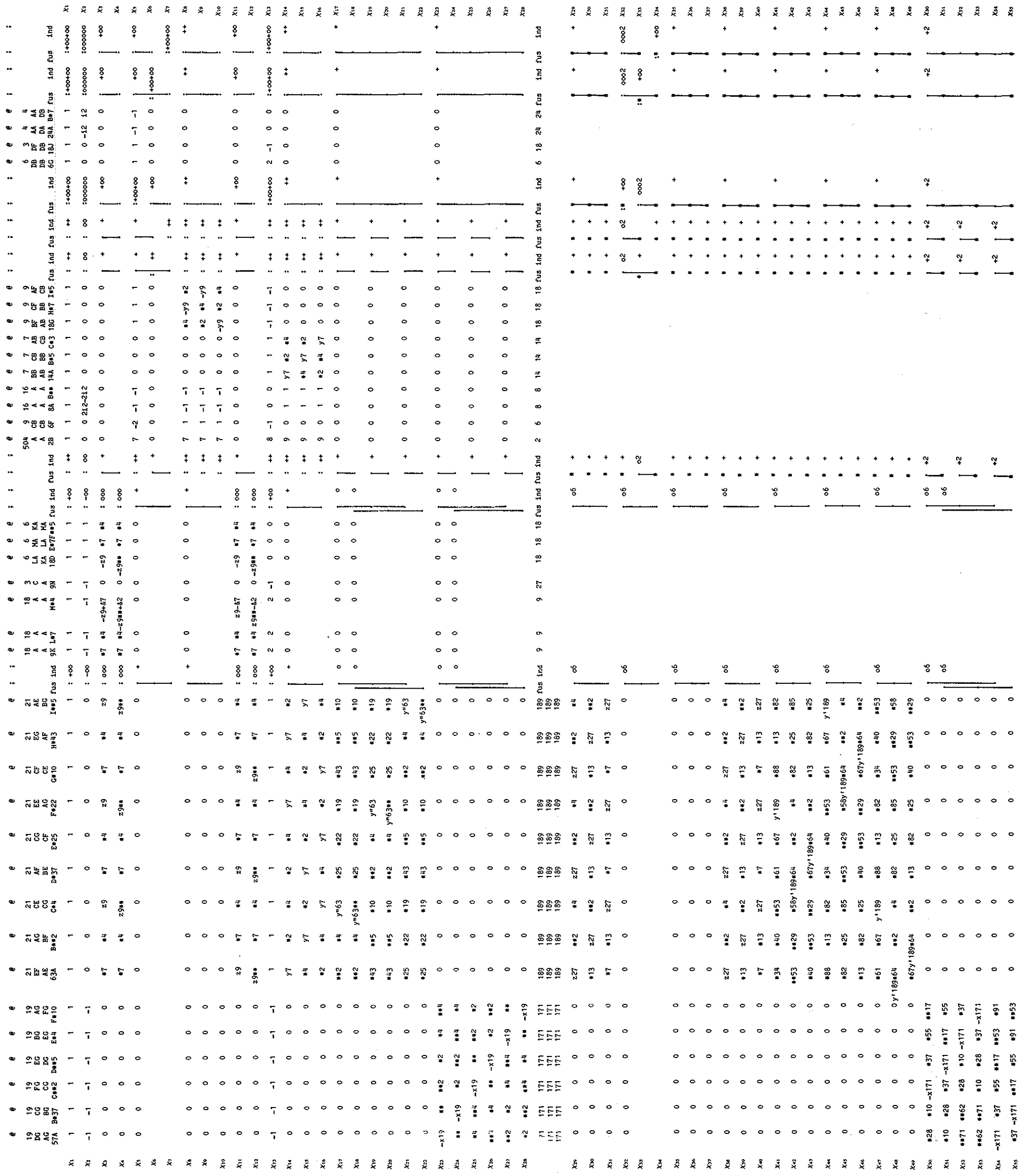
	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			14																											
	4680000		400	480	360	360	240	24	15000	15000	750	500	25	25	360	36	24	600	600	100	100	50	20	12	12	13	13	13		
	p power		A	A	A	A	A	B	A	A	A	A	A	A	BA	AA	AB	BA	AA	DA	DA	CA	DB	CB	AA	A	A	A		
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	BA	AA	AB	AA	BA	DA	DA	CA	DB	AB	BA	A	A	A		
	ind	1A	2A	2B	3A	3B	4A	4B	5A	B*	5C	5D	5E	F*	6A	6B	6C	10A	B*	10C	D*	10E	10F	12A	12B	13A	B*3	C*9		
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		χ_1
χ_2	+	13	5	1	1	-2	-3	-1	5b5+3	*	-2	3	-b5	*	2	-1	1	b5+3	*	r5	-r5	0	1	-1	0	0	0	0		χ_2
χ_3	+	13	5	1	1	-2	-3	-1	*	5b5+3	-2	3	*	-b5	2	-1	1	*	b5+3	-r5	r5	0	1	-1	0	0	0	0		χ_3
χ_4	+	40	-8	0	4	4	0	0	-10	-10	5	0	0	0	4	-2	0	2	2	2	2	-3	0	0	0	1	1	1		χ_4
χ_5	+	65	17	1	5	-1	5	1	-10	-10	0	5	0	0	-1	-1	1	2	2	-3	-3	2	1	1	-1	0	0	0		χ_5
χ_6	+	65	-7	5	-1	5	1	-1	15	15	5	0	0	0	5	-1	-1	3	3	-2	-2	3	0	-1	1	0	0	0		χ_6
χ_7	+	78	6	-6	0	3	2	0	5b5+18	*	3	3	-b5	*	3	0	0	5b5+6	*	1	1	1	-1	0	-1	0	0	0		χ_7
χ_8	+	78	6	-6	0	3	2	0	*	5b5+18	3	3	*	-b5	3	0	0	*	5b5+6	1	1	1	-1	0	-1	0	0	0		χ_8
χ_9	+	90	18	6	0	0	6	0	15	15	0	5	0	0	0	0	0	3	3	3	3	-2	1	0	0	-1	-1	-1		χ_9
χ_{10}	+	104	8	0	-4	5	4	0	-21	-21	4	4	-1	-1	5	2	0	3	3	-2	-2	-2	0	0	1	0	0	0		χ_{10}
χ_{11}	+	104	16	4	5	-4	0	2	4	4	9	-1	-1	-1	4	1	1	-4	-4	1	1	1	-1	-1	0	0	0	0		χ_{11}
χ_{12}	+	104	-16	4	5	-4	0	-2	4	4	9	-1	-1	-1	-4	-1	1	4	4	-1	-1	-1	-1	1	0	0	0	0		χ_{12}
χ_{13}	+	130	26	6	4	4	-6	0	5	5	5	5	0	0	-4	2	0	1	1	1	1	1	1	0	0	0	0	0		χ_{13}
χ_{14}	+	156	-36	4	6	0	0	0	6	6	1	11	1	1	0	0	-2	-6	-6	-1	-1	-1	-1	0	0	0	0	0		χ_{14}
χ_{15}	+	208	-16	0	4	-2	0	0	-10r5+8	10r5+8	-7	-2	-b5	*	2	2	0	-2r5+4	2r5+4	2b5	*	-1	0	0	0	0	0	0		χ_{15}
χ_{16}	+	208	-16	0	4	-2	0	0	10r5+8	-10r5+8	-7	-2	*	-b5	2	2	0	2r5+4	-2r5+4	*	2b5	-1	0	0	0	0	0	0		χ_{16}
χ_{17}	+	312	24	0	0	3	-4	0	20b5-3	*	-3	2	b5	*	3	0	0	-2r5-1	2r5-1	2b5	*	-1	0	0	-1	0	0	0		χ_{17}
χ_{18}	+	312	24	0	0	3	-4	0	*	20b5-3	-3	2	*	b5	3	0	0	2r5-1	-2r5-1	*	2b5	-1	0	0	-1	0	0	0		χ_{18}
χ_{19}	+	312	-24	0	0	-6	0	0	*	30b5+2	-3	2	b5	*	6	0	0	3r5+1	-3r5+1	*	-2b5	1	0	0	0	0	0	0		χ_{19}
χ_{20}	+	312	-24	0	0	-6	0	0	30b5+2	*	-3	2	*	b5	6	0	0	-3r5+1	3r5+1	-2b5	*	1	0	0	0	0	0	0		χ_{20}
χ_{21}	+	325	5	5	-5	-2	-3	1	-25b5	*	0	0	0	0	2	-1	-1	-5b5	*	0	0	0	0	1	0	0	0	0		χ_{21}
χ_{22}	+	325	5	5	-5	-2	-3	1	*	-25b5	0	0	0	0	2	-1	-1	*	-5b5	0	0	0	0	1	0	0	0	0		χ_{22}
χ_{23}	+	390	30	-6	0	-3	-2	0	25b5+15	*	0	5	0	0	-3	0	0	b5+3	*	r5	-r5	0	-1	0	1	0	0	0		χ_{23}
χ_{24}	+	390	30	-6	0	-3	-2	0	*	25b5+15	0	5	0	0	-3	0	0	*	b5+3	-r5	r5	0	-1	0	1	0	0	0		χ_{24}
χ_{25}	+	520	40	0	4	1	4	0	-5	-5	-10	0	0	0	1	-2	0	-5	-5	0	0	0	0	0	1	0	0	0		χ_{25}
χ_{26}	+	520	16	4	1	4	0	-2	20	20	-5	-5	0	0	4	1	1	-4	-4	1	1	1	-1	1	0	0	0	0		χ_{26}
χ_{27}	+	520	-16	4	1	4	0	2	20	20	-5	-5	0	0	-4	-1	1	4	4	-1	-1	-1	-1	-1	0	0	0	0		χ_{27}
χ_{28}	+	576	0	0	0	0	0	0	-24	-24	6	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	c13	*3	*9		χ_{28}
χ_{29}	+	576	0	0	0	0	0	0	-24	-24	6	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	*9	c13	*3		χ_{29}
χ_{30}	+	576	0	0	0	0	0	0	-24	-24	6	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	*3	*9	c13		χ_{30}
χ_{31}	+	624	0	-8	6	0	0	0	24	24	4	-6	-1	-1	0	0	-2	0	0	0	0	0	2	0	0	0	0	0		χ_{31}
χ_{32}	+	624	-48	0	0	6	0	0	-26	-26	-6	4	-1	-1	-6	0	0	2	2	2	2	2	0	0	0	0	0	0		χ_{32}
χ_{33}	+	625	25	5	-5	-5	5	-1	0	0	0	0	0	0	-5	1	-1	0	0	0	0	0	0	-1	-1	1	1	1		χ_{33}
χ_{34}	+	780	-36	-4	-6	0	0	0	30	30	5	5	0	0	0	0	2	-6	-6	-1	-1	-1	1	0	0	0	0	0		χ_{34}
	ind	1	2	4	3	3	4	8	5	5	5	5	5	5	6	6	12	10	10	10	10	10	20	24	12	13	13	13		
		2			6	6	4		10	10	10	10	10	10	6			10	10			10		24	12	26	26	26		
χ_{35}	-	12	0	0	0	-3	-2	0	*	5b5+2	-3	2	b5	*	-3	0	0	b5+3	*	0	0	-r5	0	0	1	-1	-1	-1		χ_{35}
χ_{36}	-	12	0	0	0	-3	-2	0	5b5+2	*	-3	2	*	b5	-3	0	0	*	b5+3	0	0	r5	0	0	1	-1	-1	-1		χ_{36}
χ_{37}	-	52	0	0	4	1	2	0	*	5b5-8	2	2	b5	*	-3	0	0	*	-5b5	0	0	0	0	0	-1	0	0	0		χ_{37}
χ_{38}	-	52	0	0	4	1	2	0	5b5-8	*	2	2	*	b5	-3	0	0	-5b5	*	0	0	0	0	0	-1	0	0	0		χ_{38}
χ_{39}	-	104	0	0	-4	5	4	0	-21	-21	4	4	-1	-1	-3	0	0	5	5	0	0	0	0	0	1	0	0	0		χ_{39}
χ_{40}	-	156	0	0	0	6	6	0	31	31	6	6	1	1	-6	0	0	5	5	0	0	0	0	0	0	0	0	0		χ_{40}
χ_{41}	-	208	0	0	4	-2	0	0	-10r5+8	10r5+8	-7	-2	-b5	*	-6	0	0	-2r5	2r5	0	0	-r5	0	0	0	0	0	0		χ_{41}
χ_{42}	-	208	0	0	4	-2	0	0	10r5+8	-10r5+8	-7	-2	*	-b5	-6	0	0	2r5	-2r5	0	0	r5	0	0	0	0	0	0		χ_{42}
χ_{43}	-	260	0	0	-4	-7	2	0	25b5+10	*	-5	0	0	0	-3	0	0	*	b5+3	0	0	r5	0	0	-1	0	0	0		χ_{43}
χ_{44}	-	260	0	0	-4	-7	2	0	*	25b5+10	-5	0	0	0	-3	0	0	b5+3	*	0	0	-r5	0	0	-1	0	0	0		χ_{44}
χ_{45}	-	300	0	0	0	-3	-2	0	25b5	*	0	0	0	0	-3	0	0	-5b5	*	0	0	0	0	0	1	1	1	1		χ_{45}
χ_{46}	-	300	0	0	0	-3	-2	0	*	25b5	0	0	0	0	-3	0	0	*	-5b5	0	0	0	0	0	1	1	1	1		χ_{46}
χ_{47}	-	312	0	0	0	3	-4	0	20b5-3	*	-3	2	b5	*	3	0	0	-2r5+5	2r5+5	0	0	-r5	0	0	-1	0	0	0		χ_{47}
χ_{48}	-	312	0	0	0	3	-4	0	*	20b5-3	-3	2	*	b5	3	0	0	2r5+5	-2r5+5	0	0	r5	0	0	-1	0	0	0		χ_{48}
χ_{49}	-	416	0	0	-4	8	0	0	16	16	-14	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0		χ_{49}
χ_{50}	-	468	0	0	0	0	-6	0	*	-30b5+3	3	8	-b5	*	0	0	0	3r5	-3r5	0	0	-r5	0	0	0	0	0	0		χ_{50}
χ_{51}	-	468	0	0	0	0	-6	0	-30b5+3	*	3	8	*	-b5	0	0	0	-3r5	3r5	0	0	r5	0	0	0	0	0	0		χ_{51}
χ_{52}	-	520	0	0	4	1	4	0	-5	-5	-10	0	0	0	9	0	0	5	5	0	0	0	0	0	1	0	0	0		χ_{52}
χ_{53}	-	576	0	0	0	0	0	0	-24	-24	6	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	c13	*3	*9		χ_{53}
χ_{54}	-	576	0	0	0	0	0	0	-24	-24	6	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	*9	c13	*3		χ_{54}
χ_{55}	-	576	0	0	0	0	0	0	-24	-24	6	-4	1	1	0	0	0	0	0	0	0	0	0	0	0	*3	*9	c13		χ_{55}
χ_{56}	-	624	0	0	0	6	0	0	-26	-26	-6	4	-1	-1	-6	0	0	-10	-10	0	0	0	0	0	0	0	0	0		χ_{56}
χ_{57}	-	624	0	0	-12	0	0	0	24	24	4	-6	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0		χ_{57}
χ_{58}	-	624	0	0	6	0	0	0	24	24	4	-6	-1	-1	0	0	0	0	0	0	0	0	0	r6	0	0	0	0		χ_{58}
χ_{59}	-	624	0	0	6	0	0	0	24	24	4	-6	-1	-1	0	0	0	0	0	0	0	0	0	-r6	0	0	0	0		χ_{59}
χ_{60}	-	780	0	0	0	-6	6	0	5	5	0	10	0	0	6	0	0	-5	-5	0	0	0	0	0	0	0	0	0		χ_{60}

$S_4(5)$

	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
	30	30	15	20	20	30	30			15 600	720	240	16	360	36	24	18	12	50	50	30	12	10	12	12	13	13	13	15		
	BB	AB	CA	BA	AA	BBA	AAA			A	A	B	B	AD	AD	AC	BD	A	CC	DC	CD	CC	FC	BA	BA	BC	CC	AC	CID		
	AB	BB	CA	AA	BA	AAA	BBA			A	A	A	A	AD	AD	AC	BD	A	CC	DC	CD	AC	DC	BA	BA	AC	BC	CC	CID		
	15A	B*	15C	20A	B*	30A	B*	fus	ind	2C	2D	4C	4D	6D	6E	6F	6G	8A	10G	10H	10I	12C	20C	24A	B*	26A	B*3	C*9	30C		
x_1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	-b5	*	1	*	b5	b5	*		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_2
x_3	*	-b5	1	b5	*	*	b5																							x_3
x_4	-1	-1	-1	0	0	-1	-1	:	++	12	-4	0	0	-4	2	0	2	0	-3	2	1	0	0	0	0	-1	-1	-1	1	x_4
x_5	-1	-1	0	0	0	-1	-1	:	++	13	5	1	1	5	-1	1	-1	-1	-2	3	0	1	1	-1	-1	0	0	0	0	x_5
x_6	0	0	-1	1	1	0	0	:	++	13	1	-5	-1	1	1	1	1	-1	3	-2	1	1	0	-1	-1	0	0	0	1	x_6
x_7	-b5	*	0	b5	*	-b5	*		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_7
x_8	*	-b5	0	*	b5	*	-b5																							x_8
x_9	0	0	0	1	1	0	0	:	++	12	0	6	2	0	0	0	0	0	2	-3	0	0	1	0	0	-1	-1	-1	0	x_9
x_{10}	0	0	1	-1	-1	0	0	:	++	0	8	0	0	-4	2	0	-1	-2	0	0	-2	0	0	1	1	0	0	0	1	x_{10}
x_{11}	1	1	0	0	0	-1	-1	:	++	26	6	4	0	3	3	-1	0	0	1	1	1	1	-1	0	0	0	0	0	-2	x_{11}
x_{12}	1	1	0	0	0	1	1	:	++	26	-2	-4	0	-5	1	-1	-2	0	1	1	3	-1	1	0	0	0	0	0	0	x_{12}
x_{13}	-1	-1	-1	-1	-1	1	1	:	++	26	6	4	0	6	0	2	0	0	1	1	1	-2	-1	0	0	0	0	0	1	x_{13}
x_{14}	0	0	1	0	0	0	0	:	++	26	-6	-6	2	-6	0	2	0	0	1	1	-1	0	-1	0	0	0	0	0	-1	x_{14}
x_{15}	*	-b5	-1	0	0	*	b5		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{15}
x_{16}	-b5	*	-1	0	0	b5	*																							x_{16}
x_{17}	-b5	*	0	1	1	-b5	*		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{17}
x_{18}	*	-b5	0	1	1	*	-b5																							x_{18}
x_{19}	-1	-1	0	0	0	1	1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{19}
x_{20}	-1	-1	0	0	0	1	1																							x_{20}
x_{21}	-b5	*	0	b5	*	b5	*		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{21}
x_{22}	*	-b5	0	*	b5	*	b5																							x_{22}
x_{23}	b5	*	0	*	-b5	b5	*		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{23}
x_{24}	*	b5	0	-b5	*	*	b5																							x_{24}
x_{25}	1	1	-1	-1	-1	1	1	:	++	0	8	0	0	-4	2	0	-1	2	0	0	-2	0	0	-1	-1	0	0	0	1	x_{25}
x_{26}	-1	-1	1	0	0	-1	-1	:	++	26	14	-4	0	-1	-1	-1	2	0	1	1	-1	-1	1	0	0	0	0	0	-1	x_{26}
x_{27}	-1	-1	1	0	0	1	1	:	++	26	6	4	0	-9	-3	-1	0	0	1	1	1	1	-1	0	0	0	0	0	1	x_{27}
x_{28}	0	0	0	0	0	0	0	:	++	48	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0-c13	*3	*9	0	x_{28}
x_{29}	0	0	0	0	0	0	0	:	++	48	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	*9-c13	*3	0	x_{29}
x_{30}	0	0	0	0	0	0	0	:	++	48	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	*3	*9-c13	0	x_{30}
x_{31}	0	0	1	0	0	0	0	:	++	52	-12	0	0	6	0	-2	0	0	2	2	-2	0	0	0	0	0	0	0	1	x_{31}
x_{32}	1	1	0	0	0	-1	-1	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r6	-r6	0	0	0	0	x_{32}
x_{33}	0	0	0	0	0	0	0	:	++	25	5	-5	-1	5	-1	1	-1	1	0	0	0	1	0	1	1	-1	-1	-1	0	x_{33}
x_{34}	0	0	-1	0	0	0	0	:	++	26	-6	6	-2	-6	0	2	0	0	1	1	-1	0	1	0	0	0	0	0	-1	x_{34}
	15	15	15	20	20	30	30	fus	ind	2	4	8	8	12	12	6	12	8	10	10	20	24	40	24	24	26	26	26	60		
	30	30	30	20	20	30	30					8		12			12	8				24	40	24	24	26	26	26	60		
x_{35}	b5	*	0	*	-b5	b5	*		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{35}
x_{36}	*	b5	0	-b5	*	*	b5																							x_{36}
x_{37}	b5-1	*	-1	*	b5	*	b5		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{37}
x_{38}	*	b5-1	-1	b5	*	b5	*																							x_{38}
x_{39}	0	0	1	-1	-1	2	2	:	--	0	0	0	0-4r3	0	0	r3	0	0	0	0	0	0	0	r3	-r3	0	0	0	r3	x_{39}
x_{40}	1	1	0	1	1	-1	-1	:	--	0	0	6r2	0	0	0	0	0	r2	0	0	0	0	r2	r2	r2	0	0	0	0	x_{40}
x_{41}	*	-b5	-1	0	0	*	b5+2		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{41}
x_{42}	-b5	*	-1	0	0	b5+2	*																							x_{42}
x_{43}	*	-b5	1	b5	*	*	b5		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{43}
x_{44}	-b5	*	1	*	b5	b5	*																							x_{44}
x_{45}	b5	*	0	-b5	*	b5	*		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{45}
x_{46}	*	b5	0	*	-b5	*	b5																							x_{46}
x_{47}	-b5	*	0	1	1	-b5	*		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{47}
x_{48}	*	-b5	0	1	1	*	-b5																							x_{48}
x_{49}	-2	-2	1	0	0	0	0	:	--	0	0	0	0-4r3	0	0-2r3		0	0	0	0	0	0	0	0	0	0	0	0	r3	x_{49}
x_{50}	0	0	0	-1	-1	0	0		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{50}
x_{51}	0	0	0	-1	-1	0	0																							x_{51}
x_{52}	1	1	-1	-1	-1	-1	-1	:	--	0	0	0	0-4r3	0	0	r3	0	0	0	0	0	0	0	-r3	r3	0	0	0	r3	x_{52}
x_{53}	0	0	0	0	0	0	0	:	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	113	*3	*9	0	x_{53}
x_{54}	0	0	0	0	0	0	0	:	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*9	113	*3	0	x_{54}
x_{55}	0	0	0	0	0	0	0	:	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*3	*9	113	0	x_{55}
x_{56}	1	1	0	0	0	-1	-1	:	--	0	0	0	0	0	0	0	0-2r2		0	0	0	0	0	r2	r2	0	0	0	0	x_{56}
x_{57}	0	0	-2	0	0	0	0	:	--	0	0-4r2		0	0	0	0	0	0	0	0	0	2r2	r2	0	0	0	0	0	0	x_{57}
x_{58}	0	0	1	0	0	0	0	:	--	0	0-4r2		0	6r3	0	0	0	0	0	0	0	-r2	r2	0	0	0	0	0	r3	x_{58}
x_{59}	0	0	1	0	0	0	0	:	--	0	0-4r2		0	0-6r3	0	0	0	0	0	0	0	-r2	r2	0	0	0	0	0	-r3	x_{59}
x_{60}	-1	-1	0	1	1	1	1	:	--	0	0	6r2	0	0	0	0	0	-r2	0	0	0	0	r2	-r2	-r2	0	0	0	0	x_{60}

$U_3(8)$

$U_3(8)$



$U_3(8)$ and $U_3(7)$

Unitary group $U_3(8) \cong {}^2A_2(8)$

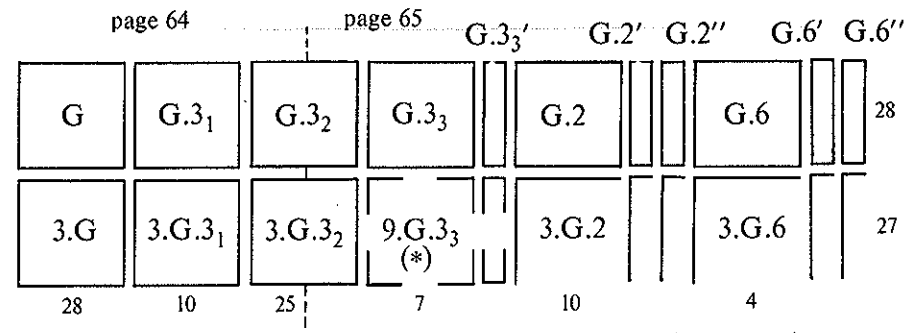
Order = 5,515,776 = $2^9.3^4.7.19$ Mult = 3 Out = $3 \times S_3$

Constructions

Unitary $GU_3(8) \cong 3.(3 \times G).3_2$: all 3×3 matrices over $\mathbf{F}_{64}$ preserving a non-singular Hermitian form;

$PGU_3(8) \cong G.3_2$; $SU_3(8) \cong 3.G$; $PSU_3(8) \cong G$

Remark : $G.3_1$ is a maximal subgroup of the Harada-Norton group, while G.6 is a maximal subgroup of the Thompson group.



(*) There is no group $3.G.3_3$ or $3.G.3_3'$. See the Introduction: 'Isoclinism'.

Maximal subgroups

Order	Index	Structure	$G.3_1$	$G.3_2$	$G.3_3$	$G.(3^2)_{1233}$	G.2	G.6	$G.S_3$	$G.(3xS_3)$	Character	Specifications Abstract	Specifications Unitary
10752	513	2^{3+6}:21	: 2^{3+6}:(7:3x3)	: 2^{3+6}:63	: 2^{3+6}:7:9	: 2^{3+6}:63:3	: 2^{3+6}:(7xS_3)	: 2^{3+6}:(7:3xS_3)	: 2^{3+6}:(7xD_{18})	: 2^{3+6}:(7xD_{18}):3	1a+512a	N($2A^3$)	isotropic point
1512	3648	3 x $L_2(8)$	: 3 x $L_2(8)$:3	: 9 x $L_2(8)$	: $L_2(8)$:9	: (9 x $L_2(8)$):3	: S_3 x $L_2(8)$	: S_3 x $L_2(8)$:3	: D_{18} x $L_2(8)$	: (D_{18} x $L_2(8)$):3	1a+133abc+399abc+512a+513abc	N(3AB)	non-isotropic point
216	25536	3^2:$2A_4$	: 3^2:$2A_4$ x 3				: 3^2:$2S_4$	: 3^2:$2S_4$ x 3				N($3C^2$), C(3D)	$U_3(2)$
216	25536	3^2:$2A_4$	: 3^2:$2A_4$ x 3									N($3C^2$), C(3E)	$U_3(2)$
216	25536	3^2:$2A_4$	: 3^2:$2A_4$ x 3									N($3C^2$), C(3F)	$U_3(2)$
162	34048	(9 x 3).S_3	: (9x3).(3xS_3)	: 9^2:S_3	: (9x3)·(3xS_3)	: 9^2.(3xS_3)	: (9x3).D_{12}	: (9x3).(6xS_3)	: 9^2:D_{12}	: 9^2.(6xS_3)		N(3C)	base
57	96768	19:3	: 19:9	: 19:3 x 3	: 19:9	: 3 x 19:9	: 19:6	: 19:18	: (19:3 x 3):2	: (19:9 x 3):2		N(19ABCDEF), C(3G)	$U_1(512)$

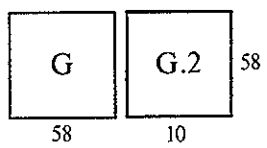
Unitary group $U_3(7) \cong {}^2A_2(7)$

Order = 5,663,616 = $2^7.3.7^3.43$ Mult = 1 Out = 2

Constructions

Unitary $GU_3(7) \cong 8 \times G$: all 3×3 matrices over $\mathbf{F}_{49}$ preserving a non-singular Hermitian form;

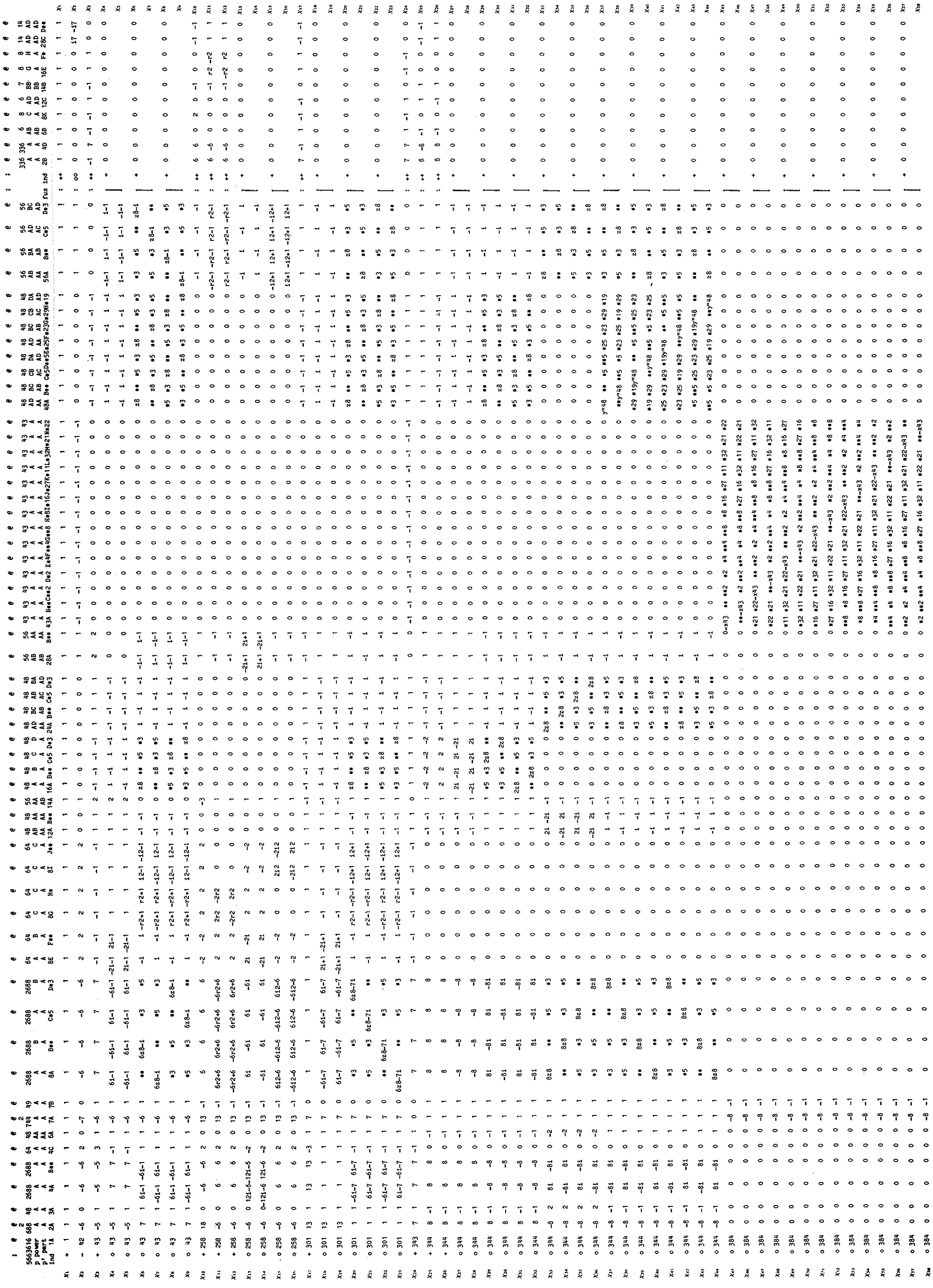
$PGU_3(7) \cong SU_3(7) \cong PSU_3(7) \cong G$



Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Specifications Unitary
16464	344	7^{1+2}:48	: 7^{1+2}:(6xD_{16})	1a+343a	N(7A)	isotropic point
2688	2107	2($L_2(7)$x4).2	: (2($L_2(7)$x4).2).2		N(2A)	non-isotropic point
384	14749	8^2:S_3	: 8^2:D_{12}		N($2A^2$)	base
336	16856	$L_2(7)$:2	: 2 x $L_2(7)$:2		N(2A,3A,4C,7B), C(2B)	$O_3(7)$
129	43904	43:3	: 43:6		N(43A-N)	$U_1(343)$

$U_3(7)$



$L_4(3)$

Linear group $L_4(3) \cong A_3(3) \cong O_6^+(3)$

Order = 6,065,280 = $2^7.3^6.5.13$ Mult = 2 Out = 2^2

G	G.2_1	G.2_2	G.2_3	29
2.G	2.G.2_1	2.G.2_2	2.G.2_3	22
29	19	23	13	

Constructions

Linear $GL_4(3) \cong 2.G.2_1$: all non-singular 4 x 4 matrices over $\mathbf{F}_3$;

$PGL_4(3) \cong G.2_1$; $SL_4(3) \cong 2.G$; $PSL_4(3) \cong G$

Orthogonal $GO_6^+(3) \cong 2 \times G.2_2$: all 6 x 6 matrices over $\mathbf{F}_3$ preserving a non-singular quadratic form of Witt defect 0,

for example $x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_5^2 - x_6^2$; $PGO_6^+(3) \cong G.2_2$; $SO_6^+(3) \cong 2 \times G$; $PSO_6^+(3) \cong O_6^+(3) \cong G$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	
			2	1	5	1	1																										5											
		6065280	880	152	832	944	944	81	1440	96	32	20	72	72	72	36	36	8	27	27	20	36	36	12	13	13	13	13	20	20			616	16	54	9	96	96	40	40	16	12	12	
p	power		A	A	A	A	A	A	A	B	A	A	BA	CA	AB	BB	CB	B	A	A	AA	AA	BA	CB	A	A	A	A	AA	AA			A	B	AC	DC	B	B	A	A	B	CB	CC	
p'	part		A	A	A	A	A	A	A	A	A	A	BA	CA	AB	BB	CB	A	A	A	AA	BA	CA	AB	A	A	A	A	AA	AA			A	A	AC	DC	A	A	A	A	A	AB	AC	
	ind	1A	2A	2B	3A	3B	3C	3D	4A	4B	4C	5A	6A	6B	6C	6D	6E	8A	9A	9B	10A	12A	12B	12C	13A	B**	C*5	D*8	20A	B**	fus	ind	2C	4D	6F	6G	8B	C**	8D	E**	8F	24A	B**	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	26	6	2	-1	-1	8	-1	4	2	0	1	3	0	-1	-1	2	0	2	-1	1	1	-2	-1	0	0	0	0	-1	-1		+	0	0	0	0	0	0	0	0	0	0	0	χ_{2}
χ_{3}	+	26	6	2	-1	8	-1	-1	4	2	0	1	0	3	-1	2	-1	0	-1	2	1	-2	1	-1	0	0	0	0	-1	-1														χ_{3}
χ_{4}	+	39	-1	7	12	3	3	3	-1	3	-1	-1	-1	-1	4	1	1	1	0	0	-1	-1	-1	0	0	0	0	0	-1	-1	:	++	13	1	4	1	3	3	-1	-1	-1	0	0	χ_{4}
χ_{5}	+	52	8	-4	-2	7	7	-2	-10	0	2	2	-1	-1	2	-1	-1	0	1	1	-2	-1	-1	0	0	0	0	0	0	0	:	oo	0	0	0	0	2i2	-2i2	i2	-i2	0	-i2	i2	χ_{5}
χ_{6}	+	65	5	-7	11	2	11	2	-5	1	-1	0	2	-1	-1	2	-1	-1	2	-1	0	-2	1	1	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{6}
χ_{7}	+	65	5	-7	11	11	2	2	-5	1	-1	0	-1	2	-1	-1	2	-1	-1	2	0	1	-2	1	0	0	0	0	0	0														χ_{7}
χ_{8}	+	90	10	10	9	9	9	0	10	-2	2	0	1	1	1	1	1	0	0	0	0	1	1	1	-1	-1	-1	-1	0	0	:	++	12	0	3	0	-2	-2	0	0	2	1	1	χ_{8}
χ_{9}	+	234	14	2	-9	18	-9	0	-4	2	0	-1	2	-1	-1	2	-1	0	0	0	-1	2	-1	-1	0	0	0	0	1	1		+	0	0	0	0	0	0	0	0	0	0	0	χ_{9}
χ_{10}	+	234	14	2	-9	-9	18	0	-4	2	0	-1	-1	2	-1	-1	2	0	0	0	-1	-1	2	-1	0	0	0	0	1	1														χ_{10}
χ_{11}	+	260	0	-4	-10	17	-1	-1	10	0	-2	0	-3	3	2	-1	-1	0	2	-1	0	1	1	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}	+	260	0	-4	-10	-1	17	-1	10	0	-2	0	3	-3	2	-1	-1	0	-1	2	0	1	1	0	0	0	0	0	0	0														χ_{12}
χ_{13}	+	260	20	4	17	-10	-10	-1	0	4	0	0	2	2	1	-2	-2	0	-1	-1	0	0	0	1	0	0	0	0	0	0	:	++	26	2	-1	-1	2	2	0	0	2	-1	-1	χ_{13}
χ_{14}	+	351	-9	15	27	0	0	0	-9	-1	-1	1	0	0	3	0	0	-1	0	0	1	0	0	-1	0	0	0	0	1	1	:	++	39	-1	3	0	-1	-1	1	1	-1	-1	-1	χ_{14}
χ_{15}	+	390	-10	-10	39	3	3	3	10	2	2	0	-1	-1	-1	-1	-1	0	0	0	0	1	1	-1	0	0	0	0	0	0	:	++	26	-2	-1	-1	4	4	0	0	0	1	1	χ_{15}
χ_{16}	+	416	16	0	-16	2	2	2	16	0	0	1	-2	-2	0	0	0	0	-1	-1	1	-2	-2	0	0	0	0	0	1	1	:	++	0	0	0	0	0	0	4	4	0	0	0	χ_{16}
χ_{17}	+	416	16	0	-16	2	2	2	-16	0	0	1	-2	-2	0	0	0	0	-1	-1	1	2	2	0	0	0	0	0	-1	-1	:	oo	0	0	0	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}	o	416	-16	0	-16	2	2	2	0	0	0	1	2	2	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	i5	-i5	:	oo	0	0	0	0	0	0	-2i2	2i2	0	0	0	χ_{18}
χ_{19}	o	416	-16	0	-16	2	2	2	0	0	0	1	2	2	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	-i5	i5	:	oo	0	0	0	0	0	0	2i2	-2i2	0	0	0	χ_{19}
χ_{20}	+	468	-8	-4	-18	9	9	0	-10	0	2	-2	1	1	2	-1	-1	0	0	0	2	-1	-1	0	0	0	0	0	0	0	:	oo	0	0	0	0	2i2	-2i2	-i2	i2	0	-i2	i2	χ_{20}
χ_{21}	+	585	5	1	18	-9	18	0	-5	-3	-1	0	-1	2	-2	1	-2	1	0	0	0	1	-2	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}	+	585	5	1	18	18	-9	0	-5	-3	-1	0	2	-1	-2	-2	1	1	0	0	0	-2	1	0	0	0	0	0	0	0														χ_{22}
χ_{23}	+	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	d13	**	*5	*8	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{23}
χ_{24}	+	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	**	d13	*8	*5	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{24}
χ_{25}	+	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	*8	*5	d13	**	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{25}
χ_{26}	+	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	*5	*8	**	d13	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{26}
χ_{27}	+	729	9	9	0	0	0	0	9	-3	1	-1	0	0	0	0	0	-1	0	0	-1	0	0	0	1	1	1	1	-1	-1	:	++	27	-1	0	0	-3	-3	-1	-1	1	0	0	χ_{27}
χ_{28}	+	780	-20	12	-3	6	6	-3	0	4	0	0	-2	-2	-3	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	:	++	26	2	-1	-1	-2	-2	0	0	-2	1	1	χ_{28}
χ_{29}	+	1040	0	-16	14	-4	-4	-4	0	0	0	0	0	0	2	2	2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	:	oo	0	0	0	0	4i2	-4i2	0	0	0	i2	-i2	χ_{29}
	ind	1	4	2	3	3	3	3	8	4	8	5	12	12	6	6	6	8	9	9	20	24	24	12	13	13	13	13	40	40	fus	ind	2	4	6	6	8	8	16	16	8	24	24	
		2			6	6	6	6	8	4		10			6			8	18	18	20	24	24	12	26	26	26	26	40	40			2		6	6	8	8			8	24	24	
χ_{30}	+	40	0	0	13	4	4	4	0	4	0	0	0	0	-3	0	0	0	1	1	0	0	0	1	1	1	1	1	0	0	:	++	12	0	3	0	4	4	0	0	0	1	1	χ_{30}
χ_{31}	o	208	0	0	-8	-8	10	1	-4i2	0	0	-2	0	0	0	0	0	0	-2	1	0	2i2	-i2	0	0	0	0	0	i2	i2		o	0	0	0	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}	o	208	0	0	-8	-8	10	1	4i2	0	0	-2	0	0	0	0	0	0	-2	1	0	-2i2	i2	0	0	0	0	0	-i2	-i2		o	0	0	0	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}	o	208	0	0	-8	10	-8	1	-4i2	0	0	-2	0	0	0	0	0	0	1	-2	0	-i2	2i2	0	0	0	0	0	i2	i2														χ_{33}
χ_{34}	o	208	0	0	-8	10	-8	1	4i2	0	0	-2	0	0	0	0	0	0	1	-2	0	i2	-2i2	0	0	0	0	0	-i2	-i2														χ_{34}
χ_{35}	o	260	0	0	17	-10	-10	-1	10i2	2	0	0	0	0	-3	0	0	i2	-1	-1	0	i2	i2	-1	0	0	0	0	0	0	:	oo	26	0	-1	-1	i2+2	-i2+2	0	0	i2	i2-1	-i2-1	χ_{35}
χ_{36}	o	260	0	0	17	-10	-10	-1	-10i2	2	0	0	0	0	-3	0	0	-i2	-1	-1	0	-i2	-i2	-1	0	0	0	0	0	0	:	oo	26	0	-1	-1	-i2+2	i2+2	0	0	-i2	-i2-1	i2-1	χ_{36}
χ_{37}	o	416	0	0	-16	2	2	2	8i2	0	0	1	0	0	0	0	0	0	-1	-1	i5	-i2	-i2	0	0	0	0	0	w40	*19	:	oo	0	0	0	0	0	0	0	0	0	0	0	χ_{37}
χ_{38}	o	416	0	0	-16	2	2	2	-8i2	0	0	1	0	0	0	0	0	0	-1	-1	-i5	i2	i2	0	0	0	0	0	*19	-w40	:	oo	0	0	0	0	0	0	0	0	0	0	0	χ_{38}
χ_{39}	o	416	0	0	-16	2	2	2	-8i2	0	0	1	0	0	0	0	0	0	-1	-1	i5	i2	i2	0	0	0	0	0	-w40	*19	:	oo	0	0	0	0	0	0	0	0	0	0	0	χ_{39}
χ_{40}	o	416	0	0	-16	2	2	2	8i2	0	0	1	0	0	0	0	0	0	-1	-1	-i5	-i2	-i2	0	0	0	0	0	*19	w40	:	oo	0	0	0	0	0	0	0	0	0	0	0	χ_{40}
χ_{41}	+	480	0	0	48	12	12	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	:	++	40	0	4	1	0	0	0	0	0	0	0	χ_{41}
χ_{42}	+	520	0	0	-20	16	16	-2	0	4	0	0	0	0	0	0	0	0	1	1	0	0	0	-2	0	0	0	0	0	0	:	oo	0	0	0	0	-2i2	2i2	0	0	-2i2	i2	-i2	χ_{42}
χ_{43}	+	520	0	0	7	-20	16	-2	0	4	0	0	0	0	3	0	0	0	1	-2	0	0	0	1	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{43}
χ_{44}	+	520	0	0	7	16	-20	-2	0	4	0	0	0	0	3	0	0	0	-2	1	0	0	0	1	0	0	0	0	0	0														χ_{44}
χ_{45}	o	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	d13	**	*5	*8	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{45}
χ_{46}	o	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	**	d13	*8	*5	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{46}
χ_{47}	o	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	*8	*5	d13	**	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{47}
χ_{48}	o	640	0	0	-8	-8	-8	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	*5	*8	**	d13	0	0	:	oo	16	0	-2	1	0	0	0	0	0	0	0	χ_{48}
χ_{49}	o	780	0	0	-3	6	6	-3	10i2	-2	0	0	0	0	-3	0	0	-i2	0	0	0	i2	i2	1	0	0	0	0	0	0	:	oo	-26	0	1	1	3i2+2	-3i2+2	0	0	-i2	-1	-1	χ_{49}
χ_{50}	o	780	0	0	-3	6	6	-3	-10i2	-2	0	0	0	0	-3	0	0	i2	0	0	0	-i2	-i2	1	0	0	0	0	0	0	:	oo	-26	0	1	1	-3i2+2	3i2+2	0	0	i2	-1	-1	χ_{50}
χ_{51}	+	1080	0	0	27	0	0	0	0	-4	0	0	0	0	3	0	0	0	0	0	0	0	0	-1	1	1	1	1	0	0	:	++	12	0	3	0	-4	-4	0	0	0	-1	-1	χ_{51}

$L_4(3)$

Maximal subgroups

Order	Index	Structure	$G.2_1$	$G.2_2$	$G.2_3$	$G.2^2$	Character	Specifications: Abstract	Linear	Orthogonal
151632	40	$3^3:L_3(3)$	$: 3^3:(L_3(3)\times 2)$	$3_+^{1+4}:(2S_4\times 2)$, $L_3(3):2$	$3_+^{1+4}:(2S_4\times 2)$, $L_3(3):2$	$3_+^{1+4}:(2S_4\times 2^2)$, $L_3(3):2\times 2$	1a+39a	$N(3A^3)$	point	isotropic plane
151632	40	$3^3:L_3(3)$	$: 3^3:(L_3(3)\times 2)$				1a+39a	$N(3A^3)$	plane	isotropic plane
51840	117	$U_4(2):2$	$2(S_4\times S_4).2$	$: U_4(2):2\times 2$	$2(S_4\times S_4).2$	$2(S_4\times S_4).2\times 2$	1a+26b+90a	C(2D)	$S_4(3)$	non-isotropic point
51840	117	$U_4(2):2$		$: U_4(2):2\times 2$			1a+26a+90a	C(2E)	$S_4(3)$	non-isotropic point
46656	130	$3^4:2(A_4\times A_4).2$	$: H.2$	$: H.2$	$: H.2$	$: H.2^2$	1a+39a+90a	$N(3^4)$	line	isotropic point
2880	2106	$(4\times A_6):2$	$: H.2$	$: D_8\times S_6$	$: H.2$	$: H.2^2$		N(2A)	$L_2(9)$	$O_2^-(3)\times O_4^-(3)$
720	8424	S_6	$: A_6\cdot 2^2$	$: A_6\cdot 2^2$	$: S_6\times 2$	$: A_6\cdot 2^2\times 2$		C(2G)	$O_4^-(3)$	$O_3(9)$
576	10530	$S_4\times S_4$	$: (S_4\times S_4):2$	$: S_4\times S_4\times 2$	$: (S_4\times S_4):2$	$: (S_4\times S_4):2\times 2$		$N(2^4)$, C(2F)		base, $O_3(3)wr2$

	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	
											51	51																																						
	13	13	13	13	20	20	20	20			840	840	576	96	96	96	648	648	216	216	108	108	72	72	9	8	10	10	12	12	12	9	9			720	48	9	96	96	16	8	8	10	10	6	12	12	
	CC	DC	BC	AC	BE	AD	BD	AE			A	A	A	B	A	A	AD	AE	CD	BE	BD	CE	BF	CF	DF	B	AD	AE	CE	AF	BG	BH	AI			A	B	DG	B	B	B	C	C	AG	AG	CH	CI	CH	
	AC	BC	CC	DC	AD	AE	AE	AD			A	A	A	A	A	A	AD	AE	CD	BE	BD	CE	BF	CF	DF	A	AD	AE	AE	BF	CG	BD	AE			A	A	DG	A	A	A	A	A	AG	AG	AH	AH	AI	
	26A	B**	C*5D**	5	40A	B**	C*21D*	19	fus	ind	2D	2E	2F	4E	4F	4G	6H	6I	6J	6K	6L	6M	6N	6O	6P	8G	10B	10C	12D	12E	12F	18A	18B	fus	ind	2G	4H	6Q	8H	I*	8J	8K	L*	10D	E*	12G	24C	D*	
χ_1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	0	0	0	0	0	0	0	0	:	++	14	6	2	-2	4	0	5	-3	2	3	-1	0	-1	2	-1	0	-1	1	1	1	0	-1	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_2
χ_3									:	++	6	14	2	-2	0	4	-3	5	3	2	0	-1	2	-1	-1	0	1	-1	1	0	1	0	-1																		χ_3
χ_4	0	0	0	0	-1	-1	-1	-1	:	++	9	9	1	-3	1	1	0	0	-3	-3	3	3	1	1	1	-1	-1	-1	0	1	1	0	0	:	++	-1	3	-1	-3	-3	1	1	1	-1	-1	0	0	0	χ_4
χ_5	0	0	0	0	i2	-i2	-i2	i2	:	++	20	-20	0	0	2	-2	2	-2	5	-5	-1	1	3	-3	0	0	0	0	0	-1	1	-1	1	:	++	0	0	0	-2r2	2r2	0	r2	-r2	0	0	0	r2	-r2	χ_5
χ_6	0	0	0	0	0	0	0	0	:	++	25	-15	-3	1	3	-1	7	3	1	0	4	-3	0	-3	0	-1	0	0	1	0	-1	1	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_6
χ_7									:	++	-15	25	-3	1	-1	3	3	7	0	1	-3	4	-3	0	0	-1	0	0	1	-1	0	0	1																		χ_7
χ_8	-1	-1	-1	-1	0	0	0	0	:	++	30	30	6	2	2	2	3	3	3	3	3	3	3	3	0	0	0	0	-1	-1	-1	0	0	:	++	0	4	0	2	2	-2	0	0	0	0	1	-1	-1	χ_8
χ_9	0	0	0	0	0	0	0	0	:	++	-6	66	6	2	0	4	3	3	-3	0	0	-3	0	-3	0	0	-1	1	-1	0	1	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_9
χ_{10}									:	++	66	-6	6	2	4	0	3	3	0	-3	-3	0	-3	0	0	0	1	-1	-1	1	0	0	0																		χ_{10}
χ_{11}	0	0	0	0	0	0	0	0	:	++	20	60	-8	0	-2	2	2	6	5	-3	-1	-3	1	1	1	0	0	0	0	1	-1	-1	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}									:	++	60	20	-8	0	2	-2	6	2	-3	5	-3	-1	1	1	1	0	0	0	0	-1	1	0	-1																		χ_{12}
χ_{13}	0	0	0	0	0	0	0	0	:	++	20	20	4	4	0	0	-7	-7	2	2	2	2	-2	-2	1	0	0	0	1	0	0	-1	-1	:	++	10	2	1	2	2	2	0	0	0	0	-1	-1	-1	χ_{13}
χ_{14}	0	0	0	0	1	1	1	1	:	++	9	9	9	1	-3	-3	9	9	0	0	0	0	0	0	0	1	-1	-1	1	0	0	0	0	:	++	9	1	0	1	1	1	-1	-1	-1	-1	1	1	1	χ_{14}
χ_{15}	0	0	0	0	0	0	0	0	:	++	-30	-30	10	-2	2	2	-3	-3	-3	-3	-3	-3	1	1	1	0	0	0	1	-1	-1	0	0	:	++	10	-2	1	-4	-4	0	0	0	0	0	1	-1	-1	χ_{15}
χ_{16}	0	0	0	0	-1	-1	-1	-1	:	++	64	64	0	0	0	0	-8	-8	4	4	-2	-2	0	0	0	0	-1	-1	0	0	0	1	1	:	++	16	0	-2	0	0	0	0	0	1	1	0	0	0	χ_{16}
χ_{17}	0	0	0	0	i5	-i5	i5	-i5	:	++	64	-64	0	0	0	0	-8	8	4	-4	-2	2	0	0	0	0	-1	1	0	0	0	1	-1	:	++	0	0	0	0	0	0	0	0	r5	-r5	0	0	0	χ_{17}
χ_{18}	0	0	0	0	w40	**	*21	*19		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{18}
χ_{19}	0	0	0	0	**	w40	*19	*21																																										χ_{19}
χ_{20}	0	0	0	0	-i2	i2	i2	-i2	:	++	60	-60	0	0	-2	2	6	-6	-3	3	-3	3	3	-3	0	0	0	0	0	1	-1	0	0	:	++	0	0	0	2r2	-2r2	0	r2	-r2	0	0	0	-r2	r2	χ_{20}
χ_{21}	0	0	0	0	0	0	0	0	:	++	-105	15	3	3	1	-3	-6	6	0	3	-3	0	3	0	0	-1	0	0	0	1	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}									:	++	15	-105	3	3	-3	1	6	-6	3	0	0	-3	0	3	0	-1	0	0	0	0	1	0	0																		χ_{22}
χ_{23}	d13	**	*5	*8	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{23}
χ_{24}	**	d13	*8	*5	0	0	0	0																																										χ_{24}
χ_{25}	*8	*5	d13	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{25}
χ_{26}	*5	*8	**	d13	0	0	0	0																																										χ_{26}
χ_{27}	1	1	1	1	-1	-1	-1	-1	:	++	81	81	9	-3	-3	-3	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	:	++	-9	3	0	-3	-3	1	-1	-1	1	1	0	0	0	χ_{27}
χ_{28}	0	0	0	0	0	0	0	0	:	++	60	60	-4	4	0	0	-3	-3	-6	-6	0	0	2	2	-1	0	0	0	1	0	0	0	0	:	++	10	2	1	-2	-2	-2	0	0	0	0	-1	1	1	χ_{28}
χ_{29}	0	0	0	0	0	0	0	0	:	++	80	-80	0	0	0	0	-10	10	-4	4	2	-2	0	0	0	0	0	0	0	0	0	-1	1	:	++	0	0	0	4r2	-4r2	0	0	0	0	0	0	r2	-r2	χ_{29}
	26	26	26	26	80	80	80	80	fus	ind	4	4	2	4	8	8	12	12	12	12	12	12	6	6	6	8	20	20	12	24	24	36	36	fus	ind	2	4	6	8	8	8	16	16	10	10	12	24	24	
																	12	12							6				12			36	36									8					12	24	24	
χ_{30}	-1	-1	-1	-1	0	0	0	0	:	++	0	0	0	0	0	0	3r3	3r3	0	0	0	0	0	0	0	0	0	0	r3	0	0	-r3	-r3	:	++	0	0	0	0	0	0	0	0	0	0	r3	-r3	r3	χ_{30}
χ_{31}	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		o	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}	0	0	0	0	0	0	0	0																											o	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}										+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		χ_{33}
χ_{34}																																																		χ_{34}
χ_{35}	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{35}
χ_{36}	0	0	0	0	0	0	0	0																																										χ_{36}
χ_{37}	0	0	0	0	w80	**	*61	*19		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{37}
χ_{38}	0	0	0	0	**	w80	*19	*61																																										χ_{38}
χ_{39}	0	0	0	0	*21	*59	w80	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{39}
χ_{40}	0	0	0	0	*59	*21	**	w80																																										χ_{40}
χ_{41}	1	1	1	1	0	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	4	0	0	0	0	0	0	0	χ_{41}
χ_{42}	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	0	6r3	-6r3	0	0	0	0	0	0	0	0	0	0	0	0	0	r3	-r3	:	++	0	0	0	0	0	0	0	0	0	0	0	r6	r6	χ_{42}
χ_{43}	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	0	-3r3	9r3	0	0	0	0	0	0	0	0	0	0	r3	0	0	r3	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{43}
χ_{44}									:	++	0	0	0	0	0	0	9r3	-3r3	0	0	0	0	0	0	0	0	0	0	r3	0	0	0	r3																		χ_{44}
χ_{45}	d13	**	*5	*8	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}	**	d13	*8	*5	0	0	0	0																																										χ_{46}
χ_{47}	*8	*5	d13	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{47}
χ_{48}	*5	*8	**	d13	0	0	0	0																																										χ_{48}
χ_{49}	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{49}
χ_{50}	0	0	0	0	0	0	0	0																																										χ_{50}
χ_{51}	-1	-1	-1	-1	0	0	0	0	:	++	0	0	0	0	0	0	9r3	9r3	0	0	0	0	0	0	0	0	0	0	-r3	0	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	r3	r3	-r3	χ_{51}

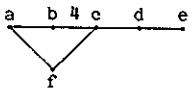
$L_5(2)$

Linear group $L_5(2) \cong A_4(2)$

Order = 9,999,360 = $2^{10}.3^2.5.7.31$ Mult = 1 Out = 2

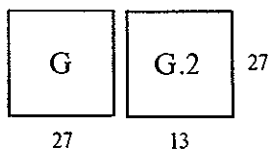
Constructions

Linear $GL_5(2) \cong PGL_5(2) \cong SL_5(2) \cong PSL_5(2) \cong G$: all 5 x 5 non-singular matrices over F_2.

Presentations $2^5{:}G \cong \langle$  $\mid (bcf)^3=(abcd)^6=1 \rangle$; adjoin $(fbacde)^5=1$ for G

Remark : There is a non-split extension $2^{5\cdot}L_5(2)$ (the Dempwolff group), which is a maximal subgroup of the Thompson group.

Maximal subgroups					Specifications	
Order	Index	Structure	G.2	Character	Abstract	Linear
322560	31	$2^4{:}L_4(2)$	$2_+^{1+6}{:}L_3(2){:}2$,	1a+30a	$N(2A^4)$	point
322560	31	$2^4{:}L_4(2)$	$L_4(2){:}2$	1a+30a	$N(2A^4)$	hyperplane
64512	155	$2^6{:}(S_3 x L_3(2))$	$2^{4+4}{:}(S_3 x S_3){:}2$,	1a+30a+124a	$N(2^6)$	line
64512	155	$2^6{:}(S_3 x L_3(2))$	$S_3 \times L_3(2){:}2$	1a+30a+124a	$N(2^6)$	plane
155	64512	31:5	: 31:10		N(31ABCDEF)	$L_1(32)$



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	
			21	1																																								
	9999360		504	536	504	180	384	128	32	15	24	12	42	42	16	12	14	14	15	15	21	21	31	31	31	31	31	31			720	48	18	18	96	96	16	5	6	8	8	12	12	
	p power		A	A	A	A	A	A	B	A	AA	BB	A	A	B	AA	AA	BA	AB	AB	BA	AA	A	A	A	A	A	A			A	B	AC	BC	A	A	A	AC	BD	A	A	AC	AB	
	p' part		A	A	A	A	A	A	A	A	AA	BB	A	A	A	AA	AA	BA	AB	AB	AA	BA	A	A	A	A	A	A			A	A	AC	BC	A	A	A	AC	BD	A	A	AC	AB	
	ind	1A	2A	2B	3A	3B	4A	4B	4C	5A	6A	6B	7A	B**	8A	12A	14A	B**	15A	B**	21A	B**	31A	B**	C*5D	**5E	**6	F*6	fus	ind	2C	4D	6C	6D	8B	C*	8D	10A	12B	16A	B*	24A	B*	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	30	14	6	6	0	6	2	2	0	2	0	2	2	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	:	++	0	0	0	0	2r2	-2r2	0	0	0	r2	-r2	-r2	r2	χ_2
χ_3	+	124	28	12	1	4	4	4	0	-1	1	0	-2	-2	0	1	0	0	-1	-1	1	1	0	0	0	0	0	0	:	++	4	0	1	-2	4	4	0	-1	0	0	0	1	1	χ_3
χ_4	+	155	27	-5	8	5	3	-5	-1	0	0	1	1	1	-1	0	-1	-1	0	0	1	1	0	0	0	0	0	0	:	++	5	1	2	-1	-3	-3	1	0	1	-1	-1	0	0	χ_4
χ_5	+	217	-7	9	7	4	-7	1	1	2	-1	0	0	0	1	-1	0	0	-1	-1	0	0	0	0	0	0	0	0	:	++	5	-3	-1	2	1	1	1	0	0	-1	-1	1	1	χ_5
χ_6	+	280	56	8	7	-5	8	0	0	0	-1	-1	0	0	0	-1	0	0	0	0	0	0	1	1	1	1	1	1	:	++	10	2	1	1	2	2	2	0	-1	0	0	-1	-1	χ_6
χ_7	o	315	-21	3	0	0	3	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	f31	**	*5	**5	**6	*6		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_7
χ_8	o	315	-21	3	0	0	3	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	**	f31	**5	*5	*6	**6																χ_8
χ_9	o	315	-21	3	0	0	3	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	**6	*6	f31	**	*5	**5		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_9
χ_{10}	o	315	-21	3	0	0	3	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	*6	**6	**	f31	**5	*5																χ_{10}
χ_{11}	o	315	-21	3	0	0	3	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	*5	**5	**6	*6	f31	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}	o	315	-21	3	0	0	3	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	**5	*5	*6	**6	**	f31																χ_{12}
χ_{13}	o	465	-31	9	3	0	1	-3	1	0	-1	0	b7	**	-1	1	-b7	**	0	0	b7	**	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}	o	465	-31	9	3	0	1	-3	1	0	-1	0	**	b7	-1	1	**	-b7	0	0	**	b7	0	0	0	0	0	0																χ_{14}
χ_{15}	o	465	17	-15	3	0	1	1	1	0	-1	0	b7	**	1	1	b7	**	0	0	b7	**	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{15}
χ_{16}	o	465	17	-15	3	0	1	1	1	0	-1	0	**	b7	1	1	**	b7	0	0	**	b7	0	0	0	0	0	0																χ_{16}
χ_{17}	+	496	48	16	-8	1	0	0	0	1	0	1	-1	-1	0	0	-1	-1	1	1	-1	-1	0	0	0	0	0	0	:	++	4	4	-2	1	0	0	0	-1	1	0	0	0	0	χ_{17}
χ_{18}	+	651	-21	-5	0	6	3	3	-1	1	0	-2	0	0	-1	0	0	0	1	1	0	0	0	0	0	0	0	0	:	++	9	-3	0	0	-3	-3	1	-1	0	1	1	0	0	χ_{18}
χ_{19}	o	651	-21	-5	0	-3	3	3	-1	1	0	1	0	0	-1	0	0	0	b15	**	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{19}
χ_{20}	o	651	-21	-5	0	-3	3	3	-1	1	0	1	0	0	-1	0	0	0	**	b15	0	0	0	0	0	0	0	0																χ_{20}
χ_{21}	+	868	-28	4	7	1	-4	4	0	-2	-1	1	0	0	0	-1	0	0	1	1	0	0	0	0	0	0	0	0	:	++	10	-2	1	1	2	2	-2	0	1	0	0	-1	-1	χ_{21}
χ_{22}	+	930	50	-6	6	0	-6	-2	-2	0	2	0	-1	-1	0	0	1	1	0	0	-1	-1	0	0	0	0	0	0	:	++	0	0	0	0	2r2	-2r2	0	0	0	-r2	r2	-r2	r2	χ_{22}
χ_{23}	o	930	-14	-6	-3	0	2	-2	2	0	1	0	2b7	**	0	-1	0	0	0	0	-b7	**	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{23}
χ_{24}	o	930	-14	-6	-3	0	2	-2	2	0	1	0	**	2b7	0	-1	0	0	0	0	**	-b7	0	0	0	0	0	0																χ_{24}
χ_{25}	+	960	64	0	-6	0	0	0	0	0	-2	0	1	1	0	0	1	1	0	0	1	1	-1	-1	-1	-1	-1	-1	:	++	0	0	0	0	4r2	-4r2	0	0	0	0	0	r2	-r2	χ_{25}
χ_{26}	+	1024	0	0	-8	4	0	0	0	-1	0	0	2	2	0	0	0	0	-1	-1	-1	-1	1	1	1	1	1	1	:	++	16	0	-2	-2	0	0	0	1	0	0	0	0	0	χ_{26}
χ_{27}	+	1240	-8	8	1	-5	-8	0	0	0	1	-1	1	1	0	1	-1	-1	0	0	1	1	0	0	0	0	0	0	:	++	10	2	1	1	-2	-2	-2	0	-1	0	0	1	1	χ_{27}

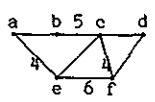
Sporadic Mathieu group M_{23}

Order = 10,200,960 = $2^7.3^2.5.7.11.23$ Mult = 1 Out = 1

Constructions

Mathieu For almost all purposes, M_{23} is best studied as the point stabilizer in M_{24} (q.v.). Thus, G is the automorphism group of the Steiner system S(4,7,23), whose 253 heptads arise from the octads of S(5,8,24) containing the fixed point.

Golay G : the automorphism group of the (unextended) binary Golay code of dimension 12, length 23, and minimal weight 7, or of its subcode of even weight words. The weight distribution of the code is $0^1 7^{253} 8^{506} 11^{1288} 12^{1288} 15^{506} 16^{253} 23^1$, and that of the cocode (words modulo the code) $0^1 1^{23} 2^{253} 3^{1771}$. The minimal representatives in the cocode are unique (i.e. the code is a perfect 3-error correcting code).

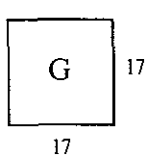
Presentation $G \cong \langle$  $\mid a=(cf)^2, b=(ef)^3, 1=(eab)^3=(bce)^5=(aecd)^4=(bcef)^4 \rangle$

Maximal subgroups

Order	Index	Structure	Character
443520	23	M_{22}	1a+22a
40320	253	$L_3(4):2_2$	1a+22a+230a
40320	253	$2^4:A_7$	1a+22a+230a
20160	506	A_8	1a+22a+230a+253a
7920	1288	M_{11}	1a+22a+230a+1035a
5760	1771	$2^4:(3xA_5):2$	1a+22a+230aa+253a+1035a
253	40320	23:11	

Specifications

Abstract	Mathieu	Golay
	point	weight 1 cocode word
	duad	weight 2 cocode word
$N(2A^4)$	heptad	weight 7 code word
	octad	weight 8 code word
	endecad, dodecad	weight 11 or 12 code word
$N(2A^4)$	triad	weight 3 cocode word
N(23AB)		



;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
			2															
10200960			688	180	32	15	12	14	14	8	11	11	14	14	15	15	23	23
p power			A	A	A	A	AA	A	A	A	A	A	AA	BA	AA	AA	A	A
p' part			A	A	A	A	AA	A	A	A	A	A	AA	BA	AA	AA	A	A
ind		1A	2A	3A	4A	5A	6A	7A	B**	8A	11A	B**	14A	B**	15A	B**	23A	B**
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	+	22	6	4	2	2	0	1	1	0	0	0	-1	-1	-1	-1	-1	-1
χ_3	o	45	-3	0	1	0	0	b7	**	-1	1	1	-b7	**	0	0	-1	-1
χ_4	o	45	-3	0	1	0	0	**	b7	-1	1	1	**	-b7	0	0	-1	-1
χ_5	+	230	22	5	2	0	1	-1	-1	0	-1	-1	1	1	0	0	0	0
χ_6	+	231	7	6	-1	1	-2	0	0	-1	0	0	0	0	1	1	1	1
χ_7	o	231	7	-3	-1	1	1	0	0	-1	0	0	0	0	b15	**	1	1
χ_8	o	231	7	-3	-1	1	1	0	0	-1	0	0	0	0	**	b15	1	1
χ_9	+	253	13	1	1	-2	1	1	1	-1	0	0	-1	-1	1	1	0	0
χ_{10}	o	770	-14	5	-2	0	1	0	0	0	0	0	0	0	0	0	b23	**
χ_{11}	o	770	-14	5	-2	0	1	0	0	0	0	0	0	0	0	0	**	b23
χ_{12}	o	896	0	-4	0	1	0	0	0	0	b11	**	0	0	1	1	-1	-1
χ_{13}	o	896	0	-4	0	1	0	0	0	0	**	b11	0	0	1	1	-1	-1
χ_{14}	o	990	-18	0	2	0	0	b7	**	0	0	0	b7	**	0	0	1	1
χ_{15}	o	990	-18	0	2	0	0	**	b7	0	0	0	**	b7	0	0	1	1
χ_{16}	+	1035	27	0	-1	0	0	-1	-1	1	1	1	-1	-1	0	0	0	0
χ_{17}	+	2024	8	-1	0	-1	-1	1	1	0	0	0	1	1	-1	-1	0	0

$U_5(2)$

Unitary group $U_5(2) \cong {}^2A_4(2)$

Order = 13,685,760 = $2^{10}.3^5.5.11$ Mult = 1 Out = 2

Constructions

Unitary $GU_5(2) \cong 3 \times G$: all 5 x 5 matrices over F_4 preserving a non-singular Hermitian form;

$PGU_5(2) \cong SU_5(2) \cong PSU_5(2) \cong G$

Quaternionic $G \times 2$: the group generated by the 165 quaternionic reflections $x \longrightarrow x - 2\langle x,m\rangle m/\langle m,m\rangle$ in the vectors $(2,0,0,0,0)^C$ (5), $(0,w,\bar{w},\bar{w},w)^C$ (160) and their images under the diagonal group $2 \times 2^{4+4} = \{\mathrm{diag}(\pm 1,\pm j,\pm k,\pm k,\pm j)^C\}$ of right multiplications, where $w = \frac{1}{2}(-1+i+j+k)$, and $\langle x,y\rangle = \Sigma x_t \overline{y_t}$. The vectors whose coordinates are in $\mathbf{Z}[w]$ form a complex lattice whose automorphism group is $6 \times U_4(2)$ (see $U_4(2)$, "biflection").

Reducing modulo 1+i shows the isomorphism with $U_5(2)$.

Complex Leech G : the stabilizer of a minimal vector in the complex Leech lattice (see Suz).

;		1	82	4					1		1																		
		13685760	944	608	77760	77760	3888	3888	944	324	152	384	96	15	1728	1728	1296	1296	432	432	288	288	216	144	144	108	108	36	
	p power		A	A	A	A	A	A	A	A	A	A	B	A	BA	AA	DA	CA	DA	CA	BB	AB	EA	DB	CB	FA	FA	FB	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	CA	DA	CA	DA	AB	BB	EA	CB	DB	FA	FA	FB	
	ind	1A	2A	2B	3A	B**	3C	D**	3E	3F	4A	4B	4C	5A	6A	B**	6C	D**	6E	F**	6G	H**	6I	6J	K**	6L	M**	6N	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	-	10	-6	2	-5	-5	1	1	4	-2	2	-2	-2	0	3	3	3	3	-3	-3	-1	-1	0	-1	-1	0	0	2	χ_2
χ_3	o	11	-5	3	-3i3+2	3i3+2	3z3-1	**	2	2	3	-1	-1	1	i3-2	-i3-2	b27	**	**	z3	-i3	i3	-2	**	z3+2	**	-2z3	0	χ_3
χ_4	o	11	-5	3	3i3+2	-3i3+2	**	3z3-1	2	2	3	-1	-1	1	-i3-2	i3-2	**	b27	z3	**	i3	-i3	-2	z3+2	**	-2z3	**	0	χ_4
χ_5	+	44	12	-4	14	14	-1	-1	5	2	4	4	0	-1	6	6	3	3	3	3	2	2	-3	-1	-1	0	0	2	χ_5
χ_6	+	55	23	7	10	10	1	1	10	1	-1	-1	3	0	2	2	5	5	5	5	-2	-2	2	1	1	-1	-1	1	χ_6
χ_7	o	55	7	-1	**	15z3-5	-2b27	**	1	1	7	3	-1	0	**	7z3+3	-2	-2	**	-2z3	**	3z3-1	1	2	2	i3-2	-i3-2	-1	χ_7
χ_8	o	55	7	-1	15z3-5	**	**	-2b27	1	1	7	3	-1	0	7z3+3	**	-2	-2	-2z3	**	3z3-1	**	1	2	2	-i3-2	i3-2	-1	χ_8
χ_9	o	66	18	10	**	9z3-6	**	3b27	3	3	2	-2	2	1	**	z3+2	**	3z3-3	**	z3-1	b27	**	3	**	b27	i3	-i3	1	χ_9
χ_{10}	o	66	18	10	9z3-6	**	3b27	**	3	3	2	-2	2	1	z3+2	**	3z3-3	**	z3-1	**	**	b27	3	b27	**	-i3	i3	1	χ_{10}
χ_{11}	-	110	-2	-10	-25	-25	2	2	2	2	6	-6	2	0	7	7	-2	-2	-2	-2	-1	-1	-2	2	2	-2	-2	2	χ_{11}
χ_{12}	o	110	30	6	-15i3-10	15i3-10	**	3z3-1	8	-4	6	2	2	0	-3i3-6	3i3-6	**	3b27	-3z3	**	-i3	i3	0	**	z3-1	0	0	0	χ_{12}
χ_{13}	o	110	30	6	15i3-10	-15i3-10	3z3-1	**	8	-4	6	2	2	0	3i3-6	-3i3-6	3b27	**	**	-3z3	i3	-i3	0	z3-1	**	0	0	0	χ_{13}
χ_{14}	o	110	-34	6	**	5b27	-3i3+2	3i3+2	11	2	-2	2	-2	0	z3-2	**	3i3+2	-3i3+2	-i3-4	i3-4	z3+2	**	-1	i3	-i3	**	2z3	0	χ_{14}
χ_{15}	o	110	-34	6	5b27	**	3i3+2	-3i3+2	11	2	-2	2	-2	0	**	z3-2	-3i3+2	3i3+2	i3-4	-i3-4	**	z3+2	-1	-i3	i3	2z3	**	0	χ_{15}
χ_{16}	+	120	24	8	30	30	12	12	3	3	8	0	0	0	6	6	6	6	0	0	2	2	3	2	2	3	3	-1	χ_{16}
χ_{17}	+	165	-27	5	30	30	3	3	3	-6	5	5	-3	0	6	6	-9	-9	3	3	2	2	3	-1	-1	0	0	2	χ_{17}
χ_{18}	+	176	-16	16	-4	-4	14	14	-4	5	0	0	0	1	-4	-4	2	2	2	2	4	4	-4	-2	-2	-1	-1	1	χ_{18}
χ_{19}	o	220	-36	-4	15i3+25	-15i3+25	**	-3z3+7	7	1	4	-4	0	0	-5i3-3	5i3-3	**	3z3-3	**	z3-1	2z3	**	3	z3-2	**	i3	-i3	-1	χ_{19}
χ_{20}	o	220	-36	-4	-15i3+25	15i3+25	-3z3+7	**	7	1	4	-4	0	0	5i3-3	-5i3-3	3z3-3	**	z3-1	**	**	2z3	3	**	z3-2	-i3	i3	-1	χ_{20}
χ_{21}	o	220	-4	12	10b27	**	**	15z3-2	-5	4	4	4	0	0	2b27	**	-b27	**	-b27	**	i3-3	-i3-3	-1	z3-1	**	2	2	0	χ_{21}
χ_{22}	o	220	-4	12	**	10b27	15z3-2	**	-5	4	4	4	0	0	**	2b27	**	-b27	**	-b27	-i3-3	i3-3	-1	**	z3-1	2	2	0	χ_{22}
χ_{23}	o	264	-24	-8	-27i3+3	27i3+3	-6	-6	3	3	8	0	0	-1	i3-9	-i3-9	**	-6z3	-2i3	2i3	-2z3	**	3	**	-2z3	i3	-i3	1	χ_{23}
χ_{24}	o	264	-24	-8	27i3+3	-27i3+3	-6	-6	3	3	8	0	0	-1	-i3-9	i3-9	-6z3	**	2i3	-2i3	**	-2z3	3	-2z3	**	-i3	i3	1	χ_{24}
χ_{25}	-	320	-64	0	-40	-40	-4	-4	14	-4	0	0	0	0	8	8	8	8	-4	-4	0	0	2	0	0	2	2	0	χ_{25}
χ_{26}	o	330	-6	2	45z3+60	**	**	9z3-3	-3	-3	10	-2	2	0	**	11z3+7	3z3	**	**	5z3+1	z3+4	**	-3	-z3	**	-i3	i3	-1	χ_{26}
χ_{27}	o	330	-6	2	**	45z3+60	9z3-3	**	-3	-3	10	-2	2	0	11z3+7	**	**	3z3	5z3+1	**	**	z3+4	-3	**	-z3	i3	-i3	-1	χ_{27}
χ_{28}	+	440	88	8	-10	-10	8	8	17	-1	-8	0	0	0	-2	-2	-2	-2	4	4	2	2	1	2	2	1	1	-1	χ_{28}
χ_{29}	o	440	-40	8	15i3-55	-15i3-55	6i3+8	-6i3+8	-1	-1	8	0	0	0	3i3+5	-3i3+5	-3i3+5	3i3+5	2	2	2z3	**	-1	**	2z3	-1	-1	-1	χ_{29}
χ_{30}	o	440	-40	8	-15i3-55	15i3-55	-6i3+8	6i3+8	-1	-1	8	0	0	0	-3i3+5	3i3+5	3i3+5	-3i3+5	2	2	**	2z3	-1	2z3	**	-1	-1	-1	χ_{30}
χ_{31}	o	495	-81	15	-45z3	**	9z3	**	9	0	-1	-1	-1	0	3z3	**	9z3	**	-3z3	**	3z3	**	-3	-3z3	**	0	0	0	χ_{31}
χ_{32}	o	495	-81	15	**	-45z3	**	9z3	9	0	-1	-1	-1	0	**	3z3	**	9z3	**	-3z3	**	3z3	-3	**	-3z3	0	0	0	χ_{32}
χ_{33}	o	495	63	-9	**	-45z3	**	9z3	9	0	-1	-5	-1	0	**	3z3	**	-9z3	**	-3z3	**	3z3	-3	**	3z3	0	0	0	χ_{33}
χ_{34}	o	495	63	-9	-45z3	**	9z3	**	9	0	-1	-5	-1	0	3z3	**	-9z3	**	-3z3	**	3z3	**	-3	3z3	**	0	0	0	χ_{34}
χ_{35}	+	660	84	20	30	30	-15	-15	3	3	-4	4	0	0	6	6	3	3	-3	-3	2	2	3	-1	-1	-3	-3	-1	χ_{35}
χ_{36}	o	704	64	0	-12i3-52	12i3-52	**	-4b27	2	2	0	0	0	-1	-8z3	**	-8	-8	4z3	**	0	0	-2	0	0	-2z3	**	0	χ_{36}
χ_{37}	o	704	64	0	12i3-52	-12i3-52	-4b27	**	2	2	0	0	0	-1	**	-8z3	-8	-8	**	4z3	0	0	-2	0	0	**	-2z3	0	χ_{37}
χ_{38}	o	880	48	16	**	-20b27	3i3+7	-3i3+7	-8	-5	0	0	0	0	-2i3-6	2i3-6	**	-6z3	i3-3	-i3-3	4z3	**	0	**	-2z3	i3	-i3	1	χ_{38}
χ_{39}	o	880	48	16	-20b27	**	-3i3+7	3i3+7	-8	-5	0	0	0	0	2i3-6	-2i3-6	-6z3	**	-i3-3	i3-3	**	4z3	0	-2z3	**	-i3	i3	1	χ_{39}
χ_{40}	+	891	27	-21	81	81	0	0	0	0	3	3	-1	1	9	9	0	0	0	0	-3	-3	0	0	0	0	0	0	χ_{40}
χ_{41}	o	891	27	-21	81z3	**	0	0	0	0	3	3	-1	1	9z3	**	0	0	0	0	-3z3	**	0	0	0	0	0	0	χ_{41}
χ_{42}	o	891	27	-21	**	81z3	0	0	0	0	3	3	-1	1	**	9z3	0	0	0	0	**	-3z3	0	0	0	0	0	0	χ_{42}
χ_{43}	o	990	-18	6	45z3	**	18z3	**	-9	0	-2	-6	-2	0	-3z3	**	0	0	-6z3	**	-3z3	**	3	0	0	0	0	0	χ_{43}
χ_{44}	o	990	-18	6	**	45z3	**	18z3	-9	0	-2	-6	-2	0	**	-3z3	0	0	**	-6z3	**	-3z3	3	0	0	0	0	0	χ_{44}
χ_{45}	+	1024	0	0	64	64	16	16	-8	4	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}	o	1215	-81	-9	0	0	0	0	0	0	-9	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{46}
χ_{47}	o	1215	-81	-9	0	0	0	0	0	0	-9	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{47}

$U_5(2)$

Maximal subgroups					Specifications		
Order	Index	Structure	G.2	Character	Abstract	Unitary	Quaternionic
82944	165	$2_-^{1+6}:3_+^{1+2}:2A_4$	$: 2_-^{1+6}:3_+^{1+2}:2S_4$	1a+44a+120a	N(2A)	isotropic point	root vector
77760	176	$3 \times U_4(2)$	$: (3 \times U_4(2)):2$	1a+55a+120a	N(3AB)	non-isotropic point	complex lattice
46080	297	$2^{4+4}:(3xA_5)$	$: 2^{4+4}:(3xA_5):2$	1a+120a+176a	$N(2^4) = N(2A_5B_{10})$	isotropic line	base
9720	1408	$3^4:S_5$	$: 3^4:(2xS_5)$		$N(3^4) = N(3(AB)_5(CD)_{10}E_{10}F_{15})$	base	
3888	3520	$S_3 \times 3_+^{1+2}:2A_4$	$: S_3 \times 3_+^{1+2}:2S_4$		N(3CD), N(3E)	non-isotropic line	
660	20736	$L_2(11)$	$: L_2(11):2$		N(2B,3F,5A,6N,11AB)		

G	G.2	47
47	13	

	16	54	54	27	27	11	11	144	144	72	72	36	24	24	24	24	15	15	18	18			720	48	18	18	96	96	16	5	6	8	8	12	12	
	B	C	D	C	D	A	A	BA	AA	FA	EA	IA	HC	GC	FB	EB	AA	AB	BC	AD			A	B	EC	FC	A	A	A	AC	ND	A	A	EB	EC	
	A	A	A	A	A	A	A	AA	BA	CA	DA	EA	AC	BC	CB	DB	AB	AA	AA	BA			A	A	EC	FC	A	A	A	AC	FD	A	A	EB	EC	
	8A	9A	B**	9C	D**	11A	B**	12A	B**	12C	D**	12E	12F	G**	12H	I**	15A	B**	18A	B**	fus	ind	2C	4D	60	6P	8B	C**	8D	10A	12J	16A	B**	24A	B**	
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	0	-2	-2	1	1	-1	-1	-1	-1	-1	-1	2	1	1	1	1	0	0	0	0	:	oo	0	0	0	0	2i2	-2i2	0	0	0	i2	-i2	-i2	i2	χ_2
χ_3	1	-z3	**	-z3	**	0	0	-i3	i3	z3-1	**	0	-1	-1	**	-z3	z3	**	z3	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_3
χ_4	1	**	-z3	**	-z3	0	0	i3	-i3	**	z3-1	0	-1	-1	-z3	**	**	z3	**	z3																χ_4
χ_5	0	2	2	-1	-1	0	0	-2	-2	1	1	1	0	0	1	1	-1	-1	0	0	:	++	4	0	1	-2	4	4	0	-1	0	0	0	1	1	χ_5
χ_6	-1	1	1	1	1	0	0	2	2	-1	-1	2	0	0	-1	-1	0	0	-1	-1	:	++	5	1	2	-1	-3	-3	1	0	1	-1	-1	0	0	χ_6
χ_7	1	**	z3	**	z3	0	0	z3	**	-2z3	**	1	-z3	**	0	0	0	0	**	z3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_7
χ_8	1	z3	**	z3	**	0	0	**	z3	**	-2z3	1	**	-z3	0	0	0	0	z3	**																χ_8
χ_9	0	0	0	0	0	0	0	**	z3-2	-z3	**	-1	**	-z3	z3	**	**	z3	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_9
χ_{10}	0	0	0	0	0	0	0	z3-2	**	**	-z3	-1	-z3	**	**	z3	z3	**	0	0																χ_{10}
χ_{11}	0	-1	-1	-1	-1	0	0	3	3	0	0	0	-1	-1	0	0	0	0	1	1	:	oo	0	0	0	0	2i2	-2i2	0	0	0	-i2	i2	-i2	i2	χ_{11}
χ_{12}	0	**	2z3	**	-z3	0	0	-i3	i3	z3+2	**	0	-1	-1	-z3	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	0	2z3	**	-z3	**	0	0	i3	-i3	**	z3+2	0	-1	-1	**	-z3	0	0	0	0																χ_{13}
χ_{14}	0	-z3	**	-z3	**	0	0	**	b27	1	1	1	z3	**	-1	-1	0	0	-z3	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	0	**	-z3	**	-z3	0	0	b27	**	1	1	1	**	z3	-1	-1	0	0	**	-z3																χ_{15}
χ_{16}	0	0	0	0	0	-1	-1	2	2	2	2	-1	0	0	0	0	0	0	0	0	:	++	10	2	1	1	2	2	2	0	-1	0	0	-1	-1	χ_{16}
χ_{17}	1	0	0	0	0	0	0	2	2	-1	-1	-1	0	0	-1	-1	0	0	0	0	:	++	5	-3	-1	2	1	1	1	0	0	-1	-1	1	1	χ_{17}
χ_{18}	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	:	++	4	4	-2	1	0	0	0	-1	1	0	0	0	0	χ_{18}
χ_{19}	0	**-2z3	**	z3	0	0	-2z3	**	z3	**	1	0	0	-z3	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{19}
χ_{20}	0-2z3	**	z3	**	0	0	**	-2z3	**	z3	1	0	0	**	-z3	0	0	0	0																	χ_{20}
χ_{21}	0	**	z3	**	z3	0	0	**	-2z3	**	z3	1	0	0	**	z3	0	0	**	-z3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}	0	z3	**	z3	**	0	0	-2z3	**	z3	**	1	0	0	z3	**	0	0	-z3	**																χ_{22}
χ_{23}	0	0	0	0	0	0	0	2z3	**	2z3	**	-1	0	0	0	0	-z3	**	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{23}
χ_{24}	0	0	0	0	0	0	0	**	2z3	**	2z3	-1	0	0	0	0	**	-z3	0	0																χ_{24}
χ_{25}	0	-1	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	:	oo	0	0	0	0	4i2	-4i2	0	0	0	0	0	i2	-i2	χ_{25}
χ_{26}	0	0	0	0	0	0	0	z3	**	z3	**	1	-z3	**	z3	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{26}
χ_{27}	0	0	0	0	0	0	0	**	z3	**	z3	1	**	-z3	**	z3	0	0	0	0																χ_{27}
χ_{28}	0	-1	-1	-1	-1	0	0	-2	-2	-2	-2	1	0	0	0	0	0	0	1	1	:	++	10	2	1	1	-2	-2	-2	0	-1	0	0	1	1	χ_{28}
χ_{29}	0	-z3	**	-z3	**	0	0	2z3	**	2z3	**	-1	0	0	0	0	0	0	-z3	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{29}
χ_{30}	0	**	-z3	**	-z3	0	0	**	2z3	**	2z3	-1	0	0	0	0	0	0	**	-z3																χ_{30}
χ_{31}	-1	0	0	0	0	0	0	-z3	**	-z3	**	-1	-z3	**	-z3	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}	-1	0	0	0	0	0	0	**	-z3	**	-z3	-1	**	-z3	**	-z3	0	0	0	0																χ_{32}
χ_{33}	1	0	0	0	0	0	0	**	-z3	**	-z3	-1	**	-z3	**	z3	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{33}
χ_{34}	1	0	0	0	0	0	0	-z3	**	-z3	**	-1	-z3	**	z3	**	0	0	0	0																χ_{34}
χ_{35}	0	0	0	0	0	0	0	2	2	-1	-1	-1	0	0	1	1	0	0	0	0	:	++	10	-2	1	1	2	2	-2	0	1	0	0	-1	-1	χ_{35}
χ_{36}	0	-z3	**	-z3	**	0	0	0	0	0	0	0	0	0	0	0	-z3	**	z3	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{36}
χ_{37}	0	**	-z3	**	-z3	0	0	0	0	0	0	0	0	0	0	0	**	-z3	**	z3																χ_{37}
χ_{38}	0-2z3	**	z3	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{38}
χ_{39}	0	**-2z3	**	z3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																χ_{39}
χ_{40}	-1	0	0	0	0	0	0	-3	-3	0	0	0	-1	-1	0	0	1	1	0	0	:	++	9	-3	0	0	-3	-3	1	-1	0	1	1	0	0	χ_{40}
χ_{41}	-1	0	0	0	0	0	0	-3z3	**	0	0	0	-z3	**	0	0	**	z3	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{41}
χ_{42}	-1	0	0	0	0	0	0	**	-3z3	0	0	0	**	-z3	0	0	z3	**	0	0																χ_{42}
χ_{43}	0	0	0	0	0	0	0	z3	**	-2z3	**	1	z3	**	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{43}
χ_{44}	0	0	0	0	0	0	0	**	z3	**	-2z3	1	**	z3	0	0	0	0	0	0																χ_{44}
χ_{45}	0	-2	-2	1	1	1	1	0	0	0	0	0	0	0	0	0	-1	-1	0	0	:	++	16	0	-2	-2	0	0	0	1	0	0	0	0	0	χ_{45}
χ_{46}	1	0	0	0	0	b11	**	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{46}
χ_{47}	1	0	0	0	0	**	b11	0	0	0	0	0	0	0	0	0	0	0	0	0																χ_{47}

$L_3(8)$ and $^2F_4(2)'$

Linear group $L_3(8) \cong A_2(8)$

Order = 16,482,816 = $2^9.3^2.7^2.73$ Mult = 1 Out = 6

Constructions

Linear $GL_3(8) \cong 7 \times G$: all non-singular 3 x 3 matrices over F_8;

$PGL_3(8) \cong SL_3(8) \cong PSL_3(8) \cong G$; $\Gamma L_3(8) \cong (7 \times G).2$; $P\Gamma L_3(8) \cong \Sigma L_3(8) \cong P\Sigma L_3(8) \cong G$

Maximal subgroups

Order	Index	Structure	G.2	G.3	G.6	Character	Specifications Abstract	Specifications Linear
225792	73	$2^6:(7xL_2(8))$	$2^{3+6}:7^2:2$,	$: 2^6:(7xL_2(8)):3$	$2^{3+6}:7^2:6$,	1a+72a	$N(2A^6)$	point
225792	73	$2^6:(7xL_2(8))$	$D_{14}xL_2(8)$	$: 2^6:(7xL_2(8)):3$	$(D_{14}xL_2(8)):3$	1a+72a	$N(2A^6)$	line
294	56064	$7^2:S_3$	$: 7^2:D_{12}$	$: 7^2:(3xS_3)$	$: 7^2:(6xS_3)$		$N(7^2)$	base
219	75264	73:3	: 73:6	: 73:9	: 73:18		N(73A-X)	$L_1(512)$
168	98112	$L_2(7)$	$: L_2(7):2$	$: L_2(7) \times 3$	$: L_2(7):2 \times 3$		C(3B)	$L_3(2)$

Abbreviated character table.

	G	G.2	G.3	G.6	72
	72	10	6	4	

;	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	;	;	@	@	@	@	@	;	;	@	@	@
	1	3																													
6482816		584	63	64	3528/6	49/2	49/3	63/3	56/6	63/6	63/18	73/24			504	9	16/2	7/3	9/3			168	8	3	4	7/2			6	3	4/2
p power		A	A	A	A	A	A	A	CA	FA	FB	A			A	AB	A	JB	BA			A	BA	A	BA	IB			BB	DA	AB
p' part		A	A	A	A	A	A	A	AA	AA	AA	A			A	AB	A	IB	AB			A	BA	A	BA	HB			BB	DB	BA
ind	1A	2A	3A	4A	7AF	7GH	7IK	9AC	14AF	21AF	63AR	73AX	fus	ind	2B	6A	8AB	14GI	18AC	fus	ind	3B	6B	9D	12A	21HI	fus	ind	6C	18D	24AB
+	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	:	+oo	1	1	1	1	1	:	+oo+oo	1	1	1
+	72	8	0	0	9	2	2	0	1	0	0	-1	:	++	0	0	2r2	0	0	:	+oo	6	2	0	0	-1	:	+oo+oo	0	0	-r2
o6	73	9	1	1	z7+9&3	b7	y7+1	1	z7&3	z7	z7	0	**	+3	0	0	0	0	0	*2	o2	0	0	0	0	0	*	+	0	0	0
o24	441	-7	0	1	0	0	0	0	0	0	0	x73	**	+12	0	0	0	0	0	*2	o8	0	0	0	0	0	**2	+4	0	0	0
+	511	-1	-2	-1	7	0	0	1	-1	-2	1	0	:	++	7	-2	-1	0	1	:	+oo	7	-1	1	-1	0	:	+oo+oo	1	1	-1
+3	511	-1	1	-1	7	0	0	-y9	-1	1	-y9	0	:	++3	7	1	-1	0	-y9	*	+	0	0	0	0	0	:*	++	0	0	0
o6	511	-1	-2	-1	7z7	0	0	1	-z7	-2z7	z7	0	**	+3	0	0	0	0	0	*2	o2	0	0	0	0	0	*	+	0	0	0
o18	511	-1	1	-1	7z7	0	0	-y9	-z7	z7	-y'63*2	0	**	+9	0	0	0	0	0	*2	o6	0	0	0	0	0	**2	+3	0	0	0
+	512	0	-1	0	8	1	1	-1	0	-1	-1	1	:	++	8	-1	0	1	-1	:	+oo	8	0	-1	0	1	:	+oo+oo	2	-1	0
o6	584	8	-1	0	8z7+9&3	b7	y7+1	-1	z7*3	-z7	-z7	0	**	+3	0	0	0	0	0	*2	o2	0	0	0	0	0	*	+	0	0	0
o2	657	17	0	1	9b7	b7+3	-1	0	b7	0	0	0	*	+	0	0	0	0	0	:	ooo2	3	-1	0	1	b7	:*	+oo	0	0	0
+3	657	17	0	1	9y7+9	-1	y7+2&3	0	y7+1	0	0	0	:	++3	9	0	1	y7	0	*	+	0	0	0	0	0	:*	++	0	0	0

Tits group = derived Ree group $R(2)' \cong {}^2F_4(2)'$

Order = 17,971,200 = $2^{11}.3^3.5^2.13$ Mult = 1 Out = 2

Constructions

Ree, Tits $^2F_4(2) \cong G.2$: the centralizer in $F_4(2)$ of an outer (graph) automorphism of order 2.

This leads to a description of G.2 as the automorphism group of a generalized octagon of order (2,4), which has 1755 "vertices" and 2925 "edges". Each vertex is incident with 5 edges, and each edge with 3 vertices. Tits established the existence of the simple subgroup G of index 2 ($^2F_4(2^n)$ is simple for n > 1).

Rudvalis $^2F_4(2) \cong G.2$: the point stabilizer in the permutation representation of the Rudvalis group (q.v.) on 4060 points, which form a rank 3 graph of valence 1755.

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Ree, Tits	Rudvalis
11232	1600	$L_3(3):2$	13:12	1a+351a+624ab			
11232	1600	$L_3(3):2$		1a+351a+624ab			
10240	1755	$2.[2^8]:5:4$	$: 2.[2^9]:5:4$	1a+78a+351a+650a+675a	N(2A)	vertex	adjacent point
7800	2304	$L_2(25)$	$: L_2(25)\cdot 2$	1a+27ab+351a+624ab+650a			non-adjacent point
6144	2925	$2^2.[2^8]:S_3$	$: 2^2.[2^9]:S_3$	1a+351a+624ab+650a+675a	$N(2A^2)$	edge	
1440	12480	$A_6\cdot 2^2$			N(2B,3A,3A,4C,5A)	$B_2(2)$	
1440	12480	$A_6\cdot 2^2$			N(2B,3A,3A,4C,5A)	$B_2(2)$	
1200	14976	$5^2:4A_4$	$: 5^2:4S_4$		$N(5A^2)$		

$^2F_4(2)'$ and A_{11}

G	G.2	22
22	12	

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	
			10	1																																		
	17971200		240	536	108	192	128	64	50	12	32	32	16	16	10	12	12	13	13	16	16	16	16			640	640	96	64	16	8	6	6	8	8	10	10	
	p power		A	A	A	B	A	B	A	AB	B	B	C	C	AA	AA	AA	A	A	A	B	A	B			A	A	B	A	B	A	AF	AF	A	B	AD	AE	
	p' part		A	A	A	A	A	A	A	AB	A	A	A	A	AA	AA	AA	A	A	A	A	A	A			A	A	A	A	A	A	AF	AF	A	A	AD	AE	
	ind	1A	2A	2B	3A	4A	4B	4C	5A	6A	8A	B**	8C	8D	10A	12A	B*	13A	B*	16A	B**	C*5D	**5	fus	ind	4D	E**	4F	4G	8E	8F	12C	D**	16E	F**	20A	B**	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	o	26	-6	2	-1	-2	-2	2	1	-1	0	0	0	0	-1	1	1	0	0	i2	-i2	-i2	i2		-	0	0	0	0	0	0	0	0	0	0	0	0	χ_2
χ_3	o	26	-6	2	-1	-2	-2	2	1	-1	0	0	0	0	-1	1	1	0	0	-i2	i2	i2	-i2															χ_3
χ_4	o	27	-5	3	0	3	-1	-1	2	0	2i-1	-2i-1	-1	-1	0	0	0	1	1	-i	i	-i	i	:	oo	4i+1	-4i+1	-3	1	-1	-1	0	0	i	-i	-i+1	i+1	χ_4
χ_5	o	27	-5	3	0	3	-1	-1	2	0	-2i-1	2i-1	-1	-1	0	0	0	1	1	i	-i	i	-i	:	oo	-4i+1	4i+1	-3	1	-1	-1	0	0	-i	i	i+1	-i+1	χ_5
χ_6	+	78	14	-2	-3	2	-2	2	3	1	2	2	0	0	-1	-1	-1	0	0	0	0	0	0	:	++	6	6	2	-2	2	0	-1	-1	0	0	1	1	χ_6
χ_7	+	300	-20	-4	3	-4	4	4	0	-1	0	0	0	0	0	-1	-1	1	1	0	0	0	0	:	oo	0	0	0	0	0	0	i3	-i3	0	0	0	0	χ_7
χ_8	+	325	5	-11	1	1	5	1	0	1	1	1	-1	-1	0	1	1	0	0	-1	-1	-1	-1	:	++	5	5	1	-3	1	-1	1	1	-1	-1	0	0	χ_8
χ_9	+	351	31	15	0	3	-1	3	1	0	-1	-1	1	1	1	0	0	0	0	-1	-1	-1	-1	:	++	9	9	-3	1	1	-1	0	0	1	1	-1	-1	χ_9
χ_{10}	o	351	-1	-9	0	3	3	-1	1	0	2i-1	-2i-1	1	1	-1	0	0	0	0	i	-i	i	-i	:	oo	4i+5	-4i+5	-3	-3	-1	1	0	0	-i	i	-i	i	χ_{10}
χ_{11}	o	351	-1	-9	0	3	3	-1	1	0	-2i-1	2i-1	1	1	-1	0	0	0	0	-i	i	-i	i	:	oo	-4i+5	4i+5	-3	-3	-1	1	0	0	i	-i	i	-i	χ_{11}
χ_{12}	+	624	-16	16	3	0	0	0	-1	1	0	0	0	0	-1	r3	-r3	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	+	624	-16	16	3	0	0	0	-1	1	0	0	0	0	-1	-r3	r3	0	0	0	0	0	0															χ_{13}
χ_{14}	+	650	10	10	2	6	2	-2	0	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	:	++	10	10	6	2	-2	0	0	0	0	0	0	0	χ_{14}
χ_{15}	+	675	35	3	0	3	3	3	0	0	-1	-1	-1	-1	0	0	0	-1	-1	1	1	1	1	:	++	5	5	-3	5	1	1	0	0	-1	-1	0	0	χ_{15}
χ_{16}	+	702	30	6	0	-6	2	-2	2	0	0	0	0	0	0	0	0	0	0	r2	r2	-r2	-r2		+	0	0	0	0	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	+	702	30	6	0	-6	2	-2	2	0	0	0	0	0	0	0	0	0	0	-r2	-r2	r2	r2															χ_{17}
χ_{18}	+	1300	20	-12	4	0	-4	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	χ_{18}
χ_{19}	+	1300	20	-12	4	0	-4	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0															χ_{19}
χ_{20}	+	1728	-64	0	0	0	0	0	3	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	:	oo	16i	-16i	0	0	0	0	0	0	0	0	i	-i	χ_{20}
χ_{21}	+	2048	0	0	-4	0	0	0	-2	0	0	0	0	0	0	0	0	0-b13	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}	+	2048	0	0	-4	0	0	0	-2	0	0	0	0	0	0	0	0	*-b13		0	0	0	0															χ_{22}

Alternating group A_{11}

Order = 19,958,400 = $2^7.3^4.5^2.7.11$ Mult = 2 Out = 2

Constructions

Alternating $S_{11} \cong G.2$: all permutations on 11 letters;

$A_{11} \cong G$: all even permutations; 2.G and 2.G.2 : the Schur double covers

Presentations $G \cong \langle x_1,\ldots x_9 | x_i^3=(x_ix_j)^2=1\rangle$; $G.2 \cong$ .....

Remark : 2.G is the involution centralizer in the Lyons group.

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Alternating
1814400	11	A_{10}	: S_{10}	1a+10a		point
362880	55	S_9	: S_9 x 2	1a+10a+44a	C(2C)	duad
120960	165	$(A_8x3):2$	: S_8 x S_3	1a+10a+44a+110a	N(3A)	triad
60480	330	$(A_7xA_4):2$	: S_7 x S_4	1a+10a+44a+110a+165a	N($2A^2$)	tetrad
43200	462	$(A_6xA_5):2$	: S_6 x S_5	1a+10a+44a+110a+132a+165a	N(2A,3A,5A)	pentad
7920	2520	M_{11}	11:10	1a+132a+462a+825a+1100a	N(2B,3C,4B,5B,8A,11AB)	S(4,5,11)
7920	2520	M_{11}		1a+132a+462a+825a+1100a	N(2B,3C,4B,5B,8A,11AB)	S(4,5,11)

G	G.2	31
2.G	2.G.2	18
31	27	

A_{11}

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		19958400	20160	1152	60480	1080	162	480	96	96	1800	25	576	288	72	36	18	84	8	9	40	11	11	48	24	12	28	45	45	20	21	21
	p power		A	A	A	A	A	A	B	A	A	A	AB	AA	BA	BB	CB	A	B	C	AA	A	A	AB	BA	CC	AA	AA	AB	AA	AA	AA
	p' part		A	A	A	A	A	A	A	A	A	A	AB	AA	BA	BB	CB	A	A	A	AA	A	A	AB	AA	BC	AA	AA	AB	AA	AA	AA
	ind	1A	2A	2B	3A	3B	3C	4A	4B	4C	5A	5B	6A	6B	6C	6D	6E	7A	8A	9A	10A	11A	B**	12A	12B	12C	14A	15A	15B	20A	21A	B*
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	+	10	6	2	7	4	1	4	2	0	5	0	-1	3	0	2	-1	3	0	1	1	-1	-1	-1	1	0	-1	2	-1	-1	0	0
χ_3	+	44	16	4	20	5	-1	6	0	2	9	-1	4	4	1	1	1	2	0	-1	1	0	0	0	0	-1	2	0	0	1	-1	-1
χ_4	+	45	13	-3	21	6	0	5	1	-3	10	0	-3	1	-2	0	0	3	-1	0	-2	1	1	1	-1	0	-1	1	1	0	0	0
χ_5	+	110	26	6	29	2	2	4	-2	0	5	0	-3	5	2	0	0	-2	0	-1	1	0	0	1	1	0	-2	-1	2	-1	1	1
χ_6	+	120	8	-8	36	6	3	0	0	0	10	0	4	-4	2	-2	1	1	0	0	-2	-1	-1	0	0	0	1	1	1	0	1	1
χ_7	o	126	-14	6	21	6	0	-4	-2	0	1	1	-3	1	-2	0	0	0	0	0	1	b11	**	1	-1	0	0	1	1	1	0	0
χ_8	o	126	-14	6	21	6	0	-4	-2	0	1	1	-3	1	-2	0	0	0	0	0	1	**	b11	1	-1	0	0	1	1	1	0	0
χ_9	+	132	20	4	6	6	-3	0	4	0	-8	2	-2	2	2	-2	1	-1	0	0	0	0	0	-2	0	0	-1	1	1	0	-1	-1
χ_{10}	+	165	29	5	21	3	3	1	1	1	-5	0	5	5	-1	-1	-1	-3	-1	0	-1	0	0	1	1	1	1	1	-2	1	0	0
χ_{11}	+	210	-14	2	42	9	3	-6	-2	2	5	0	2	-2	1	-1	-1	0	0	0	1	1	1	-2	0	-1	0	2	-1	-1	0	0
χ_{12}	+	231	35	-1	63	3	-3	5	-1	1	16	1	-1	-1	-1	-1	-1	0	1	0	0	0	0	-1	-1	1	0	-2	-2	0	0	0
χ_{13}	+	330	22	2	-6	9	-3	0	2	-4	-10	0	2	-2	1	-1	-1	1	0	0	2	0	0	2	0	-1	1	-1	-1	0	1	1
χ_{14}	+	385	21	9	49	-8	-2	-1	1	3	5	0	9	-3	0	0	0	0	-1	1	1	0	0	1	-1	0	0	-1	2	-1	0	0
χ_{15}	+	462	14	-2	-42	3	3	-6	2	2	7	2	-2	2	-1	1	1	0	0	0	-1	0	0	2	0	-1	0	-2	-2	-1	0	0
χ_{16}	+	550	50	-2	70	-5	1	0	-2	-4	0	0	-2	2	-1	1	1	-3	0	1	0	0	0	-2	0	-1	1	0	0	0	0	0
χ_{17}	+	594	6	-6	90	0	0	-4	2	0	9	-1	-6	-6	0	0	0	-1	0	0	1	0	0	2	2	0	-1	0	0	1	-1	-1
χ_{18}	+	594	6	-6	-45	0	0	-4	2	0	9	-1	3	3	0	0	0	-1	0	0	1	0	0	-1	-1	0	-1	0	0	1	-b21	*
χ_{19}	+	594	6	-6	-45	0	0	-4	2	0	9	-1	3	3	0	0	0	-1	0	0	1	0	0	-1	-1	0	-1	0	0	1	*	-b21
χ_{20}	+	660	16	-4	-36	-3	3	-6	0	-2	5	0	-4	4	1	-1	-1	2	0	0	1	0	0	0	0	1	2	-1	2	-1	-1	-1
χ_{21}	+	693	49	-3	21	6	0	-1	1	3	-17	-2	-3	1	-2	0	0	0	1	0	-1	0	0	1	-1	0	0	1	1	-1	0	0
χ_{22}	+	825	-15	9	-15	0	6	5	1	-3	0	0	9	-3	0	0	0	-1	1	0	0	0	0	1	-1	0	-1	0	0	0	-1	-1
χ_{23}	+	924	-56	4	84	9	-3	-6	0	-2	-1	-1	4	4	1	1	1	0	0	0	-1	0	0	0	0	1	0	-1	-1	-1	0	0
χ_{24}	+	990	34	6	21	-12	0	-4	-2	0	-5	0	-3	1	4	0	0	3	0	0	-1	0	0	1	-1	0	-1	1	-2	1	0	0
χ_{25}	+	990	34	6	-42	6	0	-4	-2	0	-5	0	6	-2	-2	0	0	3	0	0	-1	0	0	-2	2	0	-1	-2	1	1	0	0
χ_{26}	+	1100	-20	12	50	-10	2	0	4	0	0	0	-6	-2	-2	0	0	1	0	-1	0	0	0	-2	0	0	1	0	0	0	1	1
χ_{27}	+	1155	7	-5	-21	9	3	5	-1	1	-10	0	-5	-5	1	1	1	0	-1	0	2	0	0	-1	-1	1	0	-1	-1	0	0	0
χ_{28}	+	1232	0	-16	56	-4	-1	0	0	0	-8	2	8	0	0	2	-1	0	0	-1	0	0	0	0	0	0	0	1	1	0	0	0
χ_{29}	+	1320	8	8	-84	6	-3	0	0	0	10	0	-4	-4	2	2	-1	-3	0	0	-2	0	0	0	0	0	1	1	1	0	0	0
χ_{30}	+	1540	-56	-4	28	1	1	6	0	2	-5	0	-4	4	1	-1	-1	0	0	1	-1	0	0	0	0	-1	0	-2	1	1	0	0
χ_{31}	+	2310	-14	-2	-63	-6	-3	4	-2	0	5	0	1	1	-2	-2	1	0	0	0	1	0	0	1	1	0	0	2	-1	-1	0	0
	ind	1 2	4	2	3 6	3 6	3 6	8	4	8	5 10	5 10	6	12	12	6	6 6	7 14	8 8	9 18	20	11 22	11 22	12	24	24 24	28	15 30	15 30	40 40	21 42	21 42
χ_{32}	o	16	0	0	-8	4	-2	0	0	0	-4	1	0	0	0	0	0	2	0	1	0	b11	**	0	0	0	0	2	-1	0	-1	-1
χ_{33}	o	16	0	0	-8	4	-2	0	0	0	-4	1	0	0	0	0	0	2	0	1	0	**	b11	0	0	0	0	2	-1	0	-1	-1
χ_{34}	+	144	0	0	-48	12	0	0	0	0	-16	-1	0	0	0	0	0	4	0	0	0	1	1	0	0	0	0	2	2	0	1	1
χ_{35}	+	528	0	0	48	0	6	0	0	0	-12	-2	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	3	0	0	-1	-1
χ_{36}	+	560	0	0	-112	8	2	0	0	0	-20	0	0	0	0	0	0	0	0	-1	0	-1	-1	0	0	0	0	-2	-2	0	0	0
χ_{37}	+	616	0	0	-56	-8	4	0	0	0	-4	1	0	0	0	0	0	0	2	1	0	0	0	0	0	0	0	-1	2	0	0	0
χ_{38}	+	616	0	0	-56	-8	4	0	0	0	-4	1	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	-1	2	0	0	0
χ_{39}	+	672	0	0	0	12	6	0	0	0	12	2	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	-3	0	0	0
χ_{40}	+	880	0	0	64	4	-2	0	0	0	-10	0	0	0	0	0	0	-2	0	1	0	0	0	0	0	0	0	-1	-1	r10	1	1
χ_{41}	+	880	0	0	64	4	-2	0	0	0	-10	0	0	0	0	0	0	-2	0	1	0	0	0	0	0	0	0	-1	-1	-r10	1	1
χ_{42}	+	1200	0	0	-120	0	-6	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	1	1	0	0	0	0	0	0	0	-1	-1
χ_{43}	+	1232	0	0	56	-4	-1	0	0	0	-8	2	0	0	0	0	3	0	0	-1	0	0	0	0	0	0	0	1	1	0	0	0
χ_{44}	+	1232	0	0	56	-4	-1	0	0	0	-8	2	0	0	0	0	-3	0	0	-1	0	0	0	0	0	0	0	1	1	0	0	0
χ_{45}	+	1440	0	0	-48	12	0	0	0	0	20	0	0	0	0	0	0	-2	0	0	0	-1	-1	0	0	0	0	2	2	0	1	1
χ_{46}	+	1584	0	0	-24	-12	0	0	0	0	4	-1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	1	-2	0	-b21	*
χ_{47}	+	1584	0	0	-24	-12	0	0	0	0	4	-1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	1	-2	0	*	-b21
χ_{48}	+	1584	0	0	48	6	0	0	0	0	4	-1	0	0	0	0	0	2	0	0	0	0	0	0	0	r6	0	-2	1	0	-1	-1
χ_{49}	+	1584	0	0	48	6	0	0	0	0	4	-1	0	0	0	0	0	2	0	0	0	0	0	0	0	-r6	0	-2	1	0	-1	-1

	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
			362880	2880	1920	10080	96	32	2160	360	162	144	108	24	18	24	120	120	5	144	48	36	14	9	20	12	14	15	15
			A	A	A	A	A	B	AC	BD	CC	AD	BC	BE	CD	B	AC	AD	BE	BD	BE	CD	AC	AH	AD	AB	AD	ABF	BCG
			A	A	A	A	A	A	AC	BD	CC	AD	BC	BE	CD	A	AC	AD	BE	AD	AE	BD	AC	AC	AD	AB	AD	ABF	BCG
	fus	ind	2C	2D	2E	4D	4E	4F	6F	6G	6H	6I	6J	6K	6L	8B	10B	10C	10D	12D	12E	12F	14B	18A	20B	24A	28A	30A	30B
χ_1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	:	++	8	4	0	6	2	0	5	4	-1	1	2	0	1	2	3	-1	0	3	-1	0	1	-1	1	-1	-1	0	-1
χ_3	:	++	28	8	4	14	2	0	10	5	1	2	1	1	-1	0	3	3	-1	2	2	-1	0	1	-1	0	0	0	0
χ_4	:	++	27	3	-5	15	-1	-1	9	6	0	-3	0	-2	0	1	2	-2	0	3	-1	0	-1	0	0	1	1	-1	1
χ_5	:	++	56	12	0	14	2	0	11	0	2	3	2	0	0	-2	1	-3	0	-1	-1	2	0	-1	-1	1	0	1	0
χ_6	:	++	48	-8	0	20	-4	0	6	4	3	-2	0	0	1	0	-2	2	0	2	2	2	-1	0	0	0	-1	1	-1
χ_7	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	|																												
χ_9	:	++	48	8	0	-8	0	0	6	-4	3	2	0	0	-1	0	-2	-2	0	4	0	-2	-1	0	2	0	-1	1	1
χ_{10}	:	++	69	13	5	1	1	1	9	-5	-3	1	3	-1	1	-1	-1	3	0	1	1	1	-1	0	1	-1	1	-1	0
χ_{11}	:	++	42	-14	10	14	-2	-2	0	1	-3	4	3	1	1	0	-3	1	0	2	-2	-1	0	0	-1	0	0	0	1
χ_{12}	:	++	105	5	1	35	-1	1	15	5	-3	-1	-3	1	-1	-1	0	0	1	-1	-1	-1	0	0	0	-1	0	0	0
χ_{13}	:	++	78	2	-10	-20	0	-2	6	-1	-3	2	-3	-1	-1	0	-2	2	0	4	0	1	1	0	0	0	1	1	-1
χ_{14}	:	++	119	3	15	21	1	-1	-1	0	2	3	-4	0	0	1	-1	3	0	-3	1	0	0	-1	1	1	0	-1	0
χ_{15}	:	++	42	2	10	-14	2	-2	0	5	-3	-4	3	1	-1	0	-3	-3	0	-2	2	1	0	0	1	0	0	0	0
χ_{16}	:	++	190	10	-10	20	0	-2	10	-5	1	-2	1	-1	1	0	0	0	0	-4	0	-1	1	1	0	0	-1	0	0
χ_{17}	:	++	162	-18	-6	36	0	2	0	0	0	0	0	0	0	0	-3	-3	-1	0	0	0	1	0	1	0	1	0	0
χ_{18}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{19}	|																												
χ_{20}	:	++	84	8	-20	-14	-2	0	-6	5	3	2	3	1	-1	0	-1	3	0	-2	-2	1	0	0	1	0	0	-1	0
χ_{21}	:	++	189	9	5	-21	-1	1	9	-6	0	-3	0	2	0	1	-1	-1	0	3	-1	0	0	0	-1	1	0	-1	-1
χ_{22}	:	++	15	15	15	15	-1	-1	-15	0	6	-3	0	0	0	-1	0	0	0	3	-1	0	1	0	0	-1	1	0	0
χ_{23}	:	++	84	-16	-4	14	2	0	-6	-1	3	2	3	-1	-1	0	-1	-1	1	2	2	-1	0	0	-1	0	0	-1	-1
χ_{24}	:	++	216	12	0	-6	-2	0	-9	0	0	3	0	0	0	2	1	-3	0	-3	1	0	-1	0	-1	-1	1	1	0
χ_{25}	:	++	162	6	10	-36	0	2	0	6	0	0	0	-2	0	0	-3	1	0	0	0	0	1	0	-1	0	-1	0	1
χ_{26}	:	++	160	0	0	20	4	0	-20	0	-2	0	-2	0	0	0	0	0	0	2	-2	2	-1	1	0	0	-1	0	0
χ_{27}	:	++	147	-17	-5	-35	1	-1	9	1	3	1	-3	1	1	-1	2	-2	0	1	1	1	0	0	0	-1	0	-1	1
χ_{28}	:	++	224	-16	0	0	0	0	-4	-4	-1	-4	2	0	-1	0	4	4	0	0	0	0	0	-1	0	0	0	1	1
χ_{29}	:	++	48	-8	0	-20	4	0	6	4	3	-2	0	0	1	0	-2	2	0	-2	-2	-2	-1	0	0	0	1	1	-1
χ_{30}	:	++	28	-16	20	-14	-2	0	10	-1	1	2	1	-1	-1	0	3	-1	0	-2	-2	1	0	1	1	0	0	0	-1
χ_{31}	:	++	168	4	0	-14	-2	0	-15	4	-3	1	0	0	1	-2	3	-1	0	1	1	-2	0	0	1	1	0	0	-1
	fus	ind	2	4	2	8	8	4	6	12	6	12	6	6	12	8	10	20	10 10	24	24	24	14	18 18	40	24 24	56 56	30 30	60 60
χ_{32}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	|																												
χ_{34}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r5	0	0	0	0	0	0	0	0	0	0
χ_{35}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i15	0
χ_{36}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0
χ_{37}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	|																												
χ_{39}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r15
χ_{40}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	|																												
χ_{42}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r3	0	0	0
χ_{43}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	|																												
χ_{45}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r14	0	0
χ_{46}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	|																												
χ_{48}	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	|																												

Sz(32)

Suzuki group $Sz(32) \cong {}^2B_2(32)$

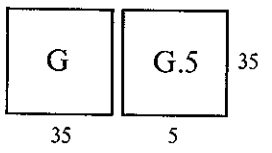
Order = 32,537,600 = $2^{10}.5^2.31.41$ Mult = 1 Out = 5

Constructions

Suzuki G : the centralizer in $S_4(32)$ of an outer (graph) automorphism of order 2

G : all 4 x 4 matrices over $\mathbf{F}_{32}$ preserving the set of vectors (t,x,y,z) for which $xy+(x^{s+2}+y^s+z)t=0$, where $s : x \rightarrow x^8$ is the automorphism of $\mathbf{F}_{32}$ with $s^2=2$; projectively this defines an oval of $32^2+1=1025$ points on which G acts doubly transitively.

Maximal subgroups					Specifications	
Order	Index	Structure	G.5	Character	Abstract	Suzuki
31744	1025	$2^{5+5}:31$	$: 2^{5+5}:31:5$	1a+1024a	$N(2A^5)$	point
164	198400	41:4	: 41:20		N(41A-J)	
100	325376	25:4	: 25:20		N(5A)	
62	524800	D_{62}	: 31:10		N(31A-O)	$L_1(32)$, point pair



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	
			1																																											
		32537600	024	64	64	25	25	25	25	25	25	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	41	41	41	41	41	41	41	41	41	41			20	4	4	4	5	
		p power	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A			A	BA	AA	AB	A	
		p' part	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A			A	BA	AA	AB	A	
	ind	1A	2A	4A	B**	5A	25A	B*6	C*11	D*16	E*21	31A	B*2	C*4	D*8	E*16	F*5	G*10	H*20	I*9	J*18	K*25	L*19	M*7	N*14	O*28	41A	B*2	C*4	D*8	E*16	F*3	G*6	H*12	I*24	J*7	fus	ind	5B	10A	20AB	*11	25F	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	: +0000	1	1	1	1	1	χ_1
χ_2	o	124	-4	4i	-4i	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	: 00000	-1	1	-i	i	-1	χ_2
χ_3	o	124	-4	-4i	4i	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	: 00000	-1	1	i	-i	-1	χ_3
χ_4	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-w41	*2	*4	*8	*16	*3	*6	*12	*24	*7	+	0	0	0	0	0	χ_4
χ_5	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*16-w41	*2	*4	*8	*7	*3	*6	*12	*24								χ_5
χ_6	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*8	*16-w41	*2	*4	*24	*7	*3	*6	*12								χ_6
χ_7	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*4	*8	*16-w41	*2	*12	*24	*7	*3	*6								χ_7
χ_8	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*2	*4	*8	*16-w41	*6	*12	*24	*7	*3								χ_8
χ_9	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*3	*6	*12	*24	*7-w41	*2	*4	*8	*16		+	0	0	0	0	0	χ_9
χ_{10}	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*7	*3	*6	*12	*24	*16-w41	*2	*4	*8								χ_{10}
χ_{11}	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*24	*7	*3	*6	*12	*8	*16-w41	*2	*4								χ_{11}
χ_{12}	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*12	*24	*7	*3	*6	*4	*8	*16-w41	*2								χ_{12}
χ_{13}	+	775	7	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*6	*12	*24	*7	*3	*2	*4	*8	*16-w41								χ_{13}
χ_{14}	+	1024	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	: +0000	4	0	0	0	-1	χ_{14}
χ_{15}	+	1025	1	1	1	0	0	0	0	0	0	y31	*2	*4	*8	*16	*5	*10	*20	*9	*18	*25	*19	*7	*14	*28	0	0	0	0	0	0	0	0	0	0	+	0	0	0	0	0	χ_{15}
χ_{16}	+	1025	1	1	1	0	0	0	0	0	0	*16	y31	*2	*4	*8	*18	*5	*10	*20	*9	*28	*25	*19	*7	*14	0	0	0	0	0	0	0	0	0	0							χ_{16}
χ_{17}	+	1025	1	1	1	0	0	0	0	0	0	*8	*16	y31	*2	*4	*9	*18	*5	*10	*20	*14	*28	*25	*19	*7	0	0	0	0	0	0	0	0	0	0							χ_{17}
χ_{18}	+	1025	1	1	1	0	0	0	0	0	0	*4	*8	*16	y31	*2	*20	*9	*18	*5	*10	*7	*14	*28	*25	*19	0	0	0	0	0	0	0	0	0	0							χ_{18}
χ_{19}	+	1025	1	1	1	0	0	0	0	0	0	*2	*4	*8	*16	y31	*10	*20	*9	*18	*5	*19	*7	*14	*28	*25	0	0	0	0	0	0	0	0	0	0							χ_{19}
χ_{20}	+	1025	1	1	1	0	0	0	0	0	0	*25	*19	*7	*14	*28	y31	*2	*4	*8	*16	*5	*10	*20	*9	*18	0	0	0	0	0	0	0	0	0	0	+	0	0	0	0	0	χ_{20}
χ_{21}	+	1025	1	1	1	0	0	0	0	0	0	*28	*25	*19	*7	*14	*16	y31	*2	*4	*8	*18	*5	*10	*20	*9	0	0	0	0	0	0	0	0	0	0							χ_{21}
χ_{22}	+	1025	1	1	1	0	0	0	0	0	0	*14	*28	*25	*19	*7	*8	*16	y31	*2	*4	*9	*18	*5	*10	*20	0	0	0	0	0	0	0	0	0	0							χ_{22}
χ_{23}	+	1025	1	1	1	0	0	0	0	0	0	*7	*14	*28	*25	*19	*4	*8	*16	y31	*2	*20	*9	*18	*5	*10	0	0	0	0	0	0	0	0	0	0							χ_{23}
χ_{24}	+	1025	1	1	1	0	0	0	0	0	0	*19	*7	*14	*28	*25	*2	*4	*8	*16	y31	*10	*20	*9	*18	*5	0	0	0	0	0	0	0	0	0	0							χ_{24}
χ_{25}	+	1025	1	1	1	0	0	0	0	0	0	*5	*10	*20	*9	*18	*25	*19	*7	*14	*28	y31	*2	*4	*8	*16	0	0	0	0	0	0	0	0	0	0	+	0	0	0	0	0	χ_{25}
χ_{26}	+	1025	1	1	1	0	0	0	0	0	0	*18	*5	*10	*20	*9	*28	*25	*19	*7	*14	*16	y31	*2	*4	*8	0	0	0	0	0	0	0	0	0	0							χ_{26}
χ_{27}	+	1025	1	1	1	0	0	0	0	0	0	*9	*18	*5	*10	*20	*14	*28	*25	*19	*7	*8	*16	y31	*2	*4	0	0	0	0	0	0	0	0	0	0							χ_{27}
χ_{28}	+	1025	1	1	1	0	0	0	0	0	0	*20	*9	*18	*5	*10	*7	*14	*28	*25	*19	*4	*8	*16	y31	*2	0	0	0	0	0	0	0	0	0	0							χ_{28}
χ_{29}	+	1025	1	1	1	0	0	0	0	0	0	*10	*20	*9	*18	*5	*19	*7	*14	*28	*25	*2	*4	*8	*16	y31	0	0	0	0	0	0	0	0	0	0							χ_{29}
χ_{30}	+	1271	-9	-1	-1	-4	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: +0000	1	1	-1	-1	1	χ_{30}
χ_{31}	+	1271	-9	-1	-1	1	-w25	*6	*11	*16	*21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+	0	0	0	0	0	χ_{31}
χ_{32}	+	1271	-9	-1	-1	1	*21	-w25	*6	*11	*16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0							χ_{32}
χ_{33}	+	1271	-9	-1	-1	1	*16	*21	-w25	*6	*11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0							χ_{33}
χ_{34}	+	1271	-9	-1	-1	1	*11	*16	*21	-w25	*6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0							χ_{34}
χ_{35}	+	1271	-9	-1	-1	1	*6	*11	*16	*21	-w25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0							χ_{35}

$L_3(9)$

Linear group $L_3(9) \cong A_2(9)$

Order = $42,456,960 = 2^7.3^6.5.7.13$ Mult = 1 Out = 2^2

Constructions

Linear $GL_3(9) \cong 8 \times G$: all non-singular 3×3 matrices over $\mathbf{F}_9$;
$PGL_3(9) \cong SL_3(9) \cong PSL_3(9) \cong G$; $\Gamma L_3(9) \cong (8 \times G).2_2$; $P\Gamma L_3(9) \cong \Sigma L_3(9) \cong P\Sigma L_3(9) \cong G.2_2$

Maximal subgroups

Order	Index	Structure	$G.2_1$	$G.2_2$	$G.2_3$	$G.2^2$	Character	Abstract	Linear
466560	91	$3^4:GL_2(9)$	$3^{2+4}:8^2.2$	: $3^4:\Gamma L_2(9)$	$3^{2+4}:8^2.2$	$3^{2+4}:8^2.2^2$	1a+90a	$N(3A^4)$	point
466560	91	$3^4:GL_2(9)$	$GL_2(9).2$	: $3^4:\Gamma L_2(9)$	$\Gamma L_2(9)$	$\Gamma L_2(9).2$	1a+90a	$N(3A^4)$	line
6048	7020	$U_3(3)$	: $U_3(3):2$	: $U_3(3):2$	: $U_3(3) \times 2$	: $U_3(3):2 \times 2$		$C(2D)$	$U_3(3)$
5616	7560	$L_3(3)$	: $L_3(3):2$	: $L_3(3) \times 2$	: $L_3(3):2$	: $L_3(3):2 \times 2$		$C(2C)$	$L_3(3)$
720	58968	$PGL_2(9)$	: $PGL_2(9) \times 2$	: $A_6{\cdot}2^2$	: $A_6{\cdot}2^2$	: $A_6{\cdot}2^2 \times 2$		$C(2B)$	$O_3(9)$
384	110565	$8^2:S_3$	: $8^2:D_{12}$	: $H.2$	: $H.2$	: $H.2^2$		$N(2A^2)$	base
273	155520	$91:3$	: $91:6$	: $(13 \times D_{14}):3$	: $(7 \times D_{26}):3$	: $(D_{14} \times D_{26}):3$		$N(7AB)$, $N(13ABCD)$	$L_1(729)$

G	$G.2_1$	$G.2_2$	$G.2_3$	90
90	12	12	14	

Abbreviated character table.

;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
	4	5	5																			
	2456960	760	832	81	5760/2	64	80/2	72	91/2	5760/4	64/2	64/2	64/2	80/2	72/2	91/4	80/4	80/4	72/4	80/8	80/16	91/24
p power		A	A	A	A	A	A	AA	A	A	A	C	C	BA	AB	A	A	BA	AC	DD	HD	CB
p' part		A	A	A	A	A	A	AA	A	A	A	A	A	AA	AA	A	A	AA	AA	AA	AA	AA
ind	1A	2A	3A	3B	4AB	4C	5AB	6A	7AB	8AD	8EF	8GH	8IJ	10AB	12AB	13AD	16AD	20AD	24AD	40AH	80AP	91AX
+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
+	90	10	9	0	10	2	0	1	-1	10	2	2	2	0	1	-1	0	0	1	0	0	-1
+	91	11	10	1	11	3	1	2	0	-9	-1	-1	-1	1	2	0	-1	1	0	1	-1	0
o2	91	11	10	1	-9	-1	1	2	0	10i-1	2i-1	1	1	1	0	0	-i	1	i-1	-1	-i	0
o4	91	-9	10	1	10i-1	1	1	0	0	10z8-i	i	r2+1	i2-1	1	i-1	0	z8*3	-1	z8-i	-i	z8*3	0
o2	640	0	-8	1	0	0	0	0	b7	0	0	0	0	0	0	3	0	0	0	0	0	b7
o4	640	0	-8	1	0	0	0	0	3	0	0	0	0	0	0	x13	0	0	0	0	0	x13
o24	640	0	-8	1	0	0	0	0	b7	0	0	0	0	0	0	x13	0	0	0	0	0	x91**2
+2	728	8	-1	-1	8	0	-b5	-1	0	8	0	0	0	-b5	-1	0	2	-b5	-1	-b5	b5	0
+2	728	8	-1	-1	8	0	-b5	-1	0	8	0	0	0	-b5	-1	0	-2	-b5	-1	-b5	-b5	0
o4	728	-8	-1	-1	8i	0	-2	1	0	8z8	0	0	0	2	-i	0	0	-2i	-z8	-2z8	0	0
o4	728	8	-1	-1	8	0	-b5	-1	0	-8	0	0	0	-b5	-1	0	2i	-b5	1	b5	m20	0
o8	728	8	-1	-1	-8	0	-b5	-1	0	8i	0	0	0	-b5	1	0	2z8	b5	-i	-m20	y'40**3	0
o16	728	-8	-1	-1	8i	0	-b5	1	0	-8z8	0	0	0	b5	-i	0	0	-m20	z8	y'40**3	y'80	0
+	729	9	0	0	9	1	-1	0	1	9	1	1	1	-1	0	1	-1	-1	0	-1	-1	1
+	819	19	9	0	19	3	-1	1	0	-1	-1	-1	-1	-1	1	0	1	-1	-1	-1	1	0
o2	819	19	9	0	-1	-1	-1	1	0	10i-9	2i-1	1	1	-1	-1	0	i	-1	i	1	i	0
o4	819	-1	9	0	10i-9	1	-1	-1	0	10z8-9i	i	r2+1	i2-1	-1	i	0	z8**	1	z8	i	z8**	0
+	910	30	19	1	-10	-2	0	3	0	10	2	-2	-2	0	-1	0	0	0	1	0	0	0
+2	910	-10	19	1	10	-2	0	-1	0	10r2+10	-2	-2r2	0	0	1	0	0	0	r2+1	0	0	0
o2	910	-10	19	1	20i-10	2	0	-1	0	10i	-2i	-2	2	0	2i-1	0	0	0	i	0	0	0
o2	910	-10	19	1	10	-2	0	-1	0	10i2-10	2	0	-2i2	0	1	0	0	0	i2-1	0	0	0

ind	;	;	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@
														5										6									
			720	720	9	8	10/2	18	18	8/2	10/2			616	48	54	9	8	6	8/2	13/4			048	96	108	9	96/2	16	12	7/2	8/2	12/2
p power			A	A	BB	C	BB	AD	AD	G	BD			A	A	AC	BC	C	AE	I	DC			A	A	AD	BD	A	C	AF	AD	E	AN
p' part			A	A	BB	A	AB	AD	AD	A	AD			A	A	AC	BC	A	AE	A	AC			A	A	AD	BD	A	A	AF	AD	A	AM
ind	fus	ind	2B	4D	6B	8K	10CD	12C	12D	16EF	20EF	fus	ind	2C	4E	6C	6D	8L	12E	16GH	26AD	fus	ind	2D	4F	6E	6F	8MN	8O	12F	14AB	16IJ	24EF
+	:	++	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1
+	:	++	0	0	0	0	0	3	-3	0	0	:	++	12	4	3	0	0	1	0	-1	:	++	6	-2	-3	0	-2	2	1	-1	0	1
+	:	++	1	9	1	1	1	0	0	-1	-1	:	++	13	-3	4	1	1	0	-1	0	:	++	7	-1	-2	1	3	-1	2	0	-1	0
o2	*	+	0	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	:	oo2	7	3	-2	1	2i-1	1	0	0	-i	-i-1
o4	**	+2	0	0	0	0	0	0	0	0	0	*3	o2	0	0	0	0	0	0	0	0	*5	o2	0	0	0	0	0	0	0	0	0	0
o2	*	+	0	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	:	oo2	32	0	-4	-1	0	0	0	-b7	0	0
o4	*	+2	0	0	0	0	0	0	0	0	0	:	oo4	16	0	-2	1	0	0	0	x13	*	+2	0	0	0	0	0	0	0	0	0	0
o24	**	+12	0	0	0	0	0	0	0	0	0	*3	o12	0	0	0	0	0	0	0	0	**3	o12	0	0	0	0	0	0	0	0	0	0
+2	:	++2	8	-8	-1	0	-b5	1	1	0	b5	*	+	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0
+2	:	++2	8	8	-1	0	-b5	-1	-1	0	-b5	*	+	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0
o4	**	+2	0	0	0	0	0	0	0	0	0	*3	o2	0	0	0	0	0	0	0	0	*5	o2	0	0	0	0	0	0	0	0	0	0
o4	**	+2	0	0	0	0	0	0	0	0	0	*3	o2	0	0	0	0	0	0	0	0	**3	o2	0	0	0	0	0	0	0	0	0	0
o8	**	+4	0	0	0	0	0	0	0	0	0	*3	o4	0	0	0	0	0	0	0	0	**3	o4	0	0	0	0	0	0	0	0	0	0
o16	**	+8	0	0	0	0	0	0	0	0	0	*3	o8	0	0	0	0	0	0	0	0	**3	o8	0	0	0	0	0	0	0	0	0	0
+	:	++	9	9	0	1	-1	0	0	1	-1	:	++	27	3	0	0	-1	0	-1	1	:	++	27	3	0	0	3	-1	0	-1	1	0
+	:	++	9	1	0	1	-1	1	1	-1	1	:	++	39	-1	3	0	-1	-1	1	0	:	++	21	5	3	0	1	1	-1	0	-1	1
o2	*	+	0	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	:	oo2	21	1	3	0	-2i-3	-1	1	0	-i	i
o4	**	+2	0	0	0	0	0	0	0	0	0	*3	o2	0	0	0	0	0	0	0	0	*5	o2	0	0	0	0	0	0	0	0	0	0
+	:	++	10	10	1	-2	0	1	1	0	0	:	++	26	2	-1	-1	2	-1	0	0	:	++	14	-2	5	-1	2	2	1	0	0	-1
+2	:	++2	10	-10	1	0	0	-1	-1	r2	0	*	+	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0
o2	*	+	0	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	:	oo2	28	-4	1	1	-4i	0	-1	0	0	-i
o2	*	+	0	0	0	0	0	0	0	0	0	:	oo2	26	-2	-1	-1	0	1	-i2	0	*	+	0	0	0	0	0	0	0	0	0	0

Unitary group $U_3(9) \cong {}^2A_2(9)$

Order = 42,573,600 = $2^5.3^6.5^2.73$ Mult = 1   Out = 4

Constructions

Unitary $GU_3(9) \cong 10 \times G$: all 3 x 3 matrices over $\mathbf{F}_{81}$ preserving a non-singular Hermitian form;

$PGU_3(9) \cong SU_3(9) \cong PSU_3(9) \cong G$

Maximal subgroups

Order	Index	Structure	G.2	G.4	Character	Abstract	Unitary
58320	730	$3^{2+4}:80$	: $3^{2+4}:80:2$	: $3^{2+4}:80:4$	1a+729a	$N(3A^2)$	isotropic point
7200	5913	$5 \times 2^{\cdot}A_6.2_2$	: $D_{10}\times 2^{\cdot}A_6.2_2$	: $(D_{10}\times 2^{\cdot}A_6.2_2)^{\cdot}2$		N(2A), N(5ABCD)	non-isotropic point
720	59130	$A_6:2_2$	: $A_6:2_2 \times 2$	: $(A_6:2_2\times 2)^{\cdot}2$		C(2B)	$O_3(9)$
600	70956	$10^2:S_3$	: $10^2:D_{12}$	: $10^2:(4\times S_3)$		$N(2A^2)$, $N(5^2)$	base
219	194400	73:3	: 73:6	: 73:12		N(73A-X)	$U_1(729)$

(Specifications: Character, Abstract, Unitary)

G	G.2	G.4
92	12	6

(92 printed to the right of G.4)

Abbreviated character table.

;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@
	4	7	7																																		
	2573600	200	290	81	80	7200/4	100/2	90	80/2	7200/4	100/2	100/4	100/4	90/4	80/4	80/4	90/4	80/8	73/24	80/16			720	720	9	8	10/2	18/2	8/2	10/2			24	24	3	4	6/2
p power		A	A	A	A	A	A	AA	A	CA	FA	CA	FA	DA	A	CA	CDA	CB	A	GD			A	A	BB	A	FB	AB	A	FB			B	B	BC	C	AD
p' part		A	A	A	A	A	A	AA	A	AA	EA	AA	EA	AA	A	AA	AAA	AA	A	AA			A	A	BB	A	EB	AB	A	EB			A	A	BC	A	AD
ind	1A	2A	3A	3B	4A	5AD	5EF	6A	8AB	10AD	10EF	10GJ	10KN	15AD	16AD	20AD	30AD	40AH	73AX	80AP	fus	ind	2B	4B	6B	8C	10OP	12AB	16EF	20EF	fus	ind	4C	8D	12C	16G	24AB
+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	:	+o+o	1	1	1	1	1
-	72	-8	-9	0	0	-8	2	1	0	-8	2	2	2	1	0	0	1	0	-1	0	:	oo	0	0	0	0	0	-3i	0	0	:	oooo	0	0	0	0	y'24
+	73	9	-8	1	1	-7	3	0	1	9	-1	-1	-1	2	-1	1	0	1	0	-1	:	++	1	9	1	1	1	0	-1	-1	:	+o+o	-1	3	-1	1	0
o4	73	-7	-8	1	1	z5-8&2	-b5	2	1	z5-8&2	-b5	z5+2&2	-b5	z5&2	1	z5	z5&2	z5	0	z5	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
o4	73	9	-8	1	1	z5-8&2	-b5	0	1	z5+8&2	b5+2	-z5	m5*3-1	z5&2	-1	z5	z5-&2	z5	0	-z5	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
+2	584	-8	17	-1	0	-8b5	-b5+1	1	0	-4r5+12	3b5+1	2	b5*	b5	0	0	-b5-2	0	0	0	:	++2	8	-8	-1	0	-b5*	1	0	b5*	:*	++	0	0	0	0	0
+2	584	24	17	-1	0	-8b5	-b5+1	-3	0	-8b5	-b5+1	2b5	-b5+1	b5	0	0	b5	0	0	0	:	++2	8	8	-1	0	-b5*	-1	0	-b5*	:*	++	0	0	0	0	0
o4	584	-8	17	-1	0	-8b5*	b5+2	1	0	8m5-8	b5	2z5	m5+2z5*3	b5*	0	0	-m5+1	0	0	0	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
o4	584	-8	17	-1	0	8z5+16&2	2b5	1	0	-8z5	2	2z5*2	2z5*3	-z5-2&2	0	0	z5	0	0	0	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
+	657	1	9	0	1	17	-3	1	1	1	1	1	1	-1	-1	1	1	1	0	-1	:	++	9	1	0	1	-1	1	-1	1	:	+o+o	3	-1	0	1	-1
o4	657	17	9	0	1	9z5+8&2	b5	-1	1	9z5+8&2	b5	-z5-2&2	b5	-z5*2	1	z5	-z5*2	z5	0	z5	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
o4	657	1	9	0	1	9z5+8&2	b5	1	1	9z5-8&2	-b5-2	z5	m5*2+1	-z5*2	-1	z5	z5*2	z5	0	-z5	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
+	729	9	0	0	1	9	-1	0	1	9	-1	-1	-1	0	1	1	0	1	-1	1	:	++	9	9	0	1	-1	0	1	-1	:	+o+o	3	3	0	-1	0
+	730	10	1	1	2	10	0	1	-2	10	0	0	0	1	0	2	1	-2	0	0	:	++	10	10	1	-2	0	1	0	0	:	+o+o	2	-2	-1	0	1
+2	730	10	1	1	-2	10	0	1	0	10	0	0	0	1	r2	-2	1	0	0	r2	:	++2	10	-10	1	0	0	-1	r2	0	:*	++	0	0	0	0	0
o4	730	10	1	1	2	10z5	0	1	-2	10z5	0	0	0	z5	0	2z5	z5	-2z5	0	0	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
o4	730	-10	1	1	0	10	0	-1	r2	-10	0	0	0	1	m16	0	-1	r2	0	m16	**	+2	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0
o8	730	10	1	1	-2	10z5	0	1	0	10z5	0	0	0	z5	r2	-2z5	z5	0	0	y"40*3	**	+4	0	0	0	0	0	0	0	0	*3	+2	0	0	0	0	0
o16	730	-10	1	1	0	10z5	0	-1	r2	-10z5	0	0	0	z5	m16	0	-z5	y"40*3	0	y"80*21	**	+8	0	0	0	0	0	0	0	0	*3	+4	0	0	0	0	0
o24	800	0	-10	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-x73	0	**	+12	0	0	0	0	0	0	0	0	*3	+6	0	0	0	0	0

HS

Sporadic Higman-Sims group HS

Order = 44,352,000 = $2^9.3^2.5^3.7.11$ Mult = 2 Out = 2

Constructions

Graph G.2 : the automorphism group of the graph of D.G.Higman and C.C.Sims, a rank 3 graph of valence 22 on 100 vertices. Any given vertex has 22 neighbours (points), and each of the remaining 77 vertices is joined to 6 of these points, and may be labelled by the corresponding hexad. Two of these 77 vertices are joined just if the corresponding hexads are disjoint. The hexads form a Steiner system S(3,6,22) and the point stabilizer is Aut(S(3,6,22)) ≅ $M_{22}.2$. The simple group G is the subgroup of even permutations.

Higman G : the automorphism group of G.Higman's "geometry" of 176 points P and 176 quadrics Q. The group acts doubly transitively on each set, and each object is incident with 50 of the other type. There are 1100 conics, each of which defines a polarity between the 8 points on it and the 8 quadrics containing it. A quadric and a point on it are polar with respect to just one conic. The intersection of two quadrics is uniquely the union of two conics, which meet in two points. The duality interchanging points and quadrics gives G.2.

Taking M_{22} as the stabilizer of ∞ and 0 in M_{24}, the points are labelled by the octads P containing ∞ but not 0, quadrics by octads Q containing 0 but not ∞, and P is incident with Q just if their intersection has size 0 or 4.

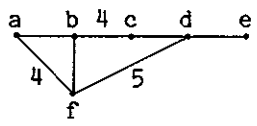
2-graph G : the automorphism group of a 2-graph on 176 points, that is, an assignment of parities to the triples of points, so that the sum of the parities of the four triples in any quadruple is even. Labelling the 176 points by octads as in the previous construction, a triple of octads is even or odd according as the sum of the sizes of their pairwise intersections is 0 or 2 mod 4.

Leech In the Leech lattice (see Co_1) G is the pointwise stabilizer of a triangle OAB of type 233, for instance $O = (0,0,0^{22})$, $A = (5,1,1^{22})$, $B = (1,5,1^{22})$. The setwise stabilizer is G.2.

The 100 vertices V of the Higman-Sims graph are represented by lattice points with VO, VA, VB all of type 2, namely $(4,4,0^{22})$ (1), $(1,1,-3,1^{21})$ (22), $(2,2,2^6 0^{16})$ (77), V and V' being incident in the graph just if type(VV') = 3.

There are 2x176 points P with type(PO) = type(PA) = 2, type(PB) = 3, falling naturally into 176 pairs $\{P_L=(2,0,2^7 0^{15}),\ P_R=(3,1,-1^7 1^{15})\}$ satisfying $P_L+P_R-A = 0$, which correspond to the points of Higman's geometry. The quadrics correspond to the pairs $\{Q_L=(0,2,2^7 0^{15}),\ Q_R=(1,3,-1^7 1^{15})\}$ obtained by interchanging the roles of A and B in the above definition. A point is incident with a quadric just if type($P_i Q_j$) is even.

In the Leech lattice mod 2 the 3-spaces containing the fixed 2-space (of type 233) become points in the quotient 22-dimensional representation over $\mathbb{F}_2$. They are described below by the types of their non-zero vectors.

Presentation G ≅ < 

| $e=(fa)^2$, $1=(cbf)^3=(fdc)^5=(bcde)^4$ >

Remark : 2.G.2 is the involution centralizer in the Harada-Norton group.

Maximal subgroups					Specifications			
Order	Index	Structure	G.2	Character	Abstract	Graph	Higman	Leech
443520	100	M_{22}	: $M_{22}:2$	1a+22a+77a		point		233-2224-point
252000	176	$U_3(5):2$	$5_+^{1+2}:[2^5]$	1a+175a			point	233-2233-point
252000	176	$U_3(5):2$		1a+175a			quadric	233-2233-point
40320	1100	$L_3(4):2_1$	: $L_3(4):2^2$	1a+22a+77a+175a+825a	N(2A,3A,4B,4C,4C,5C,7A)	edge		233-2233-point
40320	1100	S_8	: $S_8 \times 2$	1a+77a+154a+175a+693a	C(2C)		conic	233-2244-point
11520	3850	$2^4.S_6$	: $2^5.S_6$		$N(2A^4)$	non-edge		233-2244-point
10752	4125	$4^3:L_3(2)$	: $4^3:(L_3(2) \times 2)$		$N(2A^3)$			
7920	5600	M_{11}			N(2A,3A,4C,5C,6B,8B,11AB)			233-2334-point
7920	5600	M_{11}			N(2A,3A,4C,5C,6B,8C,11AB)			233-2334-point
7680	5775	$4\cdot 2^4:S_5$	: $2_+^{1+6}:S_5$		N(2A)			
2880	15400	$2 \times A_6\cdot 2^2$	: H.2		N(2B), N(2A,3A,3A,5B)		point pair	
1200	36960	$5:4 \times A_5$	: $5:4 \times S_5$		N(5B), N(2B,3A,5A)			

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			7	2		3																							40	1																		
	44352000		680	880	360	840	256	64	500	300	25	36	24	7	16	16	16	20	20	11	11	12	15	20	20				320	920	320	96	40	360	36	24	32	32	30	5	12	7	10	10	10	15	
	p	power	A	A	A	A	A	A	A	A	A	AB	AA	A	B	C	C	AA	BB	A	A	BA	BA	AA	AA				A	A	A	A	B	AC	AC	AD	B	B	BC	CD	BE	AC	BF	AD	AD	ACC	
	p'	part	A	A	A	A	A	A	A	A	A	AB	AA	A	A	A	A	AA	BB	A	A	AA	BA	AA	AA				A	A	A	A	A	AC	AC	AD	A	A	BC	CD	AE	AC	BF	AD	AD	ACC	
	ind	1A	2A	2B	3A	4A	4B	4C	5A	5B	5C	6A	6B	7A	8A	8B	8C	10A	10B	11A	B**	12A	15A	20A	B**	fus	ind		2C	2D	4D	4E	4F	6C	6D	6E	8D	8E	10C	10D	12B	14A	20C	20D	E*	30A	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	+	22	6	-2	4	-6	2	2	-3	2	2	-2	0	1	0	0	0	1	-2	0	0	0	-1	-1	-1	:	++		8	0	4	0	0	-4	2	0	2	-2	-2	0	0	1	0	-1	-1	1	x_2
x_3	+	77	13	1	5	5	5	1	2	-3	2	1	1	0	1	-1	-1	-2	1	0	0	-1	0	0	0	:	++		21	5	5	1	-1	3	3	-1	1	1	1	0	1	0	-1	0	0	-2	x_3
x_4	+	154	10	10	1	-2	6	-2	4	4	-1	1	1	0	0	0	0	0	0	0	0	1	1	-2	-2	:	++		28	4	0	4	0	1	1	1	-2	2	-2	-1	1	0	0	0	0	1	x_4
x_5	+	154	10	-10	1	-10	-2	2	4	4	-1	-1	1	0	0	2	-2	0	0	0	0	-1	1	0	0		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_5
x_6	+	154	10	-10	1	-10	-2	2	4	4	-1	-1	1	0	0	-2	2	0	0	0	0	-1	1	0	0																					x_6
x_7	+	175	15	11	4	15	-1	3	0	5	0	2	0	0	-1	1	1	0	1	-1	-1	0	-1	0	0	:	++		21	5	5	1	-1	6	0	2	1	1	1	0	-2	0	-1	0	0	1	x_7
x_8	+	231	7	-9	6	15	-1	-1	6	1	1	0	-2	0	-1	-1	-1	2	1	0	0	0	1	0	0	:	++		21	-11	5	-3	1	6	0	-2	1	1	1	-1	0	0	1	0	0	1	x_8
x_9	+	693	21	9	0	21	5	1	-7	3	-2	0	0	0	1	-1	-1	1	-1	0	0	0	0	1	1	:	++		63	15	-1	3	1	0	0	0	-1	-1	3	0	0	0	1	-1	-1	0	x_9
x_{10}	+	770	34	-10	5	-14	2	-2	-5	0	0	-1	1	0	-2	0	0	-1	0	0	0	1	0	1	1	:	++		70	-10	6	2	0	-5	1	-1	-2	-2	0	0	-1	0	0	1	1	0	x_{10}
x_{11}	o	770	-14	10	5	-10	-2	-2	-5	0	0	1	1	0	0	0	0	1	0	0	0	-1	0	i5	-i5		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{11}
x_{12}	o	770	-14	10	5	-10	-2	-2	-5	0	0	1	1	0	0	0	0	1	0	0	0	-1	0	-i5	i5																					x_{12}
x_{13}	+	825	25	9	6	-15	1	1	0	-5	0	0	-2	-1	1	1	1	0	-1	0	0	0	1	0	0	:	++		69	5	5	-3	1	-6	0	2	1	1	-1	0	0	-1	1	0	0	-1	x_{13}
x_{14}	o	896	0	16	-4	0	0	0	-4	1	1	-2	0	0	0	0	0	0	1	b11	**	0	1	0	0		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{14}
x_{15}	o	896	0	16	-4	0	0	0	-4	1	1	-2	0	0	0	0	0	0	1	**	b11	0	1	0	0																					x_{15}
x_{16}	+	1056	32	0	-6	0	0	0	6	-4	1	0	2	-1	0	0	0	2	0	0	0	0	-1	0	0	:	++		48	16	0	0	0	6	0	-2	0	0	-2	1	0	-1	0	0	0	1	x_{16}
x_{17}	+	1386	-6	18	0	6	-2	-2	11	6	1	0	0	0	0	0	0	-1	-2	0	0	0	0	1	1	:	++		0	-24	4	0	0	0	0	0	-2	2	0	1	0	0	0	-1	-1	0	x_{17}
x_{18}	+	1408	0	16	4	0	0	0	8	-7	-2	-2	0	1	0	0	0	0	1	0	0	0	-1	0	0	:	++		64	0	0	0	4	4	-2	0	0	0	-1	0	0	1	-1	0	0	-1	x_{18}
x_{19}	+	1750	-10	10	-5	-10	6	2	0	0	0	1	-1	0	-2	0	0	0	0	1	1	-1	0	0	0	:	++		70	-10	-10	2	0	-5	1	-1	2	2	0	0	-1	0	0	0	0	0	x_{19}
x_{20}	+	1925	5	-19	-1	5	5	-3	0	5	0	-1	-1	0	1	1	1	0	1	0	0	-1	-1	0	0	:	++		91	-5	-5	-5	-1	1	1	1	-1	-1	1	0	1	0	-1	0	0	1	x_{20}
x_{21}	+	1925	5	1	-1	-35	-3	1	0	5	0	1	-1	0	1	-1	-1	0	1	0	0	1	-1	0	0	:	++		21	5	5	1	-1	-9	-3	-1	1	1	1	0	1	0	-1	0	0	1	x_{21}
x_{22}	+	2520	24	0	0	24	-8	0	-5	0	0	0	0	0	0	0	0	-1	0	1	1	0	0	-1	-1	:	++		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r5	-r5	0	x_{22}
x_{23}	+	2750	-50	-10	5	10	2	2	0	0	0	-1	1	-1	0	0	0	0	0	0	0	1	0	0	0	:	++		20	-20	0	4	0	5	-1	1	2	-2	0	0	1	-1	0	0	0	0	x_{23}
x_{24}	+	3200	0	-16	-4	0	0	0	0	-5	0	2	0	1	0	0	0	0	-1	-1	-1	0	1	0	0	:	++		64	0	0	0	-4	4	-2	0	0	0	-1	0	0	1	1	0	0	-1	x_{24}
	ind	1	2	4	3	4	4	4	5	5	5	12	6	7	8	8	8	10	20	11	11	12	15	20	20	fus	ind		2	2	4	4	8	6	6	6	8	8	10	10	12	14	40	20	20	30	
		2	2		6	4			10	10	10	12	6	14	8			10		22	22	12	30	20	20														10		14	40	20	20	30	
x_{25}	+	56	8	0	2	0	0	0	6	-4	1	0	2	0	0	0	0	-2	0	1	1	0	2	0	0	:	++		0	0	0	0	0	0	0	0	0	0	0	r5	0	0	0	0	0	0	x_{25}
x_{26}	o	176	16	0	5	16i	0	0	1	6	1	3i	1	1	0	0	0	1	0	0	0	-i	0	i	i		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{26}
x_{27}	o	176	16	0	5	-16i	0	0	1	6	1	-3i	1	1	0	0	0	1	0	0	0	i	0	-i	-i																					x_{27}
x_{28}	o	616	24	0	4	16i	0	0	-9	-4	1	0	0	0	0	0	0	-1	0	0	0	2i	-1	i	i		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{28}
x_{29}	o	616	24	0	4	-16i	0	0	-9	-4	1	0	0	0	0	0	0	-1	0	0	0	-2i	-1	-i	-i																					x_{29}
x_{30}	o	924	4	0	6	24i	0	0	-1	4	-1	0	-2	0	2i	0	0	-1	0	0	0	0	1	-i	-i		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{30}
x_{31}	o	924	4	0	6	-24i	0	0	-1	4	-1	0	-2	0	-2i	0	0	-1	0	0	0	0	1	i	i																					x_{31}
x_{32}	-	1000	-40	0	10	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	-1	-1	0	0	0	0	:	oo		0	0	0	0	0	0	0	0	0	0	0	0	0	i7	0	0	0	0	x_{32}
x_{33}	o	1232	-16	0	-1	16i	0	0	7	2	2	-3i	-1	0	0	0	0	-1	0	0	0	-i	-1	i	i		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{33}
x_{34}	o	1232	-16	0	-1	-16i	0	0	7	2	2	3i	-1	0	0	0	0	-1	0	0	0	i	-1	-i	-i																					x_{34}
x_{35}	-	1792	0	0	-8	0	0	0	-8	2	2	0	0	0	0	0	0	0	0	-1	-1	0	2	0	0	:	oo		0	0	0	0	0	0	0	0	0	0	0	0	0	0	i10	0	0	0	x_{35}
x_{36}	-	1848	8	0	-6	0	0	0	-2	8	-2	0	2	0	0	0	0	-2	0	0	0	0	-1	0	0	:	oo		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i15	x_{36}
x_{37}	o	1980	36	0	0	24i	0	0	5	0	0	0	0	-1	-2i	0	0	1	0	0	0	0	0	-i	-i		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{37}
x_{38}	o	1980	36	0	0	-24i	0	0	5	0	0	0	0	-1	2i	0	0	1	0	0	0	0	0	i	i																					x_{38}
x_{39}	o	2304	0	0	0	0	0	0	4	-6	-1	0	0	1	0	0	0	0	0	b11	**	0	0	0	0		+		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{39}
x_{40}	o	2304	0	0	0	0	0	0	4	-6	-1	0	0	1	0	0	0	0	0	**	b11	0	0	0	0																					x_{40}
x_{41}	-	2520	-24	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	1	0	1	1	0	0	r5	-r5	:	oo		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	m5&3	*3	0	x_{41}
x_{42}	-	2520	-24	0	0	0	0	0	-5	0	0	0	0	0	0	0	0	1	0	1	1	0	0	-r5	r5	:	oo		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*7	m5&3	0	x_{42}

G	G.2	24
2.G	2.G.2	18
24	18	

J_3

Sporadic Janko group J_3

Order = 50,232,960 = $2^7.3^5.5.17.19$ Mult = 3 Out = 2

Constructions

Unitary (2) 3.G has a 9-dimensional unitary representation over $\mathbb{F}_4 = \{0,1,w,\bar{w}\}$ with basis vectors e(z), where z = x + iy is in $\mathbb{F}_9$. We also write $e(0) = e_\infty$, $e((1-i)^n) = e_n$, i.e.

-1+i	i	1+i
-1	0	1
-1-i	-i	1-i

or

5	2	3
4	∞	0
7	6	1

3.G is the subgroup of the unitary group (for the form $\Sigma x_n\bar{x}_n$) which preserves the subspace S of the exterior square spanned by the 18 vectors $e_ne_{n+4} + e_{n+2}e_{n+6}$, $e_\infty e_0 + e_{n-1}e_{n-3}$, and $we_ne_{n+3} + \bar{w}e_{n-1}e_{n-2}$.

3.G.2 is generated by the maps (specified by the image of e(z)):

$A_Z : w^{Xx+Yy}e(z)$ $B_Z : e(Zz)\ (Z\neq 0)$ $C_Z : e^*(Z/z)\ (Z\neq 0)$

$D_Z : W((z+Z)/z)e(z+Z)$ $E_Z : w^{(X-x)^2+(Y-y)^2}e(z)$ $F_Z : e(\bar{Z}\bar{z})$

where Z = X+iY is in $\mathbb{F}_9$, $e^*(t) = e(it)+e(-it)+e(-t)$, $W(i) = w$, $W(-i) = \bar{w}$, and otherwise W(t) = 1; the map F_Z is antilinear.

There are four orbits {0}, I, II, III on vectors, (e.g. 0, e_∞, e_0+e_4, $e_\infty+e_0+e_4$) the minimal weights of representatives being 0, 1, 2, 3 respectively. The stabilizer of $\langle e_\infty\rangle$ in 3.G is a group $(3^2xA_6).2 \cong 3 \times 3{:}PGL_2(9)$ in which $\langle E_0\rangle$ is normal and A_Z, B_Z, C_Z map to the elements $t \to Z+t$, $t \to Zt$, $t \to Z/t$ of $PGL_2(9)$. The stabilizer in 3.G of $\langle e_\infty+e_0+e_4\rangle$ is $3 \times L_2(17)$, in which the $L_2(17)$ (centralizing F_1) is generated by D_1 and C_{-1}, whose product has order 17. All non-zero isotropic vectors are of type II and the group preserves a symmetrical joining relation between isotropic vectors, the vectors joining v ($\neq 0$) forming a 3-dimensional space J(v). We define $J(U) = \langle J(u){:}u \text{ of type II in } U\rangle$. A type II vector v (e.g. $v = e_0+e_4$) is contained in five 2-spaces (projectively, lines) inside J(v), namely two <u>even</u> ones $\langle v,v'=e_3+e_5\rangle$, $\langle v,v''=e_1+e_7\rangle$ and three <u>odd</u> ones $\langle v,v'+w^nv''\rangle$. There is also a distinguished orbit of <u>special</u> 4-spaces, such as $\langle \bar{w}e_n+we_{n+2} | n=0,1,4,5\rangle$, each the disjoint union of 17 even spaces.

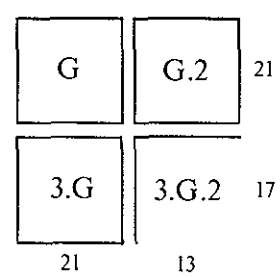
Complex 3.G has an 18-dimensional complex representation writable over $\mathbb{Q}(\sqrt{-3},\sqrt{5})$, for which explicit matrices have been computed. In characteristic 2 this becomes the representation given above on the 18-space S.

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Specifications Unitary (2)
8160	6156	$L_2(16){:}2$	: $L_2(16){:}4$	1a+323ab+324a+1140a+1215ab+1615a		special 4-space
3420	14688	$L_2(19)$	19:18			
3420	14688	$L_2(19)$				
2880	17442	$2^4{:}(3xA_5)$	: $2^4{:}(3xA_5).2$		$N(2A^4)$	even line
2448	20520	$L_2(17)$	: $L_2(17) \times 2$		C(2B)	type III point
2160	23256	$(3 \times A_6){:}2_2$	: $(3 \times M_{10}){:}2$		N(3A)	type I point
1944	25840	$3^2.(3x3^2){:}8$	: $3^2.(3x3^2){:}8.2$		$N(3B^2)$	base
1920	26163	$2_-^{1+4}{:}A_5$	: $2_-^{1+4}.S_5$		N(2A)	odd line
1152	43605	$2^{2+4}{:}(3xS_3)$	: $2^{2+4}{:}(S_3xS_3)$		$N(2A^2)$	type II (isotropic) point

J_3

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	
			1	1																					2													
	50232960		920	080	243	96	30	30	24	8	27	27	27	10	10	12	15	15	17	17	19	19			448	48	9	48	16	6	9	9	9	12	12	17	17	
	p	power	A	A	A	A	A	A	AA	A	B	B	B	BA	AA	AA	BA	AA	A	A	A	A			A	A	BB	A	A	AB	BB	CB	AB	AB	AB	AB	BB	
	p'	part	A	A	A	A	A	A	AA	A	A	A	A	AA	BA	AA	AA	BA	A	A	A	A			A	A	BB	A	A	AB	BB	BB	CB	AB	AB	AB	BB	
	ind	1A	2A	3A	3B	4A	5A	B*	6A	8A	9A	B*2	C*4	10A	B*	12A	15A	B*	17A	B*	19A	B**	fus	ind	2B	4B	6B	8B	8C	12B	18A	B*7	C*5	24A	B*	34A	B*	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	o	85	5	-5	4	1	0	0	-1	-1	1	1	1	0	0	1	0	0	0	0	b19	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	x_2
x_3	o	85	5	-5	4	1	0	0	-1	-1	1	1	1	0	0	1	0	0	0	0	**	b19																x_3
x_4	+	323	3	8	-1	3	-b5	*	0	-1	-1	-1	-1	-b5	*	0	-b5	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	x_4
x_5	+	323	3	8	-1	3	*	-b5	0	-1	-1	-1	-1	*	-b5	0	*	-b5	0	0	0	0																x_5
x_6	+	324	4	9	0	4	-1	-1	1	0	0	0	0	-1	-1	1	-1	-1	1	1	1	1	:	++	18	2	0	-2	-2	-1	0	0	0	1	1	1	1	x_6
x_7	+	646	-10	7	-2	2	-2b5	*	-1	0	1	1	1	0	0	-1	b5	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	x_7
x_8	+	646	-10	7	-2	2	*	-2b5	-1	0	1	1	1	0	0	-1	*	b5	0	0	0	0																x_8
x_9	+	816	-16	6	6	0	1	1	2	0	0	0	0	-1	-1	0	1	1	0	0	-1	-1	:	++	0	0	0	0	0	0	0	0	0	r6	*	0	0	x_9
x_{10}	+	1140	20	15	6	-4	0	0	-1	0	0	0	0	0	0	-1	0	0	1	1	0	0	:	++	18	2	0	2	2	-1	0	0	0	-1	-1	1	1	x_{10}
x_{11}	+	1215	15	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0	b17	*	-1	-1	:	++	9	-3	0	3	-1	0	0	0	0	0	0	-b17	*	x_{11}
x_{12}	+	1215	15	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0	*	b17	-1	-1	:	++	9	-3	0	3	-1	0	0	0	0	0	0	*	-b17	x_{12}
x_{13}	+	1615	15	-5	-5	-1	0	0	3	-1	1	1	1	0	0	-1	0	0	0	0	0	0	:	++	17	1	-1	1	1	1	-1	-1	-1	1	1	0	0	x_{13}
x_{14}	+	1920	0	0	3	0	0	0	0	0	*4	y9-&4	*2	0	0	0	0	0	-1	-1	1	1	:	++	16	0	1	0	0	0	-y9	*2	*4	0	0	-1	-1	x_{14}
x_{15}	+	1920	0	0	3	0	0	0	0	0	*2	*4	y9-&4	0	0	0	0	0	-1	-1	1	1	:	++	16	0	1	0	0	0	*4	-y9	*2	0	0	-1	-1	x_{15}
x_{16}	+	1920	0	0	3	0	0	0	0	0	y9-&4	*2	*4	0	0	0	0	0	-1	-1	1	1	:	++	16	0	1	0	0	0	*2	*4	-y9	0	0	-1	-1	x_{16}
x_{17}	+	1938	2	3	-6	-2	-b5	*	-1	0	0	0	0	b5	*	1	-b5	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{17}
x_{18}	+	1938	2	3	-6	-2	*	-b5	-1	0	0	0	0	*	b5	1	*	-b5	0	0	0	0																x_{18}
x_{19}	+	2432	0	-16	2	0	2	2	0	0	-1	-1	-1	0	0	0	-1	-1	1	1	0	0	:	++	16	0	-2	0	0	0	1	1	1	0	0	-1	-1	x_{19}
x_{20}	+	2754	-14	9	0	-2	-1	-1	1	0	0	0	0	1	1	1	-1	-1	0	0	-1	-1	:	++	0	4	0	2	-2	1	0	0	0	-1	-1	0	0	x_{20}
x_{21}	+	3078	-10	-9	0	2	-2	-2	-1	0	0	0	0	0	0	-1	1	1	1	1	0	0	:	++	18	-2	0	-4	0	1	0	0	0	-1	-1	1	1	x_{21}
	ind	1	2	3	3	4	5	5	6	8	9	9	9	10	10	12	15	15	17	17	19	19	fus	ind	2	4	6	8	8	12	18	18	18	24	24	34	34	
			3	6	3		12	15	15	6	24				30	30	12	15	15	51	51	57	57																
			3	6	3		12	15	15	6	24				30	30	12	15	15	51	51	57	57																
x_{22}	o2	18	2	3	0	-2	-b5	*	-1	0	0	0	0	b5	*	1	-b5	*	1	1	-1	-1		o2														x_{22}
x_{23}	o2	18	2	3	0	-2	*	-b5	-1	0	0	0	0	*	b5	1	*	-b5	1	1	-1	-1	*															x_{23}
x_{24}	o2	153	-7	3	0	1	-b5	*	-1	1	0	0	0	-b5	*	1	-b5	*	0	0	1	1		o2														x_{24}
x_{25}	o2	153	-7	3	0	1	*	-b5	-1	1	0	0	0	*	-b5	1	*	-b5	0	0	1	1	*															x_{25}
x_{26}	o2	171	11	6	0	3	1	1	2	-1	0	0	0	1	1	0	1	1	1	1	0	0	*	+														x_{26}
x_{27}	o2	171	-5	-3	0	-1	-2b5	*	1	1	0	0	0	0	0	-1	b5	*	1	1	0	0		o2														x_{27}
x_{28}	o2	171	-5	-3	0	-1	*	-2b5	1	1	0	0	0	0	0	-1	*	b5	1	1	0	0	*															x_{28}
x_{29}	o2	324	4	9	0	4	-1	-1	1	0	0	0	0	-1	-1	1	-1	-1	1	1	1	1	*	+														x_{29}
x_{30}	o2	1215	15	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0	b17	*	-1	-1	*	+														x_{30}
x_{31}	o2	1215	15	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0	*	b17	-1	-1	*	+														x_{31}
x_{32}	o2	1530	10	-15	0	-2	0	0	1	0	0	0	0	0	0	1	0	0	0	0-b19	**			+2														x_{32}
x_{33}	o2	1530	10	-15	0	-2	0	0	1	0	0	0	0	0	0	1	0	0	0	0	**-b19		*															x_{33}
x_{34}	o2	2736	-16	6	0	0	1	1	2	0	0	0	0	-1	-1	0	1	1	-1	-1	0	0	*	+														x_{34}
x_{35}	o2	2754	-14	9	0	-2	-1	-1	1	0	0	0	0	1	1	1	-1	-1	0	0	-1	-1	*	+														x_{35}
x_{36}	o2	2907	-5	-6	0	3	2	2	-2	-1	0	0	0	0	0	0	-1	-1	0	0	0	0	*	+														x_{36}
x_{37}	o2	3060	20	15	0	-4	0	0	-1	0	0	0	0	0	0	-1	0	0	0	0	1	1	*	+														x_{37}
x_{38}	o2	3078	-10	-9	0	2	-2	-2	-1	0	0	0	0	0	0	-1	1	1	1	1	0	0	*	+														x_{38}



U3(11)

Unitary group $U_3(11) \cong {}^2A_2(11)$

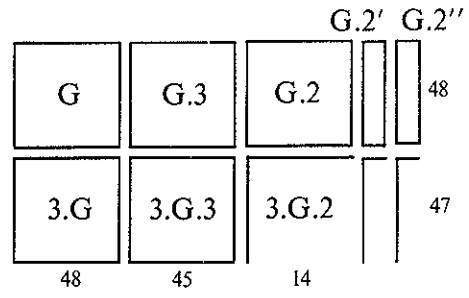
Order = $70{,}915{,}680 = 2^5.3^2.5.11^3.37$ Mult = 3 Out = S_3

Constructions

Unitary $GU_3(11) \cong 4 \times 3.G.3$: all 3×3 matrices over $\mathbf{F}_{121}$ preserving a non-singular Hermitian form;

$PGU_3(11) \cong G.3$; $SU_3(11) \cong 3.G$; $PSU_3(11) \cong G$

Remark : G.2 is a maximal subgroup of Janko's group J_4



Maximal subgroups

Order	Index	Structure	G.2	G.3	G.S_3	Character
53240	1332	11_+^{1+2}:40	: 11_+^{1+2}:(5x8:2)	: 11_+^{1+2}:120	: 11_+^{1+2}:(5x24:2))	1a+1331a
5280	13431	2(L_2(11)x2).2	: (2(L_2(11)x2).2).2	: 3 x H	: (3 x H).2	
1320	53724	L_2(11):2	: L_2(11):2 x 2			
1320	53724	L_2(11):2				
1320	53724	L_2(11):2				
360	196988	A_6	: PGL_2(9)			
360	196988	A_6				
360	196988	A_6				
288	246235	$(4^2x3):S_3$	: $(4^2x3):D_{12}$	: $12^2:S_3$	: $12^2:D_{12}$	
111	638880	37:3	: 37:6	: 111:3	: 111:6	
72	984940	$3^2:Q_8$	: $3^2:Q_8.2$	: $3^2:2A_4$	: $3^2:2S_4$	

Specifications

Abstract	Unitary
N(11A)	isotropic point
N(2A), C(3B)	non-isotropic point
C(2B)	$O_3(11)$
	$O_3(11)$
	$O_3(11)$
N(3A), N($2A^2$)	base
N(37A-L)	$U_1(1331)$
N($3A^2$)	

Abbreviated character table.

;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	;	;
		5								5																																										
70915680		280	144	5280/2	48	40/2	48	40/2	40/2	324	121	121	121	48/2	48/2	40/4	44	37/12	40/8	44/2			5280	111	5280	48	5280/2	48	48/2	48/2	40/2	40/2	40/2	44	40/4	44	37/12	40/8	44/2			320	320	12	12	10/2	12	10/2	11	12/2	22/2				
p power		A	A	A	A	A	AA	A	BA	A	A	A	A	AB	AC	BA	AA	A	CA	AB			A	A	BA	BA	BB	BC	CB	CC	BB	EB	BBB	AB	BDE	AAB	EC	CDA	ABF			A	A	AB	C	BB	AD	BD	BB	DC	AD				
p' part		A	A	A	A	A	AA	A	AA	A	A	A	A	AA	AC	AA	AA	A	AA	AA			A	A	BA	BA	BA	BC	BA	BC	AB	BA	AAB	AB	AAE	AAB	AC	AAA	AAE			A	A	AB	A	AB	AD	AD	BB	AC	AD				
ind		1A	2A	3A	4AB	4C	5AB	6A	8AB	10AB	11A	11B	11C	11D	12AB	12CD	20AD	22A	37AL	40AH	44AB	fus	ind	3B	3C	6B	6C	12EF	12G	12HI	12JK	15AB	24AB	30AB	33A	60AD	66A	111AL	120AH	132AB	fus	ind	2B	4D	6D	8C	10CD	12L	20EF	22B	24CD	44CD	fus	ind	fus	ind
+		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	+oo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	:	++	:	++
−		110	−10	2	−10	2	0	2	0	0	−11	0	0	0	2	2	0	1	−1	0	1	:	−oo	−10	−1	−10	2	−10	2	2	2	0	0	0	1	0	1	−1	0	1	:	oo	0	0	0	0	0	0	0	0	0	i11	:	oo	:	oo
+		111	−9	3	11	−1	1	3	−1	1	−10	1	1	1	−1	−1	1	2	0	−1	0	:	+oo	−9	0	−9	3	11	−1	−1	−1	1	−1	1	2	1	2	0	−1	0	:	++	11	−1	−1	1	1	−1	−1	0	1	−1	:	++	:	++
o2		111	11	3	10i−1	1	1	−1	−i	1	−10	1	1	1	−2i−1	1	−1	0	0	−i	−i−1	:	ooo2	−9	0	11	−1	10i−1	1	−2i−1	1	1	−i	1	2	−1	0	0	−i	−i−1	*	+	0	0	0	0	0	0	0	0	0	*	+	*	+
+		370	10	1	10	−2	0	1	0	0	7	7	−4	−4	1	1	0	−1	0	0	−1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	10	10	1	−2	0	1	0	−1	1	−1		+		+
+		370	10	1	10	−2	0	1	0	0	7	−4	7	−4	1	1	0	−1	0	0	−1																					+	0	0	0	0	0	0	0	0	0	0	:	++		
+		370	10	1	10	−2	0	1	0	0	7	−4	−4	7	1	1	0	−1	0	0	−1																																		:	++
+		1110	30	3	−10	2	0	3	0	0	21	−1	−1	−1	−1	−1	0	−3	0	0	1	:	+oo	0	0	0	0	20	−4	2	2	0	0	0	0	0	0	0	0	−2	:	++	10	10	1	2	0	1	0	−1	−1	−1	:	++	:	++
+		1110	−10	−6	10	2	0	2	0	0	21	−1	−1	−1	−2	2	0	1	0	0	−1	:	+oo	30	0	−10	2	10	2	−2	2	0	0	0	−3	0	1	0	0	−1	:	++	10	−10	−2	0	0	2	0	−1	0	1	:	++	:	++
+2		1110	−10	3	10	2	0	−1	0	0	21	−1	−1	−1	1−2r3	−1	0	1	0	0	−1	:	+oo2	0	0	20	2	−10r3+10	2	−r3+1	r3−1	0	0	0	0	0	−2	0	0	r3−1	:	++2	10	−10	1	0	0	−1	0	−1	−r3	1	:	++2	:	++2
o2		1110	−10	3	20i−10	−2	0	−1	0	0	21	−1	−1	−1	2i−1	1	0	1	0	0	−2i+1	:	ooo2	0	0	20	2	−10i−10	−2	−i−1	−3i+1	0	0	0	0	0	−2	0	0	i+1	*	+	0	0	0	0	0	0	0	0	0	*	+	*	+
+		1221	21	−3	1	1	1	−3	−1	1	11	0	0	0	1	1	1	−1	0	−1	1	:	+oo	21	0	21	−3	1	1	1	1	1	−1	1	−1	1	−1	0	−1	1	:	++	−1	11	−1	1	−1	−1	1	−1	1	0	:	++	:	++
o2		1221	1	−3	10i−11	−1	1	1	i	1	11	0	0	0	−2i+1	−1	−1	1	0	i	−i	:	ooo2	21	0	1	1	10i−11	−1	−2i+1	−1	1	i	1	−1	−1	1	0	i	−i	*	+	0	0	0	0	0	0	0	0	0	*	+	*	+
+		1331	11	−1	11	−1	1	−1	1	1	0	0	0	0	−1	−1	1	0	−1	1	0	:	+oo	11	−1	11	−1	11	−1	−1	−1	1	1	1	0	1	0	−1	1	0	:	++	11	11	−1	−1	1	−1	1	0	−1	0	:	++	:	++
o2		1332	−12	0	12i	0	2	0	0	−2	1	1	1	1	0	0	2i	−1	0	0	i	:	ooo2	12	0	−12	0	12i	0	0	0	2	0	−2	1	2i	−1	0	0	i	*	+	0	0	0	0	0	0	0	0	0	*	+	*	+
+2		1332	12	0	12	0	b5	0	2	b5	1	1	1	1	0	0	b5	1	0	b5	1	:	+oo2	12	0	12	0	12	0	0	0	b5	2	b5	1	b5	1	0	b5	1	:	++2	12	12	0	0	b5	0	b5	1	0	1	:	++2	:	++2
+2		1332	12	0	12	0	b5	0	−2	b5	1	1	1	1	0	0	b5	1	0	−b5	1	:	+oo2	12	0	12	0	12	0	0	0	b5	−2	b5	1	b5	1	0	−b5	1	:	++2	12	−12	0	0	b5	0	−b5	1	0	−1	:	++2	:	++2
o4		1332	12	0	−12	0	b5	0	2i	b5	1	1	1	1	0	0	−b5	1	0	m20	−1	:	ooo4	12	0	12	0	−12	0	0	0	b5	2i	b5	1	−b5	1	0	m20	−1	**	+2	0	0	0	0	0	0	0	0	0	0	**	+2	**	+2
o8		1332	−12	0	12i	0	b5	0	0	−b5	1	1	1	1	0	0	m20	−1	0	m'40**3	i	:	ooo8	12	0	−12	0	12i	0	0	0	b5	0	−b5	1	m20	−1	0	m'40**3	i	**	+4	0	0	0	0	0	0	0	0	0	0	**	+4	**	+4
o12		1440	0	0	0	0	0	0	0	0	−12	−1	−1	−1	0	0	0	0	−x37	0	0	:	ooo12	0	−3	0	0	0	0	0	0	0	0	0	0	0	0	−x37	0	0	**	+6	0	0	0	0	0	0	0	0	0	0	**	+6	**	+6
ind		1	2	3	4	4	5	6	8	10	11	11	11	11	12	12	20	22	37	40	44	fus	ind	3	9	6	6	12	12	12	12	15	24	30	33	60	66	333	120	132	fus	ind	2	4	6	8	10	12	20	22	24	44	fus	ind	fus	ind
		3	6		12	12	15	6	24	30	33	33	33	33	12	12	60	66	111	120	132			3		6	6	12	12	12	12	15	24	30	33	60	66	333	120	132																	
		3	6		12	12	15	6	24	30	33	33	33	33	12	12	60	66	111	120	132			3		6	6	12	12	12	12	15	24	30	33	60	66	333	120	132																	
o2		111	−9	0	−9	3	1	0	1	1	−10	1	1	1	0	0	1	2	0	1	2	:	ooo2	5i3+6	0	5i3+6	−i3	5i3+6	−i3	−i3	−i3	1	1	1	−z3	1	−z3	0	1	−z3	*	+													*	+	*	+
o2		111	−9	0	11	−1	1	0	−1	1	−10	1	1	1	2	2	1	2	0	−1	0	:	ooo2	5i3+6	0	5i3+6	−i3	−5i3−4	i3+2	−1	−1	1	−1	1	−z3	1	−z3	0	−1	z3+2	*	+													*	+	*	+
o4		111	11	0	10i−1	1	1	2	−i	1	−10	1	1	1	i−1	r3+1	−1	0	0	−i	−i−1	:	oooo4	5i3+6	0	−5i3−4	−1	10z12**−1	1	−z12−z3**	z12−2i+z3**	1	−i	1	−z3	−1	z3+2	0	−i	−z12**−1	**	+2													**	+2	**	+2
o2		480	0	0	0	0	0	0	0	0	−4	7	−4	−4	0	0	0	0	−1	0	0		o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*	+												o2		o2	
o2		480	0	0	0	0	0	0	0	0	−4	−4	7	−4	0	0	0	0	−1	0	0																					o2													*	+	*	
o2		480	0	0	0	0	0	0	0	0	−4	−4	−4	7	0	0	0	0	−1	0	0																																				*	+
o2		1110	30	0	−10	2	0	0	0	0	21	−1	−1	−1	2	2	0	−3	0	0	1	:	ooo2	−10i3	0	−10i3	2i3	−10	2	−2z3−2	−2z3−2	0	0	0	i3	0	i3	0	0	1	*	+													*	+	*	+
o2		1110	−10	0	10	2	0	−4	0	0	21	−1	−1	−1	−2	2	0	1	0	0	−1	:	ooo2	−10i3	0	10i3+20	2	10	2	2z3+2	−2z3−2	0	0	0	i3	0	−i3−2	0	0	−1	*	+													*	+	*	+
o4		1110	−10	0	10	2	0	2	0	0	21	−1	−1	−1	−3i+1	r3−1	0	1	0	0	−1	:	oooo4	−10i3	0	−10−2z3−2	−5r3+15i−5i3−5−2z3−2			−z12+2i+z3	z12−2i−z3	0	0	0	i3	0	1	0	0	z12−2i−z3**	**	+2													**	+2	**	+2
o4		1110	−10	0	20i−10	−2	0	2	0	0	21	−1	−1	−1	−i−1	−r3+1	0	1	0	0	−2i+1	:	oooo4	−10i3	0	−10−2z3−2	10z12−10z3**		2z3+2	−3z12+2i−z3	−z12+2i+z3	0	0	0	i3	0	1	0	0	−z12+z3**	**	+2													**	+2	**	+2
o2		1221	21	0	21	−3	1	0	1	1	11	0	0	0	0	0	1	−1	0	1	−1	:	ooo2	−5i3+6	0	−5i3+6	i3	−5i3+6	i3	i3	i3	1	1	1	z3+1	1	z3+1	0	1	z3+1	*	+													*	+	*	+
o2		1221	21	0	1	1	1	0	−1	1	11	0	0	0	−2	−2	1	−1	0	−1	1	:	ooo2	−5i3+6	0	−5i3+6	i3	5i3+16	−i3−2	1	1	1	−1	1	z3+1	1	z3+1	0	−1	−z3−1	*	+													*	+	*	+
o4		1221	1	0	10i−11	−1	1	−2	i	1	11	0	0	0	i+1	r3−1	−1	1	0	i	−i	:	oooo4	−5i3+6	0	5i3+16	1	10z12**−11	−1	−z12+z3**	z12−2i−z3**	1	i	1	z3+1	−1	−z3−1	0	i	−z12**	**	+2													**	+2	**	+2
o4		1332	−12	0	12i	0	2	0	0	−2	1	1	1	1	0	0	2i	−1	0	0	i	:	oooo4	12	0	−12	0	12i	0	0	0	2	0	−2	1	2i	−1	0	0	i	**	+2													**	+2	**	+2
o4		1332	12	0	12	0	b5	0	2	b5	1	1	1	1	0	0	b5	1	0	b5	1	:	oooo4	12	0	12	0	12	0	0	0	b5	2	b5	1	b5	1	0	b5	1	**	+2													**	+2	**	+2
o4		1332	12	0	12	0	b5	0	−2	b5	1	1	1	1	0	0	b5	1	0	−b5	1	:	oooo4	12	0	12	0	12	0	0	0	b5	−2	b5	1	b5	1	0	−b5	1	**	+2													**	+2	**	+2
o8		1332	12	0	−12	0	b5	0	2i	b5	1	1	1	1	0	0	−b5	1	0	m20	−1	:	oooo8	12	0	12	0	−12	0	0	0	b5	2i	b5	1	−b5	1	0	m20	−1	**	+4													**	+4	**	+4
o16		1332	−12	0	12i	0	b5	0	0	−b5	1	1	1	1	0	0	m20	−1	0	m'40**3	i	:	oooo16	12	0	−12	0	12i	0	0	0	b5	0	−b5	1	m20	−1	0	m'40**3	i	**	+8													**	+8	**	+8
o24		1440	0	0	0	0	0	0	0	0	−12	−1	−1	−1	0	0	0	0	−x37	0	0	:	oooo24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−x'333**46	0	0	**	+12													**	+12	**	+12

$O_8^+(2)$

Orthogonal group $O_8^+(2) \cong D_4(2)$

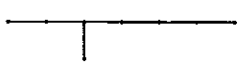
Order = 174,182,400 = $2^{12}.3^5.5^2.7$ Mult = 2^2 Out = S_3

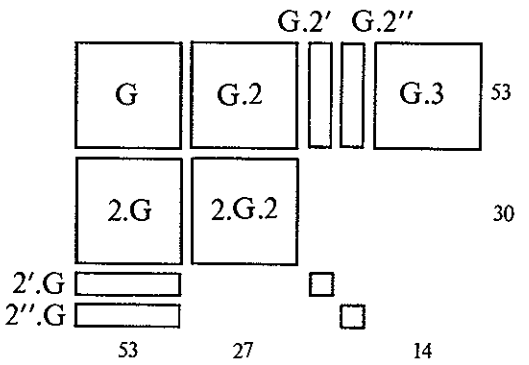
Constructions

Orthogonal $GO_8^+(2) \cong PGO_8^+(2) \cong SO_8^+(2) \cong PSO_8^+(2) \cong G.2$: all 8 x 8 matrices over F_2 preserving a non-singular quadratic form of Witt defect 0, for example $x_1x_2 + x_3x_4 + x_5x_6 + x_7x_8$; $O_8^+(2) \cong G$

Weyl 2.G.2 : the Weyl group of the E_8 root lattice, generated by the 120 reflections in the 240 root vectors, which in the standard base are $(\pm1,\pm1,0,0,0,0,0,0)$ (56) and $(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2})$ (even sign combinations) (64).
Various other bases are useful, for example that described in the "Cayley" construction below, and that in which the root vectors are $\pm(\frac{2}{3}^3,-\frac{1}{3}^6)^S$ (84) and $(1,-1,0^7)^S$ (36) on 9 coordinates adding to zero.
Reducing mod 2 shows the isomorphism with $O_8^+(2)$.

Cayley $2^2.G$: the isotopy group of the non-associative ring of Cayley integers. The real Cayley algebra is spanned by $i_\infty = 1, i_0,\ldots,i_6$ with the condition that for the sets of subscripts:
$\{\infty,1,2,4\}, \{\infty,2,3,5\}, \{\infty,3,4,6\}, \{\infty,0,4,5\}, \{\infty,1,5,6\}, \{\infty,0,2,6\}, \{\infty,0,1,3\}$
$\langle i_\infty,i_a,i_b,i_c\rangle \equiv \langle 1,i,j,k\rangle$ is a quaternion subalgebra. In this the ring of Cayley integers is the lattice (a scaled copy of the E_8 root lattice) of vectors $\Sigma a_n i_n$ ($n = \infty,0,\ldots,6$) for which all $2a_n$ are in $\mathbf{Z}$ and $\{n:a_n$ in $\mathbf{Z}\}$ is one of:
$\{\}, \{0,1,2,4\}, \{0,2,3,5\}, \{0,3,4,6\}, \{\infty,0,4,5\}, \{0,1,5,6\}, \{\infty,0,2,6\}, \{\infty,0,1,3\}$
or their complements. There are 240 unit Cayley numbers, corresponding to the E_8 root vectors.
An isotopy is a triple of maps (A,B,C) such that $xyz = 1 \Leftrightarrow x^Ay^Bz^C = 1$. There are two such isotopies (A,B,C) and (-A,-B,C) for each determinant 1 symmetry C of the lattice. The isotopy group can be extended to $2^2.G.S_3$ by adjoining the triality automorphism $(A,B,C) \rightarrow (B,C,A)$, and the duality automorphism $(A,B,C) \rightarrow (J^{-1}B^{-1}J, J^{-1}A^{-1}J, J^{-1}C^{-1}J)$ where J is the Cayley conjugation map $x \rightarrow \bar{x}$. The isotopies are generated by (L_u,R_u,B_u), where $L_u : x \rightarrow ux$, $R_u : x \rightarrow xu$, $B_u : x \rightarrow u^{-1}xu^{-1}$ and u ranges over the 240 unit Cayley numbers. The reflection in u is $-J.B_u$.

Presentation 2.G.2 ≅ 



Remark : $G.S_3$ is a maximal subgroup of the Fischer group Fi_{22}

Maximal subgroups / Specifications

Order	Index	Structure	G.2	G.3	$G.S_3$	Character	Abstract	Orthogonal	Weyl	Cayley
1451520	120	$S_6(2)$	: $S_6(2)$ x 2	$U_3(3)$:2 x 3	$U_3(3)$:2 x S_3	1a+35a+84a	C(2F)	non-isotropic point	root vector, A_1+E_7	$\langle J.B_u\rangle$
1451520	120	$S_6(2)$				1a+35b+84b				$\langle J.L_u\rangle$
1451520	120	$S_6(2)$				1a+35c+84c				$\langle J.R_u\rangle$
1290240	135	$2^6:A_8$	: $2^6:S_8$	$[2^{12}.3^2]$	$[2^{13}.3^2]$	1a+50a+84a	$N(2^6) = N(2A_{35}B_{28})$	isotropic point	standard base	congruence base mod 2
1290240	135	$2^6:A_8$	$2^{3+6}:(L_3(2)x2)$			1a+50a+84b	$N(2^6) = N(2A_{35}C_{28})$	maximal isotropic subspace		null subalgebra mod 2
1290240	135	$2^6:A_8$				1a+50a+84c	$N(2^6) = N(2A_{35}D_{28})$	maximal isotropic subspace		null subalgebra mod 2
181440	960	A_9	: S_9			1a+84a+175a+700b				
181440	960	A_9				1a+84b+175a+700c				
181440	960	A_9				1a+84c+175a+700d				
155520	1120	$(3xU_4(2)):2$	: S_3 x $U_4(2):2$	$3_+^{1+4}:2S_4$	$3_+^{1+4}:(2S_4x2)$	1a+35a+84a+300a+700b	N(3A)	minus line	A_2+E_6	
155520	1120	$(3xU_4(2)):2$				1a+35b+84b+300a+700c	N(3B)	$U_4(2)$		
155520	1120	$(3xU_4(2)):2$				1a+35c+84c+300a+700d	N(3C)	$U_4(2)$		
110592	1575	$2_+^{1+8}:(S_3xS_3xS_3)$	: H.2	: H.3	: $H.S_3$	1a+50a+84abc+300a+972a	N(2A)	isotropic line		
15552	11200	$3^4:2^3.S_4$	: S_3wrS_4	: H.3	: $H.S_3$		$N(3^4) = N(3A_4B_4C_4D_{16}E_{12})$	$O_2^-(2)wrS_4$	$A_2+A_2+A_2+A_2$	
14400	12096	$(A_5xA_5):2^2$	: S_5wr2	$5^2:4A_4$	$5^2:4S_4$		$N(2B,3A,5A)^2$	$O_4^-(2)wr2$	A_4+A_4	
14400	12096	$(A_5xA_5):2^2$					$N(2C,3B,5B)^2$	$O_4^+(4)$		
14400	12096	$(A_5xA_5):2^2$					$N(2D,3C,5C)^2$	$O_4^+(4)$		

$O_8^+(2)$

;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		174182400	110592	46080	46080	46080	3072	77760	77760	77760	1944	648	4608	512	192	192	192	64	300	300	300	1728	1728	1728	288	288	288	216	216	216	216	72	72	72	24	7	32	32	27	27	27	20	20	20	144	144	144	36	24	24	24	15	15	15	
p power			A	A	A	A	A	A	A	A	A	A	A	A	B	C	D	E	A	A	A	AA	BA	CA	AB	BC	CD	DA	EA	EA	EA	EB	EC	ED	EE	A	A	B	D	D	D	AB	BC	CD	AA	BA	CA	GA	DC	ED	FE	AA	BB	CC	
p' part			A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	CA	AB	BC	CD	DA	EA	EA	EA	EB	EC	ED	EE	A	A	A	A	A	A	AB	BC	CD	AA	BA	CA	DA	AC	BD	CE	AA	BB	CC	
ind		1A	2A	2B	2C	2D	2E	3A	3B	3C	3D	3E	4A	4B	4C	4D	4E	4F	5A	5B	5C	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	6N	7A	8A	8B	9A	9B	9C	10A	10B	10C	12A	12B	12C	12D	12E	12F	12G	15A	15B	15C	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	28	-4	4	4	4	-4	10	10	10	1	1	8	0	0	0	0	0	3	3	3	2	2	2	-2	-2	-2	5	-1	-1	-1	1	1	1	-1	0	-2	2	1	1	1	-1	-1	-1	2	2	2	-1	0	0	0	0	0	0	χ_{2}
χ_{3}	+	35	3	11	-5	-5	3	14	5	5	-1	2	7	-1	3	-1	-1	-1	5	0	0	6	-3	-3	2	1	1	3	0	0	0	2	-2	-2	0	0	1	1	2	-1	-1	1	0	0	-2	1	1	1	0	-1	-1	-1	0	0	χ_{3}
χ_{4}	+	35	3	-5	11	-5	3	5	14	5	-1	2	7	-1	-1	3	-1	-1	0	5	0	-3	6	-3	1	2	1	3	0	0	0	-2	2	-2	0	0	1	1	-1	2	-1	0	1	0	1	-2	1	1	-1	0	-1	0	-1	0	χ_{4}
χ_{5}	+	35	3	-5	-5	11	3	5	5	14	-1	2	7	-1	-1	-1	3	-1	0	0	5	-3	-3	6	1	1	2	3	0	0	0	-2	-2	2	0	0	1	1	-1	-1	2	0	0	1	1	1	-2	1	-1	-1	0	0	0	-1	χ_{5}
χ_{6}	+	50	18	10	10	10	2	5	5	5	-4	5	-2	6	2	2	2	2	0	0	0	-3	-3	-3	1	1	1	0	3	3	3	1	1	1	-1	1	0	0	-1	-1	-1	0	0	0	1	1	1	-2	-1	-1	-1	0	0	0	χ_{6}
χ_{7}	+	84	20	20	4	4	4	21	-6	-6	3	3	4	4	4	0	0	0	4	-1	-1	5	2	2	5	-2	-2	-1	5	-1	-1	-1	1	1	1	0	0	0	0	0	0	0	-1	-1	1	-2	-2	1	1	0	0	1	-1	-1	χ_{7}
χ_{8}	+	84	20	4	20	4	4	-6	21	-6	3	3	4	4	0	4	0	0	-1	4	-1	2	5	2	-2	5	-2	-1	-1	5	-1	1	-1	1	1	0	0	0	0	0	0	-1	0	-1	-2	1	-2	1	0	1	0	-1	1	-1	χ_{8}
χ_{9}	+	84	20	4	4	20	4	-6	-6	21	3	3	4	4	0	0	4	0	-1	-1	4	2	2	5	-2	-2	5	-1	-1	-1	5	1	1	-1	1	0	0	0	0	0	0	-1	-1	0	-2	-2	1	1	0	0	1	-1	-1	1	χ_{9}
χ_{10}	+	175	-17	15	15	15	-1	-5	-5	-5	13	4	-1	-1	-1	-1	-1	3	0	0	0	-5	-5	-5	3	3	3	1	-2	-2	-2	0	0	0	2	0	-1	-1	1	1	1	0	0	0	-1	-1	-1	-1	-1	-1	-1	0	0	0	χ_{10}
χ_{11}	+	210	-14	26	10	10	2	39	-15	-15	-6	3	6	-2	2	-2	-2	2	5	0	0	7	1	1	-1	1	1	-2	-5	1	1	-1	1	1	-1	0	0	0	0	0	0	1	0	0	3	-3	-3	0	-1	1	1	-1	0	0	χ_{11}
χ_{12}	+	210	-14	10	26	10	2	-15	39	-15	-6	3	6	-2	-2	2	-2	2	0	5	0	1	7	1	1	-1	1	-2	1	-5	1	1	-1	1	-1	0	0	0	0	0	0	0	1	0	-3	3	-3	0	1	-1	1	0	-1	0	χ_{12}
χ_{13}	+	210	-14	10	10	26	2	-15	-15	39	-6	3	6	-2	-2	-2	2	2	0	0	5	1	1	7	1	1	-1	-2	1	1	-5	1	1	-1	-1	0	0	0	0	0	0	0	0	1	-3	-3	3	0	1	1	-1	0	0	-1	χ_{13}
χ_{14}	+	300	12	20	20	20	12	30	30	30	3	-6	8	0	0	0	0	0	0	0	0	6	6	6	2	2	2	3	0	0	0	2	2	2	0	-1	2	-2	0	0	0	0	0	0	2	2	2	-1	0	0	0	0	0	0	χ_{14}
χ_{15}	+	350	-2	-10	-10	-10	-2	35	35	35	-1	-1	26	2	-2	-2	-2	2	0	0	0	-5	-5	-5	-1	-1	-1	7	1	1	1	-1	-1	-1	1	0	0	0	-1	-1	-1	0	0	0	-1	-1	-1	-1	1	1	1	0	0	0	χ_{15}
χ_{16}	+	525	45	5	5	5	-19	30	30	30	12	3	-7	1	-3	-3	-3	1	0	0	0	6	6	6	2	2	2	0	3	3	3	-1	-1	-1	-1	0	-1	-1	0	0	0	0	0	0	2	2	2	2	0	0	0	0	0	0	χ_{16}
χ_{17}	+	567	-9	39	-9	-9	-9	81	0	0	0	0	15	-1	-1	3	3	-1	7	-3	-3	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	-1	1	1	-3	0	0	0	-1	0	0	1	0	0	χ_{17}
χ_{18}	+	567	-9	-9	39	-9	-9	0	81	0	0	0	15	-1	3	-1	3	-1	-3	7	-3	0	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	1	-1	1	0	-3	0	0	0	-1	0	0	1	0	χ_{18}
χ_{19}	+	567	-9	-9	-9	39	-9	0	0	81	0	0	15	-1	3	3	-1	-1	-3	-3	7	0	0	9	0	0	-3	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	1	1	-1	0	0	-3	0	0	0	-1	0	0	1	χ_{19}
χ_{20}	+	700	92	20	20	20	-4	-20	-20	-20	-2	7	0	8	0	0	0	0	0	0	0	-4	-4	-4	-4	-4	-4	2	-1	-1	-1	-1	-1	-1	-1	0	2	-2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{20}
χ_{21}	+	700	-4	60	-20	-20	12	55	10	10	7	4	-4	-4	4	0	0	0	0	0	0	-1	2	2	3	-2	-2	-1	-4	2	2	0	-2	-2	0	0	0	0	-2	1	1	0	0	0	-1	2	2	-1	1	0	0	0	0	0	χ_{21}
χ_{22}	+	700	-4	-20	60	-20	12	10	55	10	7	4	-4	-4	0	4	0	0	0	0	0	2	-1	2	-2	3	-2	-1	2	-4	2	-2	0	-2	0	0	0	0	1	-2	1	0	0	0	2	-1	2	-1	0	1	0	0	0	0	χ_{22}
χ_{23}	+	700	-4	-20	-20	60	12	10	10	55	7	4	-4	-4	0	0	4	0	0	0	0	2	2	-1	-2	-2	3	-1	2	2	-4	-2	-2	0	0	0	0	0	1	1	-2	0	0	0	2	2	-1	-1	0	0	1	0	0	0	χ_{23}
χ_{24}	+	840	8	24	-40	-40	8	-24	30	30	3	3	16	0	0	0	0	0	-5	0	0	8	-10	-10	0	2	2	-1	-1	-1	-1	3	-1	-1	-1	0	0	0	0	0	0	-1	0	0	4	-2	-2	1	0	0	0	1	0	0	χ_{24}
χ_{25}	+	840	8	-40	24	-40	8	30	-24	30	3	3	16	0	0	0	0	0	0	-5	0	-10	8	-10	2	0	2	-1	-1	-1	-1	-1	3	-1	-1	0	0	0	0	0	0	0	-1	0	-2	4	-2	1	0	0	0	0	1	0	χ_{25}
χ_{26}	+	840	8	-40	-40	24	8	30	30	-24	3	3	16	0	0	0	0	0	0	0	-5	-10	-10	8	2	2	0	-1	-1	-1	-1	-1	-1	3	-1	0	0	0	0	0	0	0	0	-1	-2	-2	4	1	0	0	0	0	0	1	χ_{26}
χ_{27}	+	972	108	36	36	36	12	0	0	0	0	0	0	8	0	0	0	0	-3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-2	2	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	χ_{27}
χ_{28}	+	1050	58	50	-30	-30	-6	15	15	15	-3	6	-10	-2	2	-2	-2	-2	0	0	0	-17	7	7	-1	3	3	1	4	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	1	1	0	0	0	χ_{28}
χ_{29}	+	1050	58	-30	50	-30	-6	15	15	15	-3	6	-10	-2	-2	2	-2	-2	0	0	0	7	-17	7	3	-1	3	1	-2	4	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	-1	1	0	0	0	χ_{29}
χ_{30}	+	1050	58	-30	-30	50	-6	15	15	15	-3	6	-10	-2	-2	-2	2	-2	0	0	0	7	7	-17	3	3	-1	1	-2	-2	4	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	-1	0	0	0	χ_{30}
χ_{31}	+	1344	64	64	0	0	0	84	-24	-24	-6	-6	0	0	0	0	0	0	-1	4	4	4	-8	-8	4	0	0	-2	4	-2	-2	-2	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	1	1	χ_{31}
χ_{32}	+	1344	64	0	64	0	0	-24	84	-24	-6	-6	0	0	0	0	0	0	4	-1	4	-8	4	-8	0	4	0	-2	-2	4	-2	0	-2	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	-1	1	χ_{32}
χ_{33}	+	1344	64	0	0	64	0	-24	-24	84	-6	-6	0	0	0	0	0	0	4	4	-1	-8	-8	4	0	0	4	-2	-2	-2	4	0	0	-2	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1	-1	χ_{33}
χ_{34}	+	1400	-72	40	40	40	-8	50	50	50	-4	5	-16	0	0	0	0	0	0	0	0	-6	-6	-6	-2	-2	-2	0	-3	-3	-3	1	1	1	1	0	0	0	-1	-1	-1	0	0	0	2	2	2	2	0	0	0	0	0	0	χ_{34}
χ_{35}	+	1575	-57	15	15	15	-9	90	-45	-45	9	0	11	3	-1	-1	-1	-1	0	0	0	-6	3	3	-6	3	3	-3	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	2	-1	-1	-1	2	-1	-1	0	0	0	χ_{35}
χ_{36}	+	1575	-57	15	15	15	-9	-45	90	-45	9	0	11	3	-1	-1	-1	-1	0	0	0	3	-6	3	3	-6	3	-3	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	2	-1	-1	-1	2	-1	0	0	0	χ_{36}
χ_{37}	+	1575	-57	15	15	15	-9	-45	-45	90	9	0	11	3	-1	-1	-1	-1	0	0	0	3	3	-6	3	3	-6	-3	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	-1	2	-1	-1	-1	2	0	0	0	χ_{37}
χ_{38}	+	2100	52	-60	20	20	4	75	-60	-60	-6	3	12	-4	-4	0	0	0	0	0	0	-5	4	4	3	-4	-4	-2	1	1	1	3	-1	-1	1	0	0	0	0	0	0	0	0	0	3	0	0	0	-1	0	0	0	0	0	χ_{38}
χ_{39}	+	2100	52	20	-60	20	4	-60	75	-60	-6	3	12	-4	0	-4	0	0	0	0	0	4	-5	4	-4	3	-4	-2	1	1	1	-1	3	-1	1	0	0	0	0	0	0	0	0	0	0	3	0	0	0	-1	0	0	0	0	χ_{39}
χ_{40}	+	2100	52	20	20	-60	4	-60	-60	75	-6	3	12	-4	0	0	-4	0	0	0	0	4	4	-5	-4	-4	3	-2	1	1	1	-1	-1	3	1	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	-1	0	0	0	χ_{40}
χ_{41}	+	2240	-64	64	0	0	0	-4	-40	-40	-10	2	0	0	0	0	0	0	-5	0	0	-4	8	8	4	0	0	2	-4	2	2	-2	0	0	0	0	0	0	2	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	0	0	χ_{41}
χ_{42}	+	2240	-64	0	64	0	0	-40	-4	-40	-10	2	0	0	0	0	0	0	0	-5	0	8	-4	8	0	4	0	2	2	-4	2	0	-2	0	0	0	0	0	-1	2	-1	0	-1	0	0	0	0	0	0	0	0	0	1	0	χ_{42}
χ_{43}	+	2240	-64	0	0	64	0	-40	-40	-4	-10	2	0	0	0	0	0	0	0	0	-5	8	8	-4	0	0	4	2	2	2	-4	0	0	-2	0	0	0	0	-1	-1	2	0	0	-1	0	0	0	0	0	0	0	0	0	1	χ_{43}
χ_{44}	+	2268	-36	12	-36	-36	12	81	0	0	0	0	-12	4	-4	0	0	0	-2	3	3	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	-1	-3	0	0	0	-1	0	0	1	0	0	χ_{44}
χ_{45}	+	2268	-36	-36	12	-36	12	0	81	0	0	0	-12	4	0	-4	0	0	3	-2	3	0	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1	0	-3	0	0	0	-1	0	0	1	0	χ_{45}
χ_{46}	+	2268	-36	-36	-36	12	12	0	0	81	0	0	-12	4	0	0	-4	0	3	3	-2	0	0	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	0	0	-3	0	0	0	-1	0	0	1	χ_{46}
χ_{47}	+	2835	-45	51	-45	-45	3	-81	0	0	0	0	3	3	-5	3	3	-1	5	0	0	-9	0	0	3	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	1	0	0	3	0	0	0	1	0	0	-1	0	0	χ_{47}
χ_{48}	+	2835	-45	-45	51	-45	3	0	-81	0	0	0	3	3	3	-5	3	-1	0	5	0	0	-9	0	0	3	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	1	0	0	3	0	0	0	1	0	0	-1	0	χ_{48}
χ_{49}	+	2835	-45	-45	-45	51	3	0	0	-81	0	0	3	3	3	3	-5	-1	0	0	5	0	0	-9	0	0	3	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	1	0	0	3	0	0	0	1	0	0	-1	χ_{49}
χ_{50}	+	3200	128	0	0	0	0	-40	-40	-40	14	-4	0	0	0	0	0	0	0	0	0	8	8	8	0	0	0	2	-4	-4	-4	0	0	0	0	1	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{50}
χ_{51}	+	4096	0	0	0	0	0	64	64	64	-8	-8	0	0	0	0	0	0	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	χ_{51}
χ_{52}	+	4200	-24	40	40	40	8	-30	-30	-30	15	-3	-8	-8	0	0	0	0	0	0	0	-6	-6	-6	-2	-2	-2	3	3	3	3	1	1	1	-1	0	0	0	0	0	0	0	0	0	-2	-2	-2	1	0	0	0	0	0	0	χ_{52}
χ_{53}	+	6075	27	-45	-45	-45	-21	0	0	0	0	0	-9	-1	3	3	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{53}
	ind	1	2	2	4	4	2	3	3	3	3	3	4	4	4	8	8	4	5	5	5	6	6	6	6	12	12	6	6	6	6	6	12	12	6	7	8	8	9	9	9	10	20	20	12	12	12	12	12	24	24	15	15	15	
		2		2				6	6	6	6	6	4		4				10	10	10	6			6			6				6				14		8	18	18	18	10			12	12	12	12	12			30	30	30	
χ_{54}	+	8	0	4	0	0	0	5	-4	-4	-1	2	4	0	2	0	0	0	3	-2	-2	-3	0	0	1	0	0	3	0	0	0	-2	0	0	0	1	0	2	2	-1	-1	-1	0	0	1	-2	-2	1	-1	0	0	0	1	1	χ_{54}
χ_{55}	+	56	0	-4	0	0	0	11	-16	-16	2	2	12	0	-2	0	0	0	1	-4	-4	3	0	0	-1	0	0	6	0	0	0	2	0	0	0	0	0	2	-1	-1	-1	1	0	0	3	0	0	0	1	0	0	1	-1	-1	χ_{55}
χ_{56}	+	112	0	24	0	0	0	31	4	4	4	4	8	0	4	0	0	0	7	2	2	-9	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	0	0	-1	2	2	2	1	0	0	1	-1	-1	χ_{56}
χ_{57}	+	160	0	16	0	0	0	34	-20	-20	-2	-2	16	0	0	0	0	0	5	0	0	-6	0	0	-2	0	0	6	0	0	0	-2	0	0	0	-1	0	0	1	1	1	1	0	0	-2	-2	-2	-2	0	0	0	-1	0	0	χ_{57}
χ_{58}	+	224	0	-16	0	0	0	14	-40	-4	-1	2	16	0	0	0	0	0	-1	-6	4	6	0	0	2	0	0	3	0	0	0	2	0	0	0	0	0	0	-1	-1	2	-1	0	0	-2	4	-2	1	0	0	0	-1	0	1	χ_{58}
χ_{59}	+	224	0	-16	0	0	0	14	-4	-40	-1	2	16	0	0	0	0	0	-1	4	-6	6	0	0	2	0	0	3	0	0	0	2	0	0	0	0	0	0	-1	2	-1	-1	0	0	-2	-2	4	1	0	0	0	-1	1	0	χ_{59}
χ_{60}	+	400	0	40	0	0	0	25	-20	-20	4	10	-8	0	4	0	0	0	0	0	0	9	0	0	1	0	0	0	0	0	0	-2	0	0	0	1	0	0	-2	1	1	0	0	0	1	-2	-2	-2	1	0	0	0	0	0	χ_{60}
χ_{61}	+	448	0	32	0	0	0	16	16	16	16	-2	0	0	0	0	0	0	-2	-2	-2	0	0	0	8	0	0	0	0	0	0	2	0	0	0	0	0	0	1	1	1	2	0	0	0	0	0	0	0	0	0	1	1	1	χ_{61}
χ_{62}	+	560	0	56	0	0	0	74	20	20	-7	2	8	0	4	0	0	0	5	0	0	-6	0	0	2	0	0	-3	0	0	0	2	0	0	0	0	0	0	-1	-1	-1	1	0	0	2	2	2	-1	-2	0	0	-1	0	0	χ_{62}
χ_{63}	+	672	0	16	0	0	0	-30	-84	24	-3	6	16	0	0	0	0	0	-3	-8	2	-6	0	0	-2	0	0	-3	0	0	0	-2	0	0	0	0	0	0	0	0	0	1	0	0	-2	-2	4	1	0	0	0	0	1	-1	χ_{63}
χ_{64}	+	672	0	16	0	0	0	-30	24	-84	-3	6	16	0	0	0	0	0	-3	2	-8	-6	0	0	-2	0	0	-3	0	0	0	-2	0	0	0	0	0	0	0	0	0	1	0	0	-2	4	-2	1	0	0	0	0	-1	1	χ_{64}
χ_{65}	+	840	0	4	0	0	0	21	-60	-60	3	-6	20	0	2	0	0	0	-5	0	0	-3	0	0	1	0	0	3	0	0	0	-2	0	0	0	0	0	-2	0	0	0	-1	0	0	5	2	2	-1	-1	0	0	1	0	0	χ_{65}
χ_{66}	+	1008	0	24	0	0	0	90	36	36	9	0	8	0	-4	0	0	0	3	-2	-2	-6	0	0	-6	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	2	2	2	-1	2	0	0	0	1	1	χ_{66}
χ_{67}	+	1008	0	24	0	0	0	-45	36	-72	9	0	8	0	-4	0	0	0	3	-2	-2	3	0	0	3	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	2	-4	-1	-1	0	0	0	1	-2	χ_{67}
χ_{68}	+	1008	0	24	0	0	0	-45	-72	36	9	0	8	0	-4	0	0	0	3	-2	-2	3	0	0	3	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	-4	2	-1	-1	0	0	0	-2	1	χ_{68}
χ_{69}	+	1296	0	-24	0	0	0	81	0	0	0	0	24	0	-4	0	0	0	1	6	6	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	-3	0	0	0	-1	0	0	1	0	0	χ_{69}
χ_{70}	+	1400	0	60	0	0	0	-25	20	20	-13	8	-4	0	-2	0	0	0	0	0	0	15	0	0	3	0	0	3	0	0	0	0	0	0	0	0	0	-2	2	-1	-1	0	0	0	-1	2	2	-1	1	0	0	0	0	0	χ_{70}
χ_{71}	+	1400	0	60	0	0	0	95	-40	-40	-4	-4	-4	0	-2	0	0	0	0	0	0	-9	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	-2	-1	-1	-1	0	0	0	-1	-4	-4	2	1	0	0	0	0	0	χ_{71}
χ_{72}	+	2400	0	-80	0	0	0	60	60	60	-3	6	16	0	0	0	0	0	0	0	0	12	0	0	4	0	0	-3	0	0	0	-2	0	0	0	-1	0	0	0	0	0	0	0	0	4	-2	-2	1	0	0	0	0	0	0	χ_{72}
χ_{73}	+	2800	0	-40	0	0	0	55	-80	-80	-8	10	-24	0	-4	0	0	0	0	0	0	-9	0	0	-1	0	0	0	0	0	0	2	0	0	0	0	0	0	1	1	1	0	0	0	3	0	0	0	-1	0	0	0	0	0	χ_{73}
χ_{74}	+	2800	0	-40	0	0	0	-5	40	-140	1	-2	8	0	4	0	0	0	0	0	0	3	0	0	-1	0	0	-3	0	0	0	2	0	0	0	0	0	0	1	-2	1	0	0	0	-1	-4	2	-1	1	0	0	0	0	0	χ_{74}
χ_{75}	+	2800	0	-40	0	0	0	-5	-140	40	1	-2	8	0	4	0	0	0	0	0	0	3	0	0	-1	0	0	-3	0	0	0	2	0	0	0	0	0	0	1	1	-2	0	0	0	-1	2	-4	-1	1	0	0	0	0	0	χ_{75}
χ_{76}	+	3240	0	84	0	0	0	81	0	0	0	0	-12	0	2	0	0	0	-5	0	0	9	0	0	-3	0	0	0	0	0	0	0	0	0	0	-1	0	2	0	0	0	-1	0	0	-3	0	0	0	-1	0	0	1	0	0	χ_{76}
χ_{77}	+	3360	0	16	0	0	0	-6	-60	-60	12	-6	-16	0	0	0	0	0	5	0	0	18	0	0	-2	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	1	0	0	2	2	2	2	0	0	0	-1	0	0	χ_{77}
χ_{78}	+	3584	0	0	0	0	0	-64	80	-64	-16	-4	0	0	0	0	0	0	4	-6	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1	0	0	0	0	0	0	0	0	0	0	1	0	1	χ_{78}
χ_{79}	+	3584	0	0	0	0	0	-64	-64	80	-16	-4	0	0	0	0	0	0	4	4	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	0	0	0	0	0	0	0	0	0	0	1	1	0	χ_{79}
χ_{80}	+	4096	0	0	0	0	0	64	64	64	-8	-8	0	0	0	0	0	0	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	χ_{80}
χ_{81}	+	4200	0	20	0	0	0	-75	60	60	15	6	4	0	2	0	0	0	0	0	0	-3	0	0	-7	0	0	3	0	0	0	2	0	0	0	0	0	-2	0	0	0	0	0	0	1	-2	-2	1	-1	0	0	0	0	0	χ_{81}
χ_{82}	+	4536	0	60	0	0	0	-81	0	0	0	0	12	0	-2	0	0	0	-4	6	6	-9	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	3	0	0	0	1	0	0	-1	0	0	χ_{82}
χ_{83}	+	5600	0	-80	0	0	0	20	20	20	11	2	-16	0	0	0	0	0	0	0	0	-12	0	0	4	0	0	3	0	0	0	-2	0	0	0	0	0	0	-1	-1	-1	0	0	0	-4	2	2	-1	0	0	0	0	0	0	χ_{83}
	and	1	2	4	2	4	2	3	3	3	3	3	4	4	8	4	8	4	5	5	5	6	6	6	12	6	12	6	6	6	6	12	6	12	6	7	8	8	9	9	9	20	10	20	12	12	12	12	24	12	24	15	15	15	
	no:	:2			:2			:2	:2	:2	:2	:2	:2			:2			:2	:2	:2		:2			:2		:2					:2			:2		:2	:2	:2	:2		:2		:2	:2	:2	:2		:2		:2	:2	:2	
	and	1	2	4	4	2	2	3	3	3	3	3	4	4	8	8	4	4	5	5	5	6	6	6	12	12	6	6	6	6	6	12	12	6	6	7	8	8	9	9	9	20	20	10	12	12	12	12	24	24	12	15	15	15	
	no:	:2				:2		:2	:2	:2	:2	:2	:2				:2		:2	:2	:2			:2			:2	:2						:2		:2		:2	:2	:2	:2			:2	:2	:2	:2	:2			:2	:2	:2	:2	

	; ;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	; ;	; ;	; ;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		1451520	4608	3840	2304	384	384	128	2160	648	144	108	72	36	96	32	8	30	144	48	36	12	12	7	9	10	12	15				12096	216	192	48	24	18	96	48	32	4	6	7	8	8	
		A	A	B	B	A	B	B	AF	DF	AG	EF	DG	EG	A	B	F	AF	DH	DG	KH	KJ	GI	AF	AQ	AG	AC	ADR				A	A	FA	FE	GE	D	UA	VA	UB	WF	DG	AF	MA	OB	
		A	A	A	A	A	A	A	AF	DF	AG	EF	DG	EG	A	A	A	AF	AH	AG	EH	EJ	DI	AF	AF	AG	AC	ADR				A	A	FA	FE	GE	A	FA	FA	FB	GF	DA	AF	FA	FB	
	fus ind	2F	2G	4G	4H	4I	4J	4K	6O	6P	6Q	6R	6S	6T	8C	8D	8E	10D	12H	12I	12J	12K	12L	14A	18A	20A	24A	30A	fus ind	fus ind	fus ind	3F	3G	6U	6V	6W	9D	12M	12N	12O	12P	18B	21A	24B	24C	
χ_{1}	: ++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	: ++	: ++	: +oo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	: ++	14	-2	6	-2	2	2	-2	2	5	-2	-1	1	1	0	0	0	-1	4	0	1	-1	-1	0	-1	1	0	2	: ++	: ++	: +oo	7	-2	-1	-1	2	1	-1	-1	3	0	-1	0	1	-1	χ_{2}
χ_{3}	: ++	21	5	9	1	1	-3	1	6	3	2	0	-1	2	3	-1	-1	1	4	0	-2	0	1	0	0	-1	0	1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{3}
χ_{4}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{4}
χ_{5}																														: ++																χ_{5}
χ_{6}	: ++	20	4	0	8	0	4	0	5	2	1	-1	-2	1	-2	2	0	0	-1	3	-1	1	0	-1	-1	0	1	0	: ++	: ++	: +oo	8	-1	0	2	-1	2	4	-2	0	-1	0	1	0	0	χ_{6}
χ_{7}	: ++	42	10	10	10	2	2	2	9	-3	1	3	1	1	2	2	0	2	1	1	1	-1	-1	0	0	0	-1	-1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{7}
χ_{8}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{8}
χ_{9}																														: ++																χ_{9}
χ_{10}	: ++	35	3	-5	-5	-5	3	3	5	-1	-3	2	3	0	-1	-1	1	0	1	1	-2	0	1	0	-1	0	-1	0	: ++	: ++	: +oo	7	-2	7	-1	2	1	-1	-1	-1	0	1	0	-1	-1	χ_{10}
χ_{11}	: ++	84	4	16	-8	0	4	0	9	-6	1	-3	-2	1	2	-2	0	-1	1	1	1	1	0	0	0	1	-1	-1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{12}
χ_{13}																														: ++																χ_{13}
χ_{14}	: ++	90	10	10	2	-2	6	2	0	9	4	0	1	-2	0	0	0	0	2	-2	2	0	1	-1	0	0	0	0	: ++	: ++	: +oo	27	0	3	3	0	0	-1	-1	3	0	0	-1	-1	1	χ_{14}
χ_{15}	: ++	70	-10	10	2	2	-2	2	-5	7	-1	1	-1	-1	-4	0	0	0	5	1	-1	1	-1	0	1	0	-1	0	: ++	: ++	: +oo	14	5	-2	-2	1	-1	2	2	2	-1	1	0	0	0	χ_{15}
χ_{16}	: ++	105	-7	5	13	-3	1	-3	0	6	-4	3	2	-1	3	-1	-1	0	-2	2	1	1	0	0	0	0	0	0	: ++	: ++	: +oo	21	3	-3	-1	-1	0	5	-1	1	1	0	0	-1	-1	χ_{16}
χ_{17}	: ++	189	-3	29	-3	-3	-3	-3	9	0	-3	0	0	0	1	1	1	-1	3	-1	0	0	0	0	0	-1	1	-1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{18}
χ_{19}																														: ++																χ_{19}
χ_{20}	: ++	70	-10	-10	14	-6	2	-2	10	-2	2	-5	2	-1	0	0	0	0	2	2	-1	-1	0	0	1	0	0	0	: ++	: ++	: +oo	7	-2	-1	-1	2	1	3	3	-1	0	-1	0	-1	1	χ_{20}
χ_{21}	: ++	210	18	10	-6	2	-6	2	15	3	3	0	3	0	-2	-2	0	0	-3	1	0	0	-1	0	0	0	1	0	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{22}
χ_{23}																														: ++																χ_{23}
χ_{24}	: ++	84	20	-4	-4	4	-4	-4	-6	3	2	3	-1	-1	0	0	0	-1	2	2	-1	-1	1	0	0	1	0	-1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{24}
χ_{25}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{25}
χ_{26}																														: ++																χ_{26}
χ_{27}	: ++	162	18	-6	18	6	6	2	0	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	1	0	-1	0	0	: ++	: ++	: +oo	27	0	3	3	0	0	3	3	-1	0	0	-1	1	-1	χ_{27}
χ_{28}	: ++	210	2	-10	14	-2	-6	-2	15	3	-1	0	-1	2	-4	0	0	0	-1	-1	2	0	1	0	0	0	-1	0	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{29}
χ_{30}																														: ++																χ_{30}
χ_{31}	: ++	336	16	16	16	0	0	0	6	-6	-2	0	-2	-2	0	0	0	1	-2	-2	-2	0	0	0	0	1	0	1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{32}
χ_{33}																														: ++																χ_{33}
χ_{34}	: ++	280	-8	0	-16	0	8	0	10	10	-2	1	-2	1	0	0	0	0	-4	0	-1	-1	0	0	1	0	0	0	: ++	: ++	: +oo	56	2	0	-2	-2	2	-4	2	0	0	0	0	0	0	χ_{34}
χ_{35}	: ++	315	-21	15	-9	7	3	-1	0	-9	0	0	3	0	-3	1	-1	0	0	0	0	0	1	0	0	0	0	0	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{35}
χ_{36}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{36}
χ_{37}																														: ++																χ_{37}
χ_{38}	: ++	210	-14	10	10	-6	2	2	-15	-6	1	3	-2	1	-2	-2	0	0	1	1	1	-1	0	0	0	0	1	0	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{38}
χ_{39}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{39}
χ_{40}																														: ++																χ_{40}
χ_{41}	: ++	336	16	-16	-16	0	0	0	6	-6	-2	0	-2	-2	0	0	0	1	2	2	2	0	0	0	0	-1	0	1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{41}
χ_{42}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{42}
χ_{43}																														: ++																χ_{43}
χ_{44}	: ++	378	-6	10	-6	-6	-6	2	-9	0	3	0	0	0	2	2	0	-2	-3	1	0	0	0	0	0	0	-1	1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{44}
χ_{45}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{45}
χ_{46}																														: ++																χ_{46}
χ_{47}	: ++	189	-3	-19	-3	-3	-3	5	9	0	-3	0	0	0	1	1	-1	-1	3	-1	0	0	0	0	0	1	1	-1	+	+	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{47}
χ_{48}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	: ++																	χ_{48}
χ_{49}																														: ++																χ_{49}
χ_{50}	: ++	160	32	0	0	0	0	0	-20	-2	-4	-2	2	2	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	: ++	: ++	: +oo	8	8	8	0	0	-1	0	0	0	0	-1	1	0	0	χ_{50}
χ_{51}	: ++	512	0	0	0	0	0	0	-16	8	0	-4	0	0	0	0	0	2	0	0	0	0	0	1	-1	0	0	-1	: ++	: ++	: +oo	64	-8	0	0	0	-2	0	0	0	0	0	1	0	0	χ_{51}
χ_{52}	: ++	420	4	-20	-4	-4	4	-4	0	-3	4	3	1	1	0	0	0	0	2	-2	-1	1	-1	0	0	0	0	0	: ++	: ++	: +oo	42	6	-6	2	2	0	-2	-2	-2	0	0	0	0	0	χ_{52}
χ_{53}	: ++	405	-27	-15	9	9	-3	1	0	0	0	0	0	0	3	-1	1	0	0	0	0	0	0	-1	0	0	0	0	: ++	: ++	: +oo	27	0	3	-3	0	0	3	-3	-1	0	0	-1	1	1	χ_{53}
	fus ind	2	2	4	4	4	4	4	6	6	6	6	6	6	8	8	8	10	12	12	12	12	12	14	18	20	24	30																		
		2	2	4		4			6	6	6	6	6	6	8			10	12	12			12	14	18	20	24	30																		
χ_{54}	: ++	6	2	4	0	2	0	0	3	-3	-1	0	-1	2	2	0	0	1	-3	1	0	0	-1	-1	0	-1	-1	-2																		χ_{54}
χ_{55}	: ++	14	-6	4	0	2	0	0	-1	-4	3	2	0	0	-2	0	0	-1	-3	1	0	0	2	0	-1	-1	1	-1																		χ_{55}
χ_{56}	: ++	56	8	16	0	0	0	0	11	2	-1	2	2	2	4	0	0	1	-3	1	0	0	0	0	-1	1	1	1																		χ_{56}
χ_{57}	: ++	64	0	16	0	0	0	0	4	-8	0	-2	0	0	0	0	0	-1	-6	-2	0	0	0	1	1	1	0	-1																		χ_{57}
χ_{58}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		χ_{58}
χ_{59}																																														χ_{59}
χ_{60}	: ++	120	8	0	0	0	0	0	15	-6	-1	0	2	2	-4	0	0	0	3	3	0	0	0	1	0	0	-1	0																		χ_{60}
χ_{61}	: ++	112	16	0	0	0	0	0	4	4	4	4	4	-2	0	0	0	2	0	0	0	0	0	0	1	0	0	-1																		χ_{61}
χ_{62}	: ++	196	12	24	0	4	0	0	16	7	0	-2	-3	0	0	0	0	1	0	0	0	0	1	0	1	-1	0	1																		χ_{62}
χ_{63}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		χ_{63}
χ_{64}																																														χ_{64}
χ_{65}	: ++	126	10	4	0	2	0	0	-9	-9	-5	0	1	-2	-2	0	0	1	-3	1	0	0	-1	0	0	-1	1	1																		χ_{65}
χ_{66}	: ++	252	-12	24	0	-4	0	0	0	9	0	0	3	0	0	0	0	-3	0	0	0	0	-1	0	0	-1	0	0																		χ_{66}
χ_{67}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		χ_{67}
χ_{68}																																														χ_{68}
χ_{69}	: ++	216	-24	16	0	0	0	0	-9	0	3	0	0	0	-4	0	0	1	-3	1	0	0	0	-1	0	1	-1	1																		χ_{69}
χ_{70}	: ++	210	6	-20	0	-10	0	0	15	3	3	0	-3	0	-2	0	0	0	-3	1	0	0	-1	0	0	0	1	0																		χ_{70}
χ_{71}	: ++	350	10	20	0	-6	0	0	5	-10	1	2	-2	-2	2	0	0	0	3	-1	0	0	0	0	-1	0	-1	0																		χ_{71}
χ_{72}	: ++	120	-24	0	0	8	0	0	0	3	0	6	-3	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0																		χ_{72}
χ_{73}	: ++	280	-24	0	0	0	0	0	-5	-8	3	-2	0	0	4	0	0	0	3	3	0	0	0	0	1	0	1	0																		χ_{73}
χ_{74}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		χ_{74}
χ_{75}																																														χ_{75}
χ_{76}	: ++	594	6	-4	0	6	0	0	9	0	-3	0	0	0	-2	0	0	-1	3	-1	0	0	0	-1	0	1	1	-1																		χ_{76}
χ_{77}	: ++	336	-16	-16	0	0	0	0	6	-6	2	0	2	2	0	0	0	1	0	-4	0	0	0	0	0	-1	0	1																		χ_{77}
χ_{78}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																		χ_{78}
χ_{79}																																														χ_{79}
χ_{80}	: ++	512	0	0	0	0	0	0	-16	8	0	-4	0	0	0	0	0	2	0	0	0	0	0	1	-1	0	0	-1																		χ_{80}
χ_{81}	: ++	210	-26	-20	0	-2	0	0	15	3	-5	0	1	-2	2	0	0	0	-3	1	0	0	1	0	0	0	-1	0																		χ_{81}
χ_{82}	: ++	378	30	-20	0	6	0	0	-9	0	3	0	0	0	2	0	0	-2	-3	1	0	0	0	0	0	0	-1	1																		χ_{82}
χ_{83}	: ++	280	8	0	0	-8	0	0	-20	1	-4	4	-1	2	0	0	0	0	0	0	0	0	1	0	1	0	0	0																		χ_{83}
																													: :																
																														: :																

$O_8^-(2)$

		1A	2A	2B	2C	3A	3B	3C	4A	4B	4C	4D	5A	6A	6B	6C	6D	6E
	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		197406720	84 320	46 080	3 072	60 480	1 080	648	1 536	1 536	192	64	180	576	360	288	72	72
p power			A	A	A	A	A	A	A	A	B	C	A	AA	BA	AB	CA	BB
p' part			A	A	A	A	A	A	A	A	A	A	A	AA	BA	AB	CA	BB
ind		1A	2A	2B	2C	3A	3B	3C	4A	4B	4C	4D	5A	6A	6B	6C	6D	6E
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	+	34	2	10	2	13	1	-2	-2	6	2	2	4	5	-1	1	2	1
χ_{3}	+	51	19	11	3	6	6	-3	7	-1	3	-1	1	-2	4	2	1	2
χ_{4}	+	84	20	20	4	21	3	3	4	4	4	0	4	5	5	5	-1	-1
χ_{5}	+	204	-20	20	-4	36	3	-3	0	8	0	0	4	4	-5	-4	1	-1
χ_{6}	+	204	44	4	-4	-6	9	6	8	0	0	0	-1	2	-1	-2	2	1
χ_{7}	+	357	5	13	-11	42	-3	6	1	9	-3	1	2	2	5	-2	2	1
χ_{8}	+	476	60	20	12	14	2	8	8	0	0	0	-4	6	0	2	0	2
χ_{9}	+	476	-4	36	12	56	-4	-1	0	8	0	0	1	8	-4	0	-1	0
χ_{10}	+	595	-45	35	3	-35	10	1	-5	11	-5	-1	0	-3	0	5	-3	2
χ_{11}	+	714	-22	50	-6	63	6	3	-10	-2	2	-2	4	-1	-4	-1	-1	2
χ_{12}	+	714	106	34	10	-21	9	-6	14	6	2	2	-1	-5	1	-5	-2	1
χ_{13}	+	1020	60	60	12	15	0	3	-4	-4	4	0	0	-9	0	3	3	0
χ_{14}	+	1071	-81	31	-1	21	6	0	7	-9	-1	3	1	-3	-6	1	0	-2
χ_{15}	+	1071	111	31	-17	21	6	0	7	-9	-1	-1	1	-3	6	1	0	-2
χ_{16}	+	1071	15	-41	15	21	6	0	-5	3	-1	-1	1	-3	0	1	0	-2
χ_{17}	+	1071	15	-41	15	21	6	0	-5	3	-1	-1	1	-3	0	1	0	-2
χ_{18}	+	1190	70	-50	-10	35	5	-7	2	10	-2	2	0	-5	-5	7	1	1
χ_{19}	+	1344	64	64	0	84	-6	-6	0	0	0	0	-1	4	4	4	-2	-2
χ_{20}	+	1428	20	-28	4	63	3	6	-4	12	-4	0	-2	-1	5	-1	2	-1
χ_{21}	+	2142	126	-10	-2	-21	3	0	2	-6	-2	-2	2	3	-3	-1	0	-1
χ_{22}	+	2142	126	-10	-2	-21	3	0	2	-6	-2	-2	2	3	-3	-1	0	-1
χ_{23}	+	2142	-66	-10	14	-21	3	0	2	-6	-2	2	2	3	3	-1	0	-1
χ_{24}	+	2142	-66	-10	14	-21	3	0	2	-6	-2	2	2	3	3	-1	0	-1
χ_{25}	+	2176	128	0	0	-8	4	-2	0	0	0	0	-4	8	-4	0	2	0
χ_{26}	+	2295	-105	15	-9	90	0	0	11	3	-1	-1	0	-6	0	-6	0	0
χ_{27}	+	2295	-105	15	-9	-45	0	0	11	3	-1	-1	0	3	0	3	0	0
χ_{28}	+	2295	-105	15	-9	-45	0	0	11	3	-1	-1	0	3	0	3	0	0
χ_{29}	+	2835	-45	-45	3	0	0	0	3	3	3	-1	0	0	0	0	0	0
χ_{30}	+	2835	-45	-45	3	0	0	0	3	3	3	-1	0	0	0	0	0	0
χ_{31}	+	2835	-45	-45	3	0	0	0	3	3	3	-1	0	0	0	0	0	0
χ_{32}	+	2835	-45	-45	3	0	0	0	3	3	3	-1	0	0	0	0	0	0
χ_{33}	+	2856	-88	40	8	42	9	3	-8	-8	0	0	-4	2	-1	-2	-1	1
χ_{34}	+	2856	104	56	8	-84	-9	3	0	16	0	0	1	-4	-1	-4	-1	-1
χ_{35}	+	3264	-64	64	0	-36	-6	6	0	0	0	0	-1	-4	-4	4	2	-2
χ_{36}	+	4096	0	0	0	64	-8	-8	0	0	0	0	-4	0	0	0	0	0
χ_{37}	+	4284	60	-20	12	21	-12	0	4	-12	-4	0	4	-3	0	1	0	4
χ_{38}	+	4760	-40	40	-8	-70	5	-10	-16	0	0	0	0	2	5	-2	2	1
χ_{39}	+	5355	75	-45	-21	0	0	9	-9	-1	3	3	0	0	0	0	-3	0

	6F	7A	8A	8B	9A	10A	B*	10C	12A	12B	12C	15A	B*	15C	17A	B*2	C*3	D*6	21A	B*	30A	B*	fus	ind
	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;
	24	21	32	32	9	60	60	20	48	24	12	90	90	45	17	17	17	17	21	21	30	30		
p power	BC	A	A	B	C	AA	AA	AB	AA	CC	DB	AB	AB	AA	A	A	A	A	AA	AA	BBB	AAB		
p' part	BC	A	A	A	A	AA	AA	AB	AA	AC	CB	AB	AB	AA	A	A	A	A	AA	AA	AAB	BBB		
ind	6F	7A	8A	8B	9A	10A	B*	10C	12A	12B	12C	15A	B*	15C	17A	B*2	C*3	D*6	21A	B*	30A	B*	fus	ind
χ_{1}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++
χ_{2}	-1	-1	0	0	1	2	2	0	1	-1	0	1	1	-2	0	0	0	0	-1	-1	-1	-1	:	++
χ_{3}	0	2	1	1	0	-1	-1	1	-2	0	-1	1	1	1	0	0	0	0	-1	-1	-1	-1	:	++
χ_{4}	1	0	0	0	0	0	0	0	1	1	1	-2	-2	1	-1	-1	-1	-1	0	0	0	0	:	++
χ_{5}	-1	1	2	-2	0	0	0	0	0	0	-1	-2	-2	1	0	0	0	0	1	1	0	0	:	++
χ_{6}	-1	1	-2	2	0	-1	-1	-1	2	0	0	-1	-1	-1	0	0	0	0	1	1	-1	-1	:	++
χ_{7}	1	0	-1	-1	0	0	0	-2	-2	0	0	2	2	2	0	0	0	0	0	0	0	0	:	++
χ_{8}	0	0	2	-2	-1	0	0	0	2	0	0	2	2	-1	0	0	0	0	0	0	0	0	:	++
χ_{9}	0	0	-2	2	-1	1	1	1	0	0	-1	1	1	1	0	0	0	0	0	0	1	1	:	++
χ_{10}	0	0	-1	-1	1	0	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_{11}	0	0	0	0	0	-2	-2	0	-1	-1	1	1	1	-2	0	0	0	0	0	0	1	1	:	++
χ_{12}	1	0	0	0	0	1	1	-1	-1	-1	0	-1	-1	-1	0	0	0	0	0	0	1	1	:	++
χ_{13}	0	-2	0	0	0	0	0	0	-1	1	-1	0	0	0	0	0	0	0	1	1	0	0	:	++
χ_{14}	2	0	-1	-1	0	-1	-1	1	1	-1	0	1	1	1	0	0	0	0	0	0	-1	-1	:	++
χ_{15}	-2	0	-1	-1	0	1	1	1	1	-1	0	1	1	1	0	0	0	0	0	0	1	1	:	++
χ_{16}	0	0	1	1	0	r5	-r5	-1	1	-1	0	1	1	1	0	0	0	0	0	0	r5	-r5		+
χ_{17}	0	0	1	1	0	-r5	r5	-1	1	-1	0	1	1	1	0	0	0	0	0	0	-r5	r5		
χ_{18}	-1	0	0	0	-1	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_{19}	0	0	0	0	0	-1	-1	-1	0	0	0	-1	-1	-1	1	1	1	1	0	0	-1	-1	:	++
χ_{20}	1	0	0	0	0	0	0	2	-1	-1	0	-2	-2	-2	0	0	0	0	0	0	0	0	:	++
χ_{21}	1	0	0	0	0	-2b5	*	0	-1	1	0	3b5+2	*	-1	0	0	0	0	0	0	b5	*		+
χ_{22}	1	0	0	0	0	*	-2b5	0	-1	1	0	*	3b5+2	-1	0	0	0	0	0	0	*	b5		
χ_{23}	-1	0	0	0	0	2b5	*	0	-1	1	0	3b5+2	*	-1	0	0	0	0	0	0	-b5	*		+
χ_{24}	-1	0	0	0	0	*	2b5	0	-1	1	0	*	3b5+2	-1	0	0	0	0	0	0	*	-b5		
χ_{25}	0	-1	0	0	1	-2	-2	0	0	0	0	-1	-1	2	0	0	0	0	-1	-1	1	1	:	++
χ_{26}	0	-1	1	1	0	0	0	0	2	2	0	0	0	0	0	0	0	0	-1	-1	0	0	:	++
χ_{27}	0	-1	1	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	-b21	*	0	0		+
χ_{28}	0	-1	1	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	*	-b21	0	0		
χ_{29}	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	-d17	*2	*3	*6	0	0	0	0		+
χ_{30}	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	*2	-d17	*6	*3	0	0	0	0		
χ_{31}	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	*6	*3	-d17	*2	0	0	0	0		+
χ_{32}	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	*3	*6	*2	-d17	0	0	0	0		
χ_{33}	-1	0	0	0	0	2	2	0	-2	0	1	-1	-1	2	0	0	0	0	0	0	-1	-1	:	++
χ_{34}	-1	0	0	0	0	-1	-1	1	0	0	1	1	1	1	0	0	0	0	0	0	-1	-1	:	++
χ_{35}	0	2	0	0	0	1	1	-1	0	0	0	-1	-1	-1	0	0	0	0	-1	-1	1	1	:	++
χ_{36}	0	1	0	0	1	0	0	0	0	0	0	2	2	-1	-1	-1	-1	-1	1	1	0	0	:	++
χ_{37}	0	0	0	0	0	0	0	0	1	-1	0	-2	-2	1	0	0	0	0	0	0	0	0	:	++
χ_{38}	1	0	0	0	-1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++
χ_{39}	0	0	1	1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	:	++

	2D	2E	4E	4F	4G	4H	4I	6G	6H	6I	6J	6K	6L	8C	8D	8E	10D	12D	12E	12F	12G	12H	14A	18A	20A	24A	30C	
	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
	1451 520	4 608	3 840	2 304	384	384	128	2 160	648	144	108	72	36	96	32	8	30	144	48	36	12	12	7	9	10	12	15	
p power	A	A	B	B	A	B	B	AD	CD	AE	BD	CE	BE	A	B	D	AD	BF	BE	EF	DG	EH	AD	AH	CE	AC	CDG	
p' part	A	A	A	A	A	A	A	AD	CD	AE	BD	CE	BE	A	A	A	AD	AF	AE	BF	CG	BH	AD	AD	AE	AC	CDG	
ind	2D	2E	4E	4F	4G	4H	4I	6G	6H	6I	6J	6K	6L	8C	8D	8E	10D	12D	12E	12F	12G	12H	14A	18A	20A	24A	30C	
χ_{1}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	20	4	0	8	0	4	0	5	2	1	-1	-2	1	-2	2	0	0	-1	3	-1	0	1	-1	-1	0	1	0	χ_{2}
χ_{3}	21	5	9	1	1	-3	1	6	3	2	0	-1	2	3	-1	-1	1	4	0	-2	1	0	0	0	-1	0	1	χ_{3}
χ_{4}	42	10	10	10	2	2	2	9	-3	1	3	1	1	2	2	0	2	1	1	1	-1	-1	0	0	0	-1	-1	χ_{4}
χ_{5}	78	-2	-10	14	2	2	-2	6	-3	-2	-3	1	1	0	0	0	-2	2	2	-1	-1	-1	1	0	0	0	1	χ_{5}
χ_{6}	6	-10	6	-2	-6	2	-2	6	6	2	-3	2	-1	0	0	0	1	4	0	1	0	-1	-1	0	1	0	1	χ_{6}
χ_{7}	105	-7	5	13	-3	1	-3	0	6	-4	3	2	-1	3	-1	-1	0	-2	2	1	0	1	0	0	0	0	0	χ_{7}
χ_{8}	98	18	10	2	6	6	2	-4	8	0	2	0	0	0	0	0	-2	2	-2	2	0	0	0	-1	0	0	1	χ_{8}
χ_{9}	154	10	-6	18	-2	6	2	4	1	4	-2	1	-2	0	0	0	-1	0	0	0	1	0	0	1	-1	0	-1	χ_{9}
χ_{10}	35	3	-5	-5	-5	3	3	5	-1	-3	2	3	0	-1	-1	1	0	1	1	-2	1	0	0	-1	0	-1	0	χ_{10}
χ_{11}	210	2	-10	14	-2	-6	-2	15	3	-1	0	-1	2	-4	0	0	0	-1	-1	2	1	0	0	0	0	-1	0	χ_{11}
χ_{12}	84	4	16	-8	0	4	0	9	-6	1	-3	-2	1	2	-2	0	-1	1	1	1	0	1	0	0	1	-1	-1	χ_{12}
χ_{13}	210	18	10	-6	2	-6	2	15	3	3	0	3	0	-2	-2	0	0	-3	1	0	-1	0	0	0	0	1	0	χ_{13}
χ_{14}	189	-3	-19	-3	-3	-3	5	9	0	-3	0	0	0	1	1	-1	-1	3	-1	0	0	0	0	0	1	1	-1	χ_{14}
χ_{15}	189	-3	29	-3	-3	-3	-3	9	0	-3	0	0	0	1	1	1	-1	3	-1	0	0	0	0	0	-1	1	-1	χ_{15}
χ_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{16}
																												χ_{17}
χ_{18}	70	-10	10	2	2	-2	2	-5	7	-1	1	-1	-1	-4	0	0	0	5	1	-1	-1	1	0	1	0	-1	0	χ_{18}
χ_{19}	336	16	16	16	0	0	0	6	-6	-2	0	-2	-2	0	0	0	1	-2	-2	-2	0	0	0	0	1	0	1	χ_{19}
χ_{20}	210	-14	10	10	-6	2	2	-15	-6	1	3	-2	1	-2	-2	0	0	1	1	1	0	-1	0	0	0	1	0	χ_{20}
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
																												χ_{22}
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{23}
																												χ_{24}
χ_{25}	160	32	0	0	0	0	0	-20	-2	-4	-2	2	2	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	χ_{25}
χ_{26}	405	-27	-15	9	9	-3	1	0	0	0	0	0	0	3	-1	1	0	0	0	0	0	0	-1	0	0	0	0	χ_{26}
χ_{27}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{27}
																												χ_{28}
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{29}
																												χ_{30}
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{31}
																												χ_{32}
χ_{33}	420	4	-20	-4	-4	4	-4	0	-3	4	3	1	1	0	0	0	0	2	-2	-1	-1	1	0	0	0	0	0	χ_{33}
χ_{34}	84	20	-4	-4	4	-4	-4	-6	3	2	3	-1	-1	0	0	0	-1	2	2	-1	1	-1	0	0	1	0	-1	χ_{34}
χ_{35}	336	16	-16	-16	0	0	0	6	-6	-2	0	-2	-2	0	0	0	1	2	2	2	0	0	0	0	-1	0	1	χ_{35}
χ_{36}	512	0	0	0	0	0	0	-16	8	0	-4	0	0	0	0	0	2	0	0	0	0	0	1	-1	0	0	-1	χ_{36}
χ_{37}	378	-6	10	-6	-6	-6	2	-9	0	3	0	0	0	2	2	0	-2	-3	1	0	0	0	0	0	0	-1	1	χ_{37}
χ_{38}	280	-8	0	-16	0	8	0	10	10	-2	1	-2	1	0	0	0	0	-4	0	-1	0	-1	0	1	0	0	0	χ_{38}
χ_{39}	315	-21	15	-9	7	3	-1	0	-9	0	0	3	0	-3	1	-1	0	0	0	0	1	0	0	0	0	0	0	χ_{39}

G	G.2
39	27

39

$O_8^-(2)$ and $^3D_4(2)$

Orthogonal group $O_8^-(2) \cong {}^2D_4(2)$

Order = 197,406,720 = $2^{12}.3^4.5.7.17$ Mult = 1 Out = 2

Constructions

Orthogonal $GO_8^-(2) \cong PGO_8^-(2) \cong SO_8^-(2) \cong PSO_8^-(2) \cong G.2$: all 8 x 8 matrices over $\mathbb{F}_2$ preserving a quadratic form of Witt defect 1, for example $x_1x_2 + x_3x_4 + x_5x_6 + x_7^2 + x_7x_8 + x_8^2$; $O_8^-(2) \cong G$

Presentation See page 232.

Maximal subgroups / Specifications

Order	Index	Structure	G.2	Character	Abstract	Orthogonal
1658880	119	$2^6:U_4(2)$	: $2^6:U_4(2):2$	1a+34a+84a	$N(2^6) = N(2A_{27}B_{36})$	isotropic point
1451520	136	$S_6(2)$	: $S_6(2) \times 2$	1a+51a+84a	C(2D)	non-isotropic point
258048	765	$2^{3+6}:(L_3(2)\times 3)$	: $2^{3+6}:(L_3(2)\times S_3)$	1a+84a+204b+476a	$N(2A^3)$	isotropic plane
184320	1071	$2_+^{1+8}:(S_3\times A_5)$	: $2_+^{1+8}:(S_3\times S_5)$	1a+34a+84a+476ab	N(2A)	isotropic line
120960	1632	$(3 \times A_8):2$	: $S_3 \times S_8$	1a+84a+476ab+595a	N(3A)	$O_2^-(2)\times O_6^+(2)$
8160	24192	$L_2(16):2$	: $L_2(16):4$		N(2C,3B,5A,15AB,17ABCD)	$O_4^-(4)$
4320	45696	$(S_3\times S_3\times A_5):2$	: $(S_3\times S_3):2 \times S_5$		$N(3^2) = N(3A_2B_2)$, N(2B,3A,5A)	$O_4^+(2)\times O_4^-(2)$
168	1175040	$L_2(7)$	: $L_2(7):2$		N(2C,3C,4D,7A)	

Triality twisted group $^3D_4(2)$

Order = 211,341,312 = $2^{12}.3^4.7^2.13$ Mult = 1 Out = 3

Constructions

Steinberg G : the centralizer in $O_8^+(8)$ of an outer automorphism of order 3.

This leads to a description of G.3 as the automorphism group of a generalized hexagon of order (2,8), consisting of 819 "vertices" and 2457 "edges". Each vertex is incident with 9 edges, and each edge is incident with 3 vertices.

Jordan G.3 is a subgroup of the compact Lie group $F_4(\mathbb{R})$, the automorphism group of the 27-dimensional exceptional Jordan algebra. This consists of 3 x 3 Hermitian matrices

$$\begin{bmatrix} a & C & \bar{B} \\ \bar{C} & b & A \\ B & \bar{A} & c \end{bmatrix} = (a,b,c|A,B,C) \quad (a,b,c \text{ real})$$

over the real Cayley algebra with units i_∞, i_0, ... i_6, in which $i_\infty = 1$, $i_{n+1} \to i$, $i_{n+2} \to j$, $i_{n+4} \to k$ generate a quaternion subalgebra. G.3 is the automorphism group of the set of 819 images (roots) of the idempotents

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (3) \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} (48) \qquad \begin{bmatrix} \frac{1}{2} & \frac{1}{4}s & \frac{1}{4}s \\ \frac{1}{4}s & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4}s & \frac{1}{4} & \frac{1}{4} \end{bmatrix} (768)$$

under the group $[2^{11}].S_3$ generated by the maps taking (a,b,c,|A,B,C) to

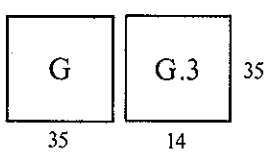
$$(a,b,c|iAi,iB,Ci),\ (b,c,a|B,C,A) \text{ and } (a,c,b|\bar{A},-\bar{C},-\bar{B}) \qquad (*)$$

where $i = \pm i_n$ and $s = \frac{1}{2}\Sigma i_n$. This group extends by $(i_0\ i_1 \ldots i_6)$ to a maximal subgroup of G, and by $(i_1\ i_2\ i_4)(i_3\ i_6\ i_5)$ to a maximal subgroup of G.3. The simple group G is generated by the "reflections" $y \to y + 4((y.r)r - y \times r)$ in the roots, where y x z is the Jordan product $\frac{1}{2}(yz + zy)$ and y.z is the natural inner product corresponding to $\mathrm{Norm}(y) = \Sigma(\mathrm{Norm}(y_{ij}))$. There are 2457 sets of 3 mutually orthogonal roots (bases), and each root is in just 9 bases. By restricting i in (*) to be ±1, we obtain the subgroup $L_2(7)$, while by restricting it to be ±1 or $\pm i_0$, we obtain the subgroup $U_3(3)$. The normalizers of these groups are maximal subgroups of G, and they are related to the complex and quaternionic reflection groups 2 x $L_3(2)$ and 2 x $U_3(3)$ respectively.

Remark : G.3 is a maximal subgroup of Thompson's group.

Maximal subgroups / Specifications

Order	Index	Structure	G.3	Character	Abstract	Steinberg	Jordan
258048	819	$2_+^{1+8}:L_2(8)$	: $2_+^{1+8}:L_2(8):3$	1a+26a+324a+468a	N(2A)	vertex	root
86016	2457	$2^2.[2^9]:(7\times S_3)$	: $2^2.[2^9]:(7:3\times S_3)$	1a+324a+468a+1664a	$N(2A^2)$	edge	base
12096	17472	$U_3(3):2$	: $U_3(3):2 \times 3$		C(3C)	$G_2(2)$	quaternionic
3024	69888	$S_3 \times L_2(8)$	: $S_3 \times L_2(8):3$		N(3A)		
2352	89856	$(7\times L_2(7)):2$	: $(7:3\times L_2(7)):2$		N(7ABC)		complex
1296	163072	$3_+^{1+2}.2S_4$	: H.3		N(3B)		
1176	179712	$7^2:2A_4$	: $7^2:(3\times 2A_4)$		$N(7^2)$		
216	978432	$3^2:2A_4$	: $3^2:2A_4 \times 3$		$N(3B^2)$, C(3D)		
52	4064256	13:4	: 13:12		N(13ABC)		



3D4(2)

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			258	3	1		3	1																														12																
	211341312		048	072	512	648	584	536	64	72	24	1176	1176	1176	49	32	32	54	54	54	12	13	13	13	56	56	56	18	18	18	21	21	21	28	28	28			096	216	192	48	24	18	96	48	32	4	6	7	8	8	
	p power		A	A	A	A	A	A	B	BA	AB	A	A	A	A	A	B	B	B	B	AB	A	A	A	BA	CA	AA	BA	CA	AA	CA	AA	BA	CA	BA	CA	AA			A	A	A	B	A	CB	CB	CA	EC	DA	DC	BB	CA		
	p' part		A	A	A	A	A	A	A	BA	AB	A	A	A	A	A	A	A	A	A	BB	A	A	A	AA	BA	CA	AA	BA	CA	AA	AA	BA	CA	AA	BA	CA			A	A	A	CB	DB	A	CB	CB	CA	DC	DA	DC	CB	CA		
	ind		1A	2A	2B	3A	3B	4A	4B	4C	6A	6B	7A	B*2	C*4	7D	8A	8B	9A	B*2	C*4	12A	13A	B*3	C*9	14A	B*5	C*3	18A	B*7	C*5	21A	B*2	C*4	28A	B*5	C*3	fus	ind	3C	3D	6C	6D	6E	9D	12B	12C	12D	12E	18D	21D	24A	24B	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	+∞	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	+	26	-6	2	-1	-1	6	-2	2	3	-1	5	5	5	-2	0	0	2	2	2	1	0	0	0	1	1	1	0	0	0	-1	-1	-1	-1	-1	-1	:	+∞	8	-1	0	2	-1	2	4	-2	0	-1	0	1	0	0	x_2
x_3	+	52	20	-4	7	-2	8	0	0	2	-1	3	3	3	3	-2	2	1	1	1	0	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0	1	1	1	:	+∞	7	-2	-1	-1	2	1	3	3	-1	0	-1	0	-1	1	x_3
x_4	+	196	-28	-4	7	7	0	8	0	-1	-1	0	0	0	0	2	-2	1	1	1	-1	1	1	1	0	0	0	-1	-1	-1	0	0	0	0	0	0	:	+∞	7	-2	-1	-1	2	1	-1	-1	3	0	-1	0	1	-1	x_4
x_5	+	273	-15	-7	-6	3	13	5	1	3	2	7	7	7	0	-1	-1	0	0	0	-1	0	0	0	-1	-1	-1	0	0	0	1	1	1	-1	-1	-1	:	+∞	21	3	-3	-1	-1	0	5	-1	1	1	0	0	-1	-1	x_5
x_6	+	324	36	12	0	0	8	0	0	0	0	9	9	9	2	2	-2	0	0	0	0	-1	-1	-1	1	1	1	0	0	0	0	0	0	1	1	1	:	+∞	27	0	3	3	0	0	3	3	-1	0	0	-1	1	-1	x_6
x_7	+	351	63	7	9	0	11	3	-1	0	1	y7+3&4	*2	*4	1	1	1	0	0	0	0	0	0	0	y7-&4	*2	*4	0	0	0	y7	*2	*4	*2	*4	-y7-1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_7
x_8	+	351	63	7	9	0	11	3	-1	0	1	*4	y7+3&4	*2	1	1	1	0	0	0	0	0	0	0	*4	y7-&4	*2	0	0	0	*4	y7	*2	-y7-1	*2	*4																		x_8
x_9	+	351	63	7	9	0	11	3	-1	0	1	*2	*4	y7+3&4	1	1	1	0	0	0	0	0	0	0	*2	*4	y7-&4	0	0	0	*2	*4	y7	*4	-y7-1	*2																		x_9
x_{10}	+	468	-12	12	0	9	0	8	0	-3	0	6	6	6	-1	-2	2	0	0	0	-1	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	:	+∞	27	0	3	3	0	0	-1	-1	3	0	0	-1	-1	1	x_{10}
x_{11}	+	637	-35	5	7	-11	-7	17	-3	1	-1	0	0	0	0	-1	-1	1	1	1	-1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	:	+∞	7	-2	7	-1	2	1	-1	-1	-1	0	1	0	-1	-1	x_{11}
x_{12}	+	1053	-99	-3	0	0	29	-3	-3	0	0	3	3	3	-4	1	1	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	1	1	1	:	+∞	27	0	3	-3	0	0	3	-3	-1	0	0	-1	1	1	x_{12}
x_{13}	+	1274	154	-14	14	5	14	-10	2	1	-2	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	:	+∞	14	5	-2	-2	1	-1	2	2	2	-1	1	0	0	0	x_{13}
x_{14}	+	1664	128	0	8	-10	0	0	0	2	0	-2	-2	-2	-2	0	0	-1	-1	-1	0	0	0	0	2	2	2	-1	-1	-1	1	1	1	0	0	0	:	+∞	8	8	8	0	0	-1	0	0	0	0	-1	1	0	0	x_{14}
x_{15}	+	1911	119	-9	0	3	7	7	-1	-1	0	0	0	0	0	-1	-1	*2	*4	y9-&2	1	0	0	0	0	0	0	y9	*2	*4	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{15}
x_{16}	+	1911	119	-9	0	3	7	7	-1	-1	0	0	0	0	0	-1	-1	y9-&2	*2	*4	1	0	0	0	0	0	0	*4	y9	*2	0	0	0	0	0	0																		x_{16}
x_{17}	+	1911	119	-9	0	3	7	7	-1	-1	0	0	0	0	0	-1	-1	*4	y9-&2	*2	1	0	0	0	0	0	0	*2	*4	y9	0	0	0	0	0	0																		x_{17}
x_{18}	+	2106	90	18	0	0	-2	6	2	0	0	6y7-3&4	*2	*4	-1	0	0	0	0	0	0	0	0	0	2y7+&4	*2	*4	0	0	0	0	0	0	*4	-y7	*2		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{18}
x_{19}	+	2106	90	18	0	0	-2	6	2	0	0	*4	6y7-3&4	*2	-1	0	0	0	0	0	0	0	0	0	*4	2y7+&4	*2	0	0	0	0	0	0	*2	*4	-y7																		x_{19}
x_{20}	+	2106	90	18	0	0	-2	6	2	0	0	*2	*4	6y7-3&4	-1	0	0	0	0	0	0	0	0	0	*2	*4	2y7+&4	0	0	0	0	0	0	-y7	*2	*4																		x_{20}
x_{21}	+	2184	-120	8	15	-3	-8	-8	0	-3	-1	7	7	7	0	0	0	0	0	0	1	0	0	0	-1	-1	-1	0	0	0	1	1	1	-1	-1	-1	:	+∞	42	6	-6	2	2	0	-2	-2	-2	0	0	0	0	0	x_{21}
x_{22}	+	2457	-135	1	9	0	5	-3	1	0	1	7y7	*2	*4	0	-1	-1	0	0	0	0	0	0	0	-y7	*2	*4	0	0	0	y7	*2	*4	-y7	*2	*4		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{22}
x_{23}	+	2457	-135	1	9	0	5	-3	1	0	1	*4	7y7	*2	0	-1	-1	0	0	0	0	0	0	0	*4	-y7	*2	0	0	0	*4	y7	*2	*4	-y7	*2																		x_{23}
x_{24}	+	2457	-135	1	9	0	5	-3	1	0	1	*2	*4	7y7	0	-1	-1	0	0	0	0	0	0	0	*2	*4	-y7	0	0	0	*2	*4	y7	*2	*4	-y7																		x_{24}
x_{25}	+	2808	-72	8	-9	0	16	0	0	0	-1	8y7+3&4	*2	*4	1	0	0	0	0	0	0	0	0	0	*4	-y7	*2	0	0	0	-y7	*2	*4	*4	y7	*2		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{25}
x_{26}	+	2808	-72	8	-9	0	16	0	0	0	-1	*4	8y7+3&4	*2	1	0	0	0	0	0	0	0	0	0	*2	*4	-y7	0	0	0	*4	-y7	*2	*2	*4	y7																		x_{26}
x_{27}	+	2808	-72	8	-9	0	16	0	0	0	-1	*2	*4	8y7+3&4	1	0	0	0	0	0	0	0	0	0	-y7	*2	*4	0	0	0	*2	*4	-y7	y7	*2	*4																		x_{27}
x_{28}	+	3822	14	6	0	6	-14	-6	-2	2	0	0	0	0	0	0	0	*4	y9-&4	*2	0	0	0	0	0	0	0	y9	*2	*4	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{28}
x_{29}	+	3822	14	6	0	6	-14	-6	-2	2	0	0	0	0	0	0	0	*2	*4	y9-&4	0	0	0	0	0	0	0	*4	y9	*2	0	0	0	0	0	0																		x_{29}
x_{30}	+	3822	14	6	0	6	-14	-6	-2	2	0	0	0	0	0	0	0	y9-&4	*2	*4	0	0	0	0	0	0	0	*2	*4	y9	0	0	0	0	0	0																		x_{30}
x_{31}	+	3969	-63	-15	0	0	-7	9	1	0	0	0	0	0	0	1	1	0	0	0	0	c13	*3	*9	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{31}
x_{32}	+	3969	-63	-15	0	0	-7	9	1	0	0	0	0	0	0	1	1	0	0	0	0	*9	c13	*3	0	0	0	0	0	0	0	0	0	0	0	0																		x_{32}
x_{33}	+	3969	-63	-15	0	0	-7	9	1	0	0	0	0	0	0	1	1	0	0	0	0	*3	*9	c13	0	0	0	0	0	0	0	0	0	0	0	0																		x_{33}
x_{34}	+	4096	0	0	-8	-8	0	0	0	0	0	8	8	8	1	0	0	-2	-2	-2	0	1	1	1	0	0	0	0	0	0	-1	-1	-1	0	0	0	:	+∞	64	-8	0	0	0	-2	0	0	0	0	0	1	0	0	x_{34}
x_{35}	+	5096	168	-8	-7	-7	0	-16	0	-3	1	0	0	0	0	0	0	2	2	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	+∞	56	2	0	-2	-2	2	-4	2	0	0	0	0	0	0	x_{35}

$L_3(11)$ and A_{12}

Linear group $L_3(11) \cong A_2(11)$

Order = 212,427,600 = $2^4.3.5^2.7.11^3.19$ Mult = 1 Out = 2

Constructions

Linear $GL_3(11) \cong 10 \times G$: all non-singular 3 x 3 matrices over F_{11};

$PGL_3(11) \cong SL_3(11) \cong PSL_3(11) \cong G$

Maximal subgroups					Specifications	
Order	Index	Structure	G.2	Character	Abstract	Linear
1597200	133	$11^2:(5x2L_2(11).2)$	$11_+^{1+2}:10^2$,	1a+132a	$N(11^2)$	point
1597200	133	$11^2:(5x2L_2(11).2)$	$2L_2(11).2.2$	1a+132a	$N(11^2)$	line
1320	160930	$L_2(11):2$	: $L_2(11):2 \times 2$			$O_3(11)$
600	354046	$10^2:S_3$	: $10^2:D_{12}$		$N(2A^2)$, $N(5^2)$	base
399	532400	133:3	: 133:6		N(7AB), N(19A-F)	$L_1(1331)$
168	1264450	$L_2(7)$	: $L_2(7):2$			

The characters of $L_3(11)$ are not printed

Alternating group A_{12}

Order = 239,500,800 = $2^9.3^5.5^2.7.11$ Mult = 2 Out = 2

Constructions

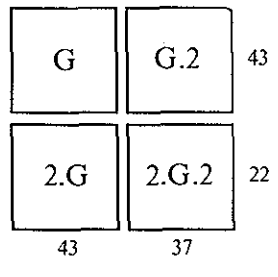
Alternating $S_{12} \cong G.2$: all permutations on 12 letters;

$A_{12} \cong G$: all even permutations; 2.G and 2.G.2 : the Schur double covers

Presentations $G \cong \langle x_1,\ldots,x_{10} | x_i^3=(x_ix_j)^2=1\rangle$; $G.2 \cong$ ———————————————

Remark : A_{12} is the largest alternating group involved in the Monster group F_1. It arises in several large groups involved in F_1, notably the Fischer group Fi_{24}' and the Harada-Norton group.

Maximal subgroups					Specifications	
Order	Index	Structure	G.2	Character	Abstract	Alternating
19958400	12	A_{11}	: S_{11}	1a+11a		point
3628800	66	S_{10}	: $S_{10} \times 2$	1a+11a+54a	C(2D)	duad
1088640	220	$(A_9x3):2$	: $S_9 \times S_3$	1a+11a+54a+154a	N(3A)	triad
518400	462	$(A_6xA_6):2^2$	: $(S_6xS_6):2$	1a+54a+132a+275a	$N(2A,3A,3B,4A,5A)^2$	bisection
483840	495	$(A_8xA_4):2$	: $S_8 \times S_4$	1a+11a+54a+154a+275a	$N(2A^2)$	tetrad
302400	792	$(A_7xA_5):2$	: $S_7 \times S_5$	1a+11a+54a+154a+275a+297a	N(2A,3A,5A)	pentad
95040	2520	M_{12}	$L_2(11):2$	1a+132a+462a+1925b		S(5,6,12)
95040	2520	M_{12}		1a+132a+462a+1925b		S(5,6,12)
41472	5775	$2^6:3^3:S_4$	: S_4wrS_3		$N(2^6) = N(2A_9B_{27}C_{27})$	trisection
23040	10395	$2^5:S_6$	: $2^6:S_6$		N(2B)	hexasection
15552	15400	$3^4:2^3.S_4$	: S_3wrS_4		$N(3^4) = N(3A_4B_{12}C_8D_{16})$	quadrisection



A_{12}

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			161	23	4	544	6			2				12			1																																
	239500800		280	040	608	320	480	972	486	880	384	192	128	600	50	440	576	144	144	144	36	18	420	16	16	27	27	27	120	10	11	11	72	72	48	24	28	180	45	20	21	60	35	35	
	p power		A	A	A	A	A	A	A	A	C	A	C	A	A	AA	AC	BA	BB	BC	CB	DC	A	A	D	D	D	D	AA	BB	A	A	AA	CA	BB	CC	AA	AA	AB	AA	AA	AAA	AA	AA	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AC	BA	BB	BC	CB	DC	A	A	A	A	A	A	AA	BB	A	A	AA	BA	AB	BC	AA	AA	AB	AA	AA	AAA	AA	AA	
	ind		1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	5A	5B	6A	6B	6C	6D	6E	6F	6G	7A	8A	8B	9A	9B	C**	10A	10B	11A	B**	12A	12B	12C	12D	14A	15A	15B	20A	21A	30A	35A	B**	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	+	11	7	-1	3	8	5	-1	2	5	3	1	-1	6	1	4	0	1	-1	3	-1	0	4	1	-1	2	-1	-1	2	-1	0	0	2	-1	0	1	0	3	0	0	1	-1	-1	-1	x_2
x_3	+	54	22	6	6	27	9	0	0	10	2	2	2	14	-1	7	3	1	3	3	0	0	5	0	0	0	0	0	2	1	-1	-1	1	1	-1	-1	1	2	-1	0	-1	2	0	0	x_3
x_4	+	55	19	-5	-1	28	10	1	1	9	3	-3	-1	15	0	4	-4	-2	-2	2	1	-1	6	-1	1	1	1	1	-1	0	0	0	0	0	0	0	-2	3	0	-1	0	-1	1	1	x_4
x_5	+	132	20	20	4	6	6	6	-3	0	4	0	4	-8	2	2	-2	2	2	-2	2	1	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	-1	1	1	0	-1	-3	-1	-1	x_5
x_6	+	154	42	-6	10	49	7	4	1	10	-2	2	-2	14	-1	9	1	3	-3	1	0	1	0	0	0	-2	1	1	2	-1	0	0	1	1	1	-1	0	-1	2	0	0	-1	0	0	x_6
x_7	+	165	21	5	-11	57	12	3	3	5	1	-3	1	20	0	-3	1	0	2	-2	-1	1	4	-1	-1	0	0	0	-4	0	0	0	-1	2	1	0	0	2	2	0	1	2	-1	-1	x_7
x_8	+	275	55	15	11	50	5	-4	5	5	-1	1	3	0	0	10	2	1	3	-1	0	-1	-5	-1	1	-1	-1	-1	0	0	0	0	2	-1	2	1	-1	0	0	0	1	0	0	0	x_8
x_9	+	297	49	-15	9	27	9	0	0	1	5	1	-3	-13	2	7	3	1	-3	-3	0	0	-4	-1	-1	0	0	0	-1	0	0	0	1	1	-1	1	0	2	-1	1	-1	2	1	1	x_9
x_{10}	+	320	64	0	0	104	14	-4	-4	16	0	0	0	35	0	4	0	-2	0	0	0	0	5	0	0	-1	-1	-1	-1	0	1	1	-2	-2	0	0	1	-1	-1	1	-1	-1	0	0	x_{10}
x_{11}	+	330	-6	10	-6	78	15	-3	6	-6	-2	2	-2	15	0	-6	6	3	1	-3	1	0	1	0	0	0	0	0	-1	0	0	0	0	-3	-2	-1	1	3	0	-1	1	-1	1	1	x_{11}
x_{12}	+	462	-42	-10	14	84	21	3	3	-14	-6	2	2	7	2	0	-4	-3	-1	-1	-1	-1	0	0	0	0	0	0	3	0	0	0	-2	1	0	-1	0	4	1	1	0	0	0	0	x_{12}
x_{13}	+	462	14	30	-2	-42	3	12	3	-6	2	2	2	7	2	2	-2	-1	-3	1	0	1	0	0	0	0	0	0	-1	0	0	0	0	-3	2	-1	0	-2	-2	-1	0	2	0	0	x_{13}
x_{14}	+	616	56	24	8	112	-5	4	-5	4	0	4	0	21	1	-4	8	-1	3	-1	0	-1	0	0	0	1	1	1	1	-1	0	0	-2	1	0	1	0	-3	0	-1	0	1	0	0	x_{14}
x_{15}	+	891	111	-9	3	162	0	0	0	9	-5	-3	-1	21	1	6	-6	0	0	0	0	0	-5	1	-1	0	0	0	1	1	0	0	0	0	-2	0	-1	-3	0	-1	1	1	0	0	x_{15}
x_{16}	+	945	49	-15	-15	189	9	0	0	1	1	1	1	35	0	-11	-3	1	-3	-3	0	0	0	1	1	0	0	0	-1	0	-1	-1	1	1	1	1	0	-1	-1	1	0	-1	0	0	x_{16}
x_{17}	o	1050	-70	10	10	105	15	-3	-3	-10	-2	-2	-2	0	0	5	1	-1	1	1	1	1	0	0	0	0	0	0	0	0	b11	**	-1	-1	1	1	0	0	0	0	0	0	0	0	x_{17}
x_{18}	o	1050	-70	10	10	105	15	-3	-3	-10	-2	-2	-2	0	0	5	1	-1	1	1	1	1	0	0	0	0	0	0	0	0	**	b11	-1	-1	1	1	0	0	0	0	0	0	0	0	x_{18}
x_{19}	+	1155	91	-5	3	21	21	-6	-6	-1	7	-1	-1	-35	0	1	-3	1	1	-3	-2	0	0	1	1	0	0	0	1	0	0	0	-1	-1	1	-1	0	1	1	-1	0	1	0	0	x_{19}
x_{20}	+	1320	56	-40	8	-48	15	6	-3	-4	0	-4	0	-15	0	-4	8	-1	-1	-1	2	-1	4	0	0	0	0	0	1	0	0	0	2	-1	0	-1	0	-3	0	1	1	1	-1	-1	x_{20}
x_{21}	o	1320	8	40	8	-84	6	-3	-3	0	0	0	0	10	0	-4	-4	2	-2	2	1	-1	-3	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	1	1	0	0	1	b35	**	x_{21}
x_{22}	o	1320	8	40	8	-84	6	-3	-3	0	0	0	0	10	0	-4	-4	2	-2	2	1	-1	-3	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	1	1	0	0	1	**	b35	x_{22}
x_{23}	+	1408	128	0	0	112	4	4	4	0	0	0	0	-22	-2	8	0	-4	0	0	0	0	-6	0	0	1	1	1	-2	0	0	0	0	0	0	0	2	2	-1	0	0	-2	-1	-1	x_{23}
x_{24}	+	1485	29	45	-3	-27	18	0	0	5	1	-3	1	-20	0	-7	-3	2	0	0	0	0	1	-1	-1	0	0	0	4	0	0	0	-1	2	1	0	1	-2	-2	0	1	-2	1	1	x_{24}
x_{25}	+	1650	50	-30	2	-15	-15	12	3	-10	-2	-2	-2	0	0	5	-7	5	3	-1	0	-1	5	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	0	0	0	-1	0	0	0	x_{25}
x_{26}	+	1728	-64	0	0	216	18	0	0	-16	0	0	0	13	-2	-4	0	2	0	0	0	0	-1	0	0	0	0	0	1	0	1	1	2	2	0	0	-1	1	-2	-1	-1	1	-1	-1	x_{26}
x_{27}	+	1925	105	-15	13	140	-25	-4	-1	-5	-3	-1	1	0	0	0	4	3	-3	1	0	1	0	-1	1	2	-1	-1	0	0	0	0	-2	1	0	-1	0	0	0	0	0	0	0	0	x_{27}
x_{28}	+	1925	-35	45	21	35	-10	8	8	5	5	-3	-3	0	0	-5	3	-2	0	0	0	0	0	1	1	-1	-1	-1	0	0	0	0	-1	2	-1	0	0	0	0	0	0	0	0	0	x_{28}
x_{29}	+	2079	7	-9	15	189	-18	0	0	-5	7	3	-1	14	-1	-11	-3	-2	0	0	0	0	0	-1	-1	0	0	0	2	1	0	0	1	-2	1	0	0	-1	2	0	0	-1	0	0	x_{29}
x_{30}	+	2112	64	0	0	-120	6	-12	6	-16	0	0	0	7	2	4	0	-2	0	0	0	0	5	0	0	0	0	0	-1	0	0	0	2	2	0	0	1	-5	1	-1	-1	-1	0	0	x_{30}
x_{31}	+	2376	56	24	-24	216	-9	0	0	-4	0	-4	0	1	1	-4	0	-1	3	3	0	0	-4	0	0	0	0	0	1	-1	0	0	2	-1	0	-1	0	1	1	1	-1	1	1	1	x_{31}
x_{32}	+	2673	117	45	9	0	0	0	0	-9	-3	3	1	-27	-2	0	0	0	0	0	0	0	6	1	-1	0	0	0	-3	0	0	0	0	0	0	0	-2	0	0	1	0	0	1	1	x_{32}
x_{33}	+	2970	34	-30	-6	-216	9	0	0	-14	6	2	-2	35	0	4	0	1	3	3	0	0	-5	0	0	0	0	0	-1	0	0	0	-2	1	0	-1	-1	-1	-1	1	1	-1	0	0	x_{33}
x_{34}	+	3080	56	-40	-24	56	11	2	2	4	0	4	0	-35	0	-4	0	-1	-1	3	2	0	0	0	0	-1	-1	-1	1	0	0	0	-2	1	0	1	0	1	1	-1	0	1	0	0	x_{34}
x_{35}	+	3520	-64	0	0	-8	10	-8	10	16	0	0	0	-15	0	-4	0	2	0	0	0	0	-1	0	0	1	1	1	1	0	0	0	-2	-2	0	0	-1	-3	0	1	-1	1	-1	-1	x_{35}
x_{36}	+	3564	-132	-36	12	162	0	0	0	0	4	0	4	-6	-1	6	-6	0	0	0	0	0	1	0	0	0	0	0	-2	-1	0	0	0	0	-2	0	1	-3	0	0	1	1	1	1	x_{36}
x_{37}	+	3696	-112	16	-16	168	6	6	-3	0	0	0	0	-14	1	8	8	2	-2	2	-2	-1	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	-2	1	0	0	-2	0	0	x_{37}
x_{38}	o	3850	-70	10	-6	-35	-5	1	-2	10	-2	2	-2	0	0	5	-3	-1	1	-3	1	0	0	0	0	1	b27	**	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	x_{38}
x_{39}	o	3850	-70	10	-6	-35	-5	1	-2	10	-2	2	-2	0	0	5	-3	-1	1	-3	1	0	0	0	0	1	**	b27	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	x_{39}
x_{40}	+	4158	14	30	-18	-189	-9	0	0	-10	2	-2	2	28	-2	11	3	-1	-3	-3	0	0	0	0	0	0	0	0	4	0	0	0	-1	-1	-1	1	0	1	1	0	0	1	0	0	x_{40}
x_{41}	+	4455	-21	-45	15	-162	0	0	0	9	-1	-3	3	15	0	-6	6	0	0	0	0	0	-4	1	-1	0	0	0	-1	0	0	0	0	0	2	0	0	3	0	-1	-1	-1	1	1	x_{41}
x_{42}	+	5632	0	0	0	64	-32	-8	-2	0	0	0	0	-8	2	0	0	0	0	0	0	0	4	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	4	-2	0	1	0	-1	-1	x_{42}
x_{43}	+	5775	35	-5	7	-210	15	6	-3	5	-5	1	-1	0	0	-10	-2	-1	1	1	-2	1	0	-1	1	0	0	0	0	0	0	0	2	-1	-2	1	0	0	0	0	0	0	0	0	x_{43}
	ind		1 2	4	4	2	3 6	3 6	3 6	3 6	8	4	8	4	5 10	5 10	12	6	12	12	6	12	6 6	7 14	8	8 8	9 18	9 18	9 18	20	20 20	11 22	11 22	24	24	12	24	28	15 30	15 30	40 40	21 42	60	35 70	35 70	
x_{44}	-	32	0	0	0	-16	8	2	-4	0	0	0	0	-8	2	0	0	0	0	0	0	0	4	0	0	2	-1	-1	0	0	-1	-1	0	0	0	0	0	4	-2	0	-2	0	-1	-1	x_{44}
x_{45}	o	160	0	0	0	-56	16	-2	-2	0	0	0	0	-20	0	0	0	0	0	0	0	0	6	0	0	1	1	1	0	0	-b11	**	0	0	0	0	0	4	1	0	0	0	1	1	x_{45}
x_{46}	o	160	0	0	0	-56	16	-2	-2	0	0	0	0	-20	0	0	0	0	0	0	0	0	6	0	0	1	1	1	0	0	**	-b11	0	0	0	0	0	4	1	0	0	0	1	1	x_{46}
x_{47}	o	704	0	0	0	-160	20	2	2	0	0	0	0	-36	-1	0	0	0	0	0	0	0	4	0	0	-1	-1	-1	0	i5	0	0	0	0	0	0	0	0	0	0	1	0	-1	-1	x_{47}
x_{48}	o	704	0	0	0	-160	20	2	2	0	0	0	0	-36	-1	0	0	0	0	0	0	0	4	0	0	-1	-1	-1	0	-i5	0	0	0	0	0	0	0	0	0	0	1	0	-1	-1	x_{48}
x_{49}	+	1408	0	0	0	112	4	4	4	0	0	0	0	-22	-2	0	0	0	0	0	0	0	-6	0	0	1	1	1	0	0	0	0	0	0	0	0	0	2	-1	r10	0	0	-1	-1	x_{49}
x_{50}	+	1408	0	0	0	112	4	4	4	0	0	0	0	-22	-2	0	0	0	0	0	0	0	-6	0	0	1	1	1	0	0	0	0	0	0	0	0	0	2	-1	-r10	0	0	-1	-1	x_{50}
x_{51}	+	1760	0	0	0	128	8	-16	-4	0	0	0	0	-20	0	0	0	0	0	0	0	0	-4	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	-2	-2	0	2	0	1	1	x_{51}
x_{52}	o	1760	0	0	0	-232	8	2	-4	0	0	0	0	-20	0	0	0	0	0	0	0	0	-4	0	0	-1	-b27	**	0	0	0	0	0	0	0	0	0	-2	-2	0	-1	0	1	1	x_{52}
x_{53}	o	1760	0	0	0	-232	8	2	-4	0	0	0	0	-20	0	0	0	0	0	0	0	0	-4	0	0	-1	**	-b27	0	0	0	0	0	0	0	0	0	-2	-2	0	-1	0	1	1	x_{53}
x_{54}	+	1760	0	0	0	104	-4	-4	5	0	0	0	0	-20	0	0	0	0	0	0	0	3	-4	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	4	1	0	-1	0	1	1	x_{54}
x_{55}	+	1760	0	0	0	104	-4	-4	5	0	0	0	0	-20	0	0	0	0	0	0	0	-3	-4	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	4	1	0	-1	0	1	1	x_{55}
x_{56}	+	1792	0	0	0	-224	-8	-8	10	0	0	0	0	-28	2	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	-1	-1	0	0	0	0	0	-4	2	0	0	0	0	0	x_{56}
x_{57}	o	2112	0	0	0	-48	24	6	6	0	0	0	0	32	2	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	0	1	0	-b35	**	x_{57}
x_{58}	o	2112	0	0	0	-48	24	6	6	0	0	0	0	32	2	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	0	1	0	**	-b35	x_{58}
x_{59}	o	2640	0	0	0	-168	12	-6	-6	0	0	0	0	20	0	0	0	0	0	0	0	0	-6	0	212	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	-1	-1	x_{59}
x_{60}	o	2640	0	0	0	-168	12	-6	-6	0	0	0	0	20	0	0	0	0	0	0	0	0	-6	0	-212	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	-1	-1	x_{60}
x_{61}	+	3840	0	0	0	96	24	-12	6	0	0	0	0	20	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	1	1	0	0	0	0	0	-4	-1	0	-2	0	-1	-1	x_{61}
x_{62}	+	5600	0	0	0	-280	-40	8	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	-1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	x_{62}
x_{63}	+	5632	0	0	0	64	-32	-8	-2	0	0	0	0	-8	2	0	0	0	0	0	0	0	4	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	4	-2	0	1	0	-1	-1	x_{63}
x_{64}	+	7392	0	0	0	336	12	12	-6	0	0	0	0	-28	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	2	0	0	0	0	0	x_{64}
x_{65}	+	7776	0	0	0	0	0	0	0	0	0	0	0	36	-4	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	1	1	x_{65}

	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
			3628	17	3	80					15	2																															
			800	280	840	640	768	384	192	64	120	160	432	432	432	162	54	54	48	96	32	600	120	50	10	720	72	48	24	6	42	9	60	12	14	30	15	21	30		
			A	A	A	A	A	A	B	C	AD	BE	AE	BD	BE	DD	CE	DE	BF	B	B	AD	AE	BD	BF	AE	CE	AG	CF	FH	AD	AM	AE	CC	AE	ACH	BDI	ABH	ABE		
			A	A	A	A	A	A	A	A	AD	BE	AE	BD	BE	DD	CE	DE	BF	A	A	AD	AE	BD	BF	AE	BE	AG	BF	CH	AD	AD	AE	AC	AE	ACH	BDI	ABH	ABE		
	fus	ind	2D	2E	2F	4E	4F	4G	4H	4I	6H	6I	6J	6K	6L	6M	6N	6O	6P	8C	8D	10C	10D	10E	10F	12E	12F	12G	12H	12I	14B	18A	20B	24A	28A	30B	30C	42A	60A		
x_1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	:	++	9	5	1	7	-1	3	-1	1	6	5	2	3	-1	0	-1	2	1	3	-1	4	0	-1	1	4	1	0	-1	-1	2	0	2	0	0	1	0	-1	-1	x_2
x_3	:	++	36	12	4	20	4	4	0	0	15	9	3	3	3	0	0	0	1	2	2	6	2	1	-1	5	-1	1	1	0	1	0	0	-1	-1	0	-1	1	0	x_3
x_4	:	++	35	7	-5	21	-3	1	1	-1	14	10	-2	2	-2	-1	1	1	-2	3	-1	5	-3	0	0	6	0	-2	0	1	0	-1	1	0	0	-1	0	0	1	x_4
x_5	:	++	48	8	0	-8	8	0	0	0	6	-4	2	0	-4	3	2	-1	0	0	0	-2	-2	-2	0	4	-2	0	2	0	-1	0	2	0	-1	1	1	-1	-1	x_5
x_6	:	++	84	20	4	28	-4	4	0	0	21	5	5	3	-1	3	2	-1	1	-2	-2	4	0	-1	-1	1	1	1	-1	0	0	0	-2	1	0	1	0	0	1	x_6
x_7	:	++	75	-5	-5	35	3	-5	-1	-1	15	10	-5	0	4	3	1	1	-2	1	1	0	0	0	0	5	2	1	0	-1	-2	0	0	1	0	0	0	1	0	x_7
x_8	:	++	125	25	5	15	7	3	3	1	20	-5	4	5	1	-1	-2	1	-1	-3	1	0	0	0	0	0	3	0	1	0	-1	-1	0	0	1	0	0	-1	0	x_8
x_9	:	++	117	21	5	-7	-7	1	-3	1	15	-9	3	3	3	0	0	0	-1	-1	-1	-3	1	2	0	5	-1	1	-1	0	-2	0	3	-1	0	0	1	1	0	x_9
x_{10}	:	++	160	16	0	64	0	0	0	0	34	16	-2	-2	-2	-2	-2	-2	0	0	0	5	1	0	0	4	-2	0	0	0	-1	1	-1	0	1	-1	1	-1	-1	x_{10}
x_{11}	:	++	90	-22	10	34	2	-6	-2	-2	6	5	2	3	-1	0	-1	2	1	0	0	-5	3	0	0	4	1	0	-1	1	-1	0	-1	0	-1	1	0	-1	-1	x_{11}
x_{12}	:	++	42	-14	10	14	-2	-2	2	-2	0	1	4	3	-5	-3	1	1	1	0	0	-3	1	2	0	2	-1	-2	1	-1	0	0	-1	0	0	0	1	0	2	x_{12}
x_{13}	:	++	42	2	10	-14	2	2	6	-2	0	5	-4	3	-1	-3	2	-1	1	0	0	-3	-3	2	0	-2	1	2	-1	0	0	0	1	0	0	0	0	0	-2	x_{13}
x_{14}	:	++	224	8	16	56	8	0	0	0	14	5	2	-7	5	-1	2	-1	1	0	0	-1	3	-1	1	-4	-1	0	-1	0	0	-1	1	0	0	-1	0	0	1	x_{14}
x_{15}	:	++	351	27	-9	69	-3	1	-3	-1	36	0	0	0	0	0	0	0	0	-3	1	1	-3	1	1	-6	0	-2	0	0	1	0	-1	0	-1	1	0	1	-1	x_{15}
x_{16}	:	++	315	-21	-5	91	-5	-5	3	3	21	9	-3	-3	-3	0	0	0	1	-1	-1	-5	-1	0	0	1	1	1	1	0	0	0	1	-1	0	1	-1	0	1	x_{16}
x_{17}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{17}
x_{18}																																								x_{18}
x_{19}	:	++	315	19	-5	-49	-1	-1	3	-1	21	-11	1	-3	1	0	-2	-2	1	1	1	-5	-1	0	0	11	-1	-1	-1	0	0	0	1	1	0	1	-1	0	1	x_{19}
x_{20}	:	++	240	8	0	-56	-8	0	0	0	6	5	2	-3	5	-3	2	-1	-3	0	0	-5	3	0	0	4	1	0	1	0	2	0	-1	0	0	1	0	-1	-1	x_{20}
x_{21}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{21}
x_{22}																																								x_{22}
x_{23}	:	++	448	32	0	0	0	0	0	0	28	-16	-4	4	-4	-2	2	2	0	0	0	-2	2	-2	0	0	0	0	0	0	0	1	0	0	0	-2	-1	0	0	x_{23}
x_{24}	:	++	225	-15	-15	-55	9	1	-3	-3	15	0	3	-6	-6	0	0	0	0	-1	-1	0	0	0	0	5	2	1	0	0	1	0	0	-1	1	0	0	1	0	x_{24}
x_{25}	:	++	300	20	-20	-20	-4	-4	0	0	-15	5	5	3	-1	3	2	-1	1	2	2	0	0	0	0	-5	1	-1	-1	0	-1	0	0	-1	1	0	0	-1	0	x_{25}
x_{26}	:	++	288	-48	0	64	0	0	0	0	-6	0	6	6	6	0	0	0	0	0	0	-7	-3	-2	0	4	-2	0	0	0	1	0	-1	0	1	-1	0	1	-1	x_{26}
x_{27}	:	++	525	25	5	35	-5	-1	3	-3	0	-5	4	-3	1	3	-2	1	-1	3	-1	0	0	0	0	-10	-1	2	1	0	0	0	0	0	0	0	0	0	0	x_{27}
x_{28}	:	++	175	15	15	35	3	3	3	-1	-35	0	-3	-2	6	4	0	0	0	-1	-1	0	0	0	0	5	2	-3	0	0	0	1	0	-1	0	0	0	0	0	x_{28}
x_{29}	:	++	441	-15	9	77	-3	5	-3	1	-21	0	3	-6	-6	0	0	0	0	1	1	-4	0	1	-1	-1	2	-1	0	0	0	0	2	1	0	-1	0	0	-1	x_{29}
x_{30}	:	++	288	16	0	-64	0	0	0	0	-6	16	-2	6	-2	0	-2	-2	0	0	0	-7	1	-2	0	-4	2	0	0	0	1	0	1	0	-1	-1	1	1	1	x_{30}
x_{31}	:	++	576	-24	-16	56	8	0	0	0	6	-9	-6	3	3	0	0	0	-1	0	0	1	1	1	-1	-4	-1	0	-1	0	2	0	1	0	0	1	1	-1	1	x_{31}
x_{32}	:	++	567	27	15	-63	9	-3	-3	3	0	0	0	0	0	0	0	0	0	3	-1	-3	-3	2	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	x_{32}
x_{33}	:	++	90	-6	10	-34	-2	6	-6	-2	6	9	-6	3	3	0	0	0	1	0	0	-5	-1	0	0	-4	-1	0	1	0	-1	0	1	0	1	1	-1	-1	1	x_{33}
x_{34}	:	++	560	-24	0	-56	-8	0	0	0	14	-9	-6	-1	3	2	0	0	3	0	0	5	1	0	0	4	1	0	1	0	0	-1	-1	0	0	-1	1	0	-1	x_{34}
x_{35}	:	++	160	-48	0	-64	0	0	0	0	34	0	6	-2	6	-2	0	0	0	0	0	5	-3	0	0	-4	2	0	0	0	-1	1	1	0	-1	-1	0	-1	1	x_{35}
x_{36}	:	++	216	0	-24	48	0	8	0	0	-36	0	0	0	0	0	0	0	0	0	0	-4	0	1	1	6	0	2	0	0	-1	0	-2	0	-1	-1	0	-1	1	x_{36}
x_{37}	:	++	336	-48	16	0	0	0	0	0	0	-6	0	6	-6	3	0	-3	-2	0	0	6	2	1	1	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	x_{37}
x_{38}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{38}
x_{39}																																								x_{39}
x_{40}	:	++	252	12	-20	-28	4	-4	0	0	-21	9	3	3	3	0	0	0	1	-2	-2	2	2	2	0	-1	-1	-1	1	0	0	0	2	1	0	-1	-1	0	-1	x_{40}
x_{41}	:	++	135	27	15	21	-3	-7	-3	-1	-36	0	0	0	0	0	0	0	0	-3	1	5	-3	0	0	6	0	2	0	0	2	0	1	0	0	-1	0	-1	1	x_{41}
x_{42}	:	++	768	0	0	0	0	0	0	0	-48	0	0	0	0	-6	0	0	0	0	0	8	0	-2	0	0	0	0	0	0	-2	0	0	0	0	2	0	1	0	x_{42}
x_{43}	:	++	525	-15	5	-105	-1	3	3	1	0	15	0	-3	-3	3	0	3	-1	-3	1	0	0	0	0	0	-3	0	-1	0	0	0	0	0	0	0	0	0	0	x_{43}
	fus	ind	2	4	2	8	8	8	8	4	6	12	12	6	12	6	12	12	6	8	8	10	20	10	10	24	24	24	24 24	24 24	14	18 18	40	24 24	56 56	30	60 60	42 42	120 120		
x_{44}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i6	0	0	0	0	0	0	0	0	0	x_{44}
x_{45}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{45}
x_{46}																																								x_{46}
x_{47}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{47}
x_{48}																																								x_{48}
x_{49}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{49}
x_{50}																																								x_{50}
x_{51}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r30	x_{51}
x_{52}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{52}
x_{53}																																								x_{53}
x_{54}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{54}
x_{55}																																								x_{55}
x_{56}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	x_{56}
x_{57}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{57}
x_{58}																																								x_{58}
x_{59}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{59}
x_{60}																																								x_{60}
x_{61}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r15	0	0	x_{61}
x_{62}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r3	0	0	0	0	0	x_{62}
x_{63}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r21	0	x_{63}
x_{64}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r6	0	0	0	0	0	0	0	0	0	0	x_{64}
x_{65}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r14	0	0	0	0	x_{65}

M_{24}

Sporadic Mathieu group M_{24}

Order = 244,823,040 = $2^{10}.3^3.5.7.11.23$ Mult = 1 Out = 1

Constructions

Golay (2) code G : the automorphism group of the (extended) binary Golay code, the unique dimension 12 length 24 code over $\mathbf{F}_2$ of minimal weight 8, which is defined in several ways below. The code and cocode (words modulo the code) have weight distributions $0^1 8^{759} 12^{2576} 16^{759} 24^1$ and $0^1 1^{24} 2^{276} 3^{2024} 4^{1771}$ respectively. The support of a code word is called a C-set, and C-sets of size 8 and 12 are called octads and dodecads respectively. A pair of complementary dodecads is a duum, and a triple of disjoint octads is a trio.

Elements of the cocode are called C*-sets. Their minimal representatives are unique except that each tetrad is one of a set of six disjoint mutually congruent tetrads (forming a sextet).

We usually arrange the coordinates in the 4 x 6 MOG array, which is particularly valuable because it simultaneously exhibits the elements of several distinct maximal subgroups.

Steiner system G : the automorphism group of the Steiner system S(5,8,24) of 759 special octads from a 24-set such that each pentad is in a unique special octad. The octads are the supports of the weight 8 words of the binary Golay code (see above). If $\{a_1,a_2,\ldots,a_8\}$ is any octad, then the number of octads (resp. dodecads) that meet $\{a_1,\ldots,a_i\}$ in precisely $\{a_1,\ldots a_j\}$ is the (j+1)'th entry in the (i+1)'th row of the left (resp. right) table below.

759

506 253

330 176 77

210 120 56 21

130 80 40 16 5

78 52 28 12 4 1

46 32 20 8 4 0 1

30 16 16 4 4 0 0 1

30 0 16 0 4 0 0 0 1

2576

1288 1288

616 672 616

280 336 336 280

120 160 176 160 120

48 72 88 88 72 48

16 32 40 48 40 32 16

0 16 16 24 24 16 16 0

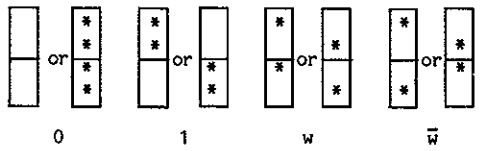
0 0 16 0 24 0 16 0 0

Hexacode In this construction it is easy to recognise codewords and to complete octads from 5 given points. The 64 hexacode words (see A_6) (ab cd ef) are obtained from

0x 0x yz yz yz yz 00 xx xx xx yy zz 00 00 00

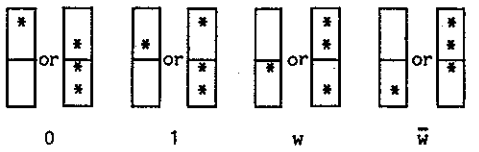
(in which (x,y,z) is any cyclic permutation of $(1,w,\bar{w})$) by freely permuting the three couples ab, cd, ef and reversing the order in any even number of them. The Golay code words are obtained from hexacode words either by giving all their digits even interpretations in any way for which the top row receives an even number of entries, or by giving the digits odd interpretations in any way for which the top row receives an odd number of entries.

even interpretations



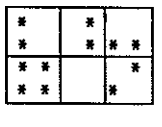
0 1 w $\bar{w}$

odd interpretations

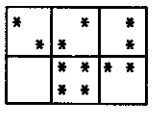


0 1 w $\bar{w}$

We give one even and one odd interpretation of (01 01 $w\bar{w}$):



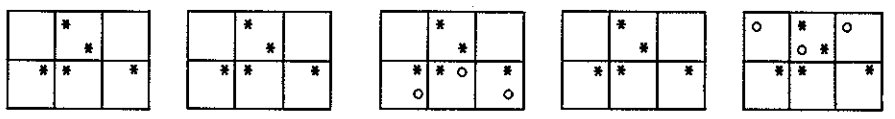
0 1 0 1 w $\bar{w}$



0 1 0 1 w $\bar{w}$

To complete an octad from 5 given points:

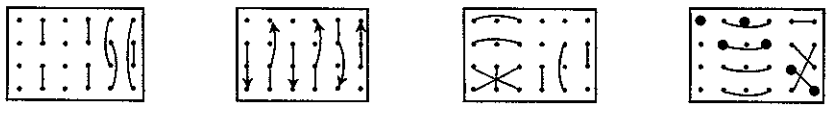
In the example (a) below, if the parity of the answer were even, we must change the three columns ? of (b). So the remaining columns are correct, and the digits below them determine the hexacodeword and trial octad of (c), whose top row has the wrong parity. So the parity is odd, and the digits in (d) determine the hexacodeword and octad as in (e).



(a) 0 ? w ? 0 ? (b) ⇒ 0 1 $\bar{w}$ w 0 1 (c) ? w ? 1 ? w (d) ⇒ 0 w $\bar{w}$ 1 0 w (e)

Using the fact that a hexacodeword is uniquely determined by any 3 correct digits, or by 5 digits of which at most one may be mistaken, the method more generally finds the unique Golay codeword nearest to any given set of odd cardinal. (The method also works for sets of even cardinal, although the answer need not be unique.)

Sextet The group fixing the sextet of MOG columns has structure $2^6{:}3{\cdot}S_6$, the stabilizer of the top row being a subgroup $3{\cdot}S_6$, and the code, as constructed above, is invariant under this group. The elements of the subgroup 2^6 correspond to hexacodewords (a), and there are other correspondences between hexacodewords and the remaining elements of $2^6{:}3$ (b) and the involutions of $3{\cdot}S_6$ (c), (d):



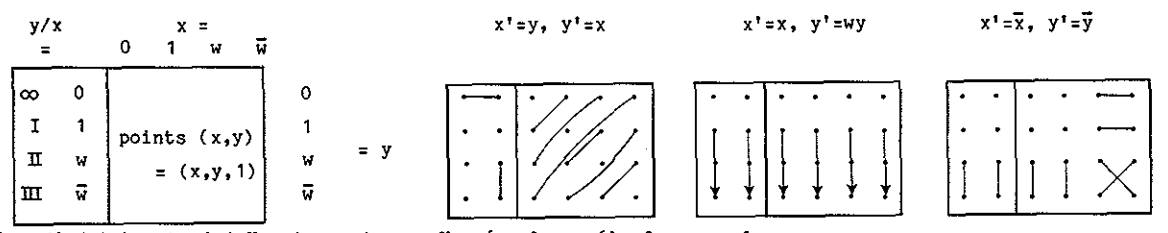
0 1 0 1 w $\bar{w}$ (a) 0 1 0 1 w $\bar{w}$ (b) 0 1 0 1 w $\bar{w}$ (c) 0 1 0 1 w $\bar{w}$ (d)

Triad In terms of the 21 point projective plane over $\mathbb{F}_4 = \{0,1,w,\bar{w}\}$ the octads of the Steiner system are

the points of a line	+	3 extra points I, II, III
a hyperconic	+	2 of I, II, III
an $\mathbb{F}_2$-subplane	+	one of I, II, III

or the points of two lines without their intersection.

Every maximal set of points with no three in line is a hyperconic, consisting of the five points of a conic and the unique point not on any chord. Hyperconics whose equations have coefficients in $\mathbb{F}_2$ mate with {II, III}, while the $\mathbb{F}_2$-subplane of all points with coordinates in $\mathbb{F}_2$ mates with I. An element of $GL_3(4)$ with determinant w effects the permutation (I II III). In fact the construction is invariant under $P\Gamma L_3(4)$, with the field automorphism effecting (II III). In the MOG array below points (x,y,0) in the left-hand octad are denoted y/x, while the points (x,y)=(x,y,1) form the right-hand square. We also show the elements of M_{24} obtained from some elements of $P\Gamma L_3(4)$:



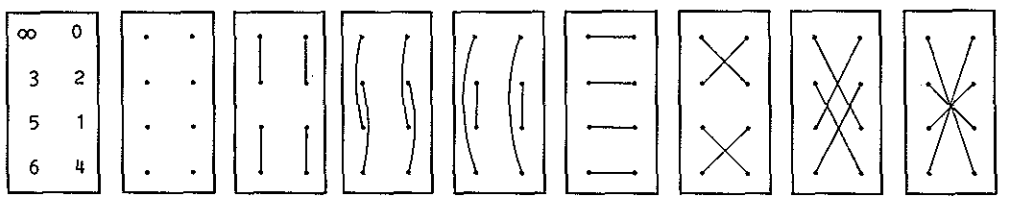
Trio We select two disjoint extended Hamming codes on V = {∞,0,...,6}, for example

(i) {}, {∞, i+3, i+5, i+6}, and complements

(ii) {}, {∞, i+1, i+2, i+4}, and complements

The binary Golay code consists of all words of the form (X+t,Y+t,Z+t) for which X, Y, Z are elements of the first Hamming code with X+Y+Z = {} or V, while t is in the second Hamming code. Using the indicated numbering for each brick of the MOG, this construction exhibits a maximal subgroup $2^6{:}(S_3 \times L_2(7))$ of M_{24} in which the typical element of 2^6 consists of three translations (see below) with trivial product, S_3 permutes the three bricks, and $L_2(7)$ acts on each brick in the same way.

The brick numbering and the translations



The eight 3-sets permuted by the $L_2(7)$ in this construction give rise to 14 dodecads by taking unions. There is one other type of family (octern) of eight 3-sets with this property, whose stabilizer is a maximal subgroup $L_2(7)$, acting transitively.

Octad This construction uses the isomorphism between A_8 and $L_4(2)$, and is only briefly sketched here. The 24 points are taken as the points of an affine 4-space V over $\mathbb{F}_2$ (whose automorphism group is $2^4{:}L_4(2) \cong 2^4{:}A_8$), together with those of the 8-point set S = {1,2,...,8} on which A_8 acts. They can be written in the MOG array thus:

1	5	0	a	b	a+b
2	6	c	a+c	b+c	a+b+c
3	7	d	a+d	b+d	a+b+d
4	8	c+d	a+c+d	b+c+d	a+b+c+d
S		V			

The stabilizer of a system of four parallel 2-spaces in V is also the stabilizer of a bisection of S into two complementary tetrads. The six 4-sets so obtained form one of a family of 35 sextets which suffice to define the code. The 759 octads of S(5,8,24) are the 1 octad S, 280 = 35x2x4 meeting S in 4 points (obtainable from the above sextets), 448 meeting S in 2 points, and the 30 affine 3-spaces in V.

Modulo 23 The binary Golay code is a quadratic residue code, modulo 23. In the projective line PL(23), let Q={0,quadratic residues}, N={∞,non-residues}. The code is spanned by the words $w_\infty = \Sigma e_i$ (all i), $w_t = \Sigma e_{n-t}$ (n in N). We have $\Sigma c_i w_i = 0 \Leftrightarrow (c_i)$ is in the code. G is generated by the elements $A : t \to t+1$, $B : t \to 2t$, $C : t \to -1/t$ and $D : t \to t^3/9$ (t in Q) or $9t^3$ (t in N), in which $\langle A,B,C\rangle \cong PSL_2(23)$ and $\langle A,B,D\rangle \cong M_{23}$.

Mathieu 12 The Mathieu group M_{12} acts on two 12-sets $L=\{n^+\}$, $R=\{n^-\}$, interchanged by its outer automorphism (duality). The stabilizer of a tetrad in one stabilizes a tetrad in the other while the stabilizer of a duad in one stabilizes a pair of complementary hexads in the other. The resulting octads splitting 4+4, 2+6, 6+2 form the Steiner system S(5,8,24). Thus (using the shuffle numbering, see M_{12}) the binary Golay code is spanned by $\{1^+2^+3^+4^+5^+6^+0^-1^-\}$, $\{0^+1^+2^+3^+7^+8^+1^-2^-\}$, $\{0^+1^+2^+4^+5^+9^+2^-3^-\}$, $\{0^+1^+3^+4^+6^+7^+3^-4^-\}$, $\{0^+1^+2^+3^+5^+10^+4^-5^-\}$, $\{0^+1^+2^+4^+6^+8^+5^-6^-\}$, $\{0^+2^+3^+4^+5^+7^+6^-7^-\}$, $\{0^+1^+2^+3^+6^+9^+7^-8^-\}$, $\{0^+1^+3^+4^+5^+8^+8^-9^-\}$, $\{0^+1^+2^+5^+6^+7^+9^-10^-\}$, $\{0^+1^+2^+3^+4^+11^+10^-11^-\}$ and $\{0^+1^+2^+3^+4^+5^+6^+7^+8^+9^+10^+11^+\}$. The shuffle and modulo 23 numberings are related by:

n^+ = whichever of n, −n is in Q n^- = whichever of 8/n, −8/n is in N.

These and the lexicographic numbering (see below) appear in the MOG array as follows:

0	∞	1	11	2	22
19	3	20	4	10	18
15	6	14	16	17	8
5	9	21	13	7	12

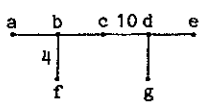
0^+	0^-	1^+	7^-	2^+	8^-
2^-	3^+	5^-	4^+	10^-	5^+
1^-	6^+	6^-	7^+	9^-	8^+
3^-	9^+	4^-	10^+	11^-	11^+

d_0	d_4	d_8	d_{12}	d_{16}	d_{20}
d_1	d_5	d_9	d_{13}	d_{17}	d_{21}
d_2	d_6	d_{10}	d_{14}	d_{18}	d_{22}
d_3	d_7	d_{11}	d_{15}	d_{19}	d_{23}

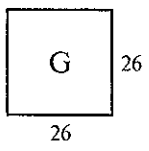
Lexicographic Each code word can be defined in succession as the lexicographically earliest word $(d_0,\ldots,d_{23})$ differing in at least 8 places from all its predecessors:

(0^{24}) $(0^{16}1^8)$ $(0^{12}1^4 0^4 1^4)$ ... (1^{24})

M_{24}

Presentation $G \cong \langle$  $\mid a=(cd)^5=(cde)^5, f=(cdg)^5, e=(bcf)^3, 1=(abf)^3 \rangle$

Maximal subgroups				Specifications		
Order	Index	Structure	Character	Abstract	Mathieu	Golay
10200960	24	M_{23}	1a+23a		point	weight 1 cocode word
887040	276	$M_{22}:2$	1a+23a+252a		duad	weight 2 cocode word
322560	759	$2^4:A_8$	1a+23a+252a+483a	$N(2A^4)$	octad	weight 8 code word
190080	1288	$M_{12}:2$	1a+252a+1035a		duum	weight 12 code words
138240	1771	$2^6:3\cdot S_6$	1a+252a+483a+1035a	$N(2^6) = N(2A_{45}B_{18})$	sextet	weight 4 cocode word
120960	2024	$L_3(4):S_3$	1a+23a+252a+483a+1265a		triad	weight 3 cocode word
64512	3795	$2^6:(L_3(2)xS_3)$	1a+252a+483a+1035a+2024a	$N(2^6) = N(2A_{21}B_{42})$	trio	
6072	40320	$L_2(23)$		N(2B,3B,4C,6B,11A,12B,23AB)	projective line	
168	1457280	$L_2(7)$		N(2B,3A,4C,7AB)	octern	



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			21	7	1																							
	244823040		504	680	080	504	384	128	96	60	24	24	42	42	16	20	11	12	12	14	14	15	15	21	21	23	23	
	p power		A	A	A	A	A	A	B	A	AA	BB	A	A	B	AB	A	AA	BC	AA	BA	AA	AA	BB	AB	A	A	
	p' part		A	A	A	A	A	A	A	A	AA	BB	A	A	A	AB	A	AA	BC	AA	BA	AA	AA	AB	BB	A	A	
	ind	1A	2A	2B	3A	3B	4A	4B	4C	5A	6A	6B	7A	B**	8A	10A	11A	12A	12B	14A	B**	15A	B**	21A	B**	23A	B**	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	23	7	-1	5	-1	-1	3	-1	3	1	-1	2	2	1	-1	1	-1	-1	0	0	0	0	-1	-1	0	0	χ_2
χ_3	o	45	-3	5	0	3	-3	1	1	0	0	-1	b7	**	-1	0	1	0	1	-b7	**	0	0	b7	**	-1	-1	χ_3
χ_4	o	45	-3	5	0	3	-3	1	1	0	0	-1	**	b7	-1	0	1	0	1	**	-b7	0	0	**	b7	-1	-1	χ_4
χ_5	o	231	7	-9	-3	0	-1	-1	3	1	1	0	0	0	-1	1	0	-1	0	0	0	b15	**	0	0	1	1	χ_5
χ_6	o	231	7	-9	-3	0	-1	-1	3	1	1	0	0	0	-1	1	0	-1	0	0	0	**	b15	0	0	1	1	χ_6
χ_7	+	252	28	12	9	0	4	4	0	2	1	0	0	0	0	2	-1	1	0	0	0	-1	-1	0	0	-1	-1	χ_7
χ_8	+	253	13	-11	10	1	-3	1	1	3	-2	1	1	1	-1	-1	0	0	1	-1	-1	0	0	1	1	0	0	χ_8
χ_9	+	483	35	3	6	0	3	3	3	-2	2	0	0	0	-1	-2	-1	0	0	0	0	1	1	0	0	0	0	χ_9
χ_{10}	o	770	-14	10	5	-7	2	-2	-2	0	1	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	b23	**	χ_{10}
χ_{11}	o	770	-14	10	5	-7	2	-2	-2	0	1	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	**	b23	χ_{11}
χ_{12}	o	990	-18	-10	0	3	6	2	-2	0	0	-1	b7	**	0	0	0	0	1	b7	**	0	0	b7	**	1	1	χ_{12}
χ_{13}	o	990	-18	-10	0	3	6	2	-2	0	0	-1	**	b7	0	0	0	0	1	**	b7	0	0	**	b7	1	1	χ_{13}
χ_{14}	+	1035	27	35	0	6	3	-1	3	0	0	2	-1	-1	1	0	1	0	0	-1	-1	0	0	-1	-1	0	0	χ_{14}
χ_{15}	o	1035	-21	-5	0	-3	3	3	-1	0	0	1	2b7	**	-1	0	1	0	-1	0	0	0	0	-b7	**	0	0	χ_{15}
χ_{16}	o	1035	-21	-5	0	-3	3	3	-1	0	0	1	**	2b7	-1	0	1	0	-1	0	0	0	0	**	-b7	0	0	χ_{16}
χ_{17}	+	1265	49	-15	5	8	-7	1	-3	0	1	0	-2	-2	1	0	0	-1	0	0	0	0	0	1	1	0	0	χ_{17}
χ_{18}	+	1771	-21	11	16	7	3	-5	-1	1	0	-1	0	0	-1	1	0	0	-1	0	0	1	1	0	0	0	0	χ_{18}
χ_{19}	+	2024	8	24	-1	8	8	0	0	-1	-1	0	1	1	0	-1	0	-1	0	1	1	-1	-1	1	1	0	0	χ_{19}
χ_{20}	+	2277	21	-19	0	6	-3	1	-3	-3	0	2	2	2	-1	1	0	0	0	0	0	0	0	-1	-1	0	0	χ_{20}
χ_{21}	+	3312	48	16	0	-6	0	0	0	-3	0	-2	1	1	0	1	1	0	0	-1	-1	0	0	1	1	0	0	χ_{21}
χ_{22}	+	3520	64	0	10	-8	0	0	0	0	-2	0	-1	-1	0	0	0	0	0	1	1	0	0	-1	-1	1	1	χ_{22}
χ_{23}	+	5313	49	9	-15	0	1	-3	-3	3	1	0	0	0	-1	-1	0	1	0	0	0	0	0	0	0	0	0	χ_{23}
χ_{24}	+	5544	-56	24	9	0	-8	0	0	-1	1	0	0	0	0	-1	0	1	0	0	0	-1	-1	0	0	1	1	χ_{24}
χ_{25}	+	5796	-28	36	-9	0	-4	4	0	1	-1	0	0	0	0	1	-1	-1	0	0	0	1	1	0	0	0	0	χ_{25}
χ_{26}	+	10395	-21	-45	0	0	3	-1	3	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{26}

$G_2(4)$

Chevalley group $G_2(4)$

Order = 251,596,800 = $2^{12}.3^3.5^2.7.13$ Mult = 2 Out = 2

Constructions

Chevalley $G_2(4) \cong G$: the adjoint Chevalley group of type G_2 over $\mathbb{F}_4$;

G.2 : the automorphism group of a generalized hexagon of order (4,4), consisting of 1365 vertices and 1365 edges, each object being incident with 5 of the other type.

Cayley G : the automorphism group of the $\mathbb{F}_4$ Cayley algebra, the tensor product of the field $\mathbb{F}_4 = \{0,1,w,\bar{w}\}$ with the non-associative ring of Cayley integers. Calculations are complicated by the fact that there is no very symmetric base for the Cayley integers. They can be generated (in $\mathbb{R}^8$) by the even sums of X_∞, X_0, ..., X_6 together with $1 = \frac{1}{2}\Sigma X_n$, with multiplication defined by the equations $2X_n^2 = X_n - 1$, $2X_0X_\infty = 1-X_1-X_2-X_4$, $2X_\infty X_0 = 1-X_3-X_5-X_6$ and their images under $L_2(7)$. Additively, this is a scaled copy of the E_8 root lattice under the norm $N(\Sigma a_n X_n) = \frac{1}{2}\Sigma a_n^2$. Each $\mathbb{F}_4$ Cayley number has the form A+wB where A and B are Cayley integers modulo doubles of Cayley integers. (Note: since X_n is not a Cayley integer, $2X_n$ is not 0 in this algebra.) There are 1 + 4095 + 2080x2 + 2016x4 elements of determinant 1, with respective orders 1, 2, 3 and 5. The elements of order 2 fall into 1365 triples of the form $\{1+X, 1+wX, 1+\bar{w}X\}$. The map $X \rightarrow A^{-1}XA$ is an automorphism of the algebra just if A has order 3 (or 1). There is a subgroup isomorphic to the Hall-Janko group J_2 containing 280x2 of these maps.

Quaternionic 2G : the automorphism group of the quaternionic Leech lattice, whose typical element is written $(q_\infty;q_0,\ldots,q_4)$. The quaternions ±1, ±i, ±j, ±k, $w = \frac{1}{2}(-1+i+j+k)$ multiplicatively generate a group $2A_4$ and additively generate the Hurwitz ring of integral quaternions. The lattice is closed under the scalar left multiplications by these numbers. There is a monomial group $2^{5+6}.(3xA_5)$ (acting on the right) generated by diag(i;i,j,k,k,j), diag(w;w,w,w,w,w), $(q_\infty)(q_0\ q_1\ \ldots\ q_4)$ and $(q_\infty\ q_0)(q_1\ q_4)$. There are 196560 = 24 x 8190 minimal vectors, which generate 8190 1-spaces, and span the lattice. They are the images under the monomial group of $(2+2i;0^5)$ (6), $(2;2,0^4)$ (120), (0;0,1+k,1+j,1+j,1+k) (1920), $(i+j+k;1^5)$ (6144), and fall naturally into 1365 congruence bases mod <1+i>. The corresponding basic decompositions, such as $\langle e_\infty\rangle\oplus\ldots\oplus\langle e_4\rangle$ each refine 5 trio decompositions such as $\langle e_\infty,e_0\rangle\oplus\langle e_1,e_4\rangle\oplus\langle e_2,e_3\rangle$, which in turn each refine to 5 basic decompositions.

The quaternionic form of a real Leech lattice vector is obtained by interpreting the columns of the MOG as the quaternion coordinates of a vector $(q_\infty;q_0,\ldots,q_4) = \Sigma q_t e_t$, according to the scheme:

q_∞	q_0	q_1	q_2	q_4	q_3
1	1	1	1	1	1
k	j	k	j	k	j
i	k	i	k	i	k
j	i	j	i	j	i

and then dividing by 2.

The unit group $2A_4$ extends in $2Co_1$ to $2A_5$, which additively generates the icosian ring of integral quaternions, and converts the lattice into a 3-dimensional module over this ring, whose automorphism group is $2J_2$ (q.v.). The subgroup $3L_3(4)$ can be written over the complex subring $\mathbb{Z}[w]$.

Graph G.2 : the automorphism group of a rank 3 graph of valence 100 on 416 points, in which the point stabilizer is $J_2.2$.

Presentations G.2 $\cong$ < a b c 8 d e f g | $a=(cd)^4,(bcde)^8=1$ >

G $\cong$ < e d 8 c b f | $a=(cd)^4, 1=(abf)^5=(abcf)^5=(abcdecbdc)^7[=(abc)^5]$ >
5
a

Remark : $G_2(4)$ is a maximal subgroup of the Suzuki group, and contains the Hall-Janko group J_2.

Maximal subgroups

Order	Index	Structure	G.2	Character
604800	416	J_2	: J_2:2	1a+65a+350a
184320	1365	$2^{2+8}:(3xA_5)$	: $2^{2+8}:(3xA_5):2$	1a+350a+364a+650a
184320	1365	$2^{4+6}:(A_5x3)$	: $2^{4+6}:(A_5x3):2$	1a+350a+364b+650a
124800	2016	$U_3(4):2$	: $U_3(4):4$	1a+650a+1365a
120960	2080	$3^{\cdot}L_3(4):2_3$	: $3^{\cdot}L_3(4):2^2$	1a+350a+364b+1365a
12096	20800	$U_3(3):2$	: $U_3(3):2 \times 2$	
3600	69888	$A_5 \times A_5$	: $(A_5xA_5):2$	
1092	230400	$L_2(13)$	: $L_2(13):2$	

Specifications

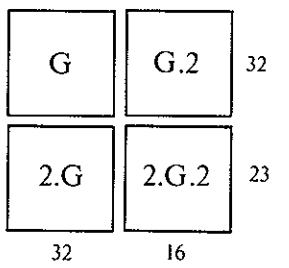
Abstract	Chevalley	Cayley	Quaternionic	Graph
			icosian ring	point
$N(2A^2)$	vertex	null subalgebra	trio decomposition	non-edge
$N(2A^4)$	edge	order 2 triple	base	
	${}^2A_2(4)$	order 5 element		
N(3A)	$A_2(4)$	order 3 element	complex subring	
C(2C)	$G_2(2)$	subfield		edge
N(2A,3B,5AB), N(2B,3A,5CD)	$D_2(4)$	quaternion subalgebra		6-point coclique
N(2B,3B,6B,7A,13AB)				

$G_2(4)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			61	3	60		1																											
	251596800		440	840	480	180	536	768	512	300	300	300	300	192	12	21	32	32	20	20	20	20	48	48	48	13	13	15	15	15	15	21	21	
	p	power	A	A	A	A	A	A	A	A	A	A	A	AA	BB	A	A	C	DA	CA	BB	AB	AA	AB	AB	A	A	BA	AA	DB	CB	AA	AA	
	p'	part	A	A	A	A	A	A	A	A	A	A	A	AA	BB	A	A	A	CA	DA	BB	AB	AA	AB	AB	A	A	AA	BA	CB	DB	AA	AA	
	ind	1A	2A	2B	3A	3B	4A	4B	4C	5A	B*	5C	D*	6A	6B	7A	8A	8B	10A	B*	10C	D*	12A	12B	C**	13A	B*	15A	B*	15C	D*	21A	B*	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	65	1	5	20	-1	9	-3	1	0	0	5	5	4	-1	2	1	1	1	1	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	χ_2
χ_3	+	78	14	-6	15	0	6	2	-2	3	3	3	3	-1	0	1	-2	2	-1	-1	-1	-1	3	-1	-1	0	0	0	0	0	0	1	1	χ_3
χ_4	o	300	-20	0	-15	0	4	8	-4	0	0	0	0	1	0	-1	0	0	0	0	0	0	1	2i3-1	-2i3-1	1	1	0	0	0	0	-1	-1	χ_4
χ_5	o	300	-20	0	-15	0	4	8	-4	0	0	0	0	1	0	-1	0	0	0	0	0	0	1	-2i3-1	2i3-1	1	1	0	0	0	0	-1	-1	χ_5
χ_6	+	350	30	10	35	5	6	2	-2	0	0	0	0	3	1	0	2	-2	0	0	0	0	3	-1	-1	-1	-1	0	0	0	0	0	0	χ_6
χ_7	+	364	-20	4	49	1	12	4	-4	-1	-1	4	4	1	1	0	0	0	0	0	-1	-1	-3	1	1	0	0	-1	-1	1	1	0	0	χ_7
χ_8	+	364	44	0	-14	4	4	8	-4	4	4	-1	-1	2	0	0	0	0	-1	-1	0	0	-2	2	2	0	0	1	1	-1	-1	0	0	χ_8
χ_9	+	378	-6	-6	0	0	-6	-6	10	3	3	3	3	0	0	0	2	-2	-1	-1	-1	-1	0	0	0	1	1	0	0	0	0	0	0	χ_9
χ_{10}	+	650	10	10	20	5	-6	-6	10	0	0	0	0	4	1	-1	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{10}
χ_{11}	+	819	-13	-9	63	0	11	-1	3	2r5-1	-2r5-1	-3b5	*	-1	0	0	-1	-1	b5	*	1	1	-1	-1	-1	0	0	*	-b5	0	0	0	0	χ_{11}
χ_{12}	+	819	-13	-9	63	0	11	-1	3	-2r5-1	2r5-1	*	-3b5	-1	0	0	-1	1	*	b5	1	1	-1	-1	-1	0	0	-b5	*	0	0	0	0	χ_{12}
χ_{13}	+	819	51	-13	0	3	3	3	3	-3b5	*	-2r5-1	2r5-1	0	-1	0	-1	-1	1	1	b5	*	0	0	0	0	0	0	0	-b5	*	0	0	χ_{13}
χ_{14}	+	819	51	-13	0	3	3	3	3	*	-3b5	2r5-1	-2r5-1	0	-1	0	-1	-1	1	1	*	b5	0	0	0	0	0	0	0	*	-b5	0	0	χ_{14}
χ_{15}	+	1300	20	20	85	-5	4	4	4	0	0	0	0	5	-1	-2	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	1	1	χ_{15}
χ_{16}	+	1365	85	21	-21	0	5	5	5	5	5	0	0	-5	0	0	1	1	0	0	1	1	-1	-1	-1	0	0	-1	-1	0	0	0	0	χ_{16}
χ_{17}	+	2925	45	-15	-45	0	5	-7	-3	0	0	0	0	3	0	-1	1	1	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	-b21	*	χ_{17}
χ_{18}	+	2925	45	-15	-45	0	5	-7	-3	0	0	0	0	3	0	-1	1	1	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	*	-b21	χ_{18}
χ_{19}	+	2925	45	-15	90	0	5	-7	-3	0	0	0	0	-6	0	-1	1	1	0	0	0	0	2	2	2	0	0	0	0	0	0	-1	-1	χ_{19}
χ_{20}	+	3276	-52	12	63	0	-4	-4	-4	*	b5+4	*	3b5	-1	0	0	0	0	*	-b5	*	b5	-1	-1	-1	0	0	-b5	*	0	0	0	0	χ_{20}
χ_{21}	+	3276	-52	12	63	0	-4	-4	-4	b5+4	*	3b5	*	-1	0	0	0	0	-b5	*	b5	*	-1	-1	-1	0	0	*	-b5	0	0	0	0	χ_{21}
χ_{22}	+	3276	12	8	0	3	-12	0	-4	3b5	*	*	b5+4	0	-1	0	0	0	*	b5	-b5	*	0	0	0	0	0	0	0	-b5	*	0	0	χ_{22}
χ_{23}	+	3276	12	8	0	3	-12	0	-4	*	3b5	b5+4	*	0	-1	0	0	0	b5	*	*	-b5	0	0	0	0	0	0	0	*	-b5	0	0	χ_{23}
χ_{24}	+	4095	-65	3	-63	0	7	-5	-1	-5b5	*	0	0	1	0	0	-1	-1	0	0	-b5	*	1	1	1	0	0	b5	*	0	0	0	0	χ_{24}
χ_{25}	+	4095	-65	3	-63	0	7	-5	-1	*	-5b5	0	0	1	0	0	-1	-1	0	0	*	-b5	1	1	1	0	0	*	b5	0	0	0	0	χ_{25}
χ_{26}	+	4095	63	-5	0	-3	-9	3	-1	0	0	-5b5	*	0	1	0	-1	-1	-b5	*	0	0	0	0	0	0	0	0	0	b5	*	0	0	χ_{26}
χ_{27}	+	4095	63	-5	0	-3	-9	3	-1	0	0	*	-5b5	0	1	0	-1	-1	*	-b5	0	0	0	0	0	0	0	0	0	*	b5	0	0	χ_{27}
χ_{28}	+	4096	0	0	64	4	0	0	0	-4	-4	-4	-4	0	0	1	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	1	1	χ_{28}
χ_{29}	+	4160	64	0	20	-4	0	0	0	0	0	-5	-5	4	0	2	0	0	-1	-1	0	0	0	0	0	0	0	0	0	1	1	-1	-1	χ_{29}
χ_{30}	+	4725	-75	-15	0	0	-3	9	5	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	b13	*	0	0	0	0	0	0	χ_{30}
χ_{31}	+	4725	-75	-15	0	0	-3	9	5	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	*	b13	0	0	0	0	0	0	χ_{31}
χ_{32}	+	5460	20	24	-84	0	12	0	4	-5	-5	0	0	-4	0	0	0	0	0	0	-1	-1	0	0	0	0	0	1	1	0	0	0	0	χ_{32}
	ind	1	2	4	3	3	4	4	4	5	5	5	5	6	12	7	8	8	10	10	20	20	12	12	12	13	13	15	15	15	15	21	21	
		2	2		6	6	4			10	10	10	10	6		14		8	10	10			12			26	26	30	30	30	30	42	42	
χ_{33}	-	12	-4	0	-6	0	-4	0	0	2	2	-3	-3	2	0	-2	0	2	1	1	0	0	2	0	0	-1	-1	-1	-1	0	0	1	1	χ_{33}
χ_{34}	-	104	8	0	-22	2	-8	0	0	2b5	*	b5-3	*	2	0	-1	0	0	*	-b5	0	0	-2	0	0	0	0	-b5	*	b5	*	-1	-1	χ_{34}
χ_{35}	-	104	8	0	-22	2	-8	0	0	*	2b5	*	b5-3	2	0	-1	0	0	-b5	*	0	0	-2	0	0	0	0	*	-b5	*	b5	-1	-1	χ_{35}
χ_{36}	-	364	-36	0	-14	4	-4	0	0	4	4	-1	-1	-6	0	0	0	2	-1	-1	0	0	2	0	0	0	0	1	1	-1	-1	0	0	χ_{36}
χ_{37}	-	560	-16	0	-70	-4	-16	0	0	0	0	-5	-5	2	0	0	0	0	-1	-1	0	0	2	0	0	1	1	0	0	1	1	0	0	χ_{37}
χ_{38}	-	1260	-36	0	0	6	12	0	0	0	0	-5	-5	0	0	0	0	-2	-1	-1	0	0	0	0	0	-1	-1	0	0	1	1	0	0	χ_{38}
χ_{39}	-	1800	40	0	-90	0	-8	0	0	0	0	0	0	-2	0	1	0	0	0	0	0	0	-2	0	0	b13	*	0	0	0	0	1	1	χ_{39}
χ_{40}	-	1800	40	0	-90	0	-8	0	0	0	0	0	0	-2	0	1	0	0	0	0	0	0	-2	0	0	*	b13	0	0	0	0	1	1	χ_{40}
χ_{41}	-	1820	-52	0	-70	2	-4	0	0	0	0	-5b5	*	2	0	0	0	-2	-b5	*	0	0	2	0	0	0	0	0	0	b5	*	0	0	χ_{41}
χ_{42}	-	1820	-52	0	-70	2	-4	0	0	0	0	*	-5b5	2	0	0	0	-2	*	-b5	0	0	2	0	0	0	0	0	0	*	b5	0	0	χ_{42}
χ_{43}	-	2016	96	0	0	6	0	0	0	6	6	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	1	1	0	0	χ_{43}
χ_{44}	-	2184	-88	0	42	0	-8	0	0	2r5+4	-2r5+4	-3b5	*	2	0	0	0	0	b5	*	0	0	-2	0	0	0	0	b5	*	0	0	0	0	χ_{44}
χ_{45}	-	2184	-88	0	42	0	-8	0	0	-2r5+4	2r5+4	*	-3b5	2	0	0	0	0	*	b5	0	0	-2	0	0	0	0	*	b5	0	0	0	0	χ_{45}
χ_{46}	-	3276	60	0	0	-6	12	0	0	6	6	-4	-4	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	χ_{46}
χ_{47}	-	3600	80	0	90	0	-16	0	0	0	0	0	0	2	0	2	0	0	0	0	0	0	2	0	0	-1	-1	0	0	0	0	-1	-1	χ_{47}
χ_{48}	-	3744	32	0	-36	0	0	0	0	-3r5-1	3r5-1	-3b5	*	-4	0	-1	0	0	b5	*	0	0	0	0	0	0	0	-1	-1	0	0	-1	-1	χ_{48}
χ_{49}	-	3744	32	0	-36	0	0	0	0	3r5-1	-3r5-1	*	-3b5	-4	0	-1	0	0	*	b5	0	0	0	0	0	0	0	-1	-1	0	0	-1	-1	χ_{49}
χ_{50}	-	3900	-20	0	-60	0	12	0	0	0	0	0	0	4	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	b21	*	χ_{50}
χ_{51}	-	3900	-20	0	-60	0	12	0	0	0	0	0	0	4	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	*	b21	χ_{51}
χ_{52}	-	4096	0	0	64	4	0	0	0	-4	-4	-4	-4	0	0	1	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	1	1	χ_{52}
χ_{53}	-	5824	-64	0	28	-2	0	0	0	2b5	*	-2r5-1	2r5-1	-4	0	0	0	0	1	1	0	0	0	0	0	0	0	-b5	*	-b5	*	0	0	χ_{53}
χ_{54}	-	5824	-64	0	28	-2	0	0	0	*	2b5	2r5-1	-2r5-1	-4	0	0	0	0	1	1	0	0	0	0	0	0	0	*	-b5	*	-b5	0	0	χ_{54}
χ_{55}	-	7488	64	0	36	0	0	0	0	-2	-2	3	3	4	0	-2	0	0	-1	-1	0	0	0	0	0	0	0	1	1	0	0	1	1	χ_{55}

$G_2(4)$

			@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
	;	;	12096A	192A	48B	216AC	18BC	96A	48B	32A	24AD	6BE	7AC	8B	8B	12AC	12BD	12CD	
			A	A	A	AC	BC	A	A	A	AD	BE	AC	A	A	AC	AD	AD	
	fus	ind	2C	4D	4E	6C	6D	8C	8D	8E	12D	12E	14A	16A	B**	24A	24B	C**	
χ_1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	:	++	7	-1	1	-2	1	3	-3	-1	2	1	0	-1	-1	0	0	0	χ_2
χ_3	:	++	6	-2	0	-3	0	2	4	-2	1	0	-1	0	0	-1	1	1	χ_3
χ_4	:	oo	6	-2	0	-3	0	-2	0	2	1	0	-1	0	0	1	i3	-i3	χ_4
χ_5	:	oo	6	-2	0	-3	0	-2	0	2	1	0	-1	0	0	1	-i3	i3	χ_5
χ_6	:	++	28	4	2	1	1	4	2	0	1	-1	0	0	0	1	-1	-1	χ_6
χ_7	:	++	14	-2	2	5	-1	2	-2	2	1	-1	0	0	0	-1	1	1	χ_7
χ_8	:	++	14	6	0	-4	2	-2	0	2	0	0	0	0	0	-2	0	0	χ_8
χ_9	:	oo	0	0	0	0	0	0	0	0	0	0	0	2i	-2i	0	0	0	χ_9
χ_{10}	:	++	20	4	4	2	-1	0	0	0	-2	1	-1	0	0	0	0	0	χ_{10}
χ_{11}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}																			χ_{12}
χ_{13}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}																			χ_{14}
χ_{15}	:	++	14	-2	-2	5	-1	2	2	2	1	1	0	0	0	-1	-1	-1	χ_{15}
χ_{16}	:	++	21	5	-3	3	0	1	1	1	-1	0	0	-1	-1	1	1	1	χ_{16}
χ_{17}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}																			χ_{18}
χ_{19}	:	++	27	3	-3	0	0	3	-3	-1	0	0	-1	1	1	0	0	0	χ_{19}
χ_{20}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{20}
χ_{21}																			χ_{21}
χ_{22}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{22}
χ_{23}																			χ_{23}
χ_{24}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{24}
χ_{25}																			χ_{25}
χ_{26}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{26}
χ_{27}																			χ_{27}
χ_{28}	:	++	64	0	0	-8	-2	0	0	0	0	0	1	0	0	0	0	0	χ_{28}
χ_{29}	:	++	56	-8	0	2	2	0	0	0	-2	0	0	0	0	0	0	0	χ_{29}
χ_{30}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{30}
χ_{31}																			χ_{31}
χ_{32}	:	++	42	2	0	6	0	-6	0	-2	2	0	0	0	0	0	0	0	χ_{32}
	fus	ind	2	4	8 8	6	6	8	8 8	8	12	24 24	14 14	16 16	16 16	24 24	24 24	24 24	
χ_{33}	:	--	0	0	0	0	0	0	2r2	0	0	0	0	r2	r2	0	-r2	-r2	χ_{33}
χ_{34}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{34}
χ_{35}																			χ_{35}
χ_{36}	:	--	0	0	0	0	0	0	2r2	0	0	0	0	-r2	-r2	0	-r2	-r2	χ_{36}
χ_{37}	:	--	0	0	0	0	0	0	4r2	0	0	0	0	0	0	0	r2	r2	χ_{37}
χ_{38}	:	oo	0	0	2i2	0	0	0	0	0	0	-i2	0	i2	-i2	0	0	0	χ_{38}
χ_{39}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{39}
χ_{40}																			χ_{40}
χ_{41}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{41}
χ_{42}																			χ_{42}
χ_{43}	:	oo	0	0	4i2	0	0	0	0	0	0	i2	0	0	0	0	0	0	χ_{43}
χ_{44}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{44}
χ_{45}																			χ_{45}
χ_{46}	:	oo	0	0	2i2	0	0	0	0	0	0	-i2	0	-i2	i2	0	0	0	χ_{46}
χ_{47}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i6	-i6	χ_{47}
χ_{48}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{48}
χ_{49}																			χ_{49}
χ_{50}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{50}
χ_{51}																			χ_{51}
χ_{52}	:	oo	0	0	0	0	0	0	0	0	0	0	i7	0	0	0	0	0	χ_{52}
χ_{53}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{53}
χ_{54}																			χ_{54}
χ_{55}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	2i3	0	0	χ_{55}



M^cL

Sporadic McLaughlin group M^cL

Order = 898,128,000 = $2^7.3^6.5^3.7.11$ Mult = 3 Out = 2

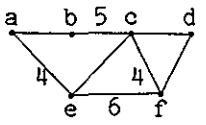
Constructions

Graph G.2 : the automorphism group of the McLaughlin graph, a rank 3 graph of valence 112 on 275 points in which the point stabilizer is $U_4(3).2$. Under the simple group G, the 2-point stabilizers are $L_3(4)$ and $3^4{:}A_6$, and there are two families of 22-point cocliques stabilized by M_{22}, and interchanged by the outer automorphism.

Leech G : the pointwise stabilizer of a triangle ABC of type 322 in the Leech lattice (see Co_1). G.2 : the setwise stabilizer. The 275 vertices V of the McLaughlin graph are represented by the lattice points for which VA, VB, VC are all of type 2, and V is joined to V' in the graph just when VV' has type 3. Taking A = $(0,0,0^{22})$, B = $(4,4,0^{22})$, C = $(5,1,1^{22})$ these points are $(4,0,4,0^{21})$ (22), $(2,2,2^6 0^{16})$ (77) and $(3,1,-1^7 1^{15})$ (176), and the containment of M_{22} is visible.

In the Leech lattice modulo 2, the 3-spaces containing the fixed 2-space (of type 223) become points in the quotient 22-dimensional representation over $\mathbb{F}_2$. They are described below by the types of their non-zero vectors. The remaining maximal subgroups arise as intersections of G with maximal subgroups of Co_1 (q.v.).

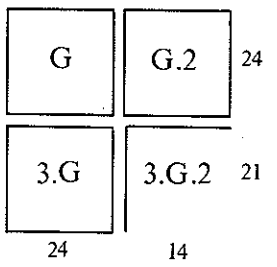
Unitary (5) 3.G has a 45-dimensional unitary representation written over $\mathbb{F}_{25}$. Explicit matrices have been computed.

Presentation 3.G ≅ <  | $a=(cf)^2, b=(ef)^3, 1=(eab)^3=(bce)^5=(aecd)^4=(cef)^{21}$ > ; for G adjoin $(cef)^7=1$

Remark : 3.G.2 is a maximal subgroup of the Lyons group.

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications: Abstract	Graph	Leech
3265920	275	$U_4(3)$	$: U_4(3){:}2_3$	1a+22a+252a		vertex	223-2223-point
443520	2025	M_{22}		1a+22a+252a+1750a		coclique	223-2234-point
443520	2025	M_{22}		1a+22a+252a+1750a		coclique	223-2234-point
126000	7128	$U_3(5)$	$: U_3(5){:}2$	1a+22a+252a+1750a+5103a	N(2A,3B,4A,5A,5B,5B,5B,7AB)		223-2333-point
58320	15400	$3_+^{1+4}{:}2S_5$	$: 3_+^{1+4}{:}4S_5$		N(3A)		$2^{27}3^{36}$-S-lattice
58320	15400	$3^4{:}M_{10}$	$: 3^4{:}(M_{10}x2)$		$N(3^4) = N(3A_{10}B_{30})$	edge	A_2^{12}-hole
40320	22275	$L_3(4){:}2_2$	$: L_3(4){:}2^2$		N(2A,3B,4A,4A,4A,5B,7AB)	non-edge	223-2333-point
40320	22275	$2{\cdot}A_8$	$: 2{\cdot}S_8$		N(2A)		octad space
40320	22275	$2^4{:}A_7$	$2^{2+4}{:}(S_3xS_3)$		$N(2A^4)$		223-2344-point
40320	22275	$2^4{:}A_7$			$N(2A^4)$		223-2344-point
7920	113400	M_{11}	$: M_{11}$ x 2		C(2B)		223-3334-point
3000	299376	$5_+^{1+2}{:}3{:}8$	: H.2		N(5A)		$2^5 3^{10}$-S-lattice



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			40	29																									7															
	898128000		320	160	972	96	750	25	360	36	14	14	8	27	27	30	11	11	12	14	14	30	30	30	30			920	720	18	48	16	5	18	18	10	10	11	11	12	12	
	p	power	A	A	A	A	A	A	AA	BA	A	A	A	A	A	AA	A	A	AA	AA	BA	AA	AA	AAA	BAA			A	A	BB	A	A	BB	AB	BB	AB	AB	BB	AB	AB	AB	
	p'	part	A	A	A	A	A	A	AA	BA	A	A	A	A	A	AA	A	A	AA	AA	BA	AA	AA	AAA	BAA			A	A	BB	A	A	BB	AB	BB	AB	AB	AB	BB	AB	AB	
	ind	1A	2A	3A	3B	4A	5A	5B	6A	6B	7A	B**	8A	9A	B**	10A	11A	B**	12A	14A	B**	15A	B**	30A	B**	fus	ind	2B	4B	6C	8B	8C	10B	12B	12C	20A	B**	22A	B**	24A	B**	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	+	22	6	-5	4	2	-3	2	3	0	1	1	0	1	1	1	0	0	-1	-1	-1	0	0	-2	-2	:	++	0	4	0	-2	2	0	1	-2	-1	-1	0	0	1	1	x_2
x_3	+	231	7	15	6	-1	6	1	7	-2	0	0	-1	0	0	2	0	0	-1	0	0	0	0	2	2	:	++	11	-5	2	-1	-1	1	1	-2	0	0	0	0	-1	-1	x_3
x_4	+	252	28	9	9	4	2	2	1	1	0	0	0	0	0	-2	-1	-1	1	0	0	-1	-1	1	1	:	++	10	10	1	2	2	0	1	1	0	0	-1	-1	-1	-1	x_4
x_5	o	770	-14	-13	5	-2	-5	0	7	1	0	0	0	-1	-1	1	0	0	1	0	0	b15	**	b15	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_5
x_6	o	770	-14	-13	5	-2	-5	0	7	1	0	0	0	-1	-1	1	0	0	1	0	0	**	b15	**	b15																	x_6
x_7	o	896	0	32	-4	0	-4	1	0	0	0	0	0	-1	-1	0	b11	**	0	0	0	2	2	0	0	:	oo	16	0	-2	0	0	1	0	0	0	0	b11	**	0	0	x_7
x_8	o	896	0	32	-4	0	-4	1	0	0	0	0	0	-1	-1	0	**	b11	0	0	0	2	2	0	0	:	oo	16	0	-2	0	0	1	0	0	0	0	**	b11	0	0	x_8
x_9	+	1750	70	-5	13	2	0	0	-5	1	0	0	0	-2	-2	0	1	1	-1	0	0	0	0	0	0	:	++	10	-10	1	4	0	0	-1	-1	0	0	-1	-1	1	1	x_9
x_{10}	+	3520	64	-44	10	0	-5	0	4	-2	-1	-1	0	1	1	-1	0	0	0	1	1	1	1	-1	-1	:	++	0	16	0	0	0	0	-2	-2	1	1	0	0	0	0	x_{10}
x_{11}	-	3520	-64	-44	10	0	-5	0	-4	2	-1	-1	0	1	1	1	0	0	0	-1	-1	1	1	1	1	:	oo	0	0	0	0	0	0	0	0	i5	-i5	0	0	0	0	x_{11}
x_{12}	+	4500	20	45	-9	4	0	0	5	-1	-1	-1	0	0	0	0	1	1	1	-1	-1	0	0	0	0	:	++	10	10	1	-2	-2	0	1	1	0	0	-1	-1	1	1	x_{12}
x_{13}	-	4752	-48	54	0	0	2	2	-6	0	-1	-1	0	0	0	2	0	0	0	1	1	-1	-1	-1	-1	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	i6	-i6	x_{13}
x_{14}	+	5103	63	0	0	3	3	-2	0	0	0	0	1	0	0	3	-1	-1	0	0	0	0	0	0	0	:	++	45	9	0	3	-1	0	0	0	-1	-1	1	1	0	0	x_{14}
x_{15}	+	5544	-56	36	9	0	19	-1	4	1	0	0	0	0	0	-1	0	0	0	0	0	1	1	-1	-1	:	++	44	-4	-1	0	0	-1	2	-1	1	1	0	0	0	0	x_{15}
x_{16}	o	8019	-45	0	0	3	-6	-1	0	0	-b7	**	-1	0	0	0	0	0	0	-b7	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{16}
x_{17}	o	8019	-45	0	0	3	-6	-1	0	0	**	-b7	-1	0	0	0	0	0	0	**	-b7	0	0	0	0																	x_{17}
x_{18}	o	8250	10	15	6	-2	0	0	-5	-2	-b7	**	0	0	0	0	0	0	1	b7	**	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{18}
x_{19}	o	8250	10	15	6	-2	0	0	-5	-2	**	-b7	0	0	0	0	0	0	1	**	b7	0	0	0	0																	x_{19}
x_{20}	+	9625	105	40	-5	-3	0	0	0	3	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	:	++	55	-5	1	-3	1	0	-2	1	0	0	0	0	0	0	x_{20}
x_{21}	o	9856	0	-80	-8	0	6	1	0	0	0	0	0	b27	**	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{21}
x_{22}	o	9856	0	-80	-8	0	6	1	0	0	0	0	0	**	b27	0	0	0	0	0	0	0	0	0	0																	x_{22}
x_{23}	o	10395	-21	27	0	-1	-5	0	3	0	0	0	1	0	0	-1	0	0	-1	0	0	b15	**-	b15	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{23}
x_{24}	o	10395	-21	27	0	-1	-5	0	3	0	0	0	1	0	0	-1	0	0	-1	0	0	**	b15	**-	b15																	x_{24}
	ind	1	2	3	3	4	5	5	6	6	7	7	8	9	9	10	11	11	12	14	14	15	15	30	30	fus	ind	2	4	6	8	8	10	12	12	20	20	22	22	24	24	
		3	6	3		12	15	15	6	6	21	21	24			30	33	33	12	42	42	15	15	30	30																		
		3	6	3		12	15	15	6	6	21	21	24			30	33	33	12	42	42	15	15	30	30																		
x_{25}	o2	126	14	-9	0	2	1	1	-1	2	0	0	0	0	0	-1	b11	**	-1	0	0	1	1	-1	-1	*	o															x_{25}
x_{26}	o2	126	14	-9	0	2	1	1	-1	2	0	0	0	0	0	-1	**	b11	-1	0	0	1	1	-1	-1	*	o															x_{26}
x_{27}	o2	792	-8	36	0	0	-8	2	4	-2	1	1	0	0	0	2	0	0	0	-1	-1	1	1	-1	-1	*	+															x_{27}
x_{28}	o2	1980	-36	9	0	4	5	0	9	0	-1	-1	0	0	0	-1	0	0	1	-1	-1	-1	-1	-1	-1	*	+															x_{28}
x_{29}	o2	2376	-24	-54	0	0	1	1	6	0	b7	**	0	0	0	1	0	0	0	-b7	**	1	1	1	1		+2															x_{29}
x_{30}	o2	2376	-24	-54	0	0	1	1	6	0	**	b7	0	0	0	1	0	0	0	**	-b7	1	1	1	1	*																x_{30}
x_{31}	o2	2520	56	-18	0	0	-5	0	2	2	0	0	0	0	0	1	1	1	0	0	0	b15	**	b15	**		+2															x_{31}
x_{32}	o2	2520	56	-18	0	0	-5	0	2	2	0	0	0	0	0	1	1	1	0	0	0	**	b15	**	b15	*																x_{32}
x_{33}	o2	2772	84	45	0	4	-3	2	-3	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	2	2	*	+															x_{33}
x_{34}	o2	4752	-48	54	0	0	2	2	-6	0	-1	-1	0	0	0	2	0	0	0	1	1	-1	-1	-1	-1	*	+															x_{34}
x_{35}	o2	5103	63	0	0	3	3	-2	0	0	0	0	1	0	0	3	-1	-1	0	0	0	0	0	0	0	*	+															x_{35}
x_{36}	o2	6336	64	-36	0	0	11	1	4	-2	1	1	0	0	0	-1	0	0	0	1	1	-1	-1	-1	-1	*	+															x_{36}
x_{37}	o2	6336	-64	-36	0	0	11	1	-4	2	1	1	0	0	0	1	0	0	0	-1	-1	-1	-1	1	1	*	+															x_{37}
x_{38}	o2	7875	35	45	0	-5	0	0	5	2	0	0	-1	0	0	0	-1	-1	1	0	0	0	0	0	0	*	+															x_{38}
x_{39}	o2	8019	-45	0	0	3	-6	-1	0	0	-b7	**	-1	0	0	0	0	0	0	-b7	**	0	0	0	0		+2															x_{39}
x_{40}	o2	8019	-45	0	0	3	-6	-1	0	0	**	-b7	-1	0	0	0	0	0	0	**	-b7	0	0	0	0	*																x_{40}
x_{41}	o2	8064	0	72	0	0	14	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	2	2	0	0	*	+															x_{41}
x_{42}	o2	10395	-21	-54	0	-1	-5	0	-6	0	0	0	1	0	0	-1	0	0	2	0	0	1	1	-1	-1	*	+															x_{42}
x_{43}	o2	10395	-21	27	0	-1	-5	0	3	0	0	0	1	0	0	-1	0	0	-1	0	0	b15	**-	b15	**		+2															x_{43}
x_{44}	o2	10395	-21	27	0	-1	-5	0	3	0	0	0	1	0	0	-1	0	0	-1	0	0	**	b15	**-	b15	*																x_{44}
x_{45}	o2	12375	55	-45	0	-1	0	0	-5	-2	-1	-1	-1	0	0	0	0	0	-1	-1	-1	0	0	0	0	*	+															x_{45}

A_{13}

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		3113510400	1451520	23040	23040	5443200	45360	1944	972	20160	1920	576	128	100800	150	8640	1152	720	648	432	144	36	36	2520	48	16	108	27	27	960	480	10	11	288	288	96	72	72	36	13	13	56	900	90	75	36	80	40	63	63	24	28	60	30	35	35	
	p power		A	A	A	A	A	A	A	A	B	A	B	A	A	AA	AB	BB	CA	BA	BC	CB	DC	A	B	D	C	C	C	AB	AA	BC	A	AA	AC	BB	EA	EC	DC	A	A	AA	AA	AB	BA	AD	AB	BA	AA	AB	CA	AA	ABA	BAC	AA	AA	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	BB	CA	BA	BC	CB	DC	A	A	A	A	A	A	AB	AA	BC	A	AA	AC	AB	BA	BC	CC	A	A	AA	AA	AB	BA	AA	AB	AA	AA	AB	AA	AA	ABA	BAC	AA	AA	
	ind	1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	5A	5B	6A	6B	6C	6D	6E	6F	6G	6H	7A	8A	8B	9A	9B	C**	10A	10B	10C	11A	12A	12B	12C	12D	12E	12F	13A	B*	14A	15A	15B	15C	18A	20A	20B	21A	21B	24A	28A	30A	30B	35A	B**	
X_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	X_{1}
X_{2}	+	12	8	4	0	9	6	3	0	6	4	2	0	7	2	5	1	4	-1	2	0	1	0	5	2	0	3	0	0	-1	3	0	1	3	-1	1	0	2	-1	-1	-1	1	4	1	-1	-1	-1	1	2	-1	-1	-1	0	-1	0	0	X_{2}
X_{3}	+	65	29	9	5	35	14	2	-1	15	5	3	1	20	0	11	3	6	2	2	2	0	-1	9	1	-1	2	-1	-1	4	4	0	-1	3	3	-1	0	0	0	0	0	1	5	-1	0	2	0	0	0	0	1	1	1	1	-1	-1	X_{3}
X_{4}	+	66	26	2	-6	36	15	3	0	14	6	-2	-2	21	1	8	-4	5	-1	-1	-3	-1	0	10	0	0	3	0	0	-3	1	-1	0	2	-2	0	-1	1	1	1	1	-2	6	0	1	-1	1	-1	1	1	0	0	-2	0	0	0	X_{4}
X_{5}	+	208	64	16	0	76	16	1	4	20	0	4	0	28	-2	16	4	4	1	4	0	1	0	5	0	0	-2	1	1	-4	4	0	-1	2	-2	0	2	-2	1	0	0	1	1	1	1	-2	0	0	-1	2	0	-1	1	-1	0	0	X_{5}
X_{6}	+	220	40	-12	0	85	22	4	4	14	4	-6	0	35	0	1	-3	0	4	-2	0	0	0	10	-2	0	1	1	1	3	-5	0	0	-1	3	1	2	0	0	-1	-1	-2	5	2	0	1	-1	-1	1	1	1	0	1	0	0	0	X_{6}
X_{7}	+	429	105	5	1	159	33	-3	-3	35	5	-1	1	64	-1	15	-1	5	-3	-3	1	-1	1	16	-1	1	0	0	0	0	0	1	0	-1	-1	-1	-1	-1	-1	0	0	0	4	-2	-1	0	0	0	-2	-2	-1	0	0	0	1	1	X_{7}
X_{8}	+	429	97	21	9	99	12	6	0	15	-3	3	1	14	-1	19	3	0	-2	4	0	0	0	-5	-1	1	-3	0	0	6	2	-1	0	3	3	3	0	0	0	0	0	-1	-1	2	-1	1	2	0	1	-2	-1	1	-1	0	0	0	X_{8}
X_{9}	+	429	69	13	5	33	15	-3	6	1	9	1	1	-21	4	9	1	-5	-3	3	-1	1	2	-5	-1	-1	0	0	0	3	-1	0	0	1	1	-3	1	1	1	0	0	-1	3	0	-2	0	-1	1	-2	1	-1	1	-1	0	0	0	X_{9}
X_{10}	+	462	-42	14	-10	84	21	3	3	-14	-6	2	2	7	2	0	-4	-1	3	-3	-1	-1	-1	0	0	0	0	0	0	-1	3	0	0	-2	2	0	1	-1	-1	-b13	*	0	4	1	-1	0	-1	1	0	0	0	0	0	-1	0	0	X_{10}
X_{11}	+	462	-42	14	-10	84	21	3	3	-14	-6	2	2	7	2	0	-4	-1	3	-3	-1	-1	-1	0	0	0	0	0	0	-1	3	0	0	-2	2	0	1	-1	-1	*	-b13	0	4	1	-1	0	-1	1	0	0	0	0	0	-1	0	0	X_{11}
X_{12}	+	495	15	-17	15	135	27	9	0	-1	-1	-1	-1	35	0	-9	7	-5	-3	3	3	1	0	5	-1	-1	0	0	0	3	-5	0	0	-1	-1	-1	-1	-1	-1	1	1	1	5	2	0	0	-1	-1	2	-1	-1	-1	1	0	0	0	X_{12}
X_{13}	+	572	104	20	0	77	14	5	-4	6	4	?	0	-13	2	17	5	-4	5	2	0	-1	0	-9	-2	0	-1	-1	-1	-5	-1	0	0	3	-1	1	0	2	-1	0	0	-1	2	-1	2	-1	-1	1	0	0	1	-1	2	1	1	1	X_{13}
X_{14}	+	792	-48	8	0	162	36	9	0	-20	-8	4	0	22	2	-6	2	-4	-3	0	0	-1	0	1	0	0	0	0	0	-2	2	0	0	-2	-2	-2	-2	-2	1	-1	-1	1	7	1	2	0	2	0	1	1	0	1	-1	1	1	1	X_{14}
X_{15}	+	936	120	8	24	216	9	-9	0	20	0	4	0	56	1	0	8	-1	3	-3	3	-1	0	5	0	0	0	0	0	8	0	-1	1	-4	4	0	-1	1	1	0	0	1	-4	-1	1	0	0	0	-1	2	0	-1	0	-1	0	0	X_{15}
X_{16}	+	1287	111	7	15	27	27	-9	0	-1	11	-1	3	-43	2	3	-5	-5	3	3	3	1	0	-1	1	1	0	0	0	-3	1	0	0	-1	-1	-1	-1	-1	-1	0	0	-1	2	2	2	0	1	-1	-1	-1	1	-1	-2	0	-1	-1	X_{16}
X_{17}	+	1365	217	13	-15	315	21	-3	0	35	-7	-1	-3	70	0	19	-5	1	1	1	-3	1	0	0	1	-1	-3	0	0	-2	2	0	1	-1	-1	-1	-1	-1	-1	0	0	0	-5	1	0	1	-2	0	0	0	1	0	-1	1	0	0	X_{17}
X_{18}	+	1430	134	-26	-10	350	35	-1	-1	22	2	-2	2	90	0	-10	-2	-5	-1	-1	-1	1	-1	9	0	0	-1	-1	-1	-6	-6	0	0	-2	-2	2	1	1	1	0	0	1	0	0	0	-1	2	2	0	0	0	1	0	0	-1	-1	X_{18}
X_{19}	+	2574	294	14	6	324	9	9	0	14	-6	-2	2	-1	-1	24	-4	-1	-3	-3	3	-1	0	-16	0	0	0	0	0	-1	-1	1	0	2	-2	0	-1	1	1	0	0	0	-1	-1	-1	0	-1	-1	2	2	0	0	-1	-1	-1	-1	X_{19}
X_{20}	+	2574	78	-2	30	-162	9	9	0	-22	2	2	2	14	4	6	-2	1	-3	-3	-3	1	0	5	0	0	0	0	0	-2	-2	0	0	2	2	2	-1	-1	-1	0	0	1	-7	-1	-2	0	2	-2	-1	2	0	-1	1	1	0	0	X_{20}
X_{21}	+	2860	268	12	-20	160	34	-2	-2	0	12	0	-4	-70	0	16	0	-6	-2	-2	-2	0	-2	-10	0	0	1	1	1	2	-2	0	0	0	0	0	0	0	0	0	0	2	5	-1	0	1	2	0	-1	-1	0	0	1	-1	0	0	X_{21}
X_{22}	+	3003	-21	-21	-5	483	42	6	-3	-21	-1	3	-1	63	-2	-21	3	-6	6	6	-2	0	1	0	1	1	0	0	0	-1	-1	0	0	3	3	-1	0	0	0	0	0	0	3	-3	-2	0	-1	-1	0	0	1	0	-1	-1	0	0	X_{22}
X_{23}	+	3432	272	24	0	414	-30	-6	0	8	-8	0	0	42	2	2	6	0	2	2	0	0	0	-5	0	0	3	0	0	-6	2	0	0	-4	0	-2	2	0	0	0	0	-1	-6	0	-1	-1	2	-2	1	-2	0	1	2	0	0	0	X_{23}
X_{24}	+	3432	120	8	-40	-168	21	3	-6	-20	0	-4	0	-8	2	0	8	-1	3	-3	-1	-1	2	9	0	0	0	0	0	8	0	0	0	4	-4	0	1	-1	-1	0	0	1	-8	1	2	0	0	0	0	0	0	1	0	-1	-1	-1	X_{24}
X_{25}	+	3432	48	-8	0	-258	12	3	12	-20	8	4	0	42	2	6	-2	4	3	0	0	1	0	-5	0	0	0	0	0	2	-2	0	0	-2	-2	2	-2	-2	1	0	0	-1	-3	-3	2	0	-2	0	1	-2	0	1	1	-1	0	0	X_{25}
X_{26}	+	3575	155	15	-45	125	-40	2	8	-15	-5	-3	-1	0	0	5	-3	0	2	8	0	0	0	5	-1	1	2	-1	-1	0	0	0	0	-3	-3	1	0	0	0	0	0	1	0	0	0	2	0	0	-1	2	-1	-1	0	0	0	0	X_{26}
X_{27}	+	3640	112	8	0	490	-14	-5	4	0	8	8	0	70	0	-26	2	-4	-5	-2	0	-1	0	0	0	0	1	1	1	-2	2	0	-1	0	-4	2	0	2	-1	0	0	0	-5	1	0	1	-2	0	0	0	0	0	-1	1	0	0	X_{27}
X_{28}	+	4004	-28	36	36	224	-28	8	8	0	12	0	-4	14	-1	-16	0	0	8	-4	0	0	0	0	0	0	-1	-1	-1	6	2	1	0	0	0	0	0	0	0	0	0	0	-1	2	-1	-1	2	0	0	0	0	0	-1	0	0	0	X_{28}
X_{29}	+	4212	216	-36	0	567	0	0	0	6	-4	-6	0	57	2	-9	-9	0	0	0	0	0	0	-9	2	0	0	0	0	9	1	0	-1	3	3	-1	0	0	0	0	0	-1	-3	0	2	0	1	1	0	0	-1	-1	1	0	1	1	X_{29}
X_{30}	+	4290	-246	34	10	510	69	-3	-3	-50	-10	-2	-2	20	0	6	-2	1	-3	-3	1	1	1	-1	0	0	0	0	0	4	4	0	0	-2	-2	2	1	1	1	0	0	-1	5	-1	0	0	0	0	-1	-1	0	-1	1	1	-1	-1	X_{30}
X_{31}	o	4290	42	2	10	-300	15	-3	-3	-14	6	2	-2	45	0	0	-4	5	-3	3	1	-1	1	-8	0	0	0	0	0	-3	-3	0	0	-2	2	0	1	-1	-1	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	b35	**	X_{31}
X_{32}	o	4290	42	2	10	-300	15	-3	-3	-14	6	2	-2	45	0	0	-4	5	-3	3	1	-1	1	-8	0	0	0	0	0	-3	-3	0	0	-2	2	0	1	-1	-1	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	**	b35	X_{32}
X_{33}	+	5005	-35	-3	45	-35	28	10	-8	21	1	-3	1	-35	0	-11	-3	0	10	4	0	0	0	0	-1	-1	1	1	1	-3	5	0	0	-3	-3	1	0	0	0	0	0	0	-5	-2	0	1	1	1	0	0	-1	0	-1	0	0	0	X_{33}
X_{34}	+	5148	264	20	0	-27	36	-9	0	-14	4	-2	0	-77	-2	-3	5	-4	3	0	0	-1	0	10	2	0	0	0	0	-5	-1	0	0	1	1	1	-2	-2	1	0	0	-2	-2	1	-2	0	-1	1	1	1	-1	0	2	1	0	0	X_{34}
X_{35}	+	5720	176	-24	0	50	50	-4	-4	8	8	0	0	-90	0	-10	-6	0	-4	2	0	0	0	1	0	0	-1	-1	-1	6	6	0	0	-4	0	2	2	0	0	0	0	1	0	0	0	-1	-2	-2	1	1	0	1	0	0	1	1	X_{35}
X_{36}	+	6006	350	22	30	252	-21	3	0	-14	-6	2	2	-49	-4	8	4	1	-1	-1	-3	1	0	0	0	0	3	0	0	7	-5	0	0	-2	2	0	1	-1	-1	0	0	0	2	-1	2	-1	-1	1	0	0	0	0	-2	1	0	0	X_{36}
X_{37}	+	6435	231	11	15	-135	-9	9	0	-35	-5	1	-1	-20	0	9	-7	-1	-3	3	3	-1	0	16	1	-1	0	0	0	-4	-4	0	0	1	1	1	1	1	1	0	0	0	-5	1	0	0	0	0	-2	-2	1	0	-1	-1	1	1	X_{37}
X_{38}	+	6864	240	-48	-16	384	6	6	6	0	0	0	0	-56	-1	0	0	6	6	-6	2	0	2	-10	0	0	0	0	0	-8	0	-1	0	0	0	0	0	0	0	0	0	2	4	1	-1	0	0	0	-1	-1	0	0	0	1	0	0	X_{38}
X_{39}	+	7371	-189	27	-45	567	0	0	0	-21	11	3	3	21	-4	-9	-9	0	0	0	0	0	0	0	-1	-1	0	0	0	-3	1	0	1	3	3	-1	0	0	0	0	0	0	-3	0	2	0	1	-1	0	0	-1	0	1	0	0	0	X_{39}
X_{40}	+	7800	-120	-40	40	600	15	-3	6	-20	0	-4	0	0	0	0	8	5	-3	3	1	-1	-2	-5	0	0	0	0	0	0	0	0	1	4	-4	0	1	-1	-1	0	0	-1	0	0	0	0	0	0	-2	1	0	1	0	0	0	0	X_{40}
X_{41}	o	8008	-56	-24	40	-224	-14	-2	1	0	0	0	0	28	-2	16	0	-6	-2	-2	-2	0	1	0	0	0	1	b27	**	-4	4	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	-1	0	0	X_{41}
X_{42}	o	8008	-56	-24	40	-224	-14	-2	1	0	0	0	0	28	-2	16	0	-6	-2	-2	-2	0	1	0	0	0	1	**	b27	-4	4	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	-1	0	0	X_{42}
X_{43}	+	8580	120	12	0	-285	48	-6	12	6	-4	-5	0	-35	0	-21	3	0	-6	0	0	0	0	5	-2	0	0	0	0	-3	5	0	0	3	3	-1	0	0	0	0	0	1	-5	-2	0	0	1	1	2	-1	1	-1	-1	0	0	0	X_{43}
X_{44}	+	9009	-231	33	9	189	0	18	0	21	9	-3	1	-21	-1	-3	-3	0	-6	0	0	0	0	0	1	1	0	0	0	3	-1	-1	0	-3	-3	-3	0	0	0	0	0	0	-6	0	-1	0	-1	1	0	0	1	0	2	0	0	0	X_{44}
X_{45}	+	9360	-384	16	0	540	36	-9	0	-20	0	-4	0	-20	0	24	4	4	3	0	0	1	0	1	0	0	0	0	0	-4	-4	0	-1	-2	2	0	-2	2	-1	0	0	1	-5	1	0	0	0	0	1	1	0	1	-1	-1	1	1	X_{45}
X_{46}	+	10296	-120	-40	-24	216	27	9	0	20	0	4	0	-64	1	0	8	5	-3	3	-3	-1	0	-1	0	0	0	0	0	0	0	1	0	-4	4	0	-1	1	1	0	0	-1	-4	2	1	0	0	0	-1	-1	0	-1	0	0	-1	-1	X_{46}
X_{47}	+	11440	64	-16	0	-140	-56	1	4	-20	0	-4	0	20	0	16	-4	-4	1	4	0	-1	0	9	0	0	-2	1	1	4	4	0	0	-2	2	0	-2	2	-1	0	0	1	5	-1	0	-2	0	0	0	0	0	1	1	1	-1	-1	X_{47}
X_{48}	+	11583	27	-9	-45	-567	0	0	0	-15	7	-3	3	78	-2	9	9	0	0	0	0	0	0	-9	1	-1	0	0	0	6	2	0	0	-3	-3	1	0	0	0	0	0	-1	3	0	-2	0	2	0	0	0	1	-1	-1	0	1	1	X_{48}
X_{49}	+	12012	168	4	0	609	-84	-3	-12	-14	4	-2	0	7	2	-15	1	4	-3	0	0	1	0	0	-2	0	0	0	0	-1	3	0	0	1	1	1	-2	-2	1	0	0	0	4	1	-1	0	-1	1	0	0	1	0	0	-1	0	0	X_{49}
X_{50}	+	12012	-56	36	0	-63	-42	6	0	14	4	-6	0	7	2	-11	9	0	-2	-2	0	0	0	0	2	0	-3	0	0	-9	-1	0	0	-1	3	1	2	0	0	0	0	0	7	-2	2	1	-1	-1	0	0	-1	0	-1	0	0	0	X_{50}
X_{51}	+	12870	30	38	30	-540	27	-9	0	14	-6	-2	2	35	0	-24	-4	5	3	3	-3	-1	0	-10	0	0	0	0	0	3	-5	0	0	2	-2	0	-1	1	1	0	0	2	5	2	0	0	-1	-1	-1	-1	0	0	1	0	0	0	X_{51}
X_{52}	+	15015	147	-17	-5	-735	21	3	-6	-35	3	1	-1	70	0	9	1	1	3	-3	1	1	-2	0	-1	1	0	0	0	-2	2	0	0	1	1	-3	1	1	1	0	0	0	-5	1	0	0	-2	0	0	0	-1	0	-1	1	0	0	X_{52}
X_{53}	+	17160	208	-8	0	-90	-6	-3	0	0	-8	8	0	-70	0	-14	-2	4	1	-2	0	1	0	10	0	0	-3	0	0	2	-2	0	0	0	-4	-2	0	2	-1	0	0	-2	5	-1	0	1	2	0	1	1	0	0	1	-1	0	0	X_{53}
X_{54}	+	20592	-384	-16	0	324	-36	-9	0	20	0	4	0	-28	2	24	-4	-4	3	0	0	-1	0	5	0	0	0	0	0	4	-4	0	0	2	-2	0	2	-2	1	0	0	1	-1	-1	-1	0	0	0	2	-1	0	-1	-1	1	0	0	X_{54}
X_{55}	+	21450	-190	10	-30	-450	15	3	0	50	-10	2	-2	0	0	-10	-2	-5	-1	-1	3	1	0	-5	0	0	3	0	0	0	0	0	0	2	2	2	-1	-1	-1	0	0	-1	0	0	0	-1	0	0	-2	1	0	1	0	0	0	0	X_{55}
	ind	1	4	2	4	3	3	3	3	8	4	8	4	5	5	12	6	6	12	12	12	6	12	7	8	8	9	9	9	10	20	20	11	24	24	12	24	24	24	13	13	28	15	15	15	36	20	40	21	21	24	56	60	30	35	35	
		2				6	6	6	6					10	10									14		8	18	18	18			20	22						24	26	26		30	30	30				42	42	24	56		30	70	70	
X_{56}	-	32	0	0	0	-16	8	-4	2	0	0	0	0	-8	2	0	0	0	0	0	0	0	0	4	0	0	2	-1	-1	0	0	0	-1	0	0	0	0	0	0	b13	*	0	4	-2	-1	0	0	0	-2	1	0	0	0	0	-1	-1	X_{56}
X_{57}	-	32	0	0	0	-16	8	-4	2	0	0	0	0	-8	2	0	0	0	0	0	0	0	0	4	0	0	2	-1	-1	0	0	0	-1	0	0	0	0	0	0	*	b13	0	4	-2	-1	0	0	0	-2	1	0	0	0	0	-1	-1	X_{57}
X_{58}	-	352	0	0	0	-128	40	-8	-2	0	0	0	0	-48	2	0	0	0	0	0	0	0	0	16	0	0	4	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	12	0	2	0	0	0	-2	-2	0	0	0	0	1	1	X_{58}
X_{59}	-	1728	0	0	0	-432	72	0	0	0	0	0	0	-112	-2	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-1	-1	0	8	2	-2	0	0	0	2	2	0	0	0	0	0	0	X_{59}
X_{60}	o	2496	0	0	0	-384	12	12	-6	0	0	0	0	-64	1	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	i5	-1	0	0	0	0	0	0	0	0	0	-4	2	1	0	0	0	1	-2	0	0	0	0	-1	-1	X_{60}
X_{61}	o	2496	0	0	0	-384	12	12	-6	0	0	0	0	-64	1	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	-i5	-1	0	0	0	0	0	0	0	0	0	-4	2	1	0	0	0	1	-2	0	0	0	0	-1	-1	X_{61}
X_{62}	-	4224	0	0	0	-96	48	12	12	0	0	0	0	64	4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	4	-2	4	0	0	0	2	-1	0	0	0	0	1	1	X_{62}
X_{63}	+	4576	0	0	0	352	16	4	-8	0	0	0	0	-64	-4	0	0	0	0	0	0	0	0	-16	0	0	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-4	2	0	0	0	2	2	0	0	0	0	-1	-1	X_{63}
X_{64}	-	4928	0	0	0	-784	56	-4	8	0	0	0	0	-112	-2	0	0	0	0	0	0	0	0	0	0	0	-4	-1	-1	0	0	0	0	0	0	0	0	0	0	1	1	0	-4	-4	1	0	0	0	0	0	0	0	0	0	0	0	X_{64}
X_{65}	-	8800	0	0	0	-800	40	-20	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-20	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	X_{65}
X_{66}	+	9152	0	0	0	272	-40	8	-16	0	0	0	0	-48	2	0	0	0	0	0	0	0	0	-4	0	0	-4	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	12	0	2	0	0	0	-1	2	0	0	0	0	1	1	X_{66}
X_{67}	o	9152	0	0	0	-736	-40	8	2	0	0	0	0	-48	2	0	0	0	0	0	0	0	0	-4	0	0	2	-b27	**	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	0	-1	0	0	0	-1	2	0	0	0	0	1	1	X_{67}
X_{68}	o	9152	0	0	0	-736	-40	8	2	0	0	0	0	-48	2	0	0	0	0	0	0	0	0	-4	0	0	2	**	-b27	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	0	-1	0	0	0	-1	2	0	0	0	0	1	1	X_{68}
X_{69}	o	9152	0	0	0	464	20	-10	-4	0	0	0	0	-48	2	0	0	0	0	0	0	0	0	-4	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	3i2	0	0	0	-6	0	-1	0	0	0	2	-1	0	0	0	0	1	1	X_{69}
X_{70}	o	9152	0	0	0	464	20	-10	-4	0	0	0	0	-48	2	0	0	0	0	0	0	0	0	-4	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	-3i2	0	0	0	-6	0	-1	0	0	0	2	-1	0	0	0	0	1	1	X_{70}
X_{71}	-	9504	0	0	0	-432	72	0	0	0	0	0	0	104	4	0	0	0	0	0	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	8	2	-2	0	0	0	2	2	0	0	0	0	-1	-1	X_{71}
X_{72}	o	11232	0	0	0	-216	-72	0	0	0	0	0	0	-8	2	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	4	-2	-1	0	0	0	1	-2	2i3	0	0	0	-1	-1	X_{72}
X_{73}	o	11232	0	0	0	-216	-72	0	0	0	0	0	0	-8	2	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	4	-2	-1	0	0	0	1	-2	-2i3	0	0	0	-1	-1	X_{73}
X_{74}	o	11232	0	0	0	432	36	0	0	0	0	0	0	-8	2	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-8	1	2	0	0	0	-2	1	0	0	0	i15	-1	-1	X_{74}
X_{75}	o	11232	0	0	0	432	36	0	0	0	0	0	0	-8	2	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-8	1	2	0	0	0	-2	1	0	0	0	-i15	-1	-1	X_{75}
X_{76}	+	13728	0	0	0	768	12	12	12	0	0	0	0	-112	-2	0	0	0	0	0	0	0	0	-20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	2	-2	0	0	0	-2	-2	0	0	0	0	0	0	X_{76}
X_{77}	o	13728	0	0	0	48	48	12	-6	0	0	0	0	88	-2	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	0	0	0	-1	-1	0	0	0	0	-b35	**	X_{77}
X_{78}	o	13728	0	0	0	48	48	12	-6	0	0	0	0	88	-2	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	0	0	0	-1	-1	0	0	0	0	**	-b35	X_{78}
X_{79}	o	16016	0	0	0	-448	-28	-4	2	0	0	0	0	56	-4	0	0	0	0	0	0	0	0	0	0	i8	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	X_{79}
X_{80}	o	16016	0	0	0	-448	-28	-4	2	0	0	0	0	56	-4	0	0	0	0	0	0	0	0	0	0	-i8	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	X_{80}
X_{81}	o	20800	0	0	0	400	-20	-8	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	0	0	-2	1	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	i14	0	0	0	0	X_{81}
X_{82}	o	20800	0	0	0	400	-20	-8	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	0	0	-2	1	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	-i14	0	0	0	0	X_{82}

A13

	;	:	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	
			399	120	11	725	1					120	15	5	2	1											3				4																							
			16800	960	520	760	920	768	192	192	960	120	760	160	728	432	324	216	144	72	54	480	32	600	240	50	30	320	324	216	96	96	24	6	168	168	18	240	80	11	24	28	90	90	30	15	18	20	21	21	30	
			A	A	A	A	A	A	B	C	AD	BE	AF	BD	AE	BE	CD	CE	BF	CF	DE	B	B	AD	AE	BD	BF	AE	DE	EE	AF	BH	EC	HI	AD	AE	AO	BE	BF	AD	CC	AE	ADI	BDL	BEJ	CGK	AH	AC	ABI	BCJ	ACG	
			A	A	A	A	A	A	A	A	AD	BE	AF	BD	AE	BE	CD	CE	BF	CF	DE	A	A	AD	AE	BD	BF	AE	CE	BE	AF	AH	BG	DI	AD	AE	AD	AE	AF	AD	AC	AE	ADI	BDL	BEJ	CGK	AE	AC	ABI	BCJ	ACG	
	fus	ind	2D	2E	2F	4E	4F	4G	4H	4I	6I	6J	6K	6L	6M	6N	6O	6P	6Q	6R	6S	8C	8D	10D	10E	10F	10G	12G	12H	12I	12J	12K	12L	12M	14B	14C	18B	20C	20D	22A	24B	28B	30C	30D	30E	30F	36A	40A	42A	42B	60A	
x_{1}	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_{1}
x_{2}	:	++	10	6	2	8	4	0	2	0	7	6	-1	4	3	0	1	3	2	-1	0	4	0	5	1	0	2	5	-1	2	1	-1	0	0	3	-1	1	3	-1	-1	1	1	2	-1	1	-1	-1	-1	0	-1	0	x_{2}
x_{3}	:	++	45	17	5	27	7	3	1	-1	21	14	5	6	5	2	0	2	2	2	-1	5	1	10	2	0	0	9	0	0	1	1	0	-1	3	3	0	2	2	1	-1	-1	1	1	-1	0	0	0	0	0	-1	x_{3}
x_{4}	:	++	44	12	-4	28	4	-4	0	0	20	15	-4	5	0	-3	-1	3	-1	-1	0	6	-2	9	-3	-1	1	10	1	1	-2	0	-1	0	2	-2	-1	3	-1	0	0	0	0	0	0	1	1	1	-1	1	0	x_{4}
x_{5}	:	++	120	32	8	48	8	0	0	0	36	14	-4	6	8	2	3	-1	2	-1	2	0	0	10	2	0	-2	6	3	0	2	0	0	0	1	-3	0	-2	-2	-1	0	-1	1	1	-1	1	0	0	1	0	1	x_{5}
x_{6}	:	++	110	2	-10	56	-4	0	-2	0	29	20	5	2	-7	2	2	2	-4	2	2	4	0	5	-3	0	0	11	2	2	-1	1	0	0	-2	2	-1	1	1	0	1	0	-1	2	0	0	-1	-1	1	-1	1	x_{6}
x_{7}	:	++	231	35	-1	105	5	1	-1	1	63	35	-1	3	-1	-1	-3	-1	-1	-1	-1	5	1	16	0	1	-1	15	-3	-3	-1	-1	1	1	0	0	0	0	0	0	-1	0	-2	-2	0	-1	0	0	0	0	0	x_{7}
x_{8}	:	++	209	45	9	43	7	3	1	3	41	0	9	8	9	0	2	0	0	0	0	-5	-1	4	0	-1	-1	1	-2	4	1	1	0	0	-1	3	-1	-2	2	0	1	1	1	-2	0	-1	1	0	-1	0	1	x_{8}
x_{9}	:	++	165	29	5	-15	1	1	1	-3	21	-13	5	3	5	-1	3	-1	-1	-1	2	-1	-1	-5	-1	0	0	9	3	-3	1	1	1	0	-3	1	0	5	1	0	-1	-1	1	-2	2	0	0	-1	0	1	-1	x_{9}
x_{10}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{10}
x_{11}	]																																																			x_{11}
x_{12}	:	++	165	-27	5	69	-11	5	-3	-3	21	15	5	3	-3	3	3	3	-1	-1	0	1	1	-5	3	0	0	9	-3	3	1	-3	-1	0	-3	1	0	-1	-1	0	1	-1	1	-2	0	0	0	1	0	1	-1	x_{12}
x_{13}	:	++	242	46	10	8	4	0	2	0	35	-14	-5	8	7	4	-1	1	-2	1	-2	-4	0	-3	1	2	0	5	-1	2	1	-1	0	0	-3	-3	-1	3	-1	0	-1	1	0	3	1	0	-1	1	0	0	0	x_{13}
x_{14}	:	++	132	-36	20	48	-8	0	-4	0	6	6	-10	6	6	-6	-3	3	2	-1	0	0	0	-8	4	2	0	6	3	0	-2	2	0	0	-1	-1	0	-2	2	0	0	-1	1	1	1	0	0	0	-1	-1	1	x_{14}
x_{15}	:	++	384	24	16	120	0	8	0	0	48	21	16	-9	0	3	-3	-3	1	1	0	0	0	4	4	-1	1	0	3	-3	0	0	-1	0	-1	3	0	0	0	-1	0	1	-2	1	1	1	0	0	-1	0	0	x_{15}
x_{16}	:	++	363	27	-5	-57	-1	7	-1	3	27	-15	-5	-3	3	-3	3	-3	1	1	0	1	1	-7	-3	-2	0	15	-3	-3	-1	-1	1	0	-1	-1	0	3	-1	0	1	-1	2	2	0	0	0	1	-1	-1	0	x_{16}
x_{17}	:	++	595	63	-5	161	5	-7	-1	-3	91	21	-5	1	3	-3	1	-3	1	1	0	-5	-1	10	-2	0	0	-1	-1	-1	-1	-1	-1	0	0	0	1	-4	0	1	1	0	1	1	1	0	-1	0	0	0	-1	x_{17}
x_{18}	:	++	550	-10	-10	190	-10	-2	2	2	70	35	-10	-5	-10	-1	1	-1	-1	-1	-1	0	0	0	0	0	0	10	1	1	2	2	1	-1	-3	-3	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	x_{18}
x_{19}	:	++	924	84	-4	84	4	4	0	0	84	-21	-4	9	0	-3	-3	3	-1	-1	0	-6	2	-1	-1	-1	1	-6	3	3	-2	0	1	0	0	0	0	-1	-1	0	0	0	-1	-1	-1	1	0	-1	0	0	-1	x_{19}
x_{20}	:	++	330	18	10	-78	2	2	-2	6	-6	21	10	9	-6	-3	-3	-3	1	1	0	0	0	-10	-2	0	0	-6	3	3	2	-2	-1	0	1	-3	0	2	2	0	0	-1	-1	-1	1	0	0	0	1	0	-1	x_{20}
x_{21}	:	++	880	72	0	-56	0	-8	0	0	64	-36	0	4	0	0	-2	0	0	0	0	0	0	-10	2	0	0	16	-2	-2	0	0	-2	0	-2	2	1	4	0	0	0	0	-1	-1	-1	0	1	0	1	-1	1	x_{21}
x_{22}	:	++	693	-91	5	189	-11	-3	1	1	21	14	5	6	5	2	0	2	2	2	-1	-1	-1	-17	-1	-2	0	9	0	0	1	1	0	1	0	0	0	-1	-1	0	-1	0	1	1	-1	0	0	-1	0	0	-1	x_{22}
x_{23}	:	++	1100	60	12	160	0	0	-4	0	50	0	-6	-10	6	6	2	0	0	0	0	0	0	0	0	0	2	-20	-2	-2	0	2	0	0	1	-3	-1	0	0	0	0	-1	0	0	0	-1	1	0	1	0	0	x_{23}
x_{24}	:	++	528	24	0	-120	0	-8	0	0	0	21	0	3	0	3	-3	-3	-3	-3	0	0	0	-12	4	-2	0	0	-3	3	0	0	1	0	3	3	0	0	0	0	0	-1	0	3	1	0	0	0	0	0	0	x_{24}
x_{25}	:	++	132	-4	20	-48	8	0	-4	0	6	14	-10	6	-10	2	-3	-1	2	-1	2	0	0	-8	-4	2	0	-6	-3	0	2	2	0	0	-1	3	0	2	-2	0	0	1	1	1	-1	0	0	0	-1	0	-1	x_{25}
x_{26}	:	++	825	45	-15	15	-5	-9	-3	3	-15	0	-15	0	9	0	6	0	0	0	0	5	1	0	0	0	0	-15	6	0	1	-3	0	0	-1	3	0	0	0	0	-1	1	0	0	0	0	0	0	-1	0	0	x_{26}
x_{27}	:	++	980	-28	20	224	0	0	4	0	14	14	-10	-16	2	-4	-1	-1	2	-1	2	0	0	-10	2	0	0	-4	-1	2	0	-2	0	0	0	0	-1	4	0	1	0	0	-1	-1	-1	0	-1	0	0	0	1	x_{27}
x_{28}	:	++	616	0	24	112	8	0	0	0	-56	0	24	-8	0	0	4	0	0	0	0	0	0	-4	0	1	-1	4	4	4	-4	0	0	0	0	0	1	2	-2	0	0	0	-1	2	0	-1	1	0	0	0	-1	x_{28}
x_{29}	:	++	1242	-18	-30	216	-4	0	2	0	63	0	15	0	-9	0	0	0	0	0	0	-4	0	-3	-3	2	0	-9	0	0	-1	-1	0	0	3	3	0	1	1	-1	-1	-1	3	0	0	0	0	1	0	0	1	x_{29}
x_{30}	:	++	330	-62	10	78	-2	-2	-2	2	-6	1	10	9	10	1	-3	1	1	1	1	0	0	-10	-2	0	0	6	-3	-3	-2	-2	1	-1	1	1	0	-2	-2	0	0	1	-1	-1	1	0	0	0	1	1	1	x_{30}
x_{31}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{31}
x_{32}	]																																																			x_{32}
x_{33}	:	++	385	-63	-15	-119	1	9	-3	-3	49	0	-15	-8	9	0	-2	0	0	0	0	-1	-1	5	-3	0	0	1	-2	4	1	-3	0	0	0	0	1	1	1	0	-1	0	-1	2	0	0	1	-1	0	0	1	x_{33}
x_{34}	:	++	1122	54	10	-168	-4	0	2	0	27	-6	-5	-6	3	6	-3	-3	-2	1	0	4	0	-13	-1	2	0	15	3	0	-1	-1	0	0	2	-2	0	-3	1	0	1	0	2	-1	-1	0	0	-1	-1	1	0	x_{34}
x_{35}	:	++	1100	-20	-20	-160	0	0	-4	0	50	-20	10	-10	-2	-2	2	-2	4	-2	-2	0	0	0	0	0	0	20	2	2	0	2	0	0	1	1	-1	0	0	0	0	1	0	0	0	0	-1	0	1	1	0	x_{35}
x_{36}	:	++	1540	84	20	-28	-4	4	0	0	28	-21	20	1	0	-3	1	3	-1	-1	0	6	-2	-5	-1	0	0	-10	-1	-1	2	0	1	0	0	0	1	-3	1	0	0	0	-2	1	-1	0	-1	1	0	0	0	x_{36}
x_{37}	:	++	1155	63	-5	-147	-7	5	3	-3	-21	21	-5	9	3	-3	3	-3	1	1	0	5	1	-10	-2	0	0	-9	-3	3	-1	3	-1	0	0	0	0	-2	-2	0	-1	0	-1	-1	1	0	0	0	0	0	1	x_{37}
x_{38}	:	++	1584	-16	-16	0	0	0	0	0	48	-34	-16	6	-16	2	0	2	2	2	2	0	0	4	4	-1	-1	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	0	-2	1	1	-1	0	0	-1	1	0	x_{38}
x_{39}	:	++	945	-63	-15	189	13	-3	1	-3	-63	0	-15	0	9	0	0	0	0	0	0	1	1	-15	-3	0	0	9	0	0	1	1	0	0	0	0	0	-1	3	-1	1	0	-3	0	0	0	0	1	0	0	-1	x_{39}
x_{40}	:	++	1200	-120	0	120	0	8	0	0	0	-15	0	15	0	3	3	-3	-3	-3	0	0	0	0	0	0	0	0	3	-3	0	0	-1	0	3	-1	0	0	0	1	0	1	0	0	0	0	0	0	0	-1	0	x_{40}
x_{41}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{41}
x_{42}	]																																																			x_{42}
x_{43}	:	++	990	-22	-10	-216	4	0	-2	0	21	20	5	-12	5	-4	0	2	-4	2	2	-4	0	-5	3	0	0	9	0	0	1	1	0	0	3	-1	0	-1	-1	0	-1	1	1	-2	0	0	0	1	0	-1	-1	x_{43}
x_{44}	:	++	231	63	-9	147	11	3	-1	3	-105	0	-9	0	-9	0	6	0	0	0	0	-1	-1	-9	3	1	1	15	-6	0	-1	-1	0	0	0	0	0	-3	1	0	-1	0	0	0	0	1	0	-1	0	0	0	x_{44}
x_{45}	:	++	120	-48	40	-48	-8	0	0	0	36	-6	-20	6	0	-6	3	-3	-2	1	0	0	0	10	2	0	0	-6	-3	0	-2	0	0	0	1	1	0	2	2	-1	0	1	1	1	-1	0	0	0	1	1	-1	x_{45}
x_{46}	:	++	1056	-120	16	-120	0	-8	0	0	48	-15	16	3	0	3	3	-3	1	1	0	0	0	16	0	1	1	0	-3	3	0	0	1	0	-1	-1	0	0	0	0	0	-1	-2	-2	0	1	0	0	-1	-1	0	x_{46}
x_{47}	:	++	1320	32	-40	-48	-8	0	0	0	-84	14	20	6	8	2	-3	-1	2	-1	2	0	0	10	2	0	0	-6	-3	0	-2	0	0	0	-3	-3	0	2	2	0	0	1	1	1	-1	0	0	0	0	0	-1	x_{47}
x_{48}	:	++	297	45	-15	27	-17	3	1	3	-63	0	-15	0	9	0	0	0	0	0	0	-5	-1	12	0	2	0	9	0	0	1	1	0	0	3	3	0	2	-2	0	1	-1	-3	0	0	0	0	0	0	0	-1	x_{48}
x_{49}	:	++	2310	-14	-2	168	4	0	-2	0	-63	-14	1	-6	1	-2	-3	1	-2	1	-2	4	0	5	1	0	-2	-15	-3	0	1	1	0	0	0	0	0	3	-1	0	1	0	2	-1	1	1	0	-1	0	0	0	x_{49}
x_{50}	:	++	1078	42	30	56	-4	0	-2	0	-119	0	-15	-2	-3	6	-2	0	0	0	0	-4	0	13	-3	-2	0	11	2	2	-1	1	0	0	0	0	1	1	1	0	-1	0	1	-2	0	0	-1	1	0	0	1	x_{50}
x_{51}	:	++	660	12	20	-84	-4	-4	0	0	-36	15	20	-3	0	-3	3	3	-1	-1	0	-6	2	5	-3	0	0	6	-3	-3	2	0	-1	0	2	-2	0	1	1	0	0	0	-1	2	0	0	0	-1	-1	1	1	x_{51}
x_{52}	:	++	1155	7	-5	-231	5	1	-1	-3	-21	49	-5	9	-5	1	3	1	1	1	-2	-5	-1	-10	2	0	0	-9	3	-3	-1	-1	1	0	0	0	0	4	0	0	1	0	-1	-1	-1	0	0	0	0	0	1	x_{52}
x_{53}	:	++	2420	-12	20	-224	0	0	4	0	-34	6	-10	-4	-6	0	-1	3	2	-1	0	0	0	10	-2	0	0	4	1	-2	0	-2	0	0	-2	2	-1	-4	0	0	0	0	1	1	1	0	1	0	1	-1	-1	x_{53}
x_{54}	:	++	1320	-48	-8	48	8	0	0	0	-84	-6	4	6	0	-6	-3	-3	-2	1	0	0	0	10	2	0	2	6	3	0	2	0	0	0	-3	1	0	-2	-2	0	0	-1	1	1	-1	-1	0	0	0	1	1	x_{54}
x_{55}	:	++	550	-90	-10	-190	10	2	2	6	70	15	-10	-5	6	3	1	3	-1	-1	0	0	0	0	0	0	0	-10	-1	-1	-2	2	-1	0	-3	1	1	0	0	0	0	-1	0	0	0	0	-1	0	0	1	0	x_{55}
	fus	ind	2	4	2	8	8	8	4	8	6	12	6	6	12	12	6	12	6	6	12	8	8	10	20	10	10	24	24	24	24	12	24 24	24 24	14	28	18	40	40	22 22	24	56	30	30	60	30 30	72 72	40 40	42 42	84 84	120 120	
x_{56}	[	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{56}
x_{57}	]																																																			x_{57}
x_{58}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{58}
x_{59}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i11	0	0	0	0	0	0	0	0	0	0	0	x_{59}
x_{60}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{60}
x_{61}	]																																																			x_{61}
x_{62}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i21	0	x_{62}
x_{63}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r30	x_{63}
x_{64}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i15	0	0	0	0	0	x_{64}
x_{65}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3i2	0	0	0	0	x_{65}
x_{66}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r21	0	0	x_{66}
x_{67}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{67}
x_{68}	]																																																			x_{68}
x_{69}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{69}
x_{70}	]																																																			x_{70}
x_{71}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2i5	0	0	0	x_{71}
x_{72}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{72}
x_{73}	]																																																			x_{73}
x_{74}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{74}
x_{75}	]																																																			x_{75}
x_{76}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{76}
x_{77}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{77}
x_{78}	]																																																			x_{78}
x_{79}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{79}
x_{80}	]																																																			x_{80}
x_{81}	[	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{81}
x_{82}	]																																																			x_{82}

A_{13} and He

Alternating group A_{13}

Order = 3,113,510,400 = $2^9.3^5.5^2.7.11.13$ Mult = 2 Out = 2

Constructions

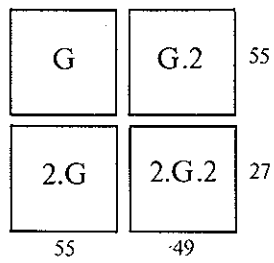
Alternating $S_{13} \cong G.2$: all permutations of 13 letters;

$A_{13} \cong G$: the even permutations; 2.G and 2.G.2 : the Schur double covers

Presentations $G \cong \langle x_1,\ldots,x_{11} | x_i^3=(x_ix_j)^2=1\rangle$; G.2 ≅

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Alternating
239500800	13	A_{12}	: S_{12}	1a+12a		point
39916800	78	S_{11}	: S_{11} x 2	1a+12a+65a	C(2D)	duad
10886400	286	$(A_{10}x3):2$	: S_{10} x S_3	1a+12a+65a+208a	N(3A)	triad
4354560	715	$(A_9xA_4):2$	: S_9 x S_4	1a+12a+65a+208a+429b	N($2A^2$)	tetrad
2419200	1287	$(A_8xA_5):2$	: S_8 x S_5	1a+12a+65a+208a+429b+572b	N(2A,3A,5A)	pentad
1814400	1716	$(A_7xA_6):2$	: S_7 x S_6	1a+12a+65a+208a+429bb+572b	N(2A,3A,3B,4A,5A)	hexad
5616	554400	$L_3(3)$			N(2B,3C,3D,...)	S(2,4,13)
5616	554400	$L_3(3)$			N(2B,3C,3D,...)	S(2,4,13)
78	39916800	13:6	: 13:12		N(13AB)	



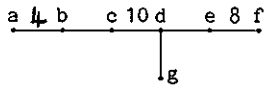
Sporadic Held group He ≅ F_7

Order = 4,030,387,200 = $2^{10}.3^3.5^2.7^3.17$ Mult = 1 Out = 2

Constructions

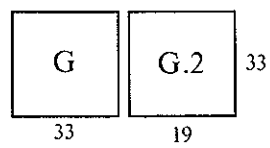
Held G : the automorphism group of a directed graph of rank 5 and out-valence 136 on 2058 points; the point stabilizer is $S_4(4).2$, and the suborbit lengths are 1, 136, 136, 425, 1360. The outer automorphism reverses the directions of the edges, thereby fusing the two orbits of size 136. The centralizer of a 2B-involution is isomorphic to the centralizers of suitable involutions in $L_5(2)$ and M_{24}.

Monster 7 x G : the centralizer in the Monster group F_1 (q.v.) of an element of class 7A. The normalizer is (7.3 x G).2. An element of order 3 in the normal subgroup 7.3 of this is centralized by $3.Fi_{24}'$, and this leads to an embedding of G.2 in Fi_{24}'. The group G has been explicitly constructed as the subgroup of Fi_{24}' stabilizing a set of 2058 3-transpositions in Fi_{24}.

Presentations G.2 ≅ <  | $g=(ef)^4=(abc)^3,a=(cd)^5,1=(cdefef)^4$ >

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract, Held
1958400	2058	$S_4(4):2$	: $S_4(4):4$, $(S_5xS_5):2$	1a+51ab+680a+1275a	point
483840	8330	$2^{2\cdot}L_3(4).S_3$	: $2^{2\cdot}L_3(4).D_{12}$	1a+51ab+680a+1275a+5272a	N($2A^2$)
138240	29155	$2^6:3\cdot S_6$	$2^{4+4}.(S_3xS_3).2$		N(2^6) = N($2A_{18}B_{45}$)
138240	29155	$2^6:3\cdot S_6$			N(2^6) = N($2A_{18}B_{45}$)
21504	187425	$2_+^{1+6}.L_3(2)$	: $2_+^{1+6}.L_3(2).2$		N(2B)
16464	244800	$7^2:2L_2(7)$	: $7^2:2L_2(7).2$		N($7C^2$)
15120	266560	$3\cdot S_7$	: $3\cdot S_7$ x 2		N(3A), C(2C)
6174	652800	$7_+^{1+2}:(S_3x3)$	: $7_+^{1+2}:(S_3x6)$		N(7C)
4032	999600	S_4 x $L_3(2)$	: S_4 x $L_3(2):2$		N($2A^2$), N(2B,3A,7AB)
3528	1142400	7:3 x $L_3(2)$	: 7:6 x $L_3(2)$		N(7AB), N(2A,3B,7C)
1200	3358656	$5^2:4A_4$	: $5^2:4S_4$		N($5A^2$)



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@		
		4	161	21	7										1																							15		7																										
	030387200		280	504	560	504	672	384	128	300	72	24	1176	1176	029	98	98	16	20	12	12	56	56	14	14	15	17	17	21	21	21	21	28	28			120	240	560	72	18	96	96	16	30	12	21	8	8	10	12	12	15	21	21		
	p power		A	A	A	A	A	B	B	A	AA	BB	A	A	A	A	A	C	AA	AA	BB	AA	BA	DB	EB	AA	A	A	CA	CA	BB	AB	AA	BA			A	A	AC	AC	BC	A	B	A	AC	AD	CC	A	A	AD	BB	BB	ABC	BEC	AEC		
	p' part		A	A	A	A	A	A	A	A	AA	BB	A	A	A	A	A	A	AA	AA	BB	AA	BA	DB	EB	AA	A	A	CA	CA	AB	BB	AA	BA			A	A	AC	AC	BC	A	C*	A	AC	AD	CC	A	A	AD	BB	BC	ABC	AEC	BEC		
\	ind	1A	2A	2B	3A	3B	4A	4B	4C	5A	6A	6B	7A	B**	7C	7D	E**	8A	10A	12A	12B	14A	B**	14C	D**	15A	17A	B*	21A	B*	21C	D**	28A	B**	fus	ind	2C	4D	6C	6D	6E	8B	C*	8D	10B	12C	14E	16A	B*	20A	24A	B*	30A	42A	B*		
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		x_1
x_2	o	51	11	3	6	0	3	3	-1	1	2	0	3b7	**	2	**	b7-1	1	1	0	0	-b7	**	b7	**	1	0	0	-1	-1	0	0	b7	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_2
x_3	o	51	11	3	6	0	3	3	-1	1	2	0	**	3b7	2	b7-1	**	1	1	0	0	**	-b7	**	b7	1	0	0	-1	-1	0	0	**	b7																										x_3
x_4	o	153	9	-7	0	3	-3	1	1	3	0	-1	b7+3	**	6	2b7	**	1	-1	0	1	b7-1	**	0	0	0	0	0	0	0	b7	**	**	-b7		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_4
x_5	o	153	9	-7	0	3	-3	1	1	3	0	-1	**	b7+3	6	**	2b7	1	-1	0	1	**	b7-1	0	0	0	0	0	0	0	**	b7	-b7	**																										x_5
x_6	+	680	56	8	14	-1	0	8	0	5	2	-1	8	8	-6	1	1	0	1	0	-1	0	0	1	1	-1	0	0	0	0	-1	-1	0	0	:	++	14	6	14	2	-1	2	2	2	-1	0	0	0	0	1	-1	-1	-1	0	0		x_6
x_7	+	1029	-35	21	21	0	-7	-3	1	4	1	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	1	-b17	*	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_7
x_8	+	1029	-35	21	21	0	-7	-3	1	4	1	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	1	*-b17	0	0	0	0	0	0	0																										x_8
x_9	+	1275	75	27	15	0	7	3	3	0	3	0	-6	-6	1	1	1	-1	0	1	0	-2	-2	-1	-1	0	0	0	1	1	0	0	0	0	:	++	15	-5	15	3	0	3	3	-1	0	1	1	-1	-1	0	0	0	0	1	1		x_9
x_{10}	o	1275	35	-21	15	0	-1	3	-1	0	-1	0	3i7-6	-3i7-6	1	-2b7	**	1	0	-1	0	i7	-i7	0	0	0	0	0	1	1	0	0	-1	-1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_{10}
x_{11}	o	1275	35	-21	15	0	-1	3	-1	0	-1	0	-3i7-6	3i7-6	1	**	-2b7	1	0	-1	0	-i7	i7	0	0	0	0	0	1	1	0	0	-1	-1																										x_{11}
x_{12}	+	1920	64	0	21	0	8	0	0	-5	1	0	9	9	9	2	2	0	-1	-1	0	1	1	0	0	1	-1	-1	0	0	0	0	1	1	:	++	36	4	-9	3	0	0	0	0	1	1	1	0	0	-1	0	0	1	-2	-2		x_{12}
x_{13}	+	4080	16	48	-6	0	0	0	0	5	-2	0	6	6	13	-1	-1	0	1	0	0	2	2	-1	-1	-1	0	0	1	1	0	0	0	0	:	++	36	4	36	0	0	0	0	0	1	-2	1	0	0	-1	0	0	1	1	1		x_{13}
x_{14}	+	4352	128	0	14	8	0	0	0	2	2	0	-2	-2	-9	-2	-2	0	-2	0	0	2	2	0	0	-1	0	0	0	0	1	1	0	0	:	++	32	0	14	2	2	0	0	0	2	0	-3	0	0	0	0	0	-1	0	0		x_{14}
x_{15}	+	6272	-64	0	35	8	-8	0	0	-3	-1	0	7	7	0	0	0	0	1	1	0	-1	-1	0	0	0	-1	-1	0	0	1	1	-1	-1	:	++	28	-4	-35	1	-2	0	0	0	3	-1	0	0	0	1	0	0	0	0	0		x_{15}
x_{16}	+	6528	64	0	-15	0	-8	0	0	3	1	0	-3	-3	11	4	4	0	-1	1	0	1	1	0	0	0	0	0	-1	-1	0	0	-1	-1	:	++	36	4	-27	-3	0	0	0	0	1	1	1	0	0	-1	0	0	-2	1	1		x_{16}
x_{17}	o	7497	81	-7	0	3	-3	1	-3	-3	0	-1	7b7	**	0	0	0	-1	1	0	1	-b7	**	0	0	0	0	0	0	0	b7	**	-b7	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_{17}
x_{18}	o	7497	81	-7	0	3	-3	1	-3	-3	0	-1	**	7b7	0	0	0	-1	1	0	1	**	-b7	0	0	0	0	0	0	0	**	b7	**	-b7																										x_{18}
x_{19}	+	7650	90	50	0	6	6	-6	-2	0	0	2	-1	-1	6	-1	-1	0	0	0	0	-1	-1	1	1	0	0	0	0	0	-1	-1	-1	-1	:	++	0	0	0	0	0	2r2	-2r2	0	0	0	0	r2	-r2	0	-r2	r2	0	0	0		x_{19}
x_{20}	o	7650	90	-14	0	-3	-6	2	-2	0	0	1	4i7-1	-4i7-1	6	2b7	**	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	-b7	**	1	1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_{20}
x_{21}	o	7650	90	-14	0	-3	-6	2	-2	0	0	1	-4i7-1	4i7-1	6	**	2b7	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	**	-b7	1	1																										x_{21}
x_{22}	+	10880	-64	0	-1	8	8	0	0	5	-1	0	-5	-5	2	2	2	0	1	-1	0	-1	-1	0	0	-1	0	0	-1	-1	1	1	1	1	:	++	44	-4	-1	-1	2	0	0	0	-1	-1	2	0	0	1	0	0	-1	-1	-1		x_{22}
x_{23}	o	11475	-45	-45	0	0	3	3	3	0	0	0	3b7	**	9	**	b7-1	-1	0	0	0	-b7	**	-b7	**	0	0	0	0	0	0	0	b7	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		x_{23}
x_{24}	o	11475	-45	-45	0	0	3	3	3	0	0	0	**	3b7	9	b7-1	**	-1	0	0	0	**	-b7	**	-b7	0	0	0	0	0	0	0	**	b7																										x_{24}
x_{25}	+	11900	140	28	20	-7	0	4	4	0	-4	1	0	0	-7	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	:	++	20	0	20	-4	-1	4	4	0	0	0	-1	0	0	0	1	1	0	-1	-1		x_{25}
x_{26}	+	13720	-56	56	-14	7	0	8	0	-5	-2	-1	0	0	0	0	0	0	-1	0	-1	0	0	0	0	1	1	1	0	0	0	0	0	0	:	++	14	6	14	2	-1	-2	-2	-2	-1	0	0	0	0	1	1	1	-1	0	0		x_{26}
x_{27}	+	14400	0	-64	0	6	0	0	0	0	0	2	8	8	-6	1	1	0	0	0	0	0	0	-1	-1	0	1	1	0	0	-1	-1	0	0	:	++	0	0	0	0	0	4r2	-4r2	0	0	0	0	0	0	0	r2	-r2	0	0	0		x_{27}
x_{28}	+	17493	21	21	-21	0	-7	-3	5	-7	3	0	0	0	0	0	0	1	1	-1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	:	++	21	1	21	-3	0	-3	-3	1	1	1	0	-1	-1	1	0	0	1	0	0		x_{28}
x_{29}	+	20825	-55	-7	35	-1	1	-7	1	0	-1	-1	-7	-7	0	0	0	1	0	1	-1	1	1	0	0	0	0	0	0	0	-1	-1	1	1	:	++	35	-5	35	-1	-1	-1	-1	-1	0	1	0	1	1	0	-1	-1	0	0	0		x_{29}
x_{30}	+	21504	0	0	-24	0	0	0	0	4	0	0	0	0	-7	0	0	0	0	0	0	0	0	0	0	1	-1	-1-b21	*	0	0	0	0	0	:	++	48	0	-24	0	0	0	0	0	-2	0	-1	0	0	0	0	0	1-b21	*		x_{30}
x_{31}	+	21504	0	0	-24	0	0	0	0	4	0	0	0	0	-7	0	0	0	0	0	0	0	0	0	0	1	-1	-1	*-b21	0	0	0	0	0	:	++	48	0	-24	0	0	0	0	0	-2	0	-1	0	0	0	0	0	1	*-b21		x_{31}
x_{32}	+	22050	90	-14	0	-6	6	-6	-2	0	0	-2	7	7	0	0	0	0	0	0	0	-1	-1	0	0	0	1	1	0	0	1	1	-1	-1	:	++	0	0	0	0	0	2r2	-2r2	0	0	0	0	-r2	r2	0	-r2	r2	0	0	0		x_{32}
x_{33}	+	23324	-196	28	14	-7	0	4	-4	-1	2	1	0	0	0	0	0	0	-1	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	:	++	14	-6	14	2	-1	-2	-2	2	-1	0	0	0	0	-1	1	1	-1	0	0		x_{33}

He

$O_7(3)$

;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		4585351680	13063680	207360	13824	629856	174960	139968	17496	17496	2916	1458	2880	1152	384	192	120	23328	3888	3888	2592	1728	864	864	432	432	432	324	216	216	108	108	54	14	16	16	324	162	27	27	40	20	144	144	72	72	48	36	36	24	13	13	14	15	108	108	54	36	20	
p power			A	A	A	A	A	A	A	A	A	A	B	C	C	B	A	AA	BA	CA	AB	CC	CB	AC	BB	BC	CC	FA	DC	EC	FC	FC	GC	A	B	C	A	A	D	E	AB	AA	EB	GB	FA	HA	GC	LB	MB	HD	A	A	AA	AB	AA	AA	BA	AD	AA	
p' part			A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	CA	AB	CC	CB	AC	BB	BC	CC	FA	DC	EC	FC	FC	GC	A	A	A	A	A	A	A	AB	AA	CB	AB	CA	BA	AC	DB	EB	BD	A	A	AA	AB	AA	AA	BA	AB	AA	
ind		1A	2A	2B	2C	3A	3B	3C	3D	3E	3F	3G	4A	4B	4C	4D	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	6N	6O	6P	7A	8A	8B	9A	9B	9C	9D	10A	10B	12A	12B	12C	12D	12E	12F	12G	12H	13A	B*	14A	15A	18A	B**	18C	18D	20A	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	78	-34	14	-2	-3	15	6	-3	-3	6	-3	-6	2	2	2	3	-7	-7	2	5	-2	2	1	-1	1	-2	2	1	1	-2	-2	1	1	0	0	3	0	0	0	-1	1	2	-1	0	-3	-1	-1	-1	-1	0	0	1	0	-1	-1	2	-1	-1	χ_{2}
χ_{3}	+	91	-21	11	-5	10	1	19	10	10	1	1	-1	3	3	-1	1	6	-3	-3	2	-5	-1	-2	5	1	1	-3	-2	-2	1	1	1	0	-1	-1	-2	4	1	1	1	-1	3	0	-1	-1	0	0	0	-1	0	0	0	1	0	0	0	2	-1	χ_{3}
χ_{4}	+	105	-35	5	1	24	15	6	-3	-3	-3	6	-5	5	1	-1	0	-8	1	-8	-4	-2	2	4	-1	1	4	1	1	1	1	1	-2	0	1	-1	3	0	0	0	0	0	2	2	-2	1	-2	-1	-1	-1	1	1	0	0	1	1	-2	-1	0	χ_{4}
χ_{5}	+	168	56	24	8	6	15	24	6	6	6	-3	4	0	0	4	3	2	11	2	6	8	0	2	3	-1	2	2	2	2	2	2	-1	0	0	0	0	3	0	0	-1	1	0	0	-2	1	0	0	0	1	-1	-1	0	0	2	2	-1	0	-1	χ_{5}
χ_{6}	+	182	70	22	6	20	29	11	-7	-7	2	2	10	2	2	2	2	16	7	7	4	3	7	0	1	3	3	-2	-3	-3	0	0	0	0	0	0	5	-1	-1	-1	2	0	-1	2	1	1	2	-1	-1	-1	0	0	0	-1	1	1	1	1	0	χ_{6}
χ_{7}	+	195	55	15	11	33	15	24	6	6	-3	6	5	3	-1	1	0	1	1	10	-3	8	0	5	3	5	2	1	2	2	-1	-1	2	-1	1	-1	0	3	0	0	0	0	0	3	2	-1	-1	0	0	1	0	0	-1	0	-2	-2	1	0	0	χ_{7}
χ_{8}	+	260	20	20	4	17	-10	-10	17	-10	8	-1	0	4	4	0	0	-7	2	2	-7	-2	2	1	2	-2	-2	2	1	-2	-2	4	1	1	0	0	-1	-1	2	-1	0	0	-2	1	0	0	1	1	-2	0	0	0	-1	0	-1	-1	-1	-1	0	χ_{8}
χ_{9}	+	260	20	20	4	17	-10	-10	-10	17	8	-1	0	4	4	0	0	-7	2	2	-7	-2	2	1	2	-2	-2	2	-2	1	4	-2	1	1	0	0	-1	-1	-1	2	0	0	-2	1	0	0	1	-2	1	0	0	0	-1	0	-1	-1	-1	-1	0	χ_{9}
χ_{10}	+	273	-91	29	-7	30	30	30	3	3	3	3	-9	1	-3	3	3	-10	-10	-10	2	-10	2	2	2	2	2	-1	-1	-1	-1	-1	-1	0	-1	1	3	3	0	0	-1	-1	-2	4	0	0	0	1	1	0	0	0	0	0	-1	-1	-1	-1	1	χ_{10}
χ_{11}	+	546	154	26	2	-21	51	-12	6	6	-3	-3	14	5	-2	-2	1	19	1	-8	-1	-4	8	-1	-1	5	-4	1	2	2	-1	-1	-1	0	0	0	3	0	0	0	1	-1	0	-3	2	-1	1	0	0	1	0	0	0	1	1	1	-2	-1	-1	χ_{11}
χ_{12}	+	819	-21	-21	19	90	9	9	9	9	9	9	-1	7	-1	-1	-1	6	-3	-3	6	1	-3	10	-3	1	1	-3	1	1	1	1	1	0	1	1	0	0	0	0	-1	-1	1	-2	-1	-1	2	1	1	-1	0	0	0	-1	0	0	0	0	-1	χ_{12}
χ_{13}	+	910	-210	30	-2	19	55	-17	19	-8	1	1	-10	2	2	-2	0	-21	-3	-3	3	7	3	-5	3	1	1	-3	-5	4	1	1	1	0	0	0	4	-2	1	-2	0	0	-1	-1	-1	-1	-1	-1	2	1	0	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}	+	910	-210	30	-2	19	55	-17	-8	19	1	1	-10	2	2	-2	0	-21	-3	-3	3	7	3	-5	3	1	1	-3	4	-5	1	1	1	0	0	0	4	-2	-2	1	0	0	-1	-1	-1	-1	-1	2	-1	1	0	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	+	1092	-140	52	4	-42	-6	30	-15	-15	12	3	0	4	4	0	2	22	-14	4	-2	-2	-2	-2	-2	-2	4	4	1	1	-2	-2	1	0	0	0	-3	0	0	0	2	0	-2	-2	0	0	-2	1	1	0	0	0	0	-1	1	1	-2	1	0	χ_{15}
χ_{16}	+	1365	245	5	-27	69	60	-3	15	15	6	6	5	1	1	-3	0	29	2	11	5	-3	5	-3	-4	-6	3	2	3	3	0	0	0	0	-1	-1	3	0	0	0	0	0	1	1	-1	2	1	1	1	0	0	0	0	0	-1	-1	2	-1	0	χ_{16}
χ_{17}	+	1365	-35	45	5	-12	-30	87	15	15	-3	-3	5	5	-3	-3	0	-8	-8	1	0	-1	3	-4	6	-4	5	1	-1	-1	-1	-1	-1	0	1	1	0	3	0	0	0	0	-1	-4	-1	2	0	-1	-1	0	0	0	0	0	-2	-2	1	0	0	χ_{17}
χ_{18}	o	1560	120	-40	-8	-60	30	12	-6	-6	3	3	0	0	0	0	0	12	-6	-6	-4	4	8	4	2	-2	-2	3	-2	-2	1	1	1	-1	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	313	-313	0	-1	0	χ_{18}
χ_{19}	o	1560	120	-40	-8	-60	30	12	-6	-6	3	3	0	0	0	0	0	12	-6	-6	-4	4	8	4	2	-2	-2	3	-2	-2	1	1	1	-1	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-313	313	0	-1	0	χ_{19}
χ_{20}	+	1638	294	54	-10	-63	45	18	-9	-9	9	0	6	2	2	-2	3	-3	15	-12	9	2	-6	5	-3	-1	-4	-3	-1	-1	-1	-1	2	0	0	0	0	0	0	0	-1	-1	2	-1	0	3	-1	-1	-1	1	0	0	0	0	0	0	0	0	1	χ_{20}
χ_{21}	+	1820	140	-20	-4	-43	20	56	11	11	-7	-7	0	4	4	0	0	5	-4	-4	-11	8	-8	5	4	-4	-4	5	-1	-1	-1	-1	-1	0	0	0	5	-1	-1	-1	0	0	4	1	0	0	1	1	1	0	0	0	0	0	-1	-1	-1	1	0	χ_{21}
χ_{22}	+	2106	-414	66	-6	-81	81	0	0	0	0	0	-14	6	-2	2	1	-9	-9	18	3	0	0	3	-3	3	-6	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	0	-3	-2	1	1	0	0	-1	0	0	-1	1	0	0	0	0	1	χ_{22}
χ_{23}	+	2184	56	24	-24	78	-21	96	24	24	6	6	-4	0	0	-4	-1	2	11	2	6	0	0	-6	3	3	-6	2	0	0	0	0	0	0	0	0	0	3	0	0	-1	1	0	0	2	-1	0	0	0	-1	0	0	0	-1	2	2	-1	0	1	χ_{23}
χ_{24}	+	2457	189	21	33	27	0	54	27	27	0	0	-9	-7	5	3	-3	27	0	0	3	6	6	3	0	0	0	0	3	3	0	0	0	0	-1	1	0	0	0	0	1	-1	2	-1	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	χ_{24}
χ_{25}	+	2730	490	90	26	-24	75	-15	3	3	12	3	10	-2	-2	2	0	4	13	-5	0	-7	-3	-4	3	5	-1	4	-1	-1	2	2	-1	0	0	0	0	-3	0	0	0	0	1	-2	1	1	-2	1	1	-1	0	0	0	0	-2	-2	1	0	0	χ_{25}
χ_{26}	+	2730	-70	10	-6	138	-15	21	-24	-24	-6	3	-10	6	6	-2	0	38	11	-7	10	-3	1	6	1	3	-3	2	0	0	0	0	-3	0	0	0	-3	0	0	0	0	0	-3	0	-1	-1	0	0	0	1	0	0	0	0	-1	-1	2	1	0	χ_{26}
χ_{27}	+	2835	315	75	3	-81	0	81	0	0	0	0	-5	3	-5	3	0	-9	18	-9	3	9	-3	3	0	-6	3	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	-3	-3	1	-2	1	0	0	0	1	1	0	0	0	0	0	0	0	χ_{27}
χ_{28}	+	4095	315	-45	-25	207	45	18	-9	-9	0	9	5	11	-1	1	0	-9	-9	18	-9	2	-6	-1	-3	-1	2	0	-1	-1	2	2	-1	0	-1	1	0	0	0	0	0	0	2	-1	2	-1	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	+	4095	-525	75	7	-36	45	72	18	18	9	0	5	-9	3	1	0	-12	-21	6	12	-8	0	4	-3	-5	-2	-3	-2	-2	1	1	-2	0	1	-1	0	0	0	0	0	0	0	0	2	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	χ_{29}
χ_{30}	+	4368	560	48	16	156	66	-24	-6	-6	-6	3	0	0	0	0	-2	20	2	20	12	-8	0	4	6	-2	4	2	-2	-2	-2	-2	1	0	0	0	0	-3	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	-1	0	0	χ_{30}
χ_{31}	+	4536	-504	-24	24	162	81	0	0	0	0	0	-4	0	0	-4	1	-18	9	-18	-6	0	0	6	-3	-3	6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2	-1	0	0	0	-1	-1	-1	0	1	0	0	0	0	1	χ_{31}
χ_{32}	+	5265	225	-15	33	162	0	81	0	0	0	0	5	-3	-3	-3	0	-18	-18	9	-6	9	-3	6	0	6	-3	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	-3	0	-1	2	0	0	0	0	0	0	1	0	0	0	0	0	0	χ_{32}
χ_{33}	+	5460	420	100	20	33	-30	96	-21	-21	-3	-3	0	4	4	0	0	-39	6	6	1	8	4	-7	-2	2	2	-3	-1	-1	-1	-1	-1	0	0	0	-6	0	0	0	0	0	4	1	0	0	1	1	1	0	0	0	0	0	0	0	0	-2	0	χ_{33}
χ_{34}	+	5460	-700	100	-12	114	60	6	-21	-21	6	6	0	-4	-4	0	0	2	-15	-16	10	5	-2	-6	4	0	0	2	3	3	0	0	0	0	0	0	-3	-3	0	0	0	0	2	2	0	0	2	-1	-1	0	0	0	0	0	-1	-1	-1	1	0	χ_{34}
χ_{35}	+	5460	-140	-140	20	-129	60	60	5	6	15	-3	0	12	-4	0	0	-5	4	4	-5	-4	4	-1	4	-4	-4	-5	2	2	-1	-1	-1	0	0	0	-3	-3	0	0	0	0	0	3	0	0	-1	0	0	0	0	0	0	0	1	1	1	1	0	χ_{35}
χ_{36}	+	5824	896	64	0	-8	154	-8	-8	-8	-8	-8	16	0	0	0	-1	32	-4	-4	-8	0	4	0	-2	0	0	-4	0	0	0	0	0	0	0	0	1	1	1	1	-1	1	0	0	-2	-2	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	χ_{36}
χ_{37}	+	5824	-896	64	0	-8	154	-8	-8	-8	-8	-8	-16	0	0	0	-1	-32	4	4	-8	0	4	0	-2	0	0	4	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0	2	2	0	0	0	0	0	0	0	-1	1	1	1	1	-1	χ_{37}
χ_{38}	+	6552	-504	-24	-8	234	45	72	18	18	0	9	4	0	0	4	-3	-18	9	-18	-6	-9	0	-2	-3	1	-2	0	-2	-2	-2	-2	1	0	0	0	0	0	0	0	1	1	0	0	-2	1	0	0	0	1	0	0	0	0	0	0	0	0	-1	χ_{38}
χ_{39}	+	7020	540	60	12	-27	0	-54	54	-27	0	0	0	4	4	0	0	-27	0	0	-3	-6	-6	-3	0	0	0	0	6	-3	0	0	0	-1	0	0	0	0	0	0	0	0	-2	1	0	0	1	-2	1	0	0	0	1	0	0	0	0	0	0	χ_{39}
χ_{40}	+	7020	540	60	12	-27	0	-54	-27	54	0	0	0	4	4	0	0	-27	0	0	-3	-6	-6	-3	0	0	0	0	-3	6	0	0	0	-1	0	0	0	0	0	0	0	0	-2	1	0	0	1	1	-2	0	0	0	1	0	0	0	0	0	0	χ_{40}
χ_{41}	+	7280	-560	80	-16	-10	-10	152	-10	-10	-10	-10	0	0	0	0	0	34	-2	-2	-10	-16	-4	2	2	2	2	-2	2	2	2	2	2	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	0	χ_{41}
χ_{42}	+	7280	-560	80	-16	-10	-10	-64	44	-10	8	-1	0	0	0	0	0	34	-2	-2	-10	8	-4	2	2	2	2	-2	-4	2	-4	2	-1	0	0	0	-1	-1	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	0	χ_{42}
χ_{43}	+	7280	-560	80	-16	-10	-10	-64	-10	44	8	-1	0	0	0	0	0	34	-2	-2	-10	8	-4	2	2	2	2	-2	2	-4	2	-4	-1	0	0	0	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	0	χ_{43}
χ_{44}	+	7371	819	51	-21	81	81	81	0	0	0	0	-1	-9	-1	-1	1	9	9	9	-3	9	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	1	1	0	0	0	0	1	-1	-3	3	-1	-1	-1	0	0	-1	0	0	0	1	0	0	0	0	-1	χ_{44}
χ_{45}	+	8190	-210	30	-18	171	-45	-45	36	-45	-9	0	10	2	2	2	0	-21	-3	-3	3	3	3	3	3	-3	-3	-3	0	3	3	-3	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	-1	2	-1	-1	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}	+	8190	-210	30	-18	171	-45	-45	-45	36	-9	0	10	2	2	2	0	-21	-3	-3	3	3	3	3	3	-3	-3	-3	3	0	-3	3	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	χ_{46}
χ_{47}	+	11648	0	128	0	-16	-124	-16	-16	-16	20	2	0	0	0	0	-2	0	0	0	-16	0	8	0	-4	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	-1	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	χ_{47}
χ_{48}	+	14742	630	102	6	162	-81	-81	0	0	0	0	-10	-6	-6	-2	2	-18	9	9	-6	-9	3	6	3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	3	0	-1	-1	0	0	0	1	0	0	0	-1	0	0	0	0	0	χ_{48}
χ_{49}	+	16380	-420	60	28	99	-90	-36	18	18	-9	-9	0	12	-4	0	0	39	-5	-5	15	4	0	-5	-6	-2	-2	3	-2	-2	1	1	1	0	0	0	0	0	0	0	0	0	0	3	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{49}
χ_{50}	+	16640	1280	0	0	-208	80	-64	8	8	-10	8	0	0	0	0	0	-16	-16	-16	0	0	0	0	0	0	0	2	0	0	0	0	0	1	0	0	-4	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	2	2	2	0	0	χ_{50}
χ_{51}	+	16640	-1280	0	0	-208	80	-64	8	8	-10	8	0	0	0	0	0	15	15	15	0	0	0	0	0	0	0	-2	0	0	0	0	0	1	0	0	-4	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-2	-2	-2	0	0	χ_{51}
χ_{52}	+	17472	-896	-64	0	-24	30	-24	-24	-24	12	-6	16	0	0	0	-3	-32	4	4	8	0	-4	0	2	0	0	4	0	0	0	0	0	0	0	0	3	3	0	0	1	-1	0	0	-2	-2	0	0	0	0	0	0	0	0	1	1	1	-1	1	χ_{52}
χ_{53}	+	17472	896	-64	0	-24	30	-24	-24	-24	12	-5	-16	0	0	0	-3	32	-4	-4	8	0	-4	0	2	0	0	-4	0	0	0	0	0	0	0	0	3	3	0	0	1	1	0	0	2	2	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	χ_{53}
χ_{54}	+	17920	0	0	0	-224	-80	64	-8	-8	-8	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-2	1	1	0	0	0	0	0	0	0	0	0	0	b13	*	0	0	0	0	0	0	0	χ_{54}
χ_{55}	+	17920	0	0	0	-224	-80	64	-8	-8	-8	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-2	1	1	0	0	0	0	0	0	0	0	0	0	*	b13	0	0	0	0	0	0	0	χ_{55}
χ_{56}	+	19683	-729	-81	27	0	0	0	0	0	0	0	9	-9	3	-3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	1	1	-1	0	0	0	0	0	-1	χ_{56}
χ_{57}	+	21840	560	-80	-48	-30	-30	24	24	24	6	-12	0	0	0	0	0	-34	2	2	10	0	4	6	-2	6	6	2	0	0	0	0	0	0	0	0	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	0	χ_{57}
χ_{58}	+	22113	189	-171	9	243	0	0	0	0	0	0	-9	-3	1	3	3	27	0	0	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	-1	-1	0	-3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	χ_{58}
ind		1 2	4	4	2	3 6	3 6	3 6	3 6	3 6	3 6	3 6	8	4 4	4	8	5 10	12	12	12	12	6	12	6 6	12	6	6	12	6 6	6 6	6	6	6 6	7 14	8	8 8	9 18	9 18	9 18	9 18	20	20	12 12	12 12	24	24	12	12 12	12 12	24	13 26	13 26	28 28	15 30	36	36	36 36	36	40 40	
χ_{59}	+	520	0	0	0	34	-20	-20	7	7	16	-2	0	4	0	0	0	0	0	0	0	0	0	6	0	0	0	0	3	3	0	0	0	2	0	2	-2	-2	1	1	0	0	-2	-2	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	χ_{59}
χ_{60}	+	560	0	0	0	74	20	20	20	-7	2	11	0	8	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	-3	0	0	3	0	0	0	2	2	2	-1	0	0	2	2	0	0	0	2	-1	0	1	1	0	0	0	0	0	0	0	χ_{60}
χ_{61}	+	560	0	0	0	74	20	20	-7	20	2	11	0	8	0	0	0	0	0	0	0	0	0	6	0	0	0	0	-3	0	0	0	-3	0	0	0	2	2	-1	2	0	0	2	2	0	0	0	-1	2	0	1	1	0	0	0	0	0	0	0	χ_{61}
χ_{62}	+	1456	0	0	0	-2	52	-56	25	25	-2	-2	0	8	0	0	-4	0	0	0	0	0	0	-6	0	0	0	0	-3	-3	0	0	0	0	0	0	-2	-2	1	1	0	0	-4	2	0	0	0	-1	-1	0	0	0	0	2	0	0	0	0	0	χ_{62}
χ_{63}	+	1456	0	0	0	-2	-56	52	-2	25	-2	-2	0	8	0	0	-4	0	0	0	0	0	0	-6	0	0	0	0	6	-3	0	0	0	0	0	0	-2	-2	-2	1	0	0	2	2	0	0	0	2	-1	0	0	0	0	-1	0	0	0	0	0	χ_{63}
χ_{64}	+	1456	0	0	0	-2	-56	52	25	-2	-2	-2	0	8	0	0	-4	0	0	0	0	0	0	-6	0	0	0	0	-3	6	0	0	0	0	0	0	-2	-2	1	-2	0	0	2	2	0	0	0	-1	2	0	0	0	0	-1	0	0	0	0	0	χ_{64}
χ_{65}	+	3120	0	0	0	-120	60	24	-12	-12	6	6	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-6	0	0	0	0	0	-4	-4	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	χ_{65}
χ_{66}	+	3640	0	0	0	-86	40	4	-5	-5	22	4	0	-4	0	0	0	0	0	0	0	0	0	6	0	0	0	0	3	3	0	0	0	0	0	-2	-8	-2	1	1	0	0	2	2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	χ_{66}
χ_{67}	+	6720	0	0	0	240	60	96	24	24	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	3r3	0	0	χ_{67}
χ_{68}	+	6720	0	0	0	240	60	95	24	24	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-3r3	0	0	χ_{68}
χ_{69}	+	7280	0	0	0	-172	80	116	17	17	8	-10	0	8	0	0	0	0	0	0	0	0	0	12	0	0	0	0	-3	-3	0	0	0	0	0	0	2	-4	-1	-1	0	0	2	-4	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	χ_{69}
χ_{70}	+	7280	0	0	0	314	80	8	-37	-37	-10	8	0	8	0	0	0	0	0	0	0	0	0	6	0	0	0	0	3	3	0	0	0	0	0	0	2	-4	-1	-1	0	0	-4	2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	χ_{70}
χ_{71}	+	7280	0	0	0	152	-100	-28	-37	17	-10	-1	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	-3	0	0	-3	0	0	0	-4	2	2	-1	0	0	2	-4	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	χ_{71}
χ_{72}	+	7280	0	0	0	152	-100	-28	17	-37	-10	-1	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	3	0	0	3	0	0	0	-4	2	-1	2	0	0	2	-4	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	χ_{72}
χ_{73}	+	10920	0	0	0	66	-60	-132	39	39	-6	-6	0	4	0	0	0	0	0	0	0	0	0	6	0	0	0	0	3	3	0	0	0	0	0	-2	6	0	0	0	0	0	-2	-2	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	χ_{73}
χ_{74}	+	11648	0	0	0	-16	-124	-16	-16	-16	20	2	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	r10	χ_{74}
χ_{75}	+	11648	0	0	0	-16	-124	-16	-16	-16	20	2	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-r10	χ_{75}
χ_{76}	+	13104	0	0	0	-18	-72	-72	9	9	18	0	0	8	0	0	4	0	0	0	0	0	0	-6	0	0	0	0	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	-4	2	0	0	0	-1	-1	0	0	0	0	-2	0	0	0	0	0	χ_{76}
χ_{77}	+	13104	0	0	0	-18	36	36	-45	36	0	-9	0	8	0	0	4	0	0	0	0	0	0	-6	0	0	0	0	3	0	0	0	3	0	0	0	0	0	0	0	0	0	2	2	0	0	0	-1	2	0	0	0	0	1	0	0	0	0	0	χ_{77}
χ_{78}	+	13104	0	0	0	-18	36	36	36	-45	0	-9	0	8	0	0	4	0	0	0	0	0	0	-6	0	0	0	0	0	3	0	0	-3	0	0	0	0	0	0	0	0	0	2	2	0	0	0	2	-1	0	0	0	0	1	0	0	0	0	0	χ_{78}
χ_{79}	+	14040	0	0	0	-54	0	-108	27	27	0	0	0	-4	0	0	0	0	0	0	0	0	0	6	0	0	0	0	3	3	0	0	0	-2	0	2	0	0	0	0	0	0	2	2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	χ_{79}
χ_{80}	+	15120	0	0	0	54	0	108	-27	54	0	0	0	-8	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	-3	6	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	1	-2	0	1	1	0	0	0	0	0	0	0	χ_{80}
χ_{81}	+	15120	0	0	0	54	0	108	54	-27	0	0	0	-8	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	6	-3	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	-2	1	0	1	1	0	0	0	0	0	0	0	χ_{81}
χ_{82}	+	16640	0	0	0	-208	80	-64	8	8	-10	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-4	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	r7	0	0	0	0	0	0	χ_{82}
χ_{83}	+	16640	0	0	0	-208	80	-64	8	8	-10	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-4	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-r7	0	0	0	0	0	0	χ_{83}
χ_{84}	+	17920	0	0	0	-224	-80	64	-8	-8	-8	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-2	1	1	0	0	0	0	0	0	0	0	0	0	b13	*	0	0	0	0	0	0	0	χ_{84}
χ_{85}	+	17920	0	0	0	-224	-80	64	-8	-8	-8	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-2	1	1	0	0	0	0	0	0	0	0	0	0	*	b13	0	0	0	0	0	0	0	χ_{85}
χ_{86}	+	21840	0	0	0	294	60	-120	-3	-3	5	6	0	-8	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	-3	-3	0	0	0	0	0	0	0	-6	0	0	0	0	4	-2	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	χ_{86}
χ_{87}	+	21840	0	0	0	132	-120	60	-3	-3	-12	-12	0	-8	0	0	0	0	0	0	0	0	0	12	0	0	0	0	-3	-3	0	0	0	0	0	0	-6	0	0	0	0	0	-2	4	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	χ_{87}
χ_{88}	+	23296	0	0	0	-32	184	-32	-32	-32	4	-14	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	χ_{88}

	fus	ind	2D	2E	2F	4E	4F	4G	4H	4I	6Q	6R	6S	6T	6U	6V	6W	6X	6Y	6Z	6A1	6B1	8C	8D	10C	10D	10E	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	18E	18F	18G	20B	24A	26A	B*	28A	B*	30A	36A	
	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			121	103	17	1		2			11	3	3	2	1													1																					
			30560	680	280	03680	12096	304	384	192	664	888	888	160	296	432	432	432	216	162	54	54	96	32	120	40	20	296	432	216	216	144	72	72	48	24	18	54	54	18	20	12	13	13	14	14	15	18	
			A	A	A	B	A	B	B	C	AD	BD	CD	BF	AE	BE	BF	CF	CE	GD	FF	GF	B	B	AF	AD	AE	DE	FE	AF	HE	DG	FG	HG	FH	GI	KF	AQ	BQ	BU	AE	AC	BD	AD	AF	AF	ACT	DI	
			A	A	A	A	A	A	A	A	AD	BD	CD	BF	AE	BE	BF	CF	CE	GD	FF	GF	A	A	AF	AD	AE	AE	CE	AF	BE	AG	CG	BG	CH	AI	FF	AD	BD	BE	AE	CC	AD	BD	AF	AF	ACT	AE	
	fus	ind	2D	2E	2F	4E	4F	4G	4H	4I	6Q	6R	6S	6T	6U	6V	6W	6X	6Y	6Z	6A1	6B1	8C	8D	10C	10D	10E	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	18E	18F	18G	20B	24A	26A	B*	28A	B*	30A	36A	
χ_{1}	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	:	++	26	-6	2	-14	6	2	2	-2	-1	-1	8	5	3	-3	-1	-4	0	-1	2	-1	0	0	-3	1	-1	-5	-2	-3	1	-1	2	-1	2	1	0	-1	2	0	1	0	0	0	-1	-1	0	1	χ_{2}
χ_{3}	:	++	39	-9	-1	-9	7	7	-1	-1	12	3	3	-1	0	3	-1	-1	-3	3	-1	-1	-1	-1	-1	-1	1	0	3	-2	-3	4	1	1	-1	2	1	0	0	0	1	-1	0	0	0	0	-1	0	χ_{3}
χ_{4}	:	++	25	-15	5	-5	7	3	-1	-3	-2	7	-2	5	-6	-3	-1	2	0	-2	-1	2	-3	1	0	0	0	4	-2	-2	1	0	0	3	2	0	1	1	-2	0	0	0	-1	-1	0	0	0	1	χ_{4}
χ_{5}	:	++	64	16	8	24	0	8	0	0	10	1	10	5	-2	1	5	2	4	1	2	-1	0	0	3	-1	1	6	0	0	3	2	2	-1	0	0	0	-2	1	1	-1	0	-1	-1	0	0	0	0	χ_{5}
χ_{6}	:	++	52	20	12	20	0	4	4	0	-2	7	7	9	2	5	3	3	-1	-2	0	0	2	2	2	2	0	2	5	0	-1	-2	1	1	1	0	0	1	1	-1	0	-1	0	0	0	0	-1	-1	χ_{6}
χ_{7}	:	++	65	25	5	15	-1	7	3	1	11	11	2	5	7	1	-1	2	4	2	-1	2	3	-1	0	0	0	-3	0	-1	3	1	-2	1	0	1	-1	2	-1	1	0	0	0	0	-1	-1	0	0	χ_{7}
χ_{8}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{8}
χ_{9}																																																	χ_{9}
χ_{10}	:	++	91	-29	7	-31	-7	9	-3	3	10	10	10	10	-2	-2	-2	-2	-2	1	1	1	-3	1	-3	1	1	-4	-4	2	-4	0	0	0	0	0	-1	1	1	1	-1	0	0	0	0	0	0	-1	χ_{10}
χ_{11}	:	++	26	-6	-14	14	-14	-2	-2	-2	-1	-1	8	1	3	-3	-5	4	0	-1	1	1	0	0	1	1	-1	5	2	-5	-1	1	-2	1	-2	1	1	-1	2	0	-1	0	0	0	0	0	1	-1	χ_{11}
χ_{12}	:	++	39	39	-1	-21	7	-5	-5	3	12	3	3	-1	12	3	-1	-1	3	3	-1	-1	1	1	-1	-1	-1	6	-3	-2	-3	-2	1	1	1	0	1	0	0	0	-1	1	0	0	0	0	-1	0	χ_{12}
χ_{13}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}																																																	χ_{14}
χ_{15}	:	++	208	0	8	-40	0	8	0	0	-8	-8	10	-4	0	0	-4	2	0	1	2	-1	0	0	-2	-2	0	-4	8	0	2	-4	2	2	0	0	0	1	-2	0	0	0	0	0	0	0	1	-1	χ_{15}
χ_{16}	:	++	65	-15	-15	25	21	-7	1	5	11	2	11	0	3	0	-6	3	-3	2	0	0	-1	-1	0	0	0	7	1	3	4	-1	-1	2	1	-1	0	-1	2	0	0	-1	0	0	0	0	0	1	χ_{16}
χ_{17}	:	++	325	5	5	-15	-7	17	1	-3	28	-8	1	-10	-4	2	-4	5	-1	1	-1	-1	1	1	0	0	0	-6	-3	2	0	2	-1	2	1	0	-1	-2	1	-1	0	1	0	0	0	0	0	0	χ_{17}
χ_{18}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{18}
χ_{19}																																																	χ_{19}
χ_{20}	:	++	234	-6	-22	66	14	2	-6	-2	-9	-9	18	5	3	-3	-1	2	0	0	-1	2	0	0	3	-1	-1	3	0	5	-3	-1	2	-1	0	1	-1	0	0	0	1	0	0	0	0	0	0	0	χ_{20}
χ_{21}	:	++	260	20	-20	20	28	4	-4	4	17	-10	-10	10	-7	2	-2	-2	2	-1	1	1	0	0	0	0	0	-7	2	1	2	1	-2	-2	2	1	1	-1	-1	-1	0	0	0	0	0	0	0	-1	χ_{21}
χ_{22}	:	++	234	-6	-6	-66	-6	-2	6	-2	-9	-9	18	9	3	-3	3	-6	0	0	0	0	0	0	-1	-1	-1	-3	0	3	3	1	-2	1	0	1	0	0	0	0	-1	0	0	0	1	1	-1	0	χ_{22}
χ_{23}	:	++	416	-16	-24	24	0	8	0	0	38	11	2	-9	2	-1	3	-6	-4	2	0	0	0	0	1	1	-1	6	0	0	3	2	2	-1	0	0	0	2	-1	-1	-1	0	0	0	0	0	1	0	χ_{23}
χ_{24}	:	++	351	39	27	21	21	-3	1	-1	27	0	0	0	3	0	0	0	6	0	0	0	-3	1	-3	1	-1	3	6	3	0	3	0	0	-2	-1	0	0	0	0	1	0	0	0	0	0	0	0	χ_{24}
χ_{25}	:	++	260	20	20	60	0	-4	4	0	-10	-1	17	5	2	5	-1	5	-1	-1	2	-1	-2	-2	0	0	0	6	-3	0	-3	2	-1	-1	1	0	0	2	-1	-1	0	1	0	0	0	0	0	0	χ_{25}
χ_{26}	:	++	260	-60	-20	20	0	4	4	0	-10	17	-1	-5	-6	3	1	-5	3	-1	-2	1	-2	-2	0	0	0	2	5	0	-1	-2	1	1	1	0	0	-1	2	0	0	1	0	0	0	0	0	-1	χ_{26}
χ_{27}	:	++	495	15	15	75	-21	11	-5	-1	9	-18	9	0	-3	0	6	-3	3	0	0	0	-1	-1	0	0	0	3	-3	-3	0	-1	-1	2	1	-1	0	0	0	0	0	-1	1	1	0	0	0	0	χ_{27}
χ_{28}	:	++	195	75	-25	-15	21	9	-3	-1	-21	15	-12	5	3	-3	-1	-4	0	-3	2	-1	3	-1	0	0	0	3	0	3	-3	3	0	3	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	:	++	585	-15	5	-105	7	-1	3	1	18	-9	18	5	-6	-3	-1	2	0	0	-1	2	3	-1	0	0	0	-6	0	-2	-3	2	2	-1	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	χ_{29}
χ_{30}	:	++	208	80	48	0	0	0	0	0	-8	10	-8	6	8	2	6	0	-4	1	0	3	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	-2	1	-1	0	0	0	0	0	0	1	0	χ_{30}
χ_{31}	:	++	144	-96	24	24	0	8	0	0	-18	9	-18	9	-6	-3	-3	6	0	0	0	0	0	0	-1	-1	-1	6	0	0	3	2	2	-1	0	0	0	0	0	0	-1	0	1	1	0	0	-1	0	χ_{31}
χ_{32}	:	++	585	105	-15	-15	-27	1	1	-3	18	18	-9	0	6	0	-6	3	3	0	0	0	-1	-1	0	0	0	-6	-3	0	0	-2	1	-2	1	0	0	0	0	0	0	-1	0	0	1	1	0	0	χ_{32}
χ_{33}	:	++	780	60	20	60	28	12	4	4	-3	6	6	-10	-3	-6	2	2	0	-3	-1	-1	0	0	0	0	0	-3	-6	1	0	-3	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	χ_{33}
χ_{34}	:	++	520	-40	0	-80	0	0	-8	0	-20	15	16	0	-4	8	0	0	2	-2	0	0	0	0	0	0	0	-8	-2	0	4	0	0	0	-2	0	0	1	1	-1	0	0	0	0	0	0	0	1	χ_{34}
χ_{35}	:	++	260	20	-20	-20	-28	-4	4	4	17	-10	-10	10	-7	2	-2	-2	2	-1	1	1	0	0	0	0	0	7	-2	-1	-2	-1	2	2	-2	1	-1	-1	-1	-1	0	0	0	0	0	0	0	1	χ_{35}
χ_{36}	:	++	416	64	16	64	0	0	0	0	-16	2	2	16	-8	4	-2	-2	-2	2	-2	-2	0	0	1	1	-1	-8	4	0	-2	0	0	0	0	0	0	-1	-1	1	-1	0	0	0	0	0	1	1	χ_{36}
χ_{37}	:	++	416	-64	16	-64	0	0	0	0	-16	2	2	16	8	-4	-2	-2	2	2	-2	-2	0	0	1	1	1	8	-4	0	2	0	0	0	0	0	0	-1	-1	-1	1	0	0	0	0	0	1	-1	χ_{37}
χ_{38}	:	++	624	-96	8	-24	0	-8	0	0	30	21	-5	5	-6	-3	5	2	0	3	2	-1	0	0	3	-1	-1	-6	0	0	-3	-2	-2	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	χ_{38}
χ_{39}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{39}
χ_{40}																																																	χ_{40}
χ_{41}	:	++	1040	-80	0	-80	0	16	0	0	14	-4	-4	0	10	4	0	0	-2	-4	0	0	0	0	0	0	0	10	4	0	-2	-2	-2	-2	0	0	0	-1	-1	1	0	0	0	0	0	0	0	1	χ_{41}
χ_{42}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{42}
χ_{43}																																																	χ_{43}
χ_{44}	:	++	819	51	-21	111	-21	-1	-1	-1	9	9	9	9	-3	-3	-3	-3	-3	0	0	0	1	1	-1	-1	1	3	3	-3	3	-1	-1	-1	-1	-1	0	0	0	0	1	1	0	0	0	0	-1	0	χ_{44}
χ_{45}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}																																																	χ_{46}
χ_{47}	:	++	832	0	32	0	0	0	0	0	-32	4	4	-16	0	0	-4	-4	0	4	2	2	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	-1	0	χ_{47}
χ_{48}	:	++	468	-60	-12	60	0	-4	4	0	-18	9	9	-9	-6	3	-3	-3	3	0	0	0	2	2	-2	-2	0	6	-3	0	-3	2	-1	-1	1	0	0	0	0	0	0	-1	0	0	0	0	1	0	χ_{48}
χ_{49}	:	++	780	60	20	-60	-28	-12	-4	4	-3	6	6	-10	-3	-6	2	2	0	-3	-1	-1	0	0	0	0	0	3	6	-1	0	3	0	0	2	1	-1	0	0	0	0	0	0	0	0	0	0	0	χ_{49}
χ_{50}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r7	-r7	0	0	χ_{50}
χ_{51}	:	++	0	0	0	0	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	1	0	0	χ_{51}
χ_{52}	:	++	416	64	-48	-64	0	0	0	0	-16	2	2	0	-8	4	6	6	-2	2	0	0	0	0	-3	1	-1	8	-4	0	2	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	-1	χ_{52}
χ_{53}	:	++	416	-64	-48	64	0	0	0	0	-16	2	2	0	8	-4	6	6	2	2	0	0	0	0	-3	1	1	-8	4	0	-2	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	1	χ_{53}
χ_{54}	:	++	1280	0	0	0	0	0	0	0	-16	-16	-16	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	b13	*	0	0	0	0	χ_{54}
χ_{55}	:	++	1280	0	0	0	0	0	0	0	-16	-16	-16	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	*	b13	0	0	0	0	χ_{55}
χ_{56}	:	++	729	81	-27	-81	27	-9	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	3	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	-1	-1	0	0	χ_{56}
χ_{57}	:	++	1040	-80	0	80	0	-16	0	0	14	-4	-4	0	10	4	0	0	-2	-4	0	0	0	0	0	0	0	-10	-4	0	2	2	2	2	0	0	0	-1	-1	1	0	0	0	0	0	0	0	-1	χ_{57}
χ_{58}	:	++	351	-9	27	9	21	-15	-3	-5	27	0	0	0	-9	0	0	0	0	0	0	0	3	-1	-3	1	1	9	0	3	0	-3	0	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	χ_{58}
	fus	ind	4	4	2	8 8	8 8	8	8	4	12	12	12	6	12	12	6	6	12	12	5	6	8 8	8	10 10	20	20	24 24	24 24	24 24	24 24	24 24	24	24	24	12	24 24	36	36	36	40 40	24 24	52 52	52 52	56 56	56 56	30 30	72 72	
χ_{59}	:	++	0	0	0	20r2	-6r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r2	0	0	0	0	-7r2	2r2	3r2	2r2	3r2	0	0	0	0	0	0	0	0	0	-r2	0	0	r2	r2	0	-r2	χ_{59}
χ_{60}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{60}
χ_{61}																																																	χ_{61}
χ_{62}	:	++	0	0	0	8r2	28r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-r2	-4r2	r2	2r2	-3r2	0	0	0	0	r2	0	0	0	-2r2	0	0	0	0	0	0	-r2	χ_{62}
χ_{63}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{63}
χ_{64}																																																	χ_{64}
χ_{65}	:	++	0	0	0	40r2	-20r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4r2	-8r2	-2r2	-2r2	0	0	0	0	0	r2	0	0	0	0	0	0	0	r2	r2	0	r2	χ_{65}
χ_{66}	:	++	0	0	0	60r2	14r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2r2	0	0	0	0	-3r2	-6r2	5r2	0	3r2	0	0	0	0	-r2	0	0	0	0	r2	0	0	0	0	0	0	χ_{66}
χ_{67}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{67}
χ_{68}																																																	χ_{68}
χ_{69}	:	++	0	0	0	80r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4r2	0	0	0	0	8r2	2r2	0	-4r2	0	0	0	0	0	0	0	0	0	0	r2	0	0	0	0	0	-r2	χ_{69}
χ_{70}	:	++	0	0	0	40r2	28r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13r2	4r2	r2	4r2	3r2	0	0	0	0	r2	0	0	0	0	0	0	0	0	0	0	r2	χ_{70}
χ_{71}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{71}
χ_{72}																																																	χ_{72}
χ_{73}	:	++	0	0	0	20r2	-14r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2r2	0	0	0	0	-7r2	2r2	-5r2	2r2	3r2	0	0	0	0	r2	0	0	0	0	r2	0	0	0	0	0	-r2	χ_{73}
χ_{74}	:	++	0	0	0	64r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r5	0	0	-8r2	4r2	0	-2r2	0	0	0	0	0	0	0	0	0	-r2	0	0	0	0	0	-r5	r2	χ_{74}
χ_{75}	:	++	0	0	0	64r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2r5	0	0	-8r2	4r2	0	-2r2	0	0	0	0	0	0	0	0	0	-r2	0	0	0	0	0	r5	r2	χ_{75}
χ_{76}	:	++	0	0	0	72r2	28r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-9r2	0	r2	0	-3r2	0	0	0	0	r2	0	0	0	2r2	0	0	0	0	0	0	0	χ_{76}
χ_{77}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{77}
χ_{78}																																																	χ_{78}
χ_{79}	:	++	0	0	0	60r2	6r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r2	0	0	0	0	-3r2	-6r2	-3r2	0	3r2	0	0	0	0	0	0	0	0	0	-r2	0	0	-r2	-r2	0	0	χ_{79}
χ_{80}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{80}
χ_{81}																																																	χ_{81}
χ_{82}	:	++	0	0	0	0	32r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4r2	0	0	0	0	0	0	-r2	0	0	0	0	0	0	0	-k56	*15	0	0	χ_{82}
χ_{83}	:	++	0	0	0	0	32r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4r2	0	0	0	0	0	0	-r2	0	0	0	0	0	0	0	*15	-k56	0	0	χ_{83}
χ_{84}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	k'52	*5	0	0	0	0	χ_{84}
χ_{85}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*21	k'52	0	0	0	0	χ_{85}
χ_{86}	:	++	0	0	0	120r2	-28r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3r2	0	-r2	6r2	-3r2	0	0	0	0	-r2	0	0	0	0	0	0	0	0	0	0	0	χ_{86}
χ_{87}	:	++	0	0	0	80r2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4r2	0	0	0	0	8r2	2r2	0	-4r2	0	0	0	0	0	0	0	0	0	0	-r2	0	0	0	0	0	-r2	χ_{87}
χ_{88}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4r5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r5	0	χ_{88}

$O_7(3)$

	ind	1A	2A	2B	2C	3A	3B	3C	3D	3E	3F	3G	4A	4B	4C	4D	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	6N	6O	6P	7A	8A	8B	9A	9B	9C	9D	10A	10B	12A	12B	12C	12D	12E	12F	12G	12H	13A	B*	14A	15A	18A	B**	18C	18D	20A	
		1	2	2	2	3	3	3	3	3	3	3	4	4	4	4	5	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	8	8	9	9	9	9	10	10	12	12	12	12	12	12	12	12	12	13	13	14	15	18	18	18	18	20	
		3	6	6	6		3						12	12	12	12	15	6	6		6		6		6	6								21	24	24	9				30	30			12	12	12			12	39	39	42	15	18	18		18	60	
		3	6	6	6		3						12	12	12	12	15	6	6		6		6		6	6								21	24	24	9				30	30			12	12	12			12	39	39	42	15	18	18		18	60	
x_{90}	o2	27	15	7	3	0	9	0	0	0	0	0	5	3	-1	1	2	6	3	0	-2	0	4	0	1	3	0	0	0	0	0	0	0	-1	1	-1	3	0	0	0	2	0	0	0	2	-1	2	0	0	1	1	1	1	-1	-13	13	0	1	0	x_{90}
x_{91}	o2	351	111	31	15	0	36	0	0	0	0	0	11	3	3	3	1	12	6	0	4	0	4	0	4	6	0	0	0	0	0	0	0	1	1	1	3	0	0	0	1	1	0	0	2	2	0	0	0	0	0	0	-1	1	13	-13	0	1	1	x_{91}
x_{92}	o2	351	99	11	-9	0	36	0	0	0	0	0	9	3	-1	-3	1	18	0	0	2	0	8	0	-4	0	0	0	0	0	0	0	0	1	-1	1	3	0	0	0	1	-1	0	0	0	0	2	0	0	0	0	0	1	1	-3	-3	0	-1	-1	x_{92}
x_{93}	o2	378	-126	34	-6	0	45	0	0	0	0	0	-14	6	-2	2	3	-18	-9	0	-2	0	4	0	1	3	0	0	0	0	0	0	0	0	0	0	6	0	0	0	-1	-1	0	0	-2	1	-2	0	0	-1	1	1	0	0	0	0	0	-2	1	x_{93}
x_{94}	o2	1728	384	64	0	0	90	0	0	0	0	0	16	0	0	0	3	24	12	0	-8	0	4	0	-2	0	0	0	0	0	0	0	0	-1	0	0	3	0	0	0	-1	-1	0	0	-2	-2	0	0	0	0	-1	-1	-1	0	-13	13	0	1	1	x_{94}
x_{95}	o2	1728	-384	64	0	0	90	0	0	0	0	0	-16	0	0	0	3	-24	-12	0	-8	0	4	0	-2	0	0	0	0	0	0	0	0	-1	0	0	3	0	0	0	-1	1	0	0	2	2	0	0	0	0	-1	-1	1	0	13	-13	0	1	-1	x_{95}
x_{96}	o2	2457	-315	77	-15	0	9	0	0	0	0	0	-5	9	-3	-1	2	36	-9	0	-4	0	-4	0	5	3	0	0	0	0	0	0	0	0	-1	1	-6	0	0	0	2	0	0	0	-2	1	0	0	0	-1	0	0	0	-1	0	0	0	2	0	x_{96}
x_{97}	o2	2457	-399	1	9	0	90	0	0	0	0	0	-11	9	1	-3	-3	-30	12	0	10	0	4	0	-2	0	0	0	0	0	0	0	0	0	1	1	3	0	0	0	1	1	0	0	-2	-2	-2	0	0	0	0	0	0	0	-13	13	0	1	-1	x_{97}
x_{98}	o2	2457	441	41	9	0	90	0	0	0	0	0	9	-3	-3	1	-3	36	0	0	-4	0	8	0	2	0	0	0	0	0	0	0	0	0	-1	-1	3	0	0	0	1	1	0	0	0	0	0	0	0	-2	0	0	0	0	3	3	0	-1	-1	x_{98}
x_{99}	o2	2808	-216	-56	24	0	45	0	0	0	0	0	-4	0	0	-4	3	0	9	0	16	0	4	0	1	-3	0	0	0	0	0	0	0	1	0	0	-3	0	0	0	-1	-1	0	0	2	-1	0	0	0	-1	0	0	1	0	3	3	0	1	1	x_{99}
x_{100}	o2	2808	456	104	24	0	45	0	0	0	0	0	4	0	0	4	3	-12	21	0	-4	0	-4	0	5	-3	0	0	0	0	0	0	0	1	0	0	-3	0	0	0	-1	1	0	0	-2	1	0	0	0	1	0	0	1	0	-13	13	0	-1	-1	x_{100}
x_{101}	o2	4914	714	74	18	0	99	0	0	0	0	0	14	6	-2	-2	-1	-6	-3	0	2	0	-4	0	-1	9	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	-1	-1	0	0	2	-1	-2	0	0	1	0	0	0	-1	13	-13	0	-1	-1	x_{101}
x_{102}	o2	4914	546	-46	-30	0	99	0	0	0	0	0	6	6	6	-2	-1	24	-15	0	8	0	8	0	-1	-3	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	-1	1	0	0	0	3	0	0	0	1	0	0	0	-1	-13	13	0	-1	1	x_{102}
x_{103}	o2	5265	-855	105	-15	0	135	0	0	0	0	0	-15	-3	5	1	0	-18	-9	0	6	0	0	0	3	3	0	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	-3	2	0	0	1	0	0	-1	0	0	0	0	0	0	x_{103}
x_{104}	o2	7020	300	140	12	0	-90	0	0	0	0	0	0	12	-4	0	0	-42	6	0	14	0	8	0	2	-6	0	0	0	0	0	0	0	-1	0	0	-3	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	-1	0	13	-13	0	-1	0	x_{104}
x_{105}	o2	7371	-441	111	3	0	-54	0	0	0	0	0	9	3	7	-3	1	18	-18	0	-6	0	0	0	6	-6	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	1	-1	0	0	0	0	-2	0	0	0	0	0	0	1	0	0	0	0	-1	x_{105}
x_{106}	o2	7371	-189	51	27	0	-54	0	0	0	0	0	-9	3	-5	-1	1	54	0	0	6	0	0	0	-6	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	-2	0	0	2	0	0	0	1	0	0	0	0	1	x_{106}
x_{107}	o2	7560	840	40	-24	0	90	0	0	0	0	0	0	0	0	0	0	12	6	0	4	0	-8	0	-2	-6	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b13	*	0	0	13	-13	0	1	0	x_{107}
x_{108}	o2	7560	840	40	-24	0	90	0	0	0	0	0	0	0	0	0	0	12	6	0	4	0	-8	0	-2	-6	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*-b13	0	0	13	-13	0	1	0	x_{108}
x_{109}	o2	9477	729	81	45	0	0	0	0	0	0	0	-9	-3	9	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	x_{109}
x_{110}	o2	9828	420	-92	36	0	36	0	0	0	0	0	0	12	-4	0	-2	6	-24	0	-2	0	-8	0	4	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	-2	0	0	0	0	0	2	0	0	0	0	0	0	1	-13	13	0	1	0	x_{110}
x_{111}	o2	12285	-735	-55	21	0	45	0	0	0	0	0	-5	9	5	-1	0	30	15	0	-10	0	-4	0	-7	3	0	0	0	0	0	0	0	0	1	-1	-3	0	0	0	0	0	0	0	-2	1	2	0	0	-1	0	0	0	0	13	-13	0	-1	0	x_{111}
x_{112}	o2	12285	-315	-155	-3	0	45	0	0	0	0	0	5	9	1	5	0	-18	9	0	-2	0	4	0	1	-3	0	0	0	0	0	0	0	0	-1	-1	-3	0	0	0	0	0	0	0	2	-1	-2	0	0	-1	0	0	0	0	-3	-3	0	1	0	x_{112}
x_{113}	o2	12285	-1155	125	-3	0	45	0	0	0	0	0	5	-3	-3	5	0	24	-15	0	8	0	-4	0	-7	-3	0	0	0	0	0	0	0	0	1	1	-3	0	0	0	0	0	0	0	2	-1	0	0	0	-1	0	0	0	0	-13	13	0	-1	0	x_{113}
x_{114}	o2	12285	525	-115	-3	0	45	0	0	0	0	0	-15	-3	-3	1	0	48	-3	0	-16	0	8	0	5	-3	0	0	0	0	0	0	0	0	1	1	-3	0	0	0	0	0	0	0	0	-3	0	0	0	1	0	0	0	0	13	-13	0	-1	0	x_{114}
x_{115}	o2	12285	945	25	21	0	45	0	0	0	0	0	-5	-15	-3	-1	0	0	-9	0	16	0	4	0	1	3	0	0	0	0	0	0	0	0	1	-1	-3	0	0	0	0	0	0	0	-2	1	0	0	0	-1	0	0	0	0	-3	-3	0	1	0	x_{115}
x_{116}	o2	14742	-1386	6	6	0	135	0	0	0	0	0	6	-6	-6	-2	-3	-36	9	0	-12	0	0	0	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0	1	x_{116}
x_{117}	o2	16848	-720	48	-48	0	-54	0	0	0	0	0	0	0	0	0	-2	36	18	0	12	0	0	0	6	6	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	x_{117}
x_{118}	o2	19656	504	-104	-24	0	-9	0	0	0	0	0	4	0	0	4	1	-36	-9	0	4	0	4	0	-5	3	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	1	-1	0	0	-2	1	0	0	0	1	0	0	0	1	3	3	0	1	-1	x_{118}
x_{119}	o2	19656	1176	56	-24	0	-9	0	0	0	0	0	-4	0	0	-4	1	-48	3	0	-16	0	-4	0	-1	3	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	1	1	0	0	2	-1	0	0	0	-1	0	0	0	1	-13	13	0	-1	1	x_{119}
x_{120}	o2	19683	-729	-81	27	0	0	0	0	0	0	0	9	-9	3	-3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	1	1	-1	0	0	0	0	0	-1	x_{120}
x_{121}	o2	22464	384	64	0	0	-126	0	0	0	0	0	-16	0	0	0	-1	24	12	0	-8	0	4	0	-2	0	0	0	0	0	0	0	0	1	0	0	3	0	0	0	-1	-1	0	0	2	2	0	0	0	0	0	0	-1	-1	-13	13	0	1	-1	x_{121}
x_{122}	o2	22464	-384	64	0	0	-126	0	0	0	0	0	16	0	0	0	-1	-24	-12	0	-8	0	4	0	-2	0	0	0	0	0	0	0	0	1	0	0	3	0	0	0	-1	1	0	0	-2	-2	0	0	0	0	0	0	1	-1	13	-13	0	1	1	x_{122}
x_{123}	o2	24192		0	-128	0	0	-36	0	0	0	0	0	0	0	0	0	2	0	0	0	16	0	-8	0	4	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	2	0	0	0	0	0	0	0	0	0	-1	-1	0	-1	0	0	0	-2	0	x_{123}

	ind	1	4	4	2	3	3	3	3	3	3	3	3	8	4	4	8	5	12	12	12	12	6	12	6	12	6	6	12	6	6	6	6	6	7	8	8	9	9	9	9	20	20	12	12	24	24	12	12	12	24	13	13	28	15	36	36	36	36	40	
			6	12	6		6	6	6	6	6	6	6	24	12	12	24	30	12	12		12			6	12	6				6	6		6	42	24	24	18	18	18	18	60	60	12	12	24	24	12	12	12	24	78	78	84	30	36	36	36	36	120	
			3	12	6		3							24	12	12	24	15	12	12		12				12	6								21	24	24	9				60	60			24	24	12			24	39	39	84	15	36	36		36	120	
			2				6								4			10																14		8	18														26	26	28	30					40	
			3				3								12			15																21		24	9														39	39	84	15					120	
			6				6								12			30																42		24	18														78	78	84	30					120	
x_{124}	o2	4536		0	0	0	0	-108	0	0	0	0	0	0	12	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	2	0	0	0	0	0	x_{124}
x_{125}	o2	12096		0	0	0	0	144	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	b13	*	0	-1	0	0	0	0	0	x_{125}
x_{126}	o2	12096		0	0	0	0	144	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	*	b13	0	-1	0	0	0	0	0	x_{126}
x_{127}	o2	14040		0	0	0	0	-180	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	-2	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{127}
x_{128}	o2	15120		0	0	0	0	180	0	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	x_{128}
x_{129}	o2	19656		0	0	0	0	72	0	0	0	0	0	0	-12	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	x_{129}
x_{130}	o2	20736		0	0	0	0	-216	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	-1	0	0	0	0	0	x_{130}
x_{131}	o2	22464		0	0	0	0	36	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r7	1	0	0	0	0	0	x_{131}
x_{132}	o2	22464		0	0	0	0	36	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-r7	1	0	0	0	0	0	x_{132}
x_{133}	o2	24192		0	0	0	0	-36	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	-1	0	0	0	0	r10	x_{133}
x_{134}	o2	24192		0	0	0	0	-36	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	-1	0	0	0	0	-r10	x_{134}
x_{135}	o2	29160		0	0	0	0	0	0	0	0	0	0	0	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	x_{135}

Orthogonal group $O_7(3) \cong B_3(3)$

Order = 4,585,351,680 = $2^9.3^9.5.7.13$ Mult = 6 Out = 2

Constructions

Orthogonal $GO_7(3) \cong 2 \times G.2$: all 7 x 7 matrices over $\mathbb{F}_3$ preserving a non-singular quadratic form;

$PGO_7(3) \cong SO_7(3) \cong PSO_7(3) \cong G.2$; $O_7(3) \cong G$

Complex 3.G has a 27-dimensional complex unitary representation over $\mathbb{Z}[w]$ ($w = z3$), in which there is an orbit of 3 x 378 root vectors. It is simplest to take 28 coordinates indexed by unordered pairs from the 8-set {1,2,3,4,5,6,7,8} and work modulo the 1-space $\langle(1^{28})\rangle$. The norm of the vector $(x_{12},\ldots,x_{78})$ is $\Sigma|x_{ij}|^2/4 - |\Sigma x_{ij}|^2/112$. The root vectors are the multiples by powers of w of the following vectors of shape $(\theta^2,1^{10},0^{16})$, where $\theta = w-\bar{w} = i3$:

(i) θ on ac, bc, and 1 on ad, bd, ae, be, af, bf, ag, bg, ah, bh

(ii) θ on ab, cd, and 1 on ac, bd, ad, bc, ef, gh, eg, fh, eh, fg

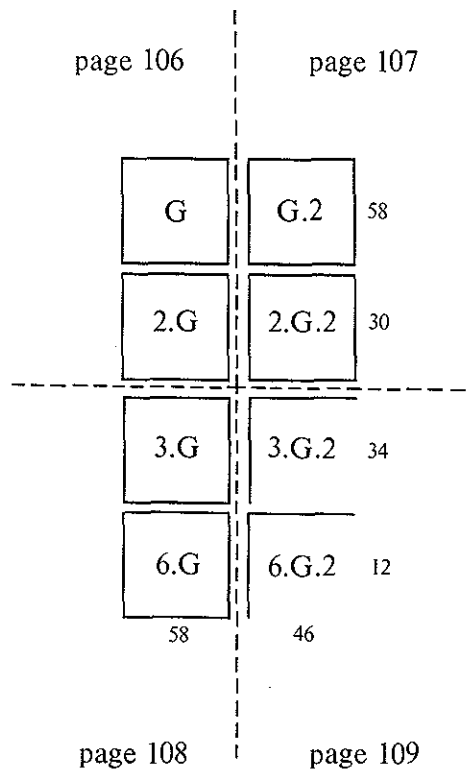
The subgroup $S_6(2)$ permutes the coordinates (see $S_6(2)$, "Hesse"). There is an invariant multiplication x*y antilinear in each coordinate, and 3.G.2 is generated by the antilinear maps ("reflections") $x \longrightarrow x^r = x*r - \langle r,x\rangle r$ as r ranges over the root vectors. The square of a coordinate vector is (2-11w)/27 times itself, and the product of two distinct coordinate vectors has the form $(A^2,B^{10},0^{16})$, where $A = (7\bar{w}-4)/216$, $B = \bar{w}/8$, and the coordinates are distributed as in the corresponding root vector. The reduction modulo 2 of this representation extends to $3Fi_{22}$ and $3\cdot({}^2E_6(2)).3$.

Remark : The Fischer group Fi_{22} contains two conjugacy classes of maximal subgroups isomorphic to $O_7(3)$.

fus	ind	-	2D	2E	2F	4E	4F	4G	4H	4I	6Q	6R	6S	6T	6U	6V	6W	6X	6Y	6Z	6A	6B	8C	8D	10C	10D	10E	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	18E	18F	18G	20B	24A	26A	B*	28A	B*	30A	36A
			2	2	2	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	6	8	8	10	10	10	12	12	12	12	12	12	12	12	12	12	18	18	18	20	24	26	26	28	28	30	36

x_{90} * +
x_{91} * +
x_{92} * +
x_{93} * +
x_{94} * +
x_{95} * +
x_{96} * +
x_{97} * +
x_{98} * +
x_{99} * +
x_{100} * +
x_{101} * +
x_{102} * +
x_{103} * +
x_{104} * +
x_{105} * +
x_{106} * +
x_{107} * +
x_{108} * +
x_{109} * +
x_{110} * +
x_{111} * +
x_{112} * +
x_{113} * +
x_{114} * +
x_{115} * +
x_{116} * +
x_{117} * +
x_{118} * +
x_{119} * +
x_{120} * +
x_{121} * +
x_{122} * +

fus	ind	4	4	2	8	8	8	8	4	12	12	12	6	12	12	6	6	12	12	6	6	8	8	10	20	20	24	24	24	24	24	24	24	24	12	24	36	36	36	40	24	52	52	56	56	30	72
					8	8																8		10			24	24	24	24	24					24				40	24		52	56	56	30	72



x_{123} * +
x_{124} * +
x_{125} * +
x_{126} * +
x_{127} * +
x_{128} * +
x_{129} * +
x_{130} * +
x_{131} * +
x_{132} * +
x_{133} * +
x_{134} * +

Presentation $3.G \cong \langle$ a b c d e f | $(abcdefg)^9=1$ $\rangle$; replacing the last relation by

g
h

$(bcdefdghd)^{10}=1$ gives 2 x G; both relations give G. See page 232.

Maximal subgroups					Specifications	
Order	Index	Structure	G.2	Character	Abstract	Orthogonal
13063680	351	$2U_4(3):2_2$	$: 2U_4(3).(2^2)_{122}$	1a+168a+182a	N(2A)	minus point
12597120	364	$3^5:U_4(2):2$	$: 3^5:(U_4(2):2x2)$	1a+168a+195a	$N(3^5) = N(3A_{40}B_{36}C_{45})$	isotropic point
12130560	378	$L_4(3):2_2$	$: L_4(3):2_2 \times 2$	1a+182a+195a	C(2D)	plus point
4245696	1080	$G_2(3)$		1a+260a+819a		
4245696	1080	$G_2(3)$		1a+260b+819a		
4094064	1120	$3^{3+3}:L_3(3)$	$: 3^{3+3}:(L_3(3)x2)$	1a+105a+195a+819a	$N(3A^3)$	isotropic plane
1451520	3159	$S_6(2)$		1a+168a+260a+2730a		
1451520	3159	$S_6(2)$		1a+168a+260b+2730a		
1259712	3640	$3_+^{1+6}:(2A_4xA_4).2$	$: 3_+^{1+6}:(2S_4xS_4)$		N(3A)	isotropic line
362880	12636	S_9				
362880	12636	S_9				
207360	22113	$(2^2 \times U_4(2)):2$	$: D_8 \times U_4(2):2$		N(2B)	non-isotropic line
161280	28431	$2^6:A_7$	$: 2^6:S_7$		$N(2^6) = N(2A_7B_{21}C_{35})$	base
17280	265356	$S_4 \times S_6$	$: 2 \times S_4 \times S_6$		$N(2B^2)$, C(2E)	$O_3(3)xO_4^-(3)$
13824	331695	$S_4x2(A_4xA_4).2$	$: S_4x2(A_4xA_4).4$		N(2C), $N(2B^2)$	$O_3(3)xO_4^+(3)$

$S_6(3)$

Abbreviated character table.

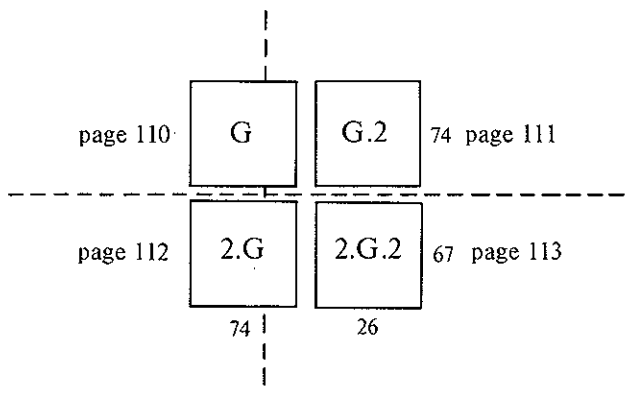
;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
	4	622	24		209	104		103	2	1							2	1																	
585351680		080	192	12597120/2	952	976	17496/2	972	680	304	152	120	155520/2	15552/2	3888/2	3888/2	592	296	648/2	432	324/2	36	14	96	32	1944/2	486/2	486/2	27/2	120	2592/2	432	288/2	288/2
p	power	A	A	A	A	A	A	A	A	A	A	A	BA	BA	CA	DA	CA	DA	FA	CB	FA	GB	A	C	C	A	A	A	E	AA	DA	IA	BC	DB
p'	part	A	A	A	A	A	A	A	A	A	A	A	AA	AA	CA	DA	CA	DA	EA	BC	EA	GB	A	A	A	A	A	A	A	AA	AA	CA	AC	AB
ind		1A	2A	2B	3AB	3C	3D	3EF	3G	4A	4B	4C	5A	6AB	6CD	6EF	6GH	5I	5J	6KL	6M	6NO	6P	7A	8A	8B	9AB	9CD	9EF	9GH	10A	12AB	12C	12DE	12FG
+		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
o2		13	-3	1	-9z3-5	-5	4	b27**	1	5	-3	1	-2	-z3-5	-3z3**	-i3	-2i3	3	0	z3+2	1	-z3-2	1	-1	-1	-1	b27	1	-2	z3	2	-b27	-1	z3**	z3-1
+		78	-2	-6	-30	15	6	-3	0	14	6	2	3	10	-2	1	-8	7	-2	1	3	1	0	1	0	0	-3	0	3	0	3	-4	-1	2	0
o2		91	11	7	-9z3-35	10	10	-3z3-5	1	11	3	-1	1	-9z3+5	-9z3-7	-4	-4	2	2	-b27	-2	-b27	1	0	1	1	3z3-2	1	1	z3	1	3z3-1	2	-z3-3	-3z3**
+		105	25	-7	-30	6	15	-3	3	5	-3	5	0	10	-2	-8	1	-2	7	1	2	1	-1	0	3	-1	-3	3	0	0	0	-4	2	2	0
+		168	8	0	60	24	15	6	0	24	8	0	3	20	8	2	11	8	-1	2	0	2	0	0	0	0	6	0	3	0	3	6	0	0	2
+		195	35	-1	50	15	24	6	3	15	-1	3	0	20	8	11	2	-1	8	2	-1	2	-1	-1	3	-1	6	3	0	0	0	6	3	0	2
o2		273	17	-7	-81z3-51	-24	3	3	3	25	-7	1	-2	-9z3-19	3z3+5	2b27	-6i3-1	8	-1	-1	2	-1	-1	0	1	1	3z3	3z3	3z3	0	2	-3z3+1	4	b27	-z3**
o2		455	-25	7	-90z3+5	-4	23	-3z3+8	5	15	-1	-1	0	30z3+5	-3i3-16	2b27**	-1	-4	-1	-3z3-4	-2	-b27**	1	0	-1	-1	3i3+5	-b27**	-b27**	-z3**	0	-3	0	-i3+2	-2i3-1
o2		546	-14	14	-27i3+87	15	-3	9z3-3	3	26	2	-2	1	13i3+7	-3i3-5	-i3-2	-2i3+7	7	1	z3-3	5	z3**	-1	0	0	0	-3z3**	3z3	-3i3	0	1	2b27	-1	-2z3	2z3**
o2		728	-8	0	90i3+26	-46	17	2b27	-4	40	-8	0	-2	-10i3-26	-8	-2z3**	8i3+1	10	1	-2z3**	0	-2z3	0	0	0	0	-3z3+2	-b27**	3z3-1	-z3**	2	3i3-5	-2	0	-2z3**
+		819	99	7	9	-18	36	9	9	-21	3	7	-1	9	9	0	0	6	12	-3	-2	3	1	0	1	1	0	0	0	0	-1	-3	-6	1	-3
o2		1170	-30	6	90b27	-45	36	9z3	0	50	-6	-2	0	15i3-45	-9i3+15	3i3	6i3	3	0	-3z3	-3	3z3	0	1	0	0	9z3**	0	0	0	0	3i3+5	-1	-2z3**	-i3-3
o2		1365	-75	-7	-135i3+15	-30	60	-3b27	3	25	9	5	0	5i3-45	6z3	-8i3	2i3	-6	0	-z3+1	2	z3-1	-1	0	-3	1	-3b27**	3	0	0	0	4b27**	-2	2z3	0
o2		1365	85	21	-135z3+15	-21	24	-9i3-12	0	5	5	5	0	25z3+15	-15z3-17	4i3+7	4z3	-5	4	i3-2	3	-i3-2	0	0	1	1	-3z3**	-3i3	3z3	0	0	-b27	-1	-b27	-b27
o2		1365	5	-7	135z3-120	24	6	-3b27	3	45	5	-7	0	55z3+40	15z3+8	2z3	4z3-8	8	2	-z3-3	2	-z3**	-1	0	-1	-1	6z3**	3z3	-3i3	0	0	-9z3	0	-z3	-z3
+		1820	60	28	200	65	11	11	-7	20	4	4	0	0	-12	9	9	9	3	3	1	-3	1	0	0	0	-7	2	2	-1	0	2	5	4	-2
o2		1820	60	28	-90i3-70	29	-7	21z3+8	5	20	4	4	0	-10i3-30	-12z3**	4i3-3	2i3-3	-3	-3	5z3+4	1	z3+2	1	0	0	0	3i3+2	-4	-1	-z3	0	2	5	4z3**	-2
+		2106	-54	6	-324	81	0	0	0	66	10	6	1	36	0	9	-18	9	0	0	-3	0	0	-1	0	0	0	0	0	0	1	-6	-3	0	-2
o2		2184	104	0	216z3-84	-12	33	6b27	0	24	8	0	-1	28i3+8	8b27	-4i3-10	-2i3-1	-4	5	2z3**	0	2z3	0	0	0	0	6z3**	-3i3	3z3	0	-1	6	0	0	2
+		2457	57	21	270	27	27	27	0	21	5	-7	-3	30	30	3	3	3	3	3	3	3	0	0	-3	1	0	0	0	0	-3	-6	3	2	2
o2		2457	-87	21	81z3-54	27	27	27z3	0	-15	-7	5	2	9z3-6	-27z3-6	3	3	3	3	3z3	3	3z3	0	0	-1	-1	0	0	0	0	-2	3b27**	3	z3-2	-b27**
+		2730	-70	14	300	57	57	30	3	10	-14	6	0	20	-16	-7	-7	-7	-7	2	5	2	-1	0	0	0	3	3	3	0	0	10	1	0	-2
+		2835	-45	-21	405	81	0	0	0	75	11	3	0	45	9	-9	18	9	0	0	-3	0	0	0	-1	-1	0	0	0	0	0	3	-3	-3	-1
o2		3640	-40	0	90i3-410	22	49	3i3-5	4	40	-8	0	0	30i3+50	8b27	3i3+5	5	-10	-7	2b27	0	2	0	0	0	0	-3z3-11	b27	b27	z3	0	-3i3-5	-2	0	-2z3
o2		4095	15	-21	135i3+450	45	45	-9z3**	0	55	-1	-5	0	-45i3+30	9i3-12	3	3	-3	-3	3z3**	-3	3z3**	0	0	1	1	-9z3	0	0	0	0	-6i3+1	1	-i3-2	-1
+		4536	216	0	-324	0	81	0	0	-24	-8	0	1	36	0	-18	9	0	9	0	0	0	0	0	0	0	0	0	0	0	1	-6	0	0	-2
+		5265	225	-27	405	0	81	0	0	-15	-7	-3	0	45	9	18	-9	0	9	0	0	0	0	1	-1	-1	0	0	0	0	0	3	0	-3	-1
+		5460	180	-28	60	123	-3	6	3	-20	-4	12	0	-60	0	3	3	3	-3	6	-1	0	-1	0	0	0	-3	-6	0	0	0	-2	-5	0	2
+		5460	20	28	600	33	33	-21	-3	60	-4	4	0	80	20	-1	-1	-7	-7	-1	1	-1	1	0	0	0	6	-3	-3	0	0	6	-3	4	2
o2		5460	20	28	540z3+60	-75	-21	-9z3+15	-3	60	-4	4	0	20b27	-4b27**	-1	6i3-1	5	-1	3z3-1	1	-b27	1	0	0	0	6z3	-3z3	-3z3	0	0	6	-3	4z3	2
+		5824	64	0	-656	100	46	-8	-8	64	0	0	-1	64	-8	-8	-8	4	-2	4	0	-2	0	0	0	0	1	1	1	1	-1	-8	4	0	0
o2		5824	64	0	-576z3-80	-44	-26	12i3-8	4	64	0	0	-1	64z3	-8	-8z3**	-8z3	4	-2	4z3**	0	-2z3	0	0	0	0	1	b27**	-3z3+1	z3	-1	-8	4	0	0
o2		6552	-72	0	270i3+234	-90	-9	18z3	0	40	-8	0	2	-30i3+6	0	6z3	6i3-3	6	3	-6z3	0	0	0	0	0	0	-9z3**	0	0	0	-2	-3i3-5	-2	0	-2z3
o2		7371	171	-21	-729z3-405	-81	0	0	0	51	3	3	1	-9z3-45	9z3**	9i3	0	-9	0	0	-3	0	0	0	-1	-1	0	0	0	0	1	-3b27	3	-3z3**	-z3+1
+		8190	-210	-14	90	63	117	9	9	-90	-2	2	0	-30	6	-3	-3	15	-3	-3	-5	-3	1	0	0	0	0	0	0	0	0	0	3	2	4
o2		9477	-27	-27	729z3	0	0	0	0	45	-3	-3	2	81z3	-27z3	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	-2	-9z3	0	-3z3	3z3
o2		10920	-120	0	270i3-150	-114	57	9i3-15	0	-40	8	0	0	10i3+30	24z3	5i3+3	-2i3+3	6	-3	-i3+3	0	i3+3	0	0	0	0	3b27	0	-3	0	0	3i3+5	2	0	2z3
+		11648	-128	0	416	-88	-52	-16	8	0	0	0	-2	64	16	-8	-8	-8	4	4	0	-2	0	0	0	0	2	-1	5	-1	2	0	0	0	0
o2		11648	-128	0	-288i3-448	56	20	24z3-4	-4	0	0	0	-2	64z3**	16	-8z3	-8z3**	-8	4	4z3	0	-2z3**	0	0	0	0	2	-b27	-3z3+2	-z3**	2	0	0	0	0
o2		12285	45	21	405i3+135	-27	54	-27z3**	0	-15	9	-7	0	-15i3-45	-18z3	-3i3	-6i3	-3	-6	-3z3**	3	3z3**	0	0	3	-1	0	0	0	0	0	-6	-3	2z3	-2i3
+		14560	-160	0	-560	-92	-56	34	4	0	0	0	0	80	-16	8	8	-4	8	2	0	-4	0	0	0	0	-2	-5	1	1	0	0	0	0	0
o2		14560	-160	0	360i3+520	52	16	-15i3+7	-8	0	0	0	0	80z3**	-16	8z3	8z3**	-4	8	2z3	0	-4z3**	0	0	0	0	-2	-3z3-5	b27**	z3	0	0	0	0	0
+		16380	60	-28	180	-117	-9	18	-9	-60	4	12	0	60	24	-3	-3	3	-9	-6	-1	0	-1	0	0	0	0	0	0	0	0	-6	3	0	-2
+		16640	0	-64	-640	80	-64	8	2	0	0	0	0	0	0	0	0	0	0	0	8	0	2	1	0	0	8	2	-4	-1	0	0	0	0	0
+		16640	0	64	-640	80	-64	8	2	0	0	0	0	0	0	0	0	0	0	0	-8	0	-2	1	0	0	8	2	-4	-1	0	0	0	0	0
+		17472	192	0	-240	12	-6	-24	0	-64	0	0	-3	0	-24	0	0	12	-6	0	0	0	0	0	0	0	3	0	6	0	-3	8	-4	0	0
+2		17920	0	0	640	64	-80	-8	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	4	-2	1	0	0	0	0	0
+		19683	243	27	0	0	0	0	0	-81	-9	-9	3	0	0	0	0	0	0	0	0	0	0	-1	-3	1	0	0	0	0	3	0	0	0	0

$S_6(3)$

@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
216	144	144/2	72	36/2	13/2	14	30/2	216/2	54/2	54/2	20	24/2	30/2	36/2			11		1																					
JA	IC	IB	JB	LC	A	AB	AB	BC	DC	FC	AA	EA	BAB	BA			232	12096	440	192	108	18	1440	96	96	32	216	36	36	24	18	10	36	36	12	12	13/2	14/2	20/2
DA	CC	CB	DB	EC	A	AB	AA	AA	CA	EA	AA	AA	AAA	AA			A	B	A	B	DC	GC	A	A	C	A	MD	IE	JE	MF	PD	AE	CC	HC	DD	IE	BC	AD	AC
12H	12I	12JK	12L	12MN	13AB	14A	15AB	18AB	18CD	18EF	20A	24AB	30AB	36AB	fus	ind	2C	4D	4E	4F	6Q	6R	8C	8D	8E	8F	12N	12O	12P	12Q	12R	20B	24C	24D	24E	24F	26AB	28AB	40AB
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	-i3	0	z3	0	1	z3	-z3+1	-i3	0	0	-z3**	-z3	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-1	-3	0	-1	0	1	0	1	-2	1	-1	0	0	-1	:	++	0	6	4	-2	0	0	-4	0	-2	2	-3	1	-2	1	0	-1	-1	2	0	1	0	-1	1
2	2	0	0	-z3**	0	0	z3-b27**		-1	-1	1	z3**	z3	-z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	2	0	-3	-1	1	0	0	1	1	-2	0	0	0	-1	:	++	1	-7	5	1	1	1	-5	-1	3	-1	2	2	-1	-2	-1	0	-2	1	-1	0	1	0	0
3	0	2	-1	0	-1	0	0	2	2	-1	-1	0	0	0	:	++	12	0	-4	0	3	0	4	4	0	0	0	2	-1	0	0	1	-2	1	1	0	-1	0	-1
0	3	-1	2	0	0	-1	0	2	-1	2	0	0	0	0	:	++	13	-1	5	-1	4	1	-5	3	1	1	-1	-1	2	-1	-1	0	1	-2	0	1	0	-1	0
1	-2	2z3**	-1	1	0	0	z3**	-z3	-z3	-z3	0	z3**	-z3**	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	2z3	-1	-z3	0	0	0	2z3	-z3	-z3	0	-1	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	1	-i3+2	-1	z3**	0	0	z3	z3**	z3	1	1	0	z3	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	-2z3**	1	0	0	0	1	b27**	z3	z3**	0	0	-1	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-2	0	0	1	0	0	-1	0	0	0	-1	1	-1	0	:	++	13	7	9	-1	4	1	9	-3	3	-1	-2	0	0	2	1	-1	0	0	0	0	0	0	-1
2	1	-i3	0	z3	0	-1	0	-3z3**	0	0	0	0	0	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	2	0	0	-z3**	0	0	0	z3+2	i3	0	0	0	0	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-1	-1	2	-1	0	0	0	z3**	1	z3	0	z3**	0	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	2z3	2	-z3**	0	0	0	2z3**	-z3	-1	0	-z3	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	1	1	1	1	0	0	0	-3	0	0	0	0	0	-1	:	++	26	14	10	-2	-1	-1	10	2	2	2	5	1	1	1	-1	0	1	1	-1	-1	0	0	0
-1	1	1	1	z3	0	0	0	i3	0	-i3	0	0	0	-1	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-3	1	-2	0	0	-1	1	0	0	0	1	0	1	0	:	++	0	6	-4	-2	0	0	-4	0	2	-2	-3	-1	2	1	0	1	-1	2	0	-1	0	-1	1
3	0	2	-1	0	0	0	-z3**	2z3**	-1	-z3	-1	0	-z3**	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-1	-1	-1	-1	0	0	0	0	0	0	1	0	0	0	:	++	39	21	-1	5	3	0	1	1	-1	-1	3	-1	-1	-1	0	-1	1	1	1	-1	0	0	1
3	-1	-1	-1	-z3	0	0	-z3**	0	0	0	0	-z3	z3**	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-3	1	1	0	0	0	0	-1	-1	-1	0	0	0	1	:	++	26	28	-10	4	-1	-1	-10	2	4	0	1	-1	-1	1	1	0	-1	-1	-1	1	0	0	0
0	-3	-1	2	0	1	0	0	0	0	0	0	-1	0	0	:	++	27	-21	-5	-5	0	0	-5	3	-1	-1	-3	1	-2	1	0	0	1	-2	0	-1	1	0	0
1	0	-2z3	1	0	0	0	0	-z3**	-z3**	-z3**	0	0	0	z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	-1	-1	z3**	0	0	0	3z3	0	0	0	1	0	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-3	0	-2	1	0	-1	0	1	0	0	0	1	0	1	0	:	++	12	0	-4	0	3	0	-4	-4	0	0	0	2	-1	0	0	1	2	-1	-1	0	-1	0	1
-3	0	2	-1	0	0	1	0	0	0	0	0	-1	0	0	:	++	39	-27	-5	-3	3	0	-5	-1	-3	1	0	-2	1	0	0	0	-2	1	-1	0	0	1	0
1	3	-1	-1	0	0	0	0	-3	0	0	0	0	0	1	:	++	26	-14	10	2	-1	-1	-10	-2	2	2	-5	1	1	-1	1	0	-1	-1	1	-1	0	0	0
-3	1	-1	-1	1	0	0	0	2	-1	-1	0	0	0	0	:	++	26	-14	10	2	-1	-1	10	2	-2	-2	-5	1	1	-1	1	0	1	1	-1	1	0	0	0
-3	1	-1	-1	z3**	0	0	0	2z3	-z3	-z3	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	0	0	0	0	0	0	-1	1	1	1	-1	0	-1	1	:	++	0	0	16	0	0	0	16	0	0	0	0	-2	-2	0	0	1	-2	-2	0	0	0	0	1
-2	0	0	0	0	0	0	-z3	1	z3	z3**	-1	0	-z3	1	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	-2z3	1	0	0	0	-1	3z3**	0	0	0	0	1	z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-3	i3	0	0	0	0	z3	0	0	0	1	-z3**	z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-3	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	:	++	26	-28	-10	-4	-1	-1	10	-2	4	0	-1	-1	-1	-1	-1	0	1	1	1	1	0	0	0
0	0	0	0	0	0	1	-z3	0	0	0	0	z3	z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	0	2z3	-1	0	0	0	0	z3-1	0	i3	0	0	0	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	-2	1	1	0	0	-1	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r10
0	0	0	0	0	0	0	z3**	-2	z3**	z3	0	0	-z3**	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	i3	0	-z3**	0	0	0	0	0	0	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	-1	-1	0	0	0	0	:	++	0	56	0	-8	0	0	0	0	0	0	2	0	0	-2	2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	-z3**	-z3	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3	1	1	0	0	0	0	0	0	0	0	0	0	0	:	++	26	14	10	-2	-1	-1	-10	-2	-2	-2	5	1	1	1	-1	0	-1	-1	1	1	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r7	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	:	++	0	64	0	0	0	0	0	0	0	0	-8	0	0	0	-2	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	3	0	0	1	0	0	-1	:	++	0	0	16	0	0	0	-16	0	0	0	0	-2	-2	0	0	1	2	2	0	0	0	0	-1
0	0	0	0	0	b13	0	0	0	0	0	0	0	0	0	:	++2	32	0	0	0	-4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b13	0	0
0	0	0	0	0	1	-1	0	0	0	0	-1	0	0	0	:	++	27	27	-9	3	0	0	9	-3	-3	1	0	0	0	0	0	1	0	0	0	0	1	-1	-1

$S_6(3)$

	1A	2A	2B	3AB	3C	3D	3EF	3G	4A	4B	4C	5A	6AB	6CD	6EF	6GH	6I	6J	6KL	6M	6NO	6P	7A	8A	8B	9AB	9CD	9EF	9GH	10A	12AB	12C	12DE	12FG
ind	1	2	4	3	3	3	3	3	4	4	4	5	6	6	6	6	6	6	6	12	6	12	7	8	8	9	9	9	9	10	12	12	12	12
	2	2		6	6	6	6	6	4		4	10	6	6	6	6	6	6	6		6		14	8		18	18	18	18	10	12	12	12	12
o2	14	-6	0	-9z3-4	-4	5	-b27	2	4	0	2	-1	z3-4	3z3	-2i3	-i3	0	-3	-z3+1	0	z3-1	0	0	-2	0	-b27**	2	-1	-z3**	-1	b27**	-2	-z3	-z3-2
-	182	18	0	-61	20	20	-7	2	20	0	2	2	21	-9	-6	-6	0	0	3	0	3	0	0	2	0	-7	2	2	-1	-2	-7	2	-1	3
o2	182	18	0	-9z3+56	20	20	-3z3+5	2	20	0	2	2	9z3+24	-9z3	6	6	0	0	-3z3**	0	-3z3**	0	0	2	0	3z3+8	2	2	-z3**	-2	3z3+8	2	-z3	-3z3
o2	364	4	0	90z3+4	-14	13	b27	4	24	0	-4	-1	30z3+4	3i3+13	-2	3i3-2	-2	1	-3z3-5	0	b27	0	0	0	0	-3i3-5	b27**	b27**	z3**	-1	6	0	2z3	-2i3
-	520	-40	0	-20	-38	-2	7	4	0	0	8	0	20	-4	2	2	-10	2	5	0	-1	0	2	0	0	-2	-5	1	1	0	0	0	-4	0
o2	546	-74	0	81z3+60	6	33	6	6	-4	0	6	1	-9z3-20	-3z3-8	4b27	-3i3-2	-2	-11	-2	0	-2	0	0	-2	0	-3z3	-3z3	-3z3	0	1	-b27**	2	-3z3	z3+2
o2	910	-70	0	90i3-35	-26	37	3i3-8	-2	20	0	2	0	10i3-25	11	8i3+2	i3+2	2	-7	-i3+2	0	i3+2	0	0	-2	0	-3z3-5	3z3+4	b27	z3	0	3i3+2	-4	-1	-i3
-	1092	-52	0	174	30	-6	-15	6	40	0	4	2	26	2	-10	8	2	2	-1	0	-1	0	0	0	0	3	-3	0	0	-2	4	-8	-2	0
o2	1092	12	0	135i3-15	-60	30	3b27	0	56	0	4	-3	5i3-45	-6z3	-2i3	8i3	0	6	-z3+1	0	z3-1	0	0	0	0	3b27**	0	-3	0	-3	-4b27**	2	-2z3	0
o2	1092	-52	0	27i3-177	48	3	3b27**	0	40	0	4	2	13i3+17	3i3+11	2i3+8	2z3-9	-4	-1	z3-2	0	-z3	0	0	0	0	3z3**	3i3	-3z3	0	-2	-3i3-5	-2	-2z3**	-i3-3
o2	1260	-60	0	-90b27**	-36	45	-9z3**	0	40	0	-4	0	-15i3-45	-9i3-15	6i3	3i3	0	-3	-3z3**	0	3z3**	0	0	0	0	-9z3	0	0	0	0	3i3-5	-2	2z3	i3-3
o2	1820	20	0	-90i3+200	-16	11	-3i3-16	2	40	0	-4	0	30i3+20	-12i3+2	2	-3i3+2	-4	-1	3i3-4	0	-1	0	0	0	0	3z3+8	-3z3-4	-3z3-4	-z3	0	3i3-5	-2	2	-i3+3
-	2730	110	0	165	-42	48	-24	0	-20	0	6	0	35	-7	8	-10	-10	-4	-4	0	2	0	0	2	0	3	0	-3	0	0	7	-2	-3	-3
-	2730	-50	0	-375	48	12	3	6	60	0	-2	0	55	-23	10	-8	4	4	1	0	1	0	0	-2	0	-6	-3	0	0	0	-3	-6	1	3
o2	2730	110	0	-135z3-240	-6	66	-9i3+3	6	-20	0	6	0	-25z3	-15z3+8	4z3-8	4i3+8	2	2	2z3**	0	2z3	0	0	2	0	3z3	-3z3**	-3i3	0	0	-3z3-8	-2	-3z3	3z3
o2	2730	-50	0	135z3+435	66	21	-9z3+12	0	60	0	-2	0	-55z3+5	15z3+7	2i3-8	i3+10	-2	1	z3	0	-z3+2	0	0	-2	0	-6z3	-3i3	-3z3**	0	0	-3b27	0	z3**	z3-1
-	3120	80	0	-120	96	-12	-12	6	0	0	0	0	-40	8	-4	-4	-16	-4	8	0	2	0	-2	0	0	6	-3	-3	0	0	0	0	0	0
o2	3276	36	0	135i3-369	36	36	9z3	0	56	0	-4	1	45i3+21	-18z3**	-6	-6	0	0	3z3	0	3z3	0	0	0	0	9z3**	0	0	0	1	-4b27	2	2z3**	0
o2	3640	40	0	90i3+130	94	4	-21z3-2	-2	0	0	8	0	-10i3-50	-6i3-14	-2i3+4	-8z3	-14	4	5z3+2	0	z3	0	0	0	0	-3i3-5	-2	1	z3	0	0	0	-4z3**	0
-	4368	112	0	-384	-24	66	-6	0	0	0	0	-2	64	-32	-8	10	-8	-2	-2	0	4	0	0	0	0	-6	0	-3	0	2	0	0	0	0
o2	4368	112	0	108i3+372	12	84	9i3+21	6	0	0	0	-2	-56z3	-8b27**	-4z3+8	-4i3-8	4	4	-2z3**	0	-2z3	0	0	0	0	-6z3	-3z3**	-3i3	0	2	0	0	0	0
o2	4914	-186	0	-81z3+135	54	54	-27z3	0	-36	0	-2	-1	9z3-15	27z3+3	-6	-6	-6	-6	3z3	0	3z3	0	0	2	0	0	0	0	0	-1	-9z3**	0	z3**	-3z3**
-	5460	-260	0	-210	6	60	-21	6	-40	0	4	0	10	10	10	10	10	-8	1	0	1	0	0	0	0	-3	-3	-3	0	0	-4	8	-2	0
o2	5460	60	0	-270i3+60	-120	-3	-9i3+6	-6	40	0	4	0	10i3	-12i3+6	2i3	-5i3	-12	-3	-i3	0	i3	0	0	0	0	-3b27	3	0	0	0	-3i3-5	-2	-2	i3+3
-	5824	64	0	-656	100	46	-8	-8	64	0	0	-1	64	-8	-8	-8	4	-2	4	0	-2	0	0	0	0	1	1	1	1	-1	-8	4	0	0
o2	5824	64	0	-576z3-80	-44	-26	12i3-8	4	64	0	0	-1	64z3	-8	-8z3**	-8z3	4	-2	4z3**	0	-2z3	0	0	0	0	1	b27**	-3z3+1	z3	-1	-8	4	0	0
-	7280	80	0	800	98	44	-10	-10	80	0	0	0	80	8	8	8	2	-4	2	0	-4	0	0	0	0	-1	-1	-1	-1	0	8	2	0	0
o2	7280	80	0	-360i3-280	-46	-28	15i3+17	2	80	0	0	0	80z3**	8	8z3	8z3**	2	-4	2z3	0	-4z3**	0	0	0	0	-1	3z3-1	-b27**	-z3	0	8	2	0	0
o2	8190	-150	0	540z3-45	-72	9	-18z3**	0	20	0	-6	0	30i3+15	3	-12z3**	3i3+6	0	-3	-6z3**	0	0	0	0	2	0	9z3	0	0	0	0	-3i3+2	-4	3	i3
o2	9828	108	0	405i3-135	-54	27	27z3	0	24	0	4	3	15i3-45	18z3**	-6i3	-3i3	6	3	-3z3	0	3z3	0	0	0	0	0	0	0	0	3	6	0	-2z3**	2i3
o2	10206	-54	0	-729z3	0	0	0	0	36	0	-6	1	81z3	27z3	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	9z3	0	3z3	3z3
o2	10920	-200	0	-270i3+390	-42	12	3b27	-6	0	0	8	0	30i3+10	4b27**	-6i3-8	-8	10	4	b27	0	-3z3+1	0	0	0	0	-6z3	3z3	3z3	0	0	0	0	-4z3	0
-	11648	-128	0	416	-88	-52	-16	8	0	0	0	-2	64	16	-8	-8	-8	4	4	0	-2	0	0	0	0	2	-1	5	-1	2	0	0	0	0
o2	11648	-128	0	288i3-448	56	20	-12i3-16	-4	0	0	0	-2	64z3	16	-8z3**	-8z3	-8	4	4z3**	0	-2z3	0	0	0	0	2	-b27**	3z3+5	-z3	2	0	0	0	0
-	14040	-120	0	-540	-54	-54	27	0	0	0	8	0	60	-12	6	6	6	6	-3	0	-3	0	-2	0	0	0	0	0	0	0	0	0	-4	0
-	14742	18	0	-81	-162	0	0	0	-60	0	-6	2	81	-9	0	0	-18	0	0	0	0	0	0	-2	0	0	0	0	0	-2	3	6	3	-3
o2	14742	18	0	729z3+405	0	81	0	0	-60	0	-6	2	-9z3-45	-9z3**	0	-9i3	0	9	0	0	0	0	0	-2	0	0	0	0	0	-2	3b27	0	3z3**	-z3+1
-2	16640	0	0	-640	80	-64	8	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	8	2	-4	-1	0	0	0	0	0
-	17472	192	0	-240	12	-6	-24	0	-64	0	0	-3	0	-24	0	0	12	-6	0	0	0	0	0	0	0	3	0	6	0	-3	8	-4	0	0
-2	17920	0	0	640	64	-80	-8	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	4	-2	1	0	0	0	0	0
-	21840	240	0	240	6	-12	24	-6	-80	0	0	0	0	24	0	0	6	-12	0	0	0	0	0	0	0	-3	-6	0	0	0	-8	-2	0	0



Symplectic group $S_6(3) \cong C_3(3)$

Order = 4,585,351,680 = $2^9.3^9.5.7.13$ Mult = 2 Out = 2

Constructions

Symplectic $Sp_6(3) \cong 2.G$: all 6 x 6 matrices over F_3 preserving a non-singular symplectic form;

$PSp_6(3) \cong S_6(3) \cong G$

Complex The automorphism group of the extraspecial group 3_+^{1+6} is $3^6{:}2S_6(3).2$. A faithful 27-dimensional representation of 3_+^{1+6} extends to a representation of $3_+^{1+6}{:}2S_6(3).2$ in which an involution in the normal subgroup $3_+^{1+6}.2$ has eigenspaces of dimensions 13 and 14. This leads to constructions of G and 2.G respectively as 13- and 14-dimensional groups of complex matrices.

$S_6(3)$

12H	12I	12JK	12L	12MN	13AB	14A	15AB	18AB	18CD	18E	20A	24AB	30AB	36AB	fus	ind	2C	4D	4E	4F	6Q	6R	8C	8D	8E	8F	12N	12O	12P	12Q	12R	20B	24C	24D	24E	24F	26AB	28AB	40AB
12	12	12	12	12	13	28	15	18	18	18	20	24	30	36	fus	ind	2	8	4	8	6	6	8	8	8	8	24	12	12	24	24	20	24	24	24	24	26	56	40
12	12		12	12	26	28	30	18	18	18	20	24	30	36				8		8			8	8	8		24			24	24	20	24	24	24	24	26	56	40
1	2	0	-i3	-z3**	1	0	-z3**	-z3-2	0	-i3	-1	z3	-z3**	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	0	0	-1	0	0	-1	-3	0	0	0	-1	1	-1	:	oo	0	7i2	0	3i2	0	0	-i2	-i2	2i2	0	-2i2	0	0	0	i2	0	-i2	-i2	-i2	-i2	0	0	-i2
2	2	0	0	-z3**	0	0	-z3**	3z3	0	0	0	-z3	z3**	-z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2	0	-i3	-z3**	0	0	-1	-2z3	z3	z3	-1	0	-1	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	-1	0	0	0	2	-1	-1	0	0	0	0	:	oo	0	6i2	0	-2i2	0	0	10i2	2i2	0	0	-3i2	0	0	i2	0	0	i2	i2	-i2	0	0	-i2	0
-1	0	0	i3	0	0	0	z3	z3	z3	z3	1	z3	z3	-z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	2	0	i3	-1	0	0	0	-b27	-z3	-z3**	0	1	0	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	-2	0	0	1	0	0	-1	-1	-1	2	0	0	1	1	:	oo	0	14i2	0	-2i2	0	0	4i2	0	0	0	5i2	0	0	i2	-i2	0	i2	-2i2	0	0	0	0	-i2
2	-2	0	0	z3**	0	0	0	-z3-2	0	-i3	1	0	0	-z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-2	0	i3	z3	0	0	-z3	-z3**	-1	-z3	0	0	z3	z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	0	-i3	-z3**	-1	0	0	3z3	0	0	0	0	0	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	0	i3	-1	0	0	0	-z3	-z3	-z3	0	0	0	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	0	0	0	0	0	0	0	-1	2	-1	0	-1	0	1	:	oo	0	21i2	0	i2	0	0	-5i2	-i2	-2i2	0	3i2	0	0	i2	0	0	-2i2	i2	-i2	i2	0	0	0
0	-2	0	0	1	0	0	0	-2	1	-2	0	1	0	0	:	oo	0	7i2	0	3i2	0	0	-5i2	3i2	-2i2	0	-2i2	0	0	0	i2	0	i2	-2i2	0	i2	0	0	0
-2	0	0	0	0	0	0	0	-z3	-z3**	-1	0	-z3	0	z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-2	0	i3	z3	0	0	0	-2z3	1	z3**	0	z3**	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	-1	-1	0	0	0	0	:	oo	0	20i2	0	4i2	0	0	0	0	0	0	2i2	0	0	-2i2	-i2	0	0	0	0	0	0	-i2	0
2	2	0	0	-z3	0	0	1-3z3**	0	0	1	0	1	-z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	-z3	0	0	0	-2z3	-2z3**	1	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	-2	-2	1	0	0	-1	0	:	oo	0	0	0	0	0	0	4i2	4i2	0	0	0	0	0	0	0	0	-2i2	i2	i2	0	0	0	-i2
0	0	0	0	0	0	0	z3	-2z3	z3**	1	0	0	-z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-2	0	0	z3	0	0	-z3	0	0	0	-1	-z3**	-z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-2	0	0	1	0	0	0	1	1	1	0	0	0	-1	:	oo	0	28i2	0	4i2	0	0	10i2	-2i2	0	0	i2	0	0	i2	i2	0	i2	i2	i2	0	0	0	0
1	-2	0	-i3	1	0	0	0	-z3+1	-i3	0	0	0	0	z3**	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	0	0	0	0	0	0	-1	1	1	1	-1	0	-1	1	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i5	0	0	0	0	0	0	i5
-2	0	0	0	0	0	0	-z3	1	z3	z3**	-1	0	-z3	1	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-4	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	-1	:	oo	0	28i2	0	-4i2	0	0	0	0	-4i2	0	i2	0	0	-i2	i2	0	0	0	0	-i2	0	0	0
-4	0	0	0	0	0	0	0	-1	-z3**	-z3	0	0	0	-1	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	0	0	-i3	0	0	0	0	-3z3	0	0	0	-1	0	-z3	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-2	0	i3	z3	0	0	0	0	0	0	-1	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	z3	0	0	0	1	-z3	z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	-z3**	0	0	0	-2z3	z3	z3	0	0	0	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	-2	1	1	0	0	-1	0	:	oo	0	0	0	0	0	0	16i2	0	0	0	0	0	0	0	0	0	-2i2	-2i2	0	0	0	0	i2
0	0	0	0	0	0	0	z3	-2	z3	z3**	0	0	-z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	-1	0	0	0	0	0	0	0	0	0	0	:	oo	0	-6i2	0	2i2	0	0	10i2	2i2	0	0	3i2	0	0	-i2	0	0	i2	i2	-i2	0	0	i2	0
0	0	0	0	0	0	0	-1	0	0	0	0	1	1	0	:	oo	0	21i2	0	i2	0	0	-9i2	3i2	2i2	0	3i2	0	0	i2	0	0	0	0	0	-i2	0	0	i2
-3	0	0	-i3	0	0	0	-z3	0	0	0	0	z3**	z3	0	*	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	r7	0	0	0	0	0	0	0	0	:	oo2	0	32i2	0	0	0	0	0	0	0	0	-4i2	0	0	0	-i2	0	0	0	0	0	0	u56	0
2	0	0	0	0	0	0	0	3	0	0	1	0	0	-1	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i5	0	0	0	0	0	0	-i5
0	0	0	0	0	b13	0	0	0	0	0	0	0	0	0	:	oo2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	k13	0	0
4	0	0	0	0	0	0	0	-3	0	0	0	0	0	1	:	oo	0	28i2	0	-4i2	0	0	0	0	4i2	0	i2	0	0	-i2	i2	0	0	0	0	i2	0	0	0

Maximal subgroups

Order	Index	Structure	G.2	Character
12597120	364	$3_+^{1+4}:2U_4(2)$	$: 3_+^{1+4}:2U_4(2).2$	1a+168a+195a
4094064	1120	$3^6:L_3(3)$	$: 3^6:(L_3(3)x2)$	1a+105a+195a+819a
1259712	3640	$3^{3+4}:2(S_4xA_4)$	$: 3^{3+4}:2(S_4xS_4)$	
622080	7371	$2(A_4xU_4(2))$	$: 2(A_4xU_4(2)).2$	
41472	110565	$2^{2+6}:3^3:S_3$	$: 2^{2+6}:3^3:D_{12}$	
29484	155520	$L_2(27):3$	$: L_2(27):6$	
24192	189540	$(2xU_3(3))'2$	$: (4xU_3(3))'2$	
11232	408240	$L_3(3):2$	$: L_3(3):2 \times 2$	
1092	4199040	$L_2(13)$		
1092	4199040	$L_2(13)$		
60	76422528	A_5	$: S_5$	

Specifications

Abstract	Symplectic
N(3AB)	point
	isotropic plane
	isotropic line
N(2A)	non-isotropic line
$N(2A^2)$	$S_2(3)wrS_3$
	$S_2(27)$
N(2B)	$U_3(3)$
C(2C)	
N(2B,3G,5A)	

$G_2(5)$

Chevalley group $G_2(5)$

Order = 5,859,000,000 = $2^6.3^3.5^6.7.31$ Mult = 1 Out = 1

Constructions

Chevalley $G_2(5) \cong G$: adjoint Chevalley group of type G_2 over $\mathbf{F}_5$;

the automorphism group of a generalized hexagon of order (5,5), consisting of 3906 "vertices" and 3906 "edges", each object being incident with 6 of the other type;

Cayley G : the automorphism group of the Cayley algebra modulo 5, consisting of vectors $\Sigma a_n i_n$, with a_n in $\mathbf{F}_5$ (subscripts modulo 7), where n runs over $\{\infty,0,\ldots,6\}$ and $i_\infty=1$, $i_{n+1} \to i$, $i_{n+2} \to j$, $i_{n+4} \to k$ form a quaternion subalgebra.

Remark : $G_2(5)$ is a maximal subgroup of the Lyons group.

Maximal subgroups / Specifications

Order	Index	Structure	Character	Abstract	Chevalley	Cayley
1500000	3906	$5_+^{1+4}:GL_2(5)$	1a+930a+1085a+1890a	N(5A)	vertex	null subalgebra
1500000	3906	$5^{2+3}:GL_2(5)$	1a+930a+1085b+1890a	N(5A^2)	edge	isotropic point
756000	7750	$3{\cdot}U_3(5):2$		N(3A)	$^2A_2(5)$	minus point
744000	7875	$L_3(5):2$			$A_2(5)$	plus point
14400	406875	$2{\cdot}(A_5 \times A_5).2$		N(2A)	$D_2(5)$	quaternion subalgebra
12096	484375	$U_3(3):2$				$G_2(\mathbf{Z})$
1344	4359375	$2^3{\cdot}L_3(2)$		N(2A^3)		base

G 44

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	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		5859000000	14400	378000	720	480	480	375000	15000	3750	1875	1250	720	720	36	21	24	24	600	600	50	50
	p power		A	A	A	A	A	A	A	A	A	A	AA	BA	BA	A	A	B	AA	BA	CA	EA
	p' part		A	A	A	A	A	A	A	A	A	A	AA	BA	BA	A	A	A	AA	BA	CA	EA
	ind	1A	2A	3A	3B	4A	4B	5A	5B	5C	5D	5E	6A	6B	6C	7A	8A	8B	10A	10B	10C	10D
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	+	124	4	-20	1	0	4	-1	4	9	-6	-1	4	1	1	-2	0	-2	-1	4	-1	-1
χ_3	+	280	-8	28	4	0	0	30	-10	0	-5	5	4	4	-2	0	0	0	2	2	2	-3
χ_4	+	651	11	21	-3	-5	-5	26	26	1	1	1	5	5	-1	0	-1	-1	6	6	1	1
χ_5	+	930	18	48	0	6	6	55	15	0	-5	5	0	0	0	-1	0	0	3	3	-2	3
χ_6	o	960	0	-48	0	0	0	-40	0	-10	-5	10	0	0	0	1	0	0	0	0	0	0
χ_7	o	960	0	-48	0	0	0	-40	0	-10	-5	10	0	0	0	1	0	0	0	0	0	0
χ_8	+	1085	5	77	-1	1	5	-40	5	0	-10	10	5	-1	-1	0	1	-1	0	5	0	0
χ_9	+	1085	5	-49	5	5	1	85	0	-10	-5	10	-1	5	-1	0	-1	1	5	0	0	0
χ_{10}	+	1240	-8	-20	4	0	0	-10	-10	15	15	-10	4	4	-2	1	0	0	2	2	-3	2
χ_{11}	+	1890	18	0	0	6	6	15	15	15	15	-10	0	0	0	0	0	0	3	3	3	-2
χ_{12}	+	2480	-16	104	-4	0	0	-20	-20	5	5	5	8	-4	2	2	0	0	4	4	-1	-1
χ_{13}	+	2604	20	84	-3	0	4	-21	-16	14	-1	4	-4	5	-1	0	0	2	-5	0	0	0
χ_{14}	+	2604	-12	21	0	4	0	104	-21	4	4	4	-3	0	0	0	-2	0	8	3	-2	-2
χ_{15}	+	2604	20	21	0	4	0	104	-21	4	4	4	5	8	2	0	2	0	0	-5	0	0
χ_{16}	+	3255	31	-21	9	-1	-5	130	5	5	5	5	-5	1	1	0	1	-1	6	1	1	1
χ_{17}	+	3255	31	105	3	-5	-1	5	10	15	0	5	1	-5	1	0	-1	1	1	6	1	1
χ_{18}	+	3906	-30	126	0	-2	6	31	36	16	1	6	6	0	0	0	0	0	-5	0	0	0
χ_{19}	+	3906	-30	0	6	6	-2	156	31	6	6	6	0	6	0	0	0	0	0	-5	0	0
χ_{20}	+	10416	-16	84	0	0	0	-84	36	-19	-4	-9	-4	8	2	0	0	0	4	4	-1	-1
χ_{21}	+	12096	0	0	0	0	0	96	-24	6	-9	-4	0	0	0	0	0	0	0	0	0	0
χ_{22}	+	12096	0	0	0	0	0	96	-24	6	-9	-4	0	0	0	0	0	0	0	0	0	0
χ_{23}	+	12096	0	0	0	0	0	96	-24	6	-9	-4	0	0	0	0	0	0	0	0	0	0
χ_{24}	+	12096	0	0	0	0	0	96	-24	6	-9	-4	0	0	0	0	0	0	0	0	0	0
χ_{25}	+	12096	0	0	0	0	0	96	-24	6	-9	-4	0	0	0	0	0	0	0	0	0	0
χ_{26}	+	13020	4	-84	9	0	4	-105	20	-5	-5	-5	4	1	1	0	0	2	-1	4	-1	-1
χ_{27}	+	13020	36	105	0	4	0	20	15	-15	0	-5	9	0	0	0	-2	0	-4	-9	1	1
χ_{28}	+	13020	4	105	0	4	0	20	15	-15	0	-5	1	-8	-2	0	2	0	4	-1	-1	-1
χ_{29}	+	15500	20	20	5	0	4	-125	0	0	0	0	-4	5	-1	2	0	-2	-5	0	0	0
χ_{30}	+	15624	-24	-252	0	0	0	124	-6	-11	4	-1	12	0	0	0	0	0	-4	6	1	1
χ_{31}	+	15624	24	126	0	-8	0	124	-6	-11	4	-1	6	0	0	0	0	0	4	-6	-1	-1
χ_{32}	+	15624	-24	126	0	0	0	124	-6	-11	4	-1	-6	0	0	0	0	0	-4	6	1	1
χ_{33}	+	15624	-24	126	0	0	0	124	-6	-11	4	-1	-6	0	0	0	0	0	-4	6	1	1
χ_{34}	+	15624	-24	0	-12	0	0	-126	4	9	-6	-1	0	12	0	0	0	0	6	-4	1	1
χ_{35}	+	15624	24	0	6	0	-8	-126	4	9	-6	-1	0	6	0	0	0	0	-6	4	-1	-1
χ_{36}	+	15624	-24	0	6	0	0	-126	4	9	-6	-1	0	-6	0	0	0	0	6	-4	1	1
χ_{37}	+	15624	-24	0	6	0	0	-126	4	9	-6	-1	0	-6	0	0	0	0	6	-4	1	1
χ_{38}	+	15625	25	-125	-5	5	5	0	0	0	0	0	-5	-5	1	1	-1	-1	0	0	0	0
χ_{39}	+	16275	35	-105	-9	-1	-1	25	25	0	0	0	-1	-1	-1	0	1	1	5	5	0	0
χ_{40}	+	17856	0	-288	0	0	0	-144	-24	-4	11	6	0	0	0	-1	0	0	0	0	0	0
χ_{41}	+	17856	0	144	0	0	0	-144	-24	-4	11	6	0	0	0	-1	0	0	0	0	0	0
χ_{42}	+	17856	0	144	0	0	0	-144	-24	-4	11	6	0	0	0	-1	0	0	0	0	0	0
χ_{43}	+	19530	-6	-126	0	-10	6	155	30	5	5	5	-6	0	0	0	0	0	-1	-6	-1	-1
χ_{44}	+	19530	-6	0	-6	6	-10	30	35	15	0	5	0	-6	0	0	0	0	-6	-1	-1	-1

	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
	24	24	750	75	75	75	30	20	20	21	21	24	24	24	24	25	30	30	31	31	31	31	31	
p power	AA	BB	AA	CA	DA	DA	BB	BA	AB	AA	AA	AA	AA	BB	BB	A	AAA	EBB	A	A	A	A	A	
p' part	AA	BB	AA	CA	DA	DA	BB	BA	AB	AA	AA	AA	AA	BB	BB	A	AAA	EBB	A	A	A	A	A	
ind	12A	12B	15A	15B	15C	D**	15E	20A	20B	21A	B*	24A	B*	24C	D*	25A	30A	30B	31A	B*2	C*4	D*8	E*16	
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	0	1	5	0	0	0	1	0	-1	1	1	0	0	1	1	-1	-1	1	0	0	0	0	0	χ_2
χ_3	0	0	3	3	-2	-2	-1	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	1	1	χ_3
χ_4	1	1	-4	1	1	1	2	0	0	0	0	-1	-1	-1	-1	1	0	0	0	0	0	0	C	χ_4
χ_5	0	0	-2	3	-2	-2	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_5
χ_6	0	0	2	2	5z3+2	**	0	0	0	1	1	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	χ_6
χ_7	0	0	2	2	**	5z3+2	0	0	0	1	1	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	χ_7
χ_8	1	-1	2	-3	2	2	-1	1	0	0	0	1	1	-1	-1	0	0	-1	0	0	0	0	0	χ_8
χ_9	-1	1	1	-4	1	1	0	0	1	0	0	-1	-1	1	1	0	-1	0	0	0	0	0	0	χ_9
χ_{10}	0	0	5	0	0	0	-1	0	0	1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	χ_{10}
χ_{11}	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	χ_{11}
χ_{12}	0	0	4	-1	-1	-1	1	0	0	-1	-1	0	0	0	0	0	-2	1	0	0	0	0	0	χ_{12}
χ_{13}	0	1	9	-1	-1	-1	2	0	-1	0	0	0	0	-1	-1	-1	1	0	0	0	0	0	0	χ_{13}
χ_{14}	1	0	-4	1	1	1	0	-1	0	0	0	1	1	0	0	-1	2	0	0	0	0	0	0	χ_{14}
χ_{15}	1	0	-4	1	1	1	0	-1	0	0	0	-1	-1	0	0	-1	0	-2	0	0	0	0	0	χ_{15}
χ_{16}	-1	1	4	-1	-1	-1	-1	-1	0	0	0	1	1	-1	-1	0	0	1	0	0	0	0	0	χ_{16}
χ_{17}	1	-1	5	0	0	0	-2	0	-1	0	0	-1	-1	1	1	0	1	0	0	0	0	0	0	χ_{17}
χ_{18}	-2	0	1	1	1	1	0	-2	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	χ_{18}
χ_{19}	0	-2	0	0	0	0	1	1	-2	0	0	0	0	0	0	1	0	1	0	0	0	0	0	χ_{19}
χ_{20}	0	0	9	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	-2	0	0	0	0	0	χ_{20}
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	e31	*2	*4	*8	*16	χ_{21}
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	*16	e31	*2	*4	*8	χ_{22}
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	*8	*16	e31	*2	*4	χ_{23}
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	*4	*8	*16	e31	*2	χ_{24}
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	*2	*4	*8	*16	e31	χ_{25}
χ_{26}	0	1	-9	1	1	1	-1	0	-1	0	0	0	0	-1	-1	0	-1	1	0	0	0	0	0	χ_{26}
χ_{27}	1	0	5	0	0	0	0	-1	0	0	0	1	1	0	0	0	-1	0	0	0	0	0	0	χ_{27}
χ_{28}	1	0	5	0	0	0	0	-1	0	0	0	-1	-1	0	0	0	1	2	0	0	0	0	0	χ_{28}
χ_{29}	0	1	-5	0	0	0	0	0	-1	-1	-1	0	0	1	1	0	1	0	0	0	0	0	0	χ_{29}
χ_{30}	0	0	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	-1	2	0	0	0	0	0	0	χ_{30}
χ_{31}	-2	0	1	1	1	1	0	2	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	χ_{31}
χ_{32}	0	0	1	1	1	1	0	0	0	0	0	r6	-r6	0	0	-1	-1	0	0	0	0	0	0	χ_{32}
χ_{33}	0	0	1	1	1	1	0	0	0	0	0	-r6	r6	0	0	-1	-1	0	0	0	0	0	0	χ_{33}
χ_{34}	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	-1	0	2	0	0	0	0	0	χ_{34}
χ_{35}	0	-2	0	0	0	0	1	0	2	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	χ_{35}
χ_{36}	0	0	0	0	0	0	1	0	0	0	0	0	0	r6	-r6	-1	0	-1	0	0	0	0	0	χ_{36}
χ_{37}	0	0	0	0	0	0	1	0	0	0	0	0	0	-r6	r6	-1	0	-1	0	0	0	0	0	χ_{37}
χ_{38}	-1	-1	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	0	0	0	1	1	1	1	1	χ_{38}
χ_{39}	-1	-1	-5	0	0	0	1	-1	-1	0	0	1	1	1	1	0	-1	-1	0	0	0	0	0	χ_{39}
χ_{40}	0	0	12	2	2	2	0	0	0	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	χ_{40}
χ_{41}	0	0	-6	-1	-1	-1	0	0	0	-b21	*	0	0	0	0	1	0	0	0	0	0	0	0	χ_{41}
χ_{42}	0	0	-6	-1	-1	-1	0	0	0	*	-b21	0	0	0	0	1	0	0	0	0	0	0	0	χ_{42}
χ_{43}	2	0	-1	-1	-1	-1	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	χ_{43}
χ_{44}	0	2	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	χ_{44}

$U_6(2)$

Unitary group $U_6(2) \cong {}^2A_5(2)$

Order = 9,196,830,720 = $2^{15}.3^6.5.7.11$ Mult = $2^2 \times 3$ Out = S_3

Constructions

Unitary $GU_6(2) \cong 3.G.3$: all 6 x 6 matrices over $\mathbb{F}_4$ preserving a non-singular Hermitian form;

$PGU_6(2) \cong G.3$; $SU_6(2) \cong 3.G$; $PSU_6(2) \cong G$

the transvections $x \longrightarrow x + (x.v)v$ (v isotropic) are the Fischer transpositions (see below).

Graphs $G.S_3$: the automorphism group of a rank 3 graph A of valence 176 on 672 points, a rank 3 graph B of valence 180 on 693 points, and a rank 4 graph C of valence 42 on 891 points. In the unitary geometry the vertices of the graphs A and B are the non-isotropic and isotropic points respectively, and are joined if they are orthogonal. The graph C is described under the Leech lattice construction below.

Leech G : the pointwise stabilizer of a triangle ABC of type 222 in the Leech lattice (see Co_1). $G.S_3$: the setwise stabilizer. There are 891 lattice points V for which VA, VB, VC all have type 2, which form a 42-valent rank 4 graph when we join orthogonal vectors. Taking $A = (0,0,0,0^{21})$, $B = (4,-4,0,0^{21})$, $C = (4,0,-4,0^{21})$, these vectors fall into four sets: $(0,-4,-4,0^{21})$ (1), $(4,0,0,\pm4,0^{20})$ (42), $(2,-2,-2,\pm2^5 0^{16})$ (336), $(3,-1,-1,\pm1^{21})$ (512), which are the orbits of a point-stabilizer $2^9{:}M_{21}$ (in G).

In the Leech lattice modulo 2, the 3-spaces containing the fixed 2-space (of type 222) become points in the quotient 22-dimensional representation over $\mathbb{F}_2$. They are described below by the types of their non-zero vectors.

Fischer $2Fi_{21} \cong 2.G$: the centralizer of a 3-transposition in Fi_{22} (q.v.);

$Fi_{21} \cong G$ is generated by a conjugacy class of 693 involutions, which are 3-transpositions; i.e. any non-commuting pair has product of order 3. These involutions form the above rank 3 graph of valence 180 when commuting involutions are joined. The automorphism group of this graph is $G.S_3$, and the point stabilizer (in G) is $2^{1+8}U_4(2)$.

Presentation See page 232.

Maximal subgroups

Order	Index	Structure	G.2	G.3	$G.S_3$	Character	Specifications: Abstract	Specifications: Unitary	Specifications: Leech	Graphs
13685760	672	$U_5(2)$	: $U_5(2){:}2$	: $U_5(2) \times 3$	: $(U_5(2)\times3){:}2$	1a+231a+440a	C(3D)	non-isotropic point		point of A
13271040	693	$2_+^{1+8}{:}U_4(2)$	: $2_+^{1+8}{:}U_4(2){:}2$	: $2_+^{1+8}{:}(U_4(2)\times3)$	: $2_+^{1+8}{:}(U_4(2)\times3){:}2$	1a+252a+440a	N(2A)	isotropic point		point of B
10321920	891	$2^9{:}L_3(4)$	: $2^9{:}L_3(4){:}2$	: $2^9{:}L_3(4){:}3$	: $2^9{:}L_3(4){:}S_3$	1a+22a+252a+616a.	N(2^9)	isotropic plane	222-2224-point	point of C
6531840	1408	$U_4(3){:}2_2$	: $U_4(3).(2^2)_{122}$	$3^5{:}S_6$	$3^5{:}(S_6\times2)$	1a+252a+1155a			222-2233-point	
6531840	1408	$U_4(3){:}2_2$				1a+252a+1155b			222-2233-point	
6531840	1408	$U_4(3){:}2_2$				1a+252a+1155c			222-2233-point	
1474560	6237	$2^{4+8}{:}(3\times A_5){:}2$	: $H.2$	: $H.3$	: $H.S_3$			isotropic line		5-point clique in B
1451520	6336	$S_6(2)$	: $S_6(2) \times 2$	$L_2(8){:}3 \times 3$	$L_2(8){:}3 \times S_3$		C(2D)	$S_6(2)$		
1451520	6336	$S_6(2)$					C(2)	$S_6(2)$		
1451520	6336	$S_6(2)$					C(2)	$S_6(2)$		
443520	20736	M_{22}	: $M_{22}{:}2$						222-2334-point	
443520	20736	M_{22}							222-2334-point	
443520	20736	M_{22}							222-2334-point	
155520	59136	$S_3 \times U_4(2)$	: $S_3 \times U_4(2){:}2$	: $H \times 3$	: $(H \times 3).2$		N(3A), C(3E)	non-isotropic line		edge of A, triad in B
93312	98560	$3^{1+4}.[2^7.3]$	: $H.2$	: $H.3$	: $H.S_3$		N(3B)	$U_3(2)wr2$	222-3333-point	
40320	228096	$L_3(4){:}2_1$	: $L_3(4){:}2^2$	: $L_3(4){:}6$	: $L_3(4){:}D_{12}$			two isotropic planes	222-3333-point	

$U_6(2)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		9	132	147	36	77	46	1	55	6	1	1	1						1																		
	196830720	71040	456	864	760	656	944	296	144	536	536	536	768	256	30	5184	5184	728	576	288	216	72	72	7	64	32	32	32	54	54	27	10	11	11	864	864	
	p power	A	A	A	A	A	A	A	A	B	B	B	B	B	A	BA	BA	AA	BB	AB	CA	CB	CC	A	B	C	D	E	B	B	B	AA	A	A	BA	AA	
	p' part	A	A	A	A	A	A	A	A	A	A	A	A	A	A	BA	BA	AA	BB	AB	CA	CB	CC	A	A	A	A	A	A	A	A	AA	A	A	BA	BA	
	ind	1A	2A	2B	2C	3A	3B	3C	4A	4B	4C	4D	4E	4F	4G	5A	6A	B**	6C	6D	6E	5F	6G	6H	7A	8A	8B	8C	8D	9A	B**	9C	10A	11A	B**	12A	B**	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	22	-10	6	-2	4	-5	4	6	-2	2	2	2	-2	2	2	-1	-1	-4	3	0	2	0	-2	1	2	0	0	0	1	1	1	0	0	0	-3	-3	χ_2
χ_3	+	231	39	7	-9	6	15	6	23	7	-1	-1	-1	-1	-1	1	3	3	6	7	-2	0	-2	0	0	3	-1	-1	-1	0	0	0	-1	0	0	5	5	χ_3
χ_4	+	252	60	28	12	9	9	9	12	-4	4	4	4	4	4	2	-3	-3	9	1	1	3	1	3	0	0	0	0	0	0	0	0	0	-1	-1	3	3	χ_4
χ_5	+	385	-95	17	-7	25	7	-2	1	9	5	5	5	-7	-3	0	-5	-5	1	-1	5	-2	2	2	0	1	-1	-1	-1	1	1	1	0	0	0	1	1	χ_5
χ_6	+	440	120	24	8	35	8	-1	24	8	0	0	0	8	0	0	12	12	3	0	3	3	3	-1	-1	0	0	0	0	2	2	-1	0	0	0	6	6	χ_6
χ_7	-	560	-80	-16	16	20	20	2	16	-16	0	0	0	0	0	0	-8	-8	4	-4	-4	-2	2	-2	0	0	0	0	0	-1	-1	2	0	-1	-1	-2	-2	χ_7
χ_8	+	616	-24	40	8	-14	-5	13	8	8	8	8	8	0	0	1	3	3	-6	-5	-2	3	1	-1	0	0	0	0	0	-2	-2	-2	1	0	0	-1	-1	χ_8
χ_9	o	770	-30	-14	10	5	-13	5	34	-6	-2	-2	-2	2	-2	0	6i3-3	-6i3-3	-3	7	1	-3	1	1	0	2	0	0	0	-1	-1	-1	0	0	0	-3i3-2	3i3-2	χ_9
χ_{10}	o	770	-30	-14	10	5	-13	5	34	-6	-2	-2	-2	2	-2	0	-6i3-3	6i3-3	-3	7	1	-3	1	1	0	2	0	0	0	-1	-1	-1	0	0	0	3i3-2	-3i3-2	χ_{10}
χ_{11}	+	1155	195	35	19	30	-6	3	-13	3	11	-5	-5	3	3	0	6	6	6	2	2	-3	-1	1	0	-1	3	-1	-1	0	0	0	0	0	0	-4	-4	χ_{11}
χ_{12}	+	1155	195	35	19	30	-6	3	-13	3	-5	11	-5	3	3	0	6	6	6	2	2	-3	-1	1	0	-1	-1	3	-1	0	0	0	0	0	0	-4	-4	χ_{12}
χ_{13}	+	1155	195	35	19	30	-6	3	-13	3	-5	-5	11	3	3	0	6	6	6	2	2	-3	-1	1	0	-1	-1	-1	3	0	0	0	0	0	0	-4	-4	χ_{13}
χ_{14}	+	1386	-246	58	-30	36	9	9	-6	2	-2	-2	-2	-6	6	1	-3	-3	-12	1	4	-3	1	-3	0	-2	0	0	0	0	0	0	-1	0	0	3	3	χ_{14}
χ_{15}	+	1540	260	4	-28	55	1	1	-12	-12	4	4	4	4	-4	0	17	17	-1	1	-5	-1	1	-1	0	0	0	0	0	1	1	1	0	0	0	-3	-3	χ_{15}
χ_{16}	o	3080	-440	40	-8	65	2	-7	40	-8	0	0	0	-8	0	0	6i3-8	-6i3-8	1	-2	1	1	1	1	0	0	0	0	0	-b27	**	-1	0	0	0	-3i3-5	3i3-5	χ_{16}
χ_{17}	o	3080	-440	40	-8	65	2	-7	40	-8	0	0	0	-8	0	0	-6i3-8	6i3-8	1	-2	1	1	1	1	0	0	0	0	0	**	-b27	-1	0	0	0	3i3-5	-3i3-5	χ_{17}
χ_{18}	+	3520	-320	64	0	10	-44	10	64	0	0	0	0	0	0	0	4	4	-14	4	-2	4	-2	0	-1	0	0	0	0	1	1	1	0	0	0	-8	-8	χ_{18}
χ_{19}	+	4620	-180	44	-36	-15	57	12	28	-20	4	4	4	-4	-4	0	9	9	9	-7	5	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{19}
χ_{20}	+	4928	320	64	64	-4	68	5	0	0	0	0	0	0	0	-2	-4	-4	-4	4	4	5	1	1	0	0	0	0	0	-1	-1	2	0	0	0	0	0	χ_{20}
χ_{21}	+	5544	-24	-56	24	9	36	9	72	24	0	0	0	-8	0	-1	12	12	9	4	1	-3	1	-3	0	0	0	0	0	0	0	0	1	0	0	0	0	χ_{21}
χ_{22}	o	6160	400	-48	-16	40	-50	4	48	16	0	0	0	0	0	0	6i3+4	-6i3+4	-8	-6	0	-2	0	2	0	0	0	0	0	b27	**	1	0	0	0	-6z3	**	χ_{22}
χ_{23}	o	6160	400	-48	-16	40	-50	4	48	16	0	0	0	0	0	0	-6i3+4	6i3+4	-8	-6	0	-2	0	2	0	0	0	0	0	**	b27	1	0	0	0	**	-6z3	χ_{23}
χ_{24}	+	6930	690	98	42	45	-36	-9	18	-6	6	6	6	10	-2	0	-12	-12	-3	-4	5	-3	-1	-3	0	2	0	0	0	0	0	0	0	0	0	0	0	χ_{24}
χ_{25}	+	8064	384	128	0	-36	-36	18	0	0	0	0	0	0	0	-1	-12	-12	12	-4	-4	0	2	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	χ_{25}
χ_{26}	+	9240	-360	88	-8	-30	33	-3	-8	-8	24	-8	-8	0	0	0	9	9	-6	1	-2	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{26}
χ_{27}	+	9240	-360	88	-8	-30	33	-3	-8	-8	-8	24	-8	0	0	0	9	9	-6	1	-2	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{27}
χ_{28}	+	9240	-360	88	-8	-30	33	-3	-8	-8	-8	-8	24	0	0	0	9	9	-6	1	-2	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{28}
χ_{29}	o	10395	315	-21	-45	0	27	0	75	-13	-1	-1	-1	3	-1	0	-9	-9	0	3	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	0	6i3+3	-6i3+3	χ_{29}
χ_{30}	o	10395	315	-21	-45	0	27	0	75	-13	-1	-1	-1	3	-1	0	-9	-9	0	3	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	0	-6i3+3	6i3+3	χ_{30}
χ_{31}	+	10395	315	-21	-45	0	27	0	-21	19	15	-1	-1	3	-1	0	-9	-9	0	3	0	0	0	0	0	-1	-3	1	1	0	0	0	0	0	0	-3	-3	χ_{31}
χ_{32}	+	10395	315	-21	-45	0	27	0	-21	19	-1	15	-1	3	-1	0	-9	-9	0	3	0	0	0	0	0	-1	1	-3	1	0	0	0	0	0	0	-3	-3	χ_{32}
χ_{33}	+	10395	315	-21	-45	0	27	0	-21	19	-1	-1	15	3	-1	0	-9	-9	0	3	0	0	0	0	0	-1	1	1	-3	0	0	0	0	0	0	-3	-3	χ_{33}
χ_{34}	+	11264	1024	0	0	104	32	-4	0	0	0	0	0	0	0	-1	16	16	-8	0	0	4	0	0	1	0	0	0	0	-1	-1	-1	-1	0	0	0	0	χ_{34}
χ_{35}	+	13860	420	100	36	-45	9	-18	84	20	4	4	4	-4	4	0	-3	-3	3	1	-5	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	3	3	χ_{35}
χ_{36}	+	14784	-1344	64	0	114	-12	6	-64	0	0	0	0	0	0	-1	-12	-12	-6	4	-2	0	-2	0	0	0	0	0	0	0	0	0	1	0	0	8	8	χ_{36}
χ_{37}	+	18711	-1161	-57	63	81	0	0	-9	15	3	3	3	-1	-5	1	0	0	9	0	-3	0	0	0	0	-1	1	1	1	0	0	0	-1	0	0	0	0	χ_{37}
χ_{38}	+	18711	1431	87	-9	81	0	0	-9	-9	-9	-9	-9	-1	-1	1	0	0	9	0	-3	0	0	0	0	-1	-1	-1	-1	0	0	0	1	0	0	0	0	χ_{38}
χ_{39}	o	20790	-810	6	-18	0	54	0	6	14	-6	-6	-6	6	2	0	18i3	-18i3	0	-6	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	6z3	**	χ_{39}
χ_{40}	o	20790	-810	6	-18	0	54	0	6	14	-6	-6	-6	6	2	0	-18i3	18i3	0	-6	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	**	6z3	χ_{40}
χ_{41}	+	24640	-960	64	-64	-20	-92	-11	0	0	0	0	0	0	0	0	12	12	12	4	4	3	1	-1	0	0	0	0	0	-2	-2	1	0	0	0	0	0	χ_{41}
χ_{42}	o	25515	-405	-117	27	0	0	0	27	-21	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	-b11	**	0	0	χ_{42}
χ_{43}	o	25515	-405	-117	27	0	0	0	27	-21	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	**	-b11	0	0	χ_{43}
χ_{44}	+	32768	0	0	0	-64	-64	8	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	1	0	0	0	0	2	2	-1	0	-1	-1	0	0	χ_{44}
χ_{45}	+	37422	270	30	54	-81	0	0	-18	6	-6	-6	-6	-2	-6	2	0	0	-9	0	3	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}	+	40095	1215	-81	-9	0	0	0	-81	-9	3	3	3	-9	3	0	0	0	0	0	0	0	0	0	-1	3	1	1	1	0	0	0	0	0	0	0	0	χ_{46}
	ind	1	2	2	2	3	3	3	4	4	4	4	4	4	4	5	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	9	10	11	11	12	12	
		2	2	2		6	6	6			4			4		10	6	6	6	6	6	6	6	6	14		8			18	18	18	10	22	22			
χ_{47}	+	56	24	8	0	11	2	2	0	0	4	0	0	4	0	1	6	6	3	2	-1	0	2	0	0	0	2	0	0	2	2	-1	-1	1	1	0	0	χ_{47}
χ_{48}	+	176	-16	16	0	-4	14	5	0	0	8	0	0	0	0	1	2	2	-4	-2	4	-1	1	3	1	0	0	0	0	-1	-1	2	-1	0	0	0	0	χ_{48}
χ_{49}	+	616	136	24	0	31	22	4	0	0	-4	0	0	4	0	1	10	10	7	6	3	4	0	0	0	0	-2	0	0	1	1	1	1	0	0	0	0	χ_{49}
χ_{50}	+	792	120	40	0	9	-18	9	0	0	4	0	0	4	0	2	-6	-6	9	-2	1	-3	1	-3	1	0	-2	0	0	0	0	0	0	0	0	0	0	χ_{50}
χ_{51}	+	1232	-240	48	0	44	-10	8	0	0	8	0	0	-8	0	2	-6	-6	-12	6	0	0	0	0	0	0	0	0	0	2	2	-1	0	0	0	0	0	χ_{51}
χ_{52}	+	2464	416	32	0	79	-20	-2	0	0	0	0	0	8	0	-1	20	20	-1	-4	-1	-4	2	0	0	0	0	0	0	1	1	1	1	0	0	0	0	χ_{52}
χ_{53}	+	3080	40	56	0	-25	-52	11	0	0	12	0	0	-4	0	0	4	4	7	-4	-1	1	-1	-3	0	0	2	0	0	-1	-1	2	0	0	0	0	0	χ_{53}
χ_{54}	o	3360	-480	32	0	75	12	-6	0	0	0	0	0	-8	0	0	-12	-12	3	-4	-1	0	2	0	0	0	0	0	0	0	0	0	0	b11	**	0	0	χ_{54}
χ_{55}	o	3360	-480	32	0	75	12	-6	0	0	0	0	0	-8	0	0	-12	-12	3	-4	-1	0	2	0	0	0	0	0	0	0	0	0	0	**	b11	0	0	χ_{55}
χ_{56}	+	5544	456	88	0	9	36	9	0	0	12	0	0	4	0	-1	-12	-12	9	4	1	3	1	3	0	0	2	0	0	0	0	0	1	0	0	0	0	χ_{56}
χ_{57}	+	6160	-560	48	0	40	58	13	0	0	-8	0	0	0	0	0	-2	-2	-8	-6	0	-5	-3	3	0	0	0	0	0	1	1	1	0	0	0	0	0	χ_{57}
χ_{58}	o	6160	-560	48	0	40	-50	4	0	0	-8	0	0	0	0	0	-12i3-2	12i3-2	-8	6	0	4	0	0	0	0	0	0	0	b27	**	1	0	0	0	0	0	χ_{58}
χ_{59}	o	6160	-560	48	0	40	-50	4	0	0	-8	0	0	0	0	0	12i3-2	-12i3-2	-8	6	0	4	0	0	0	0	0	0	0	**	b27	1	0	0	0	0	0	χ_{59}
χ_{60}	+	7776	864	96	0	81	0	0	0	0	0	0	0	8	0	1	0	0	9	0	-3	0	0	0	-1	0	0	0	0	0	0	0	-1	-1	-1	0	0	χ_{60}
χ_{61}	+	9856	-384	128	0	-44	28	10	0	0	0	0	0	0	0	1	12	12	-12	-4	-4	0	2	0	0	0	0	0	0	-2	-2	-2	1	0	0	0	0	χ_{61}
χ_{62}	+	11264	1024	0	0	104	32	-4	0	0	0	0	0	0	0	-1	16	16	-8	0	0	4	0	0	1	0	0	0	0	-1	-1	-1	-1	0	0	0	0	χ_{62}
χ_{63}	+	12320	800	32	0	35	8	8	0	0	0	0	0	-8	0	0	8	8	11	8	-1	-4	-4	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	χ_{63}
χ_{64}	o	12320	-480	-96	0	35	8	8	0	0	0	0	0	8	0	0	24z3	**	3	0	3	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	χ_{64}
χ_{65}	o	12320	-480	-96	0	35	8	8	0	0	0	0	0	8	0	0	**	24z3	3	0	3	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	χ_{65}
χ_{66}	+	13608	-1080	24	0	81	0	0	0	0	12	0	0	-4	0	-2	0	0	9	0	-3	0	0	0	0	0	-2	0	0	0	0	0	0	1	1	0	0	χ_{66}
χ_{67}	+	15840	480	-160	0	45	-36	18	0	0	0	0	0	-8	0	0	12	12	-3	-4	5	0	2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{67}
χ_{68}	+	18480	240	80	0	-60	66	-6	0	0	-8	0	0	-8	0	0	6	6	12	2	8	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{68}
χ_{69}	+	21384	-216	120	0	-81	0	0	0	0	12	0	0	4	0	-1	0	0	-9	0	3	0	0	0	-1	0	-2	0	0	0	0	0	-1	0	0	0	0	χ_{69}
χ_{70}	+	22176	1056	-32	0	36	-18	-18	0	0	16	0	0	0	0	1	-6	-6	-12	-2	4	0	-2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	χ_{70}
χ_{71}	+	24640	320	-64	0	-20	124	7	0	0	0	0	0	0	0	0	-4	-4	-4	-4	-4	5	-1	-3	0	0	0	0	0	1	1	1	0	0	0	0	0	χ_{71}
χ_{72}	o	24640	320	-64	0	-20	16	-2	0	0	0	0	0	0	0	0	**	8b27	-4	8	-4	-4	2	0	0	0	0	0	0	**	b27	1	0	0	0	0	0	χ_{72}
χ_{73}	o	24640	320	-64	0	-20	16	-2	0	0	0	0	0	0	0	0	8b27	**	-4	8	-4	-4	2	0	0	0	0	0	0	b27	**	1	0	0	0	0	0	χ_{73}
χ_{74}	+	27720	-1560	56	0	45	18	-9	0	0	-4	0	0	4	0	0	6	6	-3	2	5	-3	-1	-3	0	0	2	0	0	0	0	0	0	0	0	0	0	χ_{74}
χ_{75}	+	30240	1440	96	0	0	-54	0	0	0	-16	0	0	0	0	0	-18	-18	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	χ_{75}
χ_{76}	+	32768	0	0	0	-64	-64	8	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	1	0	0	0	0	2	2	-1	0	-1	-1	0	0	χ_{76}
χ_{77}	+	44352	-960	-64	0	-36	-36	-9	0	0	0	0	0	0	0	2	12	12	12	-4	-4	3	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{77}
	and	1	2	2	2	3	3	3	4	4	4	4	4	4	4	5	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	9	10	11	11	12	12	
	no.	:2	:2	:2		:2	:2	:2				:2		:2		:2	:2	:2	:2	:2	:2	:2	:2	:2	:2			:2		:2	:2	:2	:2	:2	:2			
	and	1	2	2	2	3	3	3	4	4	4	4	4	4	4	5	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	9	10	11	11	12	12	
	no.	:2	:2	:2		:2	:2	:2					:2	:2		:2	:2	:2	:2	:2	:2	:2	:2	:2	:2				:2	:2	:2	:2	:2	:2	:2			

$U_6(2)$

	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	;	;	
													1451	23	4	1		2																												
	144	96	96	48	48	48	24	15	18	18			520	040	608	536	384	160	648	144	108	36	96	96	64	64	30	10	144	72	48	36	12	7	16	16	9	12	12	15					
	CA	BB	AB	DC	DD	DE	EF	AA	BA	AB			A	A	B	B	B	AD	BD	AE	CD	CE	A	A	C	C	AD	AE	EH	DH	EI	GH	GJ	AD	A	A	CJ	CE	FF	ABI					
	AA	BB	BB	BC	BD	BE	AF	AA	AA	BA			A	A	A	A	A	AD	BD	AE	CD	CE	A	A	A	A	AD	AE	AH	BH	AI	CH	CJ	AD	A	A	CD	AE	BF	ABI					
	12C	12D	E**	12F	12G	12H	12I	15A	18A	B**	fus	ind	2D	2E	4H	4I	4J	6I	6J	6K	6L	6M	8E	8F	8G	8H	10B	10C	12J	12K	12L	12M	12N	14A	16A	B**	18C	24A	24B	30A	fus	ind	fus	ind	
χ_1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	:	++	χ_1
χ_2	0	1	1	-1	-1	-1	-2	-1	-1	-1	:	++	8	0	-4	4	0	-4	-1	0	2	0	0	2	2	-2	-2	0	2	-1	-2	2	0	1	0	0	-1	0	-1	1	:	++	:	++	χ_2
χ_3	2	1	1	-1	-1	-1	2	1	0	0	:	++	21	-11	5	5	-3	6	3	-2	0	-2	-3	1	1	1	1	-1	2	-1	2	2	0	0	-1	-1	0	0	1	1	:	++	:	++	χ_3
χ_4	-3	-1	-1	1	1	1	1	-1	0	0	:	++	42	10	10	10	2	9	-3	1	3	1	2	2	2	2	2	0	1	1	1	1	-1	0	0	0	0	-1	-1	-1	:	++	:	++	χ_4
χ_5	1	-3	-3	-1	-1	-1	-1	0	1	1	:	++	35	-5	-9	-1	3	5	-1	1	2	-2	-1	1	-3	1	0	0	3	3	-1	0	0	0	1	1	-1	-1	1	0	:	++	:	++	χ_5
χ_6	3	2	2	0	0	0	-1	0	0	0	:	++	20	20	4	4	4	5	2	5	-1	-1	4	0	0	0	0	0	1	-2	1	1	1	-1	0	0	-1	1	0	0	:	++	:	++	χ_6
χ_7	4	2	2	0	0	0	0	0	1	1	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2i2	-2i2	0	0	0	0	:	oo	:	oo	χ_7
χ_8	2	-1	-1	-1	-1	-1	0	1	0	0	:	++	84	4	-4	12	4	-6	3	-2	3	1	0	4	0	0	-1	-1	-4	-1	0	-1	1	0	0	0	0	0	1	-1	:	++	:	++	χ_8
χ_9	1	-i3	i3	1	1	1	-1	0	i3	-i3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+	χ_9
χ_{10}	1	i3	-i3	1	1	1	-1	0	-i3	i3																																			χ_{10}
χ_{11}	2	0	0	2	-2	-2	0	0	0	0	:	++	105	25	17	1	1	0	6	4	3	1	-3	-3	1	1	0	0	2	2	-2	-1	1	0	-1	-1	0	0	0	0		+		+	χ_{11}
χ_{12}	2	0	0	-2	2	-2	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++			χ_{12}
χ_{13}	2	0	0	-2	-2	2	0	0	0	0																																	:	++	χ_{13}
χ_{14}	0	-1	-1	1	1	1	0	1	0	0	:	++	84	-4	-16	8	-4	-6	3	2	3	-1	0	-2	2	-2	-1	1	2	-1	2	-1	-1	0	0	0	0	0	1	-1	:	++	:	++	χ_{14}
χ_{15}	3	-3	-3	1	1	1	1	0	-1	-1	:	++	70	-10	14	-2	-2	-5	7	-1	1	-1	2	2	-2	-2	0	0	5	-1	1	-1	1	0	0	0	1	-1	-1	0	:	++	:	++	χ_{15}
χ_{16}	1	-2z3	**	0	0	0	1	0	**	z3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+	χ_{16}
χ_{17}	1	**	-2z3	0	0	0	1	0	z3	**																																			χ_{17}
χ_{18}	-2	0	0	0	0	0	0	0	1	1	:	++	160	0	-16	16	0	-20	-2	0	-2	0	0	0	0	0	0	0	2	2	-2	2	0	-1	0	0	1	0	0	0	:	++	:	++	χ_{18}
χ_{19}	1	1	1	1	1	1	-1	0	0	0	:	++	210	-30	-6	10	-6	15	3	3	0	0	-2	2	-2	-2	0	0	-3	3	1	0	0	0	0	0	0	1	-1	0	:	++	:	++	χ_{19}
χ_{20}	0	0	0	0	0	0	0	1	-1	-1	:	++	168	40	8	8	8	6	6	-2	-3	1	0	0	0	0	-2	0	2	2	2	-1	-1	0	0	0	0	0	0	1	:	++	:	++	χ_{20}
χ_{21}	-3	0	0	0	0	0	1	-1	0	0	:	++	84	-44	4	4	4	9	-6	1	-3	1	-4	0	0	0	-1	1	1	-2	1	1	1	0	0	0	0	-1	0	-1	:	++	:	++	χ_{21}
χ_{22}	0	-2z3	**	0	0	0	0	0	**	z3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+	χ_{22}
χ_{23}	0	**	-2z3	0	0	0	0	0	z3	**																																			χ_{23}
χ_{24}	-3	0	0	0	0	0	1	0	0	0	:	++	210	10	22	-2	2	-15	-6	1	3	1	-2	0	-4	0	0	0	1	-2	1	1	-1	0	0	0	0	1	0	0	:	++	:	++	χ_{24}
χ_{25}	0	0	0	0	0	0	0	-1	0	0	:	++	336	16	16	16	0	6	-6	-2	0	-2	0	0	0	0	1	1	-2	-2	-2	-2	0	0	0	0	0	0	1	:	++	:	++	χ_{25}
χ_{26}	-2	1	1	-3	1	1	0	0	0	0	:	++	420	20	-20	-4	4	0	-3	-4	3	-1	0	-4	0	0	0	0	-2	1	2	1	1	0	0	0	0	0	-1	0		+		+	χ_{26}
χ_{27}	-2	1	1	1	-3	1	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++			χ_{27}
χ_{28}	-2	1	1	1	1	-3	0	0	0	0																																	:	++	χ_{28}
χ_{29}	0	-2i3-1	2i3-1	-1	-1	-1	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+	χ_{29}
χ_{30}	0	2i3-1	-2i3-1	-1	-1	-1	0	0	0	0																																			χ_{30}
χ_{31}	0	1	1	3	-1	-1	0	0	0	0	:	++	315	-45	15	-9	3	0	-9	0	0	0	3	1	1	-3	0	0	0	3	0	0	0	0	-1	-1	0	0	1	0		+		+	χ_{31}
χ_{32}	0	1	1	-1	3	-1	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++			χ_{32}
χ_{33}	0	1	1	-1	-1	3	0	0	0	0																																	:	++	χ_{33}
χ_{34}	0	0	0	0	0	0	0	-1	1	1	:	++	64	64	0	0	0	4	-8	4	-2	-2	0	0	0	0	-1	-1	0	0	0	0	0	1	0	0	1	0	0	-1	:	++	:	++	χ_{34}
χ_{35}	3	-1	-1	1	1	1	-1	0	0	0	:	++	210	50	2	2	-6	15	3	-1	0	2	2	-2	-2	-2	0	0	-1	-1	-1	2	0	0	0	0	0	-1	1	0	:	++	:	++	χ_{35}
χ_{36}	2	0	0	0	0	0	0	-1	0	0	:	++	336	-16	-32	0	0	6	-6	2	0	2	0	0	0	0	1	-1	4	-2	0	-2	0	0	0	0	0	0	0	1	:	++	:	++	χ_{36}
χ_{37}	-3	0	0	0	0	0	-1	1	0	0	:	++	189	21	-15	-7	-3	9	0	-3	0	0	1	3	-1	3	-1	1	3	0	-1	0	0	0	-1	-1	0	1	0	-1	:	++	:	++	χ_{37}
χ_{38}	-3	0	0	0	0	0	-1	1	0	0	:	++	189	-51	21	5	-3	9	0	-3	0	0	1	-3	1	1	-1	-1	3	0	-1	0	0	0	1	1	0	1	0	-1	:	++	:	++	χ_{38}
χ_{39}	0	2z3	**	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+	χ_{39}
χ_{40}	0	**	2z3	0	0	0	0	0	0	0																																			χ_{40}
χ_{41}	0	0	0	0	0	0	0	0	0	0	:	++	280	-40	-8	-8	8	10	10	2	1	-1	0	0	0	0	0	0	-2	-2	-2	1	-1	0	0	0	1	0	0	0	:	++	:	++	χ_{41}
χ_{42}	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+		+	χ_{42}
χ_{43}	0	0	0	0	0	0	0	0	0	0																																			χ_{43}
χ_{44}	0	0	0	0	0	0	0	1	0	0	:	++	512	0	0	0	0	-16	8	0	-4	0	0	0	0	0	2	0	0	0	0	0	0	1	0	0	-1	0	0	-1	:	++	:	++	χ_{44}
χ_{45}	3	0	0	0	0	0	1	-1	0	0	:	++	378	-30	6	-2	-6	-9	0	3	0	0	2	0	0	4	-2	0	-3	0	1	0	0	0	0	0	0	-1	0	1	:	++	:	++	χ_{45}
χ_{46}	0	0	0	0	0	0	0	0	0	0	:	++	405	45	9	-15	-3	0	0	0	0	0	-3	3	3	-1	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	:	++	:	++	χ_{46}
	12	12	12	12	12	12	12	15	18	18	fus	ind	2	2	4	4	4	6	6	6	6	6	8	8	8	8	10	10	12	12	12	12	12	14	16	16	18	24	24	30					
	12			12		12	30	18	18				2	2	4	4	4	6	6	6	6	6	8				10	10	12	12	12	12	12	14			18	24	24	30					
χ_{47}	3	0	0	-2	0	0	1	1	0	0	:	++	14	6	6	-2	2	-1	-4	3	2	0	-2	0	0	0	-1	1	-3	0	1	0	2	0	0	0	-1	1	0	-1					χ_{47}
χ_{48}	0	0	0	2	0	0	0	1	-1	-1	:	++	36	4	-4	-4	4	6	0	-2	3	1	0	0	0	0	1	-1	2	-4	2	-1	1	1	0	0	0	0	0	1					χ_{48}
χ_{49}	3	0	0	2	0	0	1	1	1	1	:	++	14	-26	6	-2	-6	-1	-4	-5	2	-2	2	0	0	0	-1	-1	-3	0	1	0	0	0	0	0	-1	-1	0	-1					χ_{49}
χ_{50}	-3	0	0	-2	0	0	1	-1	0	0	:	++	78	-10	14	-10	2	-9	6	-1	3	-1	-2	0	0	0	-2	0	-1	2	-1	-1	-1	1	0	0	0	1	0	1					χ_{50}
χ_{51}	0	0	0	2	0	0	-2	-1	0	0	:	++	112	0	-24	-8	0	4	4	0	4	0	0	0	0	0	2	0	-6	0	-2	0	0	0	0	0	1	0	0	-1					χ_{51}
χ_{52}	3	0	0	0	0	0	-1	-1	-1	-1	:	++	56	24	8	8	0	11	2	3	2	0	4	0	0	0	1	-1	-1	2	-1	2	0	0	0	0	-1	1	0	1					χ_{52}
χ_{53}	3	0	0	0	0	0	-1	0	1	1	:	++	210	10	2	-6	-2	-15	-6	1	3	1	2	0	0	0	0	0	5	2	-3	-1	1	0	0	0	0	-1	0	0					χ_{53}
χ_{54}	3	0	0	0	0	0	1	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0					χ_{54}
χ_{55}	3	0	0	0	0	0	1	0	0	0																																			χ_{55}
χ_{56}	-3	0	0	0	0	0	1	-1	0	0	:	++	294	46	22	-2	2	9	6	1	3	1	2	0	0	0	-1	1	1	-2	1	1	-1	0	0	0	0	-1	0	-1					χ_{56}
χ_{57}	0	0	0	-2	0	0	0	0	1	1	:	++	140	-20	-12	-12	-4	20	-4	4	-1	1	0	0	0	0	0	0	0	0	0	-3	-1	0	0	0	-1	0	0	0					χ_{57}
χ_{58}	0	0	0	-2	0	0	0	0	**	z3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0					χ_{58}
χ_{59}	0	0	0	-2	0	0	0	0	z3	**																																			χ_{59}
χ_{60}	-3	0	0	0	0	0	-1	1	0	0	:	++	216	24	24	-8	0	-9	0	3	0	0	-4	0	0	0	1	-1	-3	0	1	0	0	-1	0	0	0	-1	0	1					χ_{60}
χ_{61}	0	0	0	0	0	0	0	1	0	0	:	++	336	16	-16	-16	0	6	-6	-2	0	-2	0	0	0	0	1	1	2	2	2	2	0	0	0	0	0	0	0	1					χ_{61}
χ_{62}	0	0	0	0	0	0	0	-1	1	1	:	++	64	64	0	0	0	4	-8	4	-2	-2	0	0	0	0	-1	-1	0	0	0	0	0	1	0	0	1	0	0	-1					χ_{62}
χ_{63}	3	0	0	0	0	0	1	0	-1	-1	:	++	280	-40	24	-8	0	-5	-8	-1	-2	2	4	0	0	0	0	0	-3	0	1	0	0	0	0	0	1	1	0	0					χ_{63}
χ_{64}	3	0	0	0	0	0	-1	0	-i3	i3		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0					χ_{64}
χ_{65}	3	0	0	0	0	0	-1	0	i3	-i3																																			χ_{65}
χ_{66}	-3	0	0	0	0	0	-1	1	0	0	:	++	378	-30	-30	10	6	-9	0	3	0	0	2	0	0	0	-2	0	-3	0	1	0	0	0	0	0	0	-1	0	1					χ_{66}
χ_{67}	-3	0	0	0	0	0	1	0	0	0	:	++	120	-40	8	8	0	15	-6	-1	0	2	-4	0	0	0	0	0	-1	2	-1	2	0	1	0	0	0	-1	0	0					χ_{67}
χ_{68}	0	0	0	-2	0	0	-2	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r3	0					χ_{68}
χ_{69}	3	0	0	0	0	0	1	-1	0	0	:	++	594	-6	-6	2	6	9	0	-3	0	0	-2	0	0	0	-1	-1	3	0	-1	0	0	-1	0	0	0	1	0	-1					χ_{69}
χ_{70}	0	0	0	-2	0	0	0	1	0	0	:	++	336	16	16	16	0	6	-6	-2	0	-2	0	0	0	0	1	1	-2	-2	-2	-2	0	0	0	0	0	0	0	1					χ_{70}
χ_{71}	0	0	0	0	0	0	0	0	-1	-1	:	++	280	-40	8	8	-8	10	10	2	1	-1	0	0	0	0	0	0	2	2	2	-1	1	0	0	0	1	0	0	0					χ_{71}
χ_{72}	0	0	0	0	0	0	0	0	-z3	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0					χ_{72}
χ_{73}	0	0	0	0	0	0	0	0	**	-z3																																			χ_{73}
χ_{74}	-3	0	0	2	0	0	1	0	0	0	:	++	210	10	-14	10	-10	-15	-6	1	3	1	-2	0	0	0	0	0	1	-2	1	1	-1	0	0	0	0	1	0	0					χ_{74}
χ_{75}	0	0	0	2	0	0	0	0	0	0	:	++	0	-80	8	-8	0	0	0	4	0	-2	0	0	0	0	0	0	2	-4	-2	2	0	0	0	0	0	0	0	0					χ_{75}
χ_{76}	0	0	0	0	0	0	0	1	0	0	:	++	512	0	0	0	0	-16	8	0	-4	0	0	0	0	0	2	0	0	0	0	0	0	1	0	0	-1	0	0	-1					χ_{76}
χ_{77}	0	0	0	0	0	0	0	-1	0	0	:	++	168	40	-8	-8	-8	6	6	-2	-3	1	0	0	0	0	-2	0	-2	-2	-2	1	1	0	0	0	0	0	0	1					χ_{77}
	12	12	12	12	12	12	12	15	18	18																															:	:			
	:2				:2		:2	:2	:2	:2																															:	:			
	12	12	12	12	12	12	12	15	18	18																																	:	:	
	:2					:2	:2	:2	:2	:2																																	:	:	

$U_6(2)$

		1A	2A	2B	2C	3A	3B	3C	4A	4B	4C	4D	4E	4F	4G	5A	6A	B**	6C	6D	6E	6F	6G	6H	7A	8A	8B	8C	8D	9A	B**	9C	10A	11A	B**	12A	B**	
	ind	1	2	2	2	3	3	3	4	4	4	4	4	4	4	5	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	9	10	11	11	12	12	
		3	6	6	6	3	3		12	12	12	12	12	12	12	15	6	6	6	6	6	6	6		21	24	24	24	24	9	9		30	33	33	12	12	
		3	6	6	6	3	3		12	12	12	12	12	12	12	15	6	6	6	6	6	6	6		21	24	24	24	24	9	9		30	33	33	12	12	
χ_{78}	o2	21	-11	5	-3	6	3	0	5	-3	1	1	1	-3	1	1	-2i3+1	2i3+1	-2	-1	2	-2	2	0	0	1	-1	-1	-1	i3	-i3	0	-1	-1	-1	-i3+2	i3+2	χ_{78}
χ_{79}	o2	210	50	2	-6	15	3	0	18	10	-2	-2	-2	2	-2	0	2i3-7	-2i3-7	-1	-1	-1	2	2	0	0	2	0	0	0	**	z3-1	0	0	1	1	3i3	-3i3	χ_{79}
χ_{80}	o2	231	71	23	15	21	6	0	7	-1	3	3	3	7	3	1	2i3-4	-2i3-4	5	2	5	2	2	0	0	-1	1	1	1	z3-1	**	0	1	0	0	**	-2z3	χ_{80}
χ_{81}	o2	462	110	30	6	24	-15	0	30	6	2	2	2	6	2	2	2i3-1	-2i3-1	8	-3	0	-4	0	0	0	2	0	0	0	i3	-i3	0	0	0	0	3i3-6	-3i3-6	χ_{81}
χ_{82}	o2	924	-164	-4	12	39	-3	0	12	-4	4	4	4	-4	-4	-1	-8i3+1	8i3+1	7	5	-1	-2	2	0	0	0	0	0	0	**	-z3+1	0	1	0	0	3	3	χ_{82}
χ_{83}	o2	1155	-125	3	3	15	-24	0	35	3	3	3	3	-5	-5	0	**	-8z3	7	0	3	4	0	0	0	3	-1	-1	-1	**	z3-1	0	0	0	0	**	8z3	χ_{83}
χ_{84}	o2	1386	-54	26	18	-9	36	0	42	2	6	6	6	2	-2	1	0	0	-9	-4	-1	0	-4	0	0	2	0	0	0	0	0	0	1	0	0	6	6	χ_{84}
χ_{85}	o2	1485	45	-3	21	0	27	0	-3	-11	9	-7	-7	-3	1	0	-9	-9	0	3	0	0	0	0	1	1	3	-1	-1	0	0	0	0	0	0	-3	-3	χ_{85}
χ_{86}	o2	1485	45	-3	21	0	27	0	-3	-11	-7	9	-7	-3	1	0	-9	-9	0	3	0	0	0	0	1	1	-1	3	-1	0	0	0	0	0	0	-3	-3	χ_{86}
χ_{87}	o2	1485	45	-3	21	0	27	0	-3	-11	-7	-7	9	-3	1	0	-9	-9	0	3	0	0	0	0	1	1	-1	-1	3	0	0	0	0	0	0	-3	-3	χ_{87}
χ_{88}	o2	2310	230	-10	-18	30	33	0	22	-18	2	2	2	6	-6	0	**	-4z3+9	-10	5	2	2	2	0	0	2	0	0	0	**	-z3+1	0	0	0	0	i3-2	-i3-2	χ_{88}
χ_{89}	o2	2310	-410	54	-18	75	-21	0	6	-2	2	2	2	-10	2	0	-8i3+7	8i3+7	-5	3	3	4	0	0	0	-2	0	0	0	z3-1	**	0	0	0	0	-3	-3	χ_{89}
χ_{90}	o2	2520	-360	56	-24	45	36	0	24	8	0	0	0	-8	0	0	0	0	-3	-4	5	-6	2	0	0	0	0	0	0	0	0	0	0	1	1	6	6	χ_{90}
χ_{91}	o2	2772	-300	84	-12	9	-9	0	36	4	4	4	4	-4	4	2	6i3-3	-6i3-3	-15	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3i3	3i3	χ_{91}
χ_{92}	o2	3465	-375	9	9	45	9	0	73	-23	-7	-7	-7	1	1	0	-6i3+3	6i3+3	-3	-3	-3	0	0	0	0	1	1	1	1	0	0	0	0	0	0	-5i3+4	5i3+4	χ_{92}
χ_{93}	o2	3465	-375	9	9	45	9	0	-23	9	9	-7	-7	1	1	0	-6i3+3	6i3+3	-3	-3	-3	0	0	0	0	1	-3	1	1	0	0	0	0	0	0	i3-2	-i3-2	χ_{93}
χ_{94}	o2	3465	-375	9	9	45	9	0	-23	9	-7	9	-7	1	1	0	-6i3+3	6i3+3	-3	-3	-3	0	0	0	0	1	1	-3	1	0	0	0	0	0	0	i3-2	-i3-2	χ_{94}
χ_{95}	o2	3465	-375	9	9	45	9	0	-23	9	-7	-7	9	1	1	0	-6i3+3	6i3+3	-3	-3	-3	0	0	0	0	1	1	1	-3	0	0	0	0	0	0	i3-2	-i3-2	χ_{95}
χ_{96}	o2	3696	-16	48	-48	-24	42	0	16	-16	0	0	0	0	0	1	2i3+8	-2i3+8	8	6	0	-4	0	0	0	0	0	0	0	-i3	i3	0	-1	0	0	**	-2z3	χ_{96}
χ_{97}	o2	4620	140	-52	12	15	39	0	60	28	-4	-4	-4	-4	4	0	8i3+11	-8i3+11	-1	-1	-1	2	2	0	0	0	0	0	0	-z3+1	**	0	0	0	0	-3	-3	χ_{97}
χ_{98}	o2	4620	460	108	60	15	39	0	-4	12	4	4	4	4	4	0	-2i3+13	2i3+13	7	3	3	4	0	0	0	0	0	0	0	-z3+1	**	0	0	0	0	-i3+2	i3+2	χ_{98}
χ_{99}	o2	5544	744	72	24	99	-18	0	-24	-8	0	0	0	8	0	-1	6i3-12	-6i3-12	3	-6	3	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-6z3	**	χ_{99}
χ_{100}	o2	6720	320	-64	0	30	-12	0	64	0	0	0	0	0	0	0	-4i3-16	4i3-16	-10	-4	2	2	2	0	0	0	0	0	0	-i3	i3	0	0	-1	-1	**	-8z3	χ_{100}
χ_{101}	o2	6930	690	-30	-54	90	18	0	2	10	6	6	6	2	-2	0	6i3-12	-6i3-12	-6	6	-5	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	**	2z3	χ_{101}
χ_{102}	o2	9240	280	152	24	-60	-57	0	24	-8	8	8	8	0	0	0	-8i3-5	8i3-5	4	-1	-4	-2	2	0	0	0	0	0	0	-i3	i3	0	0	0	0	-3	-3	χ_{102}
χ_{103}	o2	9240	-40	-8	-24	-15	-30	0	88	8	0	0	0	-8	0	0	2i3-16	-2i3-16	17	-2	1	2	-2	0	0	0	0	0	0	**	-z3+1	0	0	0	0	**	-2z3	χ_{103}
χ_{104}	o2	10395	315	-21	-45	0	-54	0	75	-13	-1	-1	-1	3	-1	0	18	18	0	-6	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	0	-6	-6	χ_{104}
χ_{105}	o2	11088	720	16	48	36	-36	0	48	16	0	0	0	0	0	-2	0	0	-12	4	4	-6	-2	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	χ_{105}
χ_{106}	o2	11880	360	-24	-24	0	-27	0	-24	8	24	-8	-8	0	0	0	9	9	0	-3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3	3	χ_{106}
χ_{107}	o2	11880	360	-24	-24	0	-27	0	-24	8	-8	24	-8	0	0	0	9	9	0	-3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3	3	χ_{107}
χ_{108}	o2	11880	360	-24	-24	0	-27	0	-24	8	-8	-8	24	0	0	0	9	9	0	-3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3	3	χ_{108}
χ_{109}	o2	14784	704	64	0	-6	60	0	64	0	0	0	0	0	0	-1	-4i3+8	4i3+8	2	4	-2	2	-2	0	0	0	0	0	0	z3-1	**	0	-1	0	0	**	-8z3	χ_{109}
χ_{110}	o2	18480	-1360	112	-48	60	-6	0	-48	-16	0	0	0	0	0	0	2i3-4	-2i3-4	-4	-2	4	2	-2	0	0	0	0	0	0	-z3+1	**	0	0	0	0	6z3	**	χ_{110}
χ_{111}	o2	18711	-1161	-57	63	81	0	0	-9	15	3	3	3	-1	-5	1	0	0	9	0	-3	0	0	0	0	-1	1	1	1	0	0	0	-1	0	0	0	0	χ_{111}
χ_{112}	o2	18711	1431	87	-9	81	0	0	-9	-9	-9	-9	-9	-1	-1	1	0	0	9	0	-3	0	0	0	0	-1	-1	-1	-1	0	0	0	1	0	0	0	0	χ_{112}
χ_{113}	o2	20790	-810	6	-18	0	-108	0	6	14	-6	-6	-6	6	2	0	0	0	0	12	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	6	6	χ_{113}
χ_{114}	o2	21504	-1024	0	0	24	48	0	0	0	0	0	0	0	0	-1	**	-16z3	8	0	0	-4	0	0	0	0	0	0	0	**	z3-1	0	1	-1	-1	0	0	χ_{114}
χ_{115}	o2	25515	-405	-117	27	0	0	0	27	-21	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	-b11	**	0	0	χ_{115}
χ_{116}	o2	25515	-405	-117	27	0	0	0	27	-21	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	**	-b11	0	0	χ_{116}
χ_{117}	o2	27720	-120	-24	-72	-45	72	0	8	24	0	0	0	8	0	0	**	24z3	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	8z3	χ_{117}
χ_{118}	o2	29568	-640	128	0	-84	12	0	0	0	0	0	0	0	0	-2	8i3-4	-8i3-4	-4	-4	-4	2	2	0	0	0	0	0	0	i3	-i3	0	0	0	0	0	0	χ_{118}
χ_{119}	o2	37422	270	30	54	-81	0	0	-18	6	-6	-6	-6	-2	-6	2	0	0	-9	0	3	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	χ_{119}
χ_{120}	o2	40095	1215	-81	-9	0	0	0	-81	-9	3	3	3	-9	3	0	0	0	0	0	0	0	0	0	-1	3	1	1	1	0	0	0	0	0	0	0	0	χ_{120}
	ind	1	2	2	2	3	3	3	4	4	4	4	4	4	4	5	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	9	10	11	11	12	12	
		6	6	6	6	6	6	6	12	12	12	12	12	12	12	30	6	6	6	6	6	6	6	6	42	24	24	24	24	18	18	18	30	66	66	12	12	
		3	6	6	6	3	3		12	12	12	12	12	12	12	15	6	6	6	6	6	6	6		21	24	24	24	24	9	9		30	33	33	12	12	
		2	2	2		6	6				4			4		10	6	6	6	6	6	6	6		14		8			18	18		1	22	22			
		3	6	6		3	3				12			12		15	6	6	6	6	6	6	6		21		24			9	9		30	33	33			
		6	6	6		6	6				12			12		30	6	6	6	6	6	6	6		42		24			18	18		30	66	66			
χ_{121}	o2	120	-40	8	0	15	-6	0	0	0	4	0	0	-4	0	0	-4i3+2	4i3+2	-1	2	-1	2	2	0	1	0	-2	0	0	i3	-i3	0	0	-1	-1	0	0	χ_{121}
χ_{122}	o2	672	160	32	0	39	-12	0	0	0	0	0	0	8	0	2	4i3-8	-4i3-8	7	-4	-1	-2	2	0	0	0	0	0	0	z3-1	**	0	0	1	1	0	0	χ_{122}
χ_{123}	o2	1056	-224	32	0	51	12	0	0	0	0	0	0	-8	0	1	-8i3+4	8i3+4	-5	-4	-1	-2	2	0	-1	0	0	0	0	i3	-i3	0	1	0	0	0	0	χ_{123}
χ_{124}	o2	1848	280	8	0	51	-6	0	0	0	4	0	0	4	0	-2	4i3-14	-4i3-14	-5	2	-1	-2	2	0	0	0	2	0	0	**	z3-1	0	0	0	0	0	0	χ_{124}
χ_{125}	o2	2640	400	48	0	60	30	0	0	0	8	0	0	8	0	0	4i3-2	-4i3-2	4	6	0	4	0	0	1	0	0	0	0	i3	-i3	0	0	0	0	0	0	χ_{125}
χ_{126}	o2	3696	176	80	0	-24	42	0	0	0	8	0	0	0	0	1	-4i3+2	4i3+2	8	2	8	2	2	0	0	0	0	0	0	-i3	i3	0	1	0	0	0	0	χ_{126}
χ_{127}	o2	5544	-696	24	0	99	-18	0	0	0	-4	0	0	-4	0	-1	-12i3+6	12i3+6	3	6	3	0	0	0	0	0	2	0	0	0	0	0	-1	0	0	0	0	χ_{127}
χ_{128}	o2	6336	-576	64	0	36	72	0	0	0	0	0	0	0	0	1	0	0	-12	-8	4	-6	-2	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	χ_{128}
χ_{129}	o2	7392	-160	-96	0	24	30	0	0	0	16	0	0	0	0	2	**	8z3-6	8	6	0	-4	0	0	0	0	0	0	0	-z3+1	**	0	0	0	0	0	0	χ_{129}
χ_{130}	o2	7392	-800	96	0	69	-24	0	0	0	0	0	0	-8	0	2	**	-8z3	-11	0	-3	4	0	0	0	0	0	0	0	**	z3-1	0	0	0	0	0	0	χ_{130}
χ_{131}	o2	7776	864	96	0	81	0	0	0	0	0	0	0	8	0	1	0	0	9	0	-3	0	0	0	-1	0	0	0	0	0	0	0	-1	-1	-1	0	0	χ_{131}
χ_{132}	o2	9240	760	-24	0	75	24	0	0	0	-12	0	0	-4	0	0	4i3-20	-4i3-20	-5	0	3	4	0	0	0	0	-2	0	0	-i3	i3	0	0	0	0	0	0	χ_{132}
χ_{133}	o2	9240	-520	104	0	-15	-30	0	0	0	4	0	0	-4	0	0	8i3-10	-8i3-10	-7	2	5	2	2	0	0	0	2	0	0	**	-z3+1	0	0	0	0	0	0	χ_{133}
χ_{134}	o2	13608	-1080	24	0	81	0	0	0	0	12	0	0	-4	0	-2	0	0	9	0	-3	0	0	0	0	0	-2	0	0	0	0	0	0	1	1	0	0	χ_{134}
χ_{135}	o2	14784	1216	64	0	84	-48	0	0	0	0	0	0	0	0	-1	**	-8z3	4	-8	4	-2	-2	0	0	0	0	0	0	**	-z3+1	0	1	0	0	0	0	χ_{135}
χ_{136}	o2	15120	720	48	0	0	54	0	0	0	-8	0	0	0	0	0	18	18	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	-b11	**	0	0	χ_{136}
χ_{137}	o2	15120	720	48	0	0	54	0	0	0	-8	0	0	0	0	0	18	18	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	**	-b11	0	0	χ_{137}
χ_{138}	o2	18480	880	-112	0	60	-6	0	0	0	8	0	0	0	0	0	**	-8z3+6	-20	2	-4	-2	2	0	0	0	0	0	0	-z3+1	**	0	0	0	0	0	0	χ_{138}
χ_{139}	o2	18480	-1040	-48	0	60	-6	0	0	0	-8	0	0	8	0	0	-8i3-2	8i3-2	4	-6	0	4	0	0	0	0	0	0	0	-z3+1	**	0	0	0	0	0	0	χ_{139}
χ_{140}	o2	21120	640	128	0	-60	-84	0	0	0	0	0	0	0	0	0	-8i3+4	8i3+4	4	-4	-4	-2	2	0	1	0	0	0	0	-i3	i3	0	0	0	0	0	0	χ_{140}
χ_{141}	o2	21384	-216	120	0	-81	0	0	0	0	12	0	0	4	0	-1	0	0	-9	0	3	0	0	0	-1	0	-2	0	0	0	0	0	-1	0	0	0	0	χ_{141}
χ_{142}	o2	21504	-1024	0	0	24	48	0	0	0	0	0	0	0	0	-1	**	-16z3	8	0	0	-4	0	0	0	0	0	0	0	**	z3-1	0	1	-1	-1	0	0	χ_{142}
χ_{143}	o2	22176	-96	-96	0	-9	-72	0	0	0	0	0	0	-8	0	1	**	-24z3	15	0	3	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	χ_{143}
χ_{144}	o2	22176	-864	32	0	-9	-72	0	0	0	0	0	0	8	0	1	0	0	-9	8	-1	0	-4	0	0	0	0	0	0	0	0	0	1	0	0	0	0	χ_{144}
χ_{145}	o2	27720	360	-8	0	-45	72	0	0	0	12	0	0	-4	0	0	0	0	3	-8	-5	6	-2	0	0	0	2	0	0	0	0	0	0	0	0	0	0	χ_{145}
χ_{146}	o2	36960	-160	-160	0	-15	-12	0	0	0	0	0	0	8	0	0	-4i3+8	4i3+8	-7	-4	5	2	2	0	0	0	0	0	0	z3-1	**	0	0	0	0	0	0	χ_{146}
χ_{147}	o2	36960	-800	32	0	-60	42	0	0	0	-16	0	0	0	0	0	4i3-14	-4i3-14	4	2	-4	-2	2	0	0	0	0	0	0	**	-z3+1	0	0	0	0	0	0	χ_{147}
χ_{148}	o2	36960	1120	-32	0	-15	-12	0	0	0	0	0	0	-8	0	0	-8i3-20	8i3-20	1	4	1	-2	-2	0	0	0	0	0	0	z3-1	**	0	0	0	0	0	0	χ_{148}
	and no.	1 :6	2 :6	2 :6	2 :3	3 :6	3 :6	3 :2	4 :3	4 :3	4 :3	4 :6	4 :3	4 :6	4 :3	5 :6	6 :6	6 :6	6 :6	6 :6	6 :6	6 :6	6 :6	6 :2	7 :6	8 :3	8 :3	8 :6	8 :3	9 :6	9 :6	9 :2	10 :6	11 :6	11 :6	12 :3	12 :3	
	and no.	1 :6	2 :6	2 :6	2 :3	3 :6	3 :6	3 :2	4 :3	4 :3	4 :3	4 :3	4 :6	4 :6	4 :3	5 :6	6 :6	6 :6	6 :6	6 :6	6 :6	6 :6	6 :6	6 :2	7 :6	8 :3	8 :3	8 :3	8 :6	9 :6	9 :6	9 :2	10 :6	11 :6	11 :6	12 :3	12 :3	

$U_6(2)$

	12C	12D	E**	12F	12G	12H	12I	15A	18A	B**		
	12	12	12	12	12	12	12	15	18	18	fus	ind
	12	12	12	12	12	12	12	15	18	18		
	12	12	12	12	12	12	12	15	18	18		
χ_{78}	2	i3	-i3	1	1	1	0	1	1	1	*	+
χ_{79}	3	i3-2	-i3-2	1	1	1	-1	0	-z3	**	*	+
χ_{80}	1	**	2z3	0	0	0	1	1	**	-z3	*	+
χ_{81}	0	i3	-i3	-1	-1	-1	0	-1	-1	-1	*	+
χ_{82}	3	-1	-1	1	1	1	-1	-1	z3	**	*	+
χ_{83}	-1	0	0	0	0	0	1	0	z3	**	*	+
χ_{84}	3	2	2	0	0	0	-1	1	0	0	*	+
χ_{85}	0	1	1	3	-1	-1	0	0	0	0	*	+
χ_{86}	0	1	1	-1	3	-1	0	0	0	0		o2
χ_{87}	0	1	1	-1	-1	3	0	0	0	0	*	
χ_{88}	-2	-i3	i3	-1	-1	-1	0	0	-z3	**	*	+
χ_{89}	3	1	1	-1	-1	-1	-1	0	**	z3	*	+
χ_{90}	-3	2	2	0	0	0	1	0	0	0	*	+
χ_{91}	-3	-i3-2	i3-2	1	1	1	-1	-1	0	0	*	+
χ_{92}	1	i3-2	-i3-2	-1	-1	-1	1	0	0	0	*	+
χ_{93}	1	-i3	i3	3	-1	-1	1	0	0	0	*	+
χ_{94}	1	-i3	i3	-1	3	-1	1	0	0	0		o2
χ_{95}	1	-i3	i3	-1	-1	3	1	0	0	0	*	
χ_{96}	4	**	2z3	0	0	0	0	1	-1	-1	*	+
χ_{97}	3	1	1	-1	-1	-1	-1	0	**	-z3	*	+
χ_{98}	-1	i3	-i3	1	1	1	1	0	**	z3	*	+
χ_{99}	3	-2z3	**	0	0	0	-1	-1	0	0	*	+
χ_{100}	-2	0	0	0	0	0	0	0	-1	-1	*	+
χ_{101}	2	**	-2z3	0	0	0	2	0	0	0	*	+
χ_{102}	0	1	1	-1	-1	-1	0	0	1	1	*	+
χ_{103}	1	**	2z3	0	0	0	1	0	-z3	**	*	+
χ_{104}	0	2	2	2	2	2	0	0	0	0	*	+
χ_{105}	0	-2	-2	0	0	0	0	1	0	0	*	+
χ_{106}	0	-1	-1	-3	1	1	0	0	0	0	*	+
χ_{107}	0	-1	-1	1	-3	1	0	0	0	0		o2
χ_{108}	0	-1	-1	1	1	-3	0	0	0	0	*	
χ_{109}	-2	0	0	0	0	0	0	-1	**	-z3	*	+
χ_{110}	0	2z3	**	0	0	0	0	0	**	-z3	*	+
χ_{111}	-3	0	0	0	0	0	-1	1	0	0	*	+
χ_{112}	-3	0	0	0	0	0	-1	1	0	0	*	+
χ_{113}	0	2	2	0	0	0	0	0	0	0	*	+
χ_{114}	0	0	0	0	0	0	0	-1	-z3	**	*	+
χ_{115}	0	0	0	0	0	0	0	0	0	0		+2
χ_{116}	0	0	0	0	0	0	0	0	0	0	*	
χ_{117}	-1	0	0	0	0	0	-1	0	0	0	*	+
χ_{118}	0	0	0	0	0	0	0	1	-1	-1	*	+
χ_{119}	3	0	0	0	0	0	1	-1	0	0	*	+
χ_{120}	0	0	0	0	0	0	0	0	0	0	*	+

	2D	2E	4H	4I	4J	6I	6J	6K	6L	6M	8E	8F	8G	8H	10B	10C	12J	12K	12L	12M	12N	14A	16A	B**	18C	24A	24B	30A					
	2	2	4	4	4	6	6	6	6	6	8	8	8	8	10	10	12	12	12	12	12	14	16	16	18	24	24	30	fus	ind	fus	ind	
χ_{78}																													*	+	*	+	χ_{78}
χ_{79}																													*	+	*	+	χ_{79}
χ_{80}																													*	+	*	+	χ_{80}
χ_{81}																													*	+	*	+	χ_{81}
χ_{82}																													*	+	*	+	χ_{82}
χ_{83}																													*	+	*	+	χ_{83}
χ_{84}																													*	+	*	+	χ_{84}
χ_{85}																														o2		o2	χ_{85}
χ_{86}																													*	+	*		χ_{86}
χ_{87}																													*		*	+	χ_{87}
χ_{88}																													*	+	*	+	χ_{88}
χ_{89}																													*	+	*	+	χ_{89}
χ_{90}																													*	+	*	+	χ_{90}
χ_{91}																													*	+	*	+	χ_{91}
χ_{92}																													*	+	*	+	χ_{92}
χ_{93}																														o2		o2	χ_{93}
χ_{94}																													*	+	*		χ_{94}
χ_{95}																													*		*	+	χ_{95}
χ_{96}																													*	+	*	+	χ_{96}
χ_{97}																													*	+	*	+	χ_{97}
χ_{98}																													*	+	*	+	χ_{98}
χ_{99}																													*	+	*	+	χ_{99}
χ_{100}																													*	+	*	+	χ_{100}
χ_{101}																													*	+	*	+	χ_{101}
χ_{102}																													*	+	*	+	χ_{102}
χ_{103}																													*	+	*	+	χ_{103}
χ_{104}																													*	+	*	+	χ_{104}
χ_{105}																													*	+	*	+	χ_{105}
χ_{106}																														o2		o2	χ_{106}
χ_{107}																													*	+	*		χ_{107}
χ_{108}																													*		*	+	χ_{108}
χ_{109}																													*	+	*	+	χ_{109}
χ_{110}																													*	+	*	+	χ_{110}
χ_{111}																													*	+	*	+	χ_{111}
χ_{112}																													*	+	*	+	χ_{112}
χ_{113}																													*	+	*	+	χ_{113}
χ_{114}																													*	+	*	+	χ_{114}
χ_{115}																														+2		+2	χ_{115}
χ_{116}																													*		*		χ_{116}
χ_{117}																													*	+	*	+	χ_{117}
χ_{118}																													*	+	*	+	χ_{118}
χ_{119}																													*	+	*	+	χ_{119}
χ_{120}																													*	+	*	+	χ_{120}

	12	12	12	12	12	12	12	15	18	18	fus	ind
	12	12	12	12	12	12	12	30	18	18		
	12	12	12	12	12	12	12	15	18	18		
	12			12			12	30	18	18		
	12			12			12	15	18	18		
	12			12			12	30	18	18		
χ_{121}	3	0	0	-2	0	0	-1	0	-1	-1	*	+
χ_{122}	3	0	0	0	0	0	-1	-1	**	z3	*	+
χ_{123}	3	0	0	0	0	0	1	1	1	1	*	+
χ_{124}	3	0	0	-2	0	0	1	1	z3	**	*	+
χ_{125}	0	0	0	2	0	0	2	0	1	1	*	+
χ_{126}	0	0	0	2	0	0	0	1	-1	-1	*	+
χ_{127}	3	0	0	2	0	0	-1	-1	0	0	*	+
χ_{128}	0	0	0	0	0	0	0	1	0	0	*	+
χ_{129}	0	0	0	-2	0	0	0	-1	**	-z3	*	+
χ_{130}	-3	0	0	0	0	0	1	-1	z3	**	*	+
χ_{131}	-3	0	0	0	0	0	-1	1	0	0	*	+
χ_{132}	3	0	0	0	0	0	-1	0	1	1	*	+
χ_{133}	-3	0	0	-2	0	0	-1	0	-z3	**	*	+
χ_{134}	-3	0	0	0	0	0	-1	1	0	0	*	+
χ_{135}	0	0	0	0	0	0	0	-1	z3	**	*	+
χ_{136}	0	0	0	-2	0	0	0	0	0	0		+2
χ_{137}	0	0	0	-2	0	0	0	0	0	0	*	
χ_{138}	0	0	0	2	0	0	0	0	**	z3	*	+
χ_{139}	0	0	0	-2	0	0	2	0	**	z3	*	+
χ_{140}	0	0	0	0	0	0	0	0	1	1	*	+
χ_{141}	3	0	0	0	0	0	1	-1	0	0	*	+
χ_{142}	0	0	0	0	0	0	0	-1	-z3	**	*	+
χ_{143}	3	0	0	0	0	0	1	1	0	0	*	+
χ_{144}	3	0	0	0	0	0	-1	1	0	0	*	+
χ_{145}	3	0	0	0	0	0	-1	0	0	0	*	+
χ_{146}	-3	0	0	0	0	0	-1	0	**	-z3	*	+
χ_{147}	0	0	0	2	0	0	0	0	z3	**	*	+
χ_{148}	-3	0	0	0	0	0	1	0	**	z3	*	+
	12	12	12	12	12	12	12	15	18	18		
	:6	:3	:3	:3	:6	:3	:6	:6	:6	:6		
	12	12	12	12	12	12	12	15	18	18		
	:6	:3	:3	:3	:3	:6	:6	:6	:6	:6		

	2D	2E	4H	4I	4J	6I	6J	6K	6L	6M	8E	8F	8G	8H	10B	10C	12J	12K	12L	12M	12N	14A	16A	B**	18C	24A	24B	30A					
	2	2	4	4	4	6	6	6	6	6	8	8	8	8	10	10	12	12	12	12	12	14	16	16	18	24	24	30	fus	ind	fus	ind	
	2	2	4	4	4	6	6	6	6	6	8				10	10	12	12	12	12	12	14			18	24	24	30					
χ_{121}																																	χ_{121}
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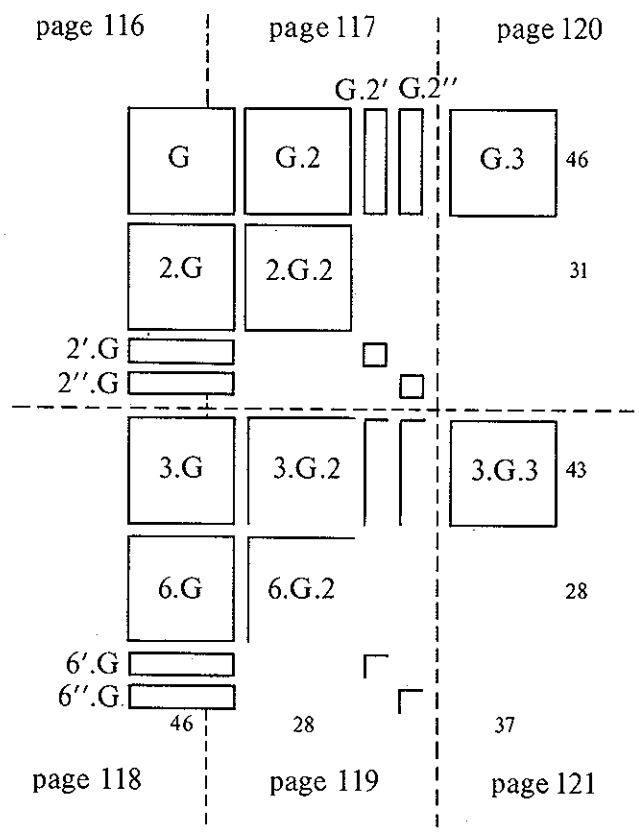
		13685760	155520	3888	512	82944	51840	4608	3456	1296	1152	576	432	192	144	24	648	81	54	1152	384	288	96	96	72	48	24	16	30	15	72	18	7	16	10	11	11	12	
		A	A	A	A	DA	EA	DB	EA	FA	EB	EB	FA	EC	FB	GC	B	B	B	NA	NB	QA	PF	QB	UA	TF	UB	TG	AE	AD	DA	FB	AG	PA	BAO	AD	BD	DA	
		A	A	A	A	DA	EA	DB	EA	FA	EB	EB	FA	EC	FB	GC	A	A	A	DA	DB	EA	DF	EB	FA	EF	FB	EG	AE	AD	DA	FA	AG	DA	BAO	AD	BD	DA	
	fus ind	3D	3E	3F	3G	6N	6O	6P	6Q	6R	6S	6T	6U	6V	6W	6X	9D	9E	9F	12O	12P	12Q	12R	12S	12T	12U	12V	12W	15B	15C	18D	18E	21A	24C	30B	33A	B*	36A	
χ_1	: +oo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	: +oo	-11	7	-2	1	5	5	-3	-1	-4	-3	3	2	1	0	1	-2	-2	1	-3	1	3	1	1	0	1	-2	-1	2	-1	2	-1	1	-1	0	0	0	0	χ_2
χ_3	: +oo	66	21	3	0	18	9	10	-3	9	1	1	3	-3	1	0	3	3	0	2	-2	5	2	1	-1	-1	1	-1	1	1	3	0	0	0	-1	0	0	-1	χ_3
χ_4	: +oo	54	27	0	0	6	15	-2	3	6	7	7	0	3	-2	0	0	0	0	6	2	3	-2	-1	0	1	2	1	2	-1	0	0	0	0	0	-1	-1	0	χ_4
χ_5	: +oo	55	10	-8	7	7	-20	-1	10	-2	-4	2	4	-4	2	-1	1	1	-2	7	3	-2	-1	0	-2	2	0	0	0	0	1	-2	0	1	0	0	0	1	χ_5
χ_6	: +oo	110	20	11	-1	30	30	6	12	3	6	0	3	2	3	-1	8	-1	2	6	2	0	2	2	3	2	-1	0	0	0	0	0	-1	0	0	0	0	0	χ_6
χ_7	: -oo	-100	-10	-1	-7	28	10	-4	-2	1	2	2	-5	-2	5	1	-7	2	-1	4	-4	-2	0	2	1	0	-1	0	0	0	1	1	0	0	0	-1	-1	1	χ_7
χ_8	: +oo	-44	46	1	7	-12	6	4	6	-3	-2	10	-3	2	1	-1	1	1	-2	-4	-4	2	0	2	-1	0	-1	0	1	1	-3	0	0	0	1	0	0	-1	χ_8
χ_9	: ooo	-55b27	15z3-10	3i3+5	-7	11z3-23	5z3+10	-13z3-7	-z3-2	-5i3+3	5z3+2	-z3+2	i3+3	b27**	-2z3**	1	-b27	-b27	2	b27	-z3+1	3z3-2	-z3-3	z3+2	-2	-z3	0	z3	0	0	z3-1	0	0	-z3**	0	0	0	z3**	χ_9
χ_{10}	: ooo	-55b27**	-15z3-25	-3i3+5	-7	-11z3-34	-5z3+5	13z3+6	z3-1	5i3+3	-5z3-3	z3+3	-i3+3	b27	-2z3	1	-b27**	-b27**	2	b27**	z3+2	-3z3-5	z3-2	-z3+1	-2	-z3**	0	z3**	0	0	-z3-2	0	0	-z3	0	0	0	z3	χ_{10}
χ_{11}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}																																							χ_{12}
χ_{13}																																							χ_{13}
χ_{14}	: +oo	-99	36	9	0	-3	24	13	-12	-3	-8	4	-3	0	1	0	0	0	0	-3	5	0	-3	-4	3	0	-1	0	1	1	0	0	0	1	-1	0	0	0	χ_{14}
χ_{15}	: +oo	-110	25	-2	7	2	-25	10	-7	2	7	1	2	-1	-2	-1	-2	-2	-2	-6	6	-3	-2	3	0	1	0	-1	0	0	2	2	0	0	0	0	0	0	χ_{15}
χ_{16}	: ooo	165i3+275	-10b27	15z3-1	-7	-35i3-5	-20i3-50	i3+7	i3+13	z3**	4i3-2	-2z3**	z3**	-2	z3**	1	9z3+8	-1	-1	i3+7	-2z3	-2z3**	-2z3	-2	z3**	-2z3	z3**	0	0	0	z3	1	0	0	0	0	0	z3	χ_{16}
χ_{17}	: ooo	-165i3+275	-10b27**	-15z3-16	-7	35i3-5	20i3-50	-i3+7	-i3+13	z3	-4i3-2	-2z3	z3	-2	z3	1	-9z3-1	-1	-1	-i3+7	-2z3**	-2z3	-2z3**	-2	z3	-2z3**	z3	0	0	0	z3**	1	0	0	0	0	0	z3**	χ_{17}
χ_{18}	: +oo	-440	100	-8	-8	40	40	-8	4	-14	-8	4	4	0	-2	0	1	1	1	-8	0	4	0	0	-2	0	0	0	0	0	1	1	-1	0	0	0	0	1	χ_{18}
χ_{19}	: +oo	330	105	15	0	-6	-15	2	-15	3	-7	5	3	-3	-1	0	-3	-3	0	10	-2	1	2	1	1	-1	1	-1	0	0	-3	0	0	0	0	0	0	1	χ_{19}
χ_{20}	: +oo	176	68	14	-7	-16	20	16	20	2	4	4	2	4	-2	1	-10	-1	-1	0	0	0	0	0	0	0	0	0	-2	1	2	-1	0	0	0	0	0	0	χ_{20}
χ_{21}	: +oo	594	54	0	0	66	-24	10	6	12	-8	-2	0	0	4	0	0	0	0	-6	6	6	-2	0	0	-2	0	0	-1	-1	0	0	0	0	1	0	0	0	χ_{21}
χ_{22}	: ooo	220b27**	30i3-20	-3z3+1	7	-26i3-62	-20i3+10	-2i3-6	-2i3+4	-z3+5	4i3-6	-2i3	-5z3**	2	-z3+1	-1	-3i3-5	b27	1	2i3+6	4z3	2i3	0	-2	z3-1	0	z3**	0	0	0	-2z3	1	0	0	0	0	0	0	χ_{22}
χ_{23}	: ooo	220b27	-30i3-20	3z3+4	7	26i3-62	20i3+10	2i3-6	2i3+4	z3+6	-4i3-6	2i3	-5z3	2	z3+2	-1	3i3-5	b27**	1	-2i3+6	4z3**	-2i3	0	-2	-z3-2	0	z3	0	0	0	-2z3**	1	0	0	0	0	0	0	χ_{23}
χ_{24}	: +oo	-495	45	18	0	33	-75	-7	-3	6	5	5	6	-3	2	0	0	0	0	-15	-3	-3	1	-3	0	1	0	1	0	0	0	0	0	-1	0	0	0	0	χ_{24}
χ_{25}	: +oo	144	144	-18	0	-48	24	-16	0	6	8	8	-6	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	1	1	0	χ_{25}
χ_{26}	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{26}
χ_{27}																																							χ_{27}
χ_{28}																																							χ_{28}
χ_{29}	: ooo	-495z3-990	-45i3	-9z3+9	0	-63z3+18	-45	9z3-6	3i3	9z3**	3	3i3	3z3-3	3	-3z3**	0	0	0	0	z3+2	5z3+6	-i3	z3+2	-1	z3-1	-i3	-z3**	-1	0	0	0	0	0	-z3	0	0	0	0	χ_{29}
χ_{30}	: ooo	495z3-495	45i3	9z3+18	0	63z3+81	-45	-9z3-15	-3i3	9z3	3	-3i3	-3z3-6	3	-3z3	0	0	0	0	-z3+1	-5z3+1	i3	-z3+1	-1	-z3-2	i3	-z3	-1	0	0	0	0	0	-z3**	0	0	0	0	χ_{30}
χ_{31}	+oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{31}
χ_{32}																																							χ_{32}
χ_{33}																																							χ_{33}
χ_{34}	: +oo	704	-16	20	8	64	64	0	16	-8	0	0	4	0	0	0	2	2	-1	0	0	0	0	0	0	0	0	0	-1	-1	-2	1	1	0	-1	0	0	0	χ_{34}
χ_{35}	: +oo	990	45	-9	0	78	15	22	-27	-3	7	1	3	3	1	0	0	0	0	6	2	-3	2	-1	-3	-1	-1	1	0	0	0	0	0	0	0	0	0	0	χ_{35}
χ_{36}	: +oo	264	84	12	0	-24	-24	-8	-12	-6	-8	4	0	0	-2	0	3	3	0	8	0	-4	0	0	2	0	0	0	-1	-1	3	0	0	0	1	0	0	-1	χ_{36}
χ_{37}	: +oo	891	81	0	0	27	-81	-21	9	0	-9	-3	0	3	0	0	0	0	0	3	3	-3	-1	3	0	-1	0	1	1	1	0	0	0	-1	-1	0	0	0	χ_{37}
χ_{38}	: +oo	891	81	0	0	27	81	-21	9	0	9	-3	0	-3	0	0	0	0	0	3	3	-3	-1	-3	0	-1	0	-1	1	1	0	0	0	-1	1	0	0	0	χ_{38}
χ_{39}	: ooo	495z3+990	45i3	9z3-9	0	-81z3-18	-45	15z3+6	-3i3	9z3**	3	-3i3	-3z3+3	3	-3z3**	0	0	0	0	-z3-2	-z3-6	i3	-z3-2	-1	-z3+1	i3	-z3**	-1	0	0	0	0	0	-z3	0	0	0	0	χ_{39}
χ_{40}	: ooo	-495z3+495	-45i3	-9z3-18	0	81z3+63	-45	-15z3-9	3i3	9z3	3	3i3	3z3+6	3	-3z3	0	0	0	0	z3-1	z3-5	-i3	z3-1	-1	z3+2	-i3	-z3	-1	0	0	0	0	0	-z3**	0	0	0	0	χ_{40}
χ_{41}	: +oo	880	-20	-2	7	48	-60	16	12	-6	4	4	-6	-4	-2	-1	-8	1	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{41}
χ_{42}	: ooo	-1215	0	0	0	81	0	9	0	0	0	0	0	0	0	0	0	0	0	9	-3	0	-3	0	0	0	0	0	0	0	0	0	0	-1	0	-b11	**	0	χ_{42}
χ_{43}	: ooo	-1215	0	0	0	81	0	9	0	0	0	0	0	0	0	0	0	0	0	9	-3	0	-3	0	0	0	0	0	0	0	0	0	0	-1	0	**	-b11	0	χ_{43}
χ_{44}	: +oo	-1024	128	-16	8	0	0	0	0	0	0	0	0	0	0	0	8	-1	2	0	0	0	0	0	0	0	0	0	-2	1	0	0	1	0	0	-1	-1	0	χ_{44}
χ_{45}	: +oo	-891	162	0	0	-27	0	21	18	0	0	-6	0	0	0	0	0	0	0	-3	-3	-6	1	0	0	-2	0	0	2	-1	0	0	0	1	0	0	0	0	χ_{45}
χ_{46}	: +oo	0	0	0	27	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	χ_{46}

Note:- The pagination is unorthodox. Consult the map!

$U_6(2)$

		3D	3E	3F	3G	6N	6O	6P	6Q	6R	6S	6T	6U	6V	6W	6X	9D	9E	9F	12O	12P	12Q	12R	12S	12T	12U	12V	12W	15B	15C	18D	18E	21A	24C	30B	33A	B*	36A	
	fus ind	3	3	3	9	6	6	6	6	6	6	6	6	6	6	18	9	9	9	12	12	12	12	12	12	12	12	12	15	15	18	18	63	24	30	33	33	36	
		3	3	3		6	6	6	6	6	6	6	6	6	6		9		9	12	12	12	12	12	12	12	12	12	15	15	18	18	63	24	30	33	33	36	
		3	3	3		6	6	6	6	6	6	6	6	6	6		9		9	12	12	12	12	12	12	12	12	12	15	15	18	18	63	24	30	33	33	36	
χ_{78}	:ooo2	513+6	-5z3-4	213	0	-313-2	-5z3-6	i3+2	3z3+4-2z3**	3z3+2	-z3	-2	-z3-2	2z3**	0-3z3**	0	i3	i3+2	-i3	-z3	-i3	-z3-2	2z3	z3+2	0	z3	1	1	z3**	1	0	1	-1	-z3	-z3	-z3**		χ_{78}
χ_{79}	:ooo2	55z3-10	513	7z3+5	0	23z3+6	513-10	7z3-2	i3-4	3z3-1	i3+2	-i3+2	-z3-3	-i3	3z3-1	0	6z3	0	i3	-z3-2	-z3+2	i3	3z3+2	i3-2	-z3+1	-1	z3**	1	0	0	2z3	-1	0	-z3	0	1	1	0	χ_{79}
χ_{80}	:ooo2	-55z3-44	-513-9	i3-6	0	-7z3-12	-10i3-4	z3+4	-513-1	-1-213-4	2z3**	i3+2	-213	-1	0 3z3**	0	z3-1-7z3-4-3z3-4-2z3**						z3	2	i3-2	-2z3	-1	0	1	1	-z3**	-z3**	0	-z3	1	0	0	z3**	χ_{80}
χ_{81}	:ooo2	-5513-66-35z3-28		-413	0	-1513-10-25z3-30		-313-6	-3z3-4	8z3**-9z3-6		-3z3	-4	-z3-2	0	0	6z3**	0	i3-313-6		-i3	-3z3	-i3	-z3-2	0	z3+2	0	-z3	2	-1	2z3**	-1	0	-1	0	0	0	0	χ_{81}
χ_{82}	:ooo2	2213-132	713-12	-213-9	0	613+28	-713+22	-213-4	313-8	i3+4-313+2	i3+2	1	-i3	-i3+2	0	-6	0	z3-1	213	213-4	-i3	2	i3+2	-i3	-1	-1	-1	-z3	-z3	-2	z3**	0	0	z3	0	0	0	χ_{82}
χ_{83}	:ooo2	11013	513-30	-5z3+5	0	-213-32	-30z3-5	6i3	i3-2	7z3+3-313+6	3i3	-z3+5	-i3-3z3**		0-3z3**	0	-z3-2	213-4		213	-i3-4	-2	-i3	-z3**	1	z3-1	1	0	0	z3**	z3	0	0	0	0	0	-z3**	χ_{83}
χ_{84}	:ooo2	3313-198	21z3+60	-313	0	913+18	-9z3	513-10	-3z3+12	-9	-z3	z3+4	-313	3z3	-1	0	0	0	0	i3-6	-i3-4	5z3+4	i3+2	-z3	i3	-z3	-1	z3	z3	z3	0	0	0	-1	z3	0	0	0	χ_{84}
χ_{85}	o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{85}
χ_{86}																																							χ_{86}
χ_{87}																																							χ_{87}
χ_{88}	:ooo2	275z3+220	-2013	-513-6	0	-13z3-36	-40z3**	3z3-4	413+8	5 8z3**	-4	i3-4	0-213-1		0	3z3	0	-i3	3z3+4-5z3-4		4	3z3	0	1	0	.3	0	0	0	-z3	-1	0	-z3	0	0	0	z3	χ_{88}
χ_{89}	:ooo2	-220z3+55	10i3+15	-7z3+1	0	36z3+7	20i3-35	213-3	-613+7	5z3-1-4i3-3	2i3+3-3z3+1		-3	z3-1	0	-6z3	0	z3+2-2i3+9		2i3+1-2i3+3		-1	1	-z3+1	-1	z3**	-1	0	0	-2z3	z3	0	1	0	0	0	0	χ_{89}
χ_{90}	:ooo2	10513+225	-3013	613-9	0	-1513+9-1513-45		-313+5	613+12	313 513-1		2	-3	-i3-3	i3+2	0	0	0	0	i3+9	2z3**	-2i3-2z3**		2z3	i3-2z3**		-1	0	0	0	0	0	0	0	z3	z3	0	χ_{90}
χ_{91}	:ooo2	-3313-297	-3z3+75	15z3+12	0	-913+15	15z3+45	-513+3	-3z3+3	3z3-6-z3-11	5z3+7		3z3	-z3+1	-z3-2	0	0	0	0	-i3-9-2z3**		z3-1	2z3**	b27	z3+2	-z3**		z3	z3**	2z3	-z3	0	0	0	0	0	0	0	χ_{91}
χ_{92}	:ooo2	-495	45	-9	0	81	45	-15	-3	-9	-3	-3	3	-3	3	0	-9	0	0	1	1	1	1	1	1	1	1	1	0	0	3	0	0	1	0	0	0	1	χ_{92}
χ_{93}	o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{93}
χ_{94}																																							χ_{94}
χ_{95}																																							χ_{95}
χ_{96}	:ooo2	8813+132	-213+72	-513-6	0	-813-28	213+8	813-12	-213-16-3i3+2		213	213	i3-4	-213	-i3	0	3	0	z3+2	4	-4	-2	0	2	1	0	-1	0	z3	z3	-1	-z3	0	0	-z3	0	0	1	χ_{96}
χ_{97}	:ooo2	-22013-330	513	7z3+14	0	-4413-10	513-10	-4i3-10	i3-4-3z3-4		i3+2	-i3+2	-z3+6	-i3-3z3-4		0 6z3**		0	-z3+1	4i3+6	-2	i3	2	i3-2	-z3-2	-1	z3	1	0	0	2z3**	-z3**	0	0	0	0	0	0	χ_{97}
χ_{98}	:ooo2	11013	513+60	-713-15	0	-213-32	15i3+10	6i3	i3+16	-8z3	3i3+6	3i3+6-2z3**		i3	0	0	-3	0	i3	2i3-4	213	-i3+2	-2	i3	2z3**	1	0	1	0	0	1	1	0	0	0	0	0	-1	χ_{98}
χ_{99}	:ooo2	-16513+99	1513+9	3z3-12	0	313-21-30i3+24		-i3-9	-18z3	9z3+6	213	-i3-3	3z3	-213	z3+2	0	0	0	0-513+3-2z3**			i3+3	2z3**	-2	-z3-2	2z3**		z3	0	-1	-1	0	0	0	0	-1	0	0	0	χ_{99}
χ_{100}	:ooo2	22013-540	10i3+30	413+6	0	441i3+20	-40	8z3	4z3-8	i3-7	8	-4z3	2i3-4	0	2z3**	0	3z3	0	z3-1	-8z3	0	4z3	0	0	-2z3	0	0	0	0	0	-z3	-z3**	0	0	0	-1	-1	z3	χ_{100}
χ_{101}	:ooo2	495	-45	9	0	63	45	-9	3	-9	-3	3	-3	-3	3	0	9	0	0	-1	-5	-1	-1	1	-1	-1	1	1	0	0	-3	0	0	1	0	0	0	-1	χ_{101}
χ_{102}	:ooo2	-11013-330-80i3-30		-7z3+1	0	1813+22-10i3-20		4z3-8	10-7z3-1-213-4-413+2-3z3+1					-213	-b27	0-6z3**		0	-i3-213-6		4z3	-213	0	-2	-z3+1	0	z3**	0	0	0-2z3**		1	0	0	0	0	0	0	χ_{102}
χ_{103}	:ooo2	-44013-330-50i3-30		413-9	0	-2413-58	10i3+20	-813-2	6i3+2-413-7		2i3+4	4z3**	-1	2i3	1	0	3	0	z3-1	-2	2	4	-2	2	1	-2	-1	0	0	0	-1	-z3**	0	0	0	0	0	1	χ_{103}
χ_{104}	:ooo2	-495z3-990	-4513	-9z3+9	0	-63z3+18	-45	9z3-6	313	9z3**	3	313	3z3-3		3-3z3**	0	0	0	0	z3+2	5z3+6	-i3	z3+2	-1	z3-1	-i3	-z3**	-1	0	0	0	0	0	-z3	0	0	0	0	χ_{104}
χ_{105}	:ooo2	-33013-594-1513+63		-613-9	0	-4213-18-1513-45		-1013-2	-6z3	313-313+7		-2z3	-3	-i3-3	i3-2	0	0	0	0	213-6	4z3**	-i3-3	0	-2z3	-i3	0	1	0	-2	1	0	0	0	0	0	0	0	0	χ_{105}
χ_{106}	o2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{106}
χ_{107}																																							χ_{107}
χ_{108}																																							χ_{108}
χ_{109}	:ooo2	35213-792	5213+48	i3-3	0	3213+8	1213-52	-8	-413+8	2z3-8z3**		4	2z3**	0	-2z3	0	3	0	-i3	-8	0	4	0	0	-2	0	0	0	-z3	-z3	-1	-1	0	0	-z3	0	0	1	χ_{109}
χ_{110}	:ooo2	11013+330	3513+75	-i3+18	0	-1813-22	-513-25	-4z3+8	-913-1-413-7		-i3-5-2z3**		3i3+2	i3-3	1	0	6z3	0	-z3-2	2i3+6	-4z3	-i3+3	0	2z3**	-i3	0	-1	0	0	0	2z3	-z3	0	0	0	0	0	0	χ_{110}
χ_{111}	:ooo2	891	81	0	0	27	-81	-21	9	0	-9	-3	0	3	0	0	0	0	0	3	3	-3	-1	3	0	-1	0	1	1	1	0	0	0	-1	-1	0	0	0	χ_{111}
χ_{112}	:ooo2	891	81	0	0	27	81	-21	9	0	9	-3	0	-3	0	0	0	0	0	3	3	-3	-1	-3	0	-1	0	-1	1	1	0	0	0	-1	1	0	0	0	χ_{112}
χ_{113}	:ooo2	495z3+990	4513	9z3-9	0	-81z3-18	-45	15z3+6	-313	9z3**	3	-313-3z3+3		3-3z3**		0	0	0	0	-z3-2	-z3-6	i3	-z3-2	-1	-z3+1	i3	-z3**	-1	0	0	0	0	0	-z3	0	0	0	0	χ_{113}
χ_{114}	:ooo2	-16013+864-40i3+24		213-6	0	-64z3**	-64	0	-16z3**	8z3	0	0-4z3**		0	0	0 6z3**		0	-z3-2	0	0	0	0	0	0	0	0	0	-1	-1	2z3**	-z3	0	0	1	-z3	-z3	0	χ_{114}
χ_{115}	:ooo2	-1215	0	0	0	81	0	9	0	0	0	0	0	0	0	0	0	0	0	9	-3	0	-3	0	0	0	0	0	0	0	0	0	0	-1	0-b11	**		0	χ_{115}
χ_{116}	:ooo2	-1215	0	0	0	81	0	9	0	0	0	0	0	0	0	0	0	0	0	9	-3	0	-3	0	0	0	0	0	0	0	0	0	0	-1	0	**-b11		0	χ_{116}
χ_{117}	:ooo2	-990	90	-18	0	18	0	-6	-6	0	0	-6	6	0	0	0	9	0	0	2	6	2	2	0	2	2	0	0	0	0	-3	0	0	0	0	0	0	-1	χ_{117}
χ_{118}	:ooo2	-35213-528	4413+36	1113-3	0	-3213-16	-40z3**	-16	8z3	313+5	8z3**	8	2z3	0	2z3	0	6	0	-z3-2	0	0	0	0	0	0	0	0	0-2z3	z3	2	-z3	0	0	0	0	0	0	χ_{118}
χ_{119}	:ooo2	-891	162	0	0	-27	0	21	18	0	0	-6	0	0	0	0	0	0	0	-3	-3	-6	1	0	0	-2	0	0	2	-1	0	0	0	1	0	0	0	0	χ_{119}
χ_{120}	:ooo2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	u'63	0	0	0	0	0	χ_{120}



R(27)

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	
		10073444472	19656	19683	1458	1458	54	54	28	81	81	81	26	26	26	26	26	26	28	28	28	19	19	19	26	26	26	26	26	26	37	37	37	37	37	37			1512	24	27	18	18	6	6	7	9	9	9	
	p power	A	A	A	A	A	CA	BA	A	A	A	A	A	A	A	A	A	A	AA	AA	AA	A	A	A	BA	CA	DA	EA	FA	AA	A	A	A	A	A	A			A	DA	A	B	C	EA	FB	AD	B	B	B	
	p' part	A	A	A	A	A	BA	CA	A	A	A	A	A	A	A	A	A	A	AA	AA	AA	A	A	A	AA	BA	CA	DA	EA	FA	A	A	A	A	A	A			A	DA	A	A	A	EA	FA	AD	A	A	A	
	ind	1A	2A	3A	3B	C**	6A	B**	7A	9A	9B	C**	13A	B*3	C*9	D*5	E*2	F*6	14A	B*3	C*9	19A	B*3	C*9	26A	B*3	C*9	D*5	E*11	F*7	37A	B*3	C*9	D*6	E*5	F*2	fus	ind	3D	6C	9D	9E	9F	18A	18B	21A	27A	B*4	C*7	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	+oo	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	703	-1	-26	1	1	-1	-1	3	1	1	1	1	1	1	1	1	1	-1	-1	-1	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	:	+oo	7	-1	-2	1	1	-1	-1	0	1	1	1	χ_2
χ_3	o	741	13	12	3b27	**	b27	**	-1	3	3z3	**	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	:	ooo	7z3-1	z3**	-i3	-i3	z3+2	1	z3	-1	0	0	0	χ_3
χ_4	o	741	13	12	**	3b27	**	b27	-1	3	**	3z3	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	:	ooo	-7z3-8	z3	i3	i3	-z3+1	1	z3**	-1	0	0	0	χ_4
χ_5	o	1443	-13	-15	3b27	**	-b27	**	1	3	3z3	**	0	0	0	0	0	0	1	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	:	ooo	7z3+8	-z3	-i3	z3-1	-i3	-z3**	-1	1	0	0	0	χ_5
χ_6	o	1443	-13	-15	**	3b27	**	-b27	1	3	**	3z3	0	0	0	0	0	0	1	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	:	ooo	-7z3+1	-z3**	i3	-z3-2	i3	-z3	-1	1	0	0	0	χ_6
χ_7	o	2184	0	-3	6b27	**	0	0	0	-3	-3z3	**	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	1	1	1	1	1	1	:	ooo	0	0	0	0	0	0	0	0	*4	*7	z9**-&5	χ_7
χ_8	o	2184	0	-3	**	6b27	0	0	0	-3	**	-3z3	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	1	1	1	1	1	1	:	ooo	0	0	0	0	0	0	0	0	*4	*7	z9-&4	χ_8
χ_9	+	13832	0	-19	8	8	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-f37	*3	*9	*6	*5	*2		+	0	0	0	0	0	0	0	0	0	0	0	χ_9
χ_{10}	+	13832	0	-19	8	8	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*9	-f37	*3	*2	*6	*5														χ_{10}
χ_{11}	+	13832	0	-19	8	8	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*3	*9	-f37	*5	*2	*6														χ_{11}
χ_{12}	+	13832	0	-19	8	8	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*6	*5	*2	-f37	*3	*9		+	0	0	0	0	0	0	0	0	0	0	0	χ_{12}
χ_{13}	+	13832	0	-19	8	8	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*2	*6	*5	*9	-f37	*3														χ_{13}
χ_{14}	+	13832	0	-19	8	8	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*5	*2	*6	*3	*9	-f37														χ_{14}
χ_{15}	+	18278	78	53	-1	-1	-3	-3	1	-1	-1	-1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	+oo	8	0	-1	2	2	0	0	1	-1	-1	-1	χ_{15}
χ_{16}	+	18278	-26	53	-1	-1	1	1	1	-1	-1	-1	0	0	0	0	0	0	-2y7-1	*3	*2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	+	18278	-26	53	-1	-1	1	1	1	-1	-1	-1	0	0	0	0	0	0	*2	-2y7-1	*3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0														χ_{17}
χ_{18}	+	18278	-26	53	-1	-1	1	1	1	-1	-1	-1	0	0	0	0	0	0	*3	*2	-2y7-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0														χ_{18}
χ_{19}	+	18981	-27	27	0	0	0	0	-3	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	:	+oo	21	-3	3	0	0	0	0	0	0	0	0	χ_{19}
χ_{20}	+	19683	27	0	0	0	0	0	-1	0	0	0	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	:	+oo	27	3	0	0	0	0	0	-1	0	0	0	χ_{20}
χ_{21}	+	19684	28	1	1	1	1	1	0	1	1	1	y13	*3	*9	*5	*2	*6	0	0	0	0	0	0	y13	*3	*9	*5	*2	*6	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{21}
χ_{22}	+	19684	28	1	1	1	1	1	0	1	1	1	*9	y13	*3	*6	*5	*2	0	0	0	0	0	0	*9	y13	*3	*6	*5	*2	0	0	0	0	0	0														χ_{22}
χ_{23}	+	19684	28	1	1	1	1	1	0	1	1	1	*3	*9	y13	*2	*6	*5	0	0	0	0	0	0	*3	*9	y13	*2	*6	*5	0	0	0	0	0	0														χ_{23}
χ_{24}	+	19684	28	1	1	1	1	1	0	1	1	1	*5	*2	*6	y13	*3	*9	0	0	0	0	0	0	*5	*2	*6	y13	*3	*9	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{24}
χ_{25}	+	19684	28	1	1	1	1	1	0	1	1	1	*6	*5	*2	*9	y13	*3	0	0	0	0	0	0	*6	*5	*2	*9	y13	*3	0	0	0	0	0	0														χ_{25}
χ_{26}	+	19684	28	1	1	1	1	1	0	1	1	1	*2	*6	*5	*3	*9	y13	0	0	0	0	0	0	*2	*6	*5	*3	*9	y13	0	0	0	0	0	0														χ_{26}
χ_{27}	+	19684	-28	1	1	1	-1	-1	0	1	1	1	y13	*3	*9	*5	*2	*6	0	0	0	0	0	0	-y13	*3	*9	*5	*2	*6	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{27}
χ_{28}	+	19684	-28	1	1	1	-1	-1	0	1	1	1	*9	y13	*3	*6	*5	*2	0	0	0	0	0	0	*9	-y13	*3	*6	*5	*2	0	0	0	0	0	0														χ_{28}
χ_{29}	+	19684	-28	1	1	1	-1	-1	0	1	1	1	*3	*9	y13	*2	*6	*5	0	0	0	0	0	0	*3	*9	-y13	*2	*6	*5	0	0	0	0	0	0														χ_{29}
χ_{30}	+	19684	-28	1	1	1	-1	-1	0	1	1	1	*5	*2	*6	y13	*3	*9	0	0	0	0	0	0	*5	*2	*6	-y13	*3	*9	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{30}
χ_{31}	+	19684	-28	1	1	1	-1	-1	0	1	1	1	*6	*5	*2	*9	y13	*3	0	0	0	0	0	0	*6	*5	*2	*9	-y13	*3	0	0	0	0	0	0														χ_{31}
χ_{32}	+	19684	-28	1	1	1	-1	-1	0	1	1	1	*2	*6	*5	*3	*9	y13	0	0	0	0	0	0	*2	*6	*5	*3	*9	-y13	0	0	0	0	0	0														χ_{32}
χ_{33}	+	26936	0	-37	-10	-10	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	-c19	*3	*9	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	χ_{33}
χ_{34}	+	26936	0	-37	-10	-10	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	*9	-c19	*3	0	0	0	0	0	0	0	0	0	0	0	0														χ_{34}
χ_{35}	+	26936	0	-37	-10	-10	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	*3	*9	-c19	0	0	0	0	0	0	0	0	0	0	0	0														χ_{35}

R(27) and $S_8(2)$

Ree group $R(27) \cong {}^2G_2(27)$

Order = 10,073,444,472 = $2^3.3^9.7.13.19.37$ Mult = 1 Out = 3

Constructions

Ree G : the centralizer in $G_2(27)$ of an outer automorphism of order 2;

G acts doubly transitively on $27^3 + 1 = 19684$ points, preserving a Steiner system S(2,28,19684) of 512487 blocks of 28 points, any two points being in a unique block.

The elements of the Sylow 3-group of R(q) can be parametrized by triples (x,y,z) of elements of $\mathbf{F}_q$ with the multiplication law

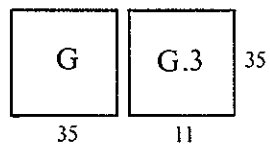
$$(x,y,z)(X,Y,Z) = (x + X , y + Y + xX^s - x^sX , z + Z + yX + x^sX^2 + xX^{s+1} - x^2X^s),$$

where s = 9 for q = 27, and $s = 3^{n+1}$ for $q = 3^{2n+1}$. The stabilizer of a point P is the Sylow 3-normalizer, and the above names can also be used for the points other than P, on which the Sylow 3-group acts by right multiplication. To obtain the normalizer, adjoin the elements taking (x,y,z) to (xt , yt^{s+1} , zt^{s+2}) for non-zero t in $\mathbf{F}_q$.

Maximal subgroups

Order	Index	Structure	G.3	Character	Abstract	Ree
511758	19684	3^{3+6}:26	: 3^{3+6}:26:3	1a+19683a	N($3A^3$)	point
19656	512487	2 x $L_2(27)$	: 2 x $L_2(27)$:3		N(2A)	$L_2(27)$, block
1512	6662331	$L_2(8)$:3	: $L_2(8)$:3 x 3		C(3D)	R(3)
222	45375876	37:6	: 37:18		N(37ABCDEF)	
168	59960979	(2^2 x D_{14}):3	: A_4 x 7:6		N($2A^2$), N(7A)	
114	88363548	19:6	: 19:18		N(19ABC)	

Specifications



Symplectic group $S_8(2) \cong B_4(2) \cong O_9(2)$

Order = 47,377,612,800 = $2^{16}.3^5.5^2.7.17$ Mult = 1 Out = 1

Constructions

Symplectic $Sp_8(2) \cong PSp_8(2) \cong S_8(2) \cong G$: all 8 x 8 matrices over $\mathbf{F}_2$ preserving a non-singular symplectic form.

The subgroup S_{10} is exhibited by taking the 8-dimensional space to be the even weight vectors in $\mathbf{F}_2^{10}$ modulo complementation, with symplectic form $\Sigma x_i y_i$.

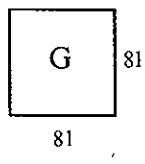
Orthogonal $GO_9(2) \cong PGO_9(2) \cong SO_9(2) \cong PSO_9(2) \cong O_9(2) \cong G$: all 9 x 9 matrices over $\mathbf{F}_2$ preserving a non-singular quadratic form. In the 10-coordinate notation above, the 120 triples {i,j,k} correspond to the minus hyperplanes $x_i+x_j+x_k=0$, while the 10+126 sets of size 1 or 5 (modulo complementation) correspond similarly to the plus hyperplanes.

Remark : G is a maximal subgroup of the Fischer group Fi_{23}. For a presentation, see page 232.

Maximal subgroups

Order	Index	Structure	Character	Abstract	Symplectic, Orthogonal
394813440	120	$O_8^-(2)$:2	1a+119a		$O_8^-(2)$, minus hyperplane
348364800	136	$O_8^+(2)$:2	1a+135a		$O_8^+(2)$, plus hyperplane
185794560	255	2^7:$S_6(2)$	1a+119a+135a	N(2A)	point
20643840	2295	2^{10}:A_8			maximal isotropic subspace
8847360	5355	2^{3+8}:(S_3xS_6)		N(2B)	isotropic line
8709120	5440	S_3 x $S_6(2)$		N(3A)	non-isotropic line
4128768	11475	2^{6+6}:(S_3x$L_3(2)$)			isotropic plane
3628800	13056	S_{10}			
1958400	24192	$S_4(4)$:2			$S_4(4)$
1036800	45696	(S_6 x S_6):2			$S_4(2)$ wr 2
2448	19353600	$L_2(17)$			

Specifications



$S_8(2)$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		47377612800	185794560	8847360	2949120	737280	147456	49152	4354560	77760	12960	3888	92160	92160	36864	12288	6144	6144	3072	3072	1024	768	512	512	3600	300	69120	13824	4608	4320	1728	1296	1152	864	864	432	288	288	288	288	144	96	
	p power		A	A	A	A	A	A	A	A	A	A	C	C	B	B	C	C	B	C	C	D	F	F	A	A	AA	AB	AC	CB	BB	DA	AE	CA	CB	DB	BD	CC	CD	CE	DE	CF	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	AC	CB	BB	DA	AE	CA	CB	DB	BD	CC	CD	CE	DE	CF	
	ind	1A	2A	2B	2C	2D	2E	2F	3A	3B	3C	3D	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	4K	4L	5A	5B	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	6N	6O	6P	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	35	-21	3	11	-5	-5	3	14	5	2	-1	-1	-9	7	-1	-5	3	-1	3	-1	-1	3	-1	5	0	-6	6	2	0	-3	-3	-2	0	0	3	1	2	-2	-2	1	0	χ_{2}
χ_{3}	+	51	-21	19	11	11	-5	3	6	6	6	-3	-9	-1	-1	7	3	-5	-1	3	-1	3	-1	3	1	1	-6	-2	2	4	-2	-3	-2	0	4	1	2	2	2	-2	1	0	χ_{3}
χ_{4}	+	85	-43	21	21	5	-11	5	22	-5	4	4	-11	-11	5	5	-3	-3	-3	5	-3	1	1	1	5	0	-10	6	6	6	3	2	-2	-4	0	0	-1	0	2	-2	-2	2	χ_{4}
χ_{5}	+	119	63	23	31	-1	15	7	35	-1	5	2	11	19	11	3	7	-1	3	7	3	-1	3	-1	9	-1	15	11	7	5	-1	0	3	3	-1	2	-1	1	-1	3	0	1	χ_{5}
χ_{6}	+	135	63	39	31	15	15	7	27	0	9	0	19	11	3	11	-1	7	3	7	3	3	-1	3	5	0	15	3	7	9	0	0	3	3	3	0	0	1	3	3	0	1	χ_{6}
χ_{7}	+	238	-98	-18	30	14	-2	-2	49	-5	4	-5	10	-22	14	-2	-6	-6	-2	2	2	-2	2	2	8	3	-11	9	-3	-6	3	1	1	4	0	3	-1	0	2	-2	1	-2	χ_{7}
χ_{8}	+	510	174	-2	46	30	14	14	69	15	-3	-3	-6	26	14	-2	10	10	-2	2	2	-2	2	2	5	0	9	13	1	-5	7	3	5	-3	1	1	3	1	3	-1	-1	-1	χ_{8}
χ_{9}	+	595	203	-13	43	-5	-5	-13	91	10	1	1	-5	35	23	-1	7	-1	-1	-1	-5	3	3	-1	10	0	11	11	-5	-1	2	5	-5	-1	-1	5	-2	1	1	1	1	-1	χ_{9}
χ_{10}	+	595	35	-45	35	51	3	3	-35	19	10	1	-5	-5	11	-5	3	3	-5	-5	3	-1	-1	7	0	5	5	-3	5	0	3	-1	-3	2	-6	-3	3	2	0	0	3	0	χ_{10}
χ_{11}	+	918	-90	150	38	54	6	6	-27	0	18	0	-22	10	6	22	-6	-6	6	2	2	6	2	2	-2	3	-15	-3	-7	0	0	0	-3	6	6	0	0	2	0	0	0	0	χ_{11}
χ_{12}	+	1190	182	166	54	70	22	22	-7	20	11	2	26	-6	6	22	10	10	6	2	2	6	2	2	-5	0	5	1	-3	1	4	2	1	-1	7	-2	4	3	1	1	-2	1	χ_{12}
χ_{13}	+	1275	195	155	35	35	-13	-21	15	15	15	6	35	-5	-9	15	-1	7	-9	-1	-5	-1	-1	3	0	0	15	-1	-1	5	-1	6	-1	-3	5	2	-1	-1	-1	-1	2	-3	χ_{13}
χ_{14}	+	1512	-504	-24	136	-24	-24	8	189	0	9	0	16	-64	24	-8	-8	8	0	8	0	0	0	0	17	-3	-39	21	1	-9	0	0	-3	3	3	0	0	1	-3	-3	0	-1	χ_{14}
χ_{15}	+	1785	-399	-103	81	65	1	-7	84	30	12	3	29	-11	-11	-3	9	-15	5	1	-3	-3	1	5	5	0	-24	-4	0	-10	-10	-3	4	0	-4	-1	2	0	2	-2	1	2	χ_{15}
χ_{16}	+	1785	441	57	121	-55	25	9	84	30	12	3	9	9	-7	-7	-15	-15	1	9	1	-3	-3	-3	5	0	36	-12	4	0	6	9	4	0	0	3	2	4	-4	4	1	0	χ_{16}
χ_{17}	+	2295	-657	87	143	15	-33	7	216	0	-9	0	-25	-65	27	3	-13	-5	3	3	-1	3	3	-1	10	0	-24	24	8	9	0	0	0	-3	-3	0	0	-1	3	3	0	1	χ_{17}
χ_{18}	+	2856	840	40	200	-24	40	8	273	-24	3	-6	0	80	24	-8	8	-8	0	8	0	0	0	0	16	1	45	25	5	-5	-8	-6	1	-3	1	-2	0	-1	-3	1	-2	-1	χ_{18}
χ_{19}	+	2856	-504	296	136	40	-24	8	21	-6	21	-6	-64	16	-8	24	8	-8	0	8	0	0	0	0	1	-4	-39	-19	1	11	2	0	-3	3	-1	2	-2	1	1	-3	0	-1	χ_{19}
χ_{20}	+	2975	-385	95	-65	15	31	-17	140	5	5	5	-25	-25	31	-1	-1	-1	7	-9	-1	-5	3	3	0	0	20	-4	4	5	5	-7	4	-7	5	5	-3	1	-3	1	1	1	χ_{20}
χ_{21}	+	2975	-665	255	135	55	-41	-1	140	5	5	5	-65	-25	-5	19	-5	-13	-5	3	-1	-1	-1	3	0	0	-20	12	12	15	-3	1	4	-5	3	-3	1	-3	1	1	1	-1	χ_{21}
χ_{22}	+	3213	693	-147	85	69	-27	-19	189	0	9	0	-35	37	9	1	-7	17	9	1	-3	-3	-3	1	8	3	21	-3	-11	-9	0	0	-3	-3	-3	0	0	1	3	3	0	-1	χ_{22}
χ_{23}	+	3213	-819	141	205	-51	-51	13	189	0	9	0	-35	-35	-3	-3	13	13	-3	13	-3	-3	-3	-3	8	3	-51	-3	13	9	0	0	-3	-3	-3	0	0	1	-3	-3	0	1	χ_{23}
χ_{24}	+	3400	840	200	200	40	72	40	190	10	-5	7	40	40	8	8	8	8	8	8	8	0	0	0	0	0	30	14	14	5	2	3	6	3	-1	-1	-2	-1	1	-3	3	1	χ_{24}
χ_{25}	+	3570	210	50	-110	66	-14	34	105	24	15	6	10	10	18	-14	2	2	-6	-6	2	6	-2	-2	0	5	-15	-7	1	5	8	-6	1	3	-1	2	0	-5	-3	1	-2	1	χ_{25}
χ_{26}	+	3570	882	178	178	-30	18	-30	231	15	6	6	50	50	2	2	-6	-6	-6	2	-6	-2	-2	-2	10	0	39	7	7	16	7	0	-9	6	-2	-2	3	-2	0	0	0	0	χ_{26}
χ_{27}	+	3570	-798	-142	98	50	34	-30	231	15	6	6	30	-50	18	2	6	-10	-6	-2	6	2	-2	-2	10	0	-21	-1	-13	-4	-1	-6	7	0	-4	2	-1	2	2	-2	-2	0	χ_{27}
χ_{28}	+	3808	-672	-32	96	-96	-32	32	112	40	7	10	16	-16	0	0	-16	16	0	0	0	0	0	0	-7	3	0	16	0	-5	-8	-6	-8	-3	1	-2	0	3	-3	1	-2	-1	χ_{28}
χ_{29}	+	4200	840	360	200	40	40	8	105	-30	15	-12	80	0	-8	24	-8	8	0	8	0	0	0	0	0	0	45	-15	5	15	-6	-6	1	-3	-3	0	-2	-1	1	1	-2	-1	χ_{29}
χ_{30}	+	4760	952	-104	56	-40	-40	-8	287	-10	-1	8	-16	64	40	8	8	-8	0	-8	0	0	0	0	5	0	-5	7	-13	1	-2	-2	-1	1	1	4	2	-1	-1	-1	2	1	χ_{30}
χ_{31}	+	5712	336	336	80	16	80	16	-84	-30	6	15	0	0	16	16	0	0	16	0	0	0	0	0	-8	-3	-36	12	-4	-6	-6	3	-4	6	-6	3	-2	2	-2	2	-1	-2	χ_{31}
χ_{32}	+	5950	-770	-130	190	-50	-34	14	-35	-35	10	10	30	30	-2	-2	6	6	6	-2	-10	2	2	2	0	0	-35	-19	13	-10	5	4	5	-2	2	2	1	-2	-2	2	-4	2	χ_{32}
χ_{33}	+	5950	-210	190	-210	30	14	14	175	-5	25	-8	-10	-10	38	-10	-2	-2	6	-10	-2	2	2	-6	0	0	15	-17	15	-5	-5	6	-1	-3	1	4	3	-3	3	-1	2	-1	χ_{33}
χ_{34}	+	7140	-252	484	4	100	-60	4	-42	-15	21	12	-4	-4	-12	20	-4	-4	-12	-4	-4	4	-4	-4	-5	0	30	22	-2	-11	1	0	6	-3	1	4	1	1	1	-3	0	1	χ_{34}
χ_{35}	+	8160	-672	224	96	160	-32	32	-48	60	-21	6	-16	16	0	0	16	-16	0	0	0	0	0	0	5	0	0	-16	0	-1	-4	-6	-8	-3	5	2	4	3	1	1	-2	-1	χ_{35}
χ_{36}	+	8925	1365	-195	245	85	37	13	105	-30	15	15	-55	-15	-23	-15	-11	13	-7	5	1	-3	1	5	0	0	45	-15	5	-15	-6	-3	1	3	3	3	-2	-1	1	1	1	1	χ_{36}
χ_{37}	+	8925	-315	125	165	-155	21	13	-210	60	15	-12	-15	25	33	-7	-19	5	9	-3	-7	-3	1	-3	0	0	-30	-10	-6	5	-4	0	6	3	-1	-4	4	3	1	-3	0	1	χ_{37}
χ_{38}	+	8960	-1792	-256	256	0	0	0	392	-40	14	-4	64	-64	0	0	0	0	0	0	0	0	0	0	5	0	-40	8	-8	-16	8	8	0	2	2	-4	0	-2	0	0	0	0	χ_{38}
χ_{39}	+	9639	567	-153	-9	-105	-9	39	0	81	0	0	-9	-9	-33	15	-9	-9	-9	-9	7	-1	7	-1	9	4	0	0	0	0	9	0	0	0	0	0	-3	0	0	0	0	0	χ_{39}
χ_{40}	+	10200	-1800	-40	120	-40	24	-8	435	30	-15	-6	0	-80	40	8	-8	8	0	-8	0	0	0	0	0	0	15	11	-9	5	-10	0	3	3	-1	2	2	-3	-1	3	0	1	χ_{40}
χ_{41}	+	11900	700	-260	60	220	-4	28	-70	65	20	-7	-20	-20	-20	-20	12	12	-4	-4	-4	4	4	4	0	0	10	10	-6	10	1	7	2	4	-8	1	1	0	-2	2	-1	-2	χ_{41}
χ_{42}	+	11900	-420	-260	220	-100	-36	-4	-280	35	5	11	20	20	44	12	4	4	-4	-12	4	4	-4	-4	0	0	0	-8	16	-5	-5	3	0	-3	1	-5	-1	1	-1	3	3	-1	χ_{42}
χ_{43}	+	13056	2304	-256	256	0	0	0	456	24	6	-12	-64	64	0	0	0	0	0	0	0	0	0	0	1	-4	24	8	-8	-16	8	0	0	-6	2	-4	0	-2	0	0	0	0	χ_{43}
χ_{44}	+	13600	160	800	160	160	32	32	-260	10	10	-8	0	0	32	32	0	0	0	0	0	0	0	0	0	0	-20	-4	-20	-10	2	-2	-4	-2	2	-4	-2	-2	-2	2	2	2	χ_{44}
χ_{45}	+	14280	840	-440	200	104	8	-24	-210	6	15	-3	-40	-40	8	8	8	8	-8	-8	8	0	0	0	0	-5	30	-2	14	-5	-2	3	-10	3	-5	1	2	-1	-1	-1	-1	3	χ_{45}
χ_{46}	+	14280	-1848	328	72	40	-56	40	294	-30	-21	-3	-40	-40	8	8	-8	-8	8	-8	-8	0	0	0	-10	0	54	22	6	1	10	-3	-2	3	-5	1	-2	3	1	1	1	1	χ_{46}
χ_{47}	+	14688	2016	96	224	-96	96	32	216	0	-9	0	-16	16	0	0	16	-16	0	0	0	0	0	0	-17	3	-24	24	8	-9	0	0	0	-3	-3	0	0	-1	-3	-3	0	-1	χ_{47}
χ_{48}	+	16065	1953	129	-31	-15	-63	-47	378	0	-9	0	41	41	33	1	1	1	9	-7	1	-3	5	5	-5	0	-30	-6	2	9	0	0	-6	3	3	0	0	-1	-3	-3	0	1	χ_{48}
χ_{49}	+	16065	1449	33	329	-135	57	33	-189	0	18	0	-19	-59	21	-19	9	-15	-3	1	-3	-3	1	-3	-5	0	39	-21	-1	0	0	0	3	6	6	0	0	2	0	0	0	0	χ_{49}
χ_{50}	+	16065	441	417	89	-135	-87	-15	-189	0	18	0	41	-31	21	-3	21	-3	-3	5	1	-3	5	1	-5	0	51	3	-13	0	0	0	3	-6	-6	0	0	2	0	0	0	0	χ_{50}
χ_{51}	+	16065	-2583	-159	329	105	-39	17	378	0	-9	0	41	-31	-3	-11	5	-19	-3	5	1	-3	-3	1	-5	0	-30	-6	2	-9	0	0	-6	3	3	0	0	-1	3	3	0	-1	χ_{51}
χ_{52}	+	16065	-1071	-447	209	225	-15	-15	-189	0	18	0	41	41	9	-7	-15	-15	9	-7	1	-3	-3	5	-5	0	-21	3	11	0	0	0	3	-6	-6	0	0	2	0	0	0	0	χ_{52}
χ_{53}	+	17850	2730	-70	170	90	-22	10	525	-15	-15	3	-10	70	10	-6	-2	14	-14	-10	-2	2	-2	-2	0	0	-15	5	-7	5	-7	3	5	3	-1	-1	-3	5	3	-1	-1	1	χ_{53}
χ_{54}	+	17850	-630	570	10	-70	74	-54	-105	30	30	3	-50	30	-22	-6	-10	6	2	-2	6	-6	-2	-2	0	0	-45	15	-5	0	6	-9	-1	0	0	3	2	-2	2	2	-1	0	χ_{54}
χ_{55}	+	18360	792	696	-104	120	-72	-40	27	0	9	0	64	-16	-24	8	-8	8	0	-8	0	0	0	0	5	0	15	3	7	-9	0	0	3	3	3	0	0	1	-3	-3	0	-1	χ_{55}
χ_{56}	+	19040	2016	352	224	160	96	32	56	20	-37	-4	16	-16	0	0	-16	16	0	0	0	0	0	0	-5	0	-24	-8	8	-5	4	0	0	-3	1	4	4	-1	1	-3	0	-1	χ_{56}
χ_{57}	+	23800	-1960	760	-40	120	-8	-40	175	40	-5	-14	-80	0	-24	8	8	-8	0	-8	0	0	0	0	0	0	35	7	11	-5	-8	2	7	5	1	-2	0	-1	-3	1	-2	-1	χ_{57}
χ_{58}	+	23800	-3080	120	120	-40	56	-40	490	-50	-5	13	-40	-40	-8	-8	8	8	8	-8	8	0	0	0	0	0	10	-6	-6	15	6	7	2	-5	3	-3	2	3	-1	-1	-1	-1	χ_{58}
χ_{59}	+	26775	-3465	-105	375	-105	-9	-9	315	45	0	-9	15	15	-41	-9	15	15	7	-1	-1	7	-1	-1	0	0	-45	-21	3	0	-3	-9	3	0	0	3	-3	0	0	0	3	0	χ_{59}
χ_{60}	+	26775	1575	-105	-345	-105	-57	39	315	45	0	-9	15	15	31	15	-9	-9	-1	-1	7	-5	-5	3	0	0	-45	-21	3	0	-3	9	3	0	0	3	-3	0	0	0	-3	0	χ_{60}
χ_{61}	+	28560	336	-368	336	80	16	16	-546	-60	12	-6	-16	-16	48	-16	0	0	0	-16	0	0	0	0	5	0	6	-2	6	10	4	-6	-2	0	4	-2	-4	0	2	-2	-2	-2	χ_{61}
χ_{62}	+	28917	-2835	405	333	-99	-99	21	0	0	0	0	-27	45	9	-15	-15	9	-15	9	5	-3	5	1	-18	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{62}
χ_{63}	+	30464	-1792	768	256	0	0	0	-280	-64	-10	8	-64	64	0	0	0	0	0	0	0	0	0	0	9	4	-40	-24	-8	0	0	8	0	2	-6	0	0	-2	0	0	0	0	χ_{63}
χ_{64}	+	32130	2898	642	82	-30	18	-14	189	0	9	0	70	-10	-30	-14	14	-2	-6	-10	-2	-6	-2	-2	-10	0	-39	21	1	9	0	0	-3	3	3	0	0	1	3	3	0	1	χ_{64}
χ_{65}	+	32130	882	258	-398	-30	18	-46	189	0	9	0	10	10	42	-6	2	2	-6	10	2	6	6	-2	-10	0	-51	-3	13	-9	0	0	-3	-3	-3	0	0	1	3	3	0	-1	χ_{65}
χ_{66}	+	32130	-2142	-318	-158	210	66	-14	189	0	9	0	10	10	18	-14	-14	-14	-6	10	2	6	-2	-2	-10	0	21	-3	-11	9	0	0	-3	-3	-3	0	0	1	-3	-3	0	1	χ_{66}
χ_{67}	+	34425	4185	-135	345	-135	-39	-39	405	0	0	0	-15	-15	-39	-7	-15	-15	9	1	1	9	1	1	0	0	45	-27	-3	0	0	0	-3	0	0	0	0	0	0	0	0	0	χ_{67}
χ_{68}	+	34425	-2295	-135	-375	-135	105	9	405	0	0	0	-15	-15	33	17	9	9	-15	1	-7	-3	-3	5	0	0	45	-27	-3	0	0	0	-3	0	0	0	0	0	0	0	0	0	χ_{68}
χ_{69}	+	34560	2304	768	256	0	0	0	-216	0	-18	0	64	-64	0	0	0	0	0	0	0	0	0	0	5	0	24	-24	-8	0	0	0	0	-6	-6	0	0	-2	0	0	0	0	χ_{69}
χ_{70}	+	38080	2240	-320	-320	64	-64	64	280	-44	10	10	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	-40	-8	-8	10	4	-10	8	2	-2	-2	4	-2	-2	2	2	-2	χ_{70}
χ_{71}	+	38556	2268	-612	-36	156	-36	-36	0	81	0	0	-36	-36	12	12	12	12	12	12	-4	-4	-4	-4	-9	1	0	0	0	0	9	0	0	0	0	0	-3	0	0	0	0	0	χ_{71}
χ_{72}	+	42525	-2835	-675	45	45	45	-3	0	0	0	0	45	45	-27	21	-3	-3	-3	-3	-3	-3	5	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{72}
χ_{73}	+	42525	-2835	-675	45	45	45	-3	0	0	0	0	45	45	-27	21	-3	-3	-3	-3	-3	-3	5	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{73}
χ_{74}	+	43520	-512	512	-512	0	0	0	176	-40	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	10	0	16	-16	16	-16	8	-8	0	4	-4	-4	0	4	0	0	0	0	χ_{74}
χ_{75}	+	47600	-1680	240	-400	-80	-16	48	140	50	20	17	0	0	-16	-16	0	0	16	0	0	0	0	0	0	0	60	12	-4	0	-6	3	-4	0	0	-3	-2	-4	4	-4	-1	0	χ_{75}
χ_{76}	+	48195	2835	-765	-45	51	-45	3	0	-81	0	0	-45	-45	-21	27	3	3	3	3	3	-5	3	-5	0	5	0	0	0	0	-9	0	0	0	0	0	3	0	0	0	0	0	χ_{76}
χ_{77}	+	51408	-1008	-48	16	144	-48	-48	-378	0	-18	0	16	16	48	-16	0	0	0	16	0	0	0	0	-7	3	30	6	-2	0	0	0	6	6	6	0	0	-2	0	0	0	0	χ_{77}
χ_{78}	+	53550	-1890	-210	30	-210	-66	30	-315	-45	0	-18	30	30	-10	6	6	6	6	-2	6	2	-6	2	0	0	45	21	-3	0	3	0	-3	0	0	6	3	0	0	0	0	0	χ_{78}
χ_{79}	+	57120	672	288	-352	160	32	32	-84	-30	6	-12	0	0	-32	-32	0	0	0	0	0	0	0	0	10	0	-36	12	-4	6	-6	6	-4	-6	6	0	-2	2	-2	2	2	2	χ_{79}
χ_{80}	+	65536	0	0	0	0	0	0	-512	64	-32	16	0	0	0	0	0	0	0	0	0	0	0	0	16	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{80}
χ_{81}	+	68850	1890	-270	-30	-270	66	-30	-405	0	0	0	-30	-30	-6	10	-6	-6	-6	2	-6	6	-2	6	0	0	-45	27	3	0	0	0	3	0	0	0	0	0	0	0	0	0	χ_{81}

$S_8(2)$

	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
	42	384	384	128	128	32	32	54	27	240	240	80	20	1	576	576	384	144	144	144	96	72	48	48	24	24	14	90	90	15	17	17	18	40	40	21	48	48	30	30		
	A	C	D	C	D	K	L	D	D	AA	AB	AC	BD	BC	CA	CB	BD	EC	LA	LB	CH	JC	LE	LF	JG	KJ	AA	AA	AC	BB	A	A	AF	CA	CB	AA	AA	DB	AAA	BBD		
	A	A	A	A	A	A	A	A	A	AA	AB	AC	BD	AC	AA	AB	AD	BC	CA	CB	AH	DC	CE	CF	DG	BJ	AA	AA	AC	BB	A	A	AA	AA	AB	AA	AA	AB	AAA	BBD		
	7A	8A	8B	8C	8D	8E	8F	9A	9B	10A	10B	10C	10D	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	14A	15A	15B	15C	17A	B*	18A	20A	20B	21A	24A	24B	30A	30B		
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	0	-3	1	1	1	-1	1	2	-1	-1	3	1	0	-2	-4	0	2	1	2	0	0	1	-2	0	-1	-1	0	-1	2	0	1	1	0	-1	1	0	0	-2	-1	0	χ_2
χ_3	2	1	-3	1	1	1	-1	0	0	-1	-1	1	1	2	0	-4	-2	2	0	2	0	-1	0	-2	-1	0	0	1	1	1	0	0	0	1	-1	-1	-2	0	-1	-1	χ_3
χ_4	1	-3	-3	1	1	-1	-1	1	1	-3	1	1	0	2	-2	-2	2	-1	-2	-2	2	2	0	0	0	1	-1	2	-1	0	0	0	-1	-1	-1	1	0	0	0	1	χ_4
χ_5	0	5	1	1	1	1	-1	2	-1	3	3	1	-1	-1	5	1	3	-1	-1	1	1	2	1	-1	0	-1	0	0	0	-1	0	0	0	1	-1	0	-1	1	0	0	χ_5
χ_6	2	1	5	1	1	-1	1	0	0	3	-1	1	0	3	1	5	-1	0	1	-1	1	0	-1	1	0	0	0	2	-1	0	-1	-1	0	-1	1	-1	1	-1	0	-1	χ_6
χ_7	0	-2	2	-2	2	0	0	1	1	2	2	0	-1	5	-5	-1	1	-1	-2	2	-1	-1	0	0	1	1	0	-1	-1	0	0	0	1	0	-2	0	1	-1	-1	-1	χ_7
χ_8	-1	2	-2	2	-2	0	0	0	0	-1	3	1	0	5	3	-1	1	-1	3	-1	-1	-1	1	1	1	1	-1	-1	2	0	0	0	0	-1	1	-1	-1	1	-1	0	χ_8
χ_9	0	1	1	-3	1	-1	1	1	1	-2	2	-2	0	-1	7	-1	-1	2	1	-1	-1	-1	1	-1	-1	0	0	1	1	0	0	0	-1	0	0	0	1	1	1	-1	χ_9
χ_{10}	0	-1	-1	-1	-1	1	1	1	1	0	0	0	1	-7	1	1	1	-1	-2	-2	1	-1	0	0	1	-1	0	0	0	-1	0	0	-1	0	0	0	-1	-1	0	0	χ_{10}
χ_{11}	1	2	-2	2	-2	0	0	0	0	0	0	-2	-1	-3	-1	-5	1	0	2	-2	-1	0	0	0	0	0	1	-2	-2	0	0	0	0	-2	0	1	-1	1	0	0	χ_{11}
χ_{12}	0	-2	2	-2	2	0	0	-1	-1	-3	1	-1	0	-3	-1	3	1	0	-1	3	-1	0	1	1	0	0	0	-2	1	0	0	0	-1	1	-1	0	1	-1	0	1	χ_{12}
χ_{13}	1	1	1	1	-3	1	-1	0	0	0	0	0	0	3	-1	7	3	3	-1	1	-1	0	-1	1	0	-1	-1	0	0	0	0	0	0	0	0	1	1	1	0	0	χ_{13}
χ_{14}	0	-4	4	0	0	0	0	0	0	1	1	1	1	-3	-5	-1	1	0	1	-1	-1	0	1	-1	0	0	0	-1	-1	0	-1	-1	0	1	1	0	-1	1	1	1	χ_{14}
χ_{15}	0	-1	3	-1	-1	1	-1	0	0	1	-3	1	0	4	2	-2	0	-2	2	-2	-2	1	0	0	-1	0	0	-1	2	0	0	0	0	-1	-1	0	2	0	1	0	χ_{15}
χ_{16}	0	-3	-3	1	1	-1	-1	0	0	1	-3	1	0	-4	0	0	-4	2	0	0	0	-1	0	0	1	0	0	-1	2	0	0	0	0	-1	-1	0	0	0	1	0	χ_{16}
χ_{17}	-1	-3	-3	1	-3	1	-1	0	0	-2	2	-2	0	0	-4	4	0	0	-1	1	0	0	-1	1	0	0	1	1	1	0	0	0	0	0	0	-1	0	0	1	-1	χ_{17}
χ_{18}	0	4	-4	0	0	0	0	0	0	0	0	0	1	-3	3	-1	1	0	-3	-1	-1	0	-1	1	0	0	0	-2	-2	1	0	0	0	0	0	0	1	-1	0	0	χ_{18}
χ_{19}	0	4	-4	0	0	0	0	0	0	1	1	1	0	1	-1	-5	-3	-2	-1	1	-1	-2	-1	1	0	0	0	1	1	-1	0	0	0	1	1	0	1	-1	1	1	χ_{19}
χ_{20}	0	3	3	-1	-1	1	1	-1	-1	0	0	0	0	4	-4	-4	-4	1	-1	-1	0	1	-1	-1	1	1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	χ_{20}
χ_{21}	0	-3	-3	-3	1	-1	1	-1	-1	0	0	0	0	4	4	-4	4	1	1	-1	0	1	1	-1	1	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	χ_{21}
χ_{22}	0	-1	-1	-1	3	1	-1	0	0	-2	-2	0	-1	9	1	1	1	0	1	1	1	0	-1	-1	0	0	0	-1	-1	0	0	0	0	0	2	0	-1	-1	1	1	χ_{22}
χ_{23}	0	1	1	1	1	1	1	0	0	-4	-4	0	-1	-3	1	1	-3	0	1	1	1	0	1	1	0	0	0	-1	-1	0	0	0	0	0	0	0	1	1	-1	-1	χ_{23}
χ_{24}	-2	0	0	0	0	0	0	-2	1	0	0	0	0	2	-2	-2	2	2	1	1	2	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	χ_{24}
χ_{25}	0	-2	-2	2	2	0	0	0	0	0	0	0	1	9	1	1	1	0	1	1	-3	0	-1	-1	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	0	0	χ_{25}
χ_{26}	0	2	2	-2	-2	0	0	0	0	2	-2	-2	0	-1	-1	-1	-1	-1	2	2	-1	2	0	0	0	1	0	1	1	0	0	0	0	0	0	0	-1	-1	-1	1	χ_{26}
χ_{27}	0	2	-2	-2	2	0	0	0	0	2	-2	-2	0	3	-3	1	-1	3	0	-2	1	0	0	2	0	-1	0	1	1	0	0	0	0	0	0	0	-1	1	-1	1	χ_{27}
χ_{28}	0	0	0	0	0	0	0	-2	1	3	3	1	-1	0	4	-4	0	0	1	-1	0	0	-1	1	0	0	0	2	2	0	0	0	0	1	-1	0	0	0	0	0	χ_{28}
χ_{29}	0	-4	4	0	0	0	0	0	0	0	0	0	0	1	-1	3	-3	-2	-1	-3	-1	-2	1	-1	0	0	0	0	0	0	1	1	0	0	0	0	-1	1	0	0	χ_{29}
χ_{30}	0	-4	4	0	0	0	0	-1	-1	-3	1	1	0	-5	5	1	-1	-2	-1	1	1	-2	-1	1	0	0	0	2	-1	0	0	0	1	-1	-1	0	-1	1	0	1	χ_{30}
χ_{31}	0	0	0	0	0	0	0	0	0	-4	-4	0	1	4	0	0	4	-2	0	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	-1	-1	χ_{31}
χ_{32}	0	2	2	-2	-2	0	0	1	1	0	0	0	0	1	-3	-3	1	1	0	0	1	-2	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	-1	-1	0	0	χ_{32}
χ_{33}	0	2	2	2	2	0	0	-2	1	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	2	1	1	0	-1	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	χ_{33}
χ_{34}	0	0	0	0	0	0	0	0	0	3	-1	-1	0	-6	2	2	2	-3	-1	-1	2	0	-1	-1	0	1	0	-2	1	0	0	0	0	1	1	0	0	0	0	-1	χ_{34}
χ_{35}	-2	0	0	0	0	0	0	0	0	3	-1	1	0	0	-4	4	0	0	-1	1	0	0	1	-1	0	0	0	2	-1	0	0	0	0	-1	1	1	0	0	0	-1	χ_{35}
χ_{36}	0	-1	-5	-1	-1	-1	1	0	0	0	0	0	0	1	-1	3	-3	-2	-1	3	-1	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	χ_{36}
χ_{37}	0	3	-1	-1	-1	-1	1	0	0	0	0	0	0	6	0	4	2	0	-3	1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	χ_{37}
χ_{38}	0	0	0	0	0	0	0	-1	-1	3	-1	1	0	0	4	-4	0	0	-2	2	0	0	0	0	0	0	0	2	-1	0	1	1	-1	-1	1	0	0	0	0	-1	χ_{38}
χ_{39}	0	3	3	-1	-1	-1	-1	0	0	-3	-3	1	0	0	0	0	0	-3	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	χ_{39}
χ_{40}	1	4	-4	0	0	0	0	0	0	0	0	0	0	-5	-3	1	-1	-2	3	1	1	-2	1	-1	0	0	-1	0	0	0	0	0	0	0	0	1	1	-1	0	0	χ_{40}
χ_{41}	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-2	-2	-2	-2	1	-2	-2	2	1	0	0	-1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	χ_{41}
χ_{42}	0	0	0	0	0	0	0	2	-1	0	0	0	0	8	-4	-4	0	-1	-1	-1	0	-1	1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{42}
χ_{43}	1	0	0	0	0	0	0	0	0	-1	-1	1	0	0	-4	4	0	0	2	-2	0	0	0	0	0	0	1	1	1	-1	0	0	0	1	-1	1	0	0	-1	-1	χ_{43}
χ_{44}	-1	0	0	0	0	0	0	1	1	0	0	0	0	-4	0	0	-4	2	0	0	0	2	0	0	0	0	-1	0	0	0	0	0	1	0	0	-1	0	0	0	0	χ_{44}
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	2	2	2	2	-1	-1	-2	-1	-1	-1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}	0	0	0	0	0	0	0	0	0	2	-2	2	0	2	2	2	2	2	-1	-1	-2	-1	1	1	-1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	-1	1	χ_{46}
χ_{47}	2	0	0	0	0	0	0	0	0	1	1	-1	-1	0	-4	4	0	0	-1	1	0	0	1	-1	0	0	0	1	1	0	0	0	0	-1	1	-1	0	0	1	1	χ_{47}
χ_{48}	0	-3	-3	1	1	-1	-1	0	0	3	-1	-1	0	6	2	2	-2	0	-1	-1	2	0	1	1	0	0	0	-2	1	0	0	0	0	1	1	0	0	0	0	-1	χ_{48}
χ_{49}	0	-5	-1	-1	-1	1	-1	0	0	-1	3	-1	0	3	5	1	-1	0	2	-2	1	0	0	0	0	0	0	1	-2	0	0	0	0	1	1	0	1	-1	-1	0	χ_{49}
χ_{50}	0	-1	-1	3	-1	-1	1	0	0	1	-3	-1	0	3	-1	-1	3	0	2	2	-1	0	0	0	0	0	0	1	-2	0	0	0	0	1	-1	0	-1	-1	1	0	χ_{50}
χ_{51}	0	3	3	3	-1	-1	1	0	0	-3	1	-1	0	6	2	2	-2	0	-1	-1	2	0	-1	-1	0	0	0	-2	1	0	0	0	0	1	-1	0	0	0	0	1	χ_{51}
χ_{52}	0	1	1	1	1	-1	-1	0	0	-1	3	-1	0	-9	-1	-1	-1	0	2	2	-1	0	0	0	0	0	0	1	-2	0	0	0	0	1	1	0	1	1	-1	0	χ_{52}
χ_{53}	0	-2	2	2	-2	0	0	0	0	0	0	0	0	1	-1	-5	-3	1	-1	1	-1	1	1	-1	1	-1	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	χ_{53}
χ_{54}	0	-2	2	2	-2	0	0	0	0	0	0	0	0	-1	1	-3	3	2	-2	0	1	-1	2	0	-1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	χ_{54}
χ_{55}	-1	4	-4	0	0	0	0	0	0	-3	1	1	0	3	1	5	-1	0	1	-1	1	0	1	-1	0	0	1	2	-1	0	0	0	0	-1	-1	-1	1	-1	0	1	χ_{55}
χ_{56}	0	0	0	0	0	0	0	2	-1	1	-3	-1	0	0	4	-4	0	0	1	-1	0	0	-1	1	0	0	0	1	-2	0	0	0	0	1	-1	0	0	0	1	0	χ_{56}
χ_{57}	0	-4	4	0	0	0	0	1	1	0	0	0	0	3	1	-3	-1	0	1	3	1	0	-1	1	0	0	0	0	0	0	0	0	-1	0	0	0	-1	1	0	0	χ_{57}
χ_{58}	0	0	0	0	0	0	0	1	1	0	0	0	0	-2	2	2	-2	-2	-1	-1	-2	1	-1	-1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	χ_{58}
χ_{59}	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	-5	3	3	3	1	0	0	-1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	χ_{59}
χ_{60}	0	-1	-1	-1	-1	1	1	0	0	0	0	0	0	-5	3	3	3	1	0	0	-1	1	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	χ_{60}
χ_{61}	0	0	0	0	0	0	0	0	0	1	-3	1	0	-6	2	2	2	0	2	2	2	0	0	0	0	0	0	-1	2	0	0	0	0	-1	-1	0	0	0	1	0	χ_{61}
χ_{62}	0	3	3	-1	3	1	-1	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	χ_{62}
χ_{63}	0	0	0	0	0	0	0	-1	-1	3	3	1	0	0	-4	4	0	0	2	-2	0	0	0	0	0	0	0	0	0	1	0	0	-1	1	-1	0	0	0	0	0	χ_{63}
χ_{64}	0	2	-2	-2	2	0	0	0	0	-2	2	2	0	-3	-5	-1	1	0	1	-1	-1	0	-1	1	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	1	1	-1	χ_{64}
χ_{65}	0	-2	-2	-2	-2	0	0	0	0	2	-2	2	0	-3	1	1	-3	0	1	1	1	0	-1	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	1	1	-1	1	χ_{65}
χ_{66}	0	2	2	-2	-2	0	0	0	0	-2	2	2	0	9	1	1	1	0	1	1	1	0	1	1	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	-1	1	-1	χ_{66}
χ_{67}	-1	1	1	1	1	1	1	0	0	0	0	0	0	-3	-3	-3	5	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	1	1	0	0	χ_{67}
χ_{68}	-1	1	1	1	1	-1	-1	0	0	0	0	0	0	-3	-3	-3	5	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	1	1	0	0	χ_{68}
χ_{69}	1	0	0	0	0	0	0	0	0	-1	3	1	0	0	4	-4	0	0	-2	2	0	0	0	0	0	0	1	-1	2	0	-1	-1	0	-1	1	1	0	0	-1	0	χ_{69}
χ_{70}	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	0	χ_{70}
χ_{71}	0	0	0	0	0	0	0	0	0	3	3	-1	1	0	0	0	0	-3	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	-1	-1	0	0	0	0	0	χ_{71}
χ_{72}	0	-3	-3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b17	*	0	0	0	0	0	0	0	0	χ_{72}
χ_{73}	0	-3	-3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*	b17	0	0	0	0	0	0	0	0	χ_{73}
χ_{74}	1	0	0	0	0	0	0	-1	-1	-2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	1	0	0	1	0	0	1	-1	χ_{74}
χ_{75}	0	0	0	0	0	0	0	2	-1	0	0	0	0	-4	0	0	-4	2	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{75}
χ_{76}	0	3	3	-1	-1	-1	-1	0	0	0	0	0	1	0	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	χ_{76}
χ_{77}	0	0	0	0	0	0	0	0	0	-3	-3	1	-1	-6	-2	-2	2	0	-2	-2	-2	0	0	0	0	0	0	2	2	0	0	0	0	1	1	0	0	0	0	0	χ_{77}
χ_{78}	0	-2	-2	-2	-2	0	0	0	0	0	0	0	0	5	-3	-3	-3	-1	0	0	1	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	χ_{78}
χ_{79}	0	0	0	0	0	0	0	0	0	2	-2	-2	0	4	0	0	4	-2	0	0	0	-2	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	1	0	χ_{79}
χ_{80}	2	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-1	1	1	0	0	0	-1	0	0	0	0	χ_{80}
χ_{81}	-2	2	2	2	2	0	0	0	0	0	0	0	0	3	3	3	-5	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	χ_{81}

Ru

Sporadic Rudvalis group Ru

Order = 145,926,144,000 = $2^{14}.3^3.5^3.7.13.29$ Mult = 2 Out = 1

Constructions

Lattice 4.G = 2.(2 x G) : the automorphism group of a certain 28-dimensional lattice over $\mathbb{Z}[i]$. There is a monomial group $(2^6{:}U_3(3)){:}2 \cong 2^6{\cdot}G_2(2)$ which may be constructed as follows. Modulo scalars, there are 28 non-zero isotropic vectors v in a unitary 3-space over $\mathbb{F}_9$. The square of a non-zero element c of $\mathbb{F}_9$ is a fourth root of unity which can be lifted to the complex number $c^{[2]} = \pm 1$ or $\pm i$, and we take coordinate vectors e_v with the understanding that $e_{cv} = c^{[2]}e_v$. Using the Hermitian form $x_1\bar{x}_3+x_2\bar{x}_2+x_3\bar{x}_1$ the 28 coordinate vectors may be taken as $e(\infty) = e_{(0,0,1)}$, $e(z,t) = e_{(1,z,z\bar{z}+it)}$ (z in $\mathbb{F}_9$, t in $\mathbb{F}_3$). The element T of $U_3(3)$ acts by $e_v \longrightarrow e_{vT}$, and the element taking $e(\infty) \longrightarrow e(\infty)$, $e(z,t) \longrightarrow e(\bar{z},-t)$ extends this to the full monomial group $2^6{\cdot}G_2(2)$. The 4x4060 minimal vectors are the images under this group of

$e_a+e_b+e_{a+b}+e_{a-b}$ (a, b, a+b, a-b distinct isotropic vectors)

$\frac{1}{2}e(\infty) + \frac{1}{2}\Sigma\,[\ e(t,0)+e(t-i,t)-e(i-t,t)-ie(t,1)-ie(t,-1)\ ]$ (summed over t in F_3)

$[\ (1-2i)e(\infty) + \Sigma e(z,t)\ ]\ /\ (2+2i)$ (summed over z in $\mathbb{F}_9$, t in $\mathbb{F}_3$)

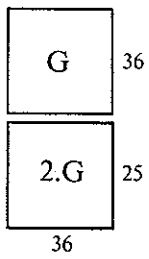
The vectors $(2+2i)e_v$ form a congruence base modulo 1+i. The group is transitive on sets of 3 minimal vectors with given inner products, except that 3 mutually orthogonal vectors may belong to a quartet, or to a dozen but no quartet, or to a base but no dozen (see below). There are 10 minimal vectors fixed by an element of class 3A, which form a decad with all inner products 1. There are also maximal heptads of 7 minimal vectors with all inner products 1. Each of these is one of a set of 50, whose sums are all congruent modulo 5, and which form a copy of the Hoffman-Singleton graph (see $U_3(5)$) when intersecting heptads are joined.

Graph G : the automorphism group of a rank 3 valence 2304 graph on 4060 points, in which the point stabilizer is ${}^2F_4(2)$. The vertices of the graph are the minimal vectors considered modulo the unit scalar factors ±1, ±i. These have norm 4; a vertex is joined (inner product ±1 or ±i) to 2304 others, and disjoined (orthogonal) to the remaining 1755. Four mutually orthogonal vertices form a quartet if any further vertex orthogonal to three of them is orthogonal to the fourth. Any pair (a_∞,b_∞) of orthogonal vertices lies in five such quartets, forming a dozen $(a_\infty,b_\infty;a_0,b_0;\ldots;a_4,b_4)$ in which any two of the pairs (a_t,b_t) form a quartet. The stabilizer of a dozen is a group $[2^{11}]PGL_2(5)$ in which $PGL_2(5)$ permutes the 6 pairs (a_t,b_t). Any quartet lies in three dozens, whose union consists of 28 mutually orthogonal vertices which can be used to define a base for the 28-space.

Orthogonal (2) Modulo 1+i the lattice yields a 28-dimensional orthogonal representation over $\mathbb{F}_2$, in which many of the vector stabilizers are maximal subgroups. In the table n(z) denotes the reduction of a sum of n minimal vectors, among which the inner product of every pair is z or $\bar{z}$.

Maximal subgroups / Specifications

Order	Index	Structure	Character	Abstract	Lattice, Graph	Orthogonal (2)
35942400	4060	${}^2F_4(2)$	1a+783a+3276a		vertex	1 or 3(0) (quartet)
774144	188500	$(2^6{:}U_3(3)){:}2$		$N(2A^6)$	congruence base	3(0) (dozen)
349440	417600	$(2^2 \times Sz(8)){:}3$		$N(2B^2)$	quaternionic	4(i)
344064	424125	$2^{3+8}{:}L_3(2)$		$N(2A^3)$	base	
252000	579072	$U_3(5){:}2$			50 heptads	
245760	593775	$2{\cdot}2^{4+6}{:}S_5$		N(2A)	dozen	2(0)
31200	4677120	$L_2(25){\cdot}2^2$			edge	2(1) or 3(-1)
20160	7238400	A_8				
12180	11980800	$L_2(29)$				
12000	12160512	$5^2{:}4S_5$		$N(5A^2)$		
4320	33779200	$3{\cdot}A_6{\cdot}2^2$		N(3A)	decad	3(1)
4000	36481536	$5_+^{1+2}{:}[2^5]$		N(5A)		
2184	66816000	$L_2(13){:}2$		N(2A,3A,6A,7A,13AB)		3(i)
1440	101337600	$A_6{\cdot}2^2$		N(2A,3A,3A,4D,5B)		
1200	121605120	$5{:}4 \times A_5$		N(5B), N(2B,3A,5A)		



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		1	245	116	2	7	3	1		1																															
	45926	144000	760	480	160	680	840	024	512	000	300	48	28	96	64	32	40	20	24	12	52	28	28	28	15	16	16	20	20	20	24	24	52	52	52	29	29		
		p power	A	A	A	A	A	A	A	A	A	AA	A	A	C	D	AA	BB	AA	AB	A	AB	AB	AB	BA	B	B	AA	AB	AB	AA	AA	AB	AB	AB	A	A		
		p' part	A	A	A	A	A	A	A	A	A	AA	A	A	A	A	AA	BB	AA	AB	A	AB	AB	AB	BA	A	A	AA	AB	AB	AA	AA	AB	AB	AB	A	A		
	ind	1A	2A	2B	3A	4A	4B	4C	4D	5A	5B	6A	7A	8A	8B	8C	10A	10B	12A	12B	13A	14A	B*3	C*5	15A	16A	B**	20A	20B	C*	24A	B*	26A	B*3	C*9	29A	B*		
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	o	378	-6	14	0	6	-6	2	-2	3	3	0	0	0	-2	0	-1	-1	0	0	1	0	0	0	0	2i	-2i	1	-1	-1	0	0	1	1	1	1	1	χ_2
χ_3	o	378	-6	14	0	6	-6	2	-2	3	3	0	0	0	-2	0	-1	-1	0	0	1	0	0	0	0	-2i	2i	1	-1	-1	0	0	1	1	1	1	1	χ_3
χ_4	+	406	22	-14	1	10	-10	-2	2	6	1	1	0	-4	-2	0	2	1	1	-1	3	0	0	0	1	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	χ_4
χ_5	+	783	15	-1	0	15	15	-1	-1	8	3	0	-1	3	3	-1	0	-1	0	0	3	-1	-1	-1	0	1	1	0	0	0	0	0	-1	-1	-1	0	0	χ_5
χ_6	+	3276	76	0	9	16	4	4	8	1	-4	1	0	2	0	2	1	0	1	1	0	0	0	0	-1	0	0	1	-1	-1	-1	-1	0	0	0	-1	-1	χ_6
χ_7	+	3654	70	14	9	-2	-10	6	-2	4	-1	1	0	-2	2	2	0	-1	1	-1	1	0	0	0	-1	0	0	-2	0	0	1	1	1	1	1	0	0	χ_7
χ_8	+	20475	-5	91	9	-5	-5	-5	-5	0	5	1	0	-1	3	-1	0	1	1	1	0	0	0	0	-1	-1	-1	0	0	0	-1	-1	0	0	0	1	1	χ_8
χ_9	+	21750	-10	-90	15	-10	-10	6	6	0	0	-1	1	2	2	-2	0	0	-1	-1	1	1	1	1	0	0	0	0	0	0	-1	-1	1	1	1	0	0	χ_9
χ_{10}	+	23751	71	91	-9	11	-1	-1	3	1	1	-1	0	1	3	1	1	1	-1	-1	0	0	0	0	1	-1	-1	1	-1	-1	1	1	0	0	0	0	0	χ_{10}
χ_{11}	+	27000	120	-40	0	0	0	8	0	0	0	0	1	0	0	0	0	0	0	0	-1-2y7-1		*4	*2	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	χ_{11}
χ_{12}	+	27000	120	-40	0	0	0	8	0	0	0	0	1	0	0	0	0	0	0	0	-1	*2-2y7-1		*4	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	χ_{12}
χ_{13}	+	27000	120	-40	0	0	0	8	0	0	0	0	1	0	0	0	0	0	0	0	-1	*4	*2-2y7-1		0	0	0	0	0	0	0	0	-1	-1	-1	1	1	χ_{13}
χ_{14}	+	27405	141	105	0	9	21	5	1	5	0	0	0	3	-3	-1	1	0	0	0	1	0	0	0	0	-1	-1	-1	1	1	0	0	1	1	1	0	0	χ_{14}
χ_{15}	+	34944	128	0	6	0	0	0	0	-6	4	2	0	0	0	0	-2	0	0	0	0	0	0	0	1	0	0	0	0	0	r6	-r6	0	0	0	-1	-1	χ_{15}
χ_{16}	+	34944	128	0	6	0	0	0	0	-6	4	2	0	0	0	0	-2	0	0	0	0	0	0	0	1	0	0	0	0	0	-r6	r6	0	0	0	-1	-1	χ_{16}
χ_{17}	+	43848	72	-56	0	0	0	-8	0	-2	3	0	0	0	0	0	2	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	2c13+1	*3	*9	0	0	χ_{17}
χ_{18}	+	43848	72	-56	0	0	0	-8	0	-2	3	0	0	0	0	0	2	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	*9	2c13+1	*3	0	0	χ_{18}
χ_{19}	+	43848	72	-56	0	0	0	-8	0	-2	3	0	0	0	0	0	2	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	*3	*9	2c13+1	0	0	χ_{19}
χ_{20}	+	45500	60	0	5	40	-20	4	0	0	0	-3	0	-2	0	-2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	-1	χ_{20}
χ_{21}	+	52780	-84	0	-5	-16	4	4	8	5	0	3	0	-2	0	-2	1	0	-1	1	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	0	0	χ_{21}
χ_{22}	+	63336	104	0	-6	16	-24	-8	0	11	-4	2	0	0	0	0	-1	0	-2	0	0	0	0	0	-1	0	0	1	1	1	0	0	0	0	0	0	0	χ_{22}
χ_{23}	+	65975	55	91	14	-5	15	-1	3	0	5	-2	0	-3	-1	1	0	1	-2	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	χ_{23}
χ_{24}	+	71253	-171	-91	0	21	21	5	5	3	3	0	0	-3	1	1	-1	-1	0	0	0	0	0	0	0	-1	-1	1	1	1	0	0	0	0	0	0	0	χ_{24}
χ_{25}	+	75400	-120	104	16	0	0	-8	0	0	-5	0	3	0	0	0	0	-1	0	0	0	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{25}
χ_{26}	+	76125	-35	105	-15	25	5	5	1	0	0	1	0	-1	1	-1	0	0	1	-1	-3	0	0	0	0	1	1	0	0	0	-1	-1	1	1	1	0	0	χ_{26}
χ_{27}	+	81432	24	-104	0	24	24	-8	-8	7	-3	0	1	0	0	0	-1	1	0	0	0	1	1	1	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	χ_{27}
χ_{28}	+	91350	-170	-14	9	10	-10	-2	2	0	5	1	0	4	-2	0	0	1	1	-1	-1	0	0	0	-1	0	0	0	0	0	1	1	-1	-1	-1	0	0	χ_{28}
χ_{29}	+	95004	-100	0	-9	8	-20	4	0	4	4	-1	0	2	0	2	0	0	-1	1	0	0	0	0	1	0	0	-2	0	0	-1	-1	0	0	0	0	0	χ_{29}
χ_{30}	+	98280	-24	0	0	-24	0	8	-8	5	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	r5	-r5	0	0	0	0	0	-1	-1	χ_{30}
χ_{31}	+	98280	-24	0	0	-24	0	8	-8	5	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	-r5	r5	0	0	0	0	0	-1	-1	χ_{31}
χ_{32}	+	102400	0	64	16	0	0	0	0	0	-5	0	-3	0	0	0	0	-1	0	0	-1	1	1	1	1	0	0	0	0	0	0	0	-1	-1	-1	1	1	χ_{32}
χ_{33}	+	105560	88	0	-10	-40	0	-8	8	10	0	-2	0	0	0	0	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{33}
χ_{34}	+	110592	0	-64	0	0	0	0	0	-8	-3	0	-1	0	0	0	0	1	0	0	1	-1	-1	-1	0	0	0	0	0	0	0	0	1	1	1-b29	*		χ_{34}
χ_{35}	+	110592	0	-64	0	0	0	0	0	-8	-3	0	-1	0	0	0	0	1	0	0	1	-1	-1	-1	0	0	0	0	0	0	0	0	1	1	1	*-b29		χ_{35}
χ_{36}	+	118784	0	64	-16	0	0	0	0	-16	-1	0	1	0	0	0	0	-1	0	0	3	1	1	1	-1	0	0	0	0	0	0	0	-1	-1	-1	0	0	χ_{36}
	ind	1	2	4	3	4	4	4	4	5	5	6	7	8	8	8	10	20	12	12	13	28	28	28	15	16	16	20	20	20	24	24	52	52	52	29	29		
		2	2		6	4	4			10	10	6	14	8	8		10		12	12	26				30	16	16	20	20	20	24	24				58	58		
χ_{37}	o	28	4	0	1	4	4i	0	0	3	-2	1	0	2i	2i	0	-1	0	1	i	2	0	0	0	1	i+1	-i+1	-1	-i	-i	-i	-i	0	0	0	-1	-1	χ_{37}
χ_{38}	o	28	4	0	1	4	-4i	0	0	3	-2	1	0	-2i	-2i	0	-1	0	1	-i	2	0	0	0	1	-i+1	i+1	-1	i	i	i	i	0	0	0	-1	-1	χ_{38}
χ_{39}	o	1248	32	0	6	0	0	0	0	-2	-2	2	2	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	i6	-i6	0	0	0	1	1	χ_{39}
χ_{40}	o	1248	32	0	6	0	0	0	0	-2	-2	2	2	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	-i6	i6	0	0	0	1	1	χ_{40}
χ_{41}	o	3276	-44	0	9	4	4i	0	0	1	-4	1	0	-2i	2i	0	1	0	1	i	0	0	0	0	-1	-i-1	i-1	-1	-i	-i	i	i	0	0	0	-1	-1	χ_{41}
χ_{42}	o	3276	-44	0	9	4	-4i	0	0	1	-4	1	0	2i	-2i	0	1	0	1	-i	0	0	0	0	-1	i-1	-i-1	-1	i	i	-i	-i	0	0	0	-1	-1	χ_{42}
χ_{43}	o	4032	64	0	9	16	16i	0	0	7	2	1	0	4i	0	0	-1	0	1	i	2	0	0	0	-1	0	0	1	i	i	i	i	0	0	0	1	1	χ_{43}
χ_{44}	o	4032	64	0	9	16	-16i	0	0	7	2	1	0	-4i	0	0	-1	0	1	-i	2	0	0	0	-1	0	0	1	-i	-i	-i	-i	0	0	0	1	1	χ_{44}
χ_{45}	o	7280	16	0	-10	-16	0	0	0	5	0	-2	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	-1	i5	-i5	0	0	0	0	0	1	1	χ_{45}
χ_{46}	o	7280	16	0	-10	-16	0	0	0	5	0	-2	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	-1	-i5	i5	0	0	0	0	0	1	1	χ_{46}
χ_{47}	o	7308	20	0	-9	20	20i	0	0	8	-2	-1	0	2i	2i	0	0	0	-1	-i	2	0	0	0	1	-i-1	i-1	0	0	0	-i	-i	0	0	0	0	0	χ_{47}
χ_{48}	o	7308	20	0	-9	20	-20i	0	0	8	-2	-1	0	-2i	-2i	0	0	0	-1	i	2	0	0	0	1	i-1	-i-1	0	0	0	i	i	0	0	0	0	0	χ_{48}
χ_{49}	-	8192	0	0	-16	0	0	0	0	-8	2	0	2	0	0	0	0	0	0	0	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	b29	*	χ_{49}
χ_{50}	-	8192	0	0	-16	0	0	0	0	-8	2	0	2	0	0	0	0	0	0	0	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	*	b29	χ_{50}
χ_{51}	o	10556	-28	0	-1	20	20i	0	0	6	-4	-1	0	-2i	2i	0	2	0	-1	-i	0	0	0	0	-1	i+1	-i+1	0	0	0	i	i	0	0	0	0	0	χ_{51}
χ_{52}	o	10556	-28	0	-1	20	-20i	0	0	6	-4	-1	0	2i	-2i	0	2	0	-1	i	0	0	0	0	-1	-i+1	i+1	0	0	0	-i	-i	0	0	0	0	0	χ_{52}
χ_{53}	-	34944	-128	0	6	32	0	0	0	-6	4	-2	0	0	0	0	2	0	2	0	0	0	0	0	1	0	0	2	0	0	0	0	0	0	0	-1	-1	χ_{53}
χ_{54}	o	38976	-64	0	15	-16	16i	0	0	1	6	-1	0	-4i	0	0	1	0	-1	i	2	0	0	0	0	0	0	-1	i	i	-i	-i	0	0	0	0	0	χ_{54}
χ_{55}	o	38976	-64	0	15	-16	-16i	0	0	1	6	-1	0	4i	0	0	1	0	-1	-i	2	0	0	0	0	0	0	-1	-i	-i	i	i	0	0	0	0	0	χ_{55}
χ_{56}	-	48256	-128	0	-20	0	0	0	0	6	6	4	-2	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{56}
χ_{57}	-	87696	240	0	0	48	0	0	0	-4	6	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	χ_{57}
χ_{58}	o	98280	216	0	0	-24	24i	0	0	5	0	0	0	0	-4i	0	1	0	0	0	0	0	0	0	0	0	0	1	-i	-i	0	0	0	0	0	-1	-1	χ_{58}
χ_{59}	o	98280	216	0	0	-24	-24i	0	0	5	0	0	0	0	4i	0	1	0	0	0	0	0	0	0	0	0	0	1	i	i	0	0	0	0	0	-1	-1	χ_{59}
χ_{60}	-	221184	0	0	0	0	0	0	0	-16	-6	0	-2	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{60}
χ_{61}	-	250560	-192	0	0	0	0	0	0	10	0	0	2	0	0	0	-2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{61}

Suz

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		4	33	161	97	34	3	46	3	1		1		3																															
		48345497600	17760	280	97760	992	240	080	072	536	288	800	300	456	1296	1296	432	72	84	192	64	32	54	54	40	20	11	288	72	48	36	24	13	13	28	45	45	15	18	18	20	21	21	24	
		p power	A	A	A	A	A	A	A	A	B	A	A	AA	BA	BA	BA	CB	A	A	B	C	B	B	AA	BB	A	AA	DA	AC	ED	DB	A	A	AB	AC	AC	BA	BB	AC	AA	AA	AA	AA	
		p' part	A	A	A	A	A	A	A	A	A	A	A	AA	BA	BA	BA	CB	A	A	A	A	A	A	AA	BB	A	AA	BA	AC	CD	BB	A	A	AB	AC	AC	BA	AA	BA	AA	AA	AA	AA	
	ind	1A	2A	2B	3A	3B	3C	4A	4B	4C	4D	5A	5B	6A	6B	C**	6D	6E	7A	8A	8B	8C	9A	B**	10A	10B	11A	12A	12B	12C	12D	12E	13A	B*	14A	15A	B*	15C	18A	B**	20A	21A	B*	24A	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	143	15	-1	35	8	-1	15	-1	-1	-1	8	3	3	6	6	0	-1	3	-1	3	-1	2	2	0	-1	0	3	0	-1	-1	2	0	0	-1	-1	-1	0	0	0	0	0	0	-1	χ_{2}
χ_{3}	+	364	44	0	-14	13	4	4	-4	8	-2	-1	4	2	-1	-1	5	0	0	0	0	2	1	1	-1	0	1	-2	1	2	-2	-1	0	0	0	-1	-1	1	-1	-1	-1	0	0	0	χ_{3}
χ_{4}	+	780	12	20	105	-3	6	28	-4	4	0	10	0	9	3	3	-3	2	3	4	0	0	0	0	2	0	-1	1	1	1	0	-1	0	0	-1	1	1	0	0	0	-2	0	0	1	χ_{4}
χ_{5}	+	1001	41	21	56	2	11	1	9	-3	-1	1	1	8	-4	-4	2	3	0	1	1	-1	-1	-1	1	1	0	4	-2	0	-1	0	0	0	0	1	1	1	-1	-1	1	0	0	-2	χ_{5}
χ_{6}	+	3432	-24	16	300	3	-6	56	8	0	0	17	-3	12	3	3	3	-2	2	0	0	0	0	0	1	1	0	-4	-1	0	0	-1	0	0	2	-1	-1	0	0	0	1	-1	-1	0	χ_{6}
χ_{7}	o	5005	13	-35	280	10	1	45	-3	-3	1	10	0	-8	6i3+4	-6i3+4	-2	1	0	-3	1	1	**	b27	-2	0	0	0	0	0	1	0	0	0	0	1	1	0	z3	**	0	0	0	0	χ_{7}
χ_{8}	o	5005	13	-35	280	10	1	45	-3	-3	1	10	0	-8	-6i3+4	6i3+4	-2	1	0	-3	1	1	b27	**	-2	0	0	0	0	0	1	0	0	0	0	1	1	0	**	z3	0	0	0	0	χ_{8}
χ_{9}	+	5940	180	36	189	27	0	20	4	4	0	0	5	-3	9	9	3	0	-3	4	0	0	0	0	0	1	0	5	-1	1	0	1	-1	-1	1	0	0	-1	0	0	0	0	0	1	χ_{9}
χ_{10}	+	10725	165	-55	330	-21	15	45	5	1	-1	0	0	-6	3	3	3	-1	1	-3	-3	-1	0	0	0	0	0	6	3	-2	-1	-1	0	0	1	0	0	0	0	0	0	1	1	0	χ_{10}
χ_{11}	+	12012	300	-56	-84	51	6	20	-4	0	2	-8	7	12	3	3	3	-2	0	0	0	-2	0	0	0	-1	0	-4	-1	0	2	-1	0	0	0	1	1	1	0	0	0	0	0	0	χ_{11}
χ_{12}	+	14300	220	20	125	44	-10	-20	12	4	0	0	0	13	4	4	4	2	-1	-4	0	0	-1	-1	0	0	0	1	4	1	0	0	0	0	-1	0	0	0	1	1	0	-1	-1	-1	χ_{12}
χ_{13}	+	15015	231	35	-105	3	3	-1	7	11	1	5	0	-9	-3	-3	3	-1	0	-1	-1	1	0	0	1	0	0	-1	-1	-1	1	1	0	0	0	-b45	*	0	0	0	-1	0	0	-1	χ_{13}
χ_{14}	+	15015	231	35	-105	3	3	-1	7	11	1	5	0	-9	-3	-3	3	-1	0	-1	-1	1	0	0	1	0	0	-1	-1	-1	1	1	0	0	0	*	-b45	0	0	0	-1	0	0	-1	χ_{14}
χ_{15}	+	15795	243	55	0	0	9	27	3	15	1	-10	0	0	0	0	0	1	3	3	-1	1	0	0	-2	0	-1	0	0	0	1	0	0	0	-1	-1	-1	0	0	0	2	0	0	0	χ_{15}
χ_{16}	+	18954	-54	54	729	0	0	66	-6	-2	0	9	-1	9	0	0	0	0	-2	2	-2	0	0	0	1	-1	1	-3	0	1	0	0	0	0	-2	0	0	-1	0	0	1	1	1	-1	χ_{16}
χ_{17}	+	25025	65	105	455	23	5	-15	1	-7	1	0	0	23	-7	-7	-1	-3	0	1	1	1	-1	-1	0	0	0	3	3	-1	1	1	0	0	0	0	0	0	-1	-1	0	0	0	1	χ_{17}
χ_{18}	o	25025	-255	-35	455	23	5	25	1	5	-1	0	0	-9	6i3-3	-6i3-3	3	1	0	1	1	-1	-1	-1	0	0	0	-5	1	-1	-1	1	0	0	0	0	0	0	-i3	i3	0	0	0	1	χ_{18}
χ_{19}	o	25025	-255	-35	455	23	5	25	1	5	-1	0	0	-9	-6i3-3	6i3-3	3	1	0	1	1	-1	-1	-1	0	0	0	-5	1	-1	-1	1	0	0	0	0	0	0	i3	-i3	0	0	0	1	χ_{19}
χ_{20}	+	40040	360	56	-406	53	-10	40	8	-8	0	0	5	-6	-9	-9	-3	2	0	0	0	0	2	2	0	1	0	-2	1	-2	0	-1	0	0	0	0	0	-1	0	0	0	0	0	0	χ_{20}
χ_{21}	o	50050	-190	70	-35	19	10	10	2	-2	0	0	0	-19	6i3-1	-6i3-1	-1	-2	0	2	2	0	b27	**	0	0	0	1	1	1	0	-1	0	0	0	0	0	0	**	-z3	0	0	0	-1	χ_{21}
χ_{22}	o	50050	-190	70	-35	19	10	10	2	-2	0	0	0	-19	-6i3-1	6i3-1	-1	-2	0	2	2	0	**	b27	0	0	0	1	1	1	0	-1	0	0	0	0	0	0	-z3	**	0	0	0	-1	χ_{22}
χ_{23}	+	54054	486	-126	189	27	0	-34	-10	10	0	9	-1	-3	-9	-9	3	0	0	-2	2	0	0	0	1	-1	0	5	-1	1	0	-1	0	0	0	0	0	-1	0	0	1	0	0	1	χ_{23}
χ_{24}	+	64064	64	0	1316	-34	-16	64	0	0	0	-1	4	4	-8	-8	-2	0	0	0	0	0	-1	-1	-1	0	0	4	-2	0	0	0	0	0	0	-1	-1	1	1	1	-1	0	0	0	χ_{24}
χ_{25}	+	64350	30	90	-45	-45	0	-10	14	2	0	0	0	3	3	3	3	0	-1	-2	2	0	0	0	0	0	0	-1	-1	-1	0	-1	0	0	-1	0	0	0	0	0	0	-b21	*	1	χ_{25}
χ_{26}	+	64350	30	90	-45	-45	0	-10	14	2	0	0	0	3	3	3	3	0	-1	-2	2	0	0	0	0	0	0	-1	-1	-1	0	-1	0	0	-1	0	0	0	0	0	0	*	-b21	1	χ_{26}
χ_{27}	+	66560	0	64	896	32	5	0	0	0	8	0	-5	0	0	0	0	1	-3	0	0	0	2	2	0	-1	-1	0	0	0	-1	0	0	0	1	0	0	1	0	0	0	0	0	0	χ_{27}
χ_{28}	+	75075	195	35	-525	15	15	35	19	3	-1	0	0	3	-3	-3	-9	-1	0	-5	-1	-1	0	0	0	0	0	-1	-1	3	-1	1	0	0	0	0	0	0	0	0	0	0	0	1	χ_{28}
χ_{29}	+	79872	0	128	384	-48	6	0	0	0	0	-8	2	0	0	0	0	2	2	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	2	1	1	-1	0	0	0	-1	-1	0	χ_{29}
χ_{30}	+	88452	324	56	0	0	18	-36	-12	0	2	7	-3	0	0	0	0	2	0	0	0	-2	0	0	-1	1	1	0	0	0	2	0	0	0	0	-2	-2	0	0	0	-1	0	0	0	χ_{30}
χ_{31}	+	93555	243	35	0	0	-9	-45	3	3	-1	10	0	0	0	0	0	-1	0	3	-1	-1	0	0	-2	0	0	0	0	0	-1	0	-b13	*	0	1	1	0	0	0	0	0	0	0	χ_{31}
χ_{32}	+	93555	243	35	0	0	-9	-45	3	3	-1	10	0	0	0	0	0	-1	0	3	-1	-1	0	0	-2	0	0	0	0	0	-1	0	*	-b13	0	1	1	0	0	0	0	0	0	0	χ_{32}
χ_{33}	+	100100	580	0	-70	38	20	-20	-12	-8	-2	0	0	10	4	4	-2	0	0	0	0	2	-1	-1	0	0	0	-2	-2	-2	-2	0	0	0	0	0	0	0	1	1	0	0	0	0	χ_{33}
χ_{34}	+	133056	-576	0	-756	54	0	64	0	0	0	-9	-4	12	0	0	6	0	0	0	0	0	0	0	-1	0	0	4	-2	0	0	0	1	1	0	0	0	-1	0	0	-1	0	0	0	χ_{34}
χ_{35}	+	146432	0	-64	1280	-16	11	0	0	0	-8	-8	-3	0	0	0	0	-1	-1	0	0	0	2	2	0	1	0	0	0	0	1	0	0	0	-1	1	1	0	0	0	0	-1	-1	0	χ_{35}
χ_{36}	+	163800	600	0	-630	-63	0	40	-8	-16	0	0	0	-6	-3	-3	9	0	0	0	0	0	0	0	0	0	-1	-2	1	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{36}
χ_{37}	+	168960	0	64	-384	48	-15	0	0	0	-8	0	-5	0	0	0	0	1	1	0	0	0	0	0	0	-1	0	0	0	0	1	0	-1	-1	1	0	0	1	0	0	0	1	1	0	χ_{37}
χ_{38}	+	189540	-540	36	729	0	0	-60	-12	4	0	0	5	9	0	0	0	0	1	-4	0	0	0	0	0	1	-1	-3	0	1	0	0	0	0	1	0	0	-1	0	0	0	1	1	-1	χ_{38}
χ_{39}	+	193050	90	-90	-945	-54	0	50	-6	14	0	0	0	15	0	0	-6	0	-3	2	2	0	0	0	0	0	0	-1	2	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	χ_{39}
χ_{40}	+	197120	512	0	560	20	-16	0	0	0	0	-10	0	-16	8	8	-4	0	0	0	0	0	-1	-1	2	0	0	0	0	0	0	0	1	1	0	-1	-1	0	-1	-1	0	0	0	0	χ_{40}
χ_{41}	+	208494	-594	90	-729	0	0	6	-18	2	0	9	4	-9	0	0	0	0	-1	-2	-2	0	0	0	1	0	0	3	0	-1	0	0	0	0	-1	0	0	1	0	0	1	-1	-1	1	χ_{41}
χ_{42}	+	243243	-405	-161	0	0	9	-45	27	-9	1	8	3	0	0	0	0	1	0	3	-1	1	0	0	0	-1	0	0	0	0	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	χ_{42}
χ_{43}	+	248832	0	-64	0	0	-9	0	0	0	8	-8	-3	0	0	0	0	-1	3	0	0	0	0	0	0	1	1	0	0	0	-1	0	-1	-1	-1	1	1	0	0	0	0	0	0	0	χ_{43}
	ind	1	2	4	3	3	3	4	4	4	8	5	5	6	6	6	6	12	7	8	8	8	9	9	10	20	11	12	12	12	24	12	13	13	28	15	15	15	18	18	20	21	21	24	
		2	2		6	6	6	4				10	10	6	6	6	6		14		8		18	18	10		22	12	12		24	12	26	26		30	30	30	18	18	20	42	42	24	
χ_{44}	-	220	12	0	-50	4	4	-20	0	0	0	-10	0	6	-6	-6	0	0	-4	0	2	0	-2	-2	2	0	0	-2	-2	0	0	0	-1	-1	0	-1	-1	0	0	0	0	-1	-1	0	χ_{44}
χ_{45}	o	364	-36	0	-14	13	4	-4	0	0	0	-1	4	-6	-3i3	3i3	-3	0	0	0	2	0	1	1	-1	0	1	2	-1	0	0	i3	0	0	0	-1	-1	1	i3	-i3	1	0	0	0	χ_{45}
χ_{46}	o	364	-36	0	-14	13	4	-4	0	0	0	-1	4	-6	3i3	-3i3	-3	0	0	0	2	0	1	1	-1	0	1	2	-1	0	0	-i3	0	0	0	-1	-1	1	-i3	i3	1	0	0	0	χ_{46}
χ_{47}	o	572	-20	0	-76	5	-4	-20	0	0	0	-8	2	4	-3i3-2	3i3-2	1	0	-2	0	2	0	-b27	**	0	0	0	4	1	0	0	i3	0	0	0	1	1	-1	**	z3	0	1	1	0	χ_{47}
χ_{48}	o	572	-20	0	-76	5	-4	-20	0	0	0	-8	2	4	3i3-2	-3i3-2	1	0	-2	0	2	0	**	-b27	0	0	0	4	1	0	0	-i3	0	0	0	1	1	-1	z3	**	0	1	1	0	χ_{48}
χ_{49}	-	4928	64	0	-364	14	-4	-64	0	0	0	-17	-2	4	-8	-8	-2	0	0	0	0	0	-1	-1	-1	0	0	-4	2	0	0	0	1	1	0	1	1	1	1	1	1	0	0	0	χ_{49}
χ_{50}	o	16016	-304	0	140	59	-4	-16	0	0	0	1	6	-4	-3i3+2	3i3+2	-1	0	0	0	0	0	**	-b27	1	0	0	-4	-1	0	0	i3	0	0	0	1	1	0	-z3	**	-1	0	0	0	χ_{50}
χ_{51}	o	16016	-304	0	140	59	-4	-16	0	0	0	1	6	-4	3i3+2	-3i3+2	-1	0	0	0	0	0	-b27	**	1	0	0	-4	-1	0	0	-i3	0	0	0	1	1	0	**	-z3	-1	0	0	0	χ_{51}
χ_{52}	-	20020	-188	0	-770	-14	4	-60	0	0	0	-10	0	22	10	10	-2	0	0	0	-2	0	1	1	2	0	0	6	0	0	0	0	0	0	0	-1	-1	0	1	1	0	0	0	0	χ_{52}
χ_{53}	-	20020	68	0	-770	-14	4	-92	0	0	0	-10	0	-10	-4	-4	2	0	0	0	-2	0	1	1	-2	0	0	-2	-2	0	0	0	0	0	0	-1	-1	0	-1	-1	-2	0	0	0	χ_{53}
χ_{54}	o	35100	-180	0	-540	27	0	-20	0	0	0	0	0	-12	-9i3	9i3	3	0	2	0	2	0	0	0	0	0	-1	4	1	0	0	-i3	0	0	0	0	0	0	0	0	0	-1	-1	0	χ_{54}
χ_{55}	o	35100	-180	0	-540	27	0	-20	0	0	0	0	0	-12	9i3	-9i3	3	0	2	0	2	0	0	0	0	0	-1	4	1	0	0	i3	0	0	0	0	0	0	0	0	0	-1	-1	0	χ_{55}
χ_{56}	-	35100	-180	0	270	-54	0	-20	0	0	0	0	0	6	0	0	-6	0	2	0	2	0	0	0	0	0	-1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	-b21	*	0	χ_{56}
χ_{57}	-	35100	-180	0	270	-54	0	-20	0	0	0	0	0	6	0	0	-6	0	2	0	2	0	0	0	0	0	-1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	*	-b21	0	χ_{57}
χ_{58}	-	60060	204	0	-420	12	12	44	0	0	0	-5	0	12	6	6	0	0	0	0	2	0	0	0	-1	0	0	-4	2	0	0	0	0	0	0	b45	*	0	0	0	-1	0	0	0	χ_{58}
χ_{59}	-	60060	204	0	-420	12	12	44	0	0	0	-5	0	12	6	6	0	0	0	0	2	0	0	0	-1	0	0	-4	2	0	0	0	0	0	0	*	b45	0	0	0	-1	0	0	0	χ_{59}
χ_{60}	-	61236	324	0	0	0	0	36	0	0	0	-9	6	0	0	0	0	0	0	0	-2	0	0	0	-1	0	-1	0	0	0	0	0	b13	*	0	0	0	0	0	0	1	0	0	0	χ_{60}
χ_{61}	-	61236	324	0	0	0	0	36	0	0	0	-9	6	0	0	0	0	0	0	0	-2	0	0	0	-1	0	-1	0	0	0	0	0	*	b13	0	0	0	0	0	0	1	0	0	0	χ_{61}
χ_{62}	-	79872	0	0	384	-48	6	0	0	0	0	-8	2	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	3r2	0	0	0	0	1	1	-1	0	0	0	-1	-1	0	χ_{62}
χ_{63}	-	79872	0	0	384	-48	6	0	0	0	0	-8	2	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	-3r2	0	0	0	0	1	1	-1	0	0	0	-1	-1	0	χ_{63}
χ_{64}	-	80080	528	0	700	52	16	-80	0	0	0	10	0	12	6	6	0	0	0	0	0	0	-2	-2	-2	0	0	4	-2	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	χ_{64}
χ_{65}	-	80080	272	0	-1190	-2	16	-48	0	0	0	10	0	2	2	2	2	0	0	0	0	0	1	1	2	0	0	-6	0	0	0	0	0	0	0	1	1	0	-1	-1	2	0	0	0	χ_{65}
χ_{66}	-	100100	-940	0	-70	38	20	20	0	0	0	0	0	2	-4	-4	-10	0	0	0	-2	0	-1	-1	0	0	0	2	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	χ_{66}
χ_{67}	-	102400	0	0	-1280	16	-20	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	-2	-2	0	0	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	1	1	0	χ_{67}
χ_{68}	-	128128	640	0	-392	40	4	0	0	0	0	8	-2	-8	-8	-8	-8	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-2	1	1	0	0	0	0	χ_{68}
χ_{69}	-	137280	-192	0	-960	-42	-24	0	0	0	0	10	0	0	6	6	-6	0	-4	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-1	-1	0	χ_{69}
χ_{70}	-	144144	336	0	126	-36	0	-16	0	0	0	9	4	6	-6	-6	0	0	0	0	0	0	0	0	1	0	0	2	2	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	2r3	χ_{70}
χ_{71}	-	144144	336	0	126	-36	0	-16	0	0	0	9	4	6	-6	-6	0	0	0	0	0	0	0	0	1	0	0	2	2	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	-2r3	χ_{71}
χ_{72}	-	192192	-576	0	924	6	-12	-64	0	0	0	7	2	12	0	0	6	0	0	0	0	0	0	0	-1	0	0	-4	2	0	0	0	0	0	0	-2	-2	-1	0	0	1	0	0	0	χ_{72}
χ_{73}	-	197120	512	0	560	20	-16	0	0	0	0	-10	0	-16	8	8	-4	0	0	0	0	0	-1	-1	2	0	0	0	0	0	0	0	1	1	0	-1	-1	0	-1	-1	0	0	0	0	χ_{73}
χ_{74}	-	228800	-320	0	-160	-52	20	0	0	0	0	0	0	-32	4	4	4	0	-2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	χ_{74}
χ_{75}	-	277200	-240	0	-630	18	0	80	0	0	0	0	0	18	-6	-6	6	0	0	0	0	0	0	0	0	0	0	2	-4	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	χ_{75}
χ_{76}	-	315392	0	0	896	32	-4	0	0	0	0	-8	-8	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	-1	-1	0	1	1	1	0	0	0	0	0	0	χ_{76}

	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
			1209	190	2		2					1																							
			600	080	304	672	160	108	72	72	23040	536	384	128	96	300	50	20	144	36	12	14	8	11	144	48	36	12	12	14	15	20	20	
			A	A	A	B	AC	BD	CC	CD	A	A	A	A	C	BC	AC	BD	AE	DE	EF	AC	B	AD	AD	AE	BD	BF	CH	AF	CCF	AD	AD	
			A	A	A	A	AC	BD	CC	CD	A	A	A	A	A	BC	AC	BD	AE	BE	CF	AC	A	AD	AD	AE	BD	BF	AH	AF	CCF	AD	AD	
	fus	ind	2C	2D	4E	4F	6F	6G	6H	6I	8D	8E	8F	8G	8H	10C	10D	10E	12F	12G	12H	14B	16A	22A	24B	24C	24D	24E	24F	28A	30A	40A	B*		
x_1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	:	++	13	-11	-3	1	-5	-2	1	1	5	5	-3	-3	1	3	-2	-1	3	0	1	-1	-1	0	-1	-1	2	0	1	1	0	0	0	x_2
x_3	:	++	14	10	6	0	-4	1	2	-2	14	-2	-2	2	0	4	-1	0	0	3	0	0	0	-1	2	-2	-1	1	0	0	1	-1	-1	x_3
x_4	:	++	50	10	2	6	5	1	2	-2	10	10	2	2	-2	0	0	0	5	-1	0	1	0	-1	1	1	1	-1	1	-1	0	0	0	x_4
x_5	:	++	49	-11	9	7	4	-2	1	1	-11	5	-3	1	3	-1	-1	-1	0	0	1	0	1	0	-2	2	-2	0	0	0	-1	-1	-1	x_5
x_6	:	++	28	44	-4	0	10	-1	-2	2	4	4	4	4	0	3	3	-1	2	-1	0	0	0	0	-2	-2	1	1	0	0	0	-1	-1	x_6
x_7		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_7
x_8																																		x_8
x_9	:	++	90	66	10	-6	9	3	0	0	10	10	2	2	2	5	0	1	1	1	0	-1	0	0	1	1	1	-1	-1	1	-1	0	0	x_9
x_{10}	:	++	125	-55	5	-1	-10	-1	-1	-1	25	9	1	-3	3	0	0	0	2	-1	-1	-1	1	0	4	0	1	1	0	-1	0	0	0	x_{10}
x_{11}	:	++	42	-66	-14	0	6	-3	0	0	50	2	2	-2	0	7	2	-1	-2	1	0	0	0	0	2	2	-1	-1	0	0	1	0	0	x_{11}
x_{12}	:	++	50	-110	-14	-6	5	-2	2	-2	-30	2	-6	2	2	0	0	0	1	-2	0	1	0	0	3	-1	0	0	-1	1	0	0	0	x_{12}
x_{13}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{13}
x_{14}																																		x_{14}
x_{15}	:	++	225	45	9	-1	0	0	3	3	45	-3	-3	1	3	0	0	0	0	0	-1	1	-1	1	0	0	0	0	0	-1	0	0	0	x_{15}
x_{16}	:	++	126	-54	6	0	-9	0	0	0	-6	10	-6	-2	-4	1	1	1	3	0	0	0	0	1	-3	1	0	0	-1	0	1	-1	-1	x_{16}
x_{17}	:	++	175	55	-1	7	-5	1	1	1	-25	7	7	-1	-1	0	0	0	-1	-1	1	0	-1	0	5	1	-1	1	-1	0	0	0	0	x_{17}
x_{18}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{18}
x_{19}																																		x_{19}
x_{20}	:	++	140	-44	-4	0	14	1	2	-2	20	-12	4	-4	0	5	0	1	2	-1	0	0	0	0	-4	0	-1	1	0	0	-1	0	0	x_{20}
x_{21}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{21}
x_{22}																																		x_{22}
x_{23}	:	++	126	66	-10	0	-9	3	0	0	-46	2	2	-2	4	1	1	1	-1	-1	0	0	0	0	-1	-1	-1	-1	1	0	1	-1	-1	x_{23}
x_{24}	:	++	336	0	-16	0	-6	0	0	0	16	16	0	0	0	-4	1	0	2	2	0	0	0	0	-2	-2	-2	0	0	0	-1	1	1	x_{24}
x_{25}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{25}
x_{26}																																		x_{26}
x_{27}	:	++	160	144	0	8	16	0	1	-3	0	0	0	0	0	-5	0	-1	0	0	-1	-1	0	1	0	0	0	0	0	1	1	0	0	x_{27}
x_{28}	:	++	175	55	-1	7	-5	1	1	1	55	-9	-1	-1	-1	0	0	0	-1	-1	1	0	1	0	1	-3	1	-1	-1	0	0	0	0	x_{28}
x_{29}	:	++	448	32	0	0	16	-4	-2	2	0	0	0	0	0	-2	-2	2	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	x_{29}
x_{30}	:	++	378	54	18	0	0	0	0	0	-54	-6	-6	-2	0	3	3	-1	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1	x_{30}
x_{31}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{31}
x_{32}																																		x_{32}
x_{33}	:	++	350	-110	6	0	-10	-2	2	-2	-10	6	6	2	0	0	0	0	-6	0	0	0	0	0	-4	0	2	0	0	0	0	0	0	x_{33}
x_{34}	:	++	336	0	-16	0	-6	0	0	0	-16	-16	0	0	0	-4	1	0	2	2	0	0	0	0	2	2	2	0	0	0	-1	-1	-1	x_{34}
x_{35}	:	++	288	-176	0	8	0	4	-3	1	0	0	0	0	0	3	-2	-1	0	0	-1	1	0	0	0	0	0	0	0	1	0	0	0	x_{35}
x_{36}	:	++	0	120	-16	0	0	3	0	0	40	8	-8	0	0	0	0	0	-4	-1	0	0	0	-1	-2	2	1	1	0	0	0	0	0	x_{36}
x_{37}	:	++	160	-176	0	-8	16	4	1	1	0	0	0	0	0	-5	0	-1	0	0	1	-1	0	0	0	0	0	0	0	-1	1	0	0	x_{37}
x_{38}	:	++	90	-54	-6	6	9	0	0	0	-30	2	-6	2	-2	5	0	1	-3	0	0	-1	0	1	3	-1	0	0	1	-1	-1	0	0	x_{38}
	:	++	300	0	4	-6	-15	0	0	0	40	-8	0	4	-2	0	0	0	1	-2	0	-1	0	0	1	1	-2	0	1	1	0	0	0	x_{39}
x_{40}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r10	*	x_{40}
x_{41}	:	++	36	0	12	-6	9	0	0	0	24	8	0	-4	-2	-4	1	0	-3	0	0	1	0	0	3	-1	0	0	1	1	-1	-1	-1	x_{41}
x_{42}	:	++	63	99	-9	-7	0	0	3	3	-45	3	3	-1	-3	3	-2	-1	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	x_{42}
x_{43}	:	++	288	144	0	-8	0	0	-3	-3	0	0	0	0	0	3	-2	-1	0	0	1	1	0	1	0	0	0	0	0	-1	0	0	0	x_{43}
	fus	ind	4	2	4	8	12	6	12	6	8	8	8	8	8	20	20	10	12	12	24	28	16	22	24	24	24	24	24	56	60	40	40	
											8		8		8	20	20						16	22	24		24	24	24	56	60	40	40	
x_{44}	:	oo	0	0	0	0	0	0	0	0	412	0	212	0	212	0	0	0	0	0	0	0	i2	0	-212	0	i2	-i2	-i2	0	0	-i2	-i2	x_{44}
x_{45}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{45}
x_{46}																																		x_{46}
x_{47}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{47}
x_{48}																																		x_{48}
x_{49}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	215	i5	0	0	0	0	0	0	0	0	0	0	0	0	0	-i5	i5	-i5	x_{49}
x_{50}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{50}
x_{51}																																		x_{51}
x_{52}	:	oo	0	0	0	0	0	0	0	0	4i2	0	212	0	-212	0	0	0	0	0	0	0	-i2	0	-212	0	i2	-i2	i2	0	0	-i2	-i2	x_{52}
x_{53}	:	oo	0	0	0	0	0	0	0	0	2012	0	-612	0	212	0	0	0	0	0	0	0	-i2	0	212	0	212	0	-i2	0	0	0	0	x_{53}
x_{54}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{54}
x_{55}																																		x_{55}
x_{56}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{56}
x_{57}																																		x_{57}
x_{58}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{58}
x_{59}																																		x_{59}
x_{60}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{60}
x_{61}																																		x_{61}
x_{62}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{62}
x_{63}																																		x_{63}
x_{64}	:	oo	0	0	0	0	0	0	0	0	5612	0	-412	0	0	0	0	0	0	0	0	0	0	0	-412	0	-i2	-i2	0	0	0	i2	i2	x_{64}
x_{65}	:	oo	0	0	0	0	0	0	0	0	4012	0	412	0	-412	0	0	0	0	0	0	0	0	0	-212	0	i2	i2	-i2	0	0	0	0	x_{65}
x_{66}	:	oo	0	0	0	0	0	0	0	0	2012	0	-612	0	-212	0	0	0	0	0	0	0	i2	0	212	0	212	0	i2	0	0	0	0	x_{66}
x_{67}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i11	0	0	0	0	0	0	0	0	0	x_{67}
x_{68}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	615	-215	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{68}
x_{69}	:	oo	0	0	0	0	0	0	0	0	1612	0	812	0	0	0	0	0	0	0	0	0	0	0	412	0	i2	-i2	0	0	0	i2	i2	x_{69}
x_{70}		-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{70}
x_{71}																																		x_{71}
x_{72}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	215	i5	0	0	0	0	0	0	0	0	0	0	0	0	0	-i5	-i5	i5	x_{72}
x_{73}	:	oo	0	0	0	0	0	0	0	0	6412	0	0	0	0	0	0	0	0	0	0	0	0	0	412	0	-212	0	0	0	0	-i2	-i2	x_{73}
x_{74}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i74	0	0	0	x_{74}
x_{75}	:	oo	0	0	0	0	0	0	0	0	4012	0	412	0	412	0	0	0	0	0	0	0	0	0	-212	0	i2	i2	i2	0	0	0	0	x_{75}
x_{76}	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	415	215	0	0	0	0	0	0	0	0	0	0	0	0	0	i5	0	0	x_{76}

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		1A	2A	2B	3A	3B	3C	4A	4B	4C	4D	5A	5B	6A	6B	C**	6D	6E	7A	8A	8B	8C	9A	B**	10A	10B	11A	12A	12B	12C	12D	12E	13A	B*	14A	15A	B*	15C	18A	B**	20A	21A	B*	24A	
	ind	1	2	2	3	3	3	4	4	4	4	5	5	6	6	6	6	6	7	8	8	8	9	9	10	10	11	12	12	12	12	12	13	13	14	15	15	15	18	18	20	21	21	24	
		3	6	6	3	3		12	12	12	12	15	15	6	6	6	6		21	24	24	24	9	9	30	30	33	12	12	12		12	39	39	42			15	18	18	60	21	21	24	
		3	6	6	3	3		12	12	12	12	15	15	6	6	6	6		21	24	24	24	9	9	30	30	33	12	12	12		12	39	39	42			15	18	18	60	21	21	24	
χ_{77}	o2	66	2	6	21	3	0	10	2	-2	0	6	1	5-2i3	-1 2i3	-1	-1	0	3	2	2	0	z3-1	**	2	1	0	1	1	1	0	-1	1	1	-1	0	0	1	**	-z3	0	0	0	-1	χ_{77}
χ_{78}	o2	78	14	-6	15	6	0	6	-2	2	0	3	3	-1	-2i3+2	2i3+2	2	0	1	-2	2	0	**	z3-1	-1	-1	1	3	0	-1	0	-2	0	0	1	0	0	0	-z3	**	1	1	1	1	χ_{78}
χ_{79}	o2	429	-19	9	69	6	0	21	-3	1	3	9	-1	5	-2i3-4	2i3-4	2	0	2	1	1	-1	i3	-i3	1	-1	0	-3	0	1	0	0	0	0	2	0	0	-1	-1	-1	1	-1	-1	1	χ_{79}
χ_{80}	o2	1365	85	21	-21	24	0	5	5	5	-3	0	5	-5	2i3-2	-2i3-2	4	0	0	1	1	1	**	z3-1	0	1	1	-1	2	-1	0	2	0	0	0	0	0	-1	z3	**	0	0	0	1	χ_{80}
χ_{81}	o2	1716	52	36	141	-3	0	20	4	4	0	6	1	13	2i3+1	-2i3+1	1	0	1	4	0	0	**	z3+2	2	1	0	5	-1	1	0	1	0	0	1	0	0	1	**	z3	0	1	1	1	χ_{81}
χ_{82}	o2	2145	-31	-15	210	3	0	49	1	1	-3	15	0	2	-2i3-7	2i3-7	-1	0	3	-3	1	1	z3+2	**	-1	0	0	-2	1	-2	0	1	0	0	-1	0	0	0	-z3	**	-1	0	0	0	χ_{82}
χ_{83}	o2	2925	45	-15	-45	-18	0	5	-3	-7	3	0	0	3	0	0	6	0	-1	1	1	-1	0	0	0	0	-1	-1	2	-1	0	0	0	0	-1	0	0	0	0	0	0-b21	*	1	χ_{83}
χ_{84}	o2	2925	45	-15	-45	-18	0	5	-3	-7	3	0	0	3	0	0	6	0	-1	1	1	-1	0	0	0	0	-1	-1	2	-1	0	0	0	0	-1	0	0	0	0	0	0	*-b21	1	χ_{84}
χ_{85}	o2	3003	59	-21	105	15	0	11	-5	-5	3	3	3	-7	4i3-7	-4i3-7	-1	0	0	-1	3	-1	**	z3+2	-1	-1	0	5	-1	1	0	1	0	0	0	0	0	0	**	-z3	1	0	0	-1	χ_{85}
χ_{86}	o2	4290	130	6	69	33	0	10	2	-2	0	0	5	-11	-4i3+7	4i3+7	1	0	-1	2	2	0	**	z3+2	0	1	0	1	1	1	0	-1	0	0	-1	0	0	-1	**	z3	0	-1	-1	-1	χ_{86}
χ_{87}	o2	5103	-81	-21	0	0	0	-9	15	3	-3	3	3	0	0	0	0	0	0	3	-1	1	0	0	-1	-1	-1	0	0	0	0	0-b13		*	0	0	0	0	0	0	1	0	0	0	χ_{87}
χ_{88}	o2	5103	-81	-21	0	0	0	-9	15	3	-3	3	3	0	0	0	0	0	0	3	-1	1	0	0	-1	-1	-1	0	0	0	0	0	*-b13		0	0	0	0	0	0	1	0	0	0	χ_{88}
χ_{89}	o2	6720	64	0	420	6	0	64	0	0	0	15	0	4	-4i3+4	4i3+4	-2	0	0	0	0	0	**	z3-1	-1	0	-1	4	-2	0	0	0	-1	-1	0	0	0	0	z3	**	-1	0	0	0	χ_{89}
χ_{90}	o2	18954	-54	54	729	0	0	66	-6	-2	0	9	-1	9	0	0	0	0	-2	2	-2	0	0	0	1	-1	1	-3	0	1	0	0	0	0	-2	0	0	-1	0	0	1	1	1	-1	χ_{90}
χ_{91}	o2	21450	10	30	615	-24	0	50	-6	-10	0	0	0	7	2i3+4	-2i3+4	4	0	2	2	-2	0	-i3	i3	0	0	0	-1	2	-1	0	0	0	0	2	0	0	0	1	1	0	-1	-1	-1	χ_{91}
χ_{92}	o2	23100	-260	0	-210	24	0	20	12	8	-6	0	0	-2	2i3+4	-2i3+4	4	0	0	0	0	-2	i3	-i3	0	0	0	2	2	2	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	χ_{92}
χ_{93}	o2	24024	88	0	-294	39	0	40	-8	-16	0	-6	4	10	2i3+1	-2i3+1	-5	0	0	0	0	0	z3-1	**	-2	0	0	-2	1	2	0	1	0	0	0	0	0	1	**	z3	0	0	0	0	χ_{93}
χ_{94}	o2	27027	531	-21	-189	54	0	19	3	11	3	-3	7	3	0	0	6	0	0	-1	-1	-1	0	0	1	-1	0	-5	-2	-1	0	0	0	0	0	0	0	1	0	0	-1	0	0	-1	χ_{94}
χ_{95}	o2	30888	-216	-48	756	27	0	56	8	0	0	3	-2	-12	-9	-9	3	0	-3	0	0	0	0	0	-1	2	0	-4	-1	0	0	-1	0	0	1	0	0	1	0	0	1	0	0	0	χ_{95}
χ_{96}	o2	42900	340	120	420	33	0	-20	4	0	6	0	0	4	-4i3+1	4i3+1	1	0	-3	0	0	2	i3	-i3	0	0	0	4	1	0	0	1	0	0	1	0	0	0	1	1	0	0	0	0	χ_{96}
χ_{97}	o2	51975	135	-105	945	-27	0	55	7	7	3	0	0	-15	9	9	-3	0	0	-5	-1	-1	0	0	0	0	0	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	1	χ_{97}
χ_{98}	o2	60060	-100	84	21	30	0	-20	12	4	0	0	-5	5	-2i3-4	2i3-4	2	0	0	-4	0	0	-i3	i3	0	-1	0	1	4	1	0	0	0	0	0	0	0	1	-1	-1	0	0	0	-1	χ_{98}
χ_{99}	o2	64350	30	90	1035	9	0	-10	14	2	0	0	0	27	6i3+3	-6i3+3	-3	0	-1	-2	2	0	0	0	0	0	0	-1	-1	-1	0	-1	0	0	-1	0	0	0	0	0	0	-1	-1	1	χ_{99}
χ_{100}	o2	75075	515	-105	735	-3	0	-5	-13	-1	-3	0	0	-1	4i3-1	-4i3-1	5	0	0	-1	-1	1	i3	-i3	0	0	0	7	1	-1	0	-1	0	0	0	0	0	0	-1	-1	0	0	0	-1	χ_{100}
χ_{101}	o2	75075	515	-105	-210	51	0	-5	-13	-1	-3	0	0	14	-2i3-1	2i3-1	-1	0	0	-1	-1	1	**	z3+2	0	0	0	-2	1	2	0	-1	0	0	0	0	0	0	**	-z3	0	0	0	2	χ_{101}
χ_{102}	o2	85800	40	120	-510	39	0	40	8	-8	0	0	0	-14	2i3+7	-2i3+7	-5	0	1	0	0	0	z3+2	**	0	0	0	-2	1	-2	0	-1	0	0	1	0	0	0	z3	**	0	1	1	0	χ_{102}
χ_{103}	o2	104247	567	111	0	0	0	-9	-9	15	3	-3	-3	0	0	0	0	0	3	3	-1	-1	0	0	-3	1	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	χ_{103}
χ_{104}	o2	108108	-180	84	945	-27	0	28	-4	4	0	-12	3	-15	-9	-9	-3	0	0	4	0	0	0	0	0	-1	0	1	1	1	0	-1	0	0	0	0	0	0	0	0	-2	0	0	1	χ_{104}
χ_{105}	o2	133056	-576	0	-756	54	0	64	0	0	0	-9	-4	12	0	0	6	0	0	0	0	0	0	0	-1	0	0	4	-2	0	0	0	1	1	0	0	0	-1	0	0	-1	0	0	0	χ_{105}
χ_{106}	o2	135135	-225	-21	756	27	0	-25	-17	3	-3	0	-5	-12	9	9	3	0	0	3	3	1	0	0	0	-1	0	-4	-1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	χ_{106}
χ_{107}	o2	139776	512	0	-336	60	0	0	0	0	0	6	-4	-16	4i3-4	-4i3-4	-4	0	0	0	0	0	**	z3-1	2	0	-1	0	0	0	0	0	0	0	0	0	0	-1	-z3	**	0	0	0	0	χ_{107}
χ_{108}	o2	160380	-324	-120	0	0	0	36	12	0	6	-15	0	0	0	0	0	0	3	0	0	2	0	0	1	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	1	0	0	0	χ_{108}
χ_{109}	o2	180180	276	0	630	9	0	-4	4	-8	-6	-15	0	6	-6i3-3	6i3-3	-3	0	0	0	0	-2	0	0	1	0	0	2	-1	-2	0	1	0	0	0	0	0	0	0	0	1	0	0	0	χ_{109}
χ_{110}	o2	189540	-540	36	729	0	0	-60	-12	4	0	0	5	9	0	0	0	0	1	-4	0	0	0	0	0	1	-1	-3	0	1	0	0	0	0	1	0	0	-1	0	0	0	1	1	-1	χ_{110}
χ_{111}	o2	193050	90	-90	-945	-54	0	50	-6	14	0	0	0	15	0	0	-6	0	-3	2	2	0	0	0	0	0	0	-1	2	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	χ_{111}
χ_{112}	o2	208494	-594	90	-729	0	0	6	-18	2	0	9	4	-9	0	0	0	0	-1	-2	-2	0	0	0	1	0	0	3	0	-1	0	0	0	0	-1	0	0	1	0	0	1	-1	-1	1	χ_{112}
χ_{113}	o2	225225	585	105	-630	-90	0	25	9	-7	-3	0	0	-6	0	0	6	0	0	-3	1	1	0	0	0	0	0	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	o2	331695	-81	-105	0	0	0	-81	15	-9	3	15	0	0	0	0	0	0	0	3	-1	-1	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	χ_{114}
	ind	1	2	4	3	3	3	4	4	4	8	5	5	6	6	6	6	12	7	8	8	8	9	9	10	20	11	12	12	12	24	12	13	13	28	15	15	15	18	18	20	21	21	24	
		6	6	12	6	6	6	12	12	12	24	30	30	6	6	6	6		42	24	24	24	18	18	30	60	66	12	12	12	24	12	78	78	84	30	30	30	18	18	60	42	42	24	
		3	6	12	3	3		12	12	12	24	15	15	6	6	6	6		21	24	24	24	9	9	30	60	33	12	12	12		12	39	39	84			15	18	18	60	21	21	24	
		2	2		6	6		4				10	10	6	6	6	6		14		8		18	18	10		22	12	12			12	26	26				30	18	18	20	42	42	24	
		3	6		3	3		12				15	15	6	6	6	6		21		24		9	9	30		33	12	12			12	39	39				15	18	18	60	21	21	24	
		6	6		6	6		12				30	30	6	6	6	6		42		24		18	18	30		66	12	12			12	78	78				30	18	18	60	42	42	24	
χ_{115}	o2	12	-4	0	-6	3	0	-4	0	0	0	-3	2	2	i3+2	-i3+2	-1	0	-2	0	2	0	-i3	i3	1	0	1	2	-1	0	0	i3	-1	-1	0	0	0	-1	-1	-1	1	1	1	0	χ_{115}
χ_{116}	o2	780	-4	0	-120	6	0	-36	0	0	0	-15	0	8	4i3+2	-4i3+2	2	0	-4	0	2	0	**	z3+2	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	**	-z3	-1	-1	-1	0	χ_{116}
χ_{117}	o2	924	-52	0	-84	15	0	-20	0	0	0	-6	4	-4	i3+8	-i3+8	-1	0	0	0	2	0	**	z3-1	-2	0	0	4	1	0	0	i3	1	1	0	0	0	1	-z3	**	0	0	0	0	χ_{117}
χ_{118}	o2	4368	-176	0	84	39	0	-16	0	0	0	3	8	4	-i3-2	i3-2	-5	0	0	0	0	0	-i3	i3	-1	0	1	-4	-1	0	0	-i3	0	0	0	0	0	-1	1	1	-1	0	0	0	χ_{118}
χ_{119}	o2	4368	80	0	-294	12	0	-48	0	0	0	-12	-2	2	4i3-4	-4i3-4	-4	0	0	0	0	0	z3+2	**	0	0	1	-6	0	0	0	0	0	0	0	0	0	1	-z3	**	2	0	0	0	χ_{119}
χ_{120}	o2	8580	-44	0	-510	-15	0	-76	0	0	0	-15	0	10	-i3+4	i3+4	1	0	-2	0	-2	0	z3-1	**	1	0	0	2	-1	0	0	-i3	0	0	0	0	0	0	**	z3	-1	1	1	0	χ_{120}
χ_{121}	o2	11088	144	0	126	18	0	-16	0	0	0	3	-2	6	0	0	-6	0	0	0	0	0	0	0	-1	0	0	2	2	0	0	0	-1	-1	0	0	0	1	0	0	-1	0	0	2r3	χ_{121}
χ_{122}	o2	11088	144	0	126	18	0	-16	0	0	0	3	-2	6	0	0	-6	0	0	0	0	0	0	0	-1	0	0	2	2	0	0	0	-1	-1	0	0	0	1	0	0	-1	0	0-2r3		χ_{122}
χ_{123}	o2	12012	-164	0	-336	6	0	-4	0	0	0	-3	2	16	-4i3-8	4i3-8	-2	0	0	0	2	0	i3	-i3	1	0	0	8	2	0	0	0	0	0	0	0	0	-1	1	1	1	0	0	0	χ_{123}
χ_{124}	o2	12012	-164	0	42	33	0	-4	0	0	0	-3	2	-14	-i3-8	i3-8	1	0	0	0	2	0	**	z3-1	1	0	0	2	-1	0	0	-i3	0	0	0	0	0	2	z3	**	1	0	0	0	χ_{124}
χ_{125}	o2	12012	92	0	-336	6	0	-36	0	0	0	-3	2	-16	8z3-6	**	2	0	0	0	2	0	i3	-i3	-3	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	χ_{125}
χ_{126}	o2	23100	-20	0	-210	24	0	-20	0	0	0	0	0	-26	-8z3+6	**	4	0	0	0	2	0	i3	-i3	0	0	0	-2	-2	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	χ_{126}
χ_{127}	o2	27456	-448	0	-120	60	0	0	0	0	0	6	6	8	4i3+8	-4i3+8	-4	0	2	0	0	0	i3	-i3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	-1	-1	0	χ_{127}
χ_{128}	o2	27456	64	0	-876	6	0	-64	0	0	0	-9	-4	4	-4i3+4	4i3+4	-2	0	2	0	0	0	**	z3-1	-1	0	0	-4	2	0	0	0	0	0	0	0	0	-1	z3	**	1	-1	-1	0	χ_{128}
χ_{129}	o2	35100	-180	0	270	-54	0	-20	0	0	0	0	0	6	0	0	-6	0	2	0	2	0	0	0	0	0	-1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0-b21	*	0	χ_{129}
χ_{130}	o2	35100	-180	0	270	-54	0	-20	0	0	0	0	0	6	0	0	-6	0	2	0	2	0	0	0	0	0	-1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	*-b21	0	χ_{130}
χ_{131}	o2	42900	-220	0	-930	6	0	-60	0	0	0	0	0	-10	4i3+2	-4i3+2	2	0	4	0	-2	0	**	z3+2	0	0	0	6	0	0	0	0	0	0	0	0	0	0	**	-z3	0	1	1	0	χ_{131}
χ_{132}	o2	61236	324	0	0	0	0	36	0	0	0	-9	6	0	0	0	0	0	0	0	-2	0	0	0	-1	0	-1	0	0	0	0	0	b13	*	0	0	0	0	0	0	1	0	0	0	χ_{132}
χ_{133}	o2	61236	324	0	0	0	0	36	0	0	0	-9	6	0	0	0	0	0	0	0	-2	0	0	0	-1	0	-1	0	0	0	0	0	*	b13	0	0	0	0	0	0	1	0	0	0	χ_{133}
χ_{134}	o2	77220	-396	0	-540	-54	0	-44	0	0	0	15	0	-12	0	0	-6	0	-4	0	-2	0	0	0	-1	0	0	4	-2	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	χ_{134}
χ_{135}	o2	105600	640	0	120	48	0	0	0	0	0	0	0	-8	-4i3+4	4i3+4	-8	0	-2	0	0	0	z3+2	**	0	0	0	0	0	0	0	0	1	1	0	0	0	0	z3	**	0	1	1	0	χ_{135}
χ_{136}	o2	112320	-576	0	540	54	0	-64	0	0	0	15	0	12	0	0	6	0	-2	0	0	0	0	0	-1	0	-1	-4	2	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	χ_{136}
χ_{137}	o2	139776	512	0	-336	60	0	0	0	0	0	6	-4	-16	4i3-4	-4i3-4	-4	0	0	0	0	0	**	z3-1	2	0	-1	0	0	0	0	0	0	0	0	0	0	-1	-z3	**	0	0	0	0	χ_{137}
χ_{138}	o2	144144	336	0	-1764	-9	0	-16	0	0	0	9	4	12	-3i3-6	3i3-6	3	0	0	0	0	0	0	0	1	0	0	-4	-1	0	0	i3	0	0	0	0	0	1	0	0	-1	0	0	0	χ_{138}
χ_{139}	o2	144144	-432	0	-630	72	0	80	0	0	0	-6	-6	18	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{139}
χ_{140}	o2	171600	400	0	1140	24	0	-80	0	0	0	0	0	4	-4i3-2	4i3-2	4	0	2	0	0	0	**	z3+2	0	0	0	4	-2	0	0	0	0	0	0	0	0	0	**	z3	0	-1	-1	0	χ_{140}
χ_{141}	o2	180180	-924	0	630	9	0	4	0	0	0	-15	0	-18	-3i3-6	3i3-6	-3	0	0	0	-2	0	0	0	1	0	0	-2	1	0	0	i3	0	0	0	0	0	0	0	0	-1	0	0	0	χ_{141}
χ_{142}	o2	300300	-260	0	-840	-39	0	60	0	0	0	0	0	-8	-i3+4	i3+4	1	0	0	0	2	0	**	z3+2	0	0	0	0	3	0	0	-i3	0	0	0	0	0	0	**	z3	0	0	0	0	χ_{142}
χ_{143}	o2	436800	320	0	840	-42	0	0	0	0	0	0	0	8	4i3+2	-4i3+2	2	0	0	0	0	0	z3-1	**	0	0	1	0	0	0	0	0	0	0	0	0	0	0	**	-z3	0	0	0	0	χ_{143}

		2C	2D	4E	4F	6F	6G	6H	6I	8D	8E	8F	8G	8H	10C	10D	10E	12F	12G	12H	14B	16A	22A	24B	24C	24D	24E	24F	28A	30A	40A	B#
fus	ind	2	2	4	4	6	6	6	6	8	8	8	8	8	10	10	10	12	12	12	14	16	22	24	24	24	24	24	28	30	40	40

X_{77} * +
X_{78} * +
X_{79} * +
X_{80} * +
X_{81} * +
X_{82} * +
X_{83} o2
X_{84} *
X_{85} * +
X_{86} * +
X_{87} o2
X_{88} *
X_{89} * +
X_{90} * +
X_{91} * +
X_{92} * +
X_{93} * +
X_{94} * +
X_{95} * +
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X_{108} * +
X_{109} * +
X_{110} * +
X_{111} * +
X_{112} * +
X_{113} * +
X_{114} * +

Sporadic Suzuki group Suz

Order = 448,345,497,600 = $2^{13}.3^7.5^2.7.11.13$ Mult = 6 Out = 2

Constructions

Graph G.2 : the automorphism group of the Suzuki graph, a rank 3 valence 416 graph on 1782 vertices, in which the point stabilizer is $G_2(4).2$. This graph is the culmination of the Suzuki chain, which includes the rank 3 graphs with automorphism groups $G_2(4).2$, $J_2.2$ and $U_3(3).2$.

Leech 6.G : the automorphism group of the complex Leech lattice. Using the modulo 11 definition of the ternary Golay code (see M_{12}), the lattice consists of all 12-dimensional vectors $(x_\infty,x_0,\ldots,x_9,x_X)$ over $Z[w]$, where $w = z3$, for which there is a Golay code word $(c_\infty,\ldots,c_X)$ and a number $m = 0$, 1 or -1, such that

(i) all $x_t \equiv m \pmod{\theta}$, where $\theta = w - \bar{w} = i3$

(ii) $x_t \equiv m + c_t.\theta \bmod 3$

(iii) $\Sigma x_t \equiv -3m \bmod 3\theta$.

Over Z the lattice becomes the 24-dimensional real Leech lattice and the standard norm $\frac{1}{9}\Sigma x_i\bar{x}_i$ becomes the type (see Co_1). The group is transitive on the minimal vectors and the stabilizer is $U_5(2)$. Modulo θ every vector is congruent to just one of:

(i) the zero vector

(ii) 3 proportional vectors $w^i v$, with v of type 2

(iii) 3x12 vectors $w^i v_1, \ldots, w^i v_{12}$, with v_i mutually orthogonal type 3 vectors.

In the real Leech lattice the stabilizer of the set of vectors $\pm w^i(3\theta,0^{11})^S$ is a group $2 \times 3^6{:}2M_{12}$ in which the subgroup $2 \times 3^6{:}2$ is generated by the maps taking (x_i) to $(-x_i)$, $(w^{c_i}x_i)$, and $(\bar{x}_i)$, for (c_i) in the Golay code. The subgroup of this group preserving the complex lattice structure (and so in 6.G) is the centralizer of $(x_i) \longrightarrow (wx_i)$, namely a group $2 \times 3^6{:}M_{11}$, in which there is a subgroup M_{11} of pure permutations acting transitively on the coordinates. The unit scalars $\pm w^i$ can be regarded as a fixed-point-free subgroup $2A_3$ of $2Co_1$, which can be extended to fixed-point-free groups $2A_4$ and $2A_5$, which yield the quaternionic and icosian Leech lattices (see $G_2(4)$ and J_2). Various maximal subgroups arise by intersecting 6G with maximal subgroups of $2Co_1$, and are labelled correspondingly below.

Presentation 3.G.2 ≅ < a b c 8 d e f g h | a=(cd)^4, (bcde)^8=1 >; for G.2 adjoin (bcdcdefgh)^{13}=1.

3.G.2 ≅ < e d 8 c b f 8 g | a=(cd)^4=(fg)^4, 1=(abcf)^5=(abcdecbdc)^7 >; for G.2 adjoin (bfgfg)^5=1.
(5, a)

fus	ind	4	2	4	8	12	6	12	6	8 8	8	8 8	8	8 8	20 20	20 20	10	12	12	24	28	16 16	22 22	24 24	24	24 24	24 24	24 24	56 56	60 60	40 40	40 40

X_{115} * +
X_{116} * +
X_{117} * +
X_{118} * +
X_{119} * +
X_{120} * +
X_{121} o2
X_{122} *
X_{123} * +
X_{124} * +
X_{125} * +
X_{126} * +
X_{127} * +
X_{128} * +
X_{129} o2
X_{130} *
X_{131} * +
X_{132} o2
X_{133} *
X_{134} * +
X_{135} * +
X_{136} * +
X_{137} * +
X_{138} * +
X_{139} * +
X_{140} * +
X_{141} * +
X_{142} * +
X_{143} * +

Maximal subgroups

Order	Index	Structure	G.2	Character
251596800	1782	$G_2(4)$	: $G_2(4){:}2$	1a+780a+1001a
19595520	22880	$3_2{\cdot}U_4(3){:}2_3$	: $3U_4(3).(2^2)_{133}$	1a+364a+780a+5940a+15795a
13685760	32760	$U_5(2)$	: $U_5(2){:}2$	1a+143a+364a+5940a+12012a+14300a
3317760	135135	$2_-^{1+6}{\cdot}U_4(2)$	: $2_-^{1+6}{\cdot}U_4(2).2$	
1924560	232960	$3^5{:}M_{11}$	: $3^5{:}(M_{11}x2)$	
1209600	370656	$J_2{:}2$	: $J_2{:}2 \times 2$	
1105920	405405	$2^{4+6}{:}3A_6$	: $2^{4+6}{:}3S_6$	
483840	926640	$(A_4xL_3(4)){:}2_1$	: $(A_4xL_3(4){:}2_3){:}2$	
368640	1216215	$2^{2+8}{:}(A_5xS_3)$	: $2^{2+8}{:}(S_5xS_3)$	
190080	2358720	$M_{12}{:}2$	: $M_{12}{:}2 \times 2$	
139968	3203200	$3^{2+4}{:}2(A_4x2^2).2$	: $3^{2+4}{:}2(S_4xD_8)$	
43200	10378368	$(A_6xA_5){:}2$	: $(A_6{:}2_2xA_5){:}2$	
25920	17297280	$(3^2{:}4xA_6){\cdot}2$	: $(3^2{:}8xA_6){\cdot}2$	
11232	39916800	$L_3(3){:}2$		
11232	39916800	$L_3(3){:}2$		
7800	57480192	$L_2(25)$	: $L_2(25){:}2$	
2520	177914880	A_7	: S_7	

Specifications

Abstract	Leech	Graph
	quaternionic	vertex
N(3A)	bisection	
	minimal vector	
N(2A)	4-space	
$N(3^5) = N(3A_{11}B_{110})$	base	
C(2C)	icosian	edge
$N(2A^4)$	sextet	
$N(2B^2)$		
$N(2A^2)$	trio	non-edge
C(2D)		
$N(3^2) = N(3A_2B_2)$	linked 3-spaces	
N(2B,3A,5A)		
$N(3C^2)$		
N(2A,3B,3C,4C,...)		
N(2A,3B,3C,4C,...)		
N(2B,3C,4D,5A,5B,...)		
N(2B,3C,3C,4D,5B,6E,7A)		

O'N

Sporadic O'Nan group O'N

Order = 460,815,505,920 = $2^9.3^4.5.7^3.11.19.31$ Mult = 3 Out = 2

Constructions

Graph G has been explicitly constructed as a permutation group. There are two permutation representations on 122760 points (black and white) which can be regarded as the vertices of a bipartite graph of valence 456, whose automorphism group is G.2, the outer automorphism interchanging black points and white points. The point stabilizer is $L_3(7):2$ and the suborbit lengths are 1, 5586, 6384, 52316, 58653 (on black points), and 456, 11172, 32928, 78204 (on white points). The edge stabilizer in G.2 is $7_+^{1+2}:(3 \times D_{16})$.

Modulo 7 3.G has two 45-dimensional representations over $\mathbb{F}_7$ on spaces V and W which are interchanged by the outer elements of 3.G.2, and spanned by vectors V_{mn} and W_{mn} (m,n in {0,1,2,3,4,5,6,7,8,9,X}) that satisfy $V_{mn} + V_{nm} = 0 = \Sigma_m V_{mn}$ and $W_{mn} + W_{nm} = 0 = \Sigma_m W_{mn}$.

The group 3.G.2 is generated by the subscript permutations

A = (0123456789X) B = (13954)(267X8) C = (3945)(26X7)

together with the involution D which takes

$$V_{ab} \longrightarrow -2W_{ab} + (-W_{ai} + 2W_{ac} - 2W_{bc} + 2W_{ic} - 2W_{jc} + W_{ec})(1-P+P^2-P^3+P^4-P^5)$$

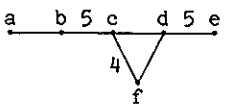
$$W_{ab} \longrightarrow 3V_{ab} + (3V_{ai} - 2V_{ac} + 2V_{bc} + 3V_{ic} - 3V_{jc} + 3V_{ec})(1-P+P^2-P^3+P^4-P^5)$$

whenever a,b,c,d,e,f,g,h,i,j,k is an arithmetic progression modulo 11 in which b-a is a quadratic residue, and P is the permutation (ab)(cdhgek)(fij). The elements A, B, C generate M_{11}, and D commutes with $\langle A,B,C^2\rangle = L_2(11)$.

Each space supports an invariant skew-symmetric trilinear form with

$[V_{qr},V_{rs},V_{qs}] = 1$	$[V_{qr},V_{rs},V_{st}] = 1$	$[V_{qr},V_{rs},V_{tu}] = 1$
$[V_{qr},V_{qs},V_{tu}] = 2$	$[V_{Zq},V_{ru},V_{st}] = 4$	$[V_{ZQ},V_{RU},V_{ST}] = 3$
$[W_{qr},W_{rs},W_{qs}] = 6$	$[W_{qr},W_{rs},W_{st}] = 6$	$[W_{qr},W_{rs},W_{tu}] = 0$
$[W_{qr},W_{qs},W_{tu}] = 3$	$[W_{Zq},W_{ru},W_{st}] = 5$	$[W_{ZQ},W_{RU},W_{ST}] = 0$

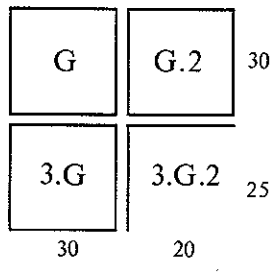
whenever (qrstu)(QRSTU)(Z) is in M_{11}, and {Q,R,S,T,U} in S(4,5,11). Values at triples of basis elements not derivable from these by the skew-symmetry of the form and the relations $V_{mn} = -V_{nm}$, $W_{mn} = -W_{nm}$, are zero.

"Presentations" 3.G ≅ <  | $(def)^5=1,(cde)^5=a=(cf)^2$, more relations? >

(3.G.2 is obtained by adjoining g such that $(fg)^4=e$, g commutes with a,b,c,d,e, and c commutes with $(dfgfg)^5$)

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract
3753792	122760	$L_3(7):2$	$7_+^{1+2}:(3xD_{16})$	1a+10944a+26752a+32395a+52668a	
3753792	122760	$L_3(7):2$		1a+10944a+26752a+32395b+52668a	
175560	2624832	J_1	: $J_1 \times 2$		C(2B)
161280	2857239	$4_2{\cdot}L_3(4):2_1$	: $4_2{\cdot}L_3(4){\cdot}2^2$		N(2A)
25920	17778376	$(3^2:4xA_6){\cdot}2$	: $(3^2:4xA_6){\cdot}2^2$		$N(3A^2)$
25920	17778376	$3^4:2_-^{1+4}D_{10}$	: H.2		$N(3A^4)$
14880	30968784	$L_2(31)$	31:30		
14880	30968784	$L_2(31)$			
10752	42858585	$4^3{\cdot}L_3(2)$	: $4^3{\cdot}(L_3(2) \times 2)$		$N(2A^3)$
7920	58183776	M_{11}			
7920	58183776	M_{11}			
2520	182863296	A_7	$L_2(7):2$, $A_6:2_2$		
2520	182863296	A_7			



	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		4	161	3	80				1																									175																				
		60815505920	280	240	640	256	180	72	372	49	32	32	20	11	36	28	45	45	16	16	16	16	19	19	19	20	20	28	28	31	31			560	30	672	672	16	30	30	7	11	12	12	15	15	19	19	19	28	28	28	28	
	p power		A	A	A	A	A	AA	A	A	B	B	AA	A	AA	AA	AA	AA	A	A	B	B	A	A	A	AA	AA	AA	AA	A	A			A	AB	A	A	B	AB	AB	BB	AB	AD	AC	BCB	ABB	BB	CB	AB	AC	AD	BC	BD	
	p' part		A	A	A	A	A	AA	A	A	A	A	AA	A	AA	AA	AA	AA	A	A	A	A	A	A	A	AA	AA	AA	AA	A	A			A	AB	A	A	A	AB	AB	BB	AB	AC	AD	ABB	BCB	AB	BB	CB	AC	AD	AC	AD	
	ind	1A	2A	3A	4A	4B	5A	6A	7A	7B	8A	8B	10A	11A	12A	14A	15A	B*	16A	B*	16C	D*	19A	B*2	C*4	20A	B**	28A	B*	31A	B**	fus	ind	2B	6B	8C	D*	8E	10B	C*	14B	22A	24A	B*	30A	B*	38A	B*3	C*9	56A	B*3	C*15	D*5	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	10944	64	9	64	0	-1	1	17	3	0	0	-1	-1	1	1	-1	-1	0	0	0	0	0	0	0	-1	-1	1	1	1	1	:	++	76	1	8	8	0	1	1	-1	-1	-1	-1	1	1	0	0	0	1	1	1	1	χ_{2}
χ_{3}	o	13376	-64	11	64	0	1	-1	-1	-1	0	0	1	0	1	-1	1	1	0	0	0	0	0	0	0	-1	-1	1	1	b31	**	┌	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{3}
χ_{4}	o	13376	-64	11	64	0	1	-1	-1	-1	0	0	1	0	1	-1	1	1	0	0	0	0	0	0	0	-1	-1	1	1	**	b31	└																						χ_{4}
χ_{5}	o	25916	-36	-4	20	4	1	0	-5	2	0	0	-1	0	2	-1	1	1	0	0	0	0	0	0	0	15	-15	-1	-1	0	0	┌	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{5}
χ_{6}	o	25916	-36	-4	20	4	1	0	-5	2	0	0	-1	0	2	-1	1	1	0	0	0	0	0	0	0	-15	15	-1	-1	0	0	└																						χ_{6}
χ_{7}	+	26752	128	22	0	0	2	2	-2	-2	0	0	-2	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	:	++	0	0	8r2	-8r2	0	0	0	0	0	-r2	r2	0	0	0	0	0	r2	-r2	r2	-r2	χ_{7}
χ_{8}	+	32395	75	-5	35	3	0	3	6	-1	3	-1	0	0	-1	-2	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	┌	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{8}
χ_{9}	+	32395	75	-5	35	3	0	3	6	-1	-1	3	0	0	-1	-2	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	└																						χ_{9}
χ_{10}	+	37696	-64	31	-64	0	1	-1	15	1	0	0	1	-1	-1	-1	1	1	0	0	0	0	0	0	0	1	1	-1	-1	0	0	:	++	76	1	-8	-8	0	1	1	-1	-1	1	1	1	1	0	0	0	-1	-1	-1	-1	χ_{10}
χ_{11}	+	52668	92	18	20	4	-2	2	-7	0	0	0	2	0	2	1	-2	-2	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r7	r7	-r7	-r7	χ_{11}
χ_{12}	+	58311	71	-9	71	7	1	-1	1	1	-1	-1	1	0	-1	1	1	1	-1	-1	-1	-1	0	0	0	1	1	1	1	0	0	:	++	209	-1	1	1	1	-1	-1	-1	0	1	1	-1	-1	0	0	0	1	1	1	1	χ_{12}
χ_{13}	+	58311	71	-9	-1	-1	1	-1	1	1	3	-1	1	0	-1	1	1	1	-1	-1	1	1	0	0	0	-1	-1	-1	-1	0	0	┌	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{13}
χ_{14}	+	58311	71	-9	-1	-1	1	-1	1	1	-1	3	1	0	-1	1	1	1	1	1	-1	-1	0	0	0	-1	-1	-1	-1	0	0	└																						χ_{14}
χ_{15}	+	58653	-35	9	-35	-3	8	1	0	0	1	1	0	1	1	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	1	1	:	++	133	1	-7	-7	1	-2	-2	0	1	-1	-1	1	1	0	0	0	0	0	0	0	χ_{15}
χ_{16}	+	64790	70	-10	-70	2	0	-2	12	-2	0	0	0	0	2	0	0	0	r2	-r2	-r2	r2	0	0	0	0	0	0	0	0	0	┌	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	+	64790	70	-10	-70	2	0	-2	12	-2	0	0	0	0	2	0	0	0	-r2	r2	r2	-r2	0	0	0	0	0	0	0	0	0	└																						χ_{17}
χ_{18}	+	85064	-56	14	-56	8	-1	-2	0	0	0	0	-1	1	-2	0	-1	-1	0	0	0	0	1	1	1	-1	-1	0	0	0	0	:	++	56	-4	0	0	0	1	1	0	1	0	0	1	1	-1	-1	-1	0	0	0	0	χ_{18}
χ_{19}	+	116963	35	-1	35	3	8	-1	0	0	-1	-1	0	0	-1	0	-1	-1	1	1	1	1	-1	-1	-1	0	0	0	0	0	0	:	++	77	-1	-7	-7	1	2	2	0	0	-1	-1	-1	-1	1	1	1	0	0	0	0	χ_{19}
χ_{20}	+	143374	14	4	126	-2	-1	-4	0	0	2	2	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	1	1	0	0	-1	-1	:	++	0	0	0	0	0	r5	-r5	0	0	0	0	r5	-r5	0	0	0	0	0	0	0	χ_{20}
χ_{21}	+	169290	90	0	-90	-2	0	0	-5	2	0	0	0	0	0	-1	0	0	r2	-r2	r2	-r2	0	0	0	0	0	1	1	-1	-1	:	++	0	0	-6r2+6	6r2+6	2	0	0	0	0	0	0	0	0	0	0	0	r2-1	-r2-1	r2-1	-r2-1	χ_{21}
χ_{22}	+	169290	90	0	-90	-2	0	0	-5	2	0	0	0	0	0	-1	0	0	-r2	r2	-r2	r2	0	0	0	0	0	1	1	-1	-1	:	++	0	0	6r2+6	-6r2+6	2	0	0	0	0	0	0	0	0	0	0	0	-r2-1	r2-1	-r2-1	r2-1	χ_{22}
χ_{23}	+	175616	0	8	0	0	-4	0	0	0	0	0	0	1	0	0	3b5+2	*	0	0	0	0	-1	-1	-1	0	0	0	0	1	1	:	++	56	2	0	0	0	-2b5	*	0	1	0	0	b5	*	-1	-1	-1	0	0	0	0	χ_{23}
χ_{24}	+	175616	0	8	0	0	-4	0	0	0	0	0	0	1	0	0	*	3b5+2	0	0	0	0	-1	-1	-1	0	0	0	0	1	1	:	++	56	2	0	0	0	*	-2b5	0	1	0	0	*	b5	-1	-1	-1	0	0	0	0	χ_{24}
χ_{25}	+	175770	90	0	90	-6	0	0	7	0	-2	-2	0	1	0	-1	0	0	0	0	0	0	1	1	1	0	0	-1	-1	0	0	:	++	210	0	6	6	-2	0	0	0	1	0	0	0	0	1	1	1	-1	-1	-1	-1	χ_{25}
χ_{26}	+	207360	0	0	0	0	0	0	-8	-1	0	0	0	-1	0	0	0	0	0	0	0	0	-c19	*2	*4	0	0	0	0	1	1	:	++	120	0	0	0	0	0	0	1	-1	0	0	0	0	c19	*2	*4	0	0	0	0	χ_{26}
χ_{27}	+	207360	0	0	0	0	0	0	-8	-1	0	0	0	-1	0	0	0	0	0	0	0	0	*4	-c19	*2	0	0	0	0	1	1	:	++	120	0	0	0	0	0	0	1	-1	0	0	0	0	*4	c19	*2	0	0	0	0	χ_{27}
χ_{28}	+	207360	0	0	0	0	0	0	-8	-1	0	0	0	-1	0	0	0	0	0	0	0	0	*2	*4	-c19	0	0	0	0	1	1	:	++	120	0	0	0	0	0	0	1	-1	0	0	0	0	*2	*4	c19	0	0	0	0	χ_{28}
χ_{29}	+	234080	-160	-10	0	0	0	2	7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	r7	-r7	-1	-1	:	++	0	0	4r2	-4r2	0	0	0	0	0	r2	-r2	0	0	0	0	0	*3	k56	*5	*15	χ_{29}
χ_{30}	+	234080	-160	-10	0	0	0	2	7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-r7	r7	-1	-1	:	++	0	0	4r2	-4r2	0	0	0	0	0	r2	-r2	0	0	0	0	0	*5	*15	*3	k56	χ_{30}
	ind	1	2	3	4	4	5	6	7	7	8	8	10	11	12	14	15	15	16	16	16	16	19	19	19	20	20	28	28	31	31	fus	ind	2	6	8	8	8	10	10	14	22	24	24	30	30	38	38	38	56	56	56	56	
		3	6		12	12	15		21	21	24	24	30	33		42			48	48	48	48	57	57	57	60	60	84	84	93	93																							
		3	6		12	12	15		21	21	24	24	30	33		42			48	48	48	48	57	57	57	60	60	84	84	93	93																							
χ_{31}	o2	342	6	0	-6	2	-3	0	-1	-1	0	0	1	1	0	-1	0	0	r2	-r2	-r2	r2	0	0	0	-1	-1	1	1	1	1	┌	o2																					χ_{31}
χ_{32}	o2	342	6	0	-6	2	-3	0	-1	-1	0	0	1	1	0	-1	0	0	-r2	r2	r2	-r2	0	0	0	-1	-1	1	1	1	1	└																						χ_{32}
χ_{33}	o2	495	15	0	15	-1	0	0	5	-2	-1	3	0	0	0	1	0	0	-1	-1	1	1	1	1	1	0	0	1	1	-1	-1	┌	o2																					χ_{33}
χ_{34}	o2	495	15	0	15	-1	0	0	5	-2	3	-1	0	0	0	1	0	0	1	1	-1	-1	1	1	1	0	0	1	1	-1	-1	└																						χ_{34}
χ_{35}	o2	5643	-21	0	51	3	3	0	8	1	-1	-1	-1	0	0	0	0	0	-1	-1	-1	-1	0	0	0	1	1	2	2	1	1	*	+																					χ_{35}
χ_{36}	o2	5643	-21	0	-21	-5	3	0	8	1	-1	3	-1	0	0	0	0	0	1	1	-1	-1	0	0	0	-1	-1	0	0	1	1	┌	o2																					χ_{36}
χ_{37}	o2	5643	-21	0	-21	-5	3	0	8	1	3	-1	-1	0	0	0	0	0	-1	-1	1	1	0	0	0	-1	-1	0	0	1	1	└																						χ_{37}
χ_{38}	o2	52668	-36	0	84	4	3	0	-7	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	-1	-1	r7	-r7	-1	-1	*	+																					χ_{38}
χ_{39}	o2	52668	-36	0	84	4	3	0	-7	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	-1	-1	-r7	r7	-1	-1	*	+																					χ_{39}
χ_{40}	o2	58311	-153	0	15	-1	6	0	1	1	-1	-1	2	0	0	1	0	0	1	1	1	1	0	0	0	0	0	1	1	0	0	*	+																					χ_{40}
χ_{41}	o2	58653	189	0	21	5	3	0	0	0	1	1	-1	1	0	0	0	0	1	1	1	1	0	0	0	1	1	0	0	1	1	*	+																					χ_{41}
χ_{42}	o2	63612	156	0	84	4	-3	0	10	3	0	0	1	-1	0	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	*	+																					χ_{42}
χ_{43}	o2	111321	57	0	-15	1	6	0	-7	0	1	1	2	1	0	1	0	0	-1	-1	-1	-1	0	0	0	0	0	-1	-1	0	0	*	+																					χ_{43}
χ_{44}	o2	116622	-114	0	126	-2	-3	0	2	2	2	2	1	0	0	-2	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	*	+																					χ_{44}
χ_{45}	o2	122760	-120	0	-120	8	0	0	15	1	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	1	0	0	-1	-1	0	0	*	+																					χ_{45}
χ_{46}	o2	169290	90	0	-90	-2	0	0	-5	2	0	0	0	0	0	-1	0	0	r2	-r2	r2	-r2	0	0	0	0	0	1	1	-1	-1	*	+																					χ_{46}
χ_{47}	o2	169290	90	0	-90	-2	0	0	-5	2	0	0	0	0	0	-1	0	0	-r2	r2	-r2	r2	0	0	0	0	0	1	1	-1	-1	*	+																					χ_{47}
χ_{48}	o2	169632	-96	0	0	0	-3	0	-6	1	0	0	-1	1	0	2	0	0	0	0	0	0	0	0	0	15	-15	0	0	0	0	┌	+2																					χ_{48}
χ_{49}	o2	169632	-96	0	0	0	-3	0	-6	1	0	0	-1	1	0	2	0	0	0	0	0	0	0	0	0	-15	15	0	0	0	0	└																						χ_{49}
χ_{50}	o2	175770	90	0	90	-6	0	0	7	0	-2	-2	0	1	0	-1	0	0	0	0	0	0	1	1	1	0	0	-1	-1	0	0	*	+																					χ_{50}
χ_{51}	o2	207360	0	0	0	0	0	0	-8	-1	0	0	0	-1	0	0	0	0	0	0	0	0	-c19	*2	*4	0	0	0	0	1	1	*	+																					χ_{51}
χ_{52}	o2	207360	0	0	0	0	0	0	-8	-1	0	0	0	-1	0	0	0	0	0	0	0	0	*4	-c19	*2	0	0	0	0	1	1	*	+																					χ_{52}
χ_{53}	o2	207360	0	0	0	0	0	0	-8	-1	0	0	0	-1	0	0	0	0	0	0	0	0	*2	*4	-c19	0	0	0	0	1	1	*	+																					χ_{53}
χ_{54}	o2	253440	0	0	0	0	0	0	12	-2	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	b31	**	┌	+2																					χ_{54}
χ_{55}	o2	253440	0	0	0	0	0	0	12	-2	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	**	b31	└																						χ_{55}

Co_3

Sporadic Conway group Co_3

Order = 495,766,656,000 = $2^{10}.3^7.5^3.7.11.23$ Mult = 1 Out = 1

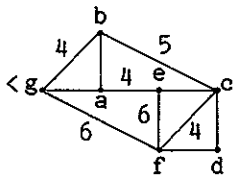
Constructions

2-graph G : the automorphism group of a 2-graph on 276 vertices, that is, an assignment of parities to the triples of vertices, so that the sum of the parities of the four triples in any quadruple is even. The 276 vertices may be taken as the 23 points and 253 heptads of S(4,7,23) (see M_{23} and M_{24}). The parity of a triple of points and/or heptads is found by counting 1 for each point not in a heptad, or pair of heptads intersecting in 1 point.

Leech G : the stabilizer of a type 3 vector V in the Leech lattice. Taking V = $(5,1^{23})$, O = $(0,0^{23})$, the 552 lattice points P with type(VP) = type(OP) = 2 fall naturally into 276 pairs $\{P_L,P_R\}$ with $P_L+P_R-V = O$, namely $\{ (4,4,0^{22}), (1,-3,1^{22}) \}$ (23) and $\{ (2,2^7 0^{16}), (3,-1^7 1^{16}) \}$ (253), forming the 2-graph described above when the parity of a triple $\{A_L,A_R\}$ $\{B_L,B_R\}$ $\{C_L,C_R\}$ is that of type(A_LB_L) + type(B_LC_L) + type(C_LA_L).

Considering the Leech lattice modulo 2, the 2-spaces containing V become points in the quotient 23-dimensional representation over $\mathbb{F}_2$. They are described below by the types of their non-zero vectors. In all cases except those marked * these vectors lift to real Leech lattice vectors of the appropriate types adding to O.

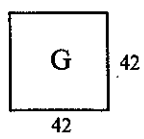
Presentation



$G \cong \langle g \mid a=(cf)^2, b=(ef)^3, d=(bg)^2=(gae)^3, 1=(eab)^3=(bce)^5=(adfg)^3=(cef)^7 \rangle$

Maximal subgroups

Order	Index	Structure	Character	Specifications: Abstract	Leech (mod 2)	2-graph
1796256000	276	McL:2	1a+275a		3-22-point	point
44352000	11178	HS			3-23-point	
13063680	37950	$U_4(3):(2^2)_{133}$			3-23*-point	point pair
10200960	48600	M_{23}			3-24-point	
3849120	128800	$3^5:(2xM_{11})$		$N(3^5) = N(3A_{55}B_{66})$	3-33-point	11 odd triples
2903040	170775	$2^{\cdot}S_6(2)$		N(2A)		
756000	655776	$U_3(5):S_3$			3-33*-point	
699840	708400	$3_+^{1+4}:4S_6$		N(3A)		
322560	1536975	$2^{4\cdot}A_8$		$N(2A^4)$	3-34-point	
241920	2049300	$L_3(4):D_{12}$			3-44-point	even triple
190080	2608200	2 x M_{12}		N(2B)	3-34*-point	
27648	17931375	$2^2.[2^7.3^2].S_3$		$N(2A^2)$		
9072	54648000	S_3 x $L_2(8):3$		N(3C)		
1440	344282400	A_4 x S_5		$N(2B^2)$, N(2A,3C,5B)		



Co_3

;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		4	29	190	349	29	4	23	1	1		4	1																																			
95766656000			03040	080	920	160	536	040	536	500	300	320	296	216	108	72	42	192	192	32	162	81	60	20	22	22	144	48	36	14	30	15	18	20	20	21	22	22	23	23	24	24	30		
p power			A	A	A	A	A	A	A	A	A	AA	AA	BA	BB	CB	A	B	B	B	A	A	AA	BB	A	A	AA	AB	CA	AA	AA	BB	AB	AA	AA	AC	BB	AB	A	A	BA	BB	AAA		
p' part			A	A	A	A	A	A	A	A	A	AA	AA	BA	BB	CB	A	A	A	A	A	A	AA	BB	A	A	AA	AB	BA	AA	AA	BB	AA	AA	AA	AC	AB	BB	A	A	AA	AB	AAA		
ind		1A	2A	2B	3A	3B	3C	4A	4B	5A	5B	6A	6B	6C	6D	6E	7A	8A	8B	8C	9A	9B	10A	10B	11A	B**	12A	12B	12C	14A	15A	15B	18A	20A	B**	21A	22A	B**	23A	B**	24A	24B	30A		
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	+	23	7	-1	-4	5	-1	-5	3	-2	3	4	-2	1	-1	-1	2	1	-3	1	-1	2	2	-1	1	1	-2	0	1	0	1	0	1	0	0	-1	-1	-1	0	0	-2	0	-1	x_2
x_3	+	253	13	-11	10	10	1	9	1	3	3	10	4	-2	-2	1	1	-1	3	-1	1	1	3	-1	0	0	0	-2	0	-1	0	0	1	-1	-1	1	0	0	0	0	2	0	0	x_3
x_4	+	253	29	-11	10	10	1	-11	5	3	3	2	2	2	-2	1	1	-3	-3	1	1	1	-1	-1	0	0	-2	2	-2	1	0	0	-1	-1	-1	1	0	0	0	0	0	0	2	x_4
x_5	+	275	35	11	5	14	-1	15	7	0	5	5	-1	2	2	-1	2	1	5	1	-1	2	0	1	0	0	3	1	0	0	0	-1	-1	0	0	-1	0	0	-1	-1	1	-1	0	x_5
x_6	o	896	0	16	32	-4	-7	0	0	-4	1	0	0	0	-2	1	0	0	0	0	2	-1	0	1	b11	**	0	0	0	0	2	1	0	0	0	0	b11	**	-1	-1	0	0	0	x_6
x_7	o	896	0	16	32	-4	-7	0	0	-4	1	0	0	0	-2	1	0	0	0	0	2	-1	0	1	**	b11	0	0	0	0	2	1	0	0	0	0	**	b11	-1	-1	0	0	0	x_7
x_8	+	1771	-21	11	-11	16	7	-5	-5	-4	1	21	-3	0	2	-1	0	-1	-1	-1	-2	-2	4	1	0	0	1	1	-2	0	-1	1	0	0	0	0	0	0	0	0	-1	-1	1	x_8
x_9	+	2024	104	0	-1	26	8	-24	8	-1	4	-1	5	2	0	0	1	4	-4	0	-1	-1	-1	0	0	0	-3	-1	0	-1	-1	1	-1	1	1	1	0	0	0	0	1	-1	-1	x_9
x_{10}	o	3520	-64	0	-44	10	-8	0	0	-5	0	-4	8	2	0	0	-1	0	0	0	1	1	1	0	0	0	0	0	0	-1	1	0	-1	15	-15	-1	0	0	1	1	0	0	1	x_{10}
x_{11}	o	3520	-64	0	-44	10	-8	0	0	-5	0	-4	8	2	0	0	-1	0	0	0	1	1	1	0	0	0	0	0	0	-1	1	0	-1	-15	15	-1	0	0	1	1	0	0	1	x_{11}
x_{12}	+	4025	105	1	-25	29	-7	-35	5	0	5	15	-3	-3	1	1	0	-1	-5	-1	2	2	0	1	-1	-1	1	-1	1	0	0	-1	0	0	0	0	1	1	0	0	-1	1	0	x_{12}
x_{13}	+	5544	168	0	-45	36	0	40	8	-6	4	3	-3	0	0	0	0	-4	4	0	0	0	-2	0	0	0	1	-1	-2	0	0	1	0	0	0	0	0	0	1	1	-1	1	-2	x_{13}
x_{14}	+	7084	-84	44	10	19	-14	-4	-4	9	-1	18	6	3	-1	2	0	0	0	0	4	-2	1	-1	0	0	2	2	-1	0	0	-1	0	1	1	0	0	0	0	0	0	0	-2	x_{14}
x_{15}	+	8855	231	55	-1	35	-7	19	11	5	0	-9	-3	3	1	1	0	5	1	1	2	-4	1	0	0	0	1	-1	1	0	-1	0	0	-1	-1	0	0	0	0	0	-1	1	1	x_{15}
x_{16}	o	9625	105	-55	40	-5	7	5	-3	0	0	0	6	3	-1	-1	0	3	-1	-1	-2	1	0	0	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	b23	**	0	2	0	x_{16}
x_{17}	o	9625	105	-55	40	-5	7	5	-3	0	0	0	6	3	-1	-1	0	3	-1	-1	-2	1	0	0	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	**	b23	0	2	0	x_{17}
x_{18}	o	20608	0	-16	-128	-20	7	0	0	8	3	0	0	0	2	-1	0	0	0	0	-2	-2	0	-1	b11	**	0	0	0	0	2	0	0	0	0	0	-b11	**	0	0	0	0	0	x_{18}
x_{19}	o	20608	0	-16	-128	-20	7	0	0	8	3	0	0	0	2	-1	0	0	0	0	-2	-2	0	-1	**	b11	0	0	0	0	2	0	0	0	0	0	**	-b11	0	0	0	0	0	x_{19}
x_{20}	+	23000	280	120	50	5	8	40	8	0	0	10	10	1	3	0	-2	0	0	0	-1	2	0	0	-1	-1	-2	2	1	0	0	0	1	0	0	1	-1	-1	0	0	0	0	0	x_{20}
x_{21}	+	26082	-126	-54	81	0	0	-6	10	7	-3	9	9	0	0	0	0	2	2	-2	0	0	-1	1	1	1	-3	1	0	0	1	0	0	-1	-1	0	1	1	0	0	-1	-1	-1	x_{21}
x_{22}	+	31625	265	-55	35	35	-1	-55	9	0	0	-5	-5	-5	-1	-1	-1	1	1	1	-1	-1	0	0	0	0	-1	3	-1	-1	0	0	1	0	0	-1	0	0	0	0	1	1	0	x_{22}
x_{23}	+	31625	-55	-55	35	35	-1	25	-7	0	0	35	-1	-1	-1	-1	-1	1	1	1	-1	-1	0	0	0	0	-5	-1	1	1	0	0	-1	0	0	-1	0	0	0	0	1	1	0	x_{23}
x_{24}	+	31625	505	-55	35	35	-1	-35	5	0	0	-5	1	7	-1	-1	-1	-5	-1	-1	-1	-1	0	0	0	0	1	-1	1	1	0	0	1	0	0	-1	0	0	0	0	1	-1	0	x_{24}
x_{25}	+	31878	294	-66	45	45	0	46	-2	3	3	-3	-3	-3	-3	0	0	2	2	-2	0	0	-1	-1	0	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	-1	-1	2	x_{25}
x_{26}	+	40250	-70	10	-115	-25	14	10	10	0	0	5	-7	-1	1	-2	0	-2	-2	-2	5	-1	0	0	1	1	1	1	1	0	0	0	-1	0	0	0	-1	-1	0	0	1	1	0	x_{26}
x_{27}	+	57960	168	120	126	45	0	-40	-8	10	0	6	6	-3	3	0	0	0	0	0	0	0	-2	0	1	1	2	-2	-1	0	1	0	0	0	0	0	-1	-1	0	0	0	0	1	x_{27}
x_{28}	+	63250	210	-110	-65	-20	22	-30	2	0	0	15	3	0	-2	-2	-2	2	2	2	4	1	0	0	0	0	3	-1	0	0	0	0	0	0	0	1	0	0	0	0	-1	-1	0	x_{28}
x_{29}	+	73600	0	144	160	16	13	0	0	0	-5	0	0	0	0	-3	2	0	0	0	4	1	0	-1	-1	-1	0	0	0	0	0	1	0	0	0	-1	1	1	0	0	0	0	0	x_{29}
x_{30}	+	80960	-448	0	176	50	8	0	0	10	0	-16	-16	2	0	0	-2	0	0	0	-1	2	2	0	0	0	0	0	0	0	1	0	-1	0	0	1	0	0	0	0	0	0	-1	x_{30}
x_{31}	+	91125	405	45	0	0	27	45	-3	0	0	0	0	0	0	3	-1	-3	-3	1	0	0	0	0	1	1	0	0	0	-1	0	0	0	0	0	-1	1	1	-1	-1	0	0	0	x_{31}
x_{32}	+	93312	0	-144	0	0	27	0	0	12	-3	0	0	0	0	3	2	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	x_{32}
x_{33}	+	129536	-512	0	-64	44	8	0	0	-14	-4	16	-8	4	0	0	1	0	0	0	-1	-1	-2	0	0	0	0	0	0	-1	1	-1	1	0	0	1	0	0	0	0	0	0	1	x_{33}
x_{34}	+	129536	512	0	-64	44	8	0	0	-14	-4	-16	8	-4	0	0	1	0	0	0	-1	-1	2	0	0	0	0	0	0	1	1	-1	-1	0	0	1	0	0	0	0	0	0	-1	x_{34}
x_{35}	+	177100	140	44	-20	-29	-14	-20	12	0	-5	20	-4	-1	-1	2	0	0	0	0	-5	1	0	-1	0	0	4	0	1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	x_{35}
x_{36}	+	184437	405	-99	0	0	-27	45	-3	12	-3	0	0	0	0	-3	1	-3	-3	1	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	x_{36}
x_{37}	+	221375	735	55	-160	-25	-7	-25	-9	0	0	0	-12	3	1	1	0	3	3	-1	2	2	0	0	0	0	-4	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{37}
x_{38}	+	226688	0	-176	320	-40	-7	0	0	-12	3	0	0	0	4	1	0	0	0	0	2	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{38}
x_{39}	+	246400	0	176	160	-56	7	0	0	0	5	0	0	0	-4	-1	0	0	0	0	-2	-2	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	0	0	0	x_{39}
x_{40}	+	249480	-504	0	-81	0	0	-24	8	5	0	-9	9	0	0	0	0	-4	4	0	0	0	1	0	0	0	-3	-1	0	0	-1	0	0	1	1	0	0	0	-1	-1	-1	1	1	x_{40}
x_{41}	+	253000	-440	0	-125	10	-8	40	8	0	0	-5	1	-2	0	0	-1	4	-4	0	1	1	0	0	0	0	1	-1	-2	1	0	0	1	0	0	-1	0	0	0	0	1	-1	0	x_{41}
x_{42}	+	255024	-336	0	-126	36	0	-16	-16	-1	4	-6	6	0	0	0	0	0	0	0	0	0	-1	0	0	0	2	2	2	0	-1	1	0	-1	-1	0	0	0	0	0	0	0	-1	x_{42}

$O_8^+(3)$

;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		49	261	261	261	331	680	188	188	188	188	188	188	472	472	472	472	52	13	13	2	2	2	1	1	1	1	46	46	46	10	7	7	7	7	7	7	5	5	5	5	5	5															
		52179814400	27360	27360	27360	776	24448	95680	95680	95680	95680	95680	95680	392	392	392	392	488	122	824	880	880	880	536	800	800	800	656	656	656	368	776	776	776	776	776	776	184	184	184	184	184	184	648	648	648	648	648	648	648	648	648	648	648	162	28	32	32	
	p power		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	D	A	B	C	D	A	A	A	AA	AB	AC	AD	BA	CB	DC	EA	FB	GC	BD	CD	DD	ED	FD	GD	LA	LB	LC	HD	ID	JD	KD	LD	LD	LD	LD	MD	A	A	E	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	AC	AD	BA	CB	DC	EA	FB	GC	BD	CD	DD	ED	FD	GD	LA	LB	LC	HD	ID	JD	KD	LD	LD	LD	LD	MD	A	A	A	
	ind	1A	2A	2B	2C	2D	3A	3B	3C	3D	3E	3F	3G	3H	3I	3J	3K	3L	3M	4A	4B	4C	4D	4E	5A	5B	5C	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	6N	6O	6P	6Q	6R	6S	6T	6U	6V	6W	6X	6Y	6Z	6A1	6B1	7A	8A	8B	
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_1
χ_2	+	260	36	20	20	4	17	44	-10	-10	17	-10	-10	17	17	-10	-10	8	-1	4	4	0	0	4	5	0	0	9	-7	-7	1	0	2	2	9	2	2	4	-2	-2	1	-2	-2	0	2	2	1	1	-2	-2	4	4	-2	-2	1	1	0	0	χ_2
χ_3	+	260	20	36	20	4	17	-10	44	-10	-10	17	-10	17	-10	17	-10	8	-1	4	0	4	0	4	0	5	0	-7	9	-7	1	2	0	2	2	9	2	-2	4	-2	-2	1	-2	2	0	2	1	-2	1	-2	4	-2	4	-2	1	1	0	0	χ_3
	+	260	20	20	36	4	17	-10	-10	44	-10	-10	17	17	-10	-10	17	8	-1	4	0	0	4	4	0	0	5	-7	-7	9	1	2	2	0	2	2	9	-2	-2	4	-2	-2	1	2	2	0	1	-2	-2	1	4	-2	-2	4	1	1	0	0	χ_4
χ_5	+	260	36	20	20	4	17	17	-10	-10	44	-10	-10	-10	-10	17	17	8	-1	4	4	0	0	4	5	0	0	9	-7	-7	1	9	2	2	0	2	2	1	-2	-2	4	-2	-2	0	2	2	-2	-2	1	1	-2	-2	4	4	1	1	0	0	χ_5
χ_6	+	260	20	36	20	4	17	-10	17	-10	-10	44	-10	-10	17	-10	17	8	-1	4	0	4	0	4	0	5	0	-7	9	-7	1	2	9	2	2	0	2	-2	1	-2	-2	4	-2	2	0	2	-2	1	-2	1	-2	4	-2	4	1	1	0	0	χ_6
χ_7	+	260	20	20	36	4	17	-10	-10	17	-10	-10	44	-10	17	17	-10	8	-1	4	0	0	4	4	0	0	5	-7	-7	9	1	2	2	9	2	2	0	-2	-2	1	-2	-2	4	2	2	0	-2	1	1	-2	-2	4	4	-2	1	1	0	0	χ_7
χ_8	+	300	20	20	20	12	57	30	30	30	30	30	30	3	3	3	3	-6	12	8	0	0	0	0	0	0	0	-7	-7	-7	9	2	2	2	2	2	2	6	6	6	6	6	6	2	2	2	3	3	3	3	0	0	0	0	0	-1	2	-2	χ_8
χ_9	+	819	91	-21	-21	19	90	90	9	9	90	9	9	9	9	9	9	9	9	7	11	-1	-1	-1	9	-1	-1	10	6	6	10	10	-3	-3	10	-3	-3	10	1	1	10	1	1	1	-3	-3	1	1	1	1	1	1	1	1	1	0	1	1	χ_9
χ_{10}	+	819	-21	91	-21	19	90	9	90	9	9	90	9	9	9	9	9	9	9	7	-1	11	-1	-1	-1	9	-1	6	10	6	10	-3	10	-3	-3	10	-3	1	10	1	1	10	1	-3	1	-3	1	1	1	1	1	1	1	1	1	0	1	1	χ_{10}
χ_{11}	+	819	-21	-21	91	19	90	9	9	90	9	9	90	9	9	9	9	9	9	7	-1	-1	11	-1	-1	-1	9	6	6	10	10	-3	-3	10	-3	-3	10	1	1	10	1	1	10	-3	-3	1	1	1	1	1	1	1	1	1	1	0	1	1	χ_{11}
χ_{12}	+	2275	35	35	35	-29	88	-20	-20	-20	115	115	115	7	34	34	34	7	7	3	-5	-5	-5	3	0	0	0	8	8	8	-8	8	8	8	-1	-1	-1	4	4	4	-5	-5	-5	-1	-1	-1	7	-2	-2	-2	1	1	1	1	1	0	-1	-1	χ_{12}
χ_{13}	+	2275	35	35	35	-29	88	-20	115	115	115	-20	-20	34	7	34	34	7	7	3	-5	-5	-5	3	0	0	0	8	8	8	-8	8	-1	-1	-1	8	8	4	-5	-5	-5	4	4	-1	-1	-1	-2	7	-2	-2	1	1	1	1	1	0	-1	-1	χ_{13}
χ_{14}	+	2275	35	35	35	-29	88	115	-20	115	-20	115	-20	34	34	7	34	7	7	3	-5	-5	-5	3	0	0	0	8	8	8	-8	-1	8	-1	8	-1	8	-5	4	-5	4	-5	4	-1	-1	-1	-2	-2	7	-2	1	1	1	1	1	0	-1	-1	χ_{14}
χ_{15}	+	2275	35	35	35	-29	88	115	115	-20	-20	-20	115	34	34	34	7	7	7	3	-5	-5	-5	3	0	0	0	8	8	8	-8	-1	-1	8	8	8	-1	-5	-5	4	4	4	-5	-1	-1	-1	-2	-2	-2	7	1	1	1	1	1	0	-1	-1	χ_{15}
χ_{16}	+	2808	120	120	120	-8	-108	54	54	54	54	54	54	-27	-27	-27	-27	27	0	8	0	0	0	8	8	8	8	12	12	12	4	-6	-6	-6	-6	-6	-6	-2	-2	-2	-2	-2	-2	3	3	3	1	1	1	1	-5	-5	-5	-5	4	1	0	0	χ_{16}
χ_{17}	+	5460	196	-140	-140	20	-129	222	60	60	-21	60	60	6	6	6	6	15	-3	12	4	0	0	-4	5	0	0	7	-5	-5	-1	-2	4	4	7	4	4	14	-4	-4	-13	-4	-4	7	-5	-5	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{17}
χ_{18}	+	5460	-140	196	-140	20	-129	60	222	60	60	-21	60	6	6	6	6	15	-3	12	0	4	0	-4	0	5	0	-5	7	-5	-1	4	-2	4	4	7	4	-4	14	-4	-4	-13	-4	-5	7	-5	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{18}
χ_{19}	+	5460	-140	-140	196	20	-129	60	60	222	60	60	-21	6	6	6	6	15	-3	12	0	0	4	-4	0	0	5	-5	-5	7	-1	4	4	-2	4	4	7	-4	-4	14	-4	-4	-13	-5	-5	7	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{19}
χ_{20}	+	5460	196	-140	-140	20	-129	-21	60	60	222	60	60	6	6	6	6	15	-3	12	4	0	0	-4	5	0	0	7	-5	-5	-1	7	4	4	-2	4	4	-13	-4	-4	14	-4	-4	7	-5	-5	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{20}
χ_{21}	+	5460	-140	196	-140	20	-129	60	-21	60	60	222	60	6	6	6	6	15	-3	12	0	4	0	-4	0	5	0	-5	7	-5	-1	4	7	4	4	-2	4	-4	-13	-4	-4	14	-4	-5	7	-5	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{21}
χ_{22}	+	5460	-140	-140	196	20	-129	60	60	-21	60	60	222	6	6	6	6	15	-3	12	0	0	4	-4	0	0	5	-5	-5	7	-1	4	4	7	4	4	-2	-4	-4	-13	-4	-4	14	-5	-5	7	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{22}
χ_{23}	+	9450	210	210	210	74	-27	135	135	135	135	135	135	54	54	54	54	27	0	-18	10	10	10	6	0	0	0	21	21	21	5	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	3	3	3	2	2	2	2	5	5	5	5	-4	0	0	0	χ_{23}
χ_{24}	+	17550	-90	-90	-90	46	783	135	135	135	135	135	135	-27	-27	-27	-27	0	27	26	-10	-10	-10	2	0	0	0	-9	-9	-9	31	-9	-9	-9	-9	-9	-9	7	7	7	7	7	7	0	0	0	1	1	1	1	4	4	4	4	-5	1	0	0	χ_{24}
χ_{25}	+	18200	280	280	280	24	461	110	110	110	110	110	110	-79	-79	-79	-79	-7	11	8	0	0	0	8	0	0	0	37	37	37	-3	10	10	10	10	10	10	6	6	6	6	6	6	1	1	1	-3	-3	-3	-3	-3	-3	-3	-3	-3	0	0	0	χ_{25}
χ_{26}	+	18200	280	280	280	24	-25	-160	-160	-160	110	110	110	137	-52	-52	-52	29	2	8	0	0	0	8	0	0	0	-17	-17	-17	-9	-8	-8	-8	10	10	10	0	0	0	6	6	6	1	1	1	-3	0	0	0	9	-3	-3	-3	0	0	0	0	χ_{26}
χ_{27}	+	18200	280	280	280	24	-25	-160	110	110	110	-160	-160	-52	137	-52	-52	29	2	8	0	0	0	8	0	0	0	-17	-17	-17	-9	-8	10	10	10	-8	-8	0	6	6	6	0	0	1	1	1	0	-3	0	0	-3	9	-3	-3	0	0	0	0	χ_{27}
χ_{28}	+	18200	280	280	280	24	-25	110	-160	110	-160	110	-160	-52	-52	137	-52	29	2	8	0	0	0	8	0	0	0	-17	-17	-17	-9	10	-8	10	-8	10	-8	6	0	6	0	6	0	1	1	1	0	0	-3	0	-3	-3	9	-3	0	0	0	0	χ_{28}
χ_{29}	+	18200	280	280	280	24	-25	110	110	-160	-160	-160	110	-52	-52	-52	137	29	2	8	0	0	0	8	0	0	0	-17	-17	-17	-9	10	10	-8	-8	-8	10	6	6	0	0	0	6	1	1	1	0	0	0	-3	-3	-3	-3	9	0	0	0	0	χ_{29}
χ_{30}	+	23400	440	120	120	40	72	720	-90	-90	-90	-90	-90	72	72	-9	-9	-9	-9	16	0	0	0	0	0	0	0	8	12	12	-8	8	-6	-6	26	-6	-6	16	-2	-2	-2	-2	-2	-1	3	3	4	4	-5	-5	1	1	1	1	1	-1	0	0	χ_{30}
χ_{31}	+	23400	120	440	120	40	72	-90	720	-90	-90	-90	-90	72	-9	72	-9	-9	-9	16	0	0	0	0	0	0	0	12	8	12	-8	-6	8	-6	-6	26	-6	-2	16	-2	-2	-2	-2	3	-1	3	4	-5	4	-5	1	1	1	1	1	-1	0	0	χ_{31}
χ_{32}	+	23400	120	120	440	40	72	-90	-90	720	-90	-90	-90	72	-9	-9	72	-9	-9	16	0	0	0	0	0	0	0	12	12	8	-8	-6	-6	8	-6	-6	26	-2	-2	16	-2	-2	-2	3	3	-1	4	-5	-5	4	1	1	1	1	1	-1	0	0	χ_{32}
χ_{33}	+	23400	440	120	120	40	72	-90	-90	-90	720	-90	-90	-9	-9	72	72	-9	-9	16	0	0	0	0	0	0	0	8	12	12	-8	26	-6	-6	8	-6	-6	-2	-2	-2	16	-2	-2	-1	3	3	-5	-5	4	4	1	1	1	1	1	-1	0	0	χ_{33}
χ_{34}	+	23400	120	440	120	40	72	-90	-90	-90	-90	720	-90	-9	72	-9	72	-9	-9	16	0	0	0	0	0	0	0	12	8	12	-8	-6	26	-6	-6	8	-6	-2	-2	-2	-2	16	-2	3	-1	3	-5	4	-5	4	1	1	1	1	1	-1	0	0	χ_{34}
χ_{35}	+	23400	120	120	440	40	72	-90	-90	-90	-90	-90	720	-9	72	72	-9	-9	-9	16	0	0	0	0	0	0	0	12	12	8	-8	-6	-6	26	-6	-6	8	-2	-2	-2	-2	-2	16	3	3	-1	-5	4	4	-5	1	1	1	1	1	-1	0	0	χ_{35}
χ_{36}	+	24192	0	0	0	128	864	216	216	216	216	216	216	54	54	54	54	0	27	0	0	0	0	0	-8	-8	-8	0	0	0	32	0	0	0	0	0	0	8	8	8	8	8	8	0	0	0	2	2	2	2	-4	-4	-4	-4	5	0	0	0	χ_{36}
χ_{37}	+	27300	140	140	140	-124	813	300	300	300	300	300	300	30	30	30	30	21	39	0	0	0	0	-8	0	0	0	5	5	5	-19	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	5	5	5	2	2	2	2	-1	-1	-1	-1	-1	0	-2	2	χ_{37}
χ_{38}	+	29120	448	0	0	-64	-40	716	-40	-40	176	-40	-40	68	68	14	14	14	-13	0	0	0	0	0	10	0	0	16	0	0	8	-20	0	0	16	0	0	-4	8	8	-16	8	8	-2	0	0	-4	-4	2	2	2	2	-4	-4	-1	0	0	0	χ_{38}
χ_{39}	+	29120	0	448	0	-64	-40	-40	716	-40	-40	176	-40	68	14	68	14	14	-13	0	0	0	0	0	0	10	0	0	16	0	8	0	-20	0	0	16	0	8	-4	8	8	-16	8	0	-2	0	-4	2	-4	2	2	-4	2	-4	-1	0	0	0	χ_{39}
χ_{40}	+	29120	0	0	448	-64	-40	-40	-40	716	-40	-40	176	68	14	14	68	14	-13	0	0	0	0	0	0	0	10	0	0	16	8	0	0	-20	0	0	16	8	8	-4	8	8	-16	0	0	-2	-4	2	2	-4	2	-4	-4	2	-1	0	0	0	χ_{40}
χ_{41}	+	29120	448	0	0	-64	-40	176	-40	-40	716	-40	-40	14	14	68	68	14	-13	0	0	0	0	0	10	0	0	16	0	0	8	16	0	0	-20	0	0	-16	8	8	-4	8	8	-2	0	0	2	2	-4	-4	-4	-4	2	2	-1	0	0	0	χ_{41}
χ_{42}	+	29120	0	448	0	-64	-40	-40	176	-40	-40	716	-40	14	68	14	68	14	-13	0	0	0	0	0	0	10	0	0	16	0	8	0	16	0	0	-20	0	8	-16	8	8	-4	8	0	-2	0	2	-4	2	-4	-4	2	-4	2	-1	0	0	0	χ_{42}
χ_{43}	+	29120	0	0	448	-64	-40	-40	-40	176	-40	-40	716	14	68	68	14	14	-13	0	0	0	0	0	0	0	10	0	0	16	8	0	0	16	0	0	-20	8	8	-16	8	8	-4	0	0	-2	2	-4	-4	2	-4	2	2	-4	-1	0	0	0	χ_{43}
χ_{44}	+	40950	350	-210	-210	-42	855	855	45	45	45	45	45	45	45	-36	-36	-9	18	10	-10	10	10	2	0	0	0	-1	-21	-21	-9	-1	-3	-3	17	-3	-3	-9	-3	-3	21	-3	-3	-1	-3	-3	-3	-3	0	0	-3	-3	3	3	0	0	0	0	χ_{44}
χ_{45}	+	40950	-210	350	-210	-42	855	45	855	45	45	45	45	45	-36	45	-36	-9	18	10	10	-10	10	2	0	0	0	-21	-1	-21	-9	-3	-1	-3	-3	17	-3	-3	-9	-3	-3	21	-3	-3	-1	-3	-3	0	-3	0	-3	3	-3	3	0	0	0	0	χ_{45}
χ_{46}	+	40950	-210	-210	350	-42	855	45	45	855	45	45	45	45	-36	-36	45	-9	18	10	10	10	-10	2	0	0	0	-21	-21	-1	-9	-3	-3	-1	-3	-3	17	-3	-3	-9	-3	-3	21	-3	-3	-1	-3	0	0	-3	-3	3	3	-3	0	0	0	0	χ_{46}
χ_{47}	+	40950	350	-210	-210	-42	855	45	45	45	855	45	45	-36	-36	45	45	-9	18	10	-10	10	10	2	0	0	0	-1	-21	-21	-9	17	-3	-3	-1	-3	-3	21	-3	-3	-9	-3	-3	-1	-3	-3	0	0	-3	-3	3	3	-3	-3	0	0	0	0	χ_{47}
χ_{48}	+	40950	-210	350	-210	-42	855	45	45	45	45	855	45	-36	45	-36	45	-9	18	10	10	-10	10	2	0	0	0	-21	-1	-21	-9	-3	17	-3	-3	-1	-3	-3	21	-3	-3	-9	-3	-3	-1	-3	0	-3	0	-3	3	-3	3	-3	0	0	0	0	χ_{48}
χ_{49}	+	40950	-210	-210	350	-42	855	45	45	45	45	45	855	-36	45	45	-36	-9	18	10	10	10	-10	2	0	0	0	-21	-21	-1	-9	-3	-3	17	-3	-3	-1	-3	-3	21	-3	-3	-9	-3	-3	-1	0	-3	-3	0	3	-3	-3	3	0	0	0	0	χ_{49}
χ_{50}	+	46592	896	0	0	0	-64	584	-64	-64	584	-64	-64	-64	-64	-64	-64	44	-10	0	16	0	0	0	17	-8	-8	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{50}
χ_{51}	+	46592	0	896	0	0	-64	-64	584	-64	-64	584	-64	-64	-64	-64	-64	44	-10	0	0	16	0	0	-8	17	-8	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{51}
χ_{52}	+	46592	0	0	896	0	-64	-64	-64	584	-64	-64	584	-64	-64	-64	-64	44	-10	0	0	0	16	0	-8	-8	17	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	χ_{52}

$O_8^+(3)$

		1A	2A	2B	2C	2D	3A	3B	3C	3D	3E	3F	3G	3H	3I	3J	3K	3L	3M
χ_{53}	+	49140	84	420	420	-12	297	216	-270	-270	-27	-270	-270	135	135	-108	-108	27	0
χ_{54}	+	49140	420	84	420	-12	297	-270	216	-270	-270	-27	-270	135	-108	135	-108	27	0
χ_{55}	+	49140	420	420	84	-12	297	-270	-270	216	-270	-270	-27	135	-108	-108	135	27	0
χ_{56}	+	49140	84	420	420	-12	297	-27	-270	-270	216	-270	-270	-108	-108	135	135	27	0
χ_{57}	+	49140	420	84	420	-12	297	-270	-27	-270	-270	216	-270	-108	135	-108	135	27	0
χ_{58}	+	49140	420	420	84	-12	297	-270	-270	-27	-270	-270	216	-108	135	135	-108	27	0
χ_{59}	+	54600	280	280	280	8	-561	-210	-210	-210	600	600	600	60	-21	-21	-21	-39	-3
χ_{60}	+	54600	280	280	280	8	-561	-210	600	600	600	-210	-210	-21	60	-21	-21	-39	-3
χ_{61}	+	54600	280	280	280	8	-561	600	-210	600	-210	600	-210	-21	-21	60	-21	-39	-3
χ_{62}	+	54600	280	280	280	8	-561	600	600	-210	-210	-210	600	-21	-21	-21	60	-39	-3
χ_{63}	+	54600	280	280	280	8	897	600	-210	-210	600	-210	-210	-21	-21	-21	-21	-39	-3
χ_{64}	+	54600	280	280	280	8	897	-210	600	-210	-210	600	-210	-21	-21	-21	-21	-39	-3
χ_{65}	+	54600	280	280	280	8	897	-210	-210	600	-210	-210	600	-21	-21	-21	-21	-39	-3
χ_{66}	+	66339	819	-189	-189	99	729	729	0	0	729	0	0	0	0	0	0	0	0
χ_{67}	+	66339	-189	819	-189	99	729	0	729	0	0	729	0	0	0	0	0	0	0
χ_{68}	+	66339	-189	-189	819	99	729	0	0	729	0	0	729	0	0	0	0	0	0
χ_{69}	+	163800	1400	-840	-840	88	1233	-630	180	180	-630	180	180	18	18	18	18	45	-9
χ_{70}	+	163800	-840	1400	-840	88	1233	180	-630	180	180	-630	180	18	18	18	18	45	-9
χ_{71}	+	163800	-840	-840	1400	88	1233	180	180	-630	180	180	-630	18	18	18	18	45	-9
χ_{72}	+	184275	315	315	315	51	567	-405	-405	-405	810	810	810	-162	81	81	81	0	0
χ_{73}	+	184275	315	315	315	51	567	-405	810	810	810	-405	-405	81	-162	81	81	0	0
χ_{74}	+	184275	315	315	315	51	567	810	-405	810	-405	810	-405	81	81	-162	81	0	0
χ_{75}	+	184275	315	315	315	51	567	810	810	-405	-405	-405	810	81	81	81	-162	0	0
χ_{76}	+	189540	1044	-540	-540	36	-729	1458	0	0	-729	0	0	0	0	0	0	0	0
χ_{77}	+	189540	-540	1044	-540	36	-729	0	1458	0	0	-729	0	0	0	0	0	0	0
χ_{78}	+	189540	-540	-540	1044	36	-729	0	0	1458	0	0	-729	0	0	0	0	0	0
χ_{79}	+	189540	1044	-540	-540	36	-729	-729	0	0	1458	0	0	0	0	0	0	0	0
χ_{80}	+	189540	-540	1044	-540	36	-729	0	-729	0	0	1458	0	0	0	0	0	0	0
χ_{81}	+	189540	-540	-540	1044	36	-729	0	0	-729	0	0	1458	0	0	0	0	0	0
χ_{82}	+	218700	-540	-540	-540	108	2187	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	+	232960	896	0	0	0	-320	1192	-320	-320	-536	-320	-320	-104	-104	112	112	4	4
χ_{84}	+	232960	0	896	0	0	-320	-320	1192	-320	-320	-536	-320	-104	112	-104	112	4	4
χ_{85}	+	232960	0	0	896	0	-320	-320	-320	1192	-320	-320	-536	-104	112	112	-104	4	4
χ_{86}	+	232960	896	0	0	0	-320	-536	-320	-320	1192	-320	-320	112	112	-104	-104	4	4
χ_{87}	+	232960	0	896	0	0	-320	-320	-536	-320	-320	1192	-320	112	-104	112	-104	4	4
χ_{88}	+	232960	0	0	896	0	-320	-320	-320	-536	-320	-320	1192	112	-104	-104	112	4	4
χ_{89}	+	245700	-420	-420	-420	-28	2943	270	270	270	270	270	270	27	27	27	27	27	27
χ_{90}	+	262080	-448	0	0	-64	-360	1584	-360	-360	-36	-360	-360	126	126	-36	-36	18	-9
χ_{91}	+	262080	0	-448	0	-64	-360	-360	1584	-360	-360	-36	-360	126	-36	126	-36	18	-9
χ_{92}	+	262080	0	0	-448	-64	-360	-360	-360	1584	-360	-360	-36	126	-36	-36	126	18	-9
χ_{93}	+	262080	-448	0	0	-64	-360	-36	-360	-360	1584	-360	-360	-36	-36	126	126	18	-9
χ_{94}	+	262080	0	-448	0	-64	-360	-360	-36	-360	-360	1584	-360	-36	126	-36	126	18	-9
χ_{95}	+	262080	0	0	-448	-64	-360	-360	-360	-36	-360	-360	1584	-36	126	126	-36	18	-9
χ_{96}	+	291200	0	0	0	-128	-2344	680	680	680	680	680	680	-22	-22	-22	-22	-40	-4
χ_{97}	+	291200	0	0	0	-128	1544	-400	-400	-400	680	680	680	194	-76	-76	-76	-4	-13
χ_{98}	+	291200	0	0	0	-128	1544	-400	680	680	680	-400	-400	-76	194	-76	-76	-4	-13
χ_{99}	+	291200	0	0	0	-128	1544	680	-400	680	-400	680	-400	-76	-76	194	-76	-4	-13
χ_{100}	+	291200	0	0	0	-128	1544	680	680	-400	-400	-400	680	-76	-76	-76	194	-4	-13
χ_{101}	+	332800	1280	1280	1280	0	-1568	160	160	160	160	160	160	-56	-56	-56	-56	34	16
χ_{102}	+	332800	1280	-1280	-1280	0	-1568	160	160	160	160	160	160	-56	-56	-56	-56	34	16
χ_{103}	+	332800	-1280	1280	-1280	0	-1568	160	160	160	160	160	160	-56	-56	-56	-56	34	16
χ_{104}	+	332800	-1280	-1280	1280	0	-1568	160	160	160	160	160	160	-56	-56	-56	-56	34	16
χ_{105}	+	419328	896	0	0	0	-576	72	-576	-576	72	-576	-576	72	72	72	72	-36	18
χ_{106}	+	419328	0	896	0	0	-576	-576	72	-576	-576	72	-576	72	72	72	72	-36	18
χ_{107}	+	419328	0	0	896	0	-576	-576	-576	72	-576	-576	72	72	72	72	72	-36	18
χ_{108}	+	465920	-1792	0	0	0	-640	656	-640	-640	656	-640	-640	8	8	8	8	8	8
χ_{109}	+	465920	0	-1792	0	0	-640	-640	656	-640	-640	656	-640	8	8	8	8	8	8
χ_{110}	+	465920	0	0	-1792	0	-640	-640	-640	656	-640	-640	656	8	8	8	8	8	8
χ_{111}	+	491400	-840	-840	-840	200	-675	540	540	540	540	540	540	54	54	54	54	-27	-27
χ_{112}	+	531441	729	729	729	81	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	+	716800	0	0	0	0	1408	-320	-320	-320	-320	-320	-320	-104	-104	-104	-104	-32	-14
χ_{114}	+	716800	0	0	0	0	1408	-320	-320	-320	-320	-320	-320	-104	-104	-104	-104	-32	-14

	4A	4B	4C	4D	4E	5A	5B	5C	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	6N	6O	6P	6Q	6R	6S	6T	6U	6V	6W	6X	6Y	6Z	6A1	6B1	7A	8A	8B	
χ_{53}	4	-4	0	0	4	-5	0	0	57	-39	-39	9	12	6	6	-15	6	6	0	-6	-6	-3	-6	-6	3	-3	-3	3	3	0	0	3	3	-3	-3	0	0	0	0	χ_{53}
χ_{54}	4	0	-4	0	4	0	-5	0	-39	57	-39	9	6	12	6	6	-15	6	-6	0	-6	-6	-3	-6	-3	3	-3	3	0	3	0	3	-3	3	-3	0	0	0	0	χ_{54}
χ_{55}	4	0	0	-4	4	0	0	-5	-39	-39	57	9	6	6	12	6	6	-15	-6	-6	0	-6	-6	-3	-3	-3	3	3	0	0	3	3	-3	-3	3	0	0	0	0	χ_{55}
χ_{56}	4	-4	0	0	4	-5	0	0	57	-39	-39	9	-15	6	6	12	6	6	-3	-6	-6	0	-6	-6	3	-3	-3	0	0	3	3	-3	-3	3	3	0	0	0	0	χ_{56}
χ_{57}	4	0	-4	0	4	0	-5	0	-39	57	-39	9	6	-15	6	6	12	6	-6	-3	-6	-6	0	-6	-3	3	-3	0	3	0	3	-3	3	-3	3	0	0	0	0	χ_{57}
χ_{58}	4	0	0	-4	4	0	0	-5	-39	-39	57	9	6	6	-15	6	6	12	-6	-6	-3	-6	-6	0	-3	-3	3	0	3	3	0	-3	3	3	-3	0	0	0	0	χ_{58}
χ_{59}	16	0	0	0	0	0	0	0	-17	-17	-17	-1	10	10	10	-8	-8	-8	-10	-10	-10	8	8	8	1	1	1	8	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	χ_{59}
χ_{60}	16	0	0	0	0	0	0	0	-17	-17	-17	-1	10	-8	-8	-8	10	10	-10	8	8	8	-10	-10	1	1	1	-1	8	-1	-1	-1	-1	-1	-1	-1	0	0	0	χ_{60}
χ_{61}	16	0	0	0	0	0	0	0	-17	-17	-17	-1	-8	10	-8	10	-8	10	8	-10	8	-10	8	-10	1	1	1	-1	-1	8	-1	-1	-1	-1	-1	-1	0	0	0	χ_{61}
χ_{62}	16	0	0	0	0	0	0	0	-17	-17	-17	-1	-8	-8	10	10	10	-8	8	8	-10	-10	-10	8	1	1	1	-1	-1	-1	8	-1	-1	-1	-1	-1	0	0	0	χ_{62}
χ_{63}	16	0	0	0	0	0	0	0	-71	37	37	17	-8	10	10	-8	10	10	8	-10	-10	8	-10	-10	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	χ_{63}
χ_{64}	16	0	0	0	0	0	0	0	37	-71	37	17	10	-8	10	10	-8	10	-10	8	-10	-10	8	-10	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	χ_{64}
χ_{65}	16	0	0	0	0	0	0	0	37	37	-71	17	10	10	-8	10	10	-8	-10	-10	8	-10	-10	8	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	χ_{65}
χ_{66}	-21	19	-9	-9	-5	-1	9	9	9	-27	-27	9	9	0	0	9	0	0	9	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{66}
χ_{67}	-21	-9	19	-9	-5	9	-1	9	-27	9	-27	9	0	9	0	0	9	0	0	9	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{67}
χ_{68}	-21	-9	-9	19	-5	9	9	-1	-27	-27	9	9	0	0	9	0	0	9	0	0	9	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{68}
χ_{69}	24	0	0	0	-8	0	0	0	-31	-3	-3	1	14	-12	-12	14	-12	-12	-14	4	4	-14	4	4	5	-3	-3	-2	-2	-2	-2	1	1	1	1	1	0	0	0	χ_{69}
χ_{70}	24	0	0	0	-8	0	0	0	-3	-31	-3	1	-12	14	-12	-12	14	-12	4	-14	4	4	-14	4	-3	5	-3	-2	-2	-2	-2	1	1	1	1	1	0	0	0	χ_{70}
χ_{71}	24	0	0	0	-8	0	0	0	-3	-3	-31	1	-12	-12	14	-12	-12	14	4	4	-14	4	4	-14	-3	-3	5	-2	-2	-2	-2	1	1	1	1	1	0	0	0	χ_{71}
χ_{72}	-25	-5	-5	-5	-1	0	0	0	-9	-9	-9	-9	-9	-9	-9	18	18	18	3	3	3	-6	-6	-6	0	0	0	6	-3	-3	-3	0	0	0	0	0	0	1	1	χ_{72}
χ_{73}	-25	-5	-5	-5	-1	0	0	0	-9	-9	-9	-9	-9	18	18	18	-9	-9	3	-6	-6	-6	3	3	0	0	0	-3	6	-3	-3	0	0	0	0	0	0	1	1	χ_{73}
χ_{74}	-25	-5	-5	-5	-1	0	0	0	-9	-9	-9	-9	18	-9	18	-9	18	-9	-6	3	-6	3	-6	3	0	0	0	-3	-3	6	-3	0	0	0	0	0	0	1	1	χ_{74}
χ_{75}	-25	-5	-5	-5	-1	0	0	0	-9	-9	-9	-9	18	18	-9	-9	-9	18	-6	-6	3	3	3	-6	0	0	0	-3	-3	-3	6	0	0	0	0	0	0	1	1	χ_{75}
χ_{76}	12	-4	0	0	-4	-5	0	0	-9	27	27	-9	18	0	0	-9	0	0	18	0	0	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{76}
χ_{77}	12	0	-4	0	-4	0	-5	0	27	-9	27	-9	0	18	0	0	-9	0	0	18	0	0	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{77}
χ_{78}	12	0	0	-4	-4	0	0	-5	27	27	-9	-9	0	0	18	0	0	-9	0	0	18	0	0	-9	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{78}
χ_{79}	12	-4	0	0	-4	-5	0	0	-9	27	27	-9	-9	0	0	18	0	0	-9	0	0	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{79}
χ_{80}	12	0	-4	0	-4	0	-5	0	27	-9	27	-9	0	-9	0	0	18	0	0	-9	0	0	18	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{80}
χ_{81}	12	0	0	-4	-4	0	0	-5	27	27	-9	-9	0	0	-9	0	0	18	0	0	-9	0	0	18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{81}
χ_{82}	0	0	0	0	8	0	0	0	27	27	27	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-2	2	χ_{82}
χ_{83}	0	-16	0	0	0	5	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{83}
χ_{84}	0	0	-16	0	0	0	5	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{84}
χ_{85}	0	0	0	-16	0	0	0	5	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	χ_{85}
χ_{86}	0	-16	0	0	0	5	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{86}
χ_{87}	0	0	-16	0	0	0	5	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{87}
χ_{88}	0	0	0	-16	0	0	0	5	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	χ_{88}
χ_{89}	-8	0	0	0	0	0	0	0	39	39	39	-1	-6	-6	-6	-6	-6	-6	-10	-10	-10	-10	-10	-10	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	2	-2	χ_{89}
χ_{90}	0	0	0	0	0	-10	0	0	-16	0	0	8	-16	0	0	20	0	0	-16	8	8	-4	8	8	2	0	0	2	2	-4	-4	-4	-4	2	2	-1	0	0	0	χ_{90}
χ_{91}	0	0	0	0	0	0	-10	0	0	-16	0	8	0	-16	0	0	20	0	8	-16	8	8	-4	8	0	2	0	2	-4	2	-4	-4	2	-4	2	-1	0	0	0	χ_{91}
χ_{92}	0	0	0	0	0	0	0	-10	0	0	-16	8	0	0	-16	0	0	20	8	8	-16	8	8	-4	0	0	2	2	-4	-4	2	-4	2	2	-4	-1	0	0	0	χ_{92}
χ_{93}	0	0	0	0	0	-10	0	0	-16	0	0	8	20	0	0	-16	0	0	-4	8	8	-15	8	8	2	0	0	-4	-4	2	2	2	2	-4	-4	-1	0	0	0	χ_{93}
χ_{94}	0	0	0	0	0	0	-10	0	0	-16	0	8	0	20	0	0	-16	0	8	-4	8	8	-16	8	0	2	0	-4	2	-4	2	2	-4	2	-4	-1	0	0	0	χ_{94}
χ_{95}	0	0	0	0	0	0	0	-10	0	0	-16	8	0	0	20	0	0	-16	8	8	-4	8	8	-16	0	0	2	-4	2	2	-4	2	-4	-4	2	-1	0	0	0	χ_{95}
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0	0	0	0	-8	-8	-8	-8	-8	-8	0	0	0	-2	-2	-2	-2	4	4	4	4	4	0	0	0	χ_{96}
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	16	16	16	-8	-8	-8	0	0	0	-2	4	4	4	-8	4	4	4	1	0	0	0	χ_{97}
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	16	-8	-6	-8	16	16	0	0	0	4	-2	4	4	4	-8	4	4	1	0	0	0	χ_{98}
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	-8	16	-8	16	-8	16	0	0	0	4	4	-2	4	4	4	-8	4	1	0	0	0	χ_{99}
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	-8	-8	16	16	16	-8	0	0	0	4	4	4	-2	4	4	4	-8	1	0	0	0	χ_{100}
χ_{101}	0	0	0	0	0	0	0	0	-16	-16	-16	0	-16	-16	-16	-16	-16	-16	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{101}
χ_{102}	0	0	0	0	0	0	0	0	-16	16	16	0	-16	16	16	-16	16	16	0	0	0	0	0	0	2	-2	-2	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{102}
χ_{103}	0	0	0	0	0	0	0	0	16	-16	16	0	16	-16	16	16	-16	16	0	0	0	0	0	0	-2	2	-2	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{103}
χ_{104}	0	0	0	0	0	0	0	0	16	16	-16	0	16	16	-16	16	16	-16	0	0	0	0	0	0	-2	-2	2	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{104}
χ_{105}	0	16	0	0	0	-7	8	8	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{105}
χ_{106}	0	0	16	0	0	8	-7	8	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{106}
χ_{107}	0	0	0	16	0	8	8	-7	0	0	32	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	χ_{107}
χ_{108}	0	0	0	0	0	10	0	0	-64	0	0	0	8	0	0	8	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{108}
χ_{109}	0	0	0	0	0	0	10	0	0	-64	0	0	0	8	0	0	8	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{109}
χ_{110}	0	0	0	0	0	0	0	10	0	0	-64	0	0	0	8	0	0	8	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	χ_{110}
χ_{111}	0	0	0	0	16	0	0	0	-3	-3	-3	-19	-12	-12	-12	-12	-12	-12	-4	-4	-4	-4	-4	-4	-3	-3	-3	2	2	2	2	-1	-1	-1	-1	-1	0	0	0	χ_{111}
χ_{112}	-27	9	9	9	-3	-9	-9	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	χ_{112}
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{114}

$O_8^+(3)$

	$\lvert C \rvert$	p power	p' part	Class	χ_{1}	χ_{2}	χ_{3}	χ_{4}	χ_{5}	χ_{6}	χ_{7}	χ_{8}	χ_{9}	χ_{10}	χ_{11}	χ_{12}	χ_{13}	χ_{14}	χ_{15}	χ_{16}	χ_{17}	χ_{18}	χ_{19}	χ_{20}	χ_{21}	χ_{22}	χ_{23}	χ_{24}	χ_{25}	χ_{26}	χ_{27}	χ_{28}	χ_{29}	χ_{30}	χ_{31}	χ_{32}	χ_{33}	χ_{34}	χ_{35}	χ_{36}	χ_{37}	χ_{38}	χ_{39}	χ_{40}	χ_{41}	χ_{42}	χ_{43}	χ_{44}	χ_{45}	χ_{46}	χ_{47}	χ_{48}	χ_{49}	χ_{50}	χ_{51}	χ_{52}
@	2916	A	A	9A	1	8	-1	-1	-1	-1	-1	3	9	0	0	-2	-2	7	7	0	6	-3	-3	-3	-3	-3	0	0	-7	-7	-7	2	2	18	0	0	-9	0	0	0	3	14	-4	-4	5	-4	-4	18	0	0	-9	0	0	-1	8	8
@	2916	A	A	9B	1	-1	8	-1	-1	-1	-1	3	0	9	0	-2	7	-2	7	0	-3	6	-3	-3	-3	-3	0	0	-7	-7	2	-7	2	0	18	0	0	-9	0	0	3	-4	14	-4	-4	5	-4	0	18	0	0	-9	0	8	-1	8
	2916	A	A	9C	1	-1	-1	8	-1	-1	-1	3	0	0	9	-2	7	7	-2	0	-3	-3	6	-3	-3	-3	0	0	-7	-7	2	2	-7	0	0	18	0	0	-9	0	3	-4	-4	14	-4	-4	5	0	0	18	0	0	-9	8	8	-1
@	2916	A	A	9D	1	-1	-1	-1	8	-1	-1	3	9	0	0	7	7	-2	-2	0	-3	-3	-3	6	-3	-3	0	0	-7	2	2	-7	-7	-9	0	0	18	0	0	0	3	5	-4	-4	14	-4	-4	-9	0	0	18	0	0	-1	8	8
@	2916	A	A	9E	1	-1	-1	-1	-1	8	-1	3	0	9	0	7	-2	7	-2	0	-3	-3	-3	-3	6	-3	0	0	-7	2	-7	2	-7	0	-9	0	0	18	0	0	3	-4	5	-4	-4	14	-4	0	-9	0	0	18	0	8	-1	8
@	2916	A	A	9F	1	-1	-1	-1	-1	-1	8	3	0	0	9	7	-2	-2	7	0	-3	-3	-3	-3	-3	6	0	0	-7	2	-7	-7	2	0	0	-9	0	0	18	0	3	-4	-4	5	-4	-4	14	0	0	-9	0	0	18	8	8	-1
@	729	A	A	9G	1	-1	-1	-1	-1	-1	-1	3	0	0	0	7	-2	-2	-2	0	-3	-3	-3	6	6	6	0	0	2	2	2	2	2	0	0	0	0	0	0	0	3	5	5	5	-4	-4	-4	0	0	0	0	0	0	-1	-1	-1
@	729	A	A	9H	1	-1	-1	-1	-1	-1	-1	3	0	0	0	-2	7	-2	-2	0	-3	6	6	6	-3	-3	0	0	2	2	2	2	2	0	0	0	0	0	0	0	3	5	-4	-4	-4	5	5	0	0	0	0	0	0	-1	-1	-1
@	729	A	A	9I	1	-1	-1	-1	-1	-1	-1	3	0	0	0	-2	-2	7	-2	0	6	-3	6	-3	6	-3	0	0	2	2	2	2	2	0	0	0	0	0	0	0	3	-4	5	-4	5	-4	5	0	0	0	0	0	0	-1	-1	-1
@	729	A	A	9J	1	-1	-1	-1	-1	-1	-1	3	0	0	0	-2	-2	-2	7	0	6	6	-3	-3	-3	6	0	0	2	2	2	2	2	0	0	0	0	0	0	0	3	-4	-4	5	5	5	-4	0	0	0	0	0	0	-1	-1	-1
@	81	H	A	9K	1	2	2	2	-1	-1	-1	0	0	0	0	-2	1	1	1	0	0	0	0	0	0	0	0	0	-1	2	-1	-1	-1	0	0	0	0	0	0	0	0	-1	-1	-1	2	2	2	0	0	0	0	0	0	-1	-1	-1
@	81	I	A	9L	1	2	-1	-1	-1	2	2	0	0	0	0	1	-2	1	1	0	0	0	0	0	0	0	0	0	-1	-1	2	-1	-1	0	0	0	0	0	0	0	0	-1	2	2	2	-1	-1	0	0	0	0	0	0	-1	-1	-1
@	81	J	A	9M	1	-1	2	-1	2	-1	2	0	0	0	0	1	1	-2	1	0	0	0	0	0	0	0	0	0	-1	-1	-1	2	-1	0	0	0	0	0	0	0	0	2	-1	2	-1	2	-1	0	0	0	0	0	0	-1	-1	-1
@	81	K	A	9N	1	-1	-1	2	2	2	-1	0	0	0	0	1	1	1	-2	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	2	0	0	0	0	0	0	0	0	2	2	-1	-1	-1	2	0	0	0	0	0	0	-1	-1	-1
@	40	AA	AA	10A	1	1	0	0	1	0	0	0	1	-1	-1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	0	0	0	0	0	0	0	1	0	0
@	40	BB	BB	10B	1	0	1	0	0	1	0	0	-1	1	-1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	0	0	0	0	0	0	0	1	0
@	40	CC	CC	10C	1	0	0	1	0	0	1	0	-1	-1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	0	0	0	0	0	0	0	1
@	1728	DA	AA	12A	1	1	1	1	1	1	1	5	10	-2	-2	0	0	0	0	-4	-9	3	3	-9	3	3	-3	-1	5	-1	-1	-1	-1	-8	4	4	-8	4	4	0	9	0	0	0	0	0	0	7	-5	-5	7	-5	-5	0	0	0
@	1728	DA	AA	12B	1	1	1	1	1	1	1	5	-2	10	-2	0	0	0	0	-4	3	-9	3	3	-9	3	-3	-1	5	-1	-1	-1	-1	4	-8	4	4	-8	4	0	9	0	0	0	0	0	0	-5	7	-5	-5	7	-5	0	0	0
@	1728	DA	AA	12C	1	1	1	1	1	1	1	5	-2	-2	10	0	0	0	0	-4	3	3	-9	3	3	-9	-3	-1	5	-1	-1	-1	-1	4	4	-8	4	4	-8	0	9	0	0	0	0	0	0	-5	-5	7	-5	-5	7	0	0	0
@	432	KA	BA	12D	1	4	-2	-2	1	-2	-2	2	-2	1	1	0	0	3	3	2	-6	0	0	3	0	0	3	-1	2	-4	-4	2	2	4	-2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	-5	1	1	1	1	1	0	0	0
@	432	LA	CA	12E	1	-2	4	-2	-2	1	-2	2	1	-2	1	0	3	0	3	2	0	-6	0	0	3	0	3	-1	2	-4	2	-4	2	-2	4	-2	-2	-2	-2	0	0	0	0	0	0	0	0	1	-5	1	1	1	1	0	0	0
@	432	MA	DA	12F	1	-2	-2	4	-2	-2	1	2	1	1	-2	0	3	3	0	2	0	0	-6	0	0	3	3	-1	2	-4	2	2	-4	-2	-2	4	-2	-2	-2	0	0	0	0	0	0	0	0	1	1	-5	1	1	1	0	0	0
@	432	NA	EA	12G	1	1	-2	-2	4	-2	-2	2	-2	1	1	3	3	0	0	2	3	0	0	-6	0	0	3	-1	2	2	2	-4	-4	-2	-2	-2	4	-2	-2	0	0	0	0	0	0	0	0	1	1	1	-5	1	1	0	0	0
@	432	OA	FA	12H	1	-2	1	-2	-2	4	-2	2	1	-2	1	3	0	3	0	2	0	3	0	0	-6	0	3	-1	2	2	-4	2	-4	-2	-2	-2	-2	4	-2	0	0	0	0	0	0	0	0	1	1	1	1	-5	1	0	0	0
@	432	PA	GA	12I	1	-2	-2	1	-2	-2	4	2	1	1	-2	3	0	0	3	2	0	0	3	0	0	-6	3	-1	2	2	-4	-4	2	-2	-2	-2	-2	-2	4	0	0	0	0	0	0	0	0	1	1	1	1	1	-5	0	0	0
@	192	DE	AE	12J	1	1	1	1	1	1	1	-3	2	2	2	0	0	0	0	-4	-1	-1	-1	-1	-1	-1	-3	-1	5	-1	-1	-1	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0
@	108	TA	HA	12K	1	1	1	1	-2	-2	-2	-1	1	1	1	3	0	0	0	-1	0	0	0	0	0	0	0	-1	-1	-1	2	2	2	-2	-2	-2	1	1	1	0	0	0	0	0	0	0	0	1	1	1	-2	-2	-2	0	0	0
@	108	UA	IA	12L	1	1	-2	-2	-2	1	1	-1	1	1	1	0	3	0	0	-1	0	0	0	0	0	0	0	-1	-1	2	-1	2	2	-2	1	1	1	-2	-2	0	0	0	0	0	0	0	0	1	-2	-2	-2	1	1	0	0	0
@	108	VA	JA	12M	1	-2	1	-2	1	-2	1	-1	1	1	1	0	0	3	0	-1	0	0	0	0	0	0	0	-1	-1	2	2	-1	2	1	-2	1	-2	1	-2	0	0	0	0	0	0	0	0	-2	1	-2	1	-2	1	0	0	0
@	108	WA	KA	12N	1	-2	-2	1	1	1	-2	-1	1	1	1	0	0	0	3	-1	0	0	0	0	0	0	0	-1	-1	2	2	2	-1	1	1	-2	-2	-2	1	0	0	0	0	0	0	0	0	-2	-2	1	1	1	-2	0	0	0
@	72	EB	BB	12O	1	-2	0	0	1	0	0	0	2	-1	-1	-2	-2	1	1	0	-2	0	0	1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	1	1	-2	0	0
@	72	FC	CC	12P	1	0	-2	0	0	1	0	0	-1	2	-1	-2	1	-2	1	0	0	-2	0	0	1	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	1	-1	1	0	-2	0
@	72	GD	DD	12Q	1	0	0	-2	0	0	1	0	-1	-1	2	-2	1	1	-2	0	0	0	-2	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	1	-1	0	0	-2
@	72	HB	EB	12R	1	1	0	0	-2	0	0	0	2	-1	-1	1	1	-2	-2	0	1	0	0	-2	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	1	1	-2	0	0
@	72	IC	FC	12S	1	0	1	0	0	-2	0	0	-1	2	-1	1	-2	1	-2	0	0	1	0	0	-2	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	1	-1	1	0	-2	0
@	72	JD	GD	12T	1	0	0	1	0	0	-2	0	-1	-1	2	1	-2	-2	1	0	0	0	1	0	0	-2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	1	-1	0	0	-2
@	13	A	A	13A	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@	13	A	A	B*	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@	28	AA	AA	14A	1	1	-1	-1	1	-1	-1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	1	1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@	28	AB	AB	14B	1	-1	1	-1	-1	1	-1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	-1	1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@	28	AC	AC	14C	1	-1	-1	1	-1	-1	1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@	45	AB	AB	15A	1	-1	0	0	2	0	0	0	0	-1	-1	0	0	0	0	-1	2	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	-1	1	1
@	45	BC	BC	15B	1	0	-1	0	0	2	0	0	-1	0	-1	0	0	0	0	-1	0	2	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	-1	1
@	45	CD	CD	15D	1	0	0	-1	0	0	2	0	-1	-1	0	0	0	0	0	-1	0	0	2	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	1	1	-1
@	45	AE	AE	15E	1	2	0	0	-1	0	0	0	0	-1	-1	0	0	0	0	-1	-1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	-1	1	1
@	45	BF	BF	15F	1	0	2	0	0	-1	0	0	-1	0	-1	0	0	0	0	-1	0	-1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	-1	1
@	45	CG	CG	15G	1	0	0	2	0	0	-1	0	-1	-1	0	0	0	0	0	-1	0	0	-1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	1	1	-1
@	108	AA	AA	18A	1	0	-1	-1	3	-1	-1	-1	1	0	0	2	2	-1	-1	0	-2	1	1	1	1	1	0	0	1	1	1	-2	-2	2	0	0	-1	0	0	0	-1	-2	0	0	1	0	0	2	0	0	-1	0	0	-1	0	0
@	108	BB	BB	18B	1	-1	0	-1	-1	3	-1	-1	0	1	0	2	-1	2	-1	0	1	-2	1	1	1	1	0	0	1	1	-2	1	-2	0	2	0	0	-1	0	0	-1	0	-2	0	0	1	0	0	2	0	0	-1	0	0	-1	0
@	108	CC	CC	18C	1	-1	-1	0	-1	-1	3	-1	0	0	1	2	-1	-1	2	0	1	1	-2	1	1	1	0	0	1	1	-2	-2	1	0	0	2	0	0	-1	0	-1	0	0	-2	0	0	1	0	0	2	0	0	-1	0	0	-1
@	108	DA	AA	18D	1	3	-1	-1	0	-1	-1	-1	1	0	0	-1	-1	2	2	0	1	1	1	-2	1	1	0	0	1	-2	-2	1	1	-1	0	0	2	0	0	0	-1	1	0	0	-2	0	0	-1	0	0	2	0	0	-1	0	0
@	108	EB	BB	18E	1	-1	3	-1	-1	0	-1	-1	0	1	0	-1	2	-1	2	0	1	1	1	1	-2	1	0	0	1	-2	1	-2	1	0	-1	0	0	2	0	0	-1	0	1	0	0	-2	0	0	-1	0	0	2	0	0	-1	0
@	108	FC	CC	18F	1	-1	-1	3	-1	-1	0	-1	0	0	1	-1	2	2	-1	0	1	1	1	1	1	-2	0	0	1	-2	1	1	-2	0	0	-1	0	0	2	0	-1	0	0	1	0	0	-2	0	0	-1	0	0	2	0	0	-1
@	20	AB	AB	20A	1	-1	0	0	-1	0	0	0	1	-1	-1	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
@	20	BC	BC	20B	1	0	-1	0	0	-1	0	0	-1	1	-1	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
@	20	CD	CD	20C	1	0	0	-1	0	0	-1	0	-1	-1	1	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

	9A	9B	9C	9D	9E	9F	9G	9H	9I	9J	9K	9L	9M	9N	10A	10B	10C	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	12S	12T	13A	B*	14A	14B	14C	15A	15B	15D	15E	15F	15G	18A	18B	18C	18D	18E	18F	20A	20B	20C	
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	1	4	-2	-2	1	-2	-2	1	1	1	-2	-2	2	0	0	-1	0	0	0	0	0	0	0	1	0	0	-2	0	0	0	0	0	0	0	0	1	0	0	χ_{53}
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	1	-2	4	-2	-2	1	-2	1	1	-2	1	-2	0	2	0	0	-1	0	0	0	0	0	0	0	1	0	0	-2	0	0	0	0	0	0	0	0	1	0	χ_{54}
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	-2	-2	4	-2	-2	1	1	1	-2	-2	1	0	0	2	0	0	-1	0	0	0	0	0	0	0	1	0	0	-2	0	0	0	0	0	0	0	0	1	χ_{55}
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	1	1	-2	-2	4	-2	-2	1	-2	-2	1	1	-1	0	0	2	0	0	0	0	0	0	0	-2	0	0	1	0	0	0	0	0	0	0	0	1	0	0	χ_{56}
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	1	-2	1	-2	-2	4	-2	1	-2	1	-2	1	0	-1	0	0	2	0	0	0	0	0	0	0	-2	0	0	1	0	0	0	0	0	0	0	0	1	0	χ_{57}
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	-2	-2	1	-2	-2	4	1	-2	1	1	-2	0	0	-1	0	0	2	0	0	0	0	0	0	0	-2	0	0	1	0	0	0	0	0	0	0	0	1	χ_{58}
χ_{59}	6	6	6	-3	-3	-3	6	-3	-3	-3	0	0	0	0	0	0	0	-5	-5	-5	-2	-2	-2	4	4	4	3	-2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	1	1	1	0	0	0	χ_{59}
χ_{60}	6	-3	-3	-3	6	6	-3	6	-3	-3	0	0	0	0	0	0	0	-5	-5	-5	-2	4	4	4	-2	-2	3	1	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	1	1	-2	-2	0	0	0	χ_{60}
χ_{61}	-3	6	-3	6	-3	6	-3	-3	6	-3	0	0	0	0	0	0	0	-5	-5	-5	4	-2	4	-2	4	-2	3	1	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	-2	1	-2	0	0	0	χ_{61}
χ_{62}	-3	-3	6	6	6	-3	-3	-3	-3	6	0	0	0	0	0	0	0	-5	-5	-5	4	4	-2	-2	-2	4	3	1	1	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-2	-2	-2	1	0	0	0	χ_{62}
χ_{63}	6	-3	-3	6	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	13	1	1	4	-2	-2	4	-2	-2	-3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	1	-2	1	1	0	0	0	χ_{63}
χ_{64}	-3	6	-3	-3	6	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	1	13	1	-2	4	-2	-2	4	-2	-3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	1	-2	1	0	0	0	χ_{64}
χ_{65}	-3	-3	6	-3	-3	6	-3	-3	-3	-3	0	0	0	0	0	0	0	1	1	13	-2	-2	4	-2	-2	4	-3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-2	1	1	-2	0	0	0	χ_{65}
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	9	-3	-3	-3	0	0	-3	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	-1	1	1	χ_{66}
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-3	9	-3	0	-3	0	0	-3	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	1	-1	1	χ_{67}
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-3	-3	9	0	0	-3	0	0	-3	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	1	1	-1	χ_{68}
χ_{69}	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	9	-3	-3	6	0	0	6	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	χ_{69}
χ_{70}	0	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	0	0	-3	9	-3	0	6	0	0	6	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	χ_{70}
χ_{71}	0	0	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	0	-3	-3	9	0	0	6	0	0	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	χ_{71}
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	2	2	2	-1	2	-1	-1	-1	1	1	1	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{72}
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	2	2	2	-1	-1	-1	-1	2	-1	-1	1	-2	-2	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{73}
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	2	-1	2	-1	2	-1	-1	-1	-1	2	-1	-2	1	-2	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{74}
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	2	2	-1	-1	-1	2	-1	-1	-1	-1	2	-2	-2	1	1	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{75}
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-9	3	3	-6	0	0	3	0	0	-1	0	0	0	0	2	0	0	-1	0	0	0	0	1	-1	-1	-2	0	0	1	0	0	0	0	0	0	0	0	1	0	0	χ_{76}
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	3	-9	3	0	-6	0	0	3	0	-1	0	0	0	0	0	2	0	0	-1	0	0	0	-1	1	-1	0	-2	0	0	1	0	0	0	0	0	0	0	0	1	0	χ_{77}
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	3	3	-9	0	0	-6	0	0	3	-1	0	0	0	0	0	0	2	0	0	-1	0	0	-1	-1	1	0	0	-2	0	0	1	0	0	0	0	0	0	0	0	1	χ_{78}
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-9	3	3	3	0	0	-6	0	0	-1	0	0	0	0	-1	0	0	2	0	0	0	0	1	-1	-1	1	0	0	-2	0	0	0	0	0	0	0	0	1	0	0	χ_{79}
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	3	-9	3	0	3	0	0	-6	0	-1	0	0	0	0	0	-1	0	0	2	0	0	0	-1	1	-1	0	1	0	0	-2	0	0	0	0	0	0	0	0	1	0	χ_{80}
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	3	3	-9	0	0	3	0	0	-6	-1	0	0	0	0	0	0	-1	0	0	2	0	0	-1	-1	1	0	0	1	0	0	-2	0	0	0	0	0	0	0	0	1	χ_{81}
χ_{82}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-9	-9	-9	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{82}
χ_{83}	-5	4	4	-5	4	4	4	4	-5	-5	1	1	-2	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	2	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	χ_{83}
χ_{84}	4	-5	4	4	-5	4	4	-5	4	-5	1	-2	1	-2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	2	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	χ_{84}
χ_{85}	4	4	-5	4	4	-5	4	-5	-5	4	1	-2	-2	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	2	0	0	-1	0	0	-1	0	0	-1	0	0	-1	χ_{85}
χ_{86}	-5	4	4	-5	4	4	-5	-5	4	4	-2	-2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	-1	0	0	2	0	0	-1	0	0	-1	0	0	-1	0	0	χ_{86}
χ_{87}	4	-5	4	4	-5	4	-5	4	-5	4	-2	1	-2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	-1	0	0	2	0	0	-1	0	0	-1	0	0	-1	0	χ_{87}
χ_{88}	4	4	-5	4	4	-5	-5	4	4	-5	-2	1	1	-2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	-1	0	0	2	0	0	-1	0	0	-1	0	0	-1	χ_{88}
χ_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	-5	-5	-2	-2	-2	-2	-2	-2	3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{89}
χ_{90}	-9	0	0	18	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	-1	0	0	2	0	0	0	0	0	χ_{90}
χ_{91}	0	-9	0	0	18	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	-1	0	0	2	0	0	0	0	χ_{91}
χ_{92}	0	0	-9	0	0	18	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	-1	0	0	2	0	0	0	χ_{92}
χ_{93}	18	0	0	-9	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	2	0	0	-1	0	0	0	0	0	χ_{93}
χ_{94}	0	18	0	0	-9	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	2	0	0	-1	0	0	0	0	χ_{94}
χ_{95}	0	0	18	0	0	-9	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	0	2	0	0	-1	0	0	0	χ_{95}
χ_{96}	-4	-4	-4	-4	-4	-4	5	5	5	5	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{96}
χ_{97}	-4	-4	-4	-4	-4	-4	5	-4	-4	-4	2	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{97}
χ_{98}	-4	-4	-4	-4	-4	-4	-4	5	-4	-4	-1	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{98}
χ_{99}	-4	-4	-4	-4	-4	-4	-4	-4	5	-4	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{99}
χ_{100}	-4	-4	-4	-4	-4	-4	-4	-4	-4	5	-1	-1	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{100}
χ_{101}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	2	2	2	2	2	2	0	0	0	χ_{101}
χ_{102}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	2	-2	-2	2	-2	-2	0	0	0	χ_{102}
χ_{103}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	0	0	0	0	0	0	-2	2	-2	-2	2	-2	0	0	0	χ_{103}
χ_{104}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	0	0	0	0	0	0	-2	-2	2	-2	-2	2	0	0	0	χ_{104}
χ_{105}	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	0	0	0	0	0	0	2	-1	-1	2	-1	-1	-1	0	0	-1	0	0	1	0	0	χ_{105}
χ_{106}	0	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	0	0	0	0	0	-1	2	-1	-1	2	-1	0	-1	0	0	-1	0	0	1	0	χ_{106}
χ_{107}	0	0	-9	0	0	-9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	0	0	0	0	-1	-1	2	-1	-1	2	0	0	-1	0	0	-1	0	0	1	χ_{107}
χ_{108}	-10	8	8	-10	8	8	-1	-1	-1	-1	-1	-1	-1	-1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2	0	0	2	0	0	0	0	0	χ_{108}
χ_{109}	8	-10	8	8	-10	8	-1	-1	-1	-1	-1	-1	-1	-1	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2	0	0	2	0	0	0	0	χ_{109}
χ_{110}	8	8	-10	8	8	-10	-1	-1	-1	-1	-1	-1	-1	-1	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2	0	0	2	0	0	0	χ_{110}
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	9	9	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{111}
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	χ_{112}
χ_{113}	4	4	4	4	4	4	4	4	4	4	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b13	*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	4	4	4	4	4	4	4	4	4	4	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*	b13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{114}

$O_8^+(3)$

Orthogonal group $O_8^+(3) \cong D_4(3)$

Order = 4,952,179,814,400 = $2^{12}.3^{12}.5^2.7.13$ Mult = 2^2 Out = S_4

Constructions

Orthogonal $GO_8^+(3) \cong 2.G.(2^2)_{122}$: all 8 x 8 matrices over $\mathbb{F}_3$ preserving a quadratic form of Witt defect 0, for example $x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2+x_7^2+x_8^2$; $PGO_8^+(3) \cong G.(2^2)_{122}$; $SO_8^+(3) \cong 2.G.2_1$; $PSO_8^+(3) \cong G.2_1$; $O_8^+(3) \cong G$

Cayley $2^2.G.(2^2)_{111}$: the isotopy group of the Cayley algebra modulo 3. An isotopy is a triple of maps (A,B,C) such that $xyz = 1 \Leftrightarrow x^A y^B z^C = 1$. The maps that arise are the linear maps that preserve or negate the quadratic form, and there are two isotopies (A,B,C) and (-A,-B,C) for each such map C. The isotopy group can be extended to $2^2.G.2^2.S_3 \cong 2^2.G.S_4$ by adjoining the triality automorphism $(x,y,z) \longrightarrow (y,z,x)$ and the duality automorphism $(x,y,z) \longrightarrow (\bar{y},\bar{x},\bar{z})$.

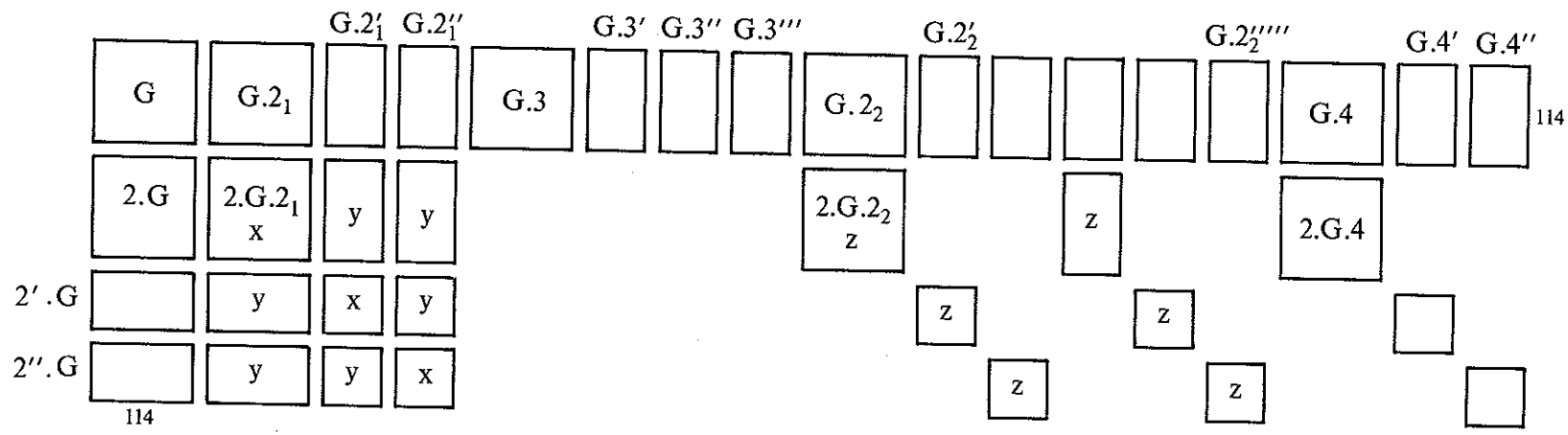
Transposition $G.2_2$ and $G.S_3$ are 3-transposition groups, that is, they are generated by a class of involutions in which every pair has product of order at most 3. The transpositions are reflections in one of the orthogonal representations.

Remark : $S_3 \times G.S_3$ and $G.S_4$ are maximal subgroups of Fi_{24} and the Baby Monster, respectively. For a presentation, see page 232.

Maximal subgroups (believed to be complete)

Order	Index	Structure	$G.2_1$	$G.(2^2)_{111}$	$G.3$	$G.A_4$	$G.2_2$	$G.(2^2)_{122}$	$G.4$	$G.D_8$	$G.S_3$	$G.S_4$	Characters	Specifications Abstract	Specifications Orthogonal
4585351680	1080	$O_7(3)$	$: O_7(3):2$	$(L_4(3)x2).2^2$	$G_2(3) \times 3$	$L_3(3):2 \times A_4$	$: H \times 2$	$: H:2 \times 2$	$L_4(3).[2^3]$	$(L_4(3) \times 2^2).2^2$	$G_2(3) \times S_3$	$L_3(3):2 \times S_4$	1a+260a+819a	C(2)	non-isotropic point
4585351680	1080	$O_7(3)$	$: O_7(3):2$				$: H : 2$	$: H:2 \times 2$					1a+260d+819a	C(2)	non-isotropic point
4585351680	1080	$O_7(3)$	$L_4(3):2 \times 2$	$(L_4(3)x2).2^2$									1a+260b+819b	C(2)	
4585351680	1080	$O_7(3)$											1a+260e+819b	C(2)	
4585351680	1080	$O_7(3)$	$L_4(3):2 \times 2$	$(L_4(3)x2).2^2$									1a+260c+819c	C(2)	
4585351680	1080	$O_7(3)$											1a+260f+819c	C(2)	
4421589120	1120	$3^6:L_4(3)$	$: 3^6:(L_4(3)x2)$	$: H.2^2$	$[2^4.3^{13}]$	$[2^6.3^{13}]$	$: H.2$	$: H.2^2$	$: H.4$	$: H.D_8$	$[2^5.3^{13}]$	$[2^7.3^{13}]$	1a+300a+819a	$N(3A_{130}B_{117}E_{117})$	isotropic point
4421589120	1120	$3^6:L_4(3)$	$: 3^6:(L_4(3)x2)$	$: H.2^2$			$3^{3+6}:(L_3(3)x2)$	$3^{3+6}:(L_3(3)x2^2)$	$3^{3+6}:(L_3(3)x4)$	$3^{3+6}:(L_3(3)xD_8)$			1a+300a+819b	$N(3A_{130}C_{117}F_{117})$	isotropic projective 3-space
4421589120	1120	$3^6:L_4(3)$	$: 3^6:(L_4(3)x2)$	$: H.2^2$									1a+300a+819c	$N(3A_{130}D_{117}G_{117})$	isotropic projective 3-space
174182400	28431	$O_8^+(2)$	2^7A_8	$[2^{14}.3]$	$: O_8^+(2):3$	$[2^{14}.3^2]$	$: O_8^+(2):2$	2^7S_8	$[2^{14}.3]$	$[2^{15}.3]$	$: O_8^+(2):S_3$	$[2^{15}.3^2]$			
174182400	28431	$O_8^+(2)$			$2^{3+6}(L_3(2)x3)$		$: O_8^+(2):2$				$2^{3+6}(L_3(2)xS_3)$				
174182400	28431	$O_8^+(2)$	2^7A_8				2^7A_8	2^7S_8							
174182400	28431	$O_8^+(2)$													
136048896	36400	$3_+^{1+8}:2(A_4xA_4xA_4).2$	$: H.2$	$: H.2^2$	$: H.3$	$: H.A_4$	$: H.2$	$: H.2^2$	$: H.4$	$: H.D_8$	$: H.S_3$	$: H.S_4$		N(3A)	isotropic line
26127360	189540	$2^{\cdot}U_4(3).2^2$	$: H.2$	$: H.2^2$	$A_4 \times U_3(3):2$	$4^2:3 \times U_3(3):2$	$: H.2$	$: H.2^2$	$: H.4$	$: H.D_8$	$S_4 \times U_3(3):2$	$4^2:S_3 \times U_3(3):2$		N(2A)	minus line
26127360	189540	$2^{\cdot}U_4(3).2^2$	$: H.2$	$: H.2^2$										N(2B)	$U_4(3)$
26127360	189540	$2^{\cdot}U_4(3).2^2$	$: H.2$	$: H.2^2$										N(2C)	$U_4(3)$
622080	7960680	$(A_4xU_4(2)):2$	$: S_4 \times U_4(2):2$				$: H \times 2$	$: H.2 \times 2$						$N(2A^2)$	$O_3(3) \times O_5(3)$
622080	7960680	$(A_4xU_4(2)):2$	$: S_4 \times U_4(2):2$				$: H \times 2$	$: H.2 \times 2$						$N(2A^2)$	$O_3(3) \times O_5(3)$
622080	7960680	$(A_4xU_4(2)):2$												$N(2B^2)$	$S_2(3) \circ S_4(3)$
622080	7960680	$(A_4xU_4(2)):2$												$N(2B^2)$	$S_2(3) \circ S_4(3)$
622080	7960680	$(A_4xU_4(2)):2$												$N(2C^2)$	$S_2(3) \circ S_4(3)$
622080	7960680	$(A_4xU_4(2)):2$												$N(2C^2)$	$S_2(3) \circ S_4(3)$
518400	9552816	$(A_6 \times A_6):2^2$	$: H \times 2$	$: H.2 \times 2$		$10^2:4A_4$	$: S_6wr2$	$: S_6wr2 \times 2$	$: H.4$	$: (S_6wr2).2 \times 2$		$10^2:4S_4$		C(2)	$O_4^-(3)$ wr 2
518400	9552816	$(A_6 \times A_6):2^2$	$: H.2$	$: H.2 \times 2$										C(2)	$O_4^+(9)$
518400	9552816	$(A_6 \times A_6):2^2$	$: H.2$	$: H.2 \times 2$										C(2)	$O_4^+(9)$
331776	14926275	$2((A_4wr2)wr2)$	$: H.2$	$: H.2^2$	$: H.3$	$: H.A_4$	$: H.2$	$: H.2^2$	$: H.4$	$: H.D_8$	$: H.S_3$	$: H.S_4$		N(2D)	$O_4^+(3)$ wr 2

$O_8^+(3)$ and $O_8^-(3)$



Note:- Only the characters of the simple group been printed (pp 136–139). The letters x,y,z indicate that there are just three isoclinism classes of groups 2.G.2, which may be expected to have essentially distinct tables.

Orthogonal group $O_8^-(3) \cong {}^2D_4(3)$

Order = 10,151,968,619,520 = $2^{10}.3^{12}.5.7.13.41$ Mult = 2 Out = 2^2

Constructions

Orthogonal $GO_8^-(3) \cong 2 \times G.2_1$: all 8 x 8 matrices over $\mathbb{F}_3$ preserving a quadratic form of Witt defect 1, for example $x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2+x_7^2-x_8^2$; $PGO_8^-(3) \cong G.2_1$; $SO_8^-(3) \cong 2 \times G$; $PSO_8^-(3) \cong O_8^-(3) \cong G$.

Remark : There is a maximal subgroup $3^{8}{\cdot}O_8^-(3){\cdot}2$ in the Monster group F_1.

Maximal subgroups (believed to be complete)

Order	Index	Structure	$G.2_1$	$G.2_2$	$G.2_3$	$G.2^2$	Specifications Abstract	Specifications Orthogonal
9523422720	1066	$3^6{:}2U_4(3).2$	: $3^6{:}2U_4(3).2^2$	: $3^6{:}2U_4(3).[2^2]$	: $3^6{:}2U_4(3).[2^2]$	: $3^6{:}2U_4(3).D_8$	$N(3^6)$	isotropic point
9170703360	1107	$O_7(3){:}2$	: $O_7(3){:}2 \times 2$	$2U_4(3).D_8$	$2U_4(3).D_8$	$2U_4(3).D_8 \times 2$	C(2D)	non-isotropic point
9170703360	1107	$O_7(3){:}2$	: $O_7(3){:}2 \times 2$				C(2E)	non-isotropic point
442158912	22960	$3^{3+6}{:}(L_3(3)\times 2^2)$	: $3^{3+6}{:}(L_3(3)\times D_8)$	: H.2	: H.2	: $H.2^2$	$N(3^3)$	isotropic plane
340122240	29848	$3_+^{1+8}{:}(2S_4\times A_6)$	: $3_+^{1+8}(2S_4\times S_6)$	: H.2	: H.2	: $H.2^2$	N(3A)	isotropic line
48522240	209223	$(4 \times L_4(3)){:}2$	: $D_8 \times L_4(3){:}2_2$	: $(4\times L_4(3){:}2){:}2$	: H.2	: $D_8 \times L_4(3).2^2$	N(2A)	minus line
1244160	8159697	$S_4 \times U_4(2){:}2$	: $H \times 2$				$N(2A^2)$, C(2F)	$O_3(3)\times O_5(3)$
1244160	8159697	$S_4 \times U_4(2){:}2$	: $H \times 2$				$N(2A^2)$, C(2G)	$O_3(3)\times O_5(3)$
531360	19105632	$L_2(81){:}2$	: $L_2(81){:}4$	: $L_2(81){:}2^2$	: $L_2(81){:}4$	: $L_2(81){:}(2\times 4)$		$O_4^-(9)$
414720	24479091	$(A_6\times 2(A_4\times A_4).2).2$	: $S_6\times 2(A_4\times A_4).2^2$	: H.2	: H.2	: $H.2^2$	N(2C)	$O_4^-(3)\times O_4^+(3)$

Note: The characters of $O_8^-(3)$ are not printed.

$O_{10}^{+}(2)$

ind		1A	2A	2B	2C	2D	3A	3B	3C	3D	4A	4B	4C	4D	4E	4F	4G	4H	4I	5A	5B	6A	6B
		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		23499295948800	990904320	371589120	11796480	1179648	592220160	362880	77760	38880	1474560	294912	92160	49152	24576	6144	6144	3072	2048	129600	300	552960	138240
p power			A	A	A	A	A	A	A	A	A	A	B	A	A	B	D	C	D	A	A	AA	AB
p' part			A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	+	155	27	43	-5	11	50	8	5	2	19	3	11	-5	3	3	7	-1	-1	15	0	18	10
χ_{3}	+	186	58	42	26	10	33	15	6	-3	2	18	10	10	2	2	-2	6	6	6	1	1	9
χ_{4}	+	340	84	84	20	20	85	19	-5	7	20	20	20	4	4	4	4	4	4	20	0	21	21
χ_{5}	+	968	-28	84	4	-12	154	7	10	-5	44	-4	4	4	-4	-4	8	0	0	23	3	26	-6
χ_{6}	+	2108	316	92	124	28	-34	29	20	2	12	44	4	28	12	4	4	12	4	-7	3	-2	-10
χ_{7}	+	5270	22	454	-10	38	629	20	-10	-13	94	-18	30	6	-2	6	10	-6	2	60	0	85	13
χ_{8}	+	6324	308	420	84	68	510	-12	24	6	60	12	20	20	12	12	8	0	0	24	-1	62	30
χ_{9}	+	7905	609	369	65	-47	408	30	30	21	-7	41	9	1	-23	-15	-3	-3	5	15	0	24	24
χ_{10}	+	9300	980	420	180	4	120	75	-15	-15	-20	92	20	20	-4	-4	0	8	8	0	0	-40	0
χ_{11}	+	11067	315	763	-69	-5	1071	21	0	6	123	-5	35	-5	-5	-13	11	3	-5	82	-3	111	31
χ_{12}	+	13020	476	924	-100	92	987	42	15	6	60	-4	60	-20	12	12	-4	-4	-4	60	0	59	51
χ_{13}	+	13888	-448	448	64	-64	1036	28	10	10	128	0	0	0	0	0	0	0	0	58	3	44	-44
χ_{14}	+	14756	1316	756	260	84	476	56	-1	-4	-4	108	36	36	12	12	0	8	8	1	-4	-4	36
χ_{15}	+	15810	1090	882	-30	18	663	78	15	-12	-22	58	50	-30	-6	-6	-10	-2	-2	30	0	-41	39
χ_{16}	+	18445	-371	637	45	29	238	49	-5	31	-43	-27	13	13	5	-11	1	-7	9	10	0	-50	22
χ_{17}	+	18445	525	-595	141	45	595	70	19	-5	125	-35	-35	-3	13	-3	5	5	-11	0	5	-45	35
χ_{18}	+	22848	1344	1344	64	64	1428	42	-30	6	64	64	64	0	0	0	0	0	0	63	3	84	84
χ_{19}	+	31620	1028	372	-220	20	-510	57	30	3	76	28	4	-28	28	-4	8	0	0	-15	0	2	-30
χ_{20}	+	31620	-764	1044	100	-12	1632	57	-15	-24	60	-20	20	4	12	-4	0	-8	8	60	0	64	-24
χ_{21}	+	36890	154	266	-70	-86	1547	-28	35	8	194	18	-30	-6	18	-6	14	-2	6	20	0	43	-13
χ_{22}	+	42160	2736	496	560	112	-680	76	-5	13	80	144	0	48	16	0	0	16	0	-20	0	24	-56
χ_{23}	+	50592	1440	672	416	160	408	-78	30	21	32	32	0	32	32	0	0	0	0	-18	-3	24	24
χ_{24}	o	55335	-1113	791	135	-9	-1071	42	0	3	111	-1	-41	-9	-17	15	-5	3	3	5	0	-15	41
χ_{25}	o	55335	-1113	791	135	-9	-1071	42	0	3	111	-1	-41	-9	-17	15	-5	3	3	5	0	-15	41
χ_{26}	+	66402	-798	1554	66	-14	3213	0	0	-18	234	-6	-6	-14	-22	2	6	6	-2	87	-3	141	-51
χ_{27}	+	73780	1204	1316	-300	196	238	28	40	34	60	12	20	-44	12	12	8	0	0	-60	0	46	14
χ_{28}	+	73780	-588	1988	20	164	2380	28	-5	7	44	-36	36	-12	-4	12	0	-8	8	15	0	108	20
χ_{29}	+	73780	-1484	1428	180	20	-833	91	-5	34	68	-28	-28	4	-12	4	-4	-4	12	-10	0	-65	63
χ_{30}	+	101184	832	1728	320	192	2652	-120	6	-12	128	0	0	0	0	0	0	0	0	-6	4	124	36
χ_{31}	+	110670	4046	126	430	-162	357	105	60	24	-138	70	-34	-2	-42	-10	-14	-6	-6	-15	0	101	21
χ_{32}	+	110670	-1330	1470	366	94	-1785	105	24	-30	150	-90	-50	14	6	-10	2	-6	10	0	5	-25	15
χ_{33}	o	110670	462	-1442	-210	62	1071	21	0	-21	102	22	-10	14	-10	-2	-6	-6	2	10	0	-81	31
χ_{34}	o	110670	462	-1442	-210	62	1071	21	0	-21	102	22	-10	14	-10	-2	-6	-6	2	10	0	-81	31
χ_{35}	+	147560	-1176	3080	40	-56	4046	77	50	5	-40	-72	40	40	-8	-8	-16	0	0	80	0	-18	-10
χ_{36}	+	147560	616	-1176	-280	-24	2618	-7	35	-13	296	40	-40	8	8	-8	8	-8	8	-20	0	-38	18
χ_{37}	+	148800	5440	960	320	-64	-1140	120	-60	30	0	128	0	0	0	0	0	0	0	0	0	-20	-60
χ_{38}	+	158100	3220	660	340	-172	2805	-30	-30	15	20	20	-60	4	4	-12	4	-12	4	0	0	85	45
χ_{39}	+	166005	-651	2709	-75	-171	5355	0	-45	9	197	-27	-3	21	5	-3	-3	5	-3	90	0	75	-45
χ_{40}	+	166005	2037	2373	-555	37	0	63	0	-18	-51	-3	21	-11	29	-19	1	9	-7	15	0	-192	0
χ_{41}	+	166005	-651	-651	-75	53	3213	0	0	36	213	21	-51	5	-27	13	-11	-3	5	15	0	-51	-51
χ_{42}	+	168640	5824	1344	960	64	-680	-26	40	-38	0	128	0	64	0	0	0	0	0	20	0	-104	-24
χ_{43}	+	208320	448	4032	-320	192	2940	42	-30	-12	-64	-64	64	0	0	0	0	0	0	-15	0	-68	60
χ_{44}	+	221340	6300	2268	-100	92	-1428	147	30	-33	92	92	20	-68	-4	4	12	-12	-4	-30	0	-84	-84
χ_{45}	+	221340	924	3164	-420	28	5355	-21	0	12	60	-4	-20	-20	12	-4	-4	12	-4	20	0	171	11
χ_{46}	+	236096	6720	2240	576	192	476	-28	-64	8	0	128	0	0	0	0	0	0	0	-64	1	60	20
χ_{47}	+	265608	2184	168	-696	104	0	63	0	36	-72	24	-24	8	24	-8	-16	0	0	-57	3	192	0

ind		6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	7A	8A	8B	8C	8D	8E	8F	9A	9B	10A	10B	10C	12A
		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		120960	9216	4320	3456	2592	1728	1728	1152	576	288	288	252	768	768	256	128	64	64	540	27	2880	480	20	4608
p power		BA	AD	DA	BA	DB	CA	BB	BC	BD	CC	DD	A	A	B	D	E	G	I	D	D	AA	AB	BC	AA
p' part		BA	AD	DA	BA	DB	CA	BB	BC	BD	CC	DD	A	A	A	A	A	A	A	A	A	AA	AB	BC	AA
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	+	6	2	6	0	-2	-3	4	-2	2	1	2	1	3	3	-1	-1	1	1	5	-1	7	3	0	-2
χ_{3}	+	13	1	1	7	-3	-2	3	5	1	2	1	4	2	2	2	2	0	0	0	0	-2	2	1	5
χ_{4}	+	21	5	3	3	3	3	3	5	5	-1	-1	4	4	4	0	0	0	0	4	1	4	4	0	5
χ_{5}	+	-7	-6	11	-1	3	2	3	1	-3	-2	3	0	-4	4	0	0	-2	2	4	1	7	-1	-1	2
χ_{6}	+	1	-2	-2	13	2	4	5	1	1	4	-2	1	0	0	4	0	0	0	-1	-1	1	-3	-1	-6
χ_{7}	+	-20	5	7	4	-5	-2	4	-4	-4	2	-1	-1	2	2	-2	2	0	0	5	-1	12	4	0	1
χ_{8}	+	14	14	2	-4	6	8	0	6	2	0	2	-4	4	-4	0	0	2	-2	0	0	8	0	-1	6
χ_{9}	+	42	-8	-3	6	9	6	-6	2	-2	2	1	2	-3	-3	-3	1	-1	-1	0	0	-1	-1	0	8
χ_{10}	+	35	-8	5	11	-3	-7	3	3	-5	-3	1	4	4	-4	0	0	2	-2	0	0	0	0	0	4
χ_{11}	+	21	-17	6	-3	-2	0	1	-3	1	0	-2	0	-1	-1	3	-1	-1	-1	6	0	10	-2	1	-9
χ_{12}	+	14	11	2	2	6	-1	6	-10	2	-1	2	0	4	4	0	0	0	0	0	0	-4	4	0	-9
χ_{13}	+	-28	-4	14	-4	-2	2	4	4	-4	-2	2	0	-8	8	0	0	0	0	1	1	2	-2	-1	8
χ_{14}	+	56	12	-4	8	0	-1	0	8	0	-1	0	0	-4	4	0	0	-2	2	-4	-1	1	1	0	8
χ_{15}	+	64	-9	4	-2	0	7	6	0	0	3	0	4	2	2	-2	-2	0	0	0	0	-10	2	0	-1
χ_{16}	+	-35	14	7	1	7	-5	1	-3	5	3	-1	0	-3	-3	1	1	-1	-1	1	1	-6	2	0	2
χ_{17}	+	0	3	15	6	-1	3	-10	0	0	3	3	0	5	5	1	1	1	1	-5	1	0	0	1	11
χ_{18}	+	84	4	-6	-6	-6	-6	-6	4	4	-2	-2	0	0	0	0	0	0	0	0	0	-1	-1	-1	4
χ_{19}	+	-1	2	-1	-7	3	-10	9	-1	-1	2	-1	1	4	-4	0	0	-2	2	0	0	-7	-3	0	10
χ_{20}	+	-71	0	-8	1	0	1	-3	1	-3	1	0	1	-4	4	0	0	2	-2	0	0	-4	4	0	12
χ_{21}	+	28	-5	16	4	-4	-5	-4	-4	4	-1	4	0	-2	-2	-2	2	0	0	-1	-1	4	-4	0	-1
χ_{22}	+	-36	-8	-3	12	1	3	4	-4	4	-1	1	-1	0	0	0	0	0	0	1	1	-4	-4	0	-16
χ_{23}	+	-6	-8	9	-6	-3	6	6	2	-2	2	1	-4	0	0	0	0	0	0	0	0	-10	2	1	8
χ_{24}	o	0	-15	-9	-6	-1	0	2	0	0	0	3	0	-1	-1	-1	-1	1	1	3	0	-3	1	0	-3
χ_{25}	o	0	-15	-9	-6	-1	0	2	0	0	0	3	0	-1	-1	-1	-1	1	1	3	0	-3	1	0	-3
χ_{26}	+	-42	13	6	0	6	0	0	6	-2	0	-2	0	-2	-2	2	-2	0	0	0	0	7	-1	1	-3
χ_{27}	+	-14	-2	-2	4	2	-8	8	-6	-2	0	-2	0	-4	4	0	0	2	-2	-5	1	4	-4	0	6
χ_{28}	+	-84	-4	-9	12	-1	3	-4	-4	-4	-1	-1	0	4	-4	0	0	-2	2	-5	1	7	3	0	8
χ_{29}	+	-35	-1	-2	-5	6	-5	3	-3	5	3	2	0	-4	-4	0	0	0	0	4	1	6	-2	0	-1
χ_{30}	+	-8	12	4	-8	0	-2	0	8	0	2	0	-1	8	-8	0	0	0	0	0	0	2	-2	0	8
χ_{31}	+	-7	21	8	17	0	-4	-3	1	-3	4	0	0	-2	-2	-2	-2	0	0	0	0	1	1	0	-3
χ_{32}	+	35	7	-10	-7	-6	8	9	3	-5	0	-2	0	-2	-2	2	2	0	0	0	0	0	0	1	-21
χ_{33}	o	-21	-1	3	-3	7	0	1	3	-1	0	-1	0	2	2	2	-2	0	0	-3	0	2	-2	0	-9
χ_{34}	o	-21	-1	3	-3	7	0	1	3	-1	0	-1	0	2	2	2	-2	0	0	-3	0	2	-2	0	-9
χ_{35}	+	-63	-2	-3	-3	-7	-6	5	1	1	-2	1	0	0	0	0	0	0	0	-1	-1	-16	0	0	2
χ_{36}	+	7	-6	19	1	3	-5	-3	-1	3	-1	3	0	0	0	0	0	0	0	-4	-1	-4	4	0	-10
χ_{37}	+	-20	-4	10	-8	6	4	0	-4	-4	-4	2	1	8	-8	0	0	0	0	0	0	0	0	0	0
χ_{38}	+	70	5	-5	10	3	10	6	-2	2	-2	-1	-2	-4	-4	0	0	0	0	0	0	0	0	0	17
χ_{39}	+	0	-21	-3	0	9	3	0	0	0	3	-3	0	1	1	-3	1	1	1	0	0	-6	-6	0	-1
χ_{40}	+	21	16	6	-9	6	0	3	-3	1	0	-2	0	-3	-3	1	1	-1	-1	0	0	7	3	0	0
χ_{41}	+	42	29	12	0	-12	0	0	-6	2	0	-4	0	1	1	1	-3	1	1	0	0	-1	-1	0	-3
χ_{42}	+	-14	-8	-2	-2	-6	-8	-6	-6	-2	0	-2	-4	0	0	0	0	0	0	4	1	4	4	0	0
χ_{43}	+	-56	12	4	10	0	10	-6	-8	0	-2	0	0	0	0	0	0	0	0	0	0	-7	-3	0	-4
χ_{44}	+	21	-4	-9	3	-9	6	3	5	5	2	-1	0	0	0	-4	0	0	0	0	0	10	-2	0	-4
χ_{45}	+	21	-5	-12	3	-4	0	-1	-3	1	0	4	0	-4	-4	0	0	0	0	-6	0	4	4	0	-21
χ_{46}	+	0	12	0	-12	8	0	-4	0	0	0	0	0	-8	8	0	0	0	0	-1	-1	0	0	1	0
χ_{47}	+	-21	-16	12	-9	-12	0	3	3	-1	0	-4	0	0	0	0	0	0	0	0	0	-1	3	-1	0

ind		12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	14A	B**	14C	15A
		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		4608	720	576	288	192	144	144	96	48	48	24	84	84	28	3240
p power		AB	EA	BC	FB	DG	IC	HA	FD	EE	IF	LH	AA	AA	AB	AD
p' part		AB	DA	AC	BB	AG	BC	CA	BD	DE	BF	CH	AA	AA	AB	AD
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	+	6	4	2	0	-2	2	1	-2	0	0	-1	-1	-1	1	-3
χ_{3}	+	-3	-1	1	3	1	1	2	1	-1	-1	0	2	2	0	-3
χ_{4}	+	5	5	5	-1	1	-1	-1	1	1	1	1	0	0	0	2
χ_{5}	+	2	-1	-2	-1	2	1	2	1	-1	-1	0	0	0	0	5
χ_{6}	+	2	0	-2	5	-2	1	0	1	0	1	0	1	1	1	2
χ_{7}	+	9	1	-3	0	1	0	-2	0	1	0	0	1	1	-1	-3
χ_{8}	+	6	0	2	0	2	2	0	2	0	0	0	0	0	0	6
χ_{9}	+	8	5	0	2	0	0	2	-2	1	0	0	0	0	-2	6
χ_{10}	+	-4	-5	-4	-1	0	-1	1	-1	-1	-1	-1	0	0	0	0
χ_{11}	+	7	6	-1	1	-1	-1	0	1	-2	-1	0	0	0	0	1
χ_{12}	+	-1	0	3	2	-1	0	3	-2	0	0	-1	0	0	0	6
χ_{13}	+	0	-4	0	0	0	0	2	0	0	0	0	0	0	0	-5
χ_{14}	+	0	-4	0	0	0	0	-1	0	0	0	-1	0	0	0	1
χ_{15}	+	-17	-4	-1	-2	-1	2	-1	0	0	0	1	-2	-2	0	3
χ_{16}	+	-6	-1	-2	-3	-2	1	-1	1	-1	1	-1	0	0	0	1
χ_{17}	+	-5	5	-5	-2	-1	-2	-1	0	1	0	-1	0	0	0	0
χ_{18}	+	4	4	4	-2	0	-2	-2	0	0	0	0	0	0	0	-9
χ_{19}	+	10	1	-2	1	2	1	-2	-1	1	-1	0	-1	-1	1	3
χ_{20}	+	4	0	-4	1	0	-1	-3	1	0	-1	1	-1	-1	1	6
χ_{21}	+	-9	-4	3	0	-1	0	-1	0	0	0	1	0	0	0	-7
χ_{22}	+	0	5	0	0	0	0	-1	0	1	0	1	-1	-1	-1	-2
χ_{23}	+	8	-1	0	2	0	0	2	2	-1	0	0	-2	-2	0	-9
χ_{24}	o	5	-3	1	2	1	-2	0	0	1	0	0	0	0	0	3b15+2
χ_{25}	o	5	-3	1	2	1	-2	0	0	1	0	0	0	0	0	**
χ_{26}	+	-3	-6	-3	0	-3	0	0	-2	2	2	0	0	0	0	-3
χ_{27}	+	6	0	2	0	2	2	0	-2	0	0	0	0	0	0	-6
χ_{28}	+	0	-1	0	0	0	0	-1	0	-1	0	1	0	0	0	-3
χ_{29}	+	-1	-4	-1	-1	-1	-1	-1	1	0	1	-1	0	0	0	-1
χ_{30}	+	0	-4	0	0	0	0	2	0	0	0	0	-1	-1	-1	3
χ_{31}	+	13	0	5	1	1	-1	0	1	0	-1	0	0	0	0	-6
χ_{32}	+	3	0	7	-3	-1	1	0	-1	0	-1	0	0	0	0	0
χ_{33}	o	7	3	-1	1	3	-1	0	-1	-1	1	0	0	0	0	3b15-2
χ_{34}	o	7	3	-1	1	3	-1	0	-1	-1	1	0	0	0	0	**
χ_{35}	+	-6	5	-2	-3	2	1	2	1	1	1	0	0	0	0	-10
χ_{36}	+	-2	-1	2	1	2	-1	-1	-1	-1	1	1	0	0	0	7
χ_{37}	+	8	0	0	-4	0	0	0	0	0	0	0	1	1	1	0
χ_{38}	+	-7	5	-3	2	1	0	2	-2	1	0	0	0	0	2	0
χ_{39}	+	-9	-1	3	0	3	0	-1	0	-1	0	-1	0	0	0	9
χ_{40}	+	0	-6	0	3	4	-3	0	1	2	-1	0	0	0	0	-3
χ_{41}	+	-3	0	-3	0	1	0	0	2	0	-2	0	0	0	0	6
χ_{42}	+	-16	0	0	2	0	0	0	-2	0	0	0	0	0	0	2
χ_{43}	+	-4	-4	4	2	0	-2	2	0	0	0	0	0	0	0	3
χ_{44}	+	-4	-1	-4	-1	0	-1	2	1	-1	1	0	0	0	0	-3
χ_{45}	+	11	0	-5	-1	-1	1	0	1	0	-1	0	0	0	0	2
χ_{46}	+	8	0	0	-4	0	0	0	0	0	0	0	0	0	0	8
χ_{47}	+	0	0	0	3	-4	-3	0	-1	0	1	0	0	0	0	6

ind		B**	15C	15D	15E	17A	B*	18A	20A	20B	21A	B**	21C	24A	24B	30A	B**	30C	D**	30E	30F	31A	B**	C*5	D**5	E**6	F*6
		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		3240	540	270	15	51	51	36	240	40	126	126	63	96	96	360	360	180	180	90	60	31	31	31	31	31	31
p power		AD	AA	AB	BC	A	A	AG	AA	BC	AB	AB	AA	AA	BB	AAE	BAE	CAA	CAA	DAC	CBB	A	A	A	A	A	A
p' part		AD	AA	AB	BC	A	A	AB	AA	AC	AB	AB	AA	AA	AB	AAE	BAE	CAA	CAA	DAC	CBB	A	A	A	A	A	A
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	+	-3	0	3	0	2	2	1	-1	1	1	1	1	0	0	1	1	-2	-2	1	0	0	0	0	0	0	0
χ_{3}	+	-3	3	0	1	-1	-1	0	2	0	1	1	-2	-1	-1	1	1	1	1	-2	-1	0	0	0	0	0	0
χ_{4}	+	2	5	-1	0	0	0	0	0	0	-2	-2	1	1	1	-2	-2	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_{5}	+	5	-1	2	0	1	1	0	-1	-1	0	0	0	2	-2	1	1	1	1	-2	-1	0	0	0	0	0	0
χ_{6}	+	2	-4	-1	0	0	0	-1	-3	-1	1	1	1	0	0	-2	-2	-2	-2	1	0	0	0	0	0	0	0
χ_{7}	+	-3	-6	0	0	0	0	1	4	0	-1	-1	-1	-1	-1	-3	-3	0	0	0	-2	0	0	0	0	0	0
χ_{8}	+	6	0	3	-1	0	0	0	0	0	2	2	-1	-2	2	2	2	2	2	-1	0	0	0	0	0	0	0
χ_{9}	+	6	3	0	0	0	0	0	3	-1	2	2	2	0	0	2	2	-1	-1	2	-1	0	0	0	0	0	0
χ_{10}	+	0	0	0	0	1	1	0	0	0	-2	-2	1	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	+	1	1	1	0	0	0	-2	-2	0	0	0	0	-1	-1	1	1	1	1	1	1	0	0	0	0	0	0
χ_{12}	+	6	-3	-3	0	-2	-2	0	0	0	0	0	0	1	1	2	2	-1	-1	-1	1	0	0	0	0	0	0
χ_{13}	+	-5	1	-2	0	-1	-1	1	-2	0	0	0	0	-2	2	-1	-1	-1	-1	2	1	0	0	0	0	0	0
χ_{14}	+	1	1	1	-1	0	0	0	1	1	0	0	0	2	-2	1	1	1	1	1	1	0	0	0	0	0	0
χ_{15}	+	3	3	3	0	0	0	0	-2	0	1	1	-2	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_{16}	+	1	-2	4	0	0	0	1	2	-2	0	0	0	0	0	-3	-3	0	0	0	2	0	0	0	0	0	0
χ_{17}	+	0	0	0	-1	0	0	-1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{18}	+	-9	3	-3	0	0	0	0	-1	-1	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_{19}	+	3	0	-3	0	0	0	0	1	-1	1	1	1	-2	2	-1	-1	2	2	-1	0	0	0	0	0	0	0
χ_{20}	+	6	-3	-3	0	0	0	0	0	0	1	1	1	2	-2	2	2	-1	-1	-1	1	0	0	0	0	0	0
χ_{21}	+	-7	2	2	0	0	0	-1	4	0	0	0	0	1	1	1	1	-2	-2	-2	2	0	0	0	0	0	0
χ_{22}	+	-2	-5	1	0	0	0	1	0	0	-1	-1	-1	0	0	2	2	-1	-1	-1	-1	0	0	0	0	0	0
χ_{23}	+	-9	3	-3	0	0	0	0	2	0	-1	-1	2	0	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_{24}	o	**	-1	2	0	0	0	-1	1	-1	0	0	0	-1	-1	**	b15-1	i15	-i15	0	1	0	0	0	0	0	0
χ_{25}	o	3b15+2	-1	2	0	0	0	-1	1	-1	0	0	0	-1	-1	b15-1	**	-i15	i15	0	1	0	0	0	0	0	0
χ_{26}	+	-3	3	0	0	0	0	0	-1	-1	0	0	0	1	1	1	1	1	1	-2	-1	0	0	0	0	0	0
χ_{27}	+	-6	3	3	0	0	0	-1	0	0	0	0	0	2	-2	-2	-2	1	1	1	-1	0	0	0	0	0	0
χ_{28}	+	-3	0	3	0	0	0	-1	-1	1	0	0	0	-2	2	1	1	-2	-2	1	0	0	0	0	0	0	0
χ_{29}	+	-1	2	-4	0	0	0	0	-2	2	0	0	0	-1	-1	3	3	0	0	0	-2	0	0	0	0	0	0
χ_{30}	+	3	-3	0	1	0	0	0	-2	0	-1	-1	-1	2	-2	-1	-1	-1	-1	2	1	0	0	0	0	0	0
χ_{31}	+	-6	-3	0	0	0	0	0	-3	1	0	0	0	1	1	-2	-2	1	1	-2	1	0	0	0	0	0	0
χ_{32}	+	0	0	0	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	o	**	1	1	0	0	0	1	2	0	0	0	0	-1	-1	3b15+2	**	-1	-1	-1	1	0	0	0	0	0	0
χ_{34}	o	3b15-2	1	1	0	0	0	1	2	0	0	0	0	-1	-1	**	3b15+2	-1	-1	-1	1	0	0	0	0	0	0
χ_{35}	+	-10	-4	2	0	0	0	-1	0	0	0	0	0	0	0	2	2	2	2	2	0	0	0	0	0	0	0
χ_{36}	+	7	-2	-2	0	0	0	0	-4	0	0	0	0	0	0	-1	-1	2	2	2	-2	0	0	0	0	0	0
χ_{37}	+	0	0	0	0	-1	-1	0	0	0	1	1	1	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	+	0	0	0	0	0	0	0	0	0	-2	-2	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	+	9	0	0	0	0	0	0	2	2	0	0	0	1	1	-3	-3	0	0	0	0	0	0	0	0	0	0
χ_{40}	+	-3	0	3	0	0	0	0	-1	1	0	0	0	0	0	1	1	-2	-2	1	0	0	0	0	0	0	0
χ_{41}	+	6	3	0	0	0	0	0	3	-1	0	0	0	1	1	2	2	-1	-1	2	-1	0	0	0	0	0	0
χ_{42}	+	2	5	-1	0	0	0	0	0	0	2	2	-1	0	0	-2	-2	1	1	1	1	0	0	0	0	0	0
χ_{43}	+	3	0	-3	0	2	2	0	1	-1	0	0	0	0	0	-1	-1	2	2	-1	0	0	0	0	0	0	0
χ_{44}	+	-3	-3	-3	0	0	0	0	2	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
χ_{45}	+	2	5	-1	0	0	0	2	0	0	0	0	0	-1	-1	-2	-2	1	1	1	1	0	0	0	0	0	0
χ_{46}	+	8	-4	2	1	0	0	-1	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	+	6	0	3	0	0	0	0	3	1	0	0	0	0	0	2	2	2	2	-1	0	0	0	0	0	0	0

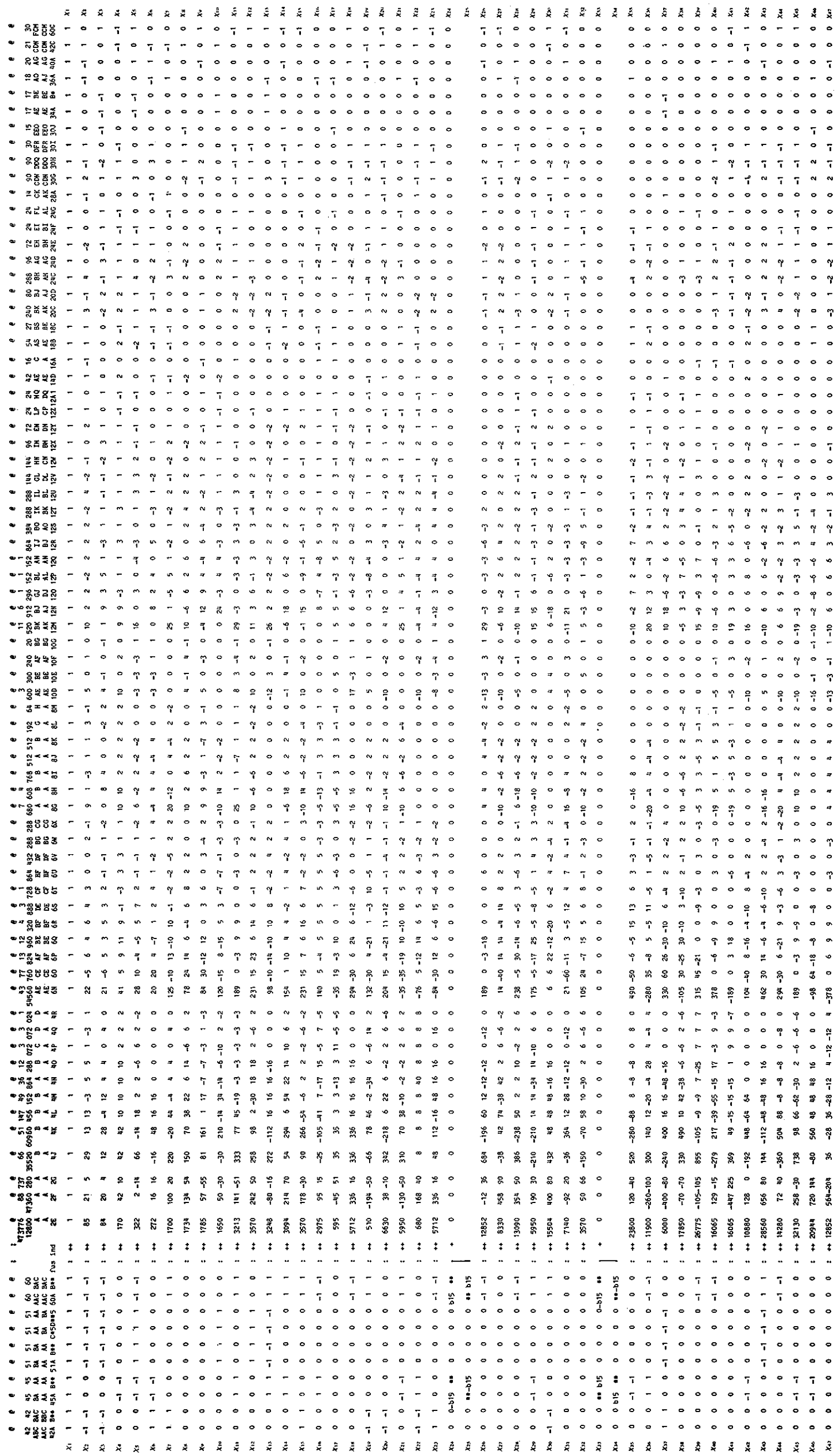
		1A	2A	2B	2C	2D	3A	3B	3C	3D	4A	4B	4C	4D	4E	4F	4G	4H	4I	5A	5B
χ_{48}	+	238809	441	-567	345	-279	0	0	81	0	273	-63	81	9	-15	9	21	5	-3	-81	4
χ_{49}	+	316200	3880	4200	40	104	2550	-15	-15	30	-120	-56	40	8	-24	8	-8	-8	-8	0	0
χ_{50}	+	332010	-3990	1722	330	26	3213	63	0	18	-222	18	-30	-22	-14	10	22	6	-2	30	0
χ_{51}	o	332010	1386	1722	-630	90	-3213	0	0	-9	162	18	-30	-6	2	-6	-10	6	-2	30	0
χ_{52}	o	332010	1386	1722	-630	90	-3213	0	0	-9	162	18	-30	-6	2	-6	-10	6	-2	30	0
χ_{53}	+	337280	1408	2176	-640	128	-2584	20	-10	-22	256	0	0	0	0	0	0	0	0	-60	0
χ_{54}	+	348160	4096	4096	0	0	5440	-104	40	-32	0	0	0	0	0	0	0	0	0	-20	0
χ_{55}	+	355725	6285	2925	-435	-339	2295	90	0	0	-195	29	5	-19	-3	5	5	-3	5	0	0
χ_{56}	+	355725	-5235	2925	525	-147	2295	90	0	0	-195	29	5	-35	45	-11	5	-3	5	0	0
χ_{57}	o	355725	525	-3555	45	189	2295	90	0	0	165	-43	5	-3	-27	-3	-7	9	1	0	0
χ_{58}	o	355725	525	-3555	45	189	2295	90	0	0	165	-43	5	-3	-27	-3	-7	9	1	0	0
χ_{59}	+	368900	1540	-1820	580	164	1190	140	65	35	-140	-172	-20	4	20	12	-4	4	-4	0	0
χ_{60}	+	439425	-4095	945	225	-111	5670	0	0	0	105	57	-15	-15	9	9	-11	-3	-3	0	0
χ_{61}	+	439425	-4095	945	225	-111	5670	0	0	0	105	57	-15	-15	9	9	-11	-3	-3	0	0
χ_{62}	o	439425	-4095	945	225	-111	-2835	0	0	0	105	57	-15	-15	9	9	-11	-3	-3	0	0
χ_{63}	o	439425	-4095	945	225	-111	-2835	0	0	0	105	57	-15	-15	9	9	-11	-3	-3	0	0
χ_{64}	o	439425	-4095	945	225	-111	-2835	0	0	0	105	57	-15	-15	9	9	-11	-3	-3	0	0
χ_{65}	o	439425	-4095	945	225	-111	-2835	0	0	0	105	57	-15	-15	9	9	-11	-3	-3	0	0
χ_{66}	+	442680	3640	-4200	-136	-40	3570	105	6	15	200	-24	-40	-8	40	8	16	0	0	0	-5
χ_{67}	+	442680	7224	-2856	760	-104	1428	105	-30	-39	-152	-56	-56	-8	8	-8	0	0	-16	15	0
χ_{68}	o	442680	-3528	280	120	88	-2142	21	0	-30	-120	40	-40	-8	-24	8	16	0	0	40	0
χ_{69}	o	442680	-3528	280	120	88	-2142	21	0	-30	-120	40	-40	-8	-24	8	16	0	0	40	0
χ_{70}	o	498015	-1953	-1953	-225	159	0	0	0	27	-129	63	-9	15	-17	-9	15	-9	-1	45	0
χ_{71}	o	498015	-1953	-1953	-225	159	0	0	0	27	-129	63	-9	15	-17	-9	15	-9	-1	45	0
χ_{72}	+	505920	3136	-3264	320	64	4692	30	-60	-6	192	-64	-64	0	0	0	0	0	0	-15	0
χ_{73}	+	539648	7168	0	1024	0	-2176	-184	20	8	0	0	0	0	0	0	0	0	0	88	3
χ_{74}	+	590240	-4704	3360	160	32	1904	98	50	-25	-160	-32	0	32	-32	0	0	0	0	-80	0
χ_{75}	+	634880	-4096	4096	0	0	-832	8	-40	-16	0	0	0	0	0	0	0	0	0	-60	0
χ_{76}	+	664020	-2604	756	-300	-12	5355	0	-45	36	68	36	-12	36	-12	-12	12	-4	-4	-90	0
χ_{77}	+	664020	-2604	3444	-300	116	-3213	-63	0	36	-60	36	-60	-28	-12	4	12	12	-4	60	0
χ_{78}	+	708288	-1344	448	-576	192	4284	-42	0	-48	-192	64	-64	0	0	0	0	0	0	-17	3
χ_{79}	o	711450	6810	-630	-390	-150	-2295	45	0	0	-30	-14	10	-22	-30	2	-2	6	6	0	0
	o	711450	6810	-630	-390	-150	-2295	45	0	0	-30	-14	10	-22	-30	2	-2	6	6	0	0
χ_{81}	o	711450	-4710	-630	570	42	-2295	45	0	0	-30	-14	10	-38	18	-14	-2	6	6	0	0
χ_{82}	o	711450	-4710	-630	570	42	-2295	45	0	0	-30	-14	10	-38	18	-14	-2	6	6	0	0
χ_{83}	o	722925	-2835	-2835	45	45	0	0	0	0	45	45	45	-3	-3	-3	-3	-3	-3	0	0
χ_{84}	o	722925	-2835	-2835	45	45	0	0	0	0	45	45	45	-3	-3	-3	-3	-3	-3	0	0
χ_{85}	o	722925	-2835	-2835	45	45	0	0	0	0	45	45	45	-3	-3	-3	-3	-3	-3	0	0
χ_{86}	o	722925	-2835	-2835	45	45	0	0	0	0	45	45	45	-3	-3	-3	-3	-3	-3	0	0
χ_{87}	o	722925	-2835	-2835	45	45	0	0	0	0	45	45	45	-3	-3	-3	-3	-3	-3	0	0
χ_{88}	o	722925	-2835	-2835	45	45	0	0	0	0	45	45	45	-3	-3	-3	-3	-3	-3	0	0
χ_{89}	+	761856	8192	0	0	0	-1920	-48	24	24	0	0	0	0	0	0	0	0	0	-24	-4
χ_{90}	+	830025	-3255	3465	-375	-183	-5355	0	45	45	265	9	-15	57	-7	-15	9	1	-7	0	0
χ_{91}	+	885360	7280	2352	240	176	-5712	-42	-15	3	208	-112	0	-16	16	0	0	-16	0	30	0
	+	1011840	-1920	2688	640	128	-408	-48	-30	42	-128	-128	0	0	0	0	0	0	0	-30	0
χ_{93}	+	1048576	0	0	0	0	4096	-128	64	-32	0	0	0	0	0	0	0	0	0	-64	-4
χ_{94}	+	1180480	-2240	2240	64	-64	-4760	70	-44	-50	0	-128	0	64	0	0	0	0	0	0	-5
χ_{95}	+	1195236	1764	-2268	-924	36	0	0	81	0	-348	36	-36	36	36	12	-12	-4	4	81	1
χ_{96}	+	1422900	2100	-1260	180	-108	2295	-180	0	0	-60	-28	20	-60	-12	-12	-4	12	12	0	0
χ_{97}	+	1494045	2205	-2835	-579	-243	0	0	-81	0	-75	-27	45	45	21	21	9	1	1	0	5

	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	6L	6M	7A	8A	8B	8C	8D	8E	8F	9A	9B
χ_{48}	0	0	0	0	0	0	0	9	0	0	0	-3	0	0	-3	-3	1	-3	-1	-1	0	0
χ_{49}	-170	30	55	-10	10	1	6	1	9	7	-1	1	2	-4	0	0	0	0	0	0	0	0
χ_{50}	-51	-51	-21	29	-6	-9	-6	0	3	3	-1	0	2	0	-2	-2	-2	2	0	0	0	0
χ_{51}	-45	-21	0	3	-9	0	3	0	0	0	0	0	3	0	-2	-2	2	-2	0	0	0	0
χ_{52}	-45	-21	0	3	-9	0	3	0	0	0	0	0	3	0	-2	-2	2	-2	0	0	0	0
χ_{53}	40	-8	-20	8	-2	4	-2	-2	4	-4	-4	2	2	-1	0	0	0	0	0	0	5	-1
χ_{54}	64	64	64	0	-8	-8	-8	-8	-8	0	0	0	0	1	0	0	0	0	0	0	1	1
χ_{55}	-105	15	90	-9	0	-6	0	0	-6	-6	-6	0	0	-1	1	1	5	1	1	1	0	0
χ_{56}	-105	15	-90	-9	0	-6	0	0	-6	6	6	0	0	-1	1	1	1	-3	1	1	0	0
χ_{57}	-105	15	0	-9	0	-6	0	0	-6	0	0	0	0	-1	1	1	-3	1	-1	-1	0	0
χ_{58}	-105	15	0	-9	0	-6	0	0	-6	0	0	0	0	-1	1	1	-3	1	-1	-1	0	0
χ_{59}	70	-50	70	-10	-5	4	7	1	-8	-2	2	1	-1	0	0	0	4	0	0	0	5	-1
χ_{60}	-90	-90	0	6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0
χ_{61}	-90	-90	0	6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0
χ_{62}	45	45	0	-3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0
χ_{63}	45	45	0	-3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0
χ_{64}	45	45	0	-3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0
χ_{65}	45	45	0	-3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0
χ_{66}	-110	90	-35	2	-5	1	3	-2	-3	5	-7	2	-1	0	0	0	0	0	0	0	0	0
χ_{67}	84	84	-63	4	-3	9	-3	-6	9	1	1	-2	1	0	0	0	0	0	0	0	0	0
χ_{68}	66	10	21	-14	6	-3	10	0	1	-3	1	0	-2	0	0	0	0	0	0	0	-3	0
χ_{69}	66	10	21	-14	6	-3	10	0	1	-3	1	0	-2	0	0	0	0	0	0	0	-3	0
χ_{70}	0	0	0	0	-9	0	-9	0	0	0	0	0	3	0	3	3	3	-1	-1	-1	0	0
χ_{71}	0	0	0	0	-9	0	-9	0	0	0	0	0	3	0	3	3	3	-1	-1	-1	0	0
χ_{72}	-44	36	28	4	-2	-2	-6	4	6	-4	4	-4	-2	2	0	0	0	0	0	0	0	0
χ_{73}	-128	0	-56	0	-8	-8	0	4	0	-8	0	4	0	4	0	0	0	0	0	0	-4	-1
χ_{74}	48	0	-42	-16	3	-6	3	-6	6	-2	2	-2	-1	0	0	0	0	0	0	0	-4	-1
χ_{75}	-64	64	-64	0	8	8	-8	8	-8	0	0	0	0	1	0	0	0	0	0	0	5	-1
χ_{76}	75	-45	0	-21	-12	0	0	3	0	0	0	3	0	0	4	4	0	0	0	0	0	0
χ_{77}	-141	51	-21	-13	12	9	-12	0	-3	3	-1	0	-4	0	-4	-4	0	0	0	0	0	0
χ_{78}	60	-20	0	12	0	6	16	0	-2	0	0	0	0	0	0	0	0	0	0	0	6	0
χ_{79}	105	-15	-45	9	0	-3	0	0	-3	3	3	0	0	-2	2	2	2	2	0	0	0	0
	105	-15	-45	9	0	-3	0	0	-3	3	3	0	0	-2	2	2	2	2	0	0	0	0
χ_{81}	105	-15	45	9	0	-3	0	0	-3	-3	-3	0	0	-2	2	2	-2	-2	0	0	0	0
χ_{82}	105	-15	45	9	0	-3	0	0	-3	-3	-3	0	0	-2	2	2	-2	-2	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1	1	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1	1	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1	1	0	0
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1	1	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1	1	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	1	1	1	0	0
χ_{89}	128	0	-64	0	8	-16	0	8	0	0	0	0	0	4	0	0	0	0	0	0	0	0
χ_{90}	-75	45	0	21	-15	0	9	-3	0	0	0	-3	-3	0	5	5	-3	1	1	1	0	0
χ_{91}	-16	-96	14	-16	-1	14	3	-7	-6	6	2	-3	-1	0	0	0	0	0	0	0	0	0
	-24	-24	36	8	6	0	6	-6	0	4	-4	-2	2	4	0	0	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	4	1
χ_{94}	40	-40	70	8	10	-2	-10	4	2	-2	2	4	2	0	0	0	0	0	0	0	-5	1
χ_{95}	0	0	0	0	0	0	0	9	0	0	0	-3	0	0	0	0	-4	0	0	0	0	0
χ_{96}	-105	15	0	-9	0	12	0	0	12	0	0	0	0	-4	4	4	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	-9	0	0	0	3	0	0	-3	-3	-3	-3	-1	-1	0	0

	10A	10B	10C	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	14A	B**	14C	15A
χ_{48}	-9	3	0	0	0	0	0	0	0	0	-3	0	0	0	-1	0	0	0	0
χ_{49}	0	0	0	-6	-14	0	-2	1	-2	1	-3	-1	0	-1	1	2	2	0	0
χ_{50}	-10	2	0	-3	-3	6	-3	3	1	-3	0	-1	-2	1	0	0	0	0	3
χ_{51}	6	2	0	15	-9	3	3	0	-1	0	0	0	-1	0	0	0	0	0	**
χ_{52}	6	2	0	15	-9	3	3	0	-1	0	0	0	-1	0	0	0	0	0	9b15+3
χ_{53}	-12	-4	0	16	0	4	0	0	0	0	-2	0	0	0	0	1	1	-1	3
χ_{54}	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-2
χ_{55}	0	0	0	3	11	0	-1	2	-1	2	0	2	0	2	0	-1	-1	-1	0
χ_{56}	0	0	0	3	11	0	-1	2	-1	2	0	-2	0	-2	0	1	1	-1	0
χ_{57}	0	0	0	3	11	0	-1	2	-1	2	0	0	0	0	0	i7	-i7	1	0
χ_{58}	0	0	0	3	11	0	-1	2	-1	2	0	0	0	0	0	-i7	i7	1	0
χ_{59}	0	0	0	10	2	-5	-2	-4	2	-2	1	-2	-1	0	1	0	0	0	0
χ_{60}	0	0	0	6	6	0	6	0	-2	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	6	6	0	6	0	-2	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	-3	-3	0	-3	0	1	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	-3	-3	0	-3	0	1	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	-3	-3	0	-3	0	1	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	-3	-3	0	-3	0	1	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	-1	-10	-18	5	2	-3	-2	-1	2	1	1	-1	0	0	0	0	0
χ_{67}	-1	-1	0	4	4	-5	4	1	0	1	-2	1	-1	1	0	0	0	0	6
χ_{68}	-8	0	0	6	-2	0	2	1	-2	-1	0	1	0	-1	0	0	0	0	-3i15-5
χ_{69}	-8	0	0	6	-2	0	2	1	-2	-1	0	1	0	-1	0	0	0	0	3i15-5
χ_{70}	-3	-3	0	0	0	-3	0	0	0	0	0	0	1	0	0	0	0	0	-9b15
χ_{71}	-3	-3	0	0	0	-3	0	0	0	0	0	0	1	0	0	0	0	0	**
χ_{72}	1	1	0	12	-4	0	-4	2	0	2	0	0	0	0	0	0	0	-2	-6
χ_{73}	8	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2
χ_{74}	16	0	0	8	-8	5	0	-2	0	0	2	2	1	0	0	0	0	0	10
χ_{75}	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	-6
χ_{76}	6	6	0	-1	-9	-4	3	0	3	0	-1	0	0	0	-1	0	0	0	-9
χ_{77}	-4	4	0	3	3	0	3	-3	3	3	0	-1	0	1	0	0	0	0	6
χ_{78}	-9	3	-1	-12	4	0	-4	-2	0	2	0	0	0	0	0	0	0	0	-8
χ_{79}	0	0	0	-3	-11	0	1	1	1	1	0	-1	0	-1	0	2b7	**	0	0
	0	0	0	-3	-11	0	1	1	1	1	0	-1	0	-1	0	**	2b7	0	0
χ_{81}	0	0	0	-3	-11	0	1	1	1	1	0	1	0	1	0	-2b7	**	0	0
χ_{82}	0	0	0	-3	-11	0	1	1	1	1	0	1	0	1	0	**	-2b7	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{89}	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	-6
χ_{90}	0	0	0	1	9	-5	-3	0	-3	0	1	0	-1	0	1	0	0	0	0
χ_{91}	-10	2	0	-8	8	1	0	2	0	0	1	2	1	0	-1	0	0	0	3
	10	-2	0	-8	-8	4	0	4	0	0	-2	0	0	0	0	-2	-2	0	-3
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8
χ_{94}	0	0	-1	0	16	0	0	-2	0	0	0	-2	0	0	0	0	0	0	0
χ_{95}	9	-3	1	0	0	0	0	0	0	0	-3	0	0	0	-1	0	0	0	0
χ_{96}	0	0	0	3	11	0	-1	-4	-1	-4	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	1	0	0	0	0	0	0	0	3	0	0	0	1	0	0	0	0

	B**	15C	15D	15E	17A	B*	18A	20A	20B	21A	B**	21C	24A	24B	30A	B**	30C	D**	30E	30F	31A	B**	C*5	D**5	E**6	F*6
χ_{48}	0	0	0	1	0	0	0	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	-1	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	3	3	3	0	0	0	0	-2	0	0	0	0	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
χ_{51}	9b15+3	-3	0	0	0	0	0	2	0	0	0	0	1	1	**	b15-1	i15	-i15	0	-1	0	0	0	0	0	0
χ_{52}	**	-3	0	0	0	0	0	2	0	0	0	0	1	1	b15-1	**	-i15	i15	0	-1	0	0	0	0	0	0
χ_{53}	3	6	0	0	0	0	1	-4	0	-1	-1	-1	0	0	3	3	0	0	0	2	0	0	0	0	0	0
χ_{54}	-2	-5	1	0	0	0	1	0	0	1	1	1	0	0	2	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{55}	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	-b17	*	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	*	-b17	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	-b17	*	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	-b17	*	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	*	-b17	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	*	-b17	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	6	3	0	0	0	0	0	3	-1	0	0	0	0	0	2	2	-1	-1	2	-1	0	0	0	0	0	0
χ_{68}	3i15-5	-2	1	0	0	0	1	0	0	0	0	0	0	0	-2b15	**	-2b15	**	1	0	0	0	0	0	0	0
χ_{69}	-3i15-5	-2	1	0	0	0	1	0	0	0	0	0	0	0	**	-2b15	**	-2b15	1	0	0	0	0	0	0	0
χ_{70}	**	0	0	0	0	0	0	1	1	0	0	0	0	0	3b15	**	0	0	0	0	0	0	0	0	0	0
χ_{71}	-9b15	0	0	0	0	0	0	1	1	0	0	0	0	0	**	3b15	0	0	0	0	0	0	0	0	0	0
χ_{72}	-6	-3	0	0	0	0	0	-3	1	2	2	2	0	0	-2	-2	1	1	-2	1	0	0	0	0	0	0
χ_{73}	-2	4	1	0	0	0	0	0	0	-2	-2	1	0	0	2	2	2	2	-1	0	0	0	0	0	0	0
χ_{74}	10	4	-2	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	-2	-2	0	0	0	0	0	0	0
χ_{75}	-6	3	3	0	-2	-2	1	0	0	1	1	1	0	0	-2	-2	1	1	1	-1	0	0	0	0	0	0
χ_{76}	-9	0	0	0	0	0	0	-2	-2	0	0	0	1	1	3	3	0	0	0	0	0	0	0	0	0	0
χ_{77}	6	-3	-3	0	0	0	0	0	0	0	0	0	-1	-1	2	2	-1	-1	-1	1	0	0	0	0	0	0
χ_{78}	-8	4	-2	0	0	0	-2	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	**	3b7+1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	3b7+1	**	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	**	3b7+1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	0	3b7+1	**	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	f31	**	*5	**5	**6	*6
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	f31	**5	*5	*6	**6
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**6	*6	f31	**	*5	**5
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*6	**6	**	f31	**5	*5
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*5	**5	**6	*6	f31	**
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**5	*5	*6	**6	**	f31
χ_{89}	-6	0	-3	-1	1	1	0	0	0	1	1	-2	0	0	-2	-2	-2	-2	1	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	3	3	3	0	0	0	0	-2	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
	-3	-3	-3	0	0	0	0	2	0	1	1	-2	0	0	1	1	1	1	1	1	0	0	0	0	0	0
χ_{93}	8	-4	2	-1	-1	-1	0	0	0	-2	-2	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1
χ_{94}	0	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	1	0	0	0	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	2	2	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$O_{10}^{+}(2)$



$O_{10}^{+}(2)$

	42A	8**	45A	8**	51A	8**	C45D**	5	60A	8**		2E	2F	2G	4J	4K	4L	4M	4N	4O	4P	4Q	4R
X_{66}	0	0	0	0	0	0	0	0	0	0	:	++	9639	-153	-105	-81	63	63	15	39	-9	-13	-9
X_{67}	-1	-1	0	0	0	0	0	0	0	0	:	++	35700	500	20	-60	420	-92	4	-12	-12	-4	-12
X_{68}	0	0	0	0	0	0	0	0	1	1	:	++	32130	-318	210	90	-406	-54	26	-30	2	6	-6
X_{69}	0	0	0	0	0	0	0	0	0-b15	##		+	0	0	0	0	0	0	0	0	0	0	0
X_{70}																							
X_{71}	0	0	0	0	0	0	0	0	##-b15														
X_{72}	1	1	0	0	0	0	0	0	-1	-1	:	++	13600	800	160	-160	-160	-32	-32	32	-32	0	0
X_{73}	1	1	0	0	0	0	0	0	0	0	:	++	43520	512	0	512	512	0	0	0	0	0	0
X_{74}	-1	-1	0	0	0	0	0	0	0	0	:	++	34425	-135	-135	-135	825	-39	-39	9	-23	9	9
X_{75}	1	1	0	0	0	0	0	0	0	0	:	++	34425	-135	-135	-135	-615	-39	57	9	25	9	-15
X_{76}	1	1	0	0	0	0	0	0	0	0	:	++	34425	-135	-135	-135	-615	-39	57	9	25	9	-15
X_{77}	17	-17	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{78}	-17	17	0	0	0	0	0	0	0	0													
X_{79}	0	0	0	0	0	0	0	0	0	0	:	++	11900	-260	220	-20	140	44	12	-20	-20	4	-4
X_{80}	0	0	0	0	0-b17-b17		#	0	0	0	:	++	42525	-675	45	405	-315	-27	21	45	-3	-15	-3
X_{81}	0	0	0	0	#	#-b17-b17		0	0	0	:	++	42525	-675	45	405	-315	-27	21	45	-3	-15	-3
X_{82}	0	0	0	0	#5	##5-s51	##	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{83}	0	0	0	0	0	##5	#5	##-s51		0													
X_{84}	0	0	0	0	0	##-s51	#5	##5	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{85}	0	0	0	0	0-s51	##	##5	#5	0	0													
X_{86}	0	0	0	0	0	0	0	0	0	0	:	++	14280	-440	104	120	280	-40	-8	8	8	0	-8
X_{87}	0	0	0	0	0	0	0	0	0	0	:	++	7140	228	-220	84	84	20	20	-28	20	-12	4
X_{88}	0	0	##	b15	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{89}	0	3	b15	##	0	0	0	0	0	0													
X_{90}	0	3	0	0	0	0	0	0	0-b15	##		+	0	0	0	0	0	0	0	0	0	0	0
X_{91}	0	0	0	0	0	0	0	0	##	b15													
X_{92}	0	0	0	0	0	0	0	0	0	0	:	++	28560	-368	80	336	336	16	16	-48	16	0	0
X_{93}	0	1	1	0	0	0	0	0	0	0	:	++	4352	256	256	0	0	0	0	0	0	0	0
X_{94}	0	0	1	1	0	0	0	0	0	0	:	++	47600	240	-80	-240	-560	80	16	-16	-16	0	16
X_{95}	-1	-1	0	0	1	1	1	0	0	0	:	++	43520	512	0	-512	-512	0	0	0	0	0	0
X_{96}	0	0	0	0	0	0	0	1	1	1	:	++	53550	-210	-210	270	-210	78	-18	14	-2	14	6
X_{97}	0	0	0	0	0	0	0	0	0	0	:	++	32130	258	-30	-558	-238	-78	-14	-30	18	-6	-6
X_{98}	0	0	1	1	0	0	0	0	0	0	:	++	51408	-48	144	144	-112	-48	-48	-48	16	0	0
X_{99}	-b7	##	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{100}	##	-b7	0	0	0	0	0	0	0	0													
X_{101}	b7	##	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{102}	##	b7	0	0	0	0	0	0	0	0													
X_{103}	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0
X_{104}	0	0	0	0	0	0	0	0	0	0													
X_{105}	0	0	0	0	0	0	0	0	0	0	:	++	21504	1024	0	0	0	0	0	0	0	0	0
X_{106}	0	0	0	0	0	0	0	0	0	0	:	++	26775	-105	-105	-585	-105	-9	7	23	7	-1	-9
X_{107}	0	0	0	0	0	0	0	0	0	0	:	++	14280	712	-120	-120	-56	-56	8	-24	-8	8	-8
X_{108}	0	0	0	0	0	0	0	0	0	0	:	++	57120	288	160	-480	-224	32	32	-32	-32	0	0
X_{109}	0	0	0	0	0	0	0	0	0	0	:	++	55536	0	0	0	0	0	0	0	0	0	0
X_{110}	0	0	0	0	0	0	0	0	0	0	:	++	38080	-320	64	-640	0	128	0	0	0	0	0
X_{111}	0	0	0	0	0	0	0	0	0	0	:	++	38556	-612	156	-324	252	-36	-36	12	12	-4	12
X_{112}	0	0	0	0	0	0	0	0	0	0	:	++	68850	-270	-270	-270	210	-78	18	18	2	18	-6
X_{113}	0	0	0	0	0	0	0	0	0	0	:	++	48195	-765	51	-405	315	27	-21	51	3	-17	3

$O_{10}^{+}(2)$

Orthogonal group $O_{10}^{+}(2) \cong D_5(2)$

Order = 23,499,295,948,800 = $2^{20}.3^5.5^2.7.17.31$ Mult = 1 Out = 2

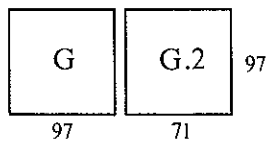
Constructions

Orthogonal $GO_{10}^{+}(2) \cong PGO_{10}^{+}(2) \cong SO_{10}^{+}(2) \cong PSO_{10}^{+}(2) \cong G.2$: all 10 x 10 matrices over $\mathbb{F}_2$ preserving a non-singular quadratic form of Witt defect 0, for example $x_1x_2+x_3x_4+x_5x_6+x_7x_8+x_9x_{10}$; $O_{10}^{+}(2) \cong G$

Remark : There is a maximal subgroup $2^{10+16}O_{10}^{+}(2)$ in the Monster group F_1.

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Orthogonal
47377612800	496	$S_8(2)$	: $S_8(2)$ x 2	1a+155a+340a	C(2E)	non-isotropic point
44590694400	527	$2^8:O_8^{+}(2)$	: $2^8:O_8^{+}(2):2$	1a+186a+340a	$N(2^8)$	isotropic point
10239344640	2295	$2^{10}:L_5(2)$	$L_5(2):2$,	1a+186a+2108a	$N(2^{10})$	maximal isotropic subspace
10239344640	2295	$2^{10}:L_5(2)$	$[2^{15}]:A_8$	1a+186a+2108a	$N(2^{10})$	maximal isotropic subspace
1184440320	19840	$(3 \times O_8^{-}(2)):2$	: $S_3 \times O_8^{-}(2):2$		N(3A)	minus line
990904320	23715	$2_{+}^{1+12}:(S_3 x A_8)$	: $2_{+}^{1+12}:(S_3 x S_8)$		N(2A)	isotropic line
198180864	118575	$2^{3+12}:(S_3 x S_3 x L_3(2))$	: $2^{3+12}:(3^2:D_8 x L_3(2))$		$N(2A^3)$	isotropic plane
3110400	7555072	$(A_5 x U_4(2)):2$	: $S_5 \times U_4(2):2$		N(2B,3A,5A)	$O_4^{-}(2) \times O_6^{-}(2)$
1451520	16189440	$(S_3 x S_3 x A_8):2$	: $(S_3 x S_3):2 \times S_8$		$N(3^2) = N(3A_2B_2)$	$O_4^{+}(2) \times O_6^{+}(2)$



$O_{10}^{-}(2)$

Orthogonal group $O_{10}^{-}(2) \cong {}^2D_5(2)$

Order = 25,015,379,558,400 = $2^{20}.3^6.5^2.7.11.17$ Mult = 1 Out = 2

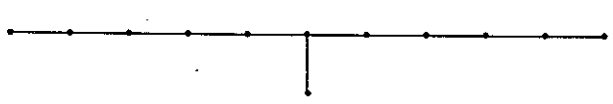
Constructions

Orthogonal $GO_{10}^{-}(2) \cong PGO_{10}^{-}(2) \cong SO_{10}^{-}(2) \cong PSO_{10}^{-}(2) \cong G.2$: all 10 x 10 matrices over $\mathbf{F}_2$ preserving a quadratic form of Witt defect 1, for example $x_1^2+x_1x_2+x_2^2+x_3x_4+x_5x_6+x_7x_8+x_9x_{10}$; $O_{10}^{-}(2) \cong G$.

By taking the 10-space as all even weight length 12 vectors over $\mathbf{F}_2$ modulo complementation, we see the inclusion of S_{12} in G.2. The reflections (a class of 3-transpositions) are the maps $S \longrightarrow S + [S,T]T$ where T is a 2-set or a 6-set, and [S,T] is the size of the intersection of S and T. (The 2-sets yield the transpositions of S_{12}.) There are 6 orbits of S_{12} on its conjugates, with intersections:

S_{12}, S_6 wreath M_2, S_4 wreath M_3, S_3 wreath M_4, S_2 wreath M_6, and M_{12},

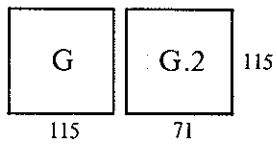
where $M_2 = S_2$, $M_3 = S_3$, $M_4 = A_4$, $M_6 = PGL_2(5)$ and M_{12} is the Mathieu group (see M_{12}, "Shuffle").

"Presentation" G.2 ≅ <  | more relations > ; see page 232.

Remark : G is a maximal subgroup of the Fischer group Fi_{24}^{1}.

Maximal subgroups

Order	Index	Structure	G.2	Character	Specifications Abstract	Specifications Orthogonal
50536120320	495	$2^8{:}O_8^{-}(2)$	: $2^8{:}O_8^{-}(2){:}2$	1a+154a+340a	$N(2^8)$	isotropic point
47377612800	528	$S_8(2)$	: $S_8(2)$ x 2	1a+187a+340a	C(2E)	non-isotropic point
1274019840	19635	$2_+^{1+12}{:}(S_3xU_4(2))$	: $2_+^{1+12}{:}(S_3xU_4(2){:}2)$		N(2A)	isotropic line
1045094400	23936	$(3 \times O_8^{+}(2)){:}2$	: $S_3 \times O_8^{+}(2){:}2$		N(3A)	minus line
990904320	25245	$2^{6+8}{:}(A_8x3)$	: $2^{6+8}{:}(A_8xS_3)$		$N(2^6)$	maximal isotropic subspace
330301440	75735	$2^{3+12}{:}(L_3(2)xA_5)$	: $2^{3+12}{:}(L_3(2)xS_5)$		$N(2^3)$	isotropic plane
239500800	104448	A_{12}	: S_{12}			
41057280	609280	$3 \times U_5(2)$	: $(3 \times U_5(2)){:}2$		N(3BC)	$U_5(2)$
2419200	10340352	$(A_5 \times A_8){:}2$	: $S_5 \times S_8$		N(2B,3A,5A)	$O_4^{-}(2)xO_6^{+}(2)$
1866240	13404160	$(S_3xS_3xU_4(2)){:}2$	: $(S_3xS_3){:}2 \times U_4(2){:}2$		$N(3A_2D_2)$	$O_4^{+}(2)xO_6^{-}(2)$



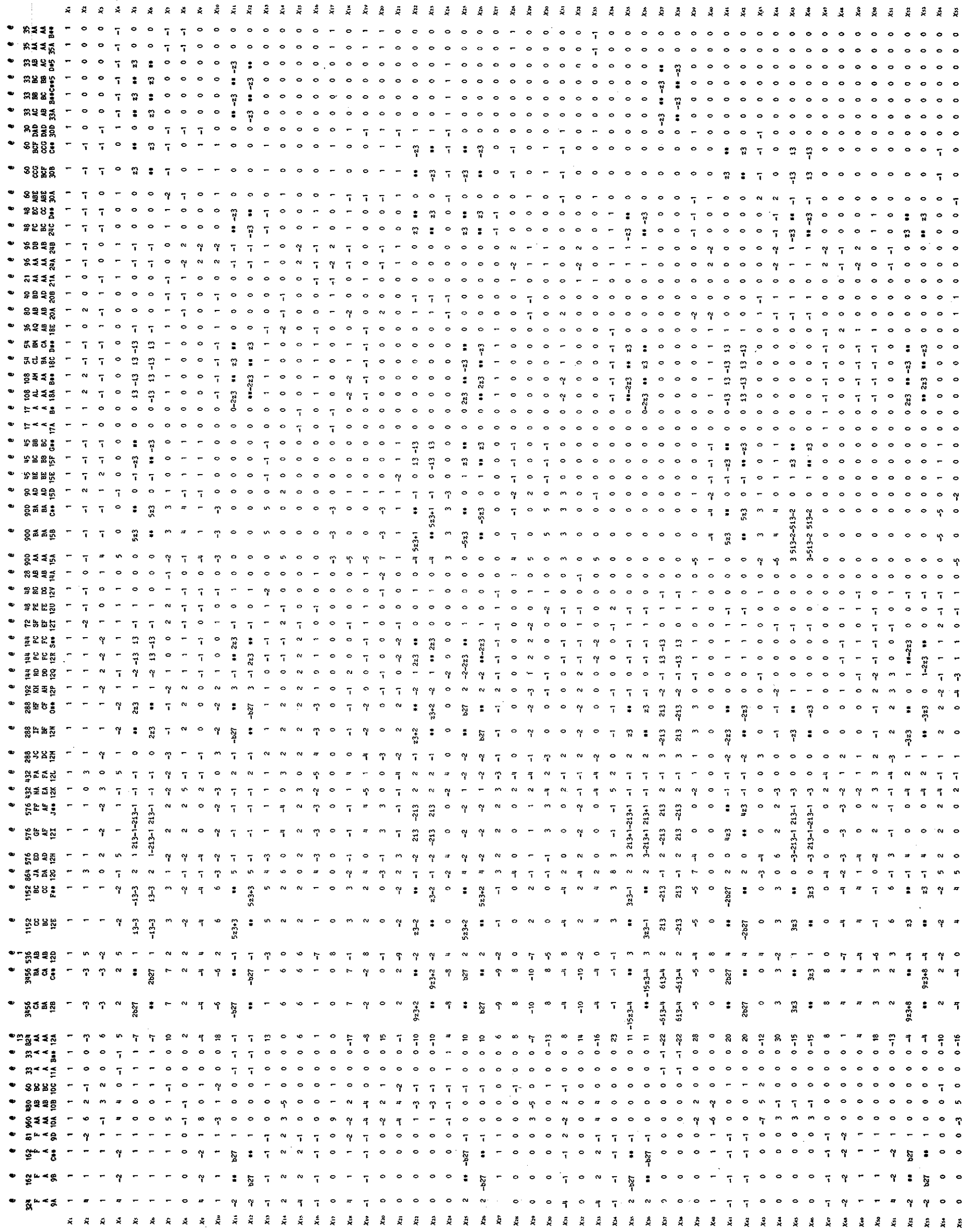
$O_{10}^{-}(2)$

		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		25015379558400	1274019840	371589120	35389440	1179648	522547200	41057280	41057280	466560	233280	23328	884736	491520	147456	92160	24576	9216	6144	6144	2048	100800	900	331776	248832	248832	155520	138240	138240	138240	13824	13824	10368	9216	7776	7776	5184	3456	2592	2592	1728	864	864	864	576	288	420	768	768	768	128	64	64	
	p power		A	A	A	A	A	A	A	A	A	A	A	A	A	B	A	C	B	D	D	A	A	AA	CA	BA	DA	AB	AC	AC	CC	BC	DA	AD	FA	FA	EA	DC	FA	FB	DB	EC	FC	FC	DD	FD	A	A	B	C	E	H	I	
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	CA	DA	AB	AC	AC	BC	CC	DA	AD	FA	FA	EA	DC	FA	FB	DB	EC	FC	FC	DD	FD	A	A	A	A	A	A	A	
	ind	1A	2A	2B	2C	2D	3A	3B	C**	3D	3E	3F	4A	4B	4C	4D	4E	4F	4G	4H	4I	5A	5B	6A	6B	C**	6D	6E	6F	G**	6H	I**	6J	6K	6L	M**	6N	6O	6P	6Q	6R	6S	6T	U**	6V	6W	7A	8A	8B	8C	8D	8E	8F	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	154	26	42	-6	10	49	-11	-11	7	4	1	18	2	10	10	2	-2	2	-2	6	14	-1	17	5	5	5	9	9	9	-3	-3	-1	1	-1	-1	-4	-3	5	-3	3	0	3	3	1	1	0	2	2	2	2	0	0	χ_{2}
χ_{3}	+	187	53	43	27	11	34	-11	-11	16	7	-2	3	19	-5	11	3	7	3	7	-1	7	2	2	5	5	14	10	-6	-6	-3	-3	8	2	-4	-4	-1	6	2	-2	4	3	0	0	2	2	5	3	3	-1	-1	1	1	χ_{3}
χ_{4}	+	340	84	84	20	20	85	10	10	19	-5	7	20	20	4	20	4	4	4	4	4	20	0	21	-6	-6	21	21	5	5	2	2	3	5	3	3	3	5	3	3	3	-1	-1	-1	5	-1	4	4	4	0	0	0	0	χ_{4}
χ_{5}	o	595	-45	35	51	3	-35	27i3-17	-27i3-17	10	19	1	11	-5	3	-5	-5	-1	3	7	-1	0	5	-3	3i3-9	-3i3-9	0	5	8i3-3	-8i3-3	-i3+3	i3+3	-6	-3	-3i3	3i3	3	0	-3	-1	2	3	i3	-i3	0	3	0	-1	-1	3	-1	1	1	χ_{5}
χ_{6}	o	595	-45	35	51	3	-35	-27i3-17	27i3-17	10	19	1	11	-5	3	-5	-5	-1	3	7	-1	0	5	-3	-3i3-9	3i3-9	0	5	-8i3-3	8i3-3	i3+3	-i3+3	-6	-3	3i3	-3i3	3	0	-3	-1	2	3	-i3	i3	0	3	0	-1	-1	3	-1	1	1	χ_{6}
χ_{7}	+	748	-20	76	44	12	118	55	55	1	10	-8	28	-4	12	4	-4	-4	4	4	4	13	3	22	7	7	-11	-2	14	14	-1	-1	1	6	-2	-2	10	5	4	4	1	2	2	2	-3	0	-1	0	0	4	0	0	0	χ_{7}
χ_{8}	+	2244	324	84	100	-12	-6	66	66	39	21	3	-4	44	4	4	-4	8	-4	8	0	-1	4	-6	18	18	9	-6	10	10	10	10	15	-6	9	9	-3	1	3	3	3	1	1	1	-3	3	4	4	-4	0	0	-2	2	χ_{8}
χ_{9}	+	5236	-12	420	-44	4	616	-44	-44	19	-14	-11	92	-20	20	20	-4	0	-4	0	8	56	1	72	-12	-12	-21	0	16	16	4	4	3	-8	-3	-3	-6	-5	9	-3	3	-2	1	1	-5	1	0	-4	4	0	0	2	-2	χ_{9}
χ_{10}	+	5236	-268	196	148	-28	322	-110	-110	25	25	-2	12	-4	-12	4	12	-8	-4	8	0	21	6	2	2	2	-25	-14	-14	-14	10	10	-7	2	2	2	-7	7	2	-2	1	1	-2	-2	-1	2	0	-4	4	0	0	-2	2	χ_{10}
χ_{11}	o	5950	190	-210	30	14	175	45z3-65	**	25	-5	-8	38	-10	-34	-10	6	2	-2	-6	2	0	0	-17	-17b27	**	-5	15	15	15	**	3z3-6	1	-1	**	-2b27	-5	3	4	6	-3	3	6z3	**	-1	2	0	2	2	2	-2	0	0	χ_{11}
χ_{12}	o	5950	190	-210	30	14	175	**	45z3-65	25	-5	-8	38	-10	-34	-10	6	2	-2	-6	2	0	0	-17	**	-17b27	-5	15	15	15	3z3-6	**	1	-1	-2b27	**	-5	3	4	6	-3	3	**	6z3	-1	2	0	2	2	2	-2	0	0	χ_{12}
χ_{13}	+	6545	145	-175	81	17	245	209	209	20	29	11	49	-15	-31	-15	1	1	1	1	1	0	5	-11	1	1	10	5	21	21	9	9	4	5	1	1	13	-6	7	-13	-4	-3	-3	-3	2	-1	0	1	1	5	-3	1	1	χ_{13}
χ_{14}	+	6545	273	385	-15	-31	560	110	110	2	20	11	57	9	17	9	-7	5	-15	-3	5	35	0	48	30	30	30	16	0	0	6	6	-6	-16	3	3	12	6	3	7	-2	0	3	3	2	-1	0	-3	-3	-3	1	-1	-1	χ_{14}
χ_{15}	+	7700	660	420	180	68	350	110	110	20	35	14	12	60	20	20	12	8	12	8	0	0	0	30	30	30	30	30	30	30	6	6	12	14	12	12	3	6	-6	6	0	3	0	0	2	2	0	-4	4	0	0	2	-2	χ_{15}
χ_{16}	+	9350	1030	470	230	54	125	-55	-55	80	-10	-19	-2	110	22	30	14	18	6	10	2	0	0	-35	-23	-23	40	5	5	5	-7	-7	16	-3	-5	-5	-2	8	1	-7	8	2	-1	-1	0	-3	5	2	2	-2	2	0	0	χ_{16}
χ_{17}	+	10692	324	756	36	84	972	0	0	0	0	0	108	-4	36	36	12	0	12	0	8	57	-3	108	0	0	0	36	36	36	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	-4	4	-4	0	0	-2	2	χ_{17}
χ_{18}	+	13090	418	882	-190	18	1015	55	55	46	25	4	58	-22	-30	50	-6	-10	-6	-10	-2	70	0	55	7	7	16	39	-25	-25	-1	-1	-2	-9	4	4	1	-16	4	0	6	5	-4	-4	0	0	0	2	2	-2	-2	0	0	χ_{18}
χ_{19}	+	14960	1136	176	240	48	-160	110	110	56	-25	29	16	80	-16	0	16	16	0	0	0	-20	0	32	-34	-34	-16	-16	0	0	6	6	8	0	11	11	-1	0	5	5	8	3	3	3	0	-3	1	0	0	0	0	0	0	χ_{19}
χ_{20}	+	15147	1323	747	171	-21	567	0	0	81	0	0	-21	107	-21	35	-21	3	-13	11	-5	22	-3	-9	0	0	81	39	-9	-9	0	0	9	-9	0	0	0	9	0	0	-3	0	0	0	-3	0	6	-1	-1	3	-1	-1	-1	χ_{20}
χ_{21}	+	15708	1116	924	28	92	651	66	66	66	3	-6	-4	60	-20	60	12	-4	12	-4	-4	28	-2	-21	18	18	54	51	-29	-29	10	10	-6	11	0	0	3	-2	6	6	6	-5	4	4	2	2	0	4	4	0	0	0	0	χ_{21}
χ_{22}	o	15708	1116	252	284	60	-294	297z3+66	**	39	30	-6	44	76	-4	4	12	12	4	4	4	-7	3	-6	**	-9z3+9	-9	-30	8i3-22	-8i3-22	25z3+10	**	15	-6	18z3	**	6	-1	-6	0	3	2	2z3	**	3	0	0	0	0	4	0	0	0	χ_{22}
χ_{23}	o	15708	1116	252	284	60	-294	**	297z3+66	39	30	-6	44	76	-4	4	12	12	4	4	4	-7	3	-6	-9z3+9	**	-9	-30	-8i3-22	8i3-22	**	25z3+10	15	-6	**	18z3	6	-1	-6	0	3	2	**	2z3	3	0	0	0	0	4	0	0	0	χ_{23}
χ_{24}	+	22848	1344	1344	64	64	1428	-120	-120	42	-30	6	64	64	0	64	0	0	0	0	0	63	3	84	-24	-24	84	84	4	4	-8	-8	-6	4	-6	-6	-6	4	-6	-6	-6	-2	-2	-2	4	-2	0	0	0	0	0	0	0	χ_{24}
χ_{25}	o	26180	-700	420	4	-28	-490	**	99z3+209	20	41	8	52	20	4	-20	-12	4	12	-4	-4	0	-5	-10	**	3z3-31	-10	30	32i3-2	-32i3-2	13z3+6	**	-4	-10	-3i3-7	3i3-7	-7	-2	-4	6	0	1	**	-2z3	2	2	0	0	0	4	0	0	0	χ_{25}
χ_{26}	o	26180	-700	420	4	-28	-490	99z3+209	**	20	41	8	52	20	4	-20	-12	4	12	-4	-4	0	-5	-10	3z3-31	**	-10	30	-32i3-2	32i3-2	**	13z3+6	-4	-10	3i3-7	-3i3-7	-7	-2	-4	6	0	1	-2z3	**	2	2	0	0	0	4	0	0	0	χ_{26}
χ_{27}	+	32725	85	805	-395	69	-350	55	55	55	55	1	45	-35	53	5	13	-7	-19	-7	1	0	0	-62	-41	-41	-5	10	10	10	7	7	7	18	-5	-5	7	-5	1	13	7	7	-5	-5	3	-3	0	-3	-3	-7	1	-1	-1	χ_{27}
χ_{28}	+	35904	-960	960	64	-64	1884	264	264	48	-6	-6	128	0	0	0	0	0	0	0	0	64	-1	60	-24	-24	-60	-60	4	4	-8	-8	0	-4	-6	-6	18	4	6	6	0	-2	-2	-2	-4	2	1	-8	8	0	0	0	0	χ_{28}
χ_{29}	+	39270	-282	630	70	-42	1785	-330	-330	-33	30	12	158	14	-10	-26	-18	10	-2	-6	2	35	0	57	6	6	15	-39	25	25	-2	-2	-9	9	6	6	-18	7	12	0	3	-2	-2	-2	3	0	0	-2	-2	6	-2	0	0	χ_{29}
χ_{30}	+	39270	-1050	630	646	22	-105	264	264	75	84	12	-34	-50	6	-10	-2	-6	-2	26	2	0	5	-9	-24	-24	-15	15	31	31	-8	-8	-21	7	-6	-6	-12	1	0	0	3	4	-2	-2	1	4	0	-2	-2	-6	2	0	0	χ_{30}
χ_{31}	+	41888	928	672	-96	160	728	-220	-220	-22	-10	29	32	32	32	0	32	0	0	0	0	-42	3	88	4	4	-14	24	24	24	-12	-12	-14	-8	1	1	-2	-6	1	-3	6	-5	-3	-3	-2	1	0	0	0	0	0	0	0	χ_{31}
χ_{32}	+	56100	420	1140	580	148	750	-330	-330	-60	75	21	-4	-20	-28	20	-4	-8	12	8	0	0	0	-18	6	6	-30	30	-50	-50	-2	-2	12	-2	-3	-3	3	10	9	-3	0	7	1	1	-2	1	-5	4	-4	0	0	2	-2	χ_{32}
χ_{33}	+	59840	-576	1344	-320	64	2360	-220	-220	14	20	-10	128	0	64	0	0	0	0	0	0	20	0	120	36	36	-54	-24	40	40	4	4	6	-8	0	0	-12	-14	-6	-6	-6	4	4	4	-2	-2	4	0	0	0	0	0	0	χ_{33}
χ_{34}	+	65450	2090	-70	330	-166	875	-385	-385	20	65	2	2	50	42	-30	2	-10	-6	14	6	0	0	11	47	47	20	35	-45	-45	-9	-9	20	11	-16	-16	-7	-12	2	2	-4	-3	0	0	-4	2	0	-2	-2	-2	2	0	0	χ_{34}
χ_{35}	o	65450	-470	1050	-310	58	-1225	**	495z3+275	50	35	-7	146	-30	26	-30	2	-2	-6	-10	-2	0	0	-41	33z3+4	**	10	15	40i3-25	-40i3-25	**	7z3-5	10	7	6i3+7	-6i3+7	-5	2	-11	-3	6	-1	2i3-1	-2i3-1	-2	1	0	-2	-2	2	-2	0	0	χ_{35}
χ_{36}	o	65450	-470	1050	-310	58	-1225	495z3+275	**	50	35	-7	146	-30	26	-30	2	-2	-6	-10	-2	0	0	-41	**	33z3+4	10	15	-40i3-25	40i3-25	7z3-5	**	10	7	-6i3+7	6i3+7	-5	2	-11	-3	6	-1	-2i3-1	2i3-1	-2	1	0	-2	-2	2	-2	0	0	χ_{36}
χ_{37}	o	71400	-1560	840	680	8	-1050	-270i3-60	270i3-60	75	30	-15	56	-40	-24	-40	-8	0	8	16	0	0	0	6	36z3+6	**	15	30	-40i3-10	40i3-10	2i3-4	-2i3-4	-21	-10	9i3+6	-9i3+6	6	-1	-3	3	3	2	i3+2	-i3+2	-1	-1	0	0	0	0	0	0	0	χ_{37}
χ_{38}	o	71400	-1560	840	680	8	-1050	270i3-60	-270i3-60	75	30	-15	56	-40	-24	-40	-8	0	8	16	0	0	0	6	**	36z3+6	15	30	40i3-10	-40i3-10	-2i3-4	2i3-4	-21	-10	-9i3+6	9i3+6	6	-1	-3	3	3	2	-i3+2	i3+2	-1	-1	0	0	0	0	0	0	0	χ_{38}
χ_{39}	+	78540	-1332	1932	460	12	2940	-165	-165	51	-30	-3	76	-52	-20	20	-20	-12	4	12	-4	70	0	60	27	27	-99	-36	-20	-20	-5	-5	3	12	9	9	-6	13	-3	-3	3	-2	1	1	-3	-3	0	0	0	-4	0	0	0	χ_{39}
χ_{40}	+	83776	3904	448	320	-64	-644	-176	-176	148	4	22	0	128	0	0	0	0	0	0	0	-14	-4	28	16	16	-20	-44	20	20	-16	-16	4	-4	-2	-2	4	-4	10	-2	4	-4	2	2	-4	2	0	8	-8	0	0	0	0	χ_{40}
χ_{41}	o	104720	-240	-560	656	80	140	297i3-517	-297i3-517	50	59	-13	-16	-80	16	0	-16	16	0	0	0	0	5	12	33i3+3	-33i3+3	30	-20	8i3-28	-8i3-28	5i3-1	-5i3-1	-6	-4	-6i3+3	6i3+3	3	-10	3	7	-2	-1	-2i3-1	2i3-1	2	-1	0	0	0	0	0	0	0	χ_{41}
χ_{42}	o	104720	-240	-560	656	80	140	-297i3-517	297i3-517	50	59	-13	-16	-80	16	0	-16	16	0	0	0	0	5	12	-33i3+3	33i3+3	30	-20	-8i3-28	8i3-28	-5i3-1	5i3-1	-6	-4	6i3+3	-6i3+3	3	-10	3	7	-2	-1	2i3-1	-2i3-1	2	-1	0	0	0	0	0	0	0	χ_{42}
χ_{43}	+	106029	-1107	2205	-243	-3	2268	0	0	81	0	0	-75	-59	-51	29	21	9	-11	9	1	49	-6	-36	0	0	-81	12	-36	-36	0	0	9	12	0	0	0	-9	0	0	-3	0	0	0	3	0	0	-3	-3	9	1	-1	-1	χ_{43}
χ_{44}	+	106029	-1107	189	525	-99	1134	891	891	0	81	0	69	-11	-3	21	21	-7	-3	1	-7	-21	4	-18	27	27	0	-18	30	30	-21	-21	0	-18	0	0	9	0	0	0	0	-3	0	0	0	0	0	1	1	5	1	-1	-1	χ_{44}
χ_{45}	o	106029	-1107	189	525	-99	-567	891z3	**	0	81	0	69	-11	-3	21	21	-7	-3	1	-7	-21	4	9	27z3	**	0	9	48z3+9	**	-21z3	**	0	9	0	0	9	0	0	0	0	-3	0	0	0	0	0	1	1	5	1	-1	-1	χ_{45}
χ_{46}	o	106029	-1107	189	525	-99	-567	**	891z3	0	81	0	69	-11	-3	21	21	-7	-3	1	-7	-21	4	9	**	27z3	0	9	**	48z3+9	**	-21z3	0	9	0	0	9	0	0	0	0	-3	0	0	0	0	0	1	1	5	1	-1	-1	χ_{46}
χ_{47}	+	123200	320	2240	320	192	3500	440	440	-100	-10	-28	128	0	0	0	0	0	0	0	0	0	0	140	-40	-40	-40	20	20	20	8	8	-4	12	-4	-4	14	8	-4	8	-4	2	-4	-4	0	0	0	8	-8	0	0	0	0	χ_{47}
χ_{48}	+	130900	1620	980	20	-108	3325	220	220	-50	-50	31	148	20	68	-60	4	-12	-12	4	4	0	0	93	-36	-36	90	5	5	5	-4	-4	6	-3	9	9	6	2	3	-1	2	2	5	5	6	3	0	-4	-4	0	0	0	0	χ_{48}
χ_{49}	+	130900	4820	1540	180	100	-1400	220	220	85	-50	-23	124	140	52	20	28	0	-4	0	8	0	0	-88	-4	-4	5	-80	0	0	12	12	-19	-8	5	5	-10	-3	-7	1	1	-6	-3	-3	1	1	0	4	-4	0	0	2	-2	χ_{49}
χ_{50}	+	130900	-940	-140	660	52	1750	715	715	40	-5	31	36	-60	20	-20	-28	20	12	-4	-4	0	0	-10	59	59	50	-50	30	30	3	3	-16	22	-13	-13	11	-6	-1	-5	4	3	3	3	-2	-5	0	0	0	4	0	0	0	χ_{50}
χ_{51}	+	130900	-2220	2100	340	52	-1925	-110	-110	115	-125	22	164	-60	36	-60	-12	-4	4	12	-4	0	0	-69	66	66	-15	75	-5	-5	10	10	3	-5	-6	-6	3	1	-6	-6	3	-5	-2	-2	1	-2	0	-4	-4	0	0	0	0	χ_{51}
χ_{52}	o	130900	2900	-1260	340	-108	700	**	495z3+55	55	-20	-14	20	20	52	-20	-12	12	-4	-12	4	0	0	-4	**	111z3+23	-55	60	-20	-20	**	15z3+7	-1	12	-3i3-7	3i3-7	-4	1	2	0	3	4	-2b27	**	-3	0	0	0	0	4	0	0	0	χ_{52}
χ_{53}	o	130900	2900	-1260	340	-108	700	495z3+55	**	55	-20	-14	20	20	52	-20	-12	12	-4	-12	4	0	0	-4	111z3+23	**	-55	60	-20	-20	15z3+7	**	-1	12	3i3-7	-3i3-7	-4	1	2	0	3	4	**	-2b27	-3	0	0	0	0	4	0	0	0	χ_{53}
χ_{54}	+	157080	-360	-840	-296	-8	2730	66	66	-15	-24	21	200	40	-104	-40	8	-16	8	16	0	0	-5	-54	18	18	45	-30	34	34	10	10	9	10	-9	-9	0	-11	9	-3	-3	4	-5	-5	1	1	0	0	0	0	0	0	0	χ_{54}
χ_{55}	+	157080	2712	-840	-40	-8	840	660	660	93	30	21	8	-24	88	-24	8	0	-8	-16	0	-35	0	72	84	84	-15	24	-40	-40	20	20	-3	-8	3	3	6	17	9	-3	-3	2	-1	-1	1	1	0	0	0	0	0	0	0	χ_{55}

$O_{10}^{-}(2)$

	1A	2A	2B	2C	2D	3A	3B	3C**	3D	3E	3F	4A	4B	4C	4D	4E	4F	4G	4H	4I	5A	5B	6A	6B	6C**	6D	6E	6F	G**	6H	I**	6J	6K	6L	M**	6N	6O	6P	6Q	6R	6S	6T	U**	6V	6W	7A	8A	8B	8C	8D	8E	8F
X_{2}	+	171072	5184	1728	576	192	972	0	0	0	0	0	128	0	0	0	0	0	0	0	0	0	-3	108	0	0	36	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	8	0	0	0	0
X_{3}	+	18174	5508	2184	612	276	-972	0	0	0	0	108	124	36	36	12	0	12	0	-21	8	-6	-108	0	0	0	-36	0	36	36	-36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	4	0	0	0	-2	2
X_{4}	0	190400	960	-2240	-320	64	1400	27013	-4190	-27013	-4190	50	50	-64	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{5}	0	190400	960	-2240	-320	64	1400	-27013	-4190	27013	-4190	50	50	-64	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{6}	0	19100	22	-1	0	1024	0	-512	880	-56	100	-32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{7}	+	2212058	8154	1386	954	-54	-567	0	162	0	0	-78	226	-54	-14	-30	6	-6	-2	-10	-7	3	9	0	0	-39	9	9	0	18	9	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-6	-2	0	0	0	0
X_{8}	+	227205	-891	3429	-315	-219	6075	0	0	0	0	117	-143	-27	-3	21	-3	-3	-3	-3	-3	90	0	27	0	-45	-45	-45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-3	1	1	1	1
X_{9}	+	25132	2496	4032	-320	192	1596	264	264	-6	-78	12	-64	-64	0	64	0	0	0	0	0	-7	8	-132	-24	-24	60	-20	-20	-8	-6	12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{10}	+	25132	2496	4032	-320	192	1596	-264	264	6	-78	12	-64	-64	0	64	0	0	0	0	0	-7	8	-132	-24	-24	60	-20	-20	-8	-6	12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{11}	o	25132	2496	1344	704	64	-2184	59413	-3330	59413	-3330	30	12	128	0	64	0	0	0	0	0	28	3	-72	-1813	-42	-6	-32	2413	-8	6	-8	1813	-42	3	-72	1813	-42	-6	-32	2413	-8	6	-8	1813	-42	3	-72	1813	-42	-6	-32	2413	-8	6	-8
X_{12}	o	25132	2496	1344	704	64	-2184	59413	-3330	59413	-3330	30	12	128	0	64	0	0	0	0	0	28	3	-72	-1813	-42	-6	-32	2413	-8	6	-8	1813	-42	3	-72	1813	-42	-6	-32	2413	-8	6	-8	1813	-42	3	-72	1813	-42	-6	-32	2413	-8	6	-8
X_{13}	o	1900	960	-2240	-320	64	1400	27013	-4190	-27013	-4190	50	50	-64	128	0	0	0	0	0	0	0	0	-72	-3013	-46	-10	0	10	-2	-2	3013	-46	-10	0	-72	3013	-46	-10	0	10	-2	-2	3013	-46	-10	0	-72	3013	-46	-10	0	10	-2	-2	3013
X_{14}	+	1403325	3645	-2835	-675	-243	0	0	0	0	-27	-75	45	45	21	9	21	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{15}	+	1403325	3645	-2835	-675	-243	0	0	0	0	-27	-75	45	45	21	9	21	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

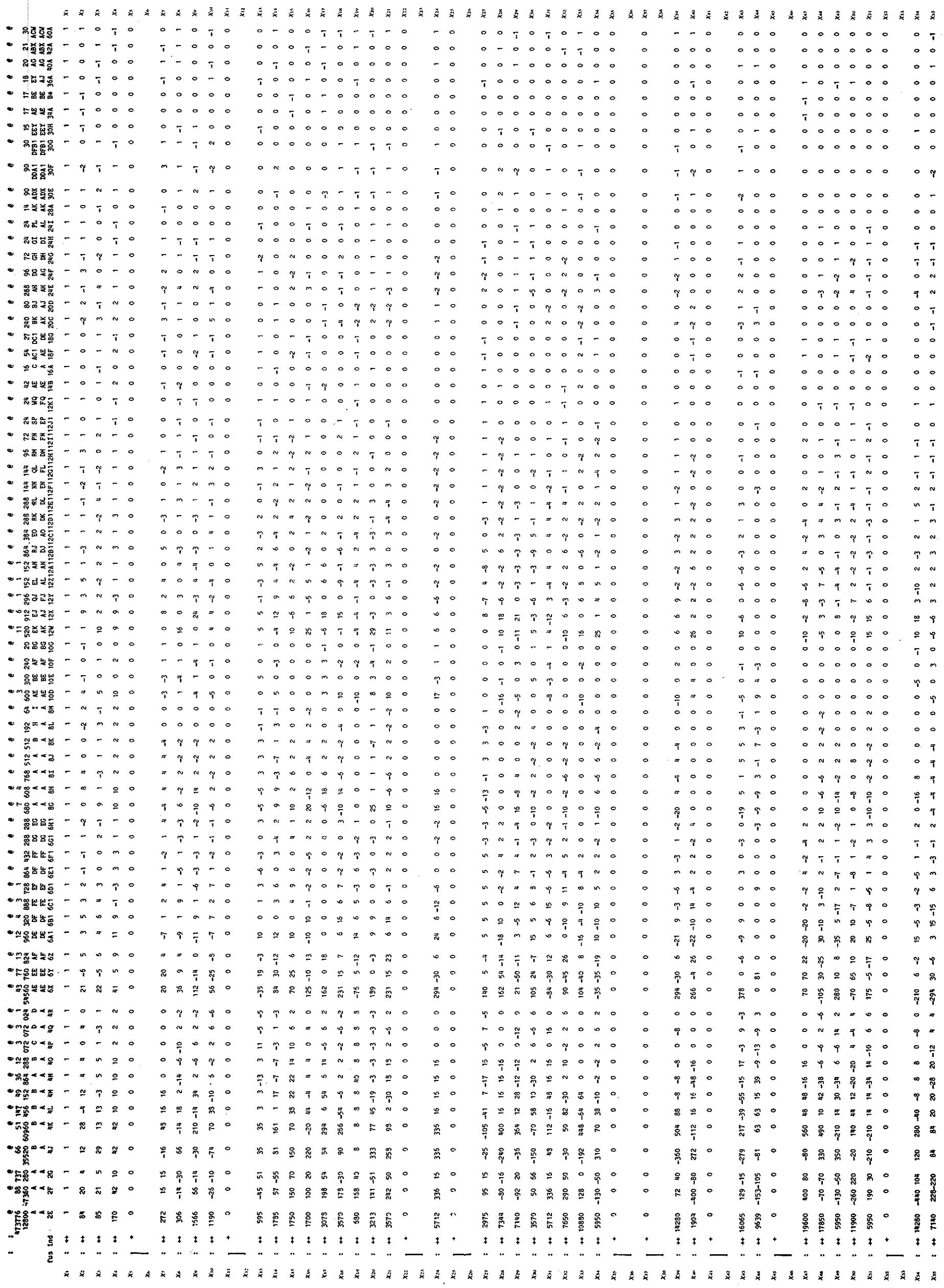
$O_{\overline{10}}(2)$



$O_{\overline{10}}(2)$

	9A	9B	C**	9D	10A	10B	10C	11A	B**	12A	12B	C**	12D	12E	F**	12G	12H	12I	J**	12K	12L	12M	12N	O**	12P	12Q	12R	S**	12T	12U	12V	14A	15A	15B	C**	15D	15E	15F	G**	17A	B*	18A	B**	18C	D**	18E	20A
X96	0	0	0	0	-5	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-3	-3	-3	0	0	0	0	0	1	0	0	0	0	0	-2
X97	0	0	0	0	3	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	3	3	3	0	0	0	0	0	0	0	0	0	0	0	-1
X98	-1	-1	-1	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X99	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X100	-1	-1	-2	1	-9	0	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	3	0	0	0	0	0	0	0	0	0	0	0	0
X101	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X102	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X103	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X104	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X105	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X106	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X107	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X108	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X109	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X111	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X112	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X113	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X114	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X115	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$O_{10}^{-}(2)$



$O_{10}^{-}(2)$

	2E	2F	2G	4J	4K	4L	4M	4N	4O	4P	4Q	4R	6X	6Y	6Z	6A1	6B1	6C1	6D1
X_{56}	++ 16848	720	144	432	48	48	48	48	16	0	0	0	-162	0	-18	0	0	0	0
X_{57}	++ 13770	522	90	378	-246	42	-6	42	2	6	-6	-2	162	0	18	0	0	0	0
X_{58}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{59}																			
X_{60}	++ 4352	256	256	0	0	0	0	0	0	0	0	0	-160	20	-32	-28	4	-4	4
X_{61}	++ 12852	-12	36	684	-196	60	12	-12	-12	0	-12	0	189	0	-3	-18	0	0	0
X_{62}	++ 34425	-135	-135	-135	825	-39	-39	9	-23	9	9	9	405	0	-27	0	0	0	0
X_{63}	++ 28560	656	80	144	-112	-48	-48	16	16	0	0	0	462	30	14	-6	-4	-6	-10
X_{64}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{65}																			
X_{66}	++ 23800	120	-40	520	-280	-88	8	-8	-8	0	8	0	490	-50	-6	-5	15	13	6
X_{67}	++ 35700	500	20	-60	420	-92	4	-12	-12	-4	-12	-4	420	15	20	15	5	6	-1
X_{68}	++ 11900	-250	-100	300	140	12	-20	-4	28	4	-4	4	-280	35	-8	5	-5	11	-5
X_{69}	++ 26775	-105	-105	855	-105	-9	-9	7	-25	7	7	7	315	45	-21	0	0	-9	-3
X_{70}	++ 26775	-105	-105	-585	-105	87	-9	7	23	7	-1	-9	315	45	-21	0	0	-9	-3
X_{71}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{72}																			
X_{73}	++ 32130	258	-30	738	98	66	-62	-30	2	-6	-6	-6	189	0	-3	9	-9	0	0
X_{74}	++ 14280	712	-120	-120	-56	-56	8	8	-24	-8	8	-8	-336	15	16	24	4	-3	7
X_{75}	++ 43520	512	0	512	512	0	0	0	0	0	0	0	176	-40	-16	-4	-16	-4	3
X_{76}	++ 38080	-320	64	-640	0	128	0	0	0	0	0	0	280	-44	-8	10	10	10	4
X_{77}	++ 28560	-368	80	336	336	16	16	-48	16	0	0	0	-546	-60	-2	12	10	-6	4
X_{78}	++ 32130	258	-30	-558	-238	-78	-14	-30	18	-6	-6	10	189	0	-3	9	-9	0	0
X_{79}	++ 38555	-612	156	-324	252	-36	-36	12	12	-4	12	-4	0	81	0	0	0	0	9
X_{80}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{81}																			
X_{82}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{83}																			
X_{84}	++ 15065	-447	225	359	49	-15	-15	-15	1	9	9	-7	-189	0	3	18	0	0	0
X_{85}	++ 43195	-765	51	-405	315	27	-21	51	3	-17	3	-1	0	-81	0	0	0	0	-9
X_{86}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{87}																			
X_{88}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{89}																			
X_{90}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{91}																			
X_{92}	++ 13600	800	160	-160	-160	-32	-32	32	-32	0	0	0	-260	10	-4	10	-10	-8	2
X_{93}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{94}																			
X_{95}	++ 21504	1024	0	0	0	0	0	0	0	0	0	0	-672	-24	-32	-24	16	12	-8
X_{96}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{97}																			
X_{98}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{99}																			
X_{100}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{101}																			
X_{102}	+ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{103}																			
X_{104}	++ 43520	512	0	-512	-512	0	0	0	0	0	0	0	176	-40	-16	-4	-16	-4	8
X_{105}	++ 57120	288	160	-480	-224	32	32	-32	-32	0	0	0	-84	-30	12	6	6	-12	-6
X_{106}	++ 12852	564	-204	36	-28	36	-28	-12	4	-12	-12	4	-378	0	6	9	9	0	0
X_{107}	++ 68850	-270	-270	-270	210	-78	18	18	2	18	-6	2	-405	0	27	0	0	0	0
X_{108}	++ 51408	-48	144	144	-112	-48	-48	-48	16	0	0	0	-378	0	6	-18	0	0	0
X_{109}	++ 47600	240	-80	-240	-560	80	16	-16	-16	0	16	0	140	50	12	20	0	17	-6
X_{110}	+ 65536	0	0	0	0	0	0	0	0	0	0	0	-512	64	0	-32	0	16	0
X_{111}	++ 32130	-318	210	90	-406	-54	26	-30	2	6	-6	6	189	0	-3	9	9	0	0
X_{112}	++ 34425	-135	-135	-135	-615	-39	57	9	25	9	-15	-7	405	0	-27	0	0	0	0
X_{113}	++ 53550	-210	-210	270	-210	78	-18	14	-2	14	6	-2	-315	-45	21	0	0	-18	3
X_{114}	++ 42525	-675	45	405	-315	-27	21	45	-3	-15	-3	1	0	0	0	0	0	0	0
X_{115}	++ 42525	-675	45	405	-315	-27	21	45	-3	-15	-3	1	0	0	0	0	0	0	0

Co_2

Sporadic Conway group Co_2

Order = 42,305,421,312,000 = $2^{18}.3^6.5^3.7.11.23$ Mult = 1   Out = 1

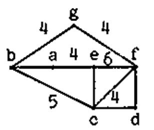
Constructions

Graph G : the automorphism group of a rank 3 graph of valence 891 on 2300 points in which the point stabilizer is $U_6(2).2$.

Leech G : the stabilizer of a type 2 vector V in the Leech lattice (see Co_1). Taking V = $(4,4,0^{22})$, O = $(0,0,0^{22})$, the 4600 points P with type(VP) = type(OP) = 2 fall naturally into 2300 pairs $\{P_L,P_R\}$ with $P_L + P_R - V = 0$, namely $\{ (4,0,\pm4,0^{21}), (0,4,\mp4,0^{21}) \}$ (44), $\{ (3,1,\pm1^{22}), (1,3,\pm1^{22}) \}$ (1024), $\{ (2,2,\pm2^6 0^{16}), (2,2,\mp2^6 0^{16}) \}$ (1232). Two pairs $\{P_L,P_R\}$ $\{Q_L,Q_R\}$ are incident in the above graph just if type(P_iQ_j) is even.

Considering the Leech lattice modulo 2, the 2-spaces containing V become points in the quotient 23-dimensional representation over $\mathbb{F}_2$ (which contains an irreducible 22-dimensional representation). They are described below by the types of their non-zero vectors. In all cases except that marked * these vectors lift to real Leech lattice vectors of the appropriate types adding to 0.

Presentation

G ≅ < 

| $a=(cf)^2, b=(ef)^3, e=(bg)^2, (aecd)^4=(cef)^7=(baefg)^3=1$ >

Maximal subgroups

Order	Index	Structure	Character	Abstract	Leech (mod 2)	Graph
				Specifications		
18393661440	2300	$U_6(2):2$	1a+275a+2024a		2-22-point	point
908328960	46575	$2^{10}:M_{22}:2$		$N(2^{10})$	2-24-point	22 edges
898128000	47104	McL			2-23-point	
743178240	56925	$2_+^{1+8}:S_6(2)$		N(2A)		
88704000	476928	HS:2			2-33-point	
41287680	1024650	$(2_+^{1+6}\times2^4).A_8$		N(2B)		
26127360	1619200	$U_4(3).D_8$			2-33*-point	non-edge
11796480	3586275	$2^{4+10}(S_5\times S_3)$		$N(2^4)$		
10200960	4147200	M_{23}			2-34-point	
933120	45337600	$3_+^{1+4}:2_-^{1+4}.S_5$		N(3A)		
12000	3525451776	$5_+^{1+2}:4S_4$		N(5A)		

G 60

60

;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		42305421312000	743178240	41287680	1474560	466560	155520	3096576	122880	73728	49152	6144	6144	1280	3000	600	5760	5184	4320	3456	576	288
	p power		A	A	A	A	A	A	B	B	A	B	B	C	A	A	AB	AA	BA	BA	BB	BC
	p' part		A	A	A	A	A	A	A	A	A	A	A	A	A	A	AB	AA	BA	BA	BB	BC
	ind	1A	2A	2B	2C	3A	3B	4A	4B	4C	4D	4E	4F	4G	5A	5B	6A	6B	6C	6D	6E	6F
χ1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ2	+	23	−9	7	−1	−4	5	7	−5	3	−1	3	−1	−1	−2	3	4	0	3	−3	1	−1
χ3	+	253	29	13	−11	10	10	29	9	1	5	1	−3	1	3	3	10	2	2	2	−2	−2
χ4	+	275	51	35	11	5	14	19	15	7	−5	7	3	−1	0	5	5	−3	6	6	2	2
χ5	+	1771	−21	−21	11	−11	16	91	−5	−5	−5	−5	3	−1	−4	1	21	−3	−6	0	0	2
χ6	+	2024	232	104	40	−1	26	8	24	24	8	8	8	0	−1	4	−1	7	4	10	2	4
χ7	+	2277	−219	133	−11	9	36	21	−35	13	5	13	−3	1	2	7	1	−3	6	−12	4	−2
χ8	+	4025	−231	105	1	−25	29	105	−35	5	1	5	1	1	0	5	15	3	9	−3	−3	1
χ9	+	7084	−84	−84	44	10	19	140	−4	−4	12	−4	−4	−4	9	−1	18	6	−9	3	3	−1
χ10	o	9625	−455	105	−15	40	−5	−7	5	29	1	−3	−7	5	0	0	0	4	−5	−5	3	3
χ11	o	9625	−455	105	−15	40	−5	−7	5	29	1	−3	−7	5	0	0	0	4	−5	−5	3	3
χ12	o	10395	315	−21	−45	27	0	−21	−9	15	19	−1	3	−5	−5	0	3	−9	0	0	0	0
χ13	o	10395	315	−21	−45	27	0	−21	−9	15	19	−1	3	−5	−5	0	3	−9	0	0	0	0
χ14	+	12650	554	330	26	−40	59	−6	50	2	10	18	2	6	0	5	0	−4	−1	11	3	−1
χ15	+	23000	600	280	120	50	5	184	40	8	24	8	8	0	0	0	10	6	15	−3	1	3

		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		56	768	768	512	512	256	64	54	120	60	40	11	864	288	288	288
	p power	A	A	C	D	D	C	E	A	AB	BA	BC	A	BA	AC	DA	EC
	p' part	A	A	A	A	A	A	A	A	AB	BA	BC	A	AA	AC	BA	BC
	ind	7A	8A	8B	8C	8D	8E	8F	9A	10A	10B	10C	11A	12A	12B	12C	12D
χ1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ2	+	2	−1	−3	3	−1	1	1	2	2	1	−1	1	−2	0	1	−3
χ3	+	1	−3	3	5	1	−1	−1	1	3	−1	−1	0	2	−2	2	4
χ4	+	2	3	5	3	−1	1	1	2	0	1	1	0	1	1	−2	4
χ5	+	0	3	−1	7	−1	−1	−1	−2	4	−1	1	0	1	1	4	−2
χ6	+	1	0	4	0	0	4	0	−1	−1	2	0	0	−1	3	2	0
χ7	+	2	−3	−7	1	1	1	1	0	−2	1	−1	0	3	1	0	−2
χ8	+	0	1	−5	5	1	−1	−1	2	0	−1	1	−1	−3	−1	−3	−7
χ9	+	0	4	0	4	4	0	0	−2	1	1	−1	0	−4	2	−1	−1
χ10	o	0	1	3	1	−3	−1	−1	1	0	0	0	0	2	−4	−1	−1
χ11	o	0	1	3	1	−3	−1	−1	1	0	0	0	0	2	−4	−1	−1
χ12	o	0	3	1	−1	3	−3	1	0	−1	0	0	0	−3	3	0	0
χ13	o	0	3	1	−1	3	−3	1	0	−1	0	0	0	−3	3	0	0
χ14	+	1	2	6	−2	−2	−2	2	−1	0	−1	1	0	−6	−4	3	−1
χ15	+	−2	8	0	0	0	0	0	2	0	0	0	−1	4	2	1	5

		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		96	96	48	48	56	28	28	30	30	30	32	32	18	20	20	23	23	24	24	28	30	30	30	
	p power	BD	EB	AE	EF	AA	AB	AB	BB	AA	AA	D	C	AB	AB	CG	A	A	CA	BB	AA	ABC	BAA	CAA	
	p' part	AD	BB	AE	BF	AA	AB	AB	BB	AA	AA	A	A	AA	AB	BG	A	A	BA	AB	AA	ABC	BAA	CAA	
	ind	12E	12F	12G	12H	14A	14B	C**	15A	15B	C**	16A	16B	18A	20A	20B	23A	B**	24A	24B	28A	30A	30B	C**	
χ1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ1
χ2	+	2	1	0	−1	−2	0	0	0	1	1	1	−1	0	0	−1	0	0	−1	0	0	−2	−1	−1	χ2
χ3	+	2	0	−2	0	1	−1	−1	0	0	0	1	−1	−1	−1	1	0	0	0	0	1	2	0	0	χ3
χ4	+	1	0	1	0	2	0	0	−1	0	0	−1	1	0	0	−1	−1	−1	0	−1	−2	1	0	0	χ4
χ5	+	1	−2	1	0	0	0	0	1	−1	−1	1	1	0	0	−1	0	0	0	−1	0	−1	1	1	χ5
χ6	+	−1	0	−1	2	1	−1	−1	1	−1	−1	0	0	1	−1	0	0	0	0	1	1	−1	−1	−1	χ6
χ7	+	−1	−2	1	0	−2	0	0	1	−1	−1	−1	−1	0	0	1	0	0	0	−1	0	1	1	1	χ7
χ8	+	1	1	−1	1	0	0	0	−1	0	0	−1	1	0	0	1	0	0	1	1	0	−1	0	0	χ8
χ9	+	0	−1	2	−1	0	0	0	−1	0	0	0	0	0	1	1	0	0	1	0	0	1	−2	−2	χ9
χ10	o	−2	−1	0	−1	0	0	0	0	0	0	1	−1	1	0	0	b23	**	1	0	0	0	0	0	χ10
χ11	o	−2	−1	0	−1	0	0	0	0	0	0	1	−1	1	0	0	**	b23	1	0	0	0	0	0	χ11
χ12	o	1	0	−1	0	0	0	0	0	b15	**	1	−1	0	1	0	−1	−1	0	1	0	0	−b15	**	χ12
χ13	o	1	0	−1	0	0	0	0	0	**	b15	1	−1	0	1	0	−1	−1	0	1	0	0	**	−b15	χ13
χ14	+	−2	−1	0	−1	1	1	1	−1	0	0	0	0	−1	0	1	0	0	−1	0	1	−1	0	0	χ14
χ15	+	0	1	2	−1	−2	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	2	0	0	0	χ15

		1A	2A	2B	2C	3A	3B	4A	4B	4C	4D	4E	4F	4G	5A	5B	6A	6B	6C	6D	6E	6F	7A	8A	8B	8C	8D	8E	8F	9A	10A	10B	10C	11A	12A	12B	12C	12D	12E	12F	12G	12H	14A	14B	C**	15A	15B	C**	16A	16B	18A	20A	20B	23A	B**	24A	24B	28A	30A	30B	C**	
x_{16}	+	31625	265	-55	-55	35	35	377	25	-7	-7	-7	1	5	0	0	35	-5	-5	-5	-1	-1	-1	-7	1	5	-3	1	1	-1	0	0	0	0	-1	-1	-1	5	-1	1	-1	1	-1	1	1	0	0	0	-1	-1	1	0	0	0	0	-1	1	-1	0	0	0	x_{16}
x_{17}	+	31625	1385	505	145	35	35	41	45	53	1	5	25	5	0	0	-5	-1	5	11	7	1	-1	1	-1	1	5	3	-1	-1	0	0	0	0	5	-1	-1	-1	1	3	-1	1	-1	1	1	0	0	0	1	-1	-1	0	0	0	0	1	-1	-1	0	0	0	x_{17}
x_{18}	+	31878	-378	518	-26	45	45	-42	-26	-26	-10	22	-2	-6	3	3	5	9	-15	-3	5	1	0	-2	-2	-6	2	-2	2	0	3	-3	-1	0	3	1	-3	1	-1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	0	0	1	1	0	0	0	0	x_{18}
x_{19}	+	37422	1134	462	-66	0	81	126	86	6	-18	6	-10	-6	-3	7	0	0	9	9	-3	-3	0	-2	6	2	2	-2	-2	0	-3	-1	-1	0	0	0	-3	3	0	-1	0	-1	0	0	0	1	0	0	0	0	0	1	-1	1	1	1	0	0	-1	0	0	x_{19}
x_{20}	+	44275	-1869	595	-29	-5	94	35	-85	59	-13	11	-5	-1	0	5	-5	3	6	-18	-2	-2	0	3	-5	-1	-1	3	-1	-2	0	1	1	0	-1	-1	2	2	-1	2	-1	-2	0	0	0	-1	0	0	1	1	0	0	-1	0	0	0	1	0	1	0	0	x_{20}
x_{21}	+	63250	-110	210	-110	-65	-20	322	-30	2	2	2	10	-10	0	0	15	7	10	4	0	-2	-2	-6	2	-2	6	2	2	1	0	0	0	0	7	-1	4	-4	-1	0	-1	-2	2	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	x_{21}
x_{22}	o	91125	405	-315	45	0	0	-27	-15	9	-51	9	-3	5	0	0	0	0	0	0	0	0	-1	-3	3	1	5	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	17	-17	0	0	0	-1	1	0	0	0	-1	-1	0	0	1	0	0	0	x_{22}
x_{23}	o	91125	405	-315	45	0	0	-27	-15	9	-51	9	-3	5	0	0	0	0	0	0	0	0	-1	-3	3	1	5	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	-17	17	0	0	0	-1	1	0	0	0	-1	-1	0	0	1	0	0	0	x_{23}
x_{24}	+	113850	954	-70	154	45	-9	138	10	10	-22	10	2	6	0	-5	5	9	9	-9	-1	1	2	2	-2	-2	6	-2	-2	0	0	-1	-1	0	3	1	3	1	-1	1	1	-1	2	0	0	1	0	0	0	0	0	0	1	0	0	-1	1	-2	-1	0	0	x_{24}
x_{25}	+	129536	512	-512	0	-64	44	512	0	0	0	0	0	0	-14	-4	16	8	-16	-4	4	0	1	0	0	0	0	0	0	-1	-2	2	0	0	8	0	-4	0	0	0	0	0	1	-1	-1	-1	1	1	0	0	-1	0	0	0	0	0	0	1	-1	1	1	x_{25}
x_{26}	+	177100	-1204	140	44	-20	-29	588	-20	12	-20	12	-20	4	0	-5	20	-16	11	11	-1	-1	0	4	0	-4	-4	0	0	1	0	1	-1	0	-6	0	3	-3	-2	1	0	1	0	0	0	1	0	0	0	0	-1	0	-1	0	0	1	0	0	1	0	0	x_{26}
x_{27}	+	184437	2997	405	261	0	0	-27	-3	45	21	-3	-3	9	12	-3	0	0	0	0	0	0	1	-3	-3	1	1	5	1	0	0	-3	1	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	-1	-1	0	2	-1	0	0	0	0	1	0	0	0	x_{27}
x_{28}	+	212520	-2520	1064	40	111	120	-56	-104	-40	8	8	8	0	-5	0	-1	-9	0	0	-4	4	0	0	-4	0	0	-4	0	0	-1	0	0	0	7	-1	4	2	-1	-2	-1	2	0	0	0	0	1	1	0	0	0	1	0	0	0	0	-1	0	0	-1	-1	x_{28}
x_{29}	+	221375	4095	735	15	-160	-25	-49	-25	87	-1	-9	7	-5	0	0	0	0	-15	15	3	-3	0	-1	-9	3	3	-1	-1	2	0	0	0	0	-4	0	-1	3	-4	-1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	-1	0	0	0	0	0	x_{29}
x_{30}	+	226688	4480	896	128	-4	68	0	64	64	0	0	0	0	13	-2	-4	-20	10	4	-4	2	0	0	0	0	0	0	0	-1	1	0	-2	0	0	4	0	-2	0	-2	0	0	0	0	0	-2	1	1	0	0	1	-1	0	0	0	0	0	0	0	1	1	x_{30}
x_{31}	o	239085	-2835	-147	45	-108	0	-147	45	45	-19	-3	-3	5	10	0	12	0	0	0	0	0	0	-3	-3	-3	-3	-3	1	0	-2	0	0	0	6	0	0	0	2	0	0	0	0	0	0	0	b15	**	1	1	0	0	0	0	0	0	0	0	b15	**	x_{31}
x_{32}	o	239085	-2835	-147	45	-108	0	-147	45	45	-19	-3	-3	5	10	0	12	0	0	0	0	0	0	-3	-3	-3	-3	-3	1	0	-2	0	0	0	6	0	0	0	2	0	0	0	0	0	0	0	**	b15	1	1	0	0	0	0	0	0	0	0	**	b15	x_{32}
x_{33}	+	245916	3996	1308	156	0	81	-36	76	12	-36	12	-4	-4	-9	1	0	0	-9	9	-3	3	-1	-4	0	-4	-4	0	0	0	3	1	1	0	0	0	-3	-3	0	1	0	-1	-1	-1	-1	1	0	0	0	0	0	1	1	0	0	-1	0	-1	1	0	0	x_{33}
x_{34}	+	253000	2120	-440	200	-125	10	104	-40	24	40	8	-24	0	0	0	-5	23	-10	2	-2	2	-1	0	4	0	0	4	0	1	0	0	0	0	5	3	2	0	1	-4	-1	0	-1	1	1	0	0	0	0	0	-1	0	0	0	0	0	1	-1	0	0	0	x_{34}
x_{35}	+	284625	-3855	1505	-55	-90	45	273	-115	5	-7	5	1	5	0	0	-10	6	15	-3	5	-1	-2	-7	3	1	-3	-1	-1	0	0	0	0	0	-6	2	-3	-1	2	-1	2	1	2	0	0	0	0	0	1	-1	0	0	0	0	0	-1	0	0	0	0	x_{35}
x_{36}	+	312984	-1512	-168	120	0	81	504	-56	-24	24	-24	8	0	9	4	0	0	-9	9	-3	3	0	8	0	0	0	0	0	0	-3	-2	0	1	0	0	-3	-3	0	1	0	-1	0	0	0	1	0	0	0	0	0	-1	0	0	0	-1	0	0	1	0	0	x_{36}
x_{37}	+	368874	810	-630	-198	0	0	378	-30	18	42	18	18	-10	24	-6	0	0	0	0	0	0	2	-6	6	-2	-2	-2	-2	0	0	0	2	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{37}
x_{38}	+	398475	-6741	1435	-61	-45	36	-21	-25	95	19	-1	-21	-9	0	-5	-5	-9	-6	-12	4	2	0	3	5	3	-1	1	1	0	0	-1	-1	0	-3	-1	0	2	1	2	-1	0	0	0	0	1	0	0	-1	1	0	0	1	0	0	0	-1	0	-1	0	0	x_{38}
x_{39}	+	398475	4011	1435	259	-45	36	-21	55	-17	19	-1	27	-1	0	-5	-5	15	6	-12	4	-2	0	3	1	3	-1	-3	1	0	0	1	-1	0	-3	-5	0	-2	1	-2	-1	0	0	0	0	1	0	0	-1	1	0	0	-1	0	0	0	1	0	1	0	0	x_{39}
x_{40}	+	430353	-5103	273	177	0	81	-63	-31	33	33	-15	-7	5	3	-2	0	0	9	9	-3	-3	0	1	-3	-3	5	-3	1	0	3	2	2	0	0	0	-3	3	0	-1	0	-1	0	0	0	1	0	0	-1	-1	0	-1	0	0	0	1	0	0	-1	0	0	x_{40}
x_{41}	+	442750	-770	1470	-130	-185	40	-210	30	-66	46	14	6	10	0	0	15	-5	-20	-8	0	-4	0	6	2	-6	2	2	-2	1	0	0	0	0	-3	3	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	x_{41}
x_{42}	+	462000	5040	560	-400	30	120	112	80	16	48	-16	-16	0	0	0	-10	18	0	0	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	4	-2	4	-2	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	x_{42}
x_{43}	+	467775	-3969	735	111	0	-81	63	55	39	15	-9	-17	-1	0	5	0	0	-9	-9	3	3	0	-1	3	-1	7	-5	-1	0	0	1	1	0	0	0	3	-3	0	1	0	1	0	0	0	-1	0	0	-1	-1	0	0	-1	1	1	-1	0	0	1	0	0	x_{43}
x_{44}	+	558900	2484	1140	276	0	-81	468	20	-12	-12	-12	4	-4	0	5	0	0	9	-9	3	-3	-1	4	0	-4	-4	0	0	0	0	-1	1	1	0	0	3	3	0	-1	0	1	-1	-1	-1	-1	0	0	0	0	0	1	0	0	1	0	-1	-1	0	0	x_{44}
x_{45}	+	637560	5880	952	280	90	-45	-168	-56	40	56	8	8	0	-15	0	10	-6	-15	3	-5	1	0	-8	0	0	0	0	0	0	-3	0	0	0	-6	-2	3	1	2	1	2	-1	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	x_{45}
x_{46}	+	664125	-1155	-35	-275	195	-30	637	85	5	13	5	-3	5	0	0	-5	15	0	-6	-2	4	0	-3	-7	-3	5	1	1	0	0	0	0	0	-11	-1	-2	2	1	-2	-1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	0	0	0	0	x_{46}
x_{47}	+	664125	-1155	-35	365	195	-30	-35	5	-11	45	5	-3	5	0	0	-5	15	0	-6	-2	-4	0	-3	5	5	-3	-3	1	0	0	0	0	0	1	7	-2	-2	-3	2	-1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	0	0	0	0	x_{47}
x_{48}	+	664125	2205	1645	-315	195	-30	77	-15	-39	37	-7	-11	5	0	0	-5	-9	0	18	10	0	0	-3	-5	1	-3	-1	-1	0	0	0	0	0	5	3	2	0	1	0	-1	-2	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	1	0	0	0	0	x_{48}
x_{49}	+	853875	7155	435	-45	135	0	-237	-45	51	-77	-13	3	-5	0	0	15	27	0	0	0	0	1	3	-1	-5	-5	-1	-1	0	0	0	0	0	-3	3	0	0	1	0	-1	0	1	1	1	0	0	0	-1	-1	0	0	0	0	0	0	-1	1	0	0	0	x_{49}
x_{50}	+	1288000	-2240	-2240	320	100	100	448	0	0	-64	0	0	0	0	0	-20	-8	-20	4	4	-4	0	0	0	0	0	0	0	1	0	0	0	-1	-2	0	4	0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	x_{50}
x_{51}	+	1291059	2835	1827	27	0	0	-189	39	-81	-69	15	-21	-5	9	-6	0	0	0	0	0	0	0	3	-3	7	3	1	1	0	-3	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-1	0	0	0	0	0	0	0	0	x_{51}
x_{52}	+	1771000	-12040	1400	-200	205	-20	-168	40	40	-40	-8	24	0	0	0	5	-7	-10	-4	-4	-2	0	0	4	0	0	4	0	1	0	0	0	0	3	1	0	-2	-1	-2	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	x_{52}
x_{53}	+	1771000	1400	-840	-40	-200	115	-168	40	-56	-8	-24	8	0	0	0	0	-4	5	-13	3	5	0	-8	0	0	0	0	0	1	0	0	0	0	-6	4	3	1	-2	1	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	x_{53}
x_{54}	+	1835008	0	0	0	-128	-128	0	0	0	0	0	0	0	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	-1	-1	0	0	0	0	0	0	x_{54}
x_{55}	+	1943040	7680	512	0	-96	-60	512	0	0	0	0	0	0	-10	0	-16	-24	0	-12	-4	0	1	0	0	0	0	0	0	0	2	0	0	0	8	0	-4	0	0	0	0	0	1	1	1	0	-1	-1	0	0	0	0	0	0	0	0	0	1	0	-1	-1	x_{55}
x_{56}	+	1992375	3255	-1225	215	180	45	-441	-25	-25	39	-9	-17	-5	0	0	20	-12	15	-3	5	-1	0	-1	3	-5	3	3	-1	0	0	0	0	0	0	-4	-3	-1	0	-1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	-1	0	0	0	0	0	x_{56}
x_{57}	+	2004750	8910	-1170	-450	0	0	-162	-90	54	-18	6	6	10	0	0	0	0	0	0	0	0	-1	6	6	2	2	-2	2	0	0	0	0	r	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	1	1	0	0	-1	0	0	0	x_{57}
x_{58}	+	2040192	-2688	896	128	-36	-36	0	-64	-64	0	0	0	0	17	2	-4	12	-6	12	-4	2	0	0	0	0	0	0	0	0	1	2	-2	0	0	-4	0	2	0	2	0	0	0	0	0	-1	-1	-1	0	0	0	1	0	0	0	0	0	0	-1	1	1	x_{58}
x_{59}	+	2072576	-8192	0	0	-160	-16	0	0	0	0	0	0	0	-24	-4	0	16	16	16	0	0	2	0	0	0	0	0	0	-1	0	-2	0	0	0	0	0	0	0	0	0	0	-2	0	0	-1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	x_{59}
x_{60}	+	2095875	-3645	-2205	-45	0	0	-189	75	27	51	27	3	-5	0	0	0	0	0	0	0	0	-2	3	-9	3	-5	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	x_{60}

CO_2

Fi_{22}

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
		64561751654400	18393661440	53084160	1769472	19595520	2519424	139968	17496	110592	46080	12288	4608	3072	600	155520	93312	10368	6912	3888	3456	3456	1728	864	432	216	42	384	384	128	32	324	162	27	60	40	22	22	864	576	576	288	144	144	144	96	96	72	36	13	13	14	30	32	32	108	108	54	36	20	21	22	22	48	48	30
		p power	A	A	A	A	A	A	A	B	B	B	C	B	A	AA	BA	BB	AB	CA	AC	BC	CC	CB	CC	DC	A	A	A	C	D	B	B	D	AA	AB	A	A	CA	DA	DA	DB	GD	GD	HD	CC	DE	IB	KD	A	A	AA	AA	C	C	AB	AB	BB	AC	BB	AA	BA	AA	BB	BA	AAA
		p' part	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	BA	BB	AB	CA	AC	BC	CC	CB	CC	DC	A	A	A	A	A	A	A	A	AA	AB	A	A	BA	AA	AA	AB	BD	BD	CD	BC	AE	CB	DD	A	A	AA	AA	A	A	AA	AA	BA	AB	AB	AA	AA	BA	AB	AA	AAA
	ind	1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	4E	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	8A	8B	8C	8D	9A	9B	9C	10A	10B	11A	B**	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	13A	B*	14A	15A	16A	B**	18A	B**	18C	18D	20A	21A	22A	B**	24A	24B	30A
χ_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	+	78	-34	14	-2	15	-3	6	-3	6	-6	-2	2	2	3	-7	-7	5	-1	2	1	1	-2	2	-2	1	1	2	-2	2	0	3	0	0	1	-1	1	1	-3	3	-3	-3	-1	-1	2	1	-1	0	-1	0	0	1	0	0	0	-1	-1	2	-1	-1	1	-1	-1	1	-1	-2
χ_3	+	429	77	45	13	6	24	15	-3	13	5	-3	5	5	4	14	-4	0	6	5	-2	4	7	3	1	1	2	1	1	1	1	0	3	0	2	0	0	0	4	-2	-2	2	2	2	-1	0	2	-1	-1	0	0	0	1	-1	-1	2	2	-1	0	0	-1	0	0	-2	-2	-1
χ_4	+	1001	-231	41	-7	56	29	2	2	1	-11	9	5	-3	1	-6	-15	5	8	-6	2	-7	2	2	2	2	0	-3	1	1	-1	2	2	-1	-1	1	0	0	1	4	-2	-2	-1	-1	2	-3	0	-2	2	0	0	0	1	1	1	0	0	0	2	-1	0	0	0	-2	0	-1
χ_5	+	1430	-154	86	6	-1	-28	26	-1	14	-6	6	10	2	5	-19	8	-4	-1	8	-3	0	-6	2	0	3	2	-2	2	2	0	-1	-1	2	1	1	0	0	-4	-1	5	-3	-2	-2	-2	0	-1	0	1	0	0	0	-1	0	0	-1	-1	-1	-1	-1	-1	0	0	-1	1	1
χ_6	+	3003	539	59	-37	105	6	15	6	11	11	-5	3	-5	3	17	26	14	-7	-1	-7	2	-1	-1	-1	2	0	-1	-1	3	-1	3	0	0	-1	-1	0	0	2	5	5	5	0	0	3	-2	1	-1	0	0	0	0	0	-1	-1	-1	-1	2	-1	1	0	0	0	-1	-1	2
χ_7	+	3080	616	136	40	119	2	20	2	24	24	8	0	8	5	31	22	10	7	4	7	-2	4	4	4	-2	0	4	4	0	0	5	-1	-1	1	1	0	0	6	3	3	3	0	0	0	2	-1	0	0	-1	-1	0	-1	0	0	1	1	1	1	-1	0	0	0	1	1	1
χ_8	+	10725	-715	165	-43	15	114	24	6	29	-15	-11	9	-7	0	5	14	-6	15	-4	5	-10	8	0	-4	2	1	1	-3	1	-1	0	3	0	0	0	0	0	2	-1	5	-3	0	0	0	-2	-1	0	0	0	0	-1	0	-1	-1	2	2	-1	0	0	1	0	0	3	1	0
χ_9	+	13650	1330	210	114	105	123	33	15	-14	10	2	10	10	0	25	7	3	9	7	9	15	9	-3	3	3	0	-2	-2	-2	2	0	3	0	0	0	-1	-1	-5	1	1	1	1	1	1	-1	1	1	1	0	0	0	0	0	0	-2	-2	1	0	0	0	-1	-1	1	1	0
χ_{10}	+	30030	1694	526	62	-21	-102	60	6	38	26	-2	18	2	5	29	-34	-14	-5	2	5	-10	-4	4	2	2	0	2	-2	2	0	-3	-3	0	-1	1	0	0	2	-1	-7	5	0	0	0	-2	-1	2	0	0	0	0	-1	0	0	-1	-1	-1	1	1	0	0	0	1	-1	-1
χ_{11}	+	32032	-2464	544	-32	91	-44	64	10	64	-24	0	0	8	7	-79	20	4	-5	2	1	4	-8	4	-2	-2	0	4	-4	0	0	1	1	-2	1	-1	0	0	-8	-5	1	-3	0	0	0	0	-1	0	0	0	0	0	1	0	0	-1	-1	-1	1	1	0	0	0	-1	1	1
χ_{12}	+	43680	-4256	416	-32	399	-60	48	-6	0	-24	0	0	8	5	-71	-44	20	-1	10	1	4	-8	-4	-2	-2	0	-4	4	0	0	3	3	0	-1	1	-1	-1	0	3	-3	-3	0	0	0	0	-1	0	0	0	0	0	-1	0	0	1	1	1	-1	1	0	1	1	1	-1	-1
χ_{13}	+	45045	4389	309	133	441	90	-18	9	29	41	5	1	1	-5	-21	42	-6	9	6	19	10	-2	6	-2	1	0	1	5	1	-1	0	0	0	-1	-1	0	0	2	5	-1	-1	-2	-2	-2	2	1	2	1	0	0	0	1	-1	-1	0	0	0	0	1	0	0	0	-1	1	-1
χ_{14}	+	48048	-1232	432	48	-84	258	6	-12	-16	0	-16	16	0	-2	-44	10	18	12	-8	-12	-6	6	6	0	0	0	0	0	0	0	0	3	0	-2	2	0	0	2	-4	-4	0	-2	-2	-2	2	0	0	-2	0	0	0	1	0	0	-2	-2	1	0	0	0	0	0	0	0	1
χ_{15}	+	50050	-5390	450	-46	595	73	10	19	10	-50	-14	-2	6	0	-35	-71	9	3	-8	5	-7	2	6	-4	-1	0	2	-2	-2	0	4	-2	1	0	0	0	0	1	7	1	1	1	1	-2	1	3	-2	1	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	1	-1	0
χ_{16}	+	50050	770	130	-126	-35	235	19	-8	-14	10	18	18	-6	0	5	-13	-5	13	5	-3	3	3	-5	-3	0	0	2	2	-2	-2	1	1	1	0	0	0	0	-5	1	1	1	3	3	3	3	-3	1	0	0	0	0	0	0	0	-1	-1	-1	1	0	0	0	0	-1	-1	0
χ_{17}	+	75075	1155	835	131	-210	150	51	-12	83	-5	19	3	11	0	30	-6	-2	-2	3	-10	2	11	7	-1	-4	0	3	3	-1	-1	-6	0	0	0	0	0	0	2	2	2	-2	0	0	3	-2	2	1	0	0	0	0	0	1	1	0	0	0	-2	0	0	0	0	0	0	0
χ_{18}	+	75075	7315	515	51	735	-93	-3	-12	-5	55	-13	7	-1	0	25	79	11	-1	-11	9	-9	-3	5	-3	0	0	-5	-1	-1	1	3	0	0	0	0	0	0	-5	7	1	1	1	1	1	-1	-1	1	-2	0	0	0	0	1	1	1	1	-2	-1	0	0	0	0	-1	1	0
χ_{19}	+	75075	-5005	-125	83	420	-12	42	15	-5	15	3	-1	-9	0	-10	-64	28	4	-10	-10	8	2	-2	2	-1	0	-1	3	3	1	-3	-3	0	0	0	0	0	4	4	-2	-6	2	2	2	0	0	0	-1	0	0	0	0	-1	-1	-1	-1	-1	1	0	0	0	0	0	2	0
χ_{20}	+	81081	3465	633	-87	0	162	81	0	33	5	-7	-3	-3	6	90	-18	-6	0	9	-6	6	9	-3	-3	0	0	-7	-3	1	-1	0	0	0	0	-2	0	0	6	0	-6	2	0	0	-3	2	0	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	2	0
χ_{21}	+	114400	-8800	480	32	685	28	-8	-26	64	-40	0	0	-8	0	-25	-52	12	-3	2	-1	-4	8	0	2	2	-1	4	-4	0	0	4	-2	1	0	0	0	0	-8	1	-5	-1	0	0	0	0	1	2	0	0	0	-1	0	0	0	2	2	2	0	0	-1	0	0	-1	1	0
χ_{22}	+	138600	-9240	360	-24	630	-153	-45	9	8	-40	8	8	-8	0	30	-33	-9	6	3	6	15	3	3	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	2	2	-1	-1	-1	-1	-2	-1	-1	-b13	*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	+	138600	-9240	360	-24	630	-153	-45	9	8	-40	8	8	-8	0	30	-33	-9	6	3	6	15	3	3	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	2	2	-1	-1	-1	-1	-2	-1	-1	*	-b13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{24}	+	150150	8470	70	54	525	57	48	-24	-34	-30	22	-6	10	0	55	73	25	13	-8	-9	9	0	-8	0	0	0	2	-2	2	0	-3	0	0	0	0	0	0	-7	5	-1	3	-3	-3	0	1	1	0	0	0	0	0	0	0	0	1	1	-2	1	0	0	0	0	1	-1	0
χ_{25}	+	205920	1056	864	160	-279	-144	72	18	64	24	0	0	-8	-5	-69	-24	0	9	12	-5	-8	-8	0	4	-2	1	-4	4	0	0	0	0	0	1	-1	0	0	-8	1	-5	3	0	0	0	0	1	0	0	0	0	-1	1	0	0	0	0	0	0	-1	1	0	0	1	-1	1
χ_{26}	+	289575	12375	615	183	405	162	-81	0	15	35	23	3	-5	0	-45	-18	-6	21	9	3	6	-9	3	-3	0	-1	-1	-5	3	1	0	0	0	0	0	0	0	6	-3	3	-1	0	0	3	2	1	-1	0	0	0	-1	0	1	1	0	0	0	0	0	-1	0	0	1	-1	0
χ_{27}	+	300300	-7700	1420	-84	-210	-291	114	-21	-4	-20	-20	20	-4	0	-50	49	-11	-2	4	6	9	-6	-2	0	3	0	0	0	-4	0	-3	-3	0	0	0	0	0	5	2	2	-2	-1	-1	2	1	2	-2	-1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
χ_{28}	+	320320	14784	1344	192	406	-116	64	-8	0	16	0	0	16	-5	114	-12	12	6	6	-6	-12	0	0	6	0	0	0	0	0	0	4	-2	1	-1	-1	0	0	0	-6	6	-2	0	0	0	0	-2	-2	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	-1
χ_{29}	+	360855	18711	1431	279	729	0	0	0	-9	39	-9	-9	7	5	81	0	0	9	0	9	0	0	0	0	0	-2	-1	-1	-1	-1	0	0	0	1	1	0	0	0	-3	-3	-3	0	0	0	0	1	0	0	1	1	0	-1	-1	-1	0	0	0	0	-1	1	0	0	-1	-1	1
χ_{30}	+	370656	-15840	1248	-96	405	324	0	0	0	-40	0	0	-8	6	-45	36	-12	21	-18	3	-12	0	0	6	0	-1	-4	4	0	0	0	0	0	0	-2	0	0	0	-3	3	-1	0	0	0	0	1	2	0	0	0	1	0	0	0	0	0	0	0	0	-1	0	0	1	-1	0
χ_{31}	+	400400	6160	1040	16	-280	-307	-10	17	-48	0	16	16	0	0	40	-23	5	8	-14	-8	1	-2	2	-2	1	0	0	0	0	0	-1	-1	-1	0	0	0	0	-3	0	0	0	7	-5	-2	1	0	0	1	0	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0
χ_{32}	+	400400	6160	1040	16	-280	-307	-10	17	-48	0	16	16	0	0	40	-23	5	8	-14	-8	1	-2	2	-2	1	0	0	0	0	0	-1	-1	-1	0	0	0	0	-3	0	0	0	-5	7	-2	1	0	0	1	0	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0
χ_{33}	o	400400	-18480	-240	80	980	98	-28	-10	16	0	-16	0	0	0	60	-66	-6	-12	6	-4	-10	-4	0	2	2	0	0	0	0	0	-4	2	-1	0	0	0	0	-2	4	4	0	0	0	0	2	0	0	0	0	0	0	0	212	-212	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	o	400400	-18480	-240	80	980	98	-28	-10	16	0	-16	0	0	0	60	-66	-6	-12	6	-4	-10	-4	0	2	2	0	0	0	0	0	-4	2	-1	0	0	0	0	-2	4	4	0	0	0	0	2	0	0	0	0	0	0	0	-212	212	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	+	450450	6930	1170	-110	-315	-72	9	9	130	10	-30	-14	-6	0	45	-36	0	21	-9	-11	4	1	-3	-5	1	0	2	2	-2	2	0	0	0	0	0	0	0	4	1	1	1	-2	-2	1	0	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0
χ_{36}	+	450450	25410	210	-350	1575	171	-18	9	-6	30	2	-2	-10	0	-15	111	3	-9	12	-23	7	-2	6	4	1	0	2	-2	-2	0	0	0	0	0	0	0	0	3	3	-3	-3	1	1	-2	-1	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0
χ_{37}	+	576576	14784	576	-320	126	180	72	18	0	-16	0	0	-16	1	114	-12	36	-18	6	10	4	-8	0	-2	-2	0	0	0	0	0	0	0	0	-1	1	0	0	0	-6	6	2	0	0	0	0	2	2	0	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	0	0	-1
χ_{38}	+	577368	-21384	216	-72	729	0	0	0	72	-24	24	0	-8	-7	81	0	0	9	0	9	0	0	0	0	0	1	-4	-4	0	0	0	0	0	1	1	0	0	0	-3	-3	-3	0	0	0	0	1	0	0	-1	-1	1	-1	0	0	0	0	0	0	1	1	0	0	-1	-1	1
χ_{39}	+	579150	-6930	1230	174	-405	324	81	0	-18	-10	30	6	-10	0	-45	36	-12	-21	9	3	-12	9	-3	-3	0	-2	2	2	2	2	0	0	0	0	0	0	0	0	3	3	-1	0	0	-3	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	-1	0
χ_{40}	o	582400	-8960	-1280	256	280	172	64	10	0	0	0	0	0	0	40	4	-20	-8	-14	-8	4	-8	4	-2	-2	0	0	0	0	0	1	1	1	0	0	b11	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	b11	**	0	0	0
χ_{41}	o	582400	-8960	-1280	256	280	172	64	10	0	0	0	0	0	0	40	4	-20	-8	-14	-8	4	-8	4	-2	-2	0	0	0	0	0	1	1	1	0	0	**	b11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	**	b11	0	0	0
χ_{42}	+	600600	21560	920	-136	525	-96	12	-42	72	40	24	0	-8	0	5	-40	-16	-19	-22	5	8	-4	-4	2	2	0	4	4	0	0	3	3	0	0	0	0	0	-3	-3	1	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	1	1	0
χ_{43}	o	600600	9240	-1000	280	210	-15	57	-15	-8	-40	-8	-8	-8	0	-30	33	17	2	-3	10	1	1	5	1	1	0	0	0	0	0	-3	0	0	0	0	0	0	1	-2	-2	2	1	1	1	1	-2	-1	1	0	0	0	0	0	0	313	-313	0	-1	0	0	0	0	0	0	0
χ_{44}	o	600600	9240	-1000	280	210	-15	57	-15	-8	-40	-8	-8	-8	0	-30	33	17	2	-3	10	1	1	5	1	1	0	0	0	0	0	-3	0	0	0	0	0	0	1	-2	-2	2	1	1	1	1	-2	-1	1	0	0	0	0	0	0	-313	313	0	-1	0	0	0	0	0	0	0
χ_{45}	+	675675	10395	-165	411	0	135	-54	54	75	-45	11	-5	3	0	0	27	15	0	0	0	3	-6	-6	0	6	0	3	3	-1	-1	0	0	0	0	0	0	0	3	0	0	0	1	1	-2	-1	0	0	-2	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	+	720720	-33264	1104	-112	1386	-18	36	-18	-80	-16	16	0	16	-5	-54	-54	6	-6	0	2	2	-4	0	-4	2	0	0	0	0	0	0	0	0	1	-1	0	0	10	-2	-2	2	0	0	0	-2	-2	2	0	0	0	0	1	0	0	0	0	0	0	-1	0	0	0	0	0	1
χ_{47}	+	800800	-12320	800	-416	-245	520	16	34	0	40	0	0	8	0	-35	-8	8	11	-8	13	-8	-8	-4	4	-2	0	4	-4	0	0	-5	1	1	0	0	0	0	3	-3	1	0	0	0	0	-1	-2	0	0	0	0	0	0	0	0	1	1	1	-1	0	0	0	0	-1	1	0
χ_{48}	+	852930	-29646	1026	18	729	0	0	0	-54	6	18	-18	-2	5	-81	0	0	9	0	-9	0	0	0	0	0	1	-2	2	-2	0	0	0	0	-1	1	1	1	0	-3	3	3	0	0	0	0	1	0	0	0	0	-1	-1	0	0	0	0	0	0	1	1	-1	-1	-1	1	-1
χ_{49}	+	938223	-2673	1647	207	-729	0	0	0	63	15	15	-9	-1	-2	-81	0	0	-9	0	-9	0	0	0	0	0	-1	-5	-5	-1	-1	0	0	0	2	2	0	0	0	3	3	3	0	0	0	0	-1	0	0	0	0	1	1	-1	-1	0	0	0	0	0	-1	0	0	1	1	-1
χ_{50}	+	972972	-24948	1836	-180	0	-243	0	0	60	-36	-20	-12	12	-3	0	81	-27	0	0	0	9	0	0	0	0	0	0	0	4	0	0	0	0	-3	1	0	0	-3	0	0	0	3	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
χ_{51}	o	982800	15120	-240	-240	0	-135	54	27	-48	0	16	-16	0	0	0	-27	-15	0	0	0	-3	6	6	0	3	0	0	0	0	0	0	0	0	0	0	b11	**	-3	0	0	0	-1	-1	2	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-b11	**	0	0	0
χ_{52}	o	982800	15120	-240	-240	0	-135	54	27	-48	0	16	-16	0	0	0	-27	-15	0	0	0	-3	6	6	0	3	0	0	0	0	0	0	0	0	0	0	**	b11	-3	0	0	0	-1	-1	2	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	**	-b11	0	0	0
χ_{53}	+	1201200	-30800	560	176	420	-30	-48	51	-16	0	-16	16	0	0	-20	34	2	-28	-2	-4	2	8	-4	2	-1	0	0	0	0	0	3	0	0	0	0	0	0	2	-4	-4	0	-2	-2	4	2	0	0	1	0	0	0	0	0	0	1	1	-2	-1	0	0	0	0	0	0	0
χ_{54}	+	1360800	30240	1440	288	0	486	0	0	-96	0	-32	0	0	0	0	-54	-18	0	0	0	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-6	0	0	0	0	0	0	-2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
χ_{55}	+	1372800	49280	640	128	1560	-312	-24	12	0	0	0	0	0	0	-40	32	-8	-8	-4	8	-16	8	-8	-4	-4	2	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	1	0	-1	0	0	0	0	0

Fi_{22}

		1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	4E	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	8A	8B
X_{56}	+	1441792	0	0	0	-512	640	64	-8	0	0	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	2	0	0
X_{57}	+	1791153	-5103	-2511	81	0	0	0	0	81	9	33	9	9	3	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3
X_{58}	+	1876446	37422	-1890	-306	729	0	0	0	54	-30	-18	18	10	-4	-81	0	0	9	0	-9	0	0	0	0	0	-2	-2	2
X_{59}	+	2027025	-24255	-1455	33	0	-324	81	0	9	85	-15	-3	-3	0	-90	-36	12	0	-9	6	12	9	-3	3	0	0	1	-3
X_{60}	+	2050048	0	2048	0	-1232	-224	-80	-8	0	0	0	0	0	-2	0	0	32	-16	0	0	0	0	8	0	0	0	0	0
X_{61}	+	2316600	-43560	-840	24	405	-162	0	0	-24	40	-8	0	-8	0	45	18	6	21	18	-3	-6	0	0	-6	0	-1	4	4
X_{62}	+	2402400	12320	-160	160	-735	-384	48	-6	-64	-40	0	0	-8	0	35	8	-16	17	8	-5	-8	-8	-4	4	-2	0	4	-4
X_{63}	+	2555904	32768	0	0	-384	192	-96	-24	0	0	0	0	0	4	-64	-64	0	0	8	0	0	0	0	0	0	1	0	0
X_{64}	+	2555904	-32768	0	0	-384	192	-96	-24	0	0	0	0	0	4	64	64	0	0	-8	0	0	0	0	0	0	1	0	0
X_{65}	+	2729376	7776	-864	-288	-729	0	0	0	0	-24	0	0	8	1	81	0	0	-9	0	9	0	0	0	0	0	-1	-4	4
Ind		1 2	2 2	2 2	2	3 6	3 6	3 6	3 6	4	4 4	4	4 4	4	5 10	6 6	6 6	6 6	6 6	6 6	6	6 6	6 6	6 6	6 6	6 6	7 14	8	8
X_{66}	+	352	0	32	0	-8	28	10	1	0	0	0	8	0	2	0	0	-4	8	0	0	0	-6	2	0	3	2	0	0
X_{67}	+	2080	384	96	0	64	-26	19	1	0	16	0	8	0	5	24	6	6	0	-3	0	6	3	3	-3	3	1	0	0
X_{68}	+	2080	-384	96	0	64	-26	19	1	0	-16	0	8	0	5	-24	-6	6	0	3	0	-6	3	3	3	3	1	0	0
X_{69}	o	5824	896	64	0	154	-8	-8	-8	0	16	0	0	0	-1	-4	32	-8	-2	-4	0	0	0	4	0	0	0	0	0
X_{70}	o	5824	-896	64	0	154	-8	-8	-8	0	-16	0	0	0	-1	4	-32	-8	-2	4	0	0	0	4	0	0	0	0	0
X_{71}	o	5824	-896	64	0	154	-8	-8	-8	0	-16	0	0	0	-1	4	-32	-8	-2	4	0	0	0	4	0	0	0	0	0
X_{72}	o	5824	896	64	0	154	-8	-8	-8	0	16	0	0	0	-1	-4	32	-8	-2	-4	0	0	0	4	0	0	0	0	0
X_{73}	+	13728	1408	224	0	120	120	39	12	0	16	0	8	0	3	40	4	8	8	13	0	-12	-9	-1	-3	0	1	0	0
X_{74}	+	13728	-1408	224	0	120	120	39	12	0	-16	0	8	0	3	-40	-4	8	8	-13	0	12	-9	-1	3	0	1	0	0
X_{75}	+	27456	0	448	0	-120	-84	60	-3	0	0	0	16	0	6	0	0	-20	-8	0	0	0	12	4	0	3	2	0	0
X_{76}	+	48048	4928	272	0	546	96	-3	15	0	40	0	4	0	-2	-4	68	-16	-10	5	0	-12	-3	5	-3	-3	0	0	0
X_{77}	+	48048	-4928	272	0	546	96	-3	15	0	-40	0	4	0	-2	4	-68	-16	-10	-5	0	12	-3	5	3	-3	0	0	0
X_{78}	o	105600	0	-640	0	120	-24	48	-24	0	0	0	0	0	0	0	0	8	8	0	0	0	0	8	0	0	-2	0	0
X_{79}	o	105600	0	-640	0	120	-24	48	-24	0	0	0	0	0	0	0	0	8	8	0	0	0	0	8	0	0	-2	0	0
X_{80}	+	123200	0	960	0	-280	404	62	-28	0	0	0	16	0	0	0	0	-12	24	0	0	0	-18	6	0	0	0	0	0
X_{81}	+	133056	0	-192	0	0	-108	-54	54	0	0	0	16	0	6	0	0	-12	0	0	0	0	-6	-6	0	-6	0	0	0
X_{82}	+	146432	11264	1024	0	776	-16	56	-16	0	64	0	0	0	7	104	32	16	-8	-4	0	0	0	4	0	0	-1	0	0
X_{83}	+	146432	-11264	1024	0	776	-16	56	-16	0	-64	0	0	0	7	-104	-32	16	-8	4	0	0	0	4	0	0	-1	0	0
X_{84}	+	228800	0	320	0	-160	380	-52	29	0	0	0	16	0	0	0	0	-4	32	0	0	0	12	-4	0	3	-2	0	0
X_{85}	+	235872	8064	1056	0	0	-324	81	0	0	16	0	24	0	-3	72	-36	12	0	-9	0	12	9	-3	3	0	0	0	0
X_{86}	+	235872	-8064	1056	0	0	-324	81	0	0	-16	0	24	0	-3	-72	36	12	0	9	0	-12	9	-3	-3	0	0	0	0
X_{87}	+	320320	9856	-576	0	406	-116	64	-8	0	-16	0	0	0	-5	-44	28	-36	18	10	0	12	0	0	-6	0	0	0	0
X_{88}	+	320320	-9856	-576	0	406	-116	64	-8	0	16	0	0	0	-5	44	-28	-36	18	-10	0	-12	0	0	6	0	0	0	0
X_{89}	+	400400	24640	560	0	1610	-64	-37	17	0	40	0	-4	0	0	-20	124	-16	14	7	0	12	3	-1	3	3	0	0	0
X_{90}	+	400400	-24640	560	0	1610	-64	-37	17	0	-40	0	-4	0	0	20	-124	-16	14	-7	0	-12	3	-1	-3	3	0	0	0
X_{91}	+	436800	17920	-320	0	840	210	-42	-33	0	0	0	16	0	0	-80	46	-14	-8	-8	0	-18	6	-2	0	-3	0	0	0
X_{92}	+	436800	-17920	-320	0	840	210	-42	-33	0	0	0	16	0	0	80	-46	-14	-8	8	0	18	6	-2	0	-3	0	0	0
X_{93}	+	480480	9856	1696	0	-336	-174	69	15	0	-16	0	-8	0	5	136	-26	-14	16	1	0	6	-3	1	-3	-3	0	0	0
X_{94}	+	480480	-9856	1696	0	-336	-174	69	15	0	16	0	-8	0	5	-136	26	-14	16	-1	0	-6	-3	1	3	-3	0	0	0
X_{95}	+	686400	28160	960	0	960	6	78	-21	0	0	0	-16	0	0	80	26	6	0	-10	0	-6	6	-6	-6	-3	1	0	0
X_{96}	+	686400	-28160	960	0	960	6	78	-21	0	0	0	-16	0	0	-80	-26	6	0	10	0	6	6	-6	6	-3	1	0	0
X_{97}	+	915200	28160	1280	0	440	548	8	8	0	0	0	0	0	0	80	-28	20	8	26	0	-12	0	-4	6	0	-1	0	0
X_{98}	+	915200	-28160	1280	0	440	548	8	8	0	0	0	0	0	0	-80	28	20	8	-26	0	12	0	-4	-6	0	-1	0	0
X_{99}	+	1029600	0	1440	0	-720	252	90	9	0	0	0	8	0	0	0	0	-36	-48	0	0	0	-6	-6	0	3	-2	0	0
X_{100}	+	1201200	24640	1680	0	-210	132	-39	24	0	40	0	-12	0	0	-20	-92	-12	-6	-11	0	-12	9	9	-3	0	0	0	0
X_{101}	+	1201200	-24640	1680	0	-210	132	-39	24	0	-40	0	-12	0	0	20	92	-12	-6	11	0	12	9	9	3	0	0	0	0
X_{102}	+	1297296	44352	1200	0	1134	-324	-81	0	0	40	0	12	0	-4	-36	-36	12	-6	-9	0	12	-9	3	3	0	0	0	0
X_{103}	+	1297296	-44352	1200	0	1134	-324	-81	0	0	-40	0	12	0	-4	36	36	12	-6	9	0	-12	-9	3	-3	0	0	0	0
X_{104}	+	1441792	0	0	0	-512	640	64	-8	0	0	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	2	0	0
X_{105}	+	1663200	0	-2400	0	0	108	54	27	0	0	0	8	0	0	0	0	12	0	0	0	0	6	6	0	-3	0	0	0
X_{106}	+	1663200	0	-2400	0	0	108	54	27	0	0	0	8	0	0	0	0	12	0	0	0	0	6	6	0	-3	0	0	0
X_{107}	+	2196480	11264	-1024	0	-456	-240	-24	-24	0	-64	0	0	0	5	104	32	-16	8	-4	0	0	0	-4	0	0	-1	0	0
X_{108}	+	2196480	-11264	-1024	0	-456	-240	-24	-24	0	64	0	0	0	5	-104	-32	-16	8	4	0	0	0	-4	0	0	-1	0	0
X_{109}	+	2358720	8064	1344	0	-1134	-324	0	0	0	16	0	0	0	-5	-36	-36	12	6	18	0	12	0	0	-6	0	0	0	0
X_{110}	+	2358720	-8064	1344	0	-1134	-324	0	0	0	-16	0	0	0	-5	36	36	12	6	-18	0	-12	0	0	6	0	0	0	0
X_{111}	+	2402400	49280	-1760	0	840	-60	66	21	0	-80	0	-8	0	0	-40	-32	4	-8	-4	0	0	-6	-2	0	3	0	0	0
X_{112}	+	2402400	-49280	-1760	0	840	-60	66	21	0	80	0	-8	0	0	40	-32	4	-8	4	0	0	-6	-2	0	3	0	0	0
X_{113}	+	2555904	32768	0	0	-384	192	-96	-24	0	0	0	0	0	4	-64	-64	0	0	8	0	0	0	0	0	0	1	0	0
X_{114}	+	2555904	-32768	0	0	-384	192	-96	-24	0	0	0	0	0	4	64	64	0	0	-8	0	0	0	0	0	0	1	0	0

	8C	8D	9A	9B	9C	10A	10B	11A	B**	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	13A	B*	14A	15A	16A	B**	18A	B**	18C	18D	20A	21A	22A	B**	24A	24B	30A
X_{56}	0	0	4	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	-2	0	0	0	0	0	0	0	-1	0	0	0	0	0
X_{57}	-3	1	0	0	0	-3	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	0	1	1	0	0	0
X_{58}	2	0	0	0	0	2	0	0	0	0	-3	3	3	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	-1	1	-1
X_{59}	1	-1	0	0	0	0	0	0	0	0	0	6	-2	0	0	-3	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	-2	0
X_{60}	0	0	-2	4	1	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	2	0	0	0	0	0	0	0
X_{61}	0	0	0	0	0	0	0	0	0	-6	-3	-3	1	0	0	0	-2	1	-2	0	0	0	1	0	0	0	0	0	0	0	0	-1	0	0	1	1	0
X_{62}	0	0	3	3	0	0	0	0	0	8	5	-1	-1	0	0	0	0	1	2	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	-1	1	0
X_{63}	0	0	0	-3	0	-2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2	2	-1	0	0	1	-1	-1	0	0	1
X_{64}	0	0	0	-3	0	2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	-2	-2	1	0	0	1	1	1	0	0	-1
X_{65}	0	0	0	0	0	1	1	1	1	0	3	-3	-3	0	0	0	0	-1	0	0	0	0	-1	1	0	0	0	0	0	0	1	-1	-1	-1	1	-1	1
Ind	8	8 8	9 18	9 18	9 18	10 10	10 10	11 22	11 22	12	12 12	12	12 12	12 12	12 12	12 12	12	12	12 12	12 12	13 26	13 26	14 14	15 30	16	16	18 18	18 18	18 18	18 18	20 20	21 42	22 22	22 22	24	24	30 30
X_{66}	0	0	-2	4	1	0	2	0	0	0	0	0	0	-2	-2	2	0	0	0	-1	1	1	0	2	0	0	0	0	0	2	0	-1	0	0	0	0	0
X_{67}	0	0	4	-2	1	-1	1	1	1	0	0	0	4	-2	4	-1	0	0	1	-1	0	0	-1	-1	0	0	0	0	0	0	1	1	-1	-1	0	0	-1
X_{68}	0	0	4	-2	1	1	1	1	1	0	0	0	-4	4	-2	-1	0	0	-1	-1	0	0	1	-1	0	0	0	0	0	0	-1	1	1	1	0	0	1
X_{69}	0	0	1	1	1	1	-1	b11	**	0	6	0	-2	0	0	0	0	0	-2	0	0	0	0	-1	0	0	-1	-1	-1	1	1	0	b11	**	0	0	1
X_{70}	0	0	1	1	1	-1	-1	**	b11	0	6	0	2	0	0	0	0	0	2	0	0	0	0	-1	0	0	1	1	1	1	-1	0	**	-b11	0	0	-1
X_{71}	0	0	1	1	1	-1	-1	b11	**	0	6	0	2	0	0	0	0	0	2	0	0	0	0	-1	0	0	1	1	1	1	-1	0	-b11	**	0	0	-1
X_{72}	0	0	1	1	1	1	-1	**	b11	0	6	0	-2	0	0	0	0	0	-2	0	0	0	0	-1	0	0	-1	-1	-1	1	1	0	**	b11	0	0	1
X_{73}	0	0	3	3	0	3	-1	0	0	0	0	0	4	-2	-2	-1	0	0	1	2	0	0	1	0	0	0	1	1	1	-1	1	1	0	0	0	0	0
X_{74}	0	0	3	3	0	-3	-1	0	0	0	0	0	-4	-2	-2	-1	0	0	-1	2	0	0	-1	0	0	0	-1	-1	-1	-1	-1	1	0	0	0	0	0
X_{75}	0	0	-6	0	0	0	-2	0	0	0	0	0	0	2	2	4	0	0	0	1	0	0	0	0	0	0	0	0	0	-2	0	-1	0	0	0	0	0
X_{76}	0	2	3	0	0	-2	2	0	0	0	6	0	-2	2	2	1	0	0	1	1	0	0	0	1	0	0	-1	-1	2	-1	0	0	0	0	0	0	1
X_{77}	0	-2	3	0	0	2	2	0	0	0	6	0	2	2	2	1	0	0	-1	1	0	0	0	1	0	0	1	1	-2	-1	0	0	0	0	0	0	-1
X_{78}	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	313	-313	0	-1	0	1	0	0	0	0	0
X_{79}	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-313	313	0	-1	0	1	0	0	0	0	0
X_{80}	0	0	-4	2	-1	0	0	0	0	0	0	0	0	2	2	-2	0	0	0	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{81}	0	0	0	0	0	0	-2	0	0	0	0	0	0	2	2	-2	0	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{82}	0	0	5	-1	-1	-1	-1	0	0	0	0	0	4	0	0	0	0	0	-2	0	0	0	1	1	0	0	-1	-1	-1	1	-1	-1	0	0	0	0	-1
X_{83}	0	0	5	-1	-1	1	-1	0	0	0	0	0	-4	0	0	0	0	0	2	0	0	0	-1	1	0	0	1	1	1	1	1	-1	0	0	0	0	1
X_{84}	0	0	2	2	-1	0	0	0	0	0	0	0	0	2	2	4	0	0	0	1	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0
X_{85}	0	0	0	0	0	-1	1	-1	-1	0	0	0	4	0	0	-3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	2
X_{86}	0	0	0	0	0	1	1	-1	-1	0	0	0	-4	0	0	-3	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1	-1	0	0	-2
X_{87}	0	0	4	-2	1	1	-1	0	0	0	-6	0	2	0	0	0	0	0	2	0	0	0	0	1	0	0	-2	-2	-2	0	-1	0	0	0	0	0	1
X_{88}	0	0	4	-2	1	-1	-1	0	0	0	-6	0	-2	0	0	0	0	0	-2	0	0	0	0	1	0	0	2	2	2	0	1	0	0	0	0	0	-1
X_{89}	0	-2	-1	-1	-1	0	0	0	0	0	6	0	-2	-2	-2	-1	0	0	1	-1	0	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0
X_{90}	0	2	-1	-1	-1	0	0	0	0	0	6	0	2	-2	-2	-1	0	0	-1	-1	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0
X_{91}	0	0	-3	0	0	0	0	1	1	0	0	0	0	-4	2	-2	0	0	0	1	0	0	0	0	0	0	1	1	-2	1	0	0	1	1	0	0	0
X_{92}	0	0	-3	0	0	0	0	1	1	0	0	0	0	2	-4	-2	0	0	0	1	0	0	0	0	0	0	-1	-1	2	1	0	0	-1	-1	0	0	0
X_{93}	0	0	-3	-3	0	1	1	0	0	0	0	0	-4	-4	2	1	0	0	-1	1	0	0	0	-1	0	0	1	1	1	1	-1	0	0	0	0	0	1
X_{94}	0	0	-3	-3	0	-1	1	0	0	0	0	0	4	2	-4	1	0	0	1	1	0	0	0	-1	0	0	-1	-1	-1	1	1	0	0	0	0	0	-1
X_{95}	0	0	0	3	0	0	0	0	0	0	0	0	0	4	-2	2	0	0	0	-1	0	0	-1	0	0	0	2	2	-1	0	0	1	0	0	0	0	0
X_{96}	0	0	0	3	0	0	0	0	0	0	0	0	0	-2	4	2	0	0	0	-1	0	0	1	0	0	0	-2	-2	1	0	0	1	0	0	0	0	0
X_{97}	0	0	-1	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	-1	-1	-1	0	-1	0	0	0	0	0
X_{98}	0	0	-1	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	-1	0	-1	0	0	0	0	0
X_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
X_{100}	0	-2	0	3	0	0	0	0	0	0	-6	0	-2	0	0	-3	0	0	1	0	0	0	0	0	0	0	-2	-2	1	0	0	0	0	0	0	0	0
X_{101}	0	2	0	3	0	0	0	0	0	0	-6	0	2	0	0	-3	0	0	-1	0	0	0	1	0	0	0	2	2	-1	0	0	0	0	0	0	0	0
X_{102}	0	2	0	0	0	2	0	0	0	0	-6	0	-2	0	0	3	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1
X_{103}	0	-2	0	0	0	-2	0	0	0	0	-6	0	2	0	0	3	0	0	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	1
X_{104}	0	0	4	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	-2	0	0	0	0	0	0	0	-1	0	0	0	0	0
X_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0	0	0	-1	b13	*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{106}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0	0	0	-1	*	b13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{107}	0	0	3	3	0	-1	1	0	0	0	0	0	-4	0	0	0	0	0	2	0	0	0	1	-1	0	0	-1	-1	-1	-1	1	-1	0	0	0	0	-1
X_{108}	0	0	3	3	0	1	1	0	0	0	0	0	4	0	0	0	0	0	-2	0	0	0	-1	-1	0	0	1	1	1	-1	-1	-1	0	0	0	0	1
X_{109}	0	0	0	0	0	-1	-1	1	1	0	6	0	-2	0	0	0	0	0	-2	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	-1
X_{110}	0	0	0	0	0	1	-1	1	1	0	6	0	2	0	0	0	0	0	2	0	0	0	0	1	0	0	0	0	0	0	-1	0	-1	-1	0	0	1
X_{111}	0	0	-3	0	0	0	0	0	0	0	0	0	4	2	2	-2	0	0	-2	1	0	0	0	0	0	0	-1	-1	2	1	0	0	0	0	0	0	0
X_{112}	0	0	-3	0	0	0	0	0	0	0	0	0	-4	2	2	-2	0	0	2	1	0	0	0	0	0	0	1	1	-2	1	0	0	0	0	0	0	0
X_{113}	0	0	0	-3	0	-2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2	2	-1	0	0	1	-1	-1	0	0	1
X_{114}	0	0	0	-3	0	2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	-2	-2	1	0	0	1	1	1	0	0	-1

Fi22

| | Ind | 1A | 2A | 2B | 2C | 3A | 3B | 3C | 3D | 4A | 4B | 4C | 4D | 4E | 5A | 6A | 6B | 6C | 6D | 6E | 6F | 6G | 6H | 6I | 6J | 6K | 7A | 7B | 8A | 8B | 8C | 8D | 9A | 9B | 9C | 10A | 10B | 11A | 8** | 12A | 12B | 12C | 12D | 12E | 12F | 12G | 12H | 12I | 12J | 12K | 13A | B* | 14A | 15A | 16A | B** | 18A | B** | 18C | 18D | 20A | 21A | 22A | B** | 24A | 24B | 30A |
|---|---|---

		1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	4E	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	8A	8B	8C	8D	9A	9B	9C	10A	10B	11A	B**	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	13A	B*	14A	15A	16A	B**	18A	B**	18C	18D	20A	21A	22A	B**	24A	24B	30A	
	ind	1	2	2	2	3	3	3	3	4	4	4	4	4	5	6	6	6	6	6	6	6	6	6	6	6	7	8	8	8	8	9	9	9	10	10	11	11	12	12	12	12	12	12	12	12	12	12	12	13	13	14	15	16	16	18	18	18	18	20	21	22	22	24	24	30	
		6	6	6	6	6	6	6	6	12	12	12	12	12	30	6	6	6	6				6	6	6	6	42	24	24	24	24	18			30	30	66	66	12	12	12	12	12	12	12	12	12	12	12	78	78	42	30	48	48	18	18	18	18	60	42	66	66	24	24	30	
		3	6	6	6	3				12	12	12	12	12	15	6	6	6	6		6		6				21					9			30	30	33	33		12	12	12	12				12	12	12	39	39	42	15	48	48	18	18		18	60	21	66	66	24	24	30	
		2	2	2		6					4		4		10	6	6	6	6				6				14				8	18			10	10	22	22		12		12	12					12		26	26	14	30			18	18		18	20	42	22	22			30	
		3	6	6		3					12		12		15	6	6	6	6				6				21				24	9			30	30	33	33		12		12	12					12		39	39	42	15			18	18		18	60	21	66	66			30	
		6	6	6		6					12		12		30	6	6	6	6				6				42				24	18			30	30	66	66		12		12	12					12		78	78	42	30			18	18		18	60	42	66	66			30	
X_{165}	o2	1728	384	64	0	90	0	0	0	0	16	0	0	0	3	12	24	-8	-2	0	0	0	0	4	0	0	-1	0	0	0	0	3	0	0	-1	-1	1	1	0	6	0	-2	0	0	0	0	0	-2	0	-1	-1	-1	0	0	0	-13	13	0	1	1	-1	-1	-1	0	0	2	X_{165}
X_{166}	o2	1728	-384	64	0	90	0	0	0	0	-16	0	0	0	3	-12	-24	-8	-2	0	0	0	0	4	0	0	-1	0	0	0	0	3	0	0	1	-1	1	1	0	6	0	2	0	0	0	0	0	2	0	-1	-1	1	0	0	0	13	-13	0	1	-1	-1	1	1	0	0	-2	X_{166}
X_{167}	o2	61776	6336	496	0	666	0	0	0	0	56	0	12	0	1	36	72	-8	-2	0	0	0	0	4	0	0	1	0	0	0	2	6	0	0	1	1	0	0	0	6	0	2	0	0	0	0	0	2	0	0	0	1	1	0	0	0	0	0	-2	1	1	0	0	0	0	1	X_{167}
X_{168}	o2	61776	-6336	496	0	666	0	0	0	0	-56	0	12	0	1	-36	-72	-8	-2	0	0	0	0	4	0	0	1	0	0	0	-2	6	0	0	-1	1	0	0	0	6	0	-2	0	0	0	0	0	-2	0	0	0	-1	1	0	0	0	0	0	-2	-1	1	0	0	0	0	-1	X_{168}
X_{169}	o2	71280	6336	336	0	594	0	0	0	0	24	0	-12	0	5	36	72	-24	6	0	0	0	0	0	0	0	-1	0	0	0	-2	0	0	0	1	1	0	0	0	6	0	-6	0	0	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	-1	-1	0	0	0	0	1	X_{169}
X_{170}	o2	71280	-6336	336	0	594	0	0	0	0	-24	0	-12	0	5	-36	-72	-24	6	0	0	0	0	0	0	0	-1	0	0	0	2	0	0	0	-1	1	0	0	0	6	0	6	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	1	-1	0	0	0	0	-1	X_{170}
X_{171}	o2	112320	8064	64	0	666	0	0	0	0	16	0	0	0	-5	-36	72	-8	-2	0	0	0	0	4	0	0	-2	0	0	0	0	-3	0	0	-1	-1	-1	-1	0	6	0	-2	0	0	0	0	0	-2	0	0	0	0	1	0	0	3	3	0	1	1	1	1	1	0	0	-1	X_{171}
X_{172}	o2	112320	-8064	64	0	666	0	0	0	0	-16	0	0	0	-5	36	-72	-8	-2	0	0	0	0	4	0	0	-2	0	0	0	0	-3	0	0	1	-1	-1	-1	0	6	0	2	0	0	0	0	0	2	0	0	0	0	1	0	0	-3	-3	0	1	-1	1	-1	-1	0	0	1	X_{172}
X_{173}	o2	123552	0	992	0	-288	0	0	0	0	0	0	24	0	2	0	0	-16	32	0	0	0	0	8	0	0	2	0	0	0	0	-6	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0	-1	0	0	0	0	0	X_{173}
X_{174}	o2	308880	14784	1456	0	414	0	0	0	0	56	0	12	0	5	84	-48	16	10	0	0	0	0	4	0	0	-2	0	0	0	-2	3	0	0	-1	1	0	0	0	-6	0	2	0	0	0	0	0	2	0	0	0	0	-1	0	0	-13	13	0	1	1	1	0	0	0	0	-1	X_{174}
X_{175}	o2	308880	-14784	1456	0	414	0	0	0	0	-56	0	12	0	5	-84	48	16	10	0	0	0	0	4	0	0	-2	0	0	0	2	3	0	0	1	1	0	0	0	-6	0	-2	0	0	0	0	0	-2	0	0	0	0	-1	0	0	13	-13	0	1	-1	1	0	0	0	0	1	X_{175}
X_{176}	o2	314496	10752	-640	0	504	0	0	0	0	0	0	0	0	-4	-96	24	8	8	0	0	0	0	8	0	0	0	0	0	0	0	-3	0	0	2	0	-b11	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-13	13	0	-1	0	0	b11	**	0	0	-1	X_{176}
X_{177}	o2	314496	-10752	-640	0	504	0	0	0	0	0	0	0	0	-4	96	-24	8	8	0	0	0	0	8	0	0	0	0	0	0	0	-3	0	0	-2	0	**-b11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	13	-13	0	-1	0	0	**-b11	0	0	1	X_{177}
X_{178}	o2	314496	-10752	-640	0	504	0	0	0	0	0	0	0	0	-4	96	-24	8	8	0	0	0	0	8	0	0	0	0	0	0	0	-3	0	0	-2	0	-b11	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	13	-13	0	-1	0	0	-b11	**	0	0	1	X_{178}
X_{179}	o2	314496	10752	-640	0	504	0	0	0	0	0	0	0	0	-4	-96	24	8	8	0	0	0	0	8	0	0	0	0	0	0	0	-3	0	0	2	0	**-b11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-13	13	0	-1	0	0	** b11	0	0	-1	X_{179}
X_{180}	o2	359424	21504	1024	0	1224	0	0	0	0	64	0	0	0	-1	24	48	16	-8	0	0	0	0	4	0	0	2	0	0	0	0	3	0	0	-1	-1	-1	-1	0	0	0	4	0	0	0	0	0	-2	0	0	0	0	-1	0	0	13	-13	0	1	-1	-1	-1	-1	0	0	-1	X_{180}
X_{181}	o2	359424	-21504	1024	0	1224	0	0	0	0	-64	0	0	0	-1	-24	-48	16	-8	0	0	0	0	4	0	0	2	0	0	0	0	3	0	0	1	-1	-1	-1	0	0	0	-4	0	0	0	0	0	2	0	0	0	0	-1	0	0	-13	13	0	1	1	-1	1	1	0	0	1	X_{181}
X_{182}	o2	432432	14784	1424	0	126	0	0	0	0	24	0	-12	0	7	84	-48	-16	-22	0	0	0	0	8	0	0	0	0	0	0	2	-3	0	0	-1	-1	0	0	0	-6	0	-6	0	0	0	0	0	0	0	0	0	0	1	0	0	-13	13	0	-1	-1	0	0	0	0	0	-1	X_{182}
X_{183}	o2	432432	-14784	1424	0	126	0	0	0	0	-24	0	-12	0	7	-84	48	-16	-22	0	0	0	0	8	0	0	0	0	0	0	-2	-3	0	0	1	-1	0	0	0	-6	0	6	0	0	0	0	0	0	0	0	0	0	1	0	0	13	-13	0	-1	1	0	0	0	0	0	1	X_{183}
X_{184}	o2	967680	21504	-1024	0	504	0	0	0	0	-64	0	0	0	5	24	48	-16	8	0	0	0	0	-4	0	0	0	0	0	0	0	-3	0	0	-1	1	-1	-1	0	0	0	-4	0	0	0	0	0	2	0	-1	-1	0	-1	0	0	13	-13	0	-1	1	0	-1	-1	0	0	-1	X_{184}
X_{185}	o2	967680	-21504	-1024	0	504	0	0	0	0	64	0	0	0	5	-24	-48	-16	8	0	0	0	0	-4	0	0	0	0	0	0	0	-3	0	0	1	1	-1	-1	0	0	0	4	0	0	0	0	0	-2	0	-1	-1	0	-1	0	0	-13	13	0	-1	-1	0	1	1	0	0	1	X_{185}
X_{186}	o2	1123200	46080	640	0	1800	0	0	0	0	0	0	0	0	0	0	72	-8	-8	0	0	0	0	-8	0	0	1	0	0	0	0	-3	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-3	-3	0	1	0	1	1	1	0	0	0	X_{186}
X_{187}	o2	1123200	-46080	640	0	1800	0	0	0	0	0	0	0	0	0	0	-72	-8	-8	0	0	0	0	-8	0	0	1	0	0	0	0	-3	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3	3	0	1	0	1	-1	-1	0	0	0	X_{187}
X_{188}	o2	1235520	38016	704	0	846	0	0	0	0	-16	0	0	0	-5	36	0	-16	26	0	0	0	0	-4	0	0	-1	0	0	0	0	3	0	0	1	-1	0	0	0	-6	0	2	0	0	0	0	0	2	0	0	0	-1	1	0	0	3	3	0	-1	-1	-1	0	0	0	0	1	X_{188}
X_{189}	o2	1235520	-38016	704	0	846	0	0	0	0	16	0	0	0	-5	-36	0	-16	26	0	0	0	0	-4	0	0	-1	0	0	0	0	3	0	0	-1	-1	0	0	0	-6	0	-2	0	0	0	0	0	-2	0	0	0	1	1	0	0	-3	-3	0	-1	1	-1	0	0	0	0	-1	X_{189}
X_{190}	o2	1729728	29568	2624	0	-630	0	0	0	0	16	0	0	0	3	60	-96	32	14	0	0	0	0	-4	0	0	0	0	0	0	0	-3	0	0	3	-1	0	0	0	6	0	-2	0	0	0	0	0	-2	0	0	0	0	0	0	0	13	-13	0	-1	1	0	0	0	0	0	0	X_{190}
X_{191}	o2	1729728	-29568	2624	0	-630	0	0	0	0	-16	0	0	0	3	-60	96	32	14	0	0	0	0	-4	0	0	0	0	0	0	0	-3	0	0	-3	-1	0	0	0	6	0	2	0	0	0	0	0	2	0	0	0	0	0	0	0	-13	13	0	-1	-1	0	0	0	0	0	0	X_{191}
X_{192}	o2	1796256	0	-2592	0	0	0	0	0	0	0	0	24	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0-b13	*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{192}
X_{193}	o2	1796256	0	-2592	0	0	0	0	0	0	0	0	24	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	*-b13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{193}
X_{194}	o2	2594592	0	2400	0	-1512	0	0	0	0	0	0	24	0	-8	0	0	-48	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	X_{194}
X_{195}	o2	2965248	50688	-768	0	216	0	0	0	0	0	0	0	0	-2	-144	-72	24	-24	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	-1	0	0	0	1	X_{195}
X_{196}	o2	2965248	-50688	-768	0	216	0	0	0	0	0	0	0	0	-2	144	72	24	-24	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	0	0	0	-1	X_{196}
X_{197}	o2	3088800	21120	-800	0	-720	0	0	0	0	-80	0	-24	0	0	120	24	-8	16	0	0	0	0	4	0	0	1	0	0	0	0	3	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	-2	0	0	0	1	0	0	0	-13	13	0	1	0	1	0	0	0	0	X_{197}
X_{198}	o2	3088800	-21120	-800	0	-720	0	0	0	0	80	0	-24	0	0	-120	-24	-8	16	0	0	0	0	4	0	0	1	0	0	0	0	3	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	2	0	0	0	-1	0	0	0	13	-13	0	1	0	1	0	0	0	0	X_{198}

Fi22

Fi_{22}

	fus	ind	2D	2E	2F	4F	4G	4H	4I	4J	6L	6M	6N	6O	6P	6Q	6R	6S
			@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
			1045094400	3317760	2903040	663552	122880	9216	6144	1536	155520	36288	11664	4320	3888	3456	1296	648
			A	A	A	B	B	B	B	C	AD	AD	BD	AF	CD	AE	BF	DD
			A	A	A	A	A	A	A	A	AD	AD	BD	AF	CD	AE	BF	DD
X_1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X_2	:	++	22	6	-6	-10	6	2	-2	2	-5	1	-5	-3	4	3	3	1
X_3	:	++	79	15	7	15	15	7	-1	-1	16	-8	-2	4	7	0	-2	1
X_4	:	++	49	-31	21	-15	1	-3	9	-3	4	-14	-5	6	4	-4	3	4
X_5	:	++	170	-6	14	-22	26	6	2	-2	-19	-1	8	-1	8	-3	-4	-1
X_6	:	++	203	-21	-21	43	11	-5	-5	3	5	-7	14	-3	5	-3	6	2
X_7	:	++	280	56	56	56	24	8	8	0	19	7	10	11	10	11	2	-2
X_8	:	++	575	15	-85	-33	15	3	-9	3	35	29	8	5	8	3	-4	2
X_9	:	++	350	30	70	30	30	6	14	-2	35	35	-1	-5	-1	3	7	-1
X_{10}	:	++	1330	66	14	82	66	6	10	6	-11	7	-20	-1	16	-3	-4	-2
X_{11}	:	++	1568	96	-56	-128	64	8	0	0	-61	-7	2	-11	20	3	-2	2
X_{12}	:	++	1120	-96	56	-128	64	-8	0	0	-59	7	-14	11	4	-3	2	-2
X_{13}	:	++	945	129	189	81	1	21	9	5	-9	21	0	9	0	15	0	3
X_{14}	:	++	448	192	-112	0	0	16	0	0	16	-56	16	-4	-2	0	-4	-2
X_{15}	:	++	1050	170	-210	-102	10	6	-14	-2	15	21	-3	-15	6	23	-3	3
X_{16}	:	++	350	-130	70	-2	30	-10	-2	6	35	-49	-1	-5	-1	11	7	-4
X_{17}	:	++	2625	225	105	33	65	25	1	9	60	0	6	0	15	-12	6	0
X_{18}	:	++	875	-165	-105	107	-5	-1	-13	-9	-25	-7	11	-15	11	-9	3	2
X_{19}	:	++	875	-165	175	-85	-5	-9	3	7	20	14	-16	10	2	-12	4	5
X_{20}	:	++	2331	-117	-105	123	75	-1	-13	-9	90	0	-18	0	9	-6	-6	0
X_{21}	:	++	2400	160	-120	-192	0	8	0	0	15	-15	-30	-15	6	7	6	-6
X_{22}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{23}																		
X_{24}	:	++	350	270	210	-2	-50	-6	6	-6	-55	35	-1	15	8	9	3	8
X_{25}	:	++	4320	-96	216	0	64	24	0	0	-99	15	0	-9	0	-3	0	6
X_{26}	:	++	225	-495	-195	33	-15	5	-23	-3	-45	-27	-18	-15	9	3	-6	0
X_{27}	:	++	4900	-60	-140	-124	100	4	-12	-4	-50	-14	13	10	4	6	13	-5
X_{28}	:	++	4480	256	336	192	64	16	0	0	34	-14	-2	6	-2	10	-6	4
X_{29}	:	++	3645	189	189	189	61	-3	-3	-3	81	-27	0	9	0	9	0	0
X_{30}	:	++	3744	-288	120	-192	0	-8	0	0	45	27	18	15	18	-3	-6	0
X_{31}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{32}																		
X_{33}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{34}																		
X_{35}	:	++	7350	150	-210	150	-10	-2	-10	-2	15	-21	6	15	15	-9	6	-3
X_{36}	:	++	1050	-150	-210	90	10	-26	18	14	15	21	-3	-15	6	-9	-3	3
X_{37}	:	++	5376	-384	-336	192	64	-16	0	0	66	42	30	-6	-6	-6	6	6
X_{38}	:	++	5832	-216	-216	-216	-56	24	24	0	81	-27	0	9	0	9	0	0
X_{39}	:	++	5850	90	210	-102	90	-14	10	-6	45	-27	18	-15	-9	-3	-6	0
X_{40}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{41}																		
X_{42}	:	++	7000	120	280	248	-40	-24	8	0	-65	7	-20	-5	16	15	-8	-2
X_{43}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{44}																		
X_{45}	:	++	4725	405	525	21	-75	-3	-11	-3	0	0	27	0	0	0	3	0
X_{46}	:	++	1680	336	-336	-48	16	16	-16	0	-66	42	6	-6	-12	6	6	6
X_{47}	:	++	5600	160	-280	-64	0	-24	0	0	-25	-7	-16	5	2	7	8	2
X_{48}	:	++	2430	-594	594	-162	46	-6	6	-6	-81	-27	0	9	0	-9	0	0
X_{49}	:	++	9477	-27	-27	-27	5	21	21	-3	-81	27	0	-9	0	-9	0	0
X_{50}	:	++	9828	324	-252	-252	36	-12	-12	12	0	0	27	0	0	0	-9	0
X_{51}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{52}																		
X_{53}	:	++	2800	240	0	-80	-80	0	-16	-16	100	28	-8	0	10	-12	0	1
X_{54}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{55}	:	++	6400	0	0	256	0	0	0	0	-80	-8	28	0	-8	0	0	-8

	6T	6U	6V	8E	8F	8G	8H	10C	12O	12P	12Q	12R	12S	12T	12U	14B
	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
	648	432	216	384	384	384	96	600	432	384	288	144	72	48	12	42
	DE	CE	CF	A	A	C	D	AD	IF	DG	DH	CH	IH	II	KJ	AD
	DE	CE	CF	A	A	A	A	AD	CF	AG	AH	BH	CH	CI	DJ	AD
X_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X_2	-3	0	0	2	-2	2	0	-3	2	-3	-1	-1	2	-2	-1	1
X_3	-3	3	1	3	3	3	-1	4	3	0	4	-2	1	-1	-1	2
X_4	-4	-4	0	-3	1	1	3	-1	0	-2	0	3	0	0	0	0
X_5	3	0	2	2	-2	6	0	-5	2	-1	-3	0	0	2	1	2
X_6	6	-3	-3	-1	-1	3	-1	3	1	5	1	-2	1	1	0	0
X_7	2	2	2	4	4	0	0	5	2	3	-1	2	2	2	0	0
X_8	6	0	-4	-5	-1	3	-3	0	0	-3	3	0	0	0	0	1
X_9	3	3	1	2	2	2	2	0	3	3	3	3	-3	-1	1	0
X_{10}	-6	0	2	-2	2	6	0	5	-2	3	-3	0	0	-2	0	0
X_{11}	6	0	-2	4	-4	0	0	-7	4	1	-1	2	2	0	0	0
X_{12}	-6	0	2	4	-4	0	0	-5	4	-5	1	-2	-2	0	0	0
X_{13}	3	0	0	-3	1	-3	3	-5	0	1	3	0	0	0	-1	0
X_{14}	-6	6	-4	0	0	0	0	-2	0	0	4	4	-2	0	0	0
X_{15}	-1	2	0	2	-2	-2	0	0	-6	1	3	-3	0	-2	1	0
X_{16}	-4	-1	1	-2	-2	2	-2	0	1	3	-1	-1	-1	1	0	0
X_{17}	0	3	3	1	1	-3	1	0	-3	-4	4	-2	1	1	0	0
X_{18}	6	3	3	-5	-1	-1	1	0	-1	1	-1	-1	-1	-1	0	0
X_{19}	-3	6	-2	-1	3	3	-3	0	-4	-2	0	0	0	0	1	0
X_{20}	0	-3	-3	3	-1	3	1	6	3	0	2	2	-1	-1	0	0
X_{21}	-2	-2	0	-4	4	0	0	0	0	-3	-1	2	2	0	0	-1
X_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{23}																
X_{24}	0	0	0	2	-2	2	0	0	-8	-5	-3	3	0	0	0	0
X_{25}	-6	0	0	-4	4	0	0	-5	0	7	-3	0	0	0	0	1
X_{26}	0	-3	3	1	5	-3	-1	0	-3	-3	-1	2	-1	1	0	1
X_{27}	3	0	-2	0	0	4	0	0	-4	-2	-2	1	-2	0	-1	0
X_{28}	4	-2	0	0	0	0	0	5	0	-2	-2	-2	-2	0	0	0
X_{29}	0	0	0	5	5	-3	-3	-5	0	1	-3	0	0	0	0	-2
X_{30}	0	-6	0	-4	4	0	0	-6	0	3	1	-2	-2	0	0	-1
X_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{32}																
X_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{34}																
X_{35}	-3	3	-3	-2	-2	-6	2	0	3	-1	-5	-2	1	-1	1	0
X_{36}	3	6	0	2	-2	-2	0	0	0	1	-5	1	-2	0	-1	0
X_{37}	-6	-6	0	0	0	0	0	1	0	-2	2	2	2	0	0	0
X_{38}	0	0	0	-4	-4	0	0	7	0	1	-3	0	0	0	0	1
X_{39}	0	3	3	-2	-2	6	2	0	-3	-3	1	-2	1	1	0	-2
X_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{41}																
X_{42}	-6	0	-2	-4	-4	0	0	0	2	-1	3	0	0	2	0	0
X_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{44}																
X_{45}	0	0	6	-3	-3	1	-3	0	6	0	0	3	0	-2	0	0
X_{46}	-6	0	0	0	0	0	0	5	6	-2	-2	-2	-2	2	0	0
X_{47}	-2	-2	2	4	-4	0	0	0	-4	9	3	0	0	0	0	0
X_{48}	0	0	0	2	-2	-6	0	5	0	1	3	0	0	0	0	1
X_{49}	0	0	0	1	1	-3	-3	2	0	-1	3	0	0	0	0	-1
X_{50}	0	0	0	0	0	-4	0	3	0	0	0	3	0	0	0	0
X_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{52}																
X_{53}	-3	6	0	0	0	0	0	0	-2	4	0	0	0	2	-1	0
X_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{55}	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	2

	14C	16C	18E	18F	18G	18H	20B	24C	24D	24E	24F	24G	30B	30C	36A	42A	
	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
	14	16	54	54	18	9	20	48	48	24	24	12	30	30	18	21	
	AF	C	AN	BN	BR	CT	BG	BE	BF	HG	HG	GH	ACL	ADO	DM	ABM	
	AF	A	AD	BD	BF	CE	AG	AE	AF	BG	BG	CH	ACL	ADO	AF	ABM	
X_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	X_1
X_2	1	0	1	-2	0	0	1	-1	1	-1	-1	0	0	2	-1	1	X_2
X_3	0	1	-2	1	1	0	0	0	0	0	0	-1	1	-1	0	-1	X_3
X_4	0	1	-2	-2	0	-1	1	0	-2	1	1	0	-1	1	0	0	X_4
X_5	0	0	-1	-1	-1	0	1	-1	1	0	0	0	1	-1	-1	-1	X_5
X_6	0	-1	-1	2	0	0	1	-1	-1	0	0	-1	0	2	1	0	X_6
X_7	0	0	1	1	-1	-1	-1	1	1	0	0	0	-1	1	-1	0	X_7
X_8	-1	1	2	-1	-1	0	0	1	-1	0	0	0	0	0	0	1	X_8
X_9	0	0	2	-1	1	0	0	-1	-1	-1	-1	-1	0	0	0	0	X_9
X_{10}	0	0	1	1	-1	0	1	1	-1	0	0	0	-1	-1	1	0	X_{10}
X_{11}	0	0	-1	-1	1	0	-1	1	-1	0	0	0	-1	-1	1	0	X_{11}
X_{12}	0	0	1	1	-1	0	-1	1	-1	0	0	0	1	1	1	0	X_{12}
X_{13}	0	-1	0	0	0	0	1	3	1	0	0	0	1	-1	0	0	X_{13}
X_{14}	0	0	-2	1	-1	0	0	0	0	0	0	0	1	1	0	0	X_{14}
X_{15}	0	0	0	0	0	-1	0	-1	1	1	1	0	0	0	0	0	X_{15}
X_{16}	0	0	-1	-1	1	-1	0	1	1	-1	-1	1	0	0	1	0	X_{16}
X_{17}	0	-1	0	0	0	0	0	-2	-2	0	0	1	0	0	0	0	X_{17}
X_{18}	0	1	-1	2	0	0	0	1	-1	-1	-1	1	0	0	-1	0	X_{18}
X_{19}	0	-1	-1	-1	1	0	0	2	0	0	0	0	0	0	-1	0	X_{19}
X_{20}	0	-1	0	0	0	0	0	0	2	0	0	1	0	0	0	0	X_{20}
X_{21}	-1	0	0	0	0	1	0	-1	1	0	0	0	0	0	0	-1	X_{21}
X_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{22}
X_{23}																	X_{23}
X_{24}	0	0	-1	2	0	0	0	-1	1	-1	-1	0	0	0	1	0	X_{24}
X_{25}	-1	0	0	0	0	0	-1	-1	1	0	0	0	1	1	0	1	X_{25}
X_{26}	1	-1	0	0	0	0	0	1	-1	0	0	-1	0	0	0	1	X_{26}
X_{27}	0	0	1	1	1	0	0	0	0	1	1	0	0	0	-1	0	X_{27}
X_{28}	0	0	-2	-2	0	1	-1	0	0	0	0	0	-1	1	0	0	X_{28}
X_{29}	0	1	0	0	0	0	1	-1	-1	0	0	0	1	-1	0	1	X_{29}
X_{30}	1	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	-1	X_{30}
X_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{31}
X_{32}																	X_{32}
X_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{33}
X_{34}																	X_{34}
X_{35}	0	0	0	0	0	0	0	1	1	0	0	-1	0	0	0	0	X_{35}
X_{36}	0	0	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	X_{36}
X_{37}	0	0	0	0	0	0	-1	0	0	0	0	0	1	-1	0	0	X_{37}
X_{38}	1	0	0	0	0	0	-1	-1	-1	0	0	0	1	-1	0	1	X_{38}
X_{39}	0	0	0	0	0	0	0	1	1	0	0	-1	0	0	0	1	X_{39}
X_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{40}
X_{41}																	X_{41}
X_{42}	0	0	1	1	1	0	0	-1	-1	0	0	0	0	0	-1	0	X_{42}
X_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{43}
X_{44}																	X_{44}
X_{45}	0	-1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	X_{45}
X_{46}	0	0	0	0	0	0	1	0	0	0	0	0	-1	-1	0	0	X_{46}
X_{47}	0	0	-1	-1	-1	1	0	1	-1	0	0	0	0	0	-1	0	X_{47}
X_{48}	-1	0	0	0	0	0	1	-1	1	0	0	0	-1	-1	0	1	X_{48}
X_{49}	1	1	0	0	0	0	0	1	1	0	0	0	-1	1	0	-1	X_{49}
X_{50}	0	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	0	X_{50}
X_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{51}
X_{52}																	X_{52}
X_{53}	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	1	0	X_{53}
X_{54}	0	0	0	0	0	0	0	0	0	2r3	-2r3	0	0	0	0	0	X_{54}
X_{55}	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	1	-1	X_{55}

			2D	2E	2F	4F	4G	4H	4I	4J	6L	6M	6N	6O	6P	6Q	6R	6S	6T	6U	6V	8E	8F	8G	8H	10C	12O	12P	12Q	12R	12S	12T	12U	14B	14C	16C	18E	18F	18G	18H	20B	24C	24D	24E	24F	24G	30B	30C	36A	42A	
x_{56}	:	++	8192	0	0	0	0	0	0	0	128	-64	-16	0	-16	0	0	8	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	0	2	0	0	2	2	0	0	0	0	0	0	0	0	-2	0	0	-1	x_{56}
x_{57}	:	++	5103	-81	567	-81	-81	-9	-33	15	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	x_{57}
x_{58}	:	++	5346	-270	-378	162	-110	30	-6	6	-81	-27	0	9	0	-9	0	0	0	0	0	2	-2	6	0	-4	0	1	3	0	0	0	0	-2	0	0	0	0	0	0	0	-1	1	0	0	0	-1	-1	0	1	x_{58}
x_{59}	:	++	7875	-45	-105	-93	-45	-17	11	-9	-90	0	18	0	-9	6	-6	0	0	3	-3	3	7	3	1	0	3	0	-2	-2	1	-1	0	0	0	-1	0	0	0	0	0	0	-2	0	0	1	0	0	0	0	x_{59}
x_{60}	:	++	10752	512	0	0	0	0	0	0	-48	0	-48	0	-12	-16	0	0	8	-4	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	2	0	0	0	x_{60}
x_{61}	:	++	1800	360	-120	-24	-120	-8	24	0	45	-27	18	-15	18	-3	6	0	0	-6	0	4	4	0	0	0	0	-3	1	-2	-2	0	0	1	-1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	x_{61}
x_{62}	:	++	5600	-480	-280	64	0	-24	0	0	-25	-7	-16	5	2	15	8	2	6	6	2	-4	4	0	0	0	4	-3	3	0	0	0	0	0	0	0	-1	-1	-1	0	0	-1	1	0	0	0	0	0	1	0	x_{62}
x_{63}	:	++	4096	0	512	0	0	0	0	0	64	64	-8	-16	-8	0	8	-8	0	0	-4	0	0	0	0	-4	0	0	0	0	0	0	0	1	1	0	-2	1	-1	0	0	0	0	0	0	0	-1	-1	0	1	x_{63}
x_{64}	:	++	4096	0	-512	0	0	0	0	0	64	64	-8	16	-8	0	-8	-8	0	0	4	0	0	0	0	-4	0	0	0	0	0	0	0	1	-1	0	-2	1	1	0	0	0	0	0	0	0	-1	1	0	1	x_{64}
x_{65}	:	++	7776	-864	216	0	-64	24	0	0	81	27	0	-9	0	9	0	0	0	0	0	4	-4	0	0	1	0	-1	-3	0	0	0	0	-1	-1	0	0	0	0	0	1	1	-1	0	0	0	1	1	0	-1	x_{65}
	fus	ind	2	2	4	4	4	4	4	4	6	6 6	6	12	6	6	12	6 6	6 6	6	12	8	8	8	8	10	12	12	12	12	12	12	12 12	14	28	16	18 18	18	36	18 18	20	24	24	24	24	24 24	30	60	36 36	42 42	
x_{66}	:	oo	0	0	0	0	0	0	0	0	0	813	0	0	0	0	0	-13	-313	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	0	0	213	0	0	13	0	0	0	0	0	0	0	0	0	13	x_{66}
x_{67}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{67}
x_{68}																																																			x_{68}
x_{69}		o	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{69}
x_{70}																																																			x_{70}
x_{71}		o	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{71}
x_{72}																																																			x_{72}
x_{73}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{73}
x_{74}																																																			x_{74}
x_{75}	:	oo	0	0	0	0	0	0	0	0	0	813	0	0	0	0	0	-13	913	0	0	0	0	0	0	0	0	0	0	0	0	0	13	0	0	0	213	0	0	0	0	0	0	0	0	0	0	0	0	13	x_{75}
x_{76}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{76}
x_{77}																																																			x_{77}
x_{78}	:	oo	0	0	0	0	0	0	0	0	0	813	0	0	0	0	0	813	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	0	0	0	0	0	0	0	0	0	3	13	x_{78}
x_{79}	:	oo	0	0	0	0	0	0	0	0	0	813	0	0	0	0	0	813	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	0	0	0	0	0	0	0	0	0	-3	13	x_{79}
x_{80}	:	oo	0	0	0	0	0	0	0	0	0	5613	0	0	0	0	0	213	-613	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	213	0	0	-13	0	0	0	0	0	0	0	0	0	0	x_{80}
x_{81}	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r3	0	0	0	0	x_{81}
x_{82}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{82}
x_{83}																																																			x_{83}
x_{84}	:	oo	0	0	0	0	0	0	0	0	0	4813	0	0	0	0	0	313	-313	0	0	0	0	0	0	0	0	0	0	0	0	0	13	0	0	0	0	0	0	13	0	0	0	0	0	0	0	0	0	-13	x_{84}
x_{85}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{85}
x_{86}																																																			x_{86}
x_{87}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{87}
x_{88}																																																			x_{88}
x_{89}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{89}
x_{90}																																																			x_{90}
x_{91}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{91}
x_{92}																																																			x_{92}
x_{93}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{93}
x_{94}																																																			x_{94}
x_{95}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{95}
x_{96}																																																			x_{96}
x_{97}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{97}
x_{98}																																																			x_{98}
x_{99}	:	oo	0	0	0	0	0	0	0	0	0	4813	0	0	0	0	0	313	913	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	x_{99}
x_{100}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{100}
x_{101}																																																			x_{101}
x_{102}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{102}
x_{103}																																																			x_{103}
x_{104}	:	oo	0	0	0	0	0	0	0	0	0	6413	0	0	0	0	0	-813	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-213	0	0	0	0	0	0	0	0	0	0	0	0	13	x_{104}
x_{105}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{105}
x_{106}																																																			x_{106}
x_{107}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{107}
x_{108}																																																			x_{108}
x_{109}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{109}
x_{110}																																																			x_{110}
x_{111}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{111}
x_{112}																																																			x_{112}
x_{113}		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{113}
x_{114}																																																			x_{114}

Fi22

[191]

Fi_{22}

			2D	2E	2F	4F	4G	4H	4I	4J	6L	6M	6N	6O	6P	6Q	6R	6S	6T	6U	6V	8E	8F	8G	8H	10C	12O	12P	12Q	12R	12S	12T	12U	14B	14C	16C	18E	18F	18G	18H	20B	24C	24D	24E	24F	24G	30B	30C	36A	42A
	fus	ind	2	2	2	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	8	8	8	8	10	12	12	12	12	12	12	12	14	14	16	18	18	18	18	20	24	24	24	24	24	30	30	36	42
X_{115}	*	+																																																	
X_{116}	*	+																																																	
X_{117}		o2																																																	
X_{118}	*																																																		
X_{119}	*	+																																																	
X_{120}	*	+																																																	
X_{121}	*	+																																																	
X_{122}	*	+																																																	
X_{123}	*	+																																																	
X_{124}	*	+																																																	
X_{125}	*	+																																																	
X_{126}	*	+																																																	
X_{127}	*	+																																																	
X_{128}	*	+																																																	
X_{129}	*	+																																																	
X_{130}	*	+																																																	
X_{131}	*	+																																																	
X_{132}		+2																																																	
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X_{134}	*	+																																																	
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X_{148}		+2																																																	
X_{149}	*																																																		
X_{150}	*	+																																																	
X_{151}	*	+																																																	
X_{152}	*	+																																																	
X_{153}	*	+																																																	
X_{154}	*	+																																																	
X_{155}		+2																																																	
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X_{157}	*	+																																																	
X_{158}	*	+																																																	
X_{159}	*	+																																																	
X_{160}	*	+																																																	
X_{161}	*	+																																																	
X_{162}	*	+																																																	
X_{163}	*	+																																																	
X_{164}	*	+																																																	

Sporadic Fischer group Fi_{22}

Order = 64,561,751,654,400 = $2^{17}.3^{9}.5^{2}.7.11.13$ Mult = 6 Out = 2

Constructions

Fischer G is generated by a conjugacy class of 3510 involutions, which are 3-transpositions; i.e. any non-commuting pair has product of order 3. Joining transpositions when they commute we obtain a rank 3 graph of valence 693, whose automorphism group is G.2. The centralizer of a transposition in G is $2^{\cdot}Fi_{21} \cong 2^{\cdot}U_6(2)$, and the centralizer of the product of two distinct transpositions is $2^2.2^8.U_4(2).2$ or $3 \times U_4(3).2$ according as they commute or not. Products of two or three mutually commuting transpositions are called bi- and tri-transpositions respectively. All maximal sets of mutually commuting transpositions (bases) are conjugate, and contain 22 transpositions, which generate an elementary Abelian group 2^{10} whose normalizer is a group $2^{10}:M_{22}$. There are three orbits of this group on transpositions: a transposition is either in the base, or commutes with a hexad of the base or with no transposition in the base. There are 22 basic, $2^5.77$ hexadic and 2^{10} anabasic transpositions. The centralizer of an anabasic transposition in $2^{10}M_{22}$ is a complementary M_{22}.

There are two classes of subgroups S_{10}, each having three orbits on transpositions, with stabilizers $2 \times S_8$, $2 \times (S_4 wr S_2)$, and $S_2 wr A_5$.

Modulo 2 3.G has a 27-dimensional unitary representation over $\mathbb{F}_4$, which exhibits it as a subgroup of $3.^2E_6(2)$. Explicit matrices have been computed.

Real G has a 78-dimensional real representation with basis $\{e_*, e_H\}$ where H ranges over the 77 hexads of S(3,6,22) (see M_{22}). It is generated by the 22 basic transpositions (i) which act as

$$e_* \to e_*, \quad e_H \to e_H \text{ (i in H)}, \quad e_H \to -e_H \text{ (i not in H)} \qquad \text{(each i)}$$

together with the anabasic transposition

$$S : 16e_* \to -5e_* + \sqrt{3}\Sigma e_H \qquad \text{(summed over all H)}$$

$$16e_H \to \sqrt{3}e_* - 7e_H - 3\Sigma e_K + \Sigma e_L \text{ (summed over K disjoint from H, and L meeting H in 2 points)}$$

The subgroup M_{22} fixes e_* and permutes the e_H. By enlarging this to $M_{22}:2$ we extend the representation to G.2.

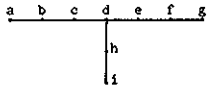
Modulo 3 G.2 has a 77-dimensional representation modulo 3, obtained by reducing the above (written in terms of $\sqrt{3}e_*$, e_H) modulo 3, and factoring by the fixed vector $\sqrt{3}e_*$. The above formulae simplify to

$$i : e_H \to e_H \text{ (i in H)}, \quad e_H \to -e_H \text{ (i not in H)}$$

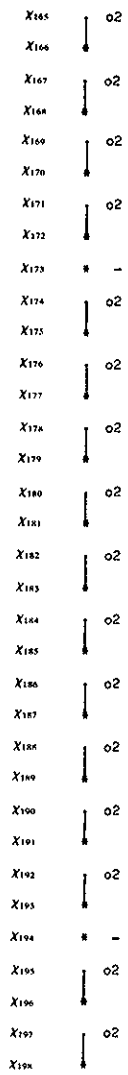
$$S : e_H \to \Sigma e_L - e_H \qquad \text{(summed over hexads L meeting H in 2 points)}$$

The 77-space is the space fixed by a transposition in the 253-dimensional representation of Fi_{23} (q.v.).

Presentation $G \cong \langle$ a b c d e f g | (abcdefh)^9=(bcdefgh)^9=(bcdefdhid)^{10}=1 $\rangle$; see page 232.



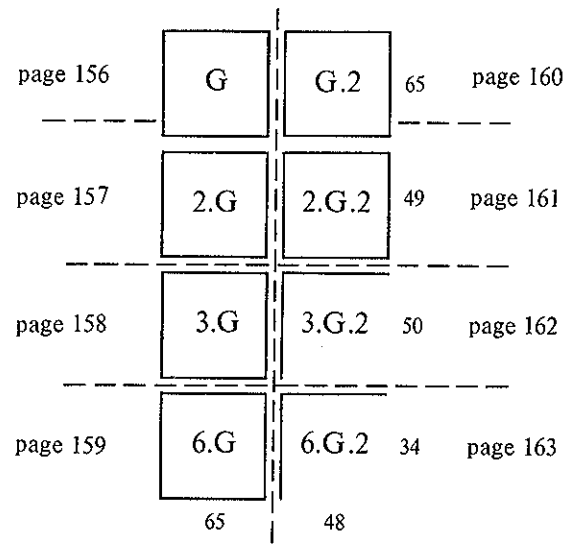
	2D	2E	2F	4F	4G	4H	4I	4J	6L	6M	6N	6O	6P	6Q	6R	6S	6T	6U	6V	8E	8F	8G	8H	10C	12O	12P	12Q	12R	12S	12T	12U	14B	14C	16C	18E	18F	18G	18H	20B	24C	24D	24E	24F	24G	30B	30C	36A	42A
fus ind	2	2	4	4	4	4	4	4	6	6	6	12	6	6	12	6	6	6	12	8	8	8	8	10	12	12	12	12	12	12	12	14	28	16	18	18	36	18	20	24	24	24	24	24	30	60	36	42
										6						6	6														12				18			18							24			36	42



Maximal subgroups (see note below)

Order	Index	Structure	G.2	Character	Specifications: Abstract	Specifications: Fischer
18393661440	3510	$2{\cdot}U_6(2)$	$: 2{\cdot}U_6(2).2$	1a+429a+3080a	N(2A)	transposition, point
4585351680	14080	$O_7(3)$	$G_2(3):2,$	1a+429a+13650a		
4585351680	14080	$O_7(3)$	$3^5:(2xU_4(2):2)$	1a+429a+13650a		
1045094400	61776	$O_8^+(2):S_3$	$: O_8^+(2):S_3 \times 2$	1a+3080a+13650a+45045a	C(2D)	
454164480	142155	$2^{10}:M_{22}$	$: 2^{10}:M_{22}:2$	1a+78a+429a+1430a+3080a+30030a+320320a+750750a	$N(2^{10})$	base
92897280	694980	$2^6:S_6(2)$	$: 2^7:S_6(2)$		$N(2B^6)$	
53084160	1216215	$(2x2_+^{1+8}:U_4(2)):2$	$: (2x2_+^{1+8}:U_4(2):2):2$		N(2B)	bi-transposition, edge
39191040	1647360	$S_3 \times U_4(3):2_2$	$: S_3 \times U_4(3).(2^2)_{122}$		N(3A)	triad
17971200	3592512	$^2F_4(2)'$	$: {}^2F_4(2)$		N(2B,2C,3D,...)	
17694720	3648645	$2^{5+8}:(S_3xA_6)$	$: 2^{5+8}:(S_3xS_6)$		$N(2B^4)$	
5038848	12812800	$3_+^{1+6}:2^{3+4}:3^2:2$	$: H.2$		N(3B)	
3628800	17791488	S_{10}				
3628800	17791488	S_{10}				
95040	679311360	M_{12}	$: M_{12}:2$		N(2B,2C,3D,3C,...)	

Note : Any other maximal subgroup of G or G.2 is the normalizer of a subgroup A_6, $L_2(7)$, $L_2(13)$ or $U_3(3)$ with trivial centralizer.



Fi_{22}

$HN = F_{5+}$

	;	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	
		2730	1774	36	544	29	15	5	3	630	500			2	1							2	1																								
	309	12000000	08000	86400	320	160	360	760	840	000	000	15000	15000	500	440	576	360	420	80	64	27	000	200	800	600	600	100	100	100	22	72	48	12	28	180	30	30	19	19	80	80	40	20	20	
		p power	A	A	A	A	B	A	B	A	A	A	A	A	AA	AB	BB	A	B	A	B	BA	AA	BB	DB	CB	EA	EB	EB	A	AB	BA	CC	AA	AA	DB	CB	A	A	CA	CA	BB	EC	DC	
		p' part	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	BB	A	A	A	A	BA	AA	BB	CB	DB	EA	EB	EB	A	AB	AA	BC	AA	AA	CB	DB	A	A	BA	BA	AB	CC	DC	
	ind	1A	2A	2B	3A	3B	4A	4B	4C	5A	5B	5C	D*	5E	6A	6B	6C	7A	8A	8B	9A	10A	10B	10C	10D	E*	10F	10G	H*	11A	12A	12B	12C	14A	15A	15B	C*	19A	B**	20A	B*	20C	20D	E*	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	+	133	21	5	7	-2	5	1	-3	-7	8	5b5+3	*	3	3	-1	2	0	-1	1	1	-4	1	0	b5+3	*	1	r5	-r5	1	1	-1	0	0	2	-b5	*	0	0	0	0	1	*	b5	x_2
x_3	+	133	21	5	7	-2	5	1	-3	-7	8	*	5b5+3	3	3	-1	2	0	-1	1	1	-4	1	0	*	b5+3	1	-r5	r5	1	1	-1	0	0	2	*	-b5	0	0	0	0	1	b5	*	x_3
x_4	+	760	56	-8	22	4	8	0	0	20	10	-10	-10	5	2	-2	4	4	0	0	1	6	-4	2	2	2	1	-3	-3	1	0	2	0	0	2	-1	-1	0	0	-2	-2	0	0	0	x_4
x_5	+	3344	176	16	41	-4	16	8	0	14	-31	4	4	9	5	1	4	5	0	0	2	1	6	1	-4	-4	1	1	1	0	-1	1	0	1	-4	1	1	0	0	1	1	-2	0	0	x_5
x_6	+	8778	154	-54	21	3	10	-10	2	28	28	5b5+18	*	3	1	-3	3	0	0	-2	0	4	4	-4	5b5+6	*	-1	1	1	0	-1	1	-1	0	1	-b5	*	0	0	0	0	0	b5	*	x_6
x_7	+	8778	154	-54	21	3	10	-10	2	28	28	*	5b5+18	3	1	-3	3	0	0	-2	0	4	4	-4	*	5b5+6	-1	1	1	0	-1	1	-1	0	1	*	-b5	0	0	0	0	0	*	b5	x_7
x_8	+	8910	286	78	27	0	14	10	6	20	35	15	15	5	7	3	0	-1	0	2	0	11	-4	3	3	3	1	3	3	0	1	-1	0	-1	2	0	0	-1	-1	-1	-1	0	1	1	x_8
x_9	+	9405	77	61	36	9	-3	9	-11	55	30	5	5	5	-4	4	1	4	-1	1	0	2	7	6	1	1	-3	1	1	0	0	0	1	0	1	-1	-1	0	0	2	2	-1	-1	-1	x_9
x_{10}	+	16929	385	33	27	0	17	1	9	29	54	-21	-21	4	7	3	0	-4	1	1	0	10	5	-2	3	3	0	-2	-2	0	1	-1	0	0	2	0	0	0	0	2	2	1	-1	-1	x_{10}
x_{11}	+	35112	616	40	84	12	24	0	0	-28	-13	10r5-8	-10r5-8	7	4	4	4	0	0	0	0	-9	-4	-5	-2r5	2r5	1	r5	-r5	0	0	0	0	0	-1	*	b5	0	0	-1	-1	0	0	0	x_{11}
x_{12}	+	35112	616	40	84	12	24	0	0	-28	-13	-10r5-8	10r5-8	7	4	4	4	0	0	0	0	-9	-4	-5	2r5	-2r5	1	-r5	r5	0	0	0	0	0	-1	b5	*	0	0	-1	-1	0	0	0	x_{12}
x_{13}	+	65835	539	-85	63	-18	11	-1	3	-105	85	25b5+15	*	5	-1	-1	2	0	1	-1	0	-11	-1	5	b5+3	*	-1	r5	-r5	0	-1	-1	0	0	3	b5	*	0	0	1	1	-1	*	-b5	x_{13}
x_{14}	+	65835	539	-85	63	-18	11	-1	3	-105	85	*	25b5+15	5	-1	-1	2	0	1	-1	0	-11	-1	5	*	b5+3	-1	-r5	r5	0	-1	-1	0	0	3	*	b5	0	0	1	1	-1	-b5	*	x_{14}
x_{15}	o	69255	-153	135	0	0	-9	-9	15	45	5	0	0	10	0	0	0	-3	-1	-1	0	-3	-3	5	0	0	2	0	0	-1	0	0	0	1	0	0	0	0	0	1	1	1	0	0	x_{15}
x_{16}	o	69255	-153	135	0	0	-9	-9	15	45	5	0	0	10	0	0	0	-3	-1	-1	0	-3	-3	5	0	0	2	0	0	-1	0	0	0	1	0	0	0	0	0	1	1	1	0	0	x_{16}
x_{17}	+	214016	1408	0	104	-40	0	16	0	161	16	-24	-24	6	4	0	0	5	4	0	-1	8	-7	0	0	0	-2	0	0	0	-2	0	0	1	-1	0	0	0	0	0	0	1	0	0	x_{17}
x_{18}	+	267520	1408	256	148	4	0	0	0	120	20	20	20	-5	4	4	4	1	0	0	-2	8	8	-4	-4	-4	3	1	1	0	0	0	0	1	3	-1	-1	0	0	0	0	0	0	0	x_{18}
x_{19}	+	270864	880	-240	189	0	16	-8	0	134	-11	24	24	4	-11	-3	0	6	0	0	0	5	-10	5	0	0	0	0	0	0	1	1	0	-2	-1	0	0	0	0	1	1	2	0	0	x_{19}
x_{20}	+	365750	2310	310	35	35	6	10	-10	0	125	0	0	0	15	-5	-5	0	0	2	-1	-15	0	5	0	0	0	0	0	0	1	3	-1	0	0	0	0	0	0	1	1	0	0	0	x_{20}
x_{21}	+	374528	0	-256	56	38	0	0	0	-112	28	-10r5+8	10r5+8	-2	0	8	2	0	0	0	-1	0	0	4	-2r5+4	2r5+4	0	2b5	*	0	0	0	0	0	-4	*	-b5	0	0	0	0	0	0	0	x_{21}
x_{22}	+	374528	0	-256	56	38	0	0	0	-112	28	10r5+8	-10r5+8	-2	0	8	2	0	0	0	-1	0	0	4	2r5+4	-2r5+4	0	*	2b5	0	0	0	0	0	-4	-b5	*	0	0	0	0	0	0	0	x_{22}
x_{23}	+	406296	2904	24	162	0	40	0	0	-84	-204	6	6	11	6	-6	0	-5	0	0	0	4	4	4	-6	-6	-1	-1	-1	0	0	-2	0	-1	-3	0	0	0	0	0	0	0	0	0	x_{23}
x_{24}	+	653125	1925	325	-41	-5	5	21	5	125	0	0	0	0	-1	7	-5	4	1	-3	1	0	5	0	0	0	0	0	0	0	3	-1	-1	0	-1	0	0	0	0	0	0	1	0	0	x_{24}
x_{25}	o	656250	-1750	250	105	15	10	-10	10	0	0	0	0	0	5	1	-5	0	0	-2	0	0	0	0	0	0	0	0	0	1	-1	1	1	0	0	0	0	b19	**	0	0	0	0	0	x_{25}
x_{26}	o	656250	-1750	250	105	15	10	-10	10	0	0	0	0	0	5	1	-5	0	0	-2	0	0	0	0	0	0	0	0	0	1	-1	1	1	0	0	0	0	**	b19	0	0	0	0	0	x_{26}
x_{27}	+	718200	2520	120	0	-27	24	0	-8	0	75	-25b5	*	0	0	0	-3	0	0	0	0	-5	0	-5	-5b5	*	0	0	0	-1	0	0	1	0	0	-b5	*	0	0	-1	-1	0	b5	*	x_{27}
x_{28}	+	718200	2520	120	0	-27	24	0	-8	0	75	*	-25b5	0	0	0	-3	0	0	0	0	-5	0	-5	*	-5b5	0	0	0	-1	0	0	1	0	0	*	-b5	0	0	-1	-1	0	*	b5	x_{28}
x_{29}	+	1053360	1232	-592	63	36	-16	8	0	210	235	10	10	10	-1	-1	-4	0	0	0	0	7	2	3	-2	-2	2	-2	-2	0	-1	-1	0	0	3	1	1	0	0	-1	-1	-2	0	0	x_{29}
x_{30}	+	1066527	2079	-225	0	0	15	-9	-9	-63	152	30b5-3	*	-3	0	0	0	0	-1	-1	0	4	9	0	-3r5	3r5	-1	-r5	r5	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	x_{30}
x_{31}	+	1066527	2079	-225	0	0	15	-9	-9	-63	152	*	30b5-3	-3	0	0	0	0	-1	-1	0	4	9	0	3r5	-3r5	-1	r5	-r5	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	x_{31}
x_{32}	+	1185030	1078	390	189	0	-26	10	30	-280	155	0	0	10	-11	-3	0	0	0	2	0	3	8	-5	0	0	-2	0	0	0	1	1	0	0	-1	0	0	0	0	-1	-1	0	0	0	x_{32}
x_{33}	+	1354320	2288	336	-27	0	16	8	0	70	-55	-30	-30	-5	-7	-3	0	-5	0	0	0	13	-2	1	6	6	3	1	1	0	-1	1	0	-1	-2	0	0	0	0	1	1	-2	0	0	x_{33}
x_{34}	+	1361920	1792	0	112	40	0	32	0	-70	-80	0	0	-10	-8	0	0	0	0	0	1	-8	2	0	0	0	2	0	0	-1	-4	0	0	0	2	0	0	0	0	0	0	2	0	0	x_{34}
x_{35}	o	1361920	-1792	0	112	40	0	0	0	-70	-80	0	0	-10	8	0	0	0	0	0	1	8	-2	0	0	0	-2	0	0	-1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	x_{35}
x_{36}	o	1361920	-1792	0	112	40	0	0	0	-70	-80	0	0	-10	8	0	0	0	0	0	1	8	-2	0	0	0	-2	0	0	-1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	x_{36}
x_{37}	+	1575936	3200	0	216	0	0	-16	0	91	-64	-24	-24	-4	-4	0	0	5	-4	0	0	0	-5	0	0	0	0	0	0	-1	2	0	0	1	1	0	0	0	0	0	0	-1	0	0	x_{37}
x_{38}	+	1625184	-1056	96	162	0	-32	0	0	204	-66	-6	-6	-1	6	-6	0	1	0	0	0	-6	4	6	6	6	-1	1	1	0	0	-2	0	1	-3	0	0	0	0	-2	-2	0	0	0	x_{38}
x_{39}	+	2031480	1848	120	-162	0	8	0	0	120	-270	0	0	10	-6	6	0	-4	0	0	0	-2	8	10	0	0	-2	0	0	0	0	2	0	0	3	0	0	0	0	-2	-2	0	0	0	x_{39}
x_{40}	+	2375000	-1000	600	-190	80	40	0	0	0	0	0	0	0	-10	-6	0	5	0	0	-1	0	0	0	0	0	0	0	0	1	0	-2	0	1	0	0	0	0	0	0	0	0	0	0	x_{40}
x_{41}	+	2407680	-1408	256	36	36	0	0	0	280	180	-20	-20	5	-4	4	4	-5	0	0	0	-8	-8	-4	-4	-4	-3	1	1	0	0	0	0	-1	1	1	1	0	0	0	0	0	0	0	x_{41}
x_{42}	+	2661120	0	768	0	-54	0	0	0	0	120	10	10	-20	0	0	6	0	0	0	0	0	0	8	-2	-2	0	-2	-2	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	x_{42}
x_{43}	+	2784375	3575	375	-189	0	-25	-1	15	-125	0	0	0	0	11	3	0	6	-1	-1	0	0	-5	0	0	0	0	0	0	0	-1	-1	0	-2	1	0	0	1	1	0	0	-1	0	0	x_{43}
x_{44}	+	2985984	4608	0	0	0	0	0	0	-36	-16	24	24	-6	0	0	0	-6	0	0	0	8	-12	0	0	0	-2	0	0	1	0	0	0	2	0	0	0	1	1	0	0	0	0	0	x_{44}
x_{45}	+	3200000	-3200	0	104	-40	0	16	0	125	0	0	0	0	4	0	0	-1	-4	0	-1	0	5	0	0	0	0	0	0	1	-2	0	0	-1	-1	0	0	1	1	0	0	1	0	0	x_{45}
x_{46}	+	3424256	0	0	-64	8	0	0	0	176	256	16	16	-4	0	0	0	-4	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	-2	-2	0	0	0	0	0	0	0	x_{46}
x_{47}	+	3878280	-2520	648	0	0	-24	0	-24	0	-95	15	15	20	0	0	0	0	0	0	0	5	0	-7	3	3	0	-2	-2	-1	0	0	0	0	0	0	0	0	0	1	1	0	1	1	x_{47}
x_{48}	+	4156250	-1750	-550	-175	5	10	30	10	0	0	0	0	0	5	-7	5	0	0	-2	-1	0	0	0	0	0	0	0	0	-1	3	1	1	0	0	0	0	0	0	0	0	0	0	0	x_{48}
x_{49}	+	4561920	-1408	0	216	0	0	-16	0	55	-80	0	0	-10	-4	0	0	-1	4	0	0	-8	7	0	0	0	2	0	0	0	2	0	0	-1	1	0	0	1	1	0	0	-1	0	0	x_{49}
x_{50}	+	4809375	175	-225	-216	0	15	-29	15	125	0	0	0	0	4	0	0	4	1	3	0	0	5	0	0	0	0	0	0	-1	-2	0	0	0	-1	0	0	0	0	0	0	1	0	0	x_{50}
x_{51}	+	5103000	2520	-360	0	0	-24	0	0	0	-125	0	0	0	0	0	0	0	0	0	0	-5	0	-5	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	-1	2r5+1	-2r5+1	0	0	0	x_{51}
x_{52}	+	5103000	2520	-360	0	0	-24	0	0	0	-125	0	0	0	0	0	0	0	0	0	0	-5	0	-5	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	-1	-2r5+1	2r5+1	0	0	0	x_{52}
x_{53}	+	5332635	-693	27	0	0	-21	-9	-21	-315	135	-15	-15	10	0	0	0	0	1	-1	0	7	-3	7	-3	-3	2	2	2	0	0	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	x_{53}
x_{54}	+	5878125	-4675	-275	36	-45	45	9	5	-125	0	0	0	0	-4	4	-5	1	-1	1	0	0	-5	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	-1	0	0	x_{54}

$HN = F_{5+}$

	@	@	@	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
														3628	887	15	1	15																																	
	21	22	25	25	60	30	30	35	35	40	40			800	04000	360	280	120	432	162	640	384	80	48	600	50	720	48	42	9	1 000	600	50	40	20	12	12	12	14	30	20	20	20	21	22	22	30		
	AA	AA	B	B	ABA	CEC	BDC	AA	AA	CA	CA			A	A	A	B	AC	AC	BC	A	A	B	C	AC	EC	AD	AE	AC	AF	BD	AD	BE	BF	AD	BF	BF	AD	AID	AE	BC	BC	ABD	AD	AD	AGD		
	AA	AA	A	A	ABA	BDC	CEC	AA	AA	AA	AA			A	A	A	A	AC	AC	BC	A	A	A	A	AC	EC	AD	AE	AC	AC	AD	AD	ED	BE	CF	BF	BF	AD	AID	AE	BC	BC	ABD	AD	AD	AGD		
	21A	22A	25A	B*	30A	30B	C*	35A	B**	40A	B**	fus	ind	2C	4D	4E	4F	6D	6E	6F	8C	8D	8E	8F	10I	10J	12D	12E	14B	18A	20F	20G	20H	20I	20J	24A	24B	C*	28A	30D	40C	40D	E*	42A	44A	B*	60A		
x_1	1	1	1	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	0	-1	-b5	*	-2	b5	*	0	0	-1	-1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_2
x_3	0	-1	*	-b5	-2	*	b5	0	0	-1	-1																																					x_3
x_4	1	1	0	0	2	-1	-1	-1	-1	0	0	:	++	16	-56	8	0	-8	4	-2	0	0	0	0	-4	1	-2	2	2	1	-6	4	-1	-2	0	0	0	0	0	2	0	0	0	-1	-1	-1	-2	x_4
x_5	-1	0	-1	-1	0	-1	-1	0	0	0	0	:	++	36	132	4	4	15	3	0	4	4	0	0	6	1	-3	1	1	0	7	2	-3	-1	-1	1	0	0	-1	0	0	-1	-1	1	0	0	2	x_5
x_6	0	0	-b5	*	1	-b5	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_6
x_7	0	0	*	-b5	1	*	-b5	0	0	0	0																																					x_7
x_8	-1	0	0	0	2	0	0	-1	-1	0	0	:	++	36	176	16	-4	15	3	0	4	-4	-2	0	6	1	5	1	1	0	1	6	1	1	1	-1	0	0	1	0	-2	-1	-1	1	0	0	0	x_8
x_9	1	0	0	0	1	1	1	-1	-1	-1	-1	:	++	51	231	7	-5	6	6	-3	-5	3	1	-1	1	1	6	-2	2	0	6	1	1	2	0	0	-1	-1	0	1	1	0	0	-1	0	0	1	x_9
x_{10}	-1	0	-1	-1	2	0	0	1	1	1	1	:	++	117	77	13	5	15	3	0	5	5	1	-3	-3	2	5	1	-2	0	2	-3	2	-2	0	-1	0	0	0	0	1	0	0	1	0	0	0	x_{10}
x_{11}	0	0	b5	*	-1	*b5+2		0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{11}
x_{12}	0	0	*	b5	-1	b5+2	*	0	0	0	0																																					x_{12}
x_{13}	0	0	0	0	-1	b5	*	0	0	1	1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{13}
x_{14}	0	0	0	0	-1	*	b5	0	0	1	1																																					x_{14}
x_{15}	0	1	0	0	0	0	0	b35	**	-1	-1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{15}
x_{16}	0	1	0	0	0	0	0	**	b35	-1	-1																																					x_{16}
x_{17}	-1	0	1	1	-1	0	0	0	0	-1	-1	:	++	160	1408	0	0	34	-2	-2	0	0	4	0	5	0	4	0	-1	1	8	-7	-2	0	0	0	0	0	1	-1	-1	0	0	-1	0	0	-1	x_{17}
x_{18}	1	0	0	0	-1	-1	-1	1	1	0	0	:	++	384	1056	32	0	6	6	6	0	0	0	0	4	-1	-6	2	-1	0	6	-4	1	2	0	0	0	0	-1	1	0	0	0	-1	0	0	-1	x_{18}
x_{19}	0	0	-1	-1	-1	0	0	1	1	0	0	:	++	252	-1540	-4	-4	-21	3	0	4	4	0	0	2	2	-1	-1	0	0	-15	-10	0	1	1	1	0	0	0	-1	0	-1	-1	0	0	0	-1	x_{19}
x_{20}	0	0	0	0	0	0	0	0	0	0	0	:	++	350	770	34	6	35	-1	-1	6	-2	0	2	0	0	5	1	0	-1	-5	0	0	-1	1	1	-1	-1	0	0	0	1	1	0	0	0	0	x_{20}
x_{21}	0	0	-b5	*	0	*	b5	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{21}
x_{22}	0	0	*	-b5	0	b5	*	0	0	0	0																																					x_{22}
x_{23}	1	0	1	1	1	0	0	0	0	0	0	:	++	216	-1056	32	8	-36	0	0	0	0	0	0	-4	1	6	2	-1	0	-6	4	-1	2	-2	0	0	0	1	-1	0	0	0	-1	0	0	1	x_{23}
x_{24}	1	0	0	0	-1	0	0	-1	-1	1	1	:	++	485	1925	5	5	-1	-1	-1	5	5	1	1	5	0	-1	-1	2	-1	0	5	0	0	0	-1	1	1	0	-1	1	0	0	-1	0	0	-1	x_{24}
x_{25}	0	-1	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{25}
x_{26}	0	-1	0	0	0	0	0	0	0	0	0																																					x_{26}
x_{27}	0	1	0	0	0	b5	*	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{27}
x_{28}	0	1	0	0	0	*	b5	0	0	0	0																																					x_{28}
x_{29}	0	0	0	0	-1	1	1	0	0	0	0	:	++	252	-2772	44	-4	-21	3	0	-4	-4	0	0	2	2	-9	-1	0	0	3	-2	-2	-1	1	-1	0	0	0	-1	0	1	1	0	0	0	1	x_{29}
x_{30}	0	0	*	b5	0	0	0	0	0	-1	-1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{30}
x_{31}	0	0	b5	*	0	0	0	0	0	-1	-1																																					x_{31}
x_{32}	0	0	0	0	-1	0	0	0	0	0	0	:	++	252	1232	-16	4	-21	3	0	-4	4	-2	0	2	2	-1	-1	0	0	7	2	2	-1	-1	1	0	0	0	-1	-2	1	1	0	0	0	-1	x_{32}
x_{33}	1	0	0	0	-2	0	0	0	0	0	0	:	++	36	1364	-44	4	15	3	0	-4	-4	0	0	6	1	5	1	1	0	-11	-6	-1	1	-1	-1	0	0	-1	0	0	1	1	1	0	0	0	x_{33}
x_{34}	0	-1	0	0	2	0	0	0	0	0	0	:	++	448	1792	0	0	28	4	-2	0	0	0	0	-2	-2	-8	0	0	1	-8	2	2	0	0	0	0	0	0	-2	0	0	0	0	-1	-1	2	x_{34}
x_{35}	0	1	0	0	-2	0	0	0	0	i10	-i10		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{35}
x_{36}	0	1	0	0	-2	0	0	0	0	-i10	i10																																					x_{36}
x_{37}	-1	-1	1	1	1	0	0	0	0	1	1	:	++	288	-3200	0	0	-6	6	0	0	0	4	0	-7	-2	4	0	1	0	0	5	0	0	0	0	0	0	-1	-1	-1	0	0	1	1	1	-1	x_{37}
x_{38}	1	0	-1	-1	1	0	0	1	1	0	0	:	++	216	2904	-40	-8	-36	0	0	0	0	0	0	-4	1	6	2	-1	0	4	4	-1	0	2	0	0	0	-1	-1	0	0	0	-1	0	0	1	x_{38}
x_{39}	-1	0	0	0	-1	0	0	1	1	0	0	:	++	432	1848	-8	0	36	0	0	0	0	0	0	-8	2	-6	-2	-2	0	-2	8	-2	2	0	0	0	0	0	1	0	0	0	1	0	0	-1	x_{39}
x_{40}	-1	1	0	0	0	0	0	0	0	0	0	:	++	400	1000	40	0	-20	-8	4	0	0	0	0	0	0	10	-2	1	1	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	-1	-1	0	x_{40}
x_{41}	1	0	0	0	1	-1	-1	0	0	0	0	:	++	384	-1056	-32	0	6	6	6	0	0	0	0	4	-1	6	-2	-1	0	-6	4	-1	-2	0	0	0	0	1	1	0	0	0	-1	0	0	1	x_{41}
x_{42}	0	0	0	0	0	1	1	0	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r6	-r6	0	0	0	0	0	0	0	0	0	x_{42}
x_{43}	0	0	0	0	1	0	0	1	1	-1	-1	:	++	315	-1925	-5	-5	21	-3	0	-5	11	-1	3	-5	0	1	1	0	0	0	-5	0	0	0	-1	0	0	0	1	-1	0	0	0	0	0	1	x_{43}
x_{44}	0	-1	-1	-1	0	0	0	-1	-1	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r11	-r11	0	x_{44}
x_{45}	-1	1	0	0	-1	0	0	-1	-1	1	1	:	++	160	-3200	0	0	34	-2	-2	0	0	-4	0	5	0	4	0	-1	1	0	5	0	0	0	0	0	0	-1	-1	1	0	0	-1	1	1	-1	x_{45}
x_{46}	-1	0	1	1	0	0	0	1	1	0	0	:	++	768	0	0	0	-48	0	-6	0	0	0	0	8	-2	0	0	-2	0	0	0	0	0	0	0	0	0	0	2	0	0	0	1	0	0	0	x_{46}
x_{47}	0	-1	0	0	0	0	0	0	0	0	0	:	++	0	2520	24	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	-5	0	0	-1	1	0	0	0	0	0	0	-1	-1	0	1	1	0	x_{47}
x_{48}	0	-1	0	0	0	0	0	0	0	0	0	:	++	350	-1750	10	-10	35	-1	-1	-10	-2	0	-2	0	0	5	1	0	-1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	-1	-1	0	x_{48}
x_{49}	-1	0	0	0	1	0	0	-1	-1	-1	-1	:	++	288	1408	0	0	-6	6	0	0	0	-4	0	-7	-2	4	0	1	0	8	-7	-2	0	0	0	0	0	1	-1	1	0	0	1	0	0	-1	x_{49}
x_{50}	1	-1	0	0	-1	0	0	-1	-1	1	1	:	++	765	-175	-15	5	6	-6	0	5	-3	-1	-3	-5	0	-4	0	2	0	0	-5	0	0	0	0	0	0	0	1	-1	0	0	-1	1	1	1	x_{50}
x_{51}	0	1	0	0	0	0	0	0	0	0	0	:	++	0	2520	24	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	-1	-1	0	0	0	0	0	0	r5	-r5	0	1	1	0	x_{51}
x_{52}	0	1	0	0	0	0	0	0	0	0	0	:	++	0	2520	24	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	0	0	-1	-1	0	0	0	0	0	0	-r5	r5	0	1	1	0	x_{52}
x_{53}	0	0	0	0	0	0	0	0	0	1	1	:	++	567	-693	-21	-1	0	0	0	-1	-9	1	3	-3	2	0	0	0	0	7	-3	2	-1	-1	0	0	0	0	0	1	-1	-1	0	0	0	0	x_{53}
x_{54}	1	0	0	0	1	0	0	1	1	-1	-1	:	++	435	-825	-25	-5	-6	-6	3	-5	3	1	-1	5	0	-6	2	1	0	0	5	0	0	0	0	-1	-1	1	-1	1	0	0	1	0	0	-1	x_{54}

HN = F_{5+}

Sporadic Harada-Norton group HN $\cong$ F $\cong$ F_5 $\cong$ F_{5+}

Order = 273,030,912,000,000 = $2^{14}.3^6.5^6.7.11.19$ Mult = 1 Out = 2

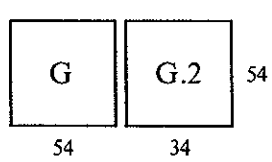
Constructions

Monster 5 x G : centralizer of a 5A-element in the Monster; the normalizer is $(D_{10} \times G).2$.

Real G has a 133-dimensional representation over $\mathbb{Q}(\sqrt{5})$, and the outer automorphism changes the sign of $\sqrt{5}$. The 133-space contains an orbit of 1140000 vectors which form a rank 12 graph of valence 462. The point stabilizer A_{12} has orbit lengths 1, 462, 2520, 2520, 10395, 16632, 30800, 69300, 166320, 166320, 311850, 362880. Orbits of equal length are fused in G.2. Explicit matrices have been computed.

Maximal subgroups

Order	Index	Structure	G.2	Specifications: Abstract	Specifications: Graph
239500800	1140000	A_{12}	: S_{12}		point
177408000	1539000	$2{\cdot}HS.2$	: $4{\cdot}HS.2$	N(2A)	
16547328	16500000	$U_3(8):3$	: $U_3(8):6$		
3686400	74064375	$2_+^{1+8}.(A_5 \times A_5).2$	: $2_+^{1+8}.(A_5 \times A_5).2^2$	N(2B)	
2520000	108345600	$(D_{10} \times U_3(5)){\cdot}2$	: $5{:}4 \times U_3(5){:}2$	N(5A)	Hoffman-Singleton graph
2000000	136515456	$5_+^{1+4}{:}2_-^{1+4}.5.4$	: H.2	N(5B)	
1658880	164587500	$2^6{\cdot}U_4(2)$	: H.2	$N(2^6) = N(2A_{36}B_{27})$	
1036800	263340000	$(A_6 \times A_6){\cdot}D_8$	: $(S_6 \times S_6){:}2^2$	$N(2A,3A,3A,4B,5A)^2$	edge
1032192	264515625	$2^3.2^2.2^6.(3 \times L_3(2))$	: H.2	$N(2B^3)$	
750000	364041216	$5^2.5.5^2.4A_5$	: H.2	$N(5B^2)$	
190080	1436400000	$M_{12}{:}2$			point pair
190080	1436400000	$M_{12}{:}2$			point pair
93312	2926000000	$3^4{:}2(A_4 \times A_4).4$	: $3^4{:}2(S_4 \times S_4).2$	$N(3^4) = N(3A_{24}B_{16})$	
58320	4681600000	$3_+^{1+4}{:}4A_5$	: $3_+^{1+4}{:}4S_5$	N(3B)	



;		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		3311126603366400	47563407360	47563407360	754974720	9437184	4354560	4354560	46656	7077888	7077888	5505024	5505024	1474560	1474560	196608	196608	65536	49152	49152	24576	16384	12288	8192	3600	69120	69120	13824	13824	5184	5184	4608	4608	1152	1152	576	1176	1176	
		p power	A	A	A	A	A	A	A	A	B	A	B	C	C	A	B	C	C	C	D	C	D	D	A	AA	BB	AB	BA	CA	CB	AC	BC	AD	BD	CD	A	A	
		p' part	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	BB	AB	BA	CA	CB	AC	BC	AD	BD	CD	A	A	
	ind	1A	2A	2B	2C	2D	3A	3B	3C	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	4K	4L	4M	4N	4O	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	7B	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	833	-63	-63	65	1	14	14	-4	25	25	-7	-7	5	-27	-7	-7	1	-3	-3	9	5	-3	1	8	-6	-6	6	6	0	0	2	2	-2	-2	4	0	0	χ_{2}
χ_{3}	+	1105	209	-47	81	17	34	7	-2	9	25	41	-7	-11	21	9	-7	1	-3	13	9	5	-3	1	5	14	-5	10	-1	2	-2	6	3	2	-1	2	-1	6	χ_{3}
χ_{4}	+	1105	-47	209	81	17	7	34	-2	25	9	-7	41	-11	21	-7	9	1	13	-3	9	5	-3	1	5	-5	14	-1	10	-2	2	3	6	-1	2	2	6	-1	χ_{4}
χ_{5}	+	1326	174	174	46	-18	21	21	3	-10	-10	22	22	26	-6	-10	-10	14	2	2	6	-6	2	-2	1	9	9	-3	-3	3	3	1	1	-3	-3	3	3	3	χ_{5}
χ_{6}	+	1377	225	225	97	33	27	27	0	9	9	41	41	37	5	9	9	1	13	13	9	5	-3	1	2	15	15	3	3	0	0	7	7	3	3	0	5	5	χ_{6}
χ_{7}	+	21658	-742	-742	410	-38	49	49	4	18	18	-14	-14	30	62	18	18	-6	-10	-10	18	-2	6	-6	8	-19	-19	-7	-7	-4	-4	5	5	1	1	4	0	0	χ_{7}
χ_{8}	+	22932	1428	1428	404	148	63	63	9	84	84	84	84	84	84	20	20	20	20	20	4	20	4	4	7	15	15	15	15	-3	-3	-1	-1	7	7	1	0	0	χ_{8}
χ_{9}	+	23205	1701	-603	421	37	84	57	12	69	-43	133	-43	-51	-51	5	-11	-11	-3	13	21	-3	-3	5	5	36	-15	-12	9	0	0	4	1	4	1	4	0	7	χ_{9}
χ_{10}	+	23205	-603	1701	421	37	57	84	12	-43	69	-43	133	-51	-51	-11	5	-11	13	-3	21	-3	-3	5	5	-15	36	9	-12	0	0	1	4	1	4	4	7	0	χ_{10}
χ_{11}	+	44200	3240	680	680	168	190	-35	1	104	8	232	8	40	40	40	8	8	8	40	8	8	8	8	0	30	5	14	-3	9	5	14	5	6	-3	-3	-5	9	χ_{11}
χ_{12}	+	44200	680	3240	680	168	-35	190	1	8	104	8	232	40	40	8	40	8	40	8	8	8	8	8	0	5	30	-3	14	5	9	5	14	-3	6	-3	9	-5	χ_{12}
χ_{13}	+	63700	980	980	-300	-44	70	70	-20	164	164	-28	-28	20	20	4	4	-12	4	4	-12	-12	-12	20	0	-10	-10	-10	-10	8	8	6	6	-2	-2	4	0	0	χ_{13}
χ_{14}	+	99450	5370	-2310	890	-70	225	90	-18	-78	18	274	-14	-130	30	-14	18	-6	-10	22	18	-2	6	-6	0	45	-30	9	-6	6	-6	5	2	-7	2	2	1	8	χ_{14}
χ_{15}	+	99450	-2310	5370	890	-70	90	225	-18	18	-78	-14	274	-130	30	18	-14	-6	22	-10	18	-2	6	-6	0	-30	45	-6	9	-6	6	2	5	2	-7	2	8	1	χ_{15}
χ_{16}	o	142884	-2268	-2268	36	36	0	0	0	-108	-108	84	84	36	36	-12	-12	4	-12	-12	-12	4	-12	20	9	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{16}
χ_{17}	o	142884	-2268	-2268	36	36	0	0	0	-108	-108	84	84	36	36	-12	-12	4	-12	-12	-12	4	-12	20	9	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}	+	162435	3843	-1533	643	-189	21	21	3	59	267	91	-21	-25	135	-5	43	-13	-17	-1	11	-9	-1	3	10	21	9	21	-3	-9	15	-11	1	-3	-3	3	0	0	χ_{18}
χ_{19}	+	162435	-1533	3843	643	-189	21	21	3	267	59	-21	91	-25	135	43	-5	-13	-1	-17	11	-9	-1	3	10	9	21	-3	21	15	-9	1	-11	-3	-3	3	0	0	χ_{19}
χ_{20}	o	183600	-720	-720	-720	48	0	0	-27	48	48	48	48	0	0	-16	-16	-16	0	0	16	0	32	-16	0	0	0	0	0	9	9	0	0	0	0	-3	-3	-3	χ_{20}
χ_{21}	o	183600	-720	-720	-720	48	0	0	-27	48	48	48	48	0	0	-16	-16	-16	0	0	16	0	32	-16	0	0	0	0	0	9	9	0	0	0	0	-3	-3	-3	χ_{21}
χ_{22}	+	216580	-252	-3836	516	4	112	49	-14	308	-108	-140	84	4	-124	-44	-12	4	-12	20	-12	4	-12	20	5	0	-11	16	9	18	-2	0	-3	-8	1	-2	0	0	χ_{22}
χ_{23}	+	216580	-3836	-252	516	4	49	112	-14	-108	308	84	-140	4	-124	-12	-44	4	20	-12	-12	4	-12	20	5	-11	0	9	16	-2	18	-3	0	1	-8	-2	0	0	χ_{23}
χ_{24}	+	249900	7980	7980	1580	172	105	105	-39	-68	-68	252	252	260	-60	-4	-4	-20	20	20	12	4	4	-4	0	45	45	-15	-15	-3	-3	5	5	1	1	1	0	0	χ_{24}
χ_{25}	+	269892	-4284	-4284	2116	-60	189	189	0	180	180	-140	-140	76	-244	-12	-12	4	-4	-4	36	12	12	-12	17	-39	-39	21	21	0	0	1	1	-3	-3	0	0	0	χ_{25}
χ_{26}	+	270725	1925	1925	-635	133	140	140	-31	37	37	-91	-91	5	5	37	37	69	-27	-27	-11	5	-11	-11	0	20	20	-4	-4	17	17	4	4	4	4	1	0	0	χ_{26}
χ_{27}	+	322218	810	810	1962	234	0	0	0	162	162	-126	-126	-18	270	-30	-30	10	6	6	18	14	6	-6	18	0	0	0	0	0	0	0	0	0	0	0	-6	-6	χ_{27}
χ_{28}	+	348075	12075	4395	1195	-149	315	-90	18	-45	51	371	83	175	15	-45	-13	-21	-9	23	3	-17	-9	-5	0	15	30	3	6	6	-6	7	-2	-5	-2	-2	0	7	χ_{28}
χ_{29}	+	348075	4395	12075	1195	-149	-90	315	18	51	-45	83	371	175	15	-13	-45	-21	23	-9	3	-17	-9	-5	0	30	15	6	3	-6	6	-2	7	-2	-5	-2	7	0	χ_{29}
χ_{30}	+	358020	10116	-3708	644	132	216	-27	0	180	-108	244	84	-124	4	84	-12	4	-12	20	-12	4	-12	20	-5	-24	-15	24	-3	0	0	8	-7	0	-3	0	5	-2	χ_{30}
χ_{31}	+	358020	-3708	10116	644	132	-27	216	0	-108	180	84	244	-124	4	-12	84	4	20	-12	-12	4	-12	20	-5	-15	-24	-3	24	0	0	-7	8	-3	0	0	-2	5	χ_{31}
χ_{32}	+	519792	3696	3696	-400	368	-84	-84	42	112	112	112	112	0	0	48	48	48	0	0	16	0	32	-16	-8	-36	-36	12	12	6	6	-4	-4	-4	-4	2	0	0	χ_{32}
χ_{33}	+	541450	17290	-630	2570	330	280	-35	-8	-62	162	546	-126	-50	110	66	-30	10	6	38	18	14	6	-6	0	40	-15	24	-11	-8	0	8	-7	0	-3	0	0	0	χ_{33}
χ_{34}	+	541450	-630	17290	2570	330	-35	280	-8	162	-62	-126	546	-50	110	-30	66	10	38	6	18	14	6	-6	0	-15	40	-11	24	0	-8	-7	8	-3	0	0	0	0	χ_{34}
χ_{35}	+	541450	3850	-5110	10	-118	-35	-35	46	362	154	42	154	-30	-30	-86	-6	-6	-14	34	-6	-14	18	10	0	-35	5	-19	13	-2	2	13	-11	5	5	2	0	0	χ_{35}
χ_{36}	+	541450	-5110	3850	10	-118	-35	-35	46	154	362	154	42	-30	-30	-6	-86	-6	34	-14	-6	-14	18	10	0	5	-35	13	-19	2	-2	-11	13	5	5	2	0	0	χ_{36}
χ_{37}	+	584766	12222	12222	1854	510	0	0	27	6	6	294	294	234	-54	70	70	30	18	18	6	10	2	-2	-9	0	0	0	0	-9	-9	0	0	0	0	3	0	0	χ_{37}
χ_{38}	+	626535	5607	-8217	2663	39	-189	216	0	351	63	63	-97	-85	-245	63	-33	7	-29	3	15	11	3	7	10	39	-24	-21	24	0	0	-1	8	3	0	0	0	-7	χ_{38}
χ_{39}	+	626535	-8217	5607	2663	39	216	-189	0	63	351	-97	63	-85	-245	-33	63	7	3	-29	15	11	3	7	10	-24	39	24	-21	0	0	8	-1	0	3	0	-7	0	χ_{39}
χ_{40}	+	812175	19215	-7665	3215	79	105	105	15	39	55	455	-105	-205	-45	-25	-41	-1	-37	43	7	3	-5	-1	0	45	-15	-15	9	-9	3	5	17	1	1	-5	0	0	χ_{40}
χ_{41}	+	812175	-7665	19215	3215	79	105	105	15	55	39	-105	455	-205	-45	-41	-25	-1	43	-37	7	3	-5	-1	0	-15	45	9	-15	3	-9	17	5	1	1	-5	0	0	χ_{41}
χ_{42}	+	877149	14301	14301	349	-483	189	189	0	-27	-27	133	133	289	1	-27	-27	61	-23	-23	21	1	9	13	-1	21	21	-3	-3	0	0	-11	-11	-3	-3	0	0	0	χ_{42}
χ_{43}	+	877149	-9891	-9891	4189	93	189	189	0	45	45	-147	-147	109	109	-51	-51	-3	-35	-35	45	13	-3	13	-1	-51	-51	-3	-3	0	0	13	13	-3	-3	0	0	0	χ_{43}
χ_{44}	+	947700	14580	14580	3060	756	0	0	0	324	324	132	132	180	180	36	36	20	36	36	-12	20	-12	20	0	0	0	0	0	0	0	0	0	0	0	0	-9	-9	χ_{44}
χ_{45}	+	1082900	-10220	-10220	2580	20	245	245	-16	-268	-268	-140	-140	100	100	52	52	20	4	4	36	-28	-12	4	0	-35	-35	-11	-11	4	4	-3	-3	5	5	8	0	0	χ_{45}
χ_{46}	+	1299480	19992	-12264	536	-360	357	-21	24	-200	408	56	280	-176	144	-40	-8	24	-16	16	-24	16	0	-8	5	-15	-9	-3	3	12	12	-7	-1	-3	3	0	0	0	χ_{46}
χ_{47}	+	1299480	-12264	19992	536	-360	-21	357	24	408	-200	280	56	-176	144	-8	-40	24	16	-16	-24	16	0	-8	5	-9	-15	3	-3	12	12	-1	-7	3	-3	0	0	0	χ_{47}
χ_{48}	+	1342575	29295	-5265	2415	-273	405	0	0	-81	-225	495	-97	-105	-105	-81	63	31	7	-9	15	-25	-9	-1	0	45	0	-27	0	0	0	-3	0	-3	0	0	-4	3	χ_{48}
χ_{49}	+	1342575	-5265	29295	2415	-273	0	405	0	-225	-81	-97	495	-105	-105	63	-81	31	-9	7	15	-25	-9	-1	0	0	45	0	-27	0	0	0	-3	0	-3	0	3	-4	χ_{49}
χ_{50}	+	1949220	35364	3108	2084	292	441	63	36	36	-252	420	-252	36	36	36	-60	-60	4	-28	-12	4	-12	-12	-5	9	15	9	15	12	12	-7	-1	1	7	4	0	0	χ_{50}
χ_{51}	+	1949220	3108	35364	2084	292	63	441	36	-252	36	-252	420	36	36	-60	36	-60	-28	4	-12	4	-12	-12	-5	15	9	15	9	12	12	-1	-7	7	1	4	0	0	χ_{51}
χ_{52}	+	2165800	15400	-2520	-2520	296	490	175	22	872	-56	-280	-56	-40	-40	-24	8	8	-8	-40	-24	-8	24	8	0	10	15	-6	-17	10	-18	-6	15	2	-1	2	0	0	χ_{52}
χ_{53}	+	2165800	-2520	15400	-2520	296	175	490	22	-56	872	-56	-280	-40	-40	8	-24	8	-40	-8	-24	-8	24	8	0	15	10	-17	-6	-18	10	15	-6	-1	2	2	0	0	χ_{53}
χ_{54}	+	2784600	-10920	-18600	1880	-40	-315	225	-18	408	504	280	-8	80	-240	-8	24	24	-48	-16	-24	16	0	-8	0	-15	45	-3	9	6	-6	-7	5	5	-7	2	0	7	χ_{54}
χ_{55}	+	2784600	-18600	-10920	1880	-40	225	-315	-18	504	408	-8	280	80	-240	24	-8	24	-16	-48	-24	16	0	-8	0	45	-15	9	-3	-6	6	5	-7	-7	5	2	7	0	χ_{55}
χ_{56}	+	2828800	43520	2560	2560	512	280	-260	-44	512	0	512	0	0	0	0	0	0	0	0	0	0	0	0	0	-40	-20	-8	-4	-4	4	-8	-20	8	-4	-4	9	2	χ_{56}
χ_{57}	+	2828800	2560	43520	2560	512	-260	280	-44	0	512	0	512	0	0	0	0	0	0	0	0	0	0	0	0	-20	-40	-4	-8	4	-4	-20	-8	-4	8	-4	2	9	χ_{57}
χ_{58}	+	3411968	-28672	-28672	4096	0	392	392	32	0	0	0	0	256	-256	0	0	0	0	0	0	0	0	0	-7	-40	-40	8	8	-16	-16	-8	-8	0	0	0	0	0	χ_{58}
χ_{59}	+	3898440	16968	-15288	2632	584	-441	504	-9	168	-120	168	-504	-120	-120	40	-56	40	8	-24	8	-24	8	8	-10	-9	24	-9	24	3	3	7	-8	-1	8	-1	0	0	χ_{59}
χ_{60}	+	3898440	-15288	16968	2632	584	504	-441	-9	-120	168	-504	168	-120	-120	-56	40	40	-24	8	8	-24	8	8	-10	24	-9	24	-9	3	3	-8	7	8	-1	-1	0	0	χ_{60}
χ_{61}	+	4331600	-5040	-40880	5200	-432	-280	350	17	-112	80	-112	336	80	80	16	80	16	-48	16	-16	16	-16	-16	0	0	-50	-8	-18	9	-11	16	-2	0	6	-3	0	0	χ_{61}
χ_{62}	+	4331600	-40880	-5040	5200	-432	350	-280	17	80	-112	336	-112	80	80	80	16	16	16	-48	-16	16	-16	-16	0	-50	0	-18	-8	-11	9	-2	16	6	0	-3	0	0	χ_{62}
χ_{63}	+	4385745	47313	-1071	209	-687	378	-189	0	-351	369	161	49	41	41	-63	-15	65	41	-7	-15	-7	9	1	-5	-30	-21	-6	3	0	0	2	11	-6	3	0	0	0	χ_{63}
χ_{64}	+	4385745	-1071	47313	209	-687	-189	378	0	369	-351	49	161	41	41	-15	-63	65	-7	41	-15	-7	9	1	-5	-21	-30	3	-6	0	0	11	2	3	-6	0	0	0	χ_{64}
χ_{65}	+	4385745	23121	-25263	4049	-111	-189	378	0	-279	441	-119	-231	-139	149	-87	-39	1	29	-19	9	5	-3	1	-5	51	-30	3	-6	0	0	-13	2	3	-6	0	0	0	χ_{65}
χ_{66}	+	4385745	-25263	23121	4049	-111	378	-189	0	441	-279	-231	-119	-139	149	-39	-87	1	-19	29	9	5	-3	1	-5	-30	51	-6	3	0	0	2	-13	-6	3	0	0	0	χ_{66}
χ_{67}	+	4526080	36864	-28672	4096	0	-56	232	16	0	0	0	0	-256	256	0	0	0	0	0	0	0	0	0	5	24	-40	8	-24	0	-16	-8	-8	0	0	0	6	-8	χ_{67}
χ_{68}	+	4526080	-28672	36864	4096	0	232	-56	16	0	0	0	0	-256	256	0	0	0	0	0	0	0	0	0	5	-40	24	-24	8	-16	0	-8	-8	0	0	0	-8	6	χ_{68}
χ_{69}	+	5012280	44856	31032	1336	-72	189	27	0	-72	-360	56	-104	304	-16	24	-72	-8	-16	16	-24	-16	0	-8	5	-39	15	21	3	0	0	1	7	-3	3	0	0	-7	χ_{69}
χ_{70}	+	5012280	31032	44856	1336	-72	27	189	0	-360	-72	-104	56	304	-16	-72	24	-8	16	-16	-24	-16	0	-8	5	15	-39	3	21	0	0	7	1	3	-3	0	-7	0	χ_{70}
χ_{71}	+	5640192	36864	36864	4096	0	-216	-216	0	0	0	0	0	256	-256	0	0	0	0	0	0	0	0	0	17	24	24	-24	-24	0	0	-8	-8	0	0	0	-2	-2	χ_{71}
χ_{72}	+	5870592	21504	21504	5120	1024	-504	-504	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	-24	-24	-24	-24	12	12	8	8	-8	-8	4	0	0	χ_{72}
χ_{73}	+	6497400	46200	-7560	2680	248	-105	-105	-42	-72	344	56	-168	-80	240	24	-8	-8	-80	16	-24	-16	0	-8	0	-45	75	15	15	-6	-18	-5	19	-1	-1	2	0	0	χ_{73}
χ_{74}	+	6497400	-7560	46200	2680	248	-105	-105	-42	344	-72	-168	56	-80	240	-8	24	-8	16	-80	-24	-16	0	-8	0	75	-45	15	15	-18	-6	19	-5	-1	-1	2	0	0	χ_{74}
χ_{75}	+	7309575	-28665	-28665	5895	-249	0	0	-27	855	-681	-105	-105	135	135	-9	119	-25	-9	-9	7	-25	7	7	0	0	0	0	0	9	9	0	0	0	0	-3	0	0	χ_{75}
χ_{76}	+	7309575	-28665	-28665	5895	-249	0	0	-27	-681	855	-105	-105	135	135	119	-9	-25	-9	-9	7	-25	7	7	0	0	0	0	0	9	9	0	0	0	0	-3	0	0	χ_{76}
χ_{77}	+	7894341	-16443	-16443	7749	261	0	0	0	189	189	-483	-483	9	-279	-99	-99	-59	33	33	-3	9	-15	-11	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{77}
χ_{78}	+	8055450	72090	-31590	2970	-486	0	0	0	-54	-486	330	90	-270	-270	42	90	-54	18	-30	-6	2	18	10	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	χ_{78}
χ_{79}	+	8055450	-31590	72090	2970	-486	0	0	0	-486	-54	90	330	-270	-270	90	42	-54	-30	18	-6	2	18	10	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	χ_{79}
χ_{80}	+	8771490	46242	-2142	-4958	-222	189	189	0	-270	450	-14	-126	10	10	18	66	-46	10	-38	18	26	-6	2	-10	-51	21	-3	-3	0	0	13	-11	-3	-3	0	0	0	χ_{80}
χ_{81}	+	8771490	-2142	46242	-4958	-222	189	189	0	450	-270	-126	-14	10	10	66	18	-46	-38	10	18	26	-6	2	-10	21	-51	-3	-3	0	0	-11	13	-3	-3	0	0	0	χ_{81}
χ_{82}	+	9398025	-2295	-36855	-6135	393	405	0	0	297	153	-279	665	-15	-15	105	-135	-7	1	-15	9	1	-15	-7	0	45	0	-27	0	0	0	-3	0	-3	0	0	-7	0	χ_{82}
χ_{83}	+	9398025	-36855	-2295	-6135	393	0	405	0	153	297	665	-279	-15	-15	-135	105	-7	-15	1	9	1	-15	-7	0	0	45	0	-27	0	0	0	-3	0	-3	0	0	-7	χ_{83}
χ_{84}	+	10740600	27000	-42120	-3720	120	-405	0	0	216	-72	216	568	-120	-120	24	-72	24	8	-24	24	-24	-24	-8	0	-45	0	27	0	0	0	3	0	3	0	0	-11	3	χ_{84}
χ_{85}	+	10740600	-42120	27000	-3720	120	0	-405	0	-72	216	568	216	-120	-120	-72	24	24	-24	8	24	-24	-24	-8	0	0	-45	0	27	0	0	0	3	0	3	0	3	-11	χ_{85}
χ_{86}	+	11609325	-2835	-2835	-2835	-531	0	0	0	621	621	-147	-147	45	45	45	45	-19	45	45	-3	-19	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{86}
χ_{87}	+	11609325	-2835	-2835	-2835	-531	0	0	0	621	621	-147	-147	45	45	45	45	-19	45	45	-3	-19	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{87}
χ_{88}	+	11880960	35328	10752	-5632	512	456	-84	-12	-512	0	-512	0	0	0	0	0	0	0	0	0	0	0	0	10	-24	-36	-24	12	12	-12	8	-4	8	-4	-4	7	0	χ_{88}
χ_{89}	+	11880960	10752	35328	-5632	512	-84	456	-12	0	-512	0	-512	0	0	0	0	0	0	0	0	0	0	0	10	-36	-24	12	-24	-12	12	-4	8	-4	8	-4	0	7	χ_{89}
χ_{90}	+	14034384	35280	35280	-560	-816	-378	-378	0	144	144	-112	-112	160	160	-48	-48	16	-32	-32	48	32	0	16	-16	30	30	6	6	0	0	-2	-2	6	6	0	0	0	χ_{90}
χ_{91}	+	14619150	23310	-57330	270	-114	0	0	-54	-162	-498	-546	462	-90	-90	-66	14	30	6	54	-18	22	6	-2	0	0	0	0	0	-18	18	0	0	0	0	6	0	0	χ_{91}
χ_{92}	+	14619150	-57330	23310	270	-114	0	0	-54	-498	-162	462	-546	-90	-90	14	-66	30	54	6	-18	22	6	-2	0	0	0	0	0	18	-18	0	0	0	0	6	0	0	χ_{92}
χ_{93}	+	15181425	-59535	-59535	-1935	369	0	0	0	-351	-351	609	609	225	225	-63	-63	-47	-15	-15	-15	1	33	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{93}
χ_{94}	+	16777216	0	0	0	0	-512	-512	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	8	8	χ_{94}
χ_{95}	+	17326400	-20160	-20160	320	320	140	140	68	-448	-448	-448	-448	0	0	64	64	64	0	0	0	0	0	0	0	60	60	12	12	0	0	-4	-4	-4	-4	-4	0	0	χ_{95}

$F_4(2)$

	8A	8B	8C	8D	8E	8F	8G	8H	8I	8J	8K	9A	9B	10A	10B	10C	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	12N	12O	12P	Q**	13A	14A	14B	15A	15B	16A	16B	16C	D**	17A	B*	18A	18B	20A	20B	21A	21B
	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
	2	2	1	1	1	1	1	1	1								1	1																																
	048	048	536	536	536	536	024	024	024	128	128	54	54	240	240	80	152	152	864	864	576	576	576	576	384	384	96	96	96	96	48	48	48	13	56	56	90	90	64	64	64	64	17	17	18	18	40	40	21	21
	I	I	C	D	A	B	I	I	I	L	O	C	C	AA	AB	AC	CB	DA	EA	FB	GE	HE	GF	HF	CD	DC	GJ	HK	EG	FH	KL	KN	KN	A	AA	BB	AA	AB	A	B	I	I	A	A	AE	BF	CE	CF	AA	BB
	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	AC	AB	BA	CA	CB	AE	BE	AF	BF	AD	BC	AJ	BK	CG	CH	CL	CN	CN	A	AA	BB	AA	AB	A	A	A	A	A	A	AA	BB	AE	AF	AA	BB
x_{1}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
x_{2}	1	-7	1	1	-3	-3	1	1	5	1	1	2	2	2	2	0	-2	-2	-2	-2	-4	-4	0	0	2	2	0	0	2	2	0	0	0	1	0	0	-1	-1	1	1	-1	-1	0	0	0	0	0	-2	0	0
x_{3}	-3	5	1	-3	5	1	-3	5	1	1	1	1	1	-1	3	1	-2	3	0	-2	4	1	0	-3	2	-1	0	1	0	2	0	0	0	0	-1	2	-1	2	-1	-1	1	1	0	0	-1	1	-1	1	-1	0
x_{4}	-3	5	-3	1	1	5	5	-3	1	1	1	1	1	3	-1	1	3	-2	-2	0	1	4	-3	0	-1	2	1	0	2	0	0	0	0	0	2	-1	2	-1	-1	-1	1	1	0	0	1	-1	-1	1	0	-1
x_{5}	6	-2	-2	-2	2	2	-2	-2	2	-2	2	0	0	-1	-1	1	5	5	-1	-1	-1	-1	3	3	1	1	-1	-1	-1	-1	3	-1	-1	0	-1	-1	1	1	-2	2	0	0	0	0	0	0	1	-1	0	0
x_{6}	9	1	5	5	1	1	1	1	-3	1	1	0	0	0	0	2	3	3	0	0	1	1	5	5	-1	-1	1	1	0	0	0	0	0	-1	1	1	2	2	1	1	-1	-1	0	0	0	0	2	0	-1	-1
x_{7}	2	10	2	2	-2	-2	-6	-6	6	-2	2	1	1	-2	-2	0	-3	-3	0	0	3	3	-1	-1	1	1	-1	-1	0	0	0	0	0	0	0	0	-1	-1	-2	2	0	0	0	0	-1	-1	0	2	0	0
x_{8}	4	4	4	4	4	4	4	4	4	0	0	0	0	3	3	-1	3	3	3	3	3	3	3	3	3	3	-1	-1	-1	-1	1	1	1	0	0	0	-2	-2	0	0	0	0	-1	-1	0	0	-1	-1	0	0
x_{9}	-7	-7	1	-3	1	-3	1	-7	1	1	1	0	0	1	-3	1	-4	9	6	2	0	-3	0	-3	-4	1	0	1	2	-2	0	0	0	0	0	-1	-1	2	-1	-1	-1	-1	0	0	0	0	-1	-1	0	1
x_{10}	-7	-7	-3	1	-3	1	-7	1	1	1	1	0	0	-3	1	1	9	-4	2	6	-3	0	-3	0	1	-4	1	0	-2	2	0	0	0	0	-1	0	2	-1	-1	-1	-1	-1	0	0	0	0	-1	-1	1	0
x_{11}	0	0	8	0	8	0	0	0	0	0	0	-2	1	0	0	0	2	-7	5	-1	-2	1	-2	1	2	1	2	1	1	-1	-1	-1	-1	0	-1	1	0	0	0	0	0	0	0	0	0	-1	0	0	1	0
x_{12}	0	0	0	8	0	8	0	0	0	0	0	1	-2	0	0	0	-7	2	-1	5	1	-2	1	-2	1	2	1	2	-1	1	-1	-1	-1	0	1	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	1
x_{13}	-4	-4	0	0	0	0	4	4	-4	4	-4	1	1	0	0	0	2	2	2	2	2	2	2	2	2	2	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0
x_{14}	-6	2	-6	2	6	-2	2	2	-2	-2	2	0	0	0	0	0	-3	6	-6	0	-1	2	3	-6	1	-2	-1	-2	-2	0	0	0	0	0	1	0	0	0	2	-2	0	0	0	0	0	0	0	0	1	-1
x_{15}	-6	2	2	-6	-2	6	2	2	-2	-2	2	0	0	0	0	0	6	-3	0	-6	2	-1	-6	3	-2	1	-2	-1	0	-2	0	0	0	0	0	1	0	0	2	-2	0	0	0	0	0	0	0	0	-1	1
x_{16}	-4	-4	0	0	0	0	4	4	-4	0	0	0	0	-3	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	4i	-4i	-1	-1	0	0	1	1	0	0
x_{17}	-4	-4	0	0	0	0	4	4	-4	0	0	0	0	-3	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-4i	4i	-1	-1	0	0	1	1	0	0
x_{18}	-1	7	-5	7	-1	-5	-1	7	3	-1	-1	0	0	-2	2	-2	-3	5	5	-3	5	-1	-3	3	-3	1	1	-1	1	1	-1	-1	-1	0	0	0	1	1	1	1	-1	-1	0	0	0	0	0	0	0	0
x_{19}	-1	7	7	-5	-5	-1	7	-1	3	-1	-1	0	0	2	-2	-2	5	-3	-3	5	-1	5	3	-3	1	-3	-1	1	1	1	-1	-1	-1	0	0	0	1	1	1	1	-1	-1	0	0	0	0	0	0	0	0
x_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	-1	-1	1	2i3-1	-2i3-1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	-1	-1	1	-2i3-1	2i3-1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{22}	4	4	8	0	-8	0	-4	-4	4	0	0	-2	1	3	-1	1	0	5	2	0	4	-5	-4	-1	0	1	0	-1	-2	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	1	-1	1	0	0
x_{23}	4	4	0	8	0	-8	-4	-4	4	0	0	1	-2	-1	3	1	5	0	0	2	-5	4	-1	-4	1	0	-1	0	0	-2	0	0	0	0	0	0	-1	2	0	0	0	0	0	0	1	0	-1	1	0	0
x_{24}	12	-4	4	4	-4	-4	-4	-4	4	0	0	0	0	0	0	0	1	1	-5	-5	-1	-1	3	3	-3	-3	-1	-1	-1	-1	-3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{25}	4	-12	4	4	-4	-4	4	4	-4	0	0	0	0	1	1	1	-3	-3	0	0	-5	-5	-1	-1	1	1	-1	-1	0	0	0	0	0	-1	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	0	0
x_{26}	5	5	-3	-3	-3	-3	5	5	5	1	1	-1	-1	0	0	0	4	4	1	1	-4	-4	-4	-4	-4	-4	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1	1	0	0	-1	-1	0	0	0	0
x_{27}	2	10	-6	-6	6	6	-6	-6	6	2	-2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	2	-2	0	0	0	0	0	0	2	0	0	0
x_{28}	11	3	-9	-1	3	-5	3	3	-1	-1	-1	0	0	0	0	0	3	-6	0	6	1	-2	-3	6	-1	2	-3	2	0	2	0	0	0	0	0	-1	0	0	-1	-1	1	1	0	0	0	0	0	0	0	1
x_{29}	11	3	-1	-9	-5	3	3	3	-1	-1	-1	0	0	0	0	0	-6	3	6	0	-2	1	6	-3	2	-1	2	-3	2	0	0	0	0	0	-1	0	0	0	-1	-1	1	1	0	0	0	0	0	0	1	0
x_{30}	4	4	-8	0	8	0	-4	-4	4	0	0	0	0	1	-3	-1	0	-3	0	0	-4	-1	4	-5	0	1	0	-1	0	0	0	0	0	0	1	2	1	-2	0	0	0	0	0	0	0	0	1	-1	-1	1
x_{31}	4	4	0	-8	0	8	-4	-4	4	0	0	0	0	-3	1	-1	-3	0	0	0	-1	-4	-5	4	1	0	-1	0	0	0	0	0	0	0	2	1	-2	1	0	0	0	0	0	0	0	0	1	-1	1	-1
x_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	-4	0	4	4	4	4	0	0	0	0	4	4	0	0	0	0	-2	2	2	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
x_{33}	-6	2	2	-6	-2	6	2	2	-2	2	-2	1	-2	0	0	0	0	1	-8	0	4	-5	-4	-1	0	-3	0	-1	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	1	0	0	0	0	0
x_{34}	-6	2	-6	2	6	-2	2	2	-2	2	-2	-2	1	0	0	0	1	0	0	-8	-5	4	-1	-4	-3	0	-1	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	1	0	0	0	0	0
x_{35}	6	6	-2	2	-2	2	-2	6	-2	-2	-2	1	1	0	0	0	1	5	2	-8	-3	-3	-3	-3	1	-3	1	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0
x_{36}	6	6	2	-2	2	-2	6	-2	-2	-2	-2	1	1	0	0	0	5	1	-8	2	-3	-3	-3	-3	-3	1	1	1	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0
x_{37}	6	-2	6	6	-6	-6	-2	-2	2	2	-2	0	0	-3	-3	-1	0	0	-3	-3	0	0	0	0	0	0	0	0	1	1	3	-1	-1	0	0	0	0	0	2	-2	0	0	0	0	0	0	-1	1	0	0
x_{38}	-1	-9	-9	-1	3	-5	-1	-1	3	-1	-1	0	0	2	-2	-2	3	0	0	0	5	-4	1	4	-1	0	1	0	0	0	0	0	0	0	0	1	1	1	-1	-1	1	1	0	0	0	0	0	0	0	-1
x_{39}	-1	-9	-1	-9	-5	3	-1	-1	3	-1	-1	0	0	-2	2	-2	0	3	0	0	-4	5	4	1	0	-1	0	1	0	0	0	0	0	0	1	0	1	1	-1	-1	1	1	0	0	0	0	0	0	-1	0
x_{40}	-5	3	-1	-5	-13	-1	-5	3	-1	-1	-1	0	0	0	0	0	1	-3	3	1	-1	5	3	-3	-3	5	-1	1	-1	1	1	1	1	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
x_{41}	-5	3	-5	-1	-1	-13	3	-5	-1	-1	-1	0	0	0	0	0	-3	1	1	3	5	-1	-3	3	5	-3	1	-1	1	-1	1	1	1	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0
x_{42}	5	-3	-7	-7	5	5	-3	-3	-7	1	1	0	0	1	1	-1	9	9	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	1	1	-1	-1	0	0	0	0	-1	1	0	0
x_{43}	5	5	1	1	1	1	-3	-3	-11	1	1	0	0	-1	-1	-1	-3	-3	0	0	1	1	1	1	-3	-3	1	1	0	0	0	0	0	0	0	0	-1	-1	1	1	1	1	0	0	0	0	-1	-1	0	0
x_{44}	-4	-4	0	0	0	0	4	4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	1	1	0	0	0	0	0	0
x_{45}	4	4	-4	-4	-4	-4	4	4	4	0	0	-1	-1	0	0	0	-7	-7	2	2	1	1	1	1	1	1	1	1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
x_{46}	8	-8	-4	4	4	-4	0	0	0	0	0	0	0	-3	1	1	-3	-5	-2	-6	7	1	3	-3	1	-1	-1	1	2	-2	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	0	-1	-1	0	0
x_{47}	8	-8	4	-4	-4	4	0	0	0	0	0	0	0	1	-3	1	-5	-3	-6	-2	1	7	-3	3	-1	1	1	-1	-2	2	0	0	0	0	0	0	-1	2	0	0	0	0	0	0	0	0	-1	-1	0	0
x_{48}	-5	-5	3	7	3	7	3	-5	3	-1	-1	0	0	0	0	0	-3	0	0	0	-3	0	-3	0	5	0	1	0	0	0	0	0	0	0	0	-1	0	0	1	1	1	1	0	0	0	0	0	0	-1	0
x_{49}	-5	-5	7	3	7	3	-5	3	3	-1	-1	0	0	0	0	0	0	-3	0	0	0	-3	0	-3	0	5	0	1	0	0	0	0	0	0	-1	0	0	0	1	1	1	1	0	0	0	0	0	0	0	-1
x_{50}	-4	-4	4	-4	4	-4	-4	-4	-4	0	0	0	0	-1	3	-1	-3	3	0	0	-3	3	-3	3	-3	3	1	-1	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	1	1	0	0
x_{51}	-4	-4	-4	4	-4	4	-4	-4	-4	0	0	0	0	3	-1	-1	3	-3	0	0	3	-3	3	-3	3	-3	-1	1	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	1	1	0	0
x_{52}	0	0	8	0	8	0	0	0	0	0	0	1	-2	0	0	0	-2	-1	8	-2	2	-1	2	-1	-2	-1	-2	-1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
x_{53}	0	0	0	8	0	8	0	0	0	0	0	-2	1	0	0	0	-1	-2	-2	8	-1	2	-1	2	-1	-2	-1	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
x_{54}	-8	8	-4	4	4	-4	0	0	0	0	0	0	0	0	0	0	-3	-3	-6	0	-1	-1	3	3	1	1	3	-1	-2	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
x_{55}	-8	8	4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	-3	-3	0	-6	-1	-1	3	3	1	1	-1	3	0	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
x_{56}	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-4	8	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	-1	1	0	0	0	-1
x_{57}	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-4	0	0	8	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	1	-1	0	0	-1	0
x_{58}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	3	3	1	0	0	0	0	4	4	-4	-4	0	0	0	0	0	0	0	0	0	1	0	0	2	2	0	0	0	0	0	0	-1	-1	1	-1	0	0
x_{59}	0	0	0	-8	0	-8	0	0	0	0	0	0	0	-2	2	2	3	0	-3	-3	3	0	3	0	3	0	-1	0	1	1	-1	-1	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
x_{60}	0	0	-8	0	-8	0	0	0	0	0	0	0	0	2	-2	2	0	3	-3	-3	0	3	0	3	0	3	0	-1	1	1	-1	-1	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
x_{61}	0	0	0	0	0	0	0	0	0	0	0	2	-1	0	0	0	8	2	5	-1	-4	2	-4	2	0	2	0	-2	1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
x_{62}	0	0	0	0	0	0	0	0	0	0	0	-1	2	0	0	0	2	8	-1	5	2	-4	2	-4	2	0	-2	0	-1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
x_{63}	-3	-3	1	-3	1	-3	-3	5	5	1	1	0	0	3	-1	-1	6	-9	0	0	2	-1	2	-1	-2	-1	2	-1	0	0	0	0	0	0	0	0	-2	1	-1	-1	-1	-1	0	0	0	0	1	1	0	0
x_{64}	-3	-3	-3	1	-3	1	5	-3	5	1	1	0	0	-1	3	-1	-9	6	0	0	-1	2	-1	2	-1	-2	-1	2	0	0	0	0	0	0	0	0	1	-2	-1	-1	-1	-1	0	0	0	0	1	1	0	0
x_{65}	-3	5	9	5	-3	-7	-3	5	1	1	1	0	0	1	-3	-1	3	6	0	0	-1	2	-1	2	3	-2	-1	2	0	0	0	0	0	0	0	0	1	-2	-1	-1	1	1	0	0	0	0	1	-1	0	0
x_{66}	-3	5	5	9	-7	-3	5	-3	1	1	1	0	0	-3	1	-1	6	3	0	0	2	-1	2	-1	-2	3	2	-1	0	0	0	0	0	0	0	0	-2	1	-1	-1	1	1	0	0	0	0	1	-1	0	0
x_{67}	0	0	0	0	0	0	0	0	0	0	0	-2	1	-1	3	1	0	0	0	0	-4	-4	4	4	0	0	0	0	0	0	0	0	0	0	2	0	-1	2	0	0	0	0	0	0	0	-1	-1	1	0	1
x_{68}	0	0	0	0	0	0	0	0	0	0	0	1	-2	3	-1	1	0	0	0	0	-4	-4	4	4	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	0	0	0	0	0	0	-1	0	-1	1	1	0
x_{69}	-8	8	-4	4	4	-4	0	0	0	0	0	0	0	1	-3	1	-3	3	0	0	-5	1	-1	5	1	-1	-1	1	0	0	0	0	0	0	0	1	-1	2	0	0	0	0	0	0	0	0	-1	-1	0	-1
x_{70}	-8	8	4	-4	-4	4	0	0	0	0	0	0	0	-3	1	1	3	-3	0	0	1	-5	5	-1	-1	1	1	-1	0	0	0	0	0	0	1	0	2	-1	0	0	0	0	0	0	0	0	-1	-1	-1	0
x_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	4	4	-4	-4	0	0	0	0	0	0	0	0	0	-1	2	2	-1	-1	0	0	0	0	0	0	0	0	1	-1	1	1
x_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-1	-1	0	0	0	0	0	0
x_{73}	8	-8	4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	-1	3	0	2	1	1	-3	-3	3	-1	1	1	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{74}	8	-8	-4	4	4	-4	0	0	0	0	0	0	0	0	0	0	3	-1	2	0	1	1	-3	-3	-1	3	1	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{75}	-1	-1	3	3	3	3	-9	7	-1	-1	-1	0	0	0	0	0	0	0	-9	3	0	0	0	0	0	0	0	0	3	-1	1	1	1	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0
x_{76}	-1	-1	3	3	3	3	7	-9	-1	-1	-1	0	0	0	0	0	0	0	3	-9	0	0	0	0	0	0	0	0	-1	3	1	1	1	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0
x_{77}	5	-3	-3	-3	9	9	5	5	9	1	1	0	0	-3	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	-1	1	0	0
x_{78}	6	6	-6	6	-6	6	6	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{79}	6	6	6	-6	6	-6	-2	6	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{80}	-2	-2	2	-2	2	-2	-2	6	6	-2	-2	0	0	2	-2	2	-3	9	0	0	1	1	1	1	-3	1	1	1	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
x_{81}	-2	-2	-2	2	-2	2	6	-2	6	-2	-2	0	0	-2	2	2	9	-3	0	0	1	1	1	1	1	-3	1	1	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
x_{82}	5	5	-3	1	-3	1	-3	5	-3	1	1	0	0	0	0	0	-3	0	0	0	-3	0	-3	0	5	0	1	0	0	0	0	0	0	0	1	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	-1	0
x_{83}	5	5	1	-3	1	-3	5	-3	-3	1	1	0	0	0	0	0	0	-3	0	0	0	-3	0	-3	0	5	0	1	0	0	0	0	0	0	0	1	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	-1
x_{84}	0	0	0	8	0	8	0	0	0	0	0	0	0	0	0	0	3	0	0	0	3	0	3	0	-5	0	-1	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	0
x_{85}	0	0	8	0	8	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	3	0	3	0	-5	0	-1	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
x_{86}	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	b17	*	0	0	0	0	0	0
x_{87}	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	*	b17	0	0	0	0	0	0
x_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	-2	0	4	-8	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	-1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0
x_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	-2	4	0	0	-8	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	1
x_{90}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	0	0	-2	-2	-2	-2	2	2	-2	-2	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0
x_{91}	2	2	6	-6	6	-6	2	-6	-6	-2	-2	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{92}	2	2	-6	6	-6	6	-6	2	-6	-2	-2	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{93}	-7	-7	-3	-3	-3	-3	1	1	9	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
x_{94}	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-2	-2	0	0	0	0	1	1	0	0	0	0	-1	-1
x_{95}	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	-4	-4	2	2	0	0	0	0	-4	-4	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	@	@	@	@	@	@	@	@	;	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
											359	20	3		1	1																								
	48	48	48	48	28	28	30	30			42400	480	072	216	280	280	384	256	192	128	128	100	24	64	64	32	16	16	20	12	12	12	13	16	16	16	16	20	20	
	ID	JC	AF	BE	AC	BD	AAA	BBB			A	C	D	CE	E	E	L	I	N	L	M	AE	KQ	I	I	A	K	K	CP	ON	QP	PP	AE	A	A	B	B	AL	AL	
	AD	BC	AF	BE	AC	BD	AAA	BBB			A	A	A	CE	A	A	A	A	A	A	A	AE	CQ	A	A	A	A	A	AP	CN	CP	CP	AE	A	A	A	A	AL	AM	
	24A	24B	24C	24D	28A	28B	30A	30B	fus	ind	2E	4P	4Q	6L	8L	8M	8N	8O	8P	8Q	8R	10D	12R	16E	16F	16G	16H	I**	20C	24E	24F	G**	26A	32A	B*	32C	D**	40A	40B	
χ_{1}	1	1	1	1	1	1	1	1	:	++	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	-2	-2	0	0	0	0	-1	-1	:	++	27	-5	3	0	-3	5	3	-1	-3	-1	1	2	0	1	-3	-1	-1	-1	0	0	0	0	1	-1	-1	1	1	2	0	χ_{2}
χ_{3}	0	1	-2	-1	-1	0	-1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{3}
χ_{4}	1	0	-1	-2	0	-1	0	-1																																χ_{4}
χ_{5}	1	1	-1	-1	1	1	-1	-1	:	++	26	-6	2	-1	4	-4	2	-2	4	-2	0	1	-1	4	0	-2	0	0	-1	-1	1	1	0	0	0	0	0	-1	1	χ_{5}
χ_{6}	-1	-1	1	1	-1	-1	0	0	:	++	27	-5	3	0	5	-3	3	-1	-3	-1	1	2	0	-3	1	-1	-1	-1	0	0	0	0	1	1	1	-1	-1	0	2	χ_{6}
χ_{7}	-1	-1	1	1	0	0	1	1	:	++	52	20	-4	-2	-2	-10	0	0	-6	4	2	2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	χ_{7}
χ_{8}	1	1	1	1	0	0	0	0	:	++	78	14	-2	-3	6	6	2	-2	2	2	-2	3	1	2	2	2	0	0	-1	-1	-1	-1	0	0	0	0	0	1	1	χ_{8}
χ_{9}	0	1	0	1	0	-1	1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{9}
χ_{10}	1	0	1	0	-1	0	0	1																																χ_{10}
χ_{11}	0	-1	0	-1	1	1	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{11}
χ_{12}	-1	0	-1	0	1	1	0	0																																χ_{12}
χ_{13}	0	0	0	0	0	0	0	0	:	oo	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2i2	-2i2	0	0	χ_{13}
χ_{14}	-1	0	1	0	1	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{14}
χ_{15}	0	-1	0	1	0	1	0	0																																χ_{15}
χ_{16}	0	0	0	0	0	0	0	0	:	oo	324	4	-12	0	-4	-4	0	4	0	0	4	-1	0	0	0	0	2i	-2i	-1	0	0	0	-1	0	0	0	0	1	1	χ_{16}
χ_{17}	0	0	0	0	0	0	0	0	:	oo	324	4	-12	0	-4	-4	0	4	0	0	4	-1	0	0	0	0	-2i	2i	-1	0	0	0	-1	0	0	0	0	1	1	χ_{17}
χ_{18}	1	1	1	-1	0	0	1	-1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{18}
χ_{19}	1	1	-1	1	0	0	-1	1																																χ_{19}
χ_{20}	0	0	0	0	-1	-1	0	0	:	oo	300	-20	-4	3	0	0	-4	4	0	4	0	0	-1	0	0	0	0	0	0	-1	-i3	i3	1	0	0	0	0	0	0	χ_{20}
χ_{21}	0	0	0	0	-1	-1	0	0	:	oo	300	-20	-4	3	0	0	-4	4	0	4	0	0	-1	0	0	0	0	0	0	-1	i3	-i3	1	0	0	0	0	0	0	χ_{21}
χ_{22}	0	-1	0	1	0	0	0	-1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{22}
χ_{23}	-1	0	1	0	0	0	-1	0																																χ_{23}
χ_{24}	1	1	-1	-1	0	0	0	0	:	++	350	30	14	-1	10	10	2	-2	-2	2	2	0	-1	-2	-2	2	0	0	0	-1	1	1	-1	0	0	0	0	0	0	χ_{24}
χ_{25}	1	1	-1	-1	0	0	1	1	:	++	378	-6	-6	0	-6	-6	6	2	6	-2	2	3	0	-2	-2	2	0	0	-1	0	0	0	1	0	0	0	0	-1	-1	χ_{25}
χ_{26}	0	0	0	0	0	0	0	0	:	++	325	5	-11	1	5	5	1	5	1	1	-3	0	1	1	1	1	-1	-1	0	1	1	1	0	-1	-1	-1	-1	0	0	χ_{26}
χ_{27}	0	0	0	0	0	0	0	0	:	++	702	30	6	0	0	8	6	2	0	2	4	2	0	0	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	-2	χ_{27}
χ_{28}	-1	0	1	0	0	-1	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{28}
χ_{29}	0	-1	0	1	-1	0	0	0																																χ_{29}
χ_{30}	0	1	0	-1	-1	0	1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{30}
χ_{31}	1	0	-1	0	0	-1	0	1																																χ_{31}
χ_{32}	0	0	0	0	0	0	-1	-1	:	++	52	-12	4	-2	0	0	-4	-4	0	4	0	2	-2	0	0	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}	0	-1	0	1	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{33}
χ_{34}	-1	0	1	0	0	0	0	0																																χ_{34}
χ_{35}	-1	1	-1	1	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{35}
χ_{36}	1	-1	1	-1	0	0	0	0																																χ_{36}
χ_{37}	0	0	0	0	0	0	0	0	:	++	624	16	8	3	14	6	4	4	2	0	2	-1	-1	2	-2	0	0	0	1	1	-1	-1	0	0	0	0	0	-1	1	χ_{37}
χ_{38}	-1	0	1	0	0	1	-1	1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{38}
χ_{39}	0	-1	0	1	1	0	1	-1																																χ_{39}
χ_{40}	1	-1	-1	-1	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{40}
χ_{41}	-1	1	-1	-1	0	0	0	0																																χ_{41}
χ_{42}	-1	-1	-1	-1	0	0	1	1	:	++	351	-1	-9	0	9	1	3	3	-3	-1	-3	1	0	-3	1	-1	1	1	-1	0	0	0	0	-1	-1	1	1	-1	1	χ_{42}
χ_{43}	1	1	1	1	0	0	-1	-1	:	++	351	31	15	0	-9	-9	3	-1	3	3	-1	1	0	-1	-1	-1	1	1	1	0	0	0	0	-1	-1	-1	-1	1	1	χ_{43}
χ_{44}	0	0	0	0	-1	-1	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r2	-2r2	0	0	0	0	χ_{44}
χ_{45}	-1	-1	-1	-1	0	0	0	0	:	++	650	10	10	2	-10	-10	6	2	-6	-2	-2	0	-2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{45}
χ_{46}	1	-1	-1	1	0	0	0	1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{46}
χ_{47}	-1	1	1	-1	0	0	1	0																																χ_{47}
χ_{48}	1	0	1	0	-2	1	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{48}
χ_{49}	0	1	0	1	1	-2	0	0																																χ_{49}
χ_{50}	-1	1	-1	1	0	0	-1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{50}
χ_{51}	1	-1	1	-1	0	0	0	-1																																χ_{51}
χ_{52}	0	-1	0	-1	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{52}
χ_{53}	-1	0	-1	0	0	0	0	0																																χ_{53}
χ_{54}	1	-1	-1	1	0	-1	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{54}
χ_{55}	-1	1	1	-1	-1	0	0	0																																χ_{55}
χ_{56}	0	0	0	0	1	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{56}
χ_{57}	0	0	0	0	0	1	0	0																																χ_{57}
χ_{58}	0	0	0	0	0	0	0	0	:	++	1728	-64	0	0	-16	16	0	0	0	0	0	3	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	-1	1	χ_{58}
χ_{59}	1	0	1	0	0	0	1	-1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{59}
χ_{60}	0	1	0	1	0	0	-1	1																																χ_{60}
χ_{61}	0	0	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{61}
χ_{62}	0	0	0	0	0	0	0	0																																χ_{62}
χ_{63}	0	1	0	1	0	0	0	-1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{63}
χ_{64}	1	0	1	0	0	0	-1	0																																χ_{64}
χ_{65}	-1	0	-1	0	0	0	1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{65}
χ_{66}	0	-1	0	-1	0	0	0	1																																χ_{66}
χ_{67}	0	0	0	0	0	0	-1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{67}
χ_{68}	0	0	0	0	0	0	0	-1																																χ_{68}
χ_{69}	1	-1	-1	1	0	1	1	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{69}
χ_{70}	-1	1	1	-1	1	0	0	1																																χ_{70}
χ_{71}	0	0	0	0	0	0	-1	-1	:	++	1728	-64	0	0	16	-16	0	0	0	0	0	3	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	1	-1	χ_{71}
χ_{72}	0	0	0	0	0	0	1	1	:	++	1248	-32	32	6	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	χ_{72}
χ_{73}	-1	1	1	-1	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{73}
χ_{74}	1	-1	-1	1	0	0	0	0																																χ_{74}
χ_{75}	0	0	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{75}
χ_{76}	0	0	0	0	0	0	0	0																																χ_{76}
χ_{77}	0	0	0	0	0	0	0	0	:	++	351	-1	-9	0	1	9	3	3	-3	-1	-3	1	0	1	-3	-1	1	1	-1	0	0	0	0	1	1	-1	-1	1	-1	χ_{77}
χ_{78}	0	0	0	0	1	-1	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{78}
χ_{79}	0	0	0	0	-1	1	0	0																																χ_{79}
χ_{80}	1	-1	1	-1	0	0	-1	1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{80}
χ_{81}	-1	1	-1	1	0	0	1	-1																																χ_{81}
χ_{82}	1	0	1	0	1	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{82}
χ_{83}	0	1	0	1	0	1	0	0																																χ_{83}
χ_{84}	-1	0	-1	0	-1	1	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{84}
χ_{85}	0	-1	0	-1	1	-1	0	0																																χ_{85}
χ_{86}	0	0	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{86}
χ_{87}	0	0	0	0	0	0	0	0																																χ_{87}
χ_{88}	0	0	0	0	-1	0	1	-1	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{88}
χ_{89}	0	0	0	0	0	-1	-1	1																																χ_{89}
χ_{90}	0	0	0	0	0	0	0	0	:	++	1404	60	12	0	0	0	-12	4	0	-4	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{90}
χ_{91}	0	0	0	0	0	0	0	0	|	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{91}
χ_{92}	0	0	0	0	0	0	0	0																																χ_{92}
χ_{93}	0	0	0	0	0	0	0	0	:	++	675	35	3	0	-5	-5	3	3	3	3	-5	0	0	-1	-1	-1	-1	-1	0	0	0	0	-1	1	1	1	1	0	0	χ_{93}
χ_{94}	0	0	0	0	0	0	0	0	:	++	4096	0	0	-8	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	χ_{94}
χ_{95}	0	0	0	0	0	0	0	0	:	++	2600	40	-24	8	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{95}

$F_4(2)$

Chevalley group $F_4(2)$

Order = 3,311,126,603,366,400 = $2^{24}.3^6.5^2.7^2.13.17$ Mult = 2 Out = 2

Constructions

Chevalley G : the adjoint Chevalley group of type F_4 over the field $\mathbb{F}_2$; the automorphism group of the exceptional 27-dimensional Jordan algebra over $\mathbb{F}_2$.

Real 2.G has a 52-dimensional real representation, in which it permutes two orbits of 2 x 69888 special vectors. These two orbits are interchanged by the outer automorphism.

The typical vector can be written $(x_*;\ldots x_S\ldots)$ where S runs over the sets of size 1, 5 or 9 in a 10-set, and the 52-space is defined by the relations $x_{S'} = -x_S$ and $x_{S(1)}+\ldots+x_{S(10)} = 0$, where S' is the complement of S, and S(1), ..., S(10) have even pairwise intersections. The norm of v is $x_*^2/3 + \Sigma x_S^2/24$ (summed over one set S from each complementary pair). There is a subgroup $S_8(2)$ (containing S_{10}) whose transvections permute the sets S by the maps $S \longrightarrow S' + [S,T]T$ defined for each set T of size 2 or 6, where [S,T] is the parity of the intersection of S and T. (The sets of size 2 yield the transpositions of S_{10}). One orbit of special vectors can be defined in terms of a 5-set A. We say that S has type m_n if it meets A in m points, A' in n points. Then the special vectors are the $S_8(2)$-images of those with

$$x_* = x_U = 2,\ x_V = 0 \qquad \text{or} \qquad x_* = x_U = x_V = 1$$

where U ranges over sets of type 1_0, 5_4, 0_5 and 4_1, and U ranges over those of type 3_2.

The above special vectors generate a lattice L which is geometrically similar to its dual lattice L*. The quotient L*/L is an elementary Abelian group of order 2^{26} which can be identified with the trace zero portion of the Jordan algebra above. After rescaling, L* contains the particular special vectors with

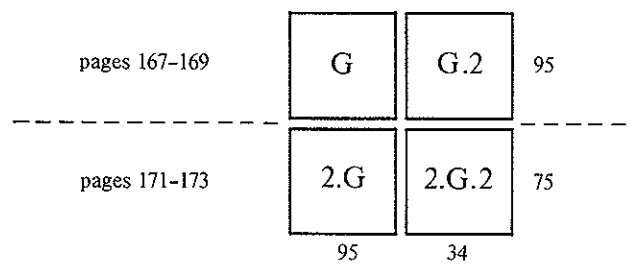
$$x_* = 6,\ x_S = 0 \text{ (all S)} \qquad \text{or} \qquad x_* = 3,\ x_A = 9,\ x_T = 1$$

where A is a fixed set, and T ranges over all other sets meeting A oddly.

Remark : $2(2^2 \times G).2$ is the normalizer of a certain dihedral group of order 8 in the Monster group F_1.

Some maximal subgroups (including all local subgroups)

Order	Index	Structure	G.2	Specification
47563407360	69615	$(2_+^{1+8} \times 2^6):S_6(2)$	$[2^{20}]:A_6{\cdot}2^2$	N(2A)
47563407360	69615	$(2_+^{1+8} \times 2^6):S_6(2)$		N(2B)
47377612800	69888	$S_8(2)$	$S_4(4):4$, $(S_6$ wr $2).2$	special vector in L
47377612800	69888	$S_8(2)$		special vector in L*
1056964608	3132675	$[2^{20}]:(S_3 \times L_3(2))$	$[2^{22}]:(S_3 \times S_3):2$	$N(2A^2)$
1056964608	3132675	$[2^{20}]:(S_3 \times L_3(2))$		$N(2B^2)$
1045094400	3168256	$O_8^+(2):S_3$		
1045094400	3168256	$O_8^+(2):S_3$		
634023936	5222400	${}^3D_4(2):3$	$7^2:(3 \times 2S_4)$	
634023936	5222400	${}^3D_4(2):3$		
35942400	92123136	${}^2F_4(2)$	: ${}^2F_4(2) \times 2$	C(2E)
12130560	272957440	$L_4(3):2_2$	: $L_4(3):2^2$	
56448	586579968	$(L_3(2) \times L_3(2)):2$	: H.2	N(2A,3A,4C,7B), N(2B,3B,4D,7A)
93312	35484467200	$3{\cdot}(3^2:Q_8 \times 3^2:Q_8).S_3$	: H.2	N(3C)



$F_4(2)$

		1A	2A	2B	2C	2D	3A	3B	3C	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	4K	4L	4M	4N	4O	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	
	ind	1 2	2 2	2 2	2 2	2 2	3 6	3 6	3 6	4	4	4 4	4 4	4 4	4 4	4	4	4 4	4 4	4 4	4 4	4	4	4	5 10	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	
χ_{96}	+	52	20	20	12	4	7	7	-2	0	0	8	8	8	0	0	0	4	4	4	4	0	0	0	2	5	5	-1	-1	2	2	3	3	1	1	-2	χ_{96}
χ_{97}	+	2380	364	-148	116	-4	49	22	4	0	0	56	-8	-24	-16	0	0	-4	-4	12	12	0	0	0	5	19	-10	-7	-2	4	-4	5	2	-1	2	-4	χ_{97}
χ_{98}	+	2380	-148	364	116	-4	22	49	4	0	0	-8	56	-24	16	0	0	-4	12	-4	12	0	0	0	5	-10	19	-2	-7	-4	4	2	5	2	-1	-4	χ_{98}
χ_{99}	+	12376	1176	664	296	56	91	1	10	0	0	112	48	64	-16	0	0	-8	8	24	8	0	0	0	1	21	19	-5	9	6	-2	11	5	5	-1	2	χ_{99}
χ_{100}	+	12376	664	1176	296	56	1	91	10	0	0	48	112	64	16	0	0	-8	24	8	8	0	0	0	1	19	21	9	-5	-2	6	5	11	-1	5	2	χ_{100}
χ_{101}	+	22100	820	820	-20	-92	50	50	14	0	0	8	8	40	0	0	0	36	-12	-12	4	0	0	0	0	10	10	10	10	10	10	-2	-2	-2	-2	-2	χ_{101}
χ_{102}	+	43316	-1260	-1260	780	4	98	98	8	0	0	-56	-56	40	0	0	0	4	-12	-12	36	0	0	0	16	-30	-30	-6	-6	0	0	6	6	-2	-2	-8	χ_{102}
χ_{103}	+	46800	2640	2640	560	16	90	90	-18	0	0	160	160	160	0	0	0	16	16	16	16	0	0	0	0	30	30	-6	-6	-6	-6	2	2	-2	-2	-2	χ_{103}
χ_{104}	+	55080	3816	-792	856	72	189	27	0	0	0	272	-48	-64	16	0	0	8	-8	40	24	0	0	0	5	51	-15	-3	3	0	0	13	7	3	-3	0	χ_{104}
χ_{105}	+	55080	-792	3816	856	72	27	189	0	0	0	-48	272	-64	-16	0	0	8	40	-8	24	0	0	0	5	-15	51	3	-3	0	0	7	13	-3	3	0	χ_{105}
χ_{106}	+	424320	15744	-2688	1664	-128	456	-57	-12	0	0	512	0	-96	-96	0	0	0	0	32	0	0	0	0	-5	24	-15	-24	-9	12	12	8	-1	-8	1	4	χ_{106}
χ_{107}	+	424320	-2688	15744	1664	-128	-57	456	-12	0	0	0	512	-96	96	0	0	0	32	0	0	0	0	0	-5	-15	24	-9	-24	12	12	-1	8	1	-8	4	χ_{107}
χ_{108}	+	433160	12488	-5432	2168	-88	161	161	-28	0	0	336	-112	-160	-80	0	0	-24	-8	8	24	0	0	0	10	71	-29	1	-7	-4	4	-7	5	-1	-1	-4	χ_{108}
χ_{109}	+	433160	-5432	12488	2168	-88	161	161	-28	0	0	-112	336	-160	80	0	0	-24	8	-8	24	0	0	0	10	-29	71	-7	1	4	-4	5	-7	-1	-1	-4	χ_{109}
χ_{110}	+	565760	4608	4608	-512	512	56	56	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	-24	-24	24	24	0	0	-8	-8	8	8	8	χ_{110}
χ_{111}	+	572832	17568	8352	2144	288	378	-27	0	0	0	448	64	256	-64	0	0	32	32	32	0	0	0	0	7	30	15	-6	-3	0	0	2	-7	6	3	0	χ_{111}
χ_{112}	+	572832	8352	17568	2144	288	-27	378	0	0	0	64	448	256	64	0	0	32	32	32	0	0	0	0	7	15	30	-3	-6	0	0	-7	2	3	6	0	χ_{112}
χ_{113}	+	716040	17352	-5688	1912	168	189	189	0	0	0	400	-176	-160	80	0	0	-24	8	-8	24	0	0	0	-10	39	-21	21	-3	0	0	1	-11	3	3	0	χ_{113}
χ_{114}	+	716040	-5688	17352	1912	168	189	189	0	0	0	-176	400	-160	-80	0	0	-24	-8	8	24	0	0	0	-10	-21	39	-3	21	0	0	-11	1	3	3	0	χ_{114}
χ_{115}	+	1082900	-13580	-13580	4140	-156	245	245	-16	0	0	-56	-56	200	0	0	0	-28	-28	-28	36	0	0	0	0	-65	-65	13	13	-8	-8	9	9	3	3	0	χ_{115}
χ_{116}	+	1082900	22260	4340	1580	-284	245	-70	38	0	0	392	-56	120	-80	0	0	-28	-28	-12	20	0	0	0	0	15	50	-19	18	-6	2	-7	-10	-5	-2	-2	χ_{116}
χ_{117}	+	1082900	4340	22260	1580	-284	-70	245	38	0	0	-56	392	120	80	0	0	-28	-12	-28	20	0	0	0	0	50	15	18	-19	2	-6	-10	-7	-2	-5	-2	χ_{117}
χ_{118}	+	1146600	10920	10920	-1640	392	315	315	-36	0	0	-112	-112	80	0	0	0	-56	-24	-24	-24	0	0	0	0	-15	-15	3	3	12	12	7	7	5	5	-4	χ_{118}
χ_{119}	+	1233792	26496	8064	3200	384	216	-189	0	0	0	512	0	160	160	0	0	0	0	32	0	0	0	0	-8	24	21	24	3	0	0	8	11	0	-3	0	χ_{119}
χ_{120}	+	1233792	8064	26496	3200	384	-189	216	0	0	0	0	512	160	-160	0	0	0	32	0	0	0	0	0	-8	21	24	3	24	0	0	11	8	-3	0	0	χ_{120}
χ_{121}	+	1299480	5208	-5544	-664	504	357	-21	24	0	0	-336	112	-64	144	0	0	56	-40	8	-24	0	0	0	5	15	21	-3	-21	24	0	-7	11	3	-3	0	χ_{121}
χ_{122}	+	1299480	-5544	5208	-664	504	-21	357	24	0	0	112	-336	-64	-144	0	0	56	8	-40	-24	0	0	0	5	21	15	-21	-3	0	24	11	-7	-3	3	0	χ_{122}
χ_{123}	+	1503684	-2268	-16092	4476	84	0	405	0	0	0	-280	-216	24	-336	0	0	20	-12	4	36	0	0	0	9	0	-45	0	-27	0	0	0	-3	0	3	0	χ_{123}
χ_{124}	+	1503684	-16092	-2268	4476	84	405	0	0	0	0	-216	-280	24	336	0	0	20	4	-12	36	0	0	0	9	-45	0	-27	0	0	0	-3	0	3	0	0	χ_{124}
χ_{125}	+	1591200	28320	-17760	3680	-224	90	225	36	0	0	448	64	-320	0	0	0	32	-32	32	0	0	0	0	0	30	-45	-6	9	-12	-12	2	5	-2	7	4	χ_{125}
χ_{126}	+	1591200	-17760	28320	3680	-224	225	90	36	0	0	64	448	-320	0	0	0	32	32	-32	0	0	0	0	0	-45	30	9	-6	-12	-12	5	2	7	-2	4	χ_{126}
χ_{127}	+	1949220	29316	-13692	284	-76	441	63	36	0	0	168	168	-216	96	0	0	-12	4	-28	-12	0	0	0	-5	-9	-15	9	15	12	-12	-7	-1	-1	-7	-4	χ_{127}
χ_{128}	+	1949220	-13692	29316	284	-76	63	441	36	0	0	168	168	-216	-96	0	0	-12	-28	4	-12	0	0	0	-5	-15	-9	15	9	-12	12	-1	-7	-7	-1	-4	χ_{128}
χ_{129}	+	2165800	8680	8680	-1960	-568	175	175	76	0	0	-112	-112	80	0	0	0	8	40	40	-24	0	0	0	0	-35	-35	7	7	4	4	11	11	-7	-7	-4	χ_{129}
χ_{130}	+	2772224	5376	-23296	4864	256	-406	224	-28	0	0	0	0	-64	64	0	0	0	-64	0	0	0	0	0	-1	6	-16	26	0	12	-4	10	16	-2	-8	4	χ_{130}
χ_{131}	+	2772224	-23296	5376	4864	256	224	-406	-28	0	0	0	0	-64	-64	0	0	0	0	-64	0	0	0	0	-1	-16	6	0	26	-4	12	16	10	-8	-2	4	χ_{131}
χ_{132}	+	2784600	41880	-4200	2600	184	225	-315	-18	0	0	496	112	-80	0	0	0	-8	-40	24	-8	0	0	0	0	-45	15	9	-3	-6	-6	5	-7	7	-5	-2	χ_{132}
χ_{133}	+	2784600	-4200	41880	2600	184	-315	225	-18	0	0	112	496	-80	0	0	0	-8	24	-40	-8	0	0	0	0	15	-45	-3	9	-6	-6	-7	5	-5	7	-2	χ_{133}
χ_{134}	+	3572100	34020	34020	1980	-108	0	0	0	0	0	168	168	360	0	0	0	-44	-12	-12	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{134}
χ_{135}	+	3572100	34020	34020	1980	-108	0	0	0	0	0	168	168	360	0	0	0	-44	-12	-12	-12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{135}
χ_{136}	+	4798080	45696	2688	-1664	-896	504	-63	-36	0	0	0	0	96	96	0	0	0	0	-32	0	0	0	0	5	-24	15	24	-15	12	-12	-8	1	-8	7	4	χ_{136}
χ_{137}	+	4798080	2688	45696	-1664	-896	-63	504	-36	0	0	0	0	96	-96	0	0	0	-32	0	0	0	0	0	5	15	-24	-15	24	-12	12	1	-8	7	-8	4	χ_{137}
χ_{138}	+	4950400	-21120	-31360	5760	-128	400	175	4	0	0	-512	0	160	160	0	0	0	0	32	0	0	0	0	0	0	-55	-16	15	12	-4	0	-9	-8	1	4	χ_{138}
χ_{139}	+	4950400	-31360	-21120	5760	-128	175	400	4	0	0	0	-512	160	-160	0	0	0	32	0	0	0	0	0	0	-55	0	15	-16	-4	12	-9	0	1	-8	4	χ_{139}
χ_{140}	+	5012280	10872	-39816	1096	-360	27	189	0	0	0	-272	560	-64	-304	0	0	-40	-8	40	-24	0	0	0	5	-15	-21	3	-3	0	0	7	-11	-3	3	0	χ_{140}
χ_{141}	+	5012280	-39816	10872	1096	-360	189	27	0	0	0	560	-272	-64	304	0	0	-40	40	-8	-24	0	0	0	5	-21	-15	-3	3	0	0	-11	7	3	-3	0	χ_{141}
χ_{142}	+	5370300	61020	-8100	2820	300	405	0	0	0	0	216	-232	-120	-240	0	0	-20	28	12	-36	0	0	0	0	-45	0	-27	0	0	0	-3	0	3	0	0	χ_{142}
χ_{143}	+	5370300	-8100	61020	2820	300	0	405	0	0	0	-232	216	-120	240	0	0	-20	12	28	-36	0	0	0	0	0	-45	0	-27	0	0	0	-3	0	3	0	χ_{143}
χ_{144}	+	5728320	28224	9792	-3136	-192	216	378	0	0	0	-128	-384	64	64	0	0	64	0	-64	0	0	0	0	-5	24	30	24	-6	0	0	8	2	0	6	0	χ_{144}
χ_{145}	+	5728320	9792	28224	-3136	-192	378	216	0	0	0	-384	-128	64	-64	0	0	64	-64	0	0	0	0	0	-5	30	24	-6	24	0	0	2	8	6	0	0	χ_{145}
χ_{146}	+	6497400	26040	-27720	-3320	472	-105	-105	-42	0	0	112	560	-160	240	0	0	24	-8	8	8	0	0	0	0	-75	-15	15	-9	-6	18	19	-17	1	1	-2	χ_{146}
χ_{147}	+	6497400	-27720	26040	-3320	472	-105	-105	-42	0	0	560	112	-160	-240	0	0	24	8	-8	8	0	0	0	0	-15	-75	-9	15	18	-6	-17	19	1	1	-2	χ_{147}
χ_{148}	+	6873984	44928	-10368	7296	384	-405	0	0	0	0	0	-512	-96	96	0	0	0	32	0	0	0	0	0	9	45	0	27	0	0	0	3	0	-3	0	0	χ_{148}
χ_{149}	+	6873984	-10368	44928	7296	384	0	-405	0	0	0	-512	0	-96	-96	0	0	0	0	32	0	0	0	0	9	0	45	0	27	0	0	0	3	0	-3	0	χ_{149}
χ_{150}	+	6930560	42112	-29568	3968	-384	-259	56	-16	0	0	0	0	-160	160	0	0	0	-32	0	0	0	0	0	10	19	24	-3	-8	-8	24	-19	8	3	0	0	χ_{150}
χ_{151}	+	6930560	-29568	42112	3968	-384	56	-259	-16	0	0	0	0	-160	-160	0	0	0	0	-32	0	0	0	0	10	24	19	-8	-3	24	-8	8	-19	0	3	0	χ_{151}
χ_{152}	+	7017192	53928	21672	920	264	-189	-189	0	0	0	336	-112	160	208	0	0	72	40	-40	24	0	0	0	-8	-39	-51	-21	3	0	0	-1	-13	-3	-3	0	χ_{152}
χ_{153}	+	7017192	21672	53928	920	264	-189	-189	0	0	0	-112	336	160	-208	0	0	72	-40	40	24	0	0	0	-8	-51	-39	3	-21	0	0	-13	-1	-3	-3	0	χ_{153}
χ_{154}	+	7429968	-45360	-45360	432	144	0	0	0	0	0	672	672	288	0	0	0	16	-48	-48	-48	0	0	0	18	0	0	0	0	0	0	0	0	0	0	0	χ_{154}
χ_{155}	+	8663200	-1120	-36960	-2720	32	385	70	-20	0	0	-448	448	0	320	0	0	32	32	32	0	0	0	0	0	35	-30	9	38	-4	12	-11	-2	-1	2	-4	χ_{155}
χ_{156}	+	8663200	-36960	-1120	-2720	32	70	385	-20	0	0	448	-448	0	-320	0	0	32	32	32	0	0	0	0	0	-30	35	38	9	12	-4	-2	-11	2	-1	-4	χ_{156}
χ_{157}	+	9052160	8192	8192	-8192	0	176	176	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	-16	-16	-16	-16	-16	-16	16	16	0	0	0	χ_{157}
χ_{158}	+	9547200	-14400	-14400	-8640	192	0	0	54	0	0	384	384	0	0	0	0	-64	0	0	64	0	0	0	0	0	0	0	0	18	18	0	0	0	0	6	χ_{158}
χ_{159}	+	10795680	38304	-26208	-1184	-480	189	-378	0	0	0	-448	448	-64	-256	0	0	32	32	-32	0	0	0	0	5	39	-30	21	6	0	0	1	-2	3	-6	0	χ_{159}
χ_{160}	+	10795680	-26208	38304	-1184	-480	-378	189	0	0	0	448	-448	-64	256	0	0	32	-32	32	0	0	0	0	5	-30	39	6	21	0	0	-2	1	-6	3	0	χ_{160}
χ_{161}	+	11456640	56448	-35712	-128	-384	-216	189	0	0	0	0	0	-160	-160	0	0	0	0	-32	0	0	0	0	-10	-24	51	-24	-3	0	0	-8	13	0	3	0	χ_{161}
χ_{162}	+	11456640	-35712	56448	-128	-384	189	-216	0	0	0	0	0	-160	160	0	0	0	-32	0	0	0	0	0	-10	51	-24	-3	-24	0	0	13	-8	3	0	0	χ_{162}
χ_{163}	+	12475008	-18816	-61824	5504	128	-504	441	36	0	0	0	0	96	96	0	0	0	0	-32	0	0	0	0	-17	24	-9	-24	9	12	-12	8	-7	8	-1	-4	χ_{163}
χ_{164}	+	12475008	-61824	-18816	5504	128	441	-504	36	0	0	0	0	96	-96	0	0	0	-32	0	0	0	0	0	-17	-9	24	9	-24	-12	12	-7	8	-1	8	-4	χ_{164}
χ_{165}	+	13861120	-1792	-30464	-256	-768	-392	-14	-32	0	0	0	0	64	64	0	0	0	0	64	0	0	0	0	-5	-40	46	-8	2	8	-8	8	2	0	6	0	χ_{165}
χ_{166}	+	13861120	-30464	-1792	-256	-768	-14	-392	-32	0	0	0	0	64	-64	0	0	0	64	0	0	0	0	0	-5	46	-40	2	-8	-8	8	2	8	6	0	0	χ_{166}
χ_{167}	+	15163200	25920	25920	-2880	576	0	0	0	0	0	-384	-384	0	0	0	0	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{167}
χ_{168}	+	15163200	25920	25920	-2880	576	0	0	0	0	0	-384	-384	0	0	0	0	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{168}
χ_{169}	+	16307200	-35840	-35840	-5120	1024	280	280	-44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	40	-8	-8	-20	-20	-8	-8	-8	-8	4	χ_{169}
χ_{170}	+	16777216	0	0	0	0	-512	-512	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	χ_{170}

$F_4(2)$

	7A	7B	8A	8B	8C	8D	8E	8F	8G	8H	8I	8J	8K	9A	9B	10A	10B	10C	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	12N	12O	12P	Q**	13A	14A	14B	15A	15B	16A	16B	16C	D**	17A	B*	18A	18B	20A	20B	21A	21B	
	7	7	8	8	8	8	8	8	8	8	8	8	8	9	9	10	10	10	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	13	14	14	15	15	16	16	16	16	17	17	18	18	20	20	21	21	
	14	14	8		8	8	8	8					8	18	18	10	10	10	12	12			12	12	12	12	12	12	12	12			12			26	14	14	30	30	16	16			34	34	18	18	20	20	42	42	
χ_{96}	3	3	4	0	2	2	2	2	0	0	0	0	2	1	1	-0	0	2	3	3	0	0	-1	-1	-3	3	-1	-1	1	1	0	0	-2	0	0	0	-1	-1	2	2	0	2	0	0	1	1	-1	-1	-2	0	0	0	χ_{96}
χ_{97}	0	7	-4	0	-2	-2	6	-2	0	0	0	0	2	1	1	-1	-3	1	-3	6	0	0	-3	0	-1	-4	1	2	-1	0	0	0	0	0	0	1	0	-1	-1	2	0	-2	0	0	0	0	1	-1	1	-1	0	1	χ_{97}
χ_{98}	7	0	-4	0	-2	-2	-2	6	0	0	0	0	2	1	1	-3	-1	1	6	-3	0	0	0	-3	4	1	2	1	0	-1	0	0	0	0	0	1	-1	0	2	-1	0	-2	0	0	0	0	-1	1	1	1	1	0	χ_{98}
	0	7	8	0	4	4	4	-4	0	0	0	0	0	-2	1	1	-1	1	3	-3	0	0	1	1	-1	5	3	1	-1	3	0	0	2	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	1	-1	-1	0	1	χ_{99}
χ_{100}	7	0	8	0	4	4	-4	4	0	0	0	0	0	1	-2	-1	1	1	-3	3	0	0	1	1	-5	1	1	3	3	-1	0	0	2	0	0	0	-1	0	1	1	0	0	0	0	0	0	1	0	-1	1	1	0	χ_{100}
χ_{101}	1	1	4	0	-6	-6	2	2	0	0	0	0	2	-1	-1	0	0	0	6	6	0	0	4	4	0	0	2	2	0	0	0	0	-2	0	0	0	1	1	0	0	0	2	0	0	0	0	1	1	0	0	1	1	χ_{101}
χ_{102}	0	0	4	0	2	2	-6	-6	0	0	0	0	2	2	2	0	0	0	-6	-6	0	0	4	4	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	-2	-2	0	2	0	0	0	0	0	0	0	0	0	0	χ_{102}
χ_{103}	5	5	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	6	0	0	-2	-2	-6	6	-2	-2	-2	-2	0	0	-2	0	0	0	1	1	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	-1	χ_{103}
χ_{104}	-3	11	-8	0	4	-4	4	4	0	0	0	0	0	0	0	1	3	1	-3	3	0	0	-1	-1	1	-5	-3	-1	1	1	0	0	0	0	0	-1	1	-1	-1	2	0	0	0	0	0	0	0	0	1	1	0	-1	χ_{104}
χ_{105}	11	-3	-8	0	-4	4	4	4	0	0	0	0	0	0	0	3	1	1	3	-3	0	0	-1	-1	5	-1	-1	-3	1	1	0	0	0	0	0	-1	-1	1	2	-1	0	0	0	0	0	0	0	0	1	-1	-1	0	χ_{105}
χ_{106}	1	8	0	0	-8	0	8	0	0	0	0	0	0	0	0	-1	-3	-1	0	-9	0	0	0	-3	0	-3	0	-1	0	-1	0	0	0	0	0	0	1	0	1	-2	0	0	0	0	0	0	0	0	-1	-1	1	-1	χ_{106}
χ_{107}	8	1	0	0	0	-8	0	8	0	0	0	0	0	0	0	-3	-1	-1	-9	0	0	0	-3	0	3	0	-1	0	-1	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	-1	1	-1	1	χ_{107}
χ_{108}	0	0	-8	0	-4	4	-4	-4	0	0	0	0	0	-1	-1	-2	-2	-2	-3	9	0	0	-1	5	1	1	5	-3	1	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	1	0	0	0	0	χ_{108}
χ_{109}	0	0	-8	0	4	-4	-4	-4	0	0	0	0	0	-1	-1	-2	-2	-2	9	-3	0	0	5	-1	-1	-1	-3	5	-1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	-1	0	0	0	0	χ_{109}
χ_{110}	6	6	0	0	0	0	0	0	0	0	0	0	0	2	2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{110}
χ_{111}	-6	1	0	0	8	0	8	0	0	0	0	0	0	0	0	3	-3	-1	6	-3	0	0	-2	1	2	5	-2	1	2	-1	0	0	0	0	0	0	-2	1	-2	-2	0	0	0	0	0	0	0	0	1	1	0	1	χ_{111}
χ_{112}	1	-6	0	0	0	8	0	8	0	0	0	0	0	0	0	-3	3	-1	-3	6	0	0	1	-2	-5	-2	1	-2	-1	2	0	0	0	0	0	0	1	-2	-2	-2	0	0	0	0	0	0	0	0	1	-1	1	0	χ_{112}
χ_{113}	3	3	-8	0	4	-4	-4	-4	0	0	0	0	0	0	0	2	2	2	-3	9	0	0	5	-1	-1	-1	1	1	-1	1	0	0	0	0	0	0	-1	3	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	3	3	-8	0	-4	4	-4	-4	0	0	0	0	0	0	0	2	2	2	9	-3	0	0	-1	5	1	1	1	1	1	-1	0	0	0	0	0	0	3	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{114}
χ_{115}	0	0	4	0	2	2	2	2	0	0	0	0	2	-1	-1	0	0	0	-3	-3	0	0	-7	-7	3	-3	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	1	1	0	0	0	0	χ_{115}
χ_{116}	0	0	4	0	-6	2	2	2	0	0	0	0	-2	2	-1	0	0	0	-3	6	0	0	-3	0	7	4	1	2	-1	0	0	0	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	-1	0	0	0	0	χ_{116}
χ_{117}	0	0	4	0	2	-6	2	2	0	0	0	0	-2	-1	2	0	0	0	6	-3	0	0	0	-3	-4	-7	2	1	0	-1	0	0	2	0	0	0	0	0	0	0	0	2	0	0	0	0	-1	0	0	0	0	0	χ_{117}
χ_{118}	0	0	-8	0	4	4	4	4	0	0	0	0	-4	0	0	0	0	0	3	3	0	0	-1	-1	-3	3	-1	-1	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	χ_{118}
χ_{119}	-7	0	0	0	8	0	-8	0	0	0	0	0	0	0	0	-4	4	0	0	-9	0	0	4	1	4	1	0	-1	0	-1	0	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	-1	0	χ_{119}
χ_{120}	0	-7	0	0	0	8	0	-8	0	0	0	0	0	0	0	4	-4	0	-9	0	0	0	1	4	-1	-4	-1	0	-1	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	-1	χ_{120}
χ_{121}	0	0	8	0	4	-4	-4	-4	0	0	0	0	0	0	0	3	1	1	-3	3	0	0	-7	5	3	-3	1	3	-1	-1	0	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	0	1	-1	0	0	χ_{121}
χ_{122}	0	0	8	0	-4	4	-4	-4	0	0	0	0	0	0	0	1	3	1	3	-3	0	0	5	-7	3	-3	3	1	-1	-1	0	0	0	0	0	0	0	0	-1	2	0	0	0	0	0	0	0	0	1	1	0	0	χ_{122}
χ_{123}	0	-7	4	0	2	-6	10	-6	0	0	0	0	-2	0	0	-3	3	1	0	-3	0	0	0	3	0	3	0	5	0	1	0	0	0	0	0	0	0	1	0	0	0	-2	0	0	0	0	0	0	-1	-1	0	-1	χ_{123}
χ_{124}	-7	0	4	0	-6	2	-6	10	0	0	0	0	-2	0	0	3	-3	1	-3	0	0	0	3	0	-3	0	5	0	1	0	0	0	0	0	0	0	1	0	0	0	0	-2	0	0	0	0	0	0	-1	1	-1	0	χ_{124}
χ_{125}	2	-5	0	0	-8	0	-8	0	0	0	0	0	0	0	0	0	0	0	6	-3	0	0	-2	1	-6	-3	-2	1	-2	-1	0	0	0	0	0	0	-2	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	χ_{125}
χ_{126}	-5	2	0	0	0	-8	0	-8	0	0	0	0	0	0	0	0	0	0	-3	6	0	0	1	-2	3	6	1	-2	-1	-2	0	0	0	0	0	0	-1	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	χ_{126}
χ_{127}	0	0	4	0	2	10	10	2	0	0	0	0	2	0	0	1	3	-1	-3	3	0	0	3	-3	-3	-3	-3	3	1	-1	0	0	0	0	0	0	0	0	1	-2	0	-2	0	0	0	0	0	0	-1	1	0	0	χ_{127}
χ_{128}	0	0	4	0	10	2	2	10	0	0	0	0	2	0	0	3	1	-1	3	-3	0	0	-3	3	3	3	3	-3	-1	1	0	0	0	0	0	0	0	0	-2	1	0	-2	0	0	0	0	0	0	-1	-1	0	0	χ_{128}
χ_{129}	0	0	-8	0	-4	-4	-4	-4	0	0	0	0	-4	1	1	0	0	0	3	3	0	0	-1	-1	-3	3	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	χ_{129}
χ_{130}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	1	-1	-1	6	0	0	0	2	-4	-2	4	6	0	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	1	-1	0	0	χ_{130}
χ_{131}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1	1	-1	0	6	0	0	-4	2	-4	2	0	6	0	2	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	-1	0	1	1	0	0	χ_{131}
χ_{132}	7	0	8	0	-4	4	-4	4	0	0	0	0	-4	0	0	0	0	0	-3	-3	0	0	1	1	3	-3	1	1	-1	3	0	0	-2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	χ_{132}
χ_{133}	0	7	8	0	4	-4	4	-4	0	0	0	0	-4	0	0	0	0	0	-3	-3	0	0	1	1	3	-3	1	1	3	-1	0	0	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	χ_{133}
χ_{134}	0	0	4	0	-6	-6	-6	-6	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-2	0	0	-b17	*	0	0	0	0	0	0	χ_{134}
χ_{135}	0	0	4	0	-6	-6	-6	-6	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-2	0	0	*	-b17	0	0	0	0	0	0	χ_{135}
χ_{136}	0	0	0	0	-8	0	8	0	0	0	0	0	0	0	0	1	3	1	0	-3	0	0	0	3	0	3	0	-3	0	1	0	0	0	0	0	1	0	0	-1	2	0	0	0	0	0	0	0	0	1	1	0	0	χ_{136}
χ_{137}	0	0	0	0	0	-8	0	8	0	0	0	0	0	0	0	3	1	1	-3	0	0	0	3	0	-3	0	-3	0	1	0	0	0	0	0	0	1	0	0	2	-1	0	0	0	0	0	0	0	0	1	-1	0	0	χ_{137}
χ_{138}	7	0	0	0	8	0	-8	0	0	0	0	0	0	-2	1	0	0	0	0	3	0	0	4	1	4	1	0	-5	0	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	χ_{138}
χ_{139}	0	7	0	0	0	8	0	-8	0	0	0	0	0	1	-2	0	0	0	3	0	0	0	1	4	-1	-4	-5	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	χ_{139}
χ_{140}	-7	0	8	0	4	4	-4	4	0	0	0	0	0	0	0	-3	-1	1	3	9	0	0	-1	-1	5	-1	-1	1	1	1	0	0	0	0	0	0	1	0	2	-1	0	0	0	0	0	0	0	0	1	1	-1	0	χ_{140}
χ_{141}	0	-7	8	0	4	4	4	-4	0	0	0	0	0	0	0	-1	-3	1	9	3	0	0	-1	-1	1	-5	1	-1	1	1	0	0	0	0	0	0	0	1	-1	2	0	0	0	0	0	0	0	0	1	-1	0	-1	χ_{141}
χ_{142}	5	-9	-4	0	6	-10	6	-2	0	0	0	0	2	0	0	0	0	0	-3	0	0	0	3	0	-3	0	5	0	1	0	0	0	0	0	0	0	1	-1	0	0	0	2	0	0	0	0	0	0	0	0	-1	0	χ_{142}
χ_{143}	-9	5	-4	0	-10	6	-2	6	0	0	0	0	2	0	0	0	0	0	0	-3	0	0	0	3	0	3	0	5	0	1	0	0	0	0	0	0	-1	1	0	0	0	2	0	0	0	0	0	0	0	0	0	-1	χ_{143}
χ_{144}	-4	3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-3	-1	0	6	0	0	4	-2	4	-2	0	-2	0	2	0	0	0	0	0	0	0	-1	1	-2	0	0	0	0	0	0	0	0	-1	-1	-1	0	χ_{144}
χ_{145}	3	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-1	-1	6	0	0	0	-2	4	2	-4	-2	0	2	0	0	0	0	0	0	0	-1	0	-2	1	0	0	0	0	0	0	0	0	-1	1	0	-1	χ_{145}
χ_{146}	0	0	8	0	-4	-4	4	-4	0	0	0	0	0	0	0	0	0	0	3	3	0	0	-1	5	-3	-3	-1	-5	1	-1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{146}
χ_{147}	0	0	8	0	-4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	3	3	0	0	5	-1	3	3	-5	-1	-1	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{147}
χ_{148}	-2	-9	0	0	0	-8	0	8	0	0	0	0	0	0	0	3	-3	1	3	0	0	0	-3	0	3	0	-5	0	-1	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	χ_{148}
χ_{149}	-9	-2	0	0	-8	0	8	0	0	0	0	0	0	0	0	-3	3	1	0	3	0	0	0	-3	0	-3	0	-5	0	-1	0	0	0	0	0	0	-1	2	0	0	0	0	0	0	0	0	0	0	-1	-1	0	1	χ_{149}
χ_{150}	0	0	0	0	0	8	0	-8	0	0	0	0	0	-1	2	2	2	-2	-3	0	0	0	-1	-4	1	4	-3	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	χ_{150}
χ_{151}	0	0	0	0	8	0	-8	0	0	0	0	0	0	2	-1	2	2	-2	0	-3	0	0	-4	-1	-4	-1	0	-3	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	χ_{151}
χ_{152}	0	0	-8	0	4	4	-4	4	0	0	0	0	0	0	0	-2	2	0	3	3	0	0	-5	1	1	1	-1	3	1	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	-2	0	0	χ_{152}
χ_{153}	0	0	-8	0	4	4	4	-4	0	0	0	0	0	0	0	2	-2	0	3	3	0	0	1	-5	-1	-1	3	-1	-1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	2	0	0	χ_{153}
χ_{154}	0	0	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-2	0	0	0	χ_{154}
χ_{155}	0	0	0	0	0	-8	0	-8	0	0	0	0	0	1	-2	0	0	0	-3	-6	0	0	-3	-6	-1	2	1	2	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	χ_{155}
χ_{156}	0	0	0	0	-8	0	-8	0	0	0	0	0	0	-2	1	0	0	0	-6	-3	0	0	-6	-3	-2	1	2	1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	χ_{156}
χ_{157}	-2	-2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1	1	0	0	0	0	0	0	-1	-1	0	0	1	1	χ_{157}
χ_{158}	-9	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{158}
χ_{159}	0	0	0	0	0	8	0	8	0	0	0	0	0	0	0	-1	-3	1	-3	-6	0	0	5	2	-1	2	1	2	-1	-2	0	0	0	0	0	-1	0	0	-1	2	0	0	0	0	0	0	0	0	1	-1	0	0	χ_{159}
χ_{160}	0	0	0	0	8	0	8	0	0	0	0	0	0	0	0	-3	-1	1	-6	-3	0	0	2	5	-2	1	2	1	-2	-1	0	0	0	0	0	-1	0	0	2	-1	0	0	0	0	0	0	0	0	1	1	0	0	χ_{160}
χ_{161}	-8	6	0	0	8	0	-8	0	0	0	0	0	0	0	0	-2	-2	2	0	-3	0	0	-4	-1	-4	-1	0	-3	0	1	0	0	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	1	0	χ_{161}
χ_{162}	6	-8	0	0	0	8	0	-8	0	0	0	0	0	0	0	-2	-2	2	-3	0	0	0	-1	-4	1	4	-3	0	1	0	0	0	0	0	0	0	2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	χ_{162}
χ_{163}	0	0	0	0	-8	0	8	0	0	0	0	0	0	0	0	-1	1	-1	0	-3	0	0	0	3	0	3	0	-3	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	χ_{163}
χ_{164}	0	0	0	0	0	-8	0	8	0	0	0	0	0	0	0	1	-1	-1	-3	0	0	0	3	0	-3	0	-3	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	χ_{164}
χ_{165}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	3	1	-1	0	6	0	0	4	-2	4	-2	0	6	0	-2	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	-1	1	-1	-1	0	0	χ_{165}
χ_{166}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	3	-1	6	0	0	0	-2	4	2	-4	6	0	-2	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	1	-1	-1	1	0	0	χ_{166}
χ_{167}	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	4r2	0	0	0	-1	-1	0	0	0	0	0	0	χ_{167}
χ_{168}	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-4r2	0	0	0	-1	-1	0	0	0	0	0	0	χ_{168}
χ_{169}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	χ_{169}
χ_{170}	8	8	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-2	-2	0	0	0	0	1	1	0	0	0	0	-1	-1	χ_{170}

$F_4(2)$

	24A	24B	24C	24D	28A	28B	30A	30B			2E	4P	4Q	6L	8L	8M	8N	8O	8P	8Q	8R	10D	12R	16E	16F	16G	16H	I**	20C	24E	24F	G**	26A	32A	B*	32C	D**	40A	40B	
	24	24	24	24	28	28	30	30	fus	ind	2	4	4	6	8	8	8	8	8	8	8	10	12	16	16	16	16	16	20	24	24	24	26	32	32	32	32	40	40	
	24	24	24	24	28	28	30	30									8		8					16			16	16		24	24	24	26	32	32	32	32	40	40	
x_{96}	-1	-1	-1	-1	1	1	0	0	:	∞	0	0	0	0	0	0	0	0	2i2	0	0	0	0	2i2	0	0	i2	-i2	0	0	-i2	-i2	0	0	0	i2	-i2	0	0	x_{96}
x_{97}	1	-2	1	0	0	-1	1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{97}
x_{98}	-2	1	0	1	-1	0	0	-1	│																															x_{98}
x_{99}	1	1	-1	1	0	-1	-1	-1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{99}
x_{100}	1	1	1	-1	-1	0	1	1	│																															x_{100}
x_{101}	0	0	2	2	1	1	0	0	:	∞	0	0	0	0	0	0	0	0	2i2	0	0	0	0	2i2	0	0	-i2	i2	0	0	-i2	-i2	0	0	0	-i2	i2	0	0	x_{101}
x_{102}	2	2	0	0	0	0	0	0	:	∞	0	0	0	0	0	0	0	0	6i2	0	0	0	0	-2i2	0	0	i2	-i2	0	0	0	0	0	0	0	-i2	i2	0	0	x_{102}
x_{103}	0	0	0	0	-1	-1	0	0	:	∞	0	0	0	0	0	0	0	0	4i2	0	0	0	0	4i2	0	0	0	0	0	0	i2	i2	0	0	0	0	0	0	0	x_{103}
x_{104}	-1	1	1	1	-1	1	-1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{104}
x_{105}	1	-1	1	1	1	-1	0	1	│																															x_{105}
x_{106}	0	1	0	-1	1	0	1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{106}
x_{107}	1	0	-1	0	0	1	0	-1	│																															x_{107}
x_{108}	1	-1	-1	-1	0	0	-1	1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{108}
x_{109}	-1	1	-1	-1	0	0	-1	1	│																															x_{109}
x_{110}	0	0	0	0	0	0	-1	1	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r10	r10		x_{110}
x_{111}	0	-1	0	-1	0	1	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{111}
x_{112}	-1	0	-1	0	1	0	0	0	│																															x_{112}
x_{113}	-1	1	-1	-1	1	-1	1	-1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{113}
x_{114}	1	-1	-1	-1	-1	1	1	-1	│																															x_{114}
x_{115}	-1	-1	-1	-1	0	0	0	0	:	∞	0	0	0	0	0	0	0	0	6i2	0	0	0	0	-2i2	0	0	-i2	i2	0	0	0	0	0	0	0	i2	-i2	0	0	x_{115}
x_{116}	-1	0	-1	2	0	0	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{116}
x_{117}	0	-1	2	-1	0	0	0	0	│																															x_{117}
x_{118}	1	1	1	1	0	0	0	0	:	++	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	2	0	0	x_{118}
x_{119}	0	-1	0	1	1	0	1	1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{119}
x_{120}	-1	0	1	0	0	1	-1	-1	│																															x_{120}
x_{121}	-1	1	-1	-1	0	0	0	1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{121}
x_{122}	1	-1	-1	-1	0	0	-1	0	│																															x_{122}
x_{123}	0	-1	0	1	0	1	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{123}
x_{124}	-1	0	1	0	1	0	0	0	│																															x_{124}
x_{125}	0	1	0	1	0	1	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{125}
x_{126}	1	0	1	0	1	0	0	0	│																															x_{126}
x_{127}	1	-1	-1	1	0	0	-1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{127}
x_{128}	-1	1	1	-1	0	0	0	1	│																															x_{128}
x_{129}	-1	-1	-1	-1	0	0	0	0	:	++	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	-2	-2	0	0	x_{129}
x_{130}	0	0	0	0	0	0	-1	-1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{130}
x_{131}	0	0	0	0	0	0	1	1	│																															x_{131}
x_{132}	1	-1	1	-1	-1	0	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{132}
x_{133}	-1	1	-1	1	0	-1	0	0	│																															x_{133}
x_{134}	0	0	0	0	0	0	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{134}
x_{135}	0	0	0	0	0	0	0	0	│																															x_{135}
x_{136}	0	1	0	-1	0	0	-1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{136}
x_{137}	1	0	-1	0	0	0	0	1	│																															x_{137}
x_{138}	0	-1	0	1	-1	0	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{138}
x_{139}	-1	0	1	0	0	-1	0	0	│																															x_{139}
x_{140}	1	1	1	-1	1	0	0	-1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{140}
x_{141}	1	1	-1	1	0	1	1	0	│																															x_{141}
x_{142}	-1	0	1	0	-1	-1	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{142}
x_{143}	0	-1	0	1	-1	-1	0	0	│																															x_{143}
x_{144}	0	0	0	0	-2	1	1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{144}
x_{145}	0	0	0	0	1	-2	0	-1	│																															x_{145}
x_{146}	-1	-1	-1	1	0	0	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{146}
x_{147}	-1	-1	1	-1	0	0	0	0	│																															x_{147}
x_{148}	1	0	-1	0	0	-1	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{148}
x_{149}	0	1	0	-1	-1	0	0	0	│																															x_{149}
x_{150}	-1	0	1	0	0	0	1	-1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{150}
x_{151}	0	-1	0	1	0	0	1	-1	│																															x_{151}
x_{152}	1	1	1	-1	0	0	-1	-1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{152}
x_{153}	1	1	-1	1	0	0	1	1	│																															x_{153}
x_{154}	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r2	2r2	0	0	0	0	0	0	0	0	0	0	0	x_{154}
x_{155}	1	0	1	0	0	0	0	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{155}
x_{156}	0	1	0	1	0	0	0	0	│																															x_{156}
x_{157}	0	0	0	0	0	0	1	-1	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r10	-r10		x_{157}
x_{158}	0	0	0	0	-1	-1	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r6	-r6	0	0	0	0	0	0	0	x_{158}
x_{159}	-1	0	-1	0	0	0	1	0	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{159}
x_{160}	0	-1	0	-1	0	0	0	-1	│																															x_{160}
x_{161}	0	-1	0	1	0	0	-1	1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{161}
x_{162}	-1	0	1	0	0	0	-1	1	│																															x_{162}
x_{163}	0	1	0	-1	0	0	1	1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{163}
x_{164}	1	0	-1	0	0	0	-1	-1	│																															x_{164}
x_{165}	0	0	0	0	0	0	0	1	┐	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{165}
x_{166}	0	0	0	0	0	0	-1	0	│																															x_{166}
x_{167}	0	0	0	0	1	1	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2y16	*3	0	0	0	0	x_{167}
x_{168}	0	0	0	0	1	1	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*5	2y16	0	0	0	0	x_{168}
x_{169}	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	x_{169}
x_{170}	0	0	0	0	0	0	0	0	:	++	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r13	0	0	0	0	0	0	x_{170}

Ly

Sporadic Lyons group Ly

Order = 51,765,179,004,000,000 = $2^8.3^7.5^6.7.11.31.37.67$ Mult = 1 Out = 1

Constructions

Graphs G has been constructed as a group of permutations on 8835156 points, which can be regarded as the vertices of a rank 5 graph A. The point stabilizer in G is $G_2(5)$, which has orbit lengths 1, 19530, 968750, 2034375, 5812500. The corresponding 2-point stabilizers are $5^{1+4}:4S_4$, $U_3(3)$, $2(A_5xA_4).2$, and $(3xL_2(7)):2$ respectively.

There is also a permutation representation on 9606125 points forming a rank 5 graph B. The point stabilizer is $3^{\cdot}$McL:2, which has orbit lengths 1, 15400, 534600, 1871100, 7185024. The corresponding 2-point stabilizers are $3^{2+4}.4S_5$, $2S_7$, $4S_6$ and $5^{1+2}:S_3$ respectively.

The joint stabilizers of a point in each of the above graphs are $3^{\cdot}U_3(5):2$, $5^{1+2}:4S_3.2$, $U_3(3):2$ and $(2A_5 \times 3).2$.

Modulo 5 G has a 111-dimensional orthogonal representation over $\mathbb{F}_5$, on a space spanned by vectors $[\infty]$ and $[z,m,n]$, where $z = x + y\theta$ (x, y in $\mathbb{F}_5$, $\theta^2 = -3$) is the typical element of $\mathbb{F}_{25}$, and m, n are in $\{0,1,2,3,4,5,6,7,8,9,X,\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6},\bar{7},\bar{8},\bar{9},\bar{X}\}$. These vectors satisfy the relations

$$[z,m,n] = x[1,m,n] + y[\theta,m,n], \qquad [\bar{z},m,n] = [z,\bar{m},n] = [z,m,\bar{n}], \qquad [z,\bar{m},\bar{n}] = [z,m,n] = -[z,n,m]$$

where $\bar{z} = x - y\theta$. An orthogonal base consists of the 111 vectors $[\infty]$, $[1,a,b]$, $[\theta,a,b]$ ($b-a$ a quadratic residue modulo 11), of the respective norms 3, 1, 3.

G is generated by its subgroup $\langle A_m,B_P,C\rangle \cong 3^5:(M_{11} \times 2)$ together with the element D of order 3 inverted by C, where:

$$A_m : [\infty] \to [\infty], \quad [z,a,b] \to [w^{E(a,b,m)}z,a,b] \qquad (2w = -1 + \theta)$$

$$B_P : [\infty] \to [\infty], \quad [z,a,b] \to [z,P(a),P(b)]$$

$$C : [\infty] \to -[\infty], \quad [z,a,b] \to [\bar{z},a,b]$$

$$D : [\infty] \to [\infty] + \sum[wp_{mn},m,n]$$

$$[1,a,b] \to [\infty] + \sum[q_{mn}+\theta r_{mn},m,n]$$

$$[\theta,a,b] \to 3[\infty] + \sum[s_{mn}+\theta t_{mn},m,n]$$

whenever a,b,c,d,e,f,g,h,i,j,k is an arithmetic progression modulo 11 in which $b-a$ is a quadratic residue;

$E(a,b,m) = 0$ ($m = a,b$), 1 ($m = c,d,e,g,h,k$), 2 ($m = f,i,j$);

P is one of the monomial permutations (generating M_{11}):

(0123456789X), (13954)(267X8), $(0\bar{0})(1\bar{1})(3\bar{9}4\bar{5})(26X7)$;

the sums in the formulae for D are over the values below, and p_{mn} is the Legendre symbol $(m-n|11)$:

m =	a	b	c	d	e	f	g	h	i	j
n =	bcdefghijk	cdefghijk	defghijk	efghijk	fghijk	ghijk	hijk	ijk	jk	k
q_{mn} =	0044022000	023002113	14421344	2344201	034133	30234	3214	033	21	1
r_{mn} =	4233330233	414242244	13333230	2141302	222410	21244	4011	140	12	3
s_{mn} =	3211122112	440112344	14244232	2411122	242003	30101	4344	403	42	2
t_{mn} =	1400033104	122412332	34421032	3004210	031343	41212	4121	031	33	3

Maximal subgroups

Order	Index	Structure	Character	Specifications: Abstract	Specifications: Graphs
5859000000	8835156	$G_2(5)$	1a+45694a+1534500a+3028266a+4226695a		point of A
5388768000	9606125	$3^{\cdot}$McL:2	1a+45694a+1534500a+3028266a+4997664a	N(3A)	point of B
46500000	1113229656	$5^{3\cdot}L_3(5)$		$N(5A^3)$	
29916800	1296826875	$2^{\cdot}A_{11}$		N(2A)	
9000000	5751686556	$5_+^{1+4}:4S_6$		N(5A)	
3849120	13448575000	$3^5:(2 \times M_{11})$		$N(3^5) = N(3A_{11}B_{110})$	
699840	73967162500	$3^{2+4}:2A_5.D_8$		$N(3^2) = N(3A_2B_2)$	edge of B
1474	35118846000000	67:22		N(67ABC)	
666	77725494000000	37:18		N(37AB)	

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		5176	399	26943	174	20	22	3	120	2						3							2																																
		5179004000000	16800	84000	960	160	50000	750	960	160	324	168	480	96	54	600	50	66	66	288	72	168	250	90	75	18	40	42	42	22	22	24	24	24	25	28	90	90	31	31	31	31	31	33	33	37	37	40	40	42	42	67	67	67	
	p power		A	A	A	A	A	A	AA	BA	BA	A	A	A	B	AA	BA	A	A	AA	BA	AA	AA	AB	BA	AC	AA	AA	AA	BA	AA	AA	BB	BB	A	AA	AAA	BAB	A	A	A	A	A	AA	BA	A	A	AA	AA	BAA	AAA	A	A	A	
	p' part		A	A	A	A	A	A	AA	BA	BA	A	A	A	A	AA	BA	A	A	AA	BA	AA	AA	AB	BA	AA	AA	AA	AA	AA	BA	AA	BB	BB	A	AA	AAA	BAB	A	A	A	A	A	AA	BA	A	A	AA	AA	AAA	BAA	A	A	A	
	ind	1A	2A	3A	3B	4A	5A	5B	6A	6B	6C	7A	8A	8B	9A	10A	10B	11A	B**	12A	12B	14A	15A	15B	15C	18A	20A	21A	B*	22A	B**	24A	24B	C*	25A	28A	30A	30B	31A	B*2	C*4	D*8	E*16	33A	B**	37A	B*	40A	B*	42A	B*	67A	B*2	C*4	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	o	2480	-16	104	-4	0	-20	5	8	-4	2	2	0	0	-1	4	-1	b11	**	0	0	-2	4	1	-1	-1	0	-1	-1	-b11	**	0	0	0	0	0	-2	1	0	0	0	0	0	b11	**	1	1	0	0	1	1	1	1	1	χ_{2}
χ_{3}	o	2480	-16	104	-4	0	-20	5	8	-4	2	2	0	0	-1	4	-1	**	b11	0	0	-2	4	1	-1	-1	0	-1	-1	**	-b11	0	0	0	0	0	-2	1	0	0	0	0	0	**	b11	1	1	0	0	1	1	1	1	1	χ_{3}
χ_{4}	+	45694	110	253	10	26	69	-6	29	2	2	-2	4	0	1	5	0	0	0	5	2	-2	3	0	3	-1	1	1	1	0	0	1	0	0	-1	-2	-1	2	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1	0	0	0	χ_{4}
χ_{5}	o	48174	126	357	6	-14	49	-1	21	6	0	0	-4	0	0	1	1	b11	**	1	-2	0	7	1	2	0	1	0	0	b11	**	-1	0	0	-1	0	1	1	0	0	0	0	0	b11	**	0	0	1	1	0	0	1	1	1	χ_{5}
χ_{6}	o	48174	126	357	6	-14	49	-1	21	6	0	0	-4	0	0	1	1	**	b11	1	-2	0	7	1	2	0	1	0	0	**	b11	-1	0	0	-1	0	1	1	0	0	0	0	0	**	b11	0	0	1	1	0	0	1	1	1	χ_{6}
χ_{7}	o	120064	0	-896	-32	0	64	14	0	0	0	0	0	0	-2	0	0	2b11	**	0	0	0	4	-2	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	1	1	1	1	-b11	**	-1	-1	0	0	0	0	0	0	0	χ_{7}
χ_{8}	o	120064	0	-896	-32	0	64	14	0	0	0	0	0	0	-2	0	0	**	2b11	0	0	0	4	-2	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	1	1	1	1	**	-b11	-1	-1	0	0	0	0	0	0	0	χ_{8}
χ_{9}	+	381766	-154	-770	67	14	-109	-9	14	11	-1	0	-6	2	1	11	1	0	0	2	-1	0	5	2	0	-1	-1	0	0	0	0	0	-1	-1	1	0	-1	-4	1	1	1	1	1	0	0	0	0	-1	-1	0	0	0	0	0	χ_{9}
χ_{10}	+	1152735	-417	2751	51	-1	235	10	63	3	-3	3	-1	-1	0	3	-2	1	1	-1	-1	3	1	1	1	0	-1	0	0	1	1	-1	-1	-1	0	-1	3	3	0	0	0	0	0	1	1	0	0	-1	-1	0	0	0	0	0	χ_{10}
χ_{11}	+	1534500	660	1980	117	16	125	0	-36	-3	3	2	6	2	0	5	0	0	0	4	1	2	5	2	0	0	1	-1	-1	0	0	0	-1	-1	0	2	-1	2	0	0	0	0	0	0	0	-1	-1	1	1	-1	-1	-1	-1	-1	χ_{11}
χ_{12}	+	3028266	1242	5103	0	6	141	16	63	0	0	3	4	0	0	-3	2	-1	-1	3	0	3	3	0	-2	0	1	0	0	-1	-1	1	0	0	1	-1	3	0	0	0	0	0	0	-1	-1	1	1	-1	-1	0	0	0	0	0	χ_{12}
χ_{13}	+	3073960	-1112	5356	10	8	210	10	-20	10	4	1	0	0	1	18	-2	-1	-1	-4	2	1	6	0	1	1	-2	1	1	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	χ_{13}
χ_{14}	+	4226695	759	-209	115	35	-180	-5	111	3	3	4	5	1	-2	4	-1	0	0	-1	-1	-4	-9	0	1	0	0	1	1	0	0	-1	1	1	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	χ_{14}
χ_{15}	+	4997664	672	8064	-36	0	164	14	0	12	6	0	0	0	0	12	2	1	1	0	0	0	14	-1	-1	0	0	0	0	1	1	0	0	0	-1	0	0	-3	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	0	χ_{15}
χ_{16}	+	5379430	-826	7294	31	14	55	5	14	-1	-7	0	6	-2	1	-1	-1	1	1	2	-1	0	19	1	-1	-1	-1	0	0	-1	-1	0	1	1	0	0	-1	-1	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	χ_{16}
χ_{17}	+	10758860	-308	-9604	-46	28	110	10	28	-14	-2	0	0	0	-1	22	2	2	2	4	-2	0	-4	-1	1	1	-2	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	χ_{17}
χ_{18}	+	11834746	-1078	-1001	106	14	371	-4	119	2	2	0	-4	0	1	-13	2	0	0	-1	2	0	-1	-4	-1	-1	-1	0	0	0	0	-1	0	0	1	0	-1	2	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	χ_{18}
χ_{19}	+	16906780	44	6292	-107	16	-95	5	20	5	-1	2	-6	-2	1	9	-1	0	0	4	1	2	-8	-2	2	-1	1	-1	-1	0	0	0	1	1	0	2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	χ_{19}
χ_{20}	+	18395586	594	-16038	0	6	-39	11	90	0	0	-1	-4	0	0	9	-1	0	0	-6	0	-1	12	0	2	0	1	-1	-1	0	0	2	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	-1	χ_{20}
χ_{21}	+	18395586	594	8019	0	6	-39	11	-45	0	0	-1	-4	0	0	9	-1	0	0	3	0	-1	-6	0	-1	0	1	-b21	*	0	0	-1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	-b21	*	-1	-1	-1	χ_{21}
χ_{22}	+	18395586	594	8019	0	6	-39	11	-45	0	0	-1	-4	0	0	9	-1	0	0	3	0	-1	-6	0	-1	0	1	*	-b21	0	0	-1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	*	-b21	-1	-1	-1	χ_{22}
χ_{23}	+	19212250	330	-5270	49	22	-250	0	-6	9	-3	1	0	-4	-2	-10	0	2	2	-2	1	1	5	-1	0	0	2	1	1	0	0	0	-1	-1	0	1	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	χ_{23}
χ_{24}	+	21312500	-1100	2750	50	20	0	0	-50	10	-2	-1	0	0	-1	0	0	0	0	2	2	-1	0	0	0	1	0	-1	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-b37	*	0	0	-1	-1	1	1	1	χ_{24}
χ_{25}	+	21312500	-1100	2750	50	20	0	0	-50	10	-2	-1	0	0	-1	0	0	0	0	2	2	-1	0	0	0	1	0	-1	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	*	-b37	0	0	-1	-1	1	1	1	χ_{25}
χ_{26}	+	22609664	0	-9856	80	0	-336	14	0	0	0	0	0	0	2	0	0	0	0	0	0	0	-6	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-c67	*2	*4	χ_{26}
χ_{27}	+	22609664	0	-9856	80	0	-336	14	0	0	0	0	0	0	2	0	0	0	0	0	0	0	-6	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*4	-c67	*2	χ_{27}
χ_{28}	+	22609664	0	-9856	80	0	-336	14	0	0	0	0	0	0	2	0	0	0	0	0	0	0	-6	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*2	*4	-c67	χ_{28}
χ_{29}	+	27252720	1584	2376	108	0	220	-5	-24	-12	0	-2	0	0	0	4	-1	0	0	0	0	2	1	-2	1	0	0	b21	*	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	-b21	*	1	1	1	χ_{29}
χ_{30}	+	27252720	1584	2376	108	0	220	-5	-24	-12	0	-2	0	0	0	4	-1	0	0	0	0	2	1	-2	1	0	0	*	b21	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	*	-b21	1	1	1	χ_{30}
χ_{31}	+	27252720	1584	-4752	-54	0	220	-5	48	6	0	-2	0	0	0	4	-1	0	0	0	0	2	-2	1	-2	0	0	1	1	0	0	0	r6	-r6	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	χ_{31}
χ_{32}	+	27252720	1584	-4752	-54	0	220	-5	48	6	0	-2	0	0	0	4	-1	0	0	0	0	2	-2	1	-2	0	0	1	1	0	0	0	-r6	r6	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	χ_{32}
χ_{33}	+	28787220	-924	-2772	63	56	345	-5	-84	-9	3	0	-6	-2	0	1	1	0	0	-4	-1	0	3	3	-2	0	1	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	χ_{33}
χ_{34}	+	29586865	385	15169	-32	21	-635	-10	49	-8	-2	0	-1	3	1	5	0	0	0	-3	0	0	19	-2	-1	1	1	0	0	0	0	-1	0	0	0	0	-1	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	χ_{34}
χ_{35}	+	30739600	-1200	6040	-8	0	-400	0	120	0	6	-4	0	0	-2	0	0	1	1	0	0	4	-10	2	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0	0	χ_{35}
χ_{36}	+	33813560	2552	11396	2	8	-190	10	-28	2	-4	-3	0	0	-1	2	2	0	0	-4	2	-3	-4	2	1	-1	-2	0	0	0	0	0	0	0	0	1	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{36}
χ_{37}	+	36887520	-1440	16752	12	0	20	20	48	-12	0	-2	0	0	0	-20	0	-1	-1	0	0	2	2	2	2	0	0	1	1	1	1	0	0	0	0	0	-2	-2	0	0	0	0	0	-1	-1	0	0	0	0	-1	-1	0	0	0	χ_{37}
χ_{38}	+	38734375	-825	-10625	40	15	0	0	15	0	-6	1	-5	3	-2	0	0	-2	-2	3	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	-1	-1	-1	-1	-1	1	1	0	0	0	0	1	1	0	0	0	χ_{38}
χ_{39}	+	43110144	0	0	0	0	144	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-e31	*2	*4	*8	*16	0	0	1	1	0	0	0	0	-1	-1	-1	χ_{39}
χ_{40}	+	43110144	0	0	0	0	144	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	*16	-e31	*2	*4	*8	0	0	1	1	0	0	0	0	-1	-1	-1	χ_{40}
χ_{41}	+	43110144	0	0	0	0	144	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	*8	*16	-e31	*2	*4	0	0	1	1	0	0	0	0	-1	-1	-1	χ_{41}
χ_{42}	+	43110144	0	0	0	0	144	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	*4	*8	*16	-e31	*2	0	0	1	1	0	0	0	0	-1	-1	-1	χ_{42}
χ_{43}	+	43110144	0	0	0	0	144	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	*2	*4	*8	*16	-e31	0	0	1	1	0	0	0	0	-1	-1	-1	χ_{43}
χ_{44}	+	44159500	-1540	1540	-161	56	125	0	-28	-1	-1	0	6	2	1	5	0	0	0	-4	-1	0	-10	-1	0	-1	1	0	0	0	0	0	-1	-1	0	0	2	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	χ_{44}
χ_{45}	+	45648306	-990	-20790	-54	-34	181	6	42	-6	0	-3	4	0	0	5	0	0	0	2	2	-3	10	1	0	0	1	0	0	0	0	-2	0	0	1	1	2	-1	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	χ_{45}
χ_{46}	+	45648306	-990	10395	108	-34	181	6	-21	12	0	-3	4	0	0	5	0	0	0	-1	-4	-3	-5	-2	0	0	1	0	0	0	0	1	0	0	1	1	-1	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	χ_{46}
χ_{47}	+	45694000	880	3520	-44	0	250	0	64	4	-2	2	0	0	1	-10	0	0	0	0	0	-2	-5	1	0	1	0	-1	-1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	-1	r10	-r10	1	1	0	0	0	χ_{47}
χ_{48}	+	45694000	880	3520	-44	0	250	0	64	4	-2	2	0	0	1	-10	0	0	0	0	0	-2	-5	1	0	1	0	-1	-1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	-1	-r10	r10	1	1	0	0	0	χ_{48}
χ_{49}	+	52994655	-945	5103	0	-21	-345	-20	63	0	0	0	1	-3	0	15	0	-1	-1	3	0	0	3	0	-2	0	-1	0	0	1	1	1	0	0	0	0	3	0	0	0	0	0	0	-1	-1	-1	-1	1	1	0	0	0	0	0	χ_{49}
χ_{50}	+	53765625	-375	8625	-15	-35	0	0	-15	-15	3	4	-5	-1	0	0	0	1	1	1	1	-4	0	0	0	0	0	1	1	-1	-1	1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-1	-1	1	1	1	χ_{50}
χ_{51}	+	56022921	297	-5103	0	-15	-204	-4	-63	0	0	3	5	-3	0	12	2	-2	-2	-3	0	3	-3	0	2	0	0	0	0	0	0	-1	0	0	1	-1	-3	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	χ_{51}
χ_{52}	+	64906250	-550	-1750	5	-50	0	0	-70	5	-1	3	0	4	-1	0	0	2	2	-2	1	3	0	0	0	-1	0	0	0	0	0	0	1	1	0	-1	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	χ_{52}
χ_{53}	+	71008476	924	-10164	-66	28	-274	1	-84	6	6	0	0	0	0	-26	-1	0	0	4	-2	0	11	-1	1	0	-2	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{53}

G 53

53

Th = $F_{3|3}$

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		9074	928	127	472	174	387	7	3	2	1		1			5																																		
		5943887872000	97280	37088	392	960	072	680	000	160	728	648	176	384	96	832	729	162	120	288	288	108	24	39	56	30	30	72	18	19	20	21	24	24	24	24	27	27	27	28	30	30	31	31	36	36	36	39	39	
		p power	A	A	A	A	A	A	A	CA	AA	BA	A	A	B	B	B	B	AA	BA	BA	CA	AB	A	AA	AC	AC	AC	CC	A	AB	AA	AA	BA	DB	DB	B	B	B	AA	AAA	BAA	A	A	AC	AC	AC	AA	AA	
		p' part	A	A	A	A	A	A	A	CA	AA	BA	A	A	A	A	A	A	AA	AA	AA	BA	CB	A	AA	AC	AC	AA	CA	A	AB	AA	AA	AA	CB	CB	A	A	A	AA	AAA	BAA	A	A	AA	AA	AA	AA	AA	
	ind	1A	2A	3A	3B	3C	4A	4B	5A	6A	6B	6C	7A	8A	8B	9A	9B	9C	10A	12A	B**	12C	12D	13A	14A	15A	B**	18A	18B	19A	20A	21A	24A	B**	24C	D**	27A	27B	C**	28A	30A	B**	31A	B**	36A	36B	C**	39A	B**	
χ_{1}	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	χ_{1}
χ_{2}	+	248	-8	14	5	-4	8	0	-2	4	-2	1	3	0	0	5	-4	2	2	2	2	-1	0	1	-1	1	1	1	-2	1	0	0	0	0	0	0	2	-1	-1	1	-1	-1	0	0	-1	-1	-1	1	1	χ_{2}
χ_{3}	+	4123	27	64	-8	1	27	-5	-2	9	0	0	7	3	-1	-8	1	4	2	0	0	0	1	2	-1	1	1	0	0	0	0	1	0	0	-1	-1	-2	1	1	-1	-1	-1	0	0	0	0	0	-1	-1	χ_{3}
χ_{4}	o	27000	120	-27	27	0	8	0	0	0	-3	3	1	0	0	0	0	0	0	2i3-1	-2i3-1	-1	0	-1	1	0	0	0	0	1	0	1	-i3	i3	0	0	0	0	0	1	0	0	-1	-1	2	2z3	**	-1	-1	χ_{4}
χ_{5}	o	27000	120	-27	27	0	8	0	0	0	-3	3	1	0	0	0	0	0	0	-2i3-1	2i3-1	-1	0	-1	1	0	0	0	0	1	0	1	i3	-i3	0	0	0	0	0	1	0	0	-1	-1	2	**	2z3	-1	-1	χ_{5}
χ_{6}	+	30628	-92	91	10	10	36	4	3	10	-5	-2	3	-4	0	10	10	1	3	3	3	0	-2	0	-1	0	0	-2	1	0	-1	0	-1	-1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	χ_{6}
χ_{7}	+	30875	155	104	14	5	27	-5	0	5	8	2	5	3	-1	14	5	2	0	0	0	0	1	0	1	0	0	2	2	0	0	-1	0	0	-1	-1	2	-1	-1	-1	0	0	-1	-1	0	0	0	0	0	χ_{7}
χ_{8}	+	61256	72	182	20	20	56	0	6	12	6	0	6	0	0	-7	-7	2	2	2	2	2	0	0	2	0	0	-3	0	0	0	0	0	0	0	0	-1	-1	-1	0	2	2	0	0	-1	-1	-1	0	0	χ_{8}
χ_{9}	o	85995	-21	0	-27	27	-21	11	-5	3	0	-3	0	3	-1	0	0	0	-1	0	0	-3	-1	0	0	b15	**	0	0	1	1	0	0	0	-1	-1	0	0	0	0	-b15	**	1	1	0	0	0	0	0	χ_{9}
χ_{10}	o	85995	-21	0	-27	27	-21	11	-5	3	0	-3	0	3	-1	0	0	0	-1	0	0	-3	-1	0	0	**	b15	0	0	1	1	0	0	0	-1	-1	0	0	0	0	**	-b15	1	1	0	0	0	0	0	χ_{10}
χ_{11}	+	147250	50	181	-8	-35	34	10	0	5	5	-4	5	2	-2	19	-8	1	0	1	1	-2	1	-1	1	0	0	-1	-1	0	0	-1	-1	-1	1	1	1	1	1	-1	0	0	0	0	1	1	1	-1	-1	χ_{11}
χ_{12}	o	767637	405	0	0	0	-27	-3	12	0	0	0	3	-3	-3	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	2	0	0	0	0	0	0	0	0	1	0	0	b31	**	0	0	0	0	0	χ_{12}
χ_{13}	o	767637	405	0	0	0	-27	-3	12	0	0	0	3	-3	-3	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	2	0	0	0	0	0	0	0	0	1	0	0	**	b31	0	0	0	0	0	χ_{13}
χ_{14}	o	779247	-273	-189	-54	0	63	-9	-3	0	3	6	0	-1	3	0	0	0	-3	3	3	0	0	1	0	0	0	0	0	0	1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	b39	**	χ_{14}
χ_{15}	o	779247	-273	-189	-54	0	63	-9	-3	0	3	6	0	-1	3	0	0	0	-3	3	3	0	0	1	0	0	0	0	0	0	1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b39	χ_{15}
χ_{16}	+	957125	-315	650	-52	-25	133	5	0	15	-6	0	8	-3	1	-25	2	2	0	-2	-2	-2	-1	0	0	0	0	3	0	0	0	-1	0	0	1	1	-1	-1	-1	0	0	0	0	0	1	1	1	0	0	χ_{16}
χ_{17}	o	1707264	-768	0	-54	54	0	0	14	-6	0	6	6	0	0	0	0	0	2	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	i6	-i6	0	0	0	0	-1	-1	1	1	0	0	0	0	0	χ_{17}
χ_{18}	o	1707264	-768	0	-54	54	0	0	14	-6	0	6	6	0	0	0	0	0	2	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	-i6	i6	0	0	0	0	-1	-1	1	1	0	0	0	0	0	χ_{18}
χ_{19}	+	2450240	832	260	71	44	64	0	-10	4	4	-5	-5	0	0	17	-10	-1	2	4	4	1	0	0	-1	-1	-1	1	1	0	0	1	0	0	0	0	-1	-1	-1	1	-1	-1	0	0	1	1	1	0	0	χ_{19}
χ_{20}	+	2572752	-1072	624	111	84	48	0	2	-4	-16	-1	7	0	0	30	3	3	-2	0	0	3	0	0	-1	-1	-1	2	-1	0	0	1	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	χ_{20}
χ_{21}	+	3376737	609	819	9	9	161	1	-13	9	3	-3	0	1	1	9	9	0	-1	-1	-1	-1	1	0	0	-1	-1	-3	0	0	1	0	1	1	1	1	0	0	0	0	-1	-1	0	0	-1	-1	-1	0	0	χ_{21}
χ_{22}	o	4096000	0	64	-8	-80	0	0	0	0	0	0	-8	0	0	-8	1	4	0	0	0	0	0	-1	0	0	0	0	0	-1	0	1	0	0	0	0	1	b27	**	0	0	0	1	1	0	0	0	-1	-1	χ_{22}
χ_{23}	o	4096000	0	64	-8	-80	0	0	0	0	0	0	-8	0	0	-8	1	4	0	0	0	0	0	-1	0	0	0	0	0	-1	0	1	0	0	0	0	1	**	b27	0	0	0	1	1	0	0	0	-1	-1	χ_{23}
χ_{24}	+	4123000	120	118	19	-80	8	0	0	0	6	3	-7	0	0	19	1	4	0	2	2	-1	0	-2	1	0	0	3	0	0	0	-1	0	0	0	0	-2	1	1	1	0	0	0	0	-1	-1	-1	1	1	χ_{24}
χ_{25}	+	4881384	1512	729	0	0	72	24	9	0	9	0	4	8	0	0	0	0	-3	-3	-3	0	0	1	0	0	0	0	0	-1	-1	1	-1	-1	0	0	0	0	0	2	0	0	0	0	0	0	0	1	1	χ_{25}
χ_{26}	+	4936750	-210	637	-38	-65	126	-10	0	15	-3	6	0	-2	2	16	-11	-2	0	-3	-3	0	-1	0	0	0	0	0	0	-1	0	0	1	1	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	χ_{26}
χ_{27}	o	6669000	-1080	-351	108	0	56	0	0	0	9	0	2	0	0	0	0	0	0	2i3-1	-2i3-1	2	0	0	-2	0	0	0	0	0	0	-1	i3	-i3	0	0	0	0	0	0	0	0	1	1	2	2z3	**	0	0	χ_{27}
χ_{28}	o	6669000	-1080	-351	108	0	56	0	0	0	9	0	2	0	0	0	0	0	0	-2i3-1	2i3-1	2	0	0	-2	0	0	0	0	0	0	-1	-i3	i3	0	0	0	0	0	0	0	0	1	1	2	**	2z3	0	0	χ_{28}
χ_{29}	o	6696000	-960	-378	135	0	64	0	0	0	6	3	3	0	0	0	0	0	0	4i3-2	-4i3-2	1	0	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-2	-2z3	**	-1	-1	χ_{29}
χ_{30}	o	6696000	-960	-378	135	0	64	0	0	0	6	3	3	0	0	0	0	0	0	-4i3-2	4i3-2	1	0	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-2	**	-2z3	-1	-1	χ_{30}
χ_{31}	+	10822875	-805	924	141	-75	91	-5	0	5	-4	5	0	3	-1	-21	6	-3	0	4	4	1	1	-2	0	0	0	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	1	1	1	χ_{31}
χ_{32}	+	11577384	552	351	135	0	-120	24	9	0	15	3	7	-8	0	0	0	0	-3	3	3	-3	0	0	-1	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	χ_{32}
χ_{33}	+	16539120	2544	0	297	-54	48	16	-5	-6	0	-3	3	0	0	0	0	0	-1	0	0	3	-2	0	3	1	1	0	0	0	1	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	χ_{33}
χ_{34}	+	18154500	1540	-273	213	-30	-28	20	0	10	-17	1	0	-4	0	-3	-3	-3	0	-1	-1	-1	2	0	0	0	0	1	1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	0	0	χ_{34}
χ_{35}	o	21326760	168	0	-135	-108	-168	0	10	12	0	-3	0	0	0	0	0	0	-2	0	0	3	0	0	0	b15	**	0	0	1	0	0	0	0	0	0	0	0	0	0	b15	**	0	0	0	0	0	0	0	χ_{35}
χ_{36}	o	21326760	168	0	-135	-108	-168	0	10	12	0	-3	0	0	0	0	0	0	-2	0	0	3	0	0	0	**	b15	0	0	1	0	0	0	0	0	0	0	0	0	0	**	b15	0	0	0	0	0	0	0	χ_{36}
χ_{37}	+	28861000	840	1078	-110	160	56	0	0	0	6	-6	0	0	0	-29	-2	-2	0	2	2	2	0	-1	0	0	0	3	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	-1	-1	-1	-1	-1	χ_{37}
χ_{38}	+	30507008	0	896	-184	32	0	0	8	0	0	0	0	0	0	32	5	-4	0	0	0	0	0	-1	0	2	2	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	1	1	0	0	0	-1	-1	χ_{38}
χ_{39}	+	40199250	3410	-78	3	165	-62	10	0	5	2	-1	-7	-6	2	3	3	3	0	-2	-2	1	1	0	1	0	0	-1	-1	0	0	-1	0	0	-1	-1	0	0	0	1	0	0	0	0	1	1	1	0	0	χ_{39}
χ_{40}	+	44330496	3584	168	6	-156	0	0	-4	-4	8	2	0	0	0	6	6	-3	4	0	0	0	0	2	0	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	-1	-1	χ_{40}
χ_{41}	+	51684750	2190	0	108	135	-162	-10	0	15	0	12	-9	6	-2	0	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	χ_{41}
χ_{42}	+	72925515	-2997	0	0	0	27	51	15	0	0	0	-9	3	3	0	0	0	3	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	0	0	-1	-1	0	0	0	0	0	χ_{42}
χ_{43}	+	76271625	-2295	729	0	0	153	-15	0	0	9	0	-11	-7	-3	0	0	0	0	-3	-3	0	0	1	1	0	0	0	0	1	0	1	-1	-1	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1	χ_{43}
χ_{44}	+	77376000	2560	1560	-60	-60	0	0	0	-20	-8	4	2	0	0	-6	-6	3	0	0	0	0	0	0	-2	0	0	-2	1	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{44}
χ_{45}	+	81153009	-783	-729	0	0	225	9	9	0	-9	0	-7	1	-3	0	0	0	-3	3	3	0	0	2	1	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	-1	χ_{45}
χ_{46}	+	91171899	315	0	243	0	-21	-45	24	0	0	-9	0	3	3	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{46}
χ_{47}	+	111321000	3240	-1728	-216	0	216	0	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	-2	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1	χ_{47}
χ_{48}	+	190373976	-3240	0	0	0	-216	0	-24	0	0	0	9	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	χ_{48}

G 48

48

Sporadic Thompson group Th ≅ E ≅ F_3 ≅ $F_{3|3}$

Order = 90,745,943,887,872,000 = $2^{15}.3^{10}.5^3.7^2.13.19.31$ Mult = 1 Out = 1

Constructions

Real G has a 248-dimensional real representation, which when reduced modulo 3 exhibits it as a subgroup of $E_8(3)$. Explicit matrices have been constructed using the subgroup $2^5{\cdot}L_5(2)$ (the Dempwolff group).

Monster 3 x G : the centralizer of an element of class 3C in the Monster group F_1; the normalizer is S_3 x G.

Some subgroups (including all p-local subgroups)

Order	Index	Structure	Specification	Status
634023936	143127000	$^3D_4(2):3$		maximal
319979520	283599225	$2^5{\cdot}L_5(2)$	$N(2A^5)$	maximal
92897280	976841775	$2_+^{1+8}{\cdot}A_9$	N(2A)	maximal
33094656	2742012000	$U_3(8):6$		maximal
25474176	3562272000	$(3 \times G_2(3)):2$	N(3A)	maximal
944784	96049408000	$[3^9].2S_4$	N(3B)	maximal
944784	96049408000	$3^2.[3^7].2S_4$	$N(3B^2)$	maximal
349920	259333401600	$3^5{:}2S_6$	N(3C)	maximal
12000	7562161990656	$5_+^{1+2}{:}4S_4$	N(5A)	maximal
12000	7562161990656	$5^2{:}GL_2(5)$	$N(5A^2)$	maximal
7056	12860819712000	$7^2{:}(3x2S_4)$	$N(7A^2)$	maximal
? 6840	13266950860800	$L_2(19):2$	N(2A,3B,5A,9C,10A,19A)	maximal if it exists
720	126036033177600	$M_{10} \cong A_6{\cdot}2_3$	N(2A,3B,3B,4B,5A)	maximal
465	195152567500800	31:15	N(31AB)	maximal
120	756216199065600	S_5	N(2A,3B,5A)	may not be maximal

Sporadic Fischer group Fi_{23}

Order = 4,089,470,473,293,004,800 = $2^{18}.3^{13}.5^2.7.11.13.17.23$ Mult = 1 Out = 1

Constructions

Fischer 2 x G : the centralizer of a transposition in Fi_{24}, the simple group G being the intersection with Fi_{24}'. For almost all purposes Fi_{23} is best regarded as a subgroup of Fi_{24} (q.v.). Thus, G is a 3-transposition group, generated by a conjugacy class of 31671 involutions, which form a rank 3 graph of valence 3510 when commuting transpositions are joined. The centralizer in G of a transposition is $2{\cdot}Fi_{22}$, and the centralizer of the product of two distinct transpositions is $2^2{\cdot}Fi_{21}.2 \cong 2^2{\cdot}U_6(2).2$ or 3 x $O_7(3)$ according as they do or do not commute. Products of 2 or 3 mutually commuting transpositions are called bi- or tri-transpositions respectively. All maximal sets of mutually commuting transpositions (bases) are conjugate and contain 23 transpositions, which generate an elementary Abelian group 2^{11} whose normalizer is a non-split extension $2^{11}{\cdot}M_{23}$. There are three orbits of $2^{11}{\cdot}M_{23}$ on transpositions: a transposition is either in the base, or commutes with a heptad or with a monad (singleton) of transpositions in the base. There are 23 basic transpositions, $2^5.253$ heptadic, and $2^{10}.23$ monadic.

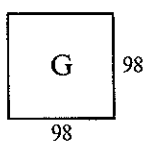
There is a subgroup S_{12} which has four orbits on transpositions, with stabilizers 2 x S_{10}, S_4wrS_3, S_2wrA_6, and M_{12}.

Modulo 3 G has a 253-dimensional irreducible representation over $\mathbb{F}_3$. In the 783-dimensional 3-modular representation of Fi_{24} the 253-space is the space negated by a transposition. Coordinate vectors are the e_A for which [A] is an octad containing ∞.

Presentation See page 232.

Some maximal subgroups (including all p-locals)

Order	Index	Structure	Character	Specifications: Abstract	Specifications: Fischer
129123503308800	31671	$2{\cdot}Fi_{22}$	1a+782a+30888a	N(2A)	transposition, point
29713078886400	137632	$O_8^+(3):S_3$	1a+30888a+106743a		
73574645760	55582605	$2^2{\cdot}U_6(2).2$		N(2B)	bi-transposition, edge
47377612800	86316516	$S_8(2)$			
27512110080	148642560	$S_3 \times O_7(3)$		N(3A)	triad
20891566080	195747435	$2^{11}{\cdot}M_{23}$		$N(2^{11})$	base
3265173504	1252451200	$3_+^{1+8}.2_-^{1+6}.3_+^{1+2}.2S_4$		N(3B)	
663238368	6165913600	$3^3.[3^7].(2 \times L_3(3))$		$N(3B^3)$	
479001600	8537488128	S_{12}			
318504960	12839581755	$(2^2x2_+^{1+8}).(3xU_4(2)).2$		N(2C)	tri-transposition
247726080	16508033685	$2^{6+8}{:}(A_7xS_3)$		$N(2^6)$	heptad
34836480	117390461760	$S_4 \times S_6(2)$		$N(2B^2)$	
3916800	1044084577536	$S_4(4):4$			
4896	835267662028800	$L_2(17):2$			



Fi23

| | ; | @ | @ | @

Fi23

	@2 592 DA BA 12D	@2 592 HB BB 12E	@1 152 FD BD 12F	@1 152 ID AD 12G	@ 864 LB CB 12H	@ 576 CC AC 12I	@ 432 KA CA 12J	@ 288 HD BD 12K	@ 288 KC CC 12L	@ 216 OB D8 12M	@ 144 KC CC 12N	@ 72 OD DD 120	@ 78 A A 13A	@ 78 A A B*	@ 84 AA AA 14A	@ 56 AB AB 14B	@ 360 AA AA 15A	@ 90 AC AC 15B	@ 32 C C 16A	@ 32 C A B**	@ 17 A A 17A	@ 648 DH DC 18A	@ 324 CB CA 18B	@ 216 BB BA 18C	@ 216 CH CC 18D	@ 216 BE BB 18E	@ 216 BE BB 18F	@ 54 BH BC 18G	@ 54 EJ EA 18H	@ 120 BA AA 20A	@ 40 BB AB 20B	@ 42 AA AA 21A	@ 44 AB AB 22A	@ 44 AA AA 22B	@ 44 AA AA C**	@ 23 A A 23A	@ 23 A A B**	@ 48 CB AB 24A	@ 48 FC BC 24B	@ 48 GA AA 24C	@ 26 BA AA 26A	@ 26 AA BA B*	@ 27 A A 27A	@ 28 BA AA 28A	@ 120 ABC ABC 30A	@ 60 AAA AAA 30B	@ 30 BCG BCG 30C	@ 35 AA AA 35A	@ 36 DE CB 36A	@ 36 ED BA 36B	@ 39 AA AA 39A	@ 39 BA BA B*	@ 42 AAA AAA 42A	@ 60 AAA AAA 60A	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	-3	5	1	-1	2	3	0	1	-2	-1	0	1	2	2	1	1	4	1	0	0	0	3	0	-1	-1	0	3	-1	0	1	1	-1	1	1	1	0	0	-1	-1	-1	0	0	-1	-1	0	0	-1	0	-1	0	-1	-1	1	-2	x_2
x_3	-1	-7	-5	2	-1	-2	-1	1	1	-1	1	1	0	0	2	0	-2	1	0	0	1	-2	1	-2	1	1	-2	1	1	-3	-1	1	-2	0	0	0	0	0	1	0	0	0	0	0	-2	0	-1	-1	-1	-1	0	0	-1	0	x_3
x_4	0	2	4	5	2	1	0	-2	4	-1	-2	1	0	0	-1	1	-1	-1	-1	-1	0	-1	2	-1	-1	2	-1	2	-1	2	0	1	1	-1	-1	0	0	-1	0	-1	0	0	1	-1	3	1	1	1	-1	0	0	0	-1	-1	x_4
x_5	3	3	-5	3	3	-1	-3	-5	1	3	-1	1	1	1	-2	0	3	0	0	0	0	1	-2	1	-2	0	3	1	-2	-2	0	1	0	0	0	0	0	1	-1	1	-1	-1	0	0	-1	-1	0	-1	0	0	1	1	1	1	x_5
x_6	2	6	2	3	0	3	2	2	2	3	0	-1	0	0	2	0	1	-2	0	0	-1	3	3	3	0	1	1	0	0	1	-1	1	0	0	0	-1	-1	1	0	1	0	0	0	0	1	1	0	-1	0	-1	0	0	-1	1	x_6
x_7	0	-2	4	-2	1	-2	3	-2	-5	1	1	1	0	0	-1	1	-3	0	0	0	0	-2	-2	2	-1	-2	-2	-1	1	1	-1	-1	1	-1	-1	0	0	0	-2	0	0	0	0	0	1	1	-1	0	0	1	0	0	0	-1	1	x_7
x_8	1	-3	1	0	0	2	4	1	-2	3	2	1	0	0	0	0	2	2	-1	-1	0	-3	3	1	4	-1	-1	1	0	1	1	0	-1	-1	-1	0	0	2	-1	0	0	0	0	0	-2	0	0	0	0	1	0	0	0	-2	x_8
x_9	-7	1	1	1	-2	3	-4	1	-2	4	0	-2	0	0	-1	1	1	1	0	0	0	-2	-5	2	-1	1	-2	-1	1	-1	1	1	0	0	0	0	0	-1	1	1	0	0	1	-1	1	-1	-1	1	1	-1	0	0	-1	-1	x_9
x_{10}	3	3	-1	-3	0	5	0	-1	2	-3	2	-1	0	0	-1	1	4	1	0	0	0	-3	0	-3	0	2	-1	0	0	-1	1	-1	-1	-1	-1	0	0	-1	1	-1	0	0	0	1	0	0	1	0	0	0	0	0	-1	2	x_{10}
x_{11}	3	7	3	-2	-2	-2	0	3	0	-2	-2	0	0	0	0	-2	3	0	-1	-1	0	4	-2	0	3	-4	2	0	1	1	-1	0	0	0	0	0	0	0	1	0	0	0	1	0	-1	-1	0	1	1	0	0	0	0	1	x_{11}
x_{12}	-6	2	-2	2	-1	-4	-3	-2	1	-4	-1	-2	1	1	0	0	0	0	-1	-1	0	-2	-2	2	-1	0	0	-1	1	0	0	0	0	0	0	0	0	2	0	0	1	1	-1	0	0	0	0	0	-1	0	1	1	0	0	x_{12}
x_{13}	4	-10	-4	2	2	-2	-2	2	2	-1	-2	-1	0	0	1	1	-1	-1	0	0	-1	0	3	0	0	1	-2	0	0	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	0	2	1	0	0	1	-1	x_{13}
x_{14}	1	3	5	3	-3	-5	1	-1	5	-3	1	-1	-1	-1	0	0	-1	2	1	1	0	0	-3	0	0	-1	2	0	0	2	0	0	0	0	0	0	0	1	1	1	-1	-1	0	0	-1	-1	0	0	0	1	-1	-1	0	-1	x_{14}
x_{15}	0	-2	-4	4	-2	0	0	2	2	1	0	-1	0	0	0	0	0	0	212	-212	1	-1	2	-1	-1	-2	1	-1	-1	0	0	0	1	-1	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	0	x_{15}
x_{16}	0	-2	-4	4	-2	0	0	2	2	1	0	-1	0	0	0	0	0	0	-212	212	1	-1	2	-1	-1	-2	1	-1	-1	0	0	0	1	-1	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	0	x_{16}
x_{17}	7	3	-1	-3	-3	-1	1	-1	-1	0	-1	2	0	0	0	0	0	0	0	0	0	1	1	1	-2	-1	-1	1	1	0	0	0	0	0	0	b23	**	-1	1	1	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	x_{17}
x_{18}	7	3	-1	-3	-3	-1	1	-1	-1	0	-1	2	0	0	0	0	0	0	0	0	0	1	1	1	-2	-1	-1	1	1	0	0	0	0	0	0	**	b23	-1	1	1	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	x_{18}
x_{19}	-3	-1	5	-7	5	-5	3	-1	-1	-1	1	-1	0	0	1	1	3	0	0	0	0	-1	2	-1	-1	2	-1	-1	-1	1	-1	1	0	0	0	0	0	-1	-1	1	0	0	0	1	-1	-1	0	1	-1	0	0	0	1	1	x_{19}
x_{20}	0	-16	0	-4	-4	0	0	0	0	2	0	0	0	0	0	-2	-2	1	0	0	0	-2	-2	2	-1	0	0	-1	-2	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	2	2	1	1	-1	0	0	0	0	0	x_{20}
x_{21}	-6	6	2	3	0	-1	0	2	2	3	2	-1	0	0	1	-1	2	2	0	0	0	-3	0	1	-2	2	-1	1	0	0	0	-1	0	0	0	0	0	-1	0	-1	0	0	-1	1	2	0	0	-1	0	0	0	0	1	0	x_{21}
x_{22}	-7	-3	-3	0	-6	0	2	-3	0	0	0	0	-1	-1	-2	0	3	0	0	0	0	3	3	3	0	1	1	0	0	-1	-1	-1	0	0	0	0	0	0	1	0	1	1	0	0	-1	1	0	1	0	-1	-1	-1	1	-1	x_{22}
x_{23}	0	12	0	-3	3	5	-3	0	-3	0	-1	0	0	0	0	0	-1	-1	0	0	0	-1	2	-1	2	2	-1	-1	-1	1	1	0	0	0	0	0	0	1	2	1	0	0	-1	0	-1	1	-1	0	0	0	0	0	0	1	x_{23}
x_{24}	3	9	3	-3	-3	1	-3	-3	-3	0	1	0	-2	-2	1	-1	-1	-1	0	0	1	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	-1	-1	-1	0	0	0	-1	-1	-1	1	0	0	0	1	1	1	-1	x_{24}
x_{25}	-2	0	6	-3	6	-3	-2	0	0	0	0	0	0	0	2	0	-3	0	0	0	0	-2	1	2	2	-1	2	-1	1	-1	1	1	0	0	0	0	0	-1	0	-1	0	0	0	0	1	1	0	-1	0	1	0	0	-1	-1	x_{25}
x_{26}	-8	0	0	0	0	4	-2	0	0	0	-2	0	0	0	-1	1	-5	1	0	0	0	5	-1	1	-2	-1	-1	1	-1	-1	-1	-1	0	0	0	1	1	0	0	0	0	0	0	1	-1	1	-1	0	0	1	0	0	-1	-1	x_{26}
x_{27}	-8	0	0	-6	0	2	-2	0	0	0	2	0	0	0	-1	-1	-1	-1	0	0	0	-3	-3	-3	0	1	1	0	0	1	-1	-1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	1	1	-1	0	1	0	0	-1	1	x_{27}
x_{28}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	-4	2	0	0	2	2	0	-1	0	0	0	1	i	i11	-i11	0	0	0	0	0	0	0	1	0	-2	0	-1	0	0	0	0	0	0	0	x_{28}
x_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	-4	2	0	0	2	2	0	-1	0	0	0	1	-i	i11	i11	0	0	0	0	0	0	0	1	0	-2	0	-1	0	0	0	0	0	0	0	x_{29}
x_{30}	-6	0	-6	3	3	-5	3	0	-3	0	1	0	0	0	0	0	-2	-2	1	1	0	-3	0	-3	0	2	-1	0	0	1	-1	0	0	0	0	0	0	1	0	1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	-2	x_{30}
x_{31}	-4	-10	4	8	-1	4	-1	-2	1	-1	1	1	0	0	0	0	0	0	0	0	0	1	1	-3	0	-1	-1	0	1	0	0	0	-1	1	1	0	0	0	-2	0	0	0	0	0	0	0	0	0	2	-1	0	0	0	0	x_{31}
x_{32}	6	-4	-2	2	5	-2	-3	4	1	-1	1	1	0	0	-2	0	-1	-1	0	0	0	0	0	0	3	0	0	0	0	-2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	3	-1	-1	1	-1	0	0	0	1	1	x_{32}
x_{33}	1	3	-3	3	0	3	4	3	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	-3	0	0	-1	2	0	0	1	1	1	0	0	0	0	0	-1	-1	-1	0	0	0	1	-1	1	-1	0	0	1	0	0	-1	1	x_{33}
x_{34}	8	0	0	-6	0	-2	2	0	0	0	-2	0	0	0	1	-1	-3	0	0	0	0	5	-1	-3	0	-1	-1	0	-1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	-1	0	-1	0	0	1	-1	x_{34}
x_{35}	6	0	6	-3	-3	1	3	0	3	0	1	0	1	1	-2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	0	-1	0	-1	1	1	0	0	-1	-1	-1	-1	0	0	1	1	1	-1	x_{35}
x_{36}	-7	3	5	0	-3	2	5	-1	-1	-3	-1	-1	1	1	0	0	0	0	1	1	0	3	3	3	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	-2	-1	-1	0	0	0	0	0	0	0	-1	1	1	0	0	x_{36}
x_{37}	7	-3	3	9	-9	-1	1	-3	-3	0	-1	0	0	0	0	0	2	-1	0	0	0	-3	-3	-3	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	-1	0	0	0	0	2	0	-1	0	0	1	0	0	0	0	x_{37}
x_{38}	-10	8	-6	2	5	-2	-1	0	3	2	1	0	0	0	0	0	0	0	0	0	0	2	-1	-2	1	-3	0	1	2	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	x_{38}
x_{39}	6	-6	-2	0	3	2	-3	-2	1	3	-1	1	0	0	2	0	3	0	1	1	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	-2	0	0	0	0	-1	-1	0	1	0	0	0	0	-1	1	x_{39}
x_{40}	0	-8	0	1	4	3	0	0	0	-2	0	0	0	0	1	-1	1	1	0	0	0	0	0	0	-3	-2	-2	0	0	1	-1	1	0	0	0	0	0	-1	0	1	0	0	0	-1	1	1	-1	0	1	0	0	0	1	1	x_{40}
x_{41}	8	0	0	3	0	1	2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	-7	-1	1	4	-3	-3	1	-1	0	0	0	1	1	1	0	0	1	0	-1	0	0	-1	0	0	0	0	0	0	-1	0	0	0	0	x_{41}
x_{42}	3	1	-5	4	4	0	6	1	-2	1	0	1	0	0	0	0	0	0	-1	-1	0	3	0	-1	-1	4	1	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	x_{42}
x_{43}	7	7	-1	1	-5	-7	-5	-1	-1	1	-1	-1	0	0	0	0	1	1	0	0	0	0	3	0	3	3	0	0	0	1	-1	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	1	1	-1	0	1	1	0	0	0	1	x_{43}
x_{44}	8	0	0	3	0	1	2	0	0	0	-2	0	0	0	0	0	-2	1	0	0	0	-1	-1	-1	-4	3	3	-1	2	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0	-2	0	1	0	0	-1	0	0	0	0	x_{44}
x_{45}	-8	-8	0	1	4	-1	-2	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	5	-1	1	1	1	1	1	-1	0	0	0	0	0	0	-1	-1	1	0	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	x_{45}
x_{46}	-7	3	-3	0	-3	-4	-1	3	-3	0	-1	0	0	0	0	0	1	1	0	0	0	0	3	0	0	-1	2	0	0	-2	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	1	1	0	0	-1	0	0	0	1	x_{46}
x_{47}	-4	-6	4	-3	0	5	-4	-2	-2	-3	2	1	0	0	0	0	0	0	0	0	0	0	3	0	0	-1	2	0	0	0	0	0	1	-1	-1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	x_{47}
x_{48}	0	8	0	5	-4	3	0	0	0	2	0	0	0	0	1	1	-5	1	0	0	0	0	0	0	3	0	0	0	0	1	-1	-1	0	0	0	0	0	-1	0	1	0	0	0	-1	-1	-1	1	0	-1	0	0	0	1	1	x_{48}
x_{49}	0	0	0	3	0	-3	0	0	0	0	0	0	2	2	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	-1	0	1	0	0	0	1	-1	-1	1	0	0	0	-1	-1	-1	-1	x_{49}
x_{50}	-6	-10	2	2	-4	-2	0	2	2	2	-2	2	0	0	0	0	3	0	0	0	0	-6	0	2	-1	0	0	-1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	-1	0	0	0	0	1	x_{50}
x_{51}	-6	-10	2	2	2	-2	0	2	-4	-1	-2	-1	0	0	0	0	3	0	0	0	0	3	0	-1	-1	0	-3	2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-1	0	0	0	0	1	x_{51}
x_{52}	8	0	0	0	0	-4	2	0	0	0	2	0	0	0	-1	1	-1	2	0	0	0	5	-1	-3	0	-1	-1	0	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	0	1	0	-1	0	0	-1	1	x_{52}
x_{53}	-3	-3	9	3	0	-3	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	2	-1	2	-2	1	-1	-1	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	x_{53}
x_{54}	10	2	2	-1	-4	3	-2	2	2	-1	0	-1	0	0	0	0	0	0	0	0	0	4	1	0	-3	1	-2	0	1	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	x_{54}
x_{55}	0	6	8	0	6	0	0	2	2	3	0	-1	0	0	-2	0	-2	-2	0	0	0,	0	0	0	0	0	0	0	0	0	0	-1	-2	0	0	0	0	0	0	0	0	0	0	-2	0	0	1	0	0	0	0	1	0	x_{55}
x_{56}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	2	0	4	-2	0	0	0	4	-2	0	0	0	0	0	-2	0	0	1	0	0	0	1	1	0	0	0	1	1	0	0	0	-2	0	-1	0	0	-1	-1	-1	0	x_{56}
x_{57}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-2	0	4	-2	0	0	0	-4	2	0	0	0	0	0	2	0	0	1	0	0	0	1	1	0	0	0	-1	-1	0	0	0	2	0	-1	0	0	-1	-1	1	0	x_{57}
x_{58}	10	-4	-10	2	5	2	1	-4	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	-2	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	1	0	0	0	0	0	x_{58}
x_{59}	6	12	-2	6	0	2	0	4	-2	-3	2	1	0	0	0	0	1	1	0	0	0	-1	-4	3	0	2	-1	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	0	0	0	0	0	0	-1	x_{59}
x_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	-1	0	0	0	0	0	-2	4	0	0	0	0	-2	0	0	0	0	0	0	0	0	1	0	2	0	0	-1	0	0	0	0	0	0	x_{60}
x_{61}	9	-9	9	0	0	0	0	3	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-1	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	x_{61}
x_{62}	0	0	0	0	0	0	0	0	0	0	0	0	b13	*	0	0	0	0	0	0	1	3	0	3	0	2	-1	0	0	0	0	0	0	0	0	0	0	1	0	b13	*	0	0	0	0	0	0	0	0	b13	*	0	0	x_{62}
x_{63}	0	0	0	0	0	0	0	0	0	0	0	0	*	b13	0	0	0	0	0	0	1	3	0	3	0	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	*	b13	0	0	0	0	0	0	0	0	*	b13	0	0	x_{63}
x_{64}	-8	0	0	0	0	0	4	0	0	0	0	0	0	0	-1	1	-3	0	0	0	0	0	3	0	0	-1	2	0	0	-2	0	1	-1	1	1	0	0	0	0	0	0	0	-1	-1	1	-1	0	1	0	1	0	0	-1	1	x_{64}
x_{65}	8	0	0	0	0	0	-4	0	0	0	0	0	0	0	1	1	-3	0	0	0	0	0	-3	0	0	-1	2	0

$Co_1 = F_{2-}$

Sporadic Conway group Co_1

Order = 4,157,776,806,543,360,000 = $2^{21}.3^9.5^4.7^2.11.13.23$ Mult = 2 Out = 1

Constructions

Leech 2.G : the automorphism group of the Leech lattice, the unique 24-dimensional even (i.e. all vector norms even) unimodular (i.e. determinant 1) lattice with no vector of norm 2. We construct this in terms of the Mathieu group M_{24} (q.v.) and the associated Golay code C. The lattice consists of all 24-dimensional vectors (x_i) over $\mathbb{Z}$ for which there is a Golay code word (c_i) and a number m = 0 or 1, such that

(i) all $x_i \equiv m \pmod 2$

(ii) $x_i \equiv m + 2c_i \pmod 4$

(iii) $\Sigma x_i \equiv 4m \pmod 8$

The unimodular norm is $\frac{1}{8}\Sigma x_i^2$, and the type of a vector is half this norm. There are 196560 = 2 x 98280 minimal vectors (norm 4, type 2):

1104 vectors	$(\pm 4^2 0^{22})$	(all possibilities)
97152 vectors	$(\pm 2^8 0^{16})$	(the support an octad of S(5,8,24), even signs)
98304 vectors	$(\mp 3 \pm 1^{23})$	(upper sign on a C-set, ∓ 3 anywhere)

The monomial group $2^{12}:M_{24}$ (becoming $2^{11}:M_{24}$ in G) is generated by the sign changes on C-sets, and permutations in M_{24}.

Orthogonal (2) Every vector of the Leech lattice is congruent modulo 2 to just one of:

the zero vector,

$\pm v$ (v of type 2),

$\pm v$ (v of type 3),

or $\{\pm v_1, \ldots \pm v_{24}\}$ (a coordinate frame of mutually orthogonal type 4 vectors).

The standard coordinate frame consists of the 48 vectors $(\pm 8, 0^{23})$.

G is the automorphism group of the 24-dimensional space over $\mathbb{F}_2$ equipped with the corresponding type function taking values 0, 2, 3, 4. The group is transitive on vectors of each type, and the type functon mod 2 is a quadratic form. There are 15 orbits of G on 2-dimensional subspaces (projective lines) which are described below by the types of their non-zero vectors. In all cases except those marked *, the non-zero vectors lift to real Leech lattice vectors of the appropriate types adding to zero. We give the pointwise stabilizers in Co_1.

Type	Stabilizer	Type	Stabilizer	Type	Stabilizer
222	$U_6(2)$	223	M^cL	224	$2^{10}M_{22}.2$
233	HS	233*	$U_4(3).2^2$	234	M_{23}
244	$2^{1+8}A_8$	333	3^5M_{11}	333*	$U_3(5)$
334*	2 x M_{12}	334	2^4A_8	344	$L_3(4).S_3$
444	$2^{4+12}.3S_6$	444	$M_{12}.2$	444	$[2^{12}].L_3(2)$

Representatives of the three orbits of spaces of type 444 are generated by $(8,0^{23})$ together with $(4^4,0^{20})$, $(4,0^{11};2^{12-})$, or $(4^2,0^{14};2^8)$, respectively. Here and elsewhere k^{n+} (resp. k^{n-}) represents a set of n coordinates $\pm k$ with positive (resp. negative) product. The coordinates to one side of a semicolon form a C-set.

S-lattices Here is a method for classifying low-dimensional sublattices L of the Leech lattice. For any u in L there are Leech lattice vectors v and w such that u = v + 2w, with type(v) at most 4. If any such v has type 4, then we can choose coordinates so that $v = (8,0^{23})$, and then the pointwise stabilizer Stab(L) of L is contained in the monomial group $2^{12}:M_{24}$. Otherwise v is uniquely determined up to sign, and so Stab<L,w> has index at most 2 in Stab(L). By repeated adjunctions of such vectors w, we obtain a lattice L' such that either Stab(L') is in $2^{12}:M_{24}$ or L' is an S-lattice, i.e. for each u in L' the corresponding v and w are again in L', and v has type at most 3. An n-dimensional S-lattice which contains (up to sign) p type 2 vectors and q type 3 vectors has type 2^p3^q (with $p+q = 2^n-1$). The table describes the 12 orbits of S-lattices and their stabilizers in $2Co_1$. In every case the setwise stabilizer has the form Stab(L).Aut(L).

Dimension	Type	Stab(L)	Aut(L)	Dimension	Type	Stab(L)	Aut(L)
0	2^03^0	$2Co_1$	1	2	$333 = 2^03^3$	3^5M_{11}	2 x S_3
1	$2 = 2^13^0$	Co_2	2	3	2^53^2	$U_4(3)$	2 x D_8
1	$3 = 2^03^1$	Co_3	2	3	2^33^4	$U_3(5)$	2 x S_3
2	$222 = 2^33^0$	$U_6(2)$	2 x S_3	4	2^93^6	3^4A_6	2 x $(S_3xS_3).2$
2	$223 = 2^23^1$	M^cL	2^2	4	2^53^{10}	$5^{1+2}.4$	2 x S_5
2	$233 = 2^13^2$	HS	2^2	6	$2^{27}3^{36}$	$3^{1+4}.2$	2 x $U_4(2).2$

Vectors

The table below classifies vectors of type up to 16. If V is congruent modulo 2 to a vector of type 4 then the "shape" column gives the expression for V in the corresponding coordinate frame, and the stabilizing group is a subgroup of $2^{12}{:}M_{24}$. Otherwise the shape-symbol mu_a+nv_b (w_c) means that there are vectors u_a, v_b, w_c (of types a, b, c) satisfying $u_a+v_b+w_c=0$, with $V = mu_a+nv_b$, and the stabilizing group is the appropriate G_{abc} or $G_{abc}.2$ from the table to the right below. The numbers given provide a check on the group orders. The total number of vectors of type n is $65520(s_{11}(n)-t(n))/691$, where $s_{11}(n)$ is the sum of the 11th powers of the divisors of n, and t(n) is Ramanujan's tau function.

Type	Shape	Number/65520	Type	Shape	Number/65520		
2	v_2	3	13	u_3-2v_2 (w_4)	124 41600	abc	G_{abc}
3	v_3	256	13	u_3+2v_3 (w_5)	10072 71936	2	Co_2
4	$(8,0^{23})$	6075	13	u_3+2v_4 (w_4)	15738 62400	3	Co_3
5	u_3+2v_2 (w_2)	70656	14	$(10,-6,2^{22})$	2861 56800	222	$U_6(2)$
6	(2^{24})	5 18400	14	$(-6^4,2^{20})$	55085 18400	223	M^cL
6	u_2+2v_2 (w_2)	6900	14	$3u_2+2v_2$ (w_2)	13800	233	HS
7	u_3+2v_2 (w_3)	28 61568	14	u_2+2v_3 (w_5)	194 30400	333	3^5M_{11}
8	$2v_2$	3	14	u_2+2v_4 (w_4)	491 83200	224	$2^{10}M_{22}$
8	$(4^{8-};0^{16})$	122 95800	15	u_3-2v_2 (w_3)	28 61568	234	M_{23}
8	u_2+2v_2 (w_3)	1 41312	15	u_3+2v_3 $(w_6)a$	13353 98400	334	2^4A_8
9	u_3+2v_2 (w_4)	124 41600	15	u_3+2v_3 $(w_6)b$	194 30400	244	$2^{1+8}A_7$
9	u_3+2v_3 (w_3)	329 72800	15	u_3+2v_4 $(w_5)a$	80123 90400	344	$M_{21}.2$
10	$(-6^2,2^{22})$	1430 78400	15	u_3+2v_4 $(w_5)b$	31477 24800	235	$U_4(3).2$
10	u_2+2v_2 (w_4)	2 79450	16	$2v_4 = (16,0^{23})$	6075	335	$U_3(5)$
10	u_2+2v_3 (w_3)	14 30784	16	u_2-2v_3 (w_4)	124 41600	245	M_{22}
11	u_3+2v_2 (w_5)	194 30400	16	u_2+2v_4 (w_5)	2861 56800	345a	$2 \times M_{11}$
11	u_3+2v_4 (w_4)	3934 65600	16	$(12,4^{7+};0^{16})$	983 66400	345b	2^4A_7
12	$2v_3$	256	16	$(4^{8-};8^2,0^{14})$	59019 84000	336a	M_{12}
12	u_2-2v_2 (w_3)	1 41312	16	$(4^{12+};8,0^{11})$	160247 80800	336b	$U_4(3).2$
12	u_2+2v_3 (w_4)	124 41600	16	$(4^{16-};0^8)$	31477 24800		
12	$(8^3,0^{21})$	20 49300					
12	$(8,0^{15};4^{8+})$	3934 65600					
12	$(4^{12-};0^{12})$	6676 99200					

Suzuki chain

The centralizer of an element of class 3D in Co_1 is $3 \times A_9$. The groups A_n (becoming $2A_n$ in $2Co_1$) obtained by fixing all but n points in this A_9 have normalizers

$NA_2 = Co_1$ $\quad NA_3 = 3.Suz.2$ $\quad NA_4 = (A_4 \times G_2(4)).2$ $\quad NA_5 = (A_5 \times J_2).2$

$NA_6 = (A_6 \times U_3(3)).2$ $\quad NA_7 = (A_7 \times L_2(7)).2$ $\quad NA_8 = A_8 \times S_4$ $\quad NA_9 = A_9 \times S_3$

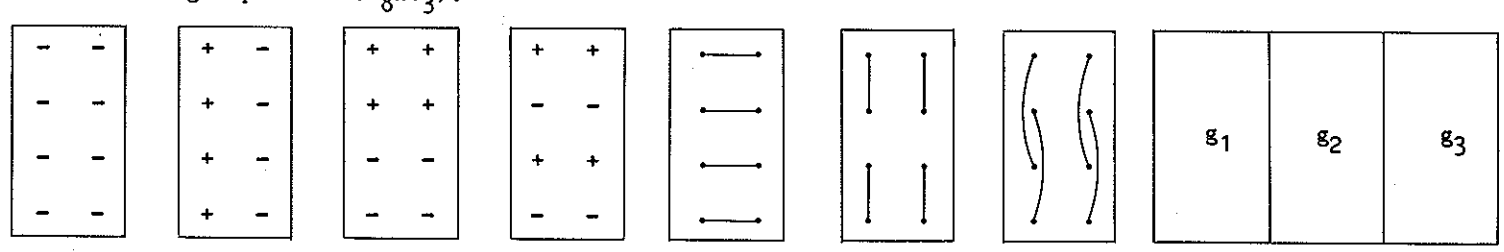
The cases n = 3,4,5 give rise to the complex and quaternionic constructions for the Leech lattice (see Suz, $G_2(4)$, J_2). Some other groups arise as normalizers of direct products of such groups A_n:

$NA_3^2 = 3^2U_4(3).2$, $\quad NA_4^2 = ((A_4 \times A_4).2 \times 2^4A_5).2$, (not maximal),

$NA_5^2 = ((A_5 \times A_5).2 \times D_{10}).2$, $\quad NA_3^3 = 3^{3+4}{:}2(S_4 \times S_4)$.

Code groups

Some subgroups arise in connection with "codes" whose "digits" may form a non-Abelian group. For example the seven elements at the left below generate a group 2_+^{1+6} acting on an 8-space, and the "codewords" (g_1,g_2,g_3) with $g_1g_2g_3 = \pm 1$ (acting on a 24-space as shown at the right) generate in $2Co_1 = 2.G$ a group 2^{3+12} whose normalizer is a maximal subgroup $2^{3+12}{\cdot}(A_8 \times S_3)$.

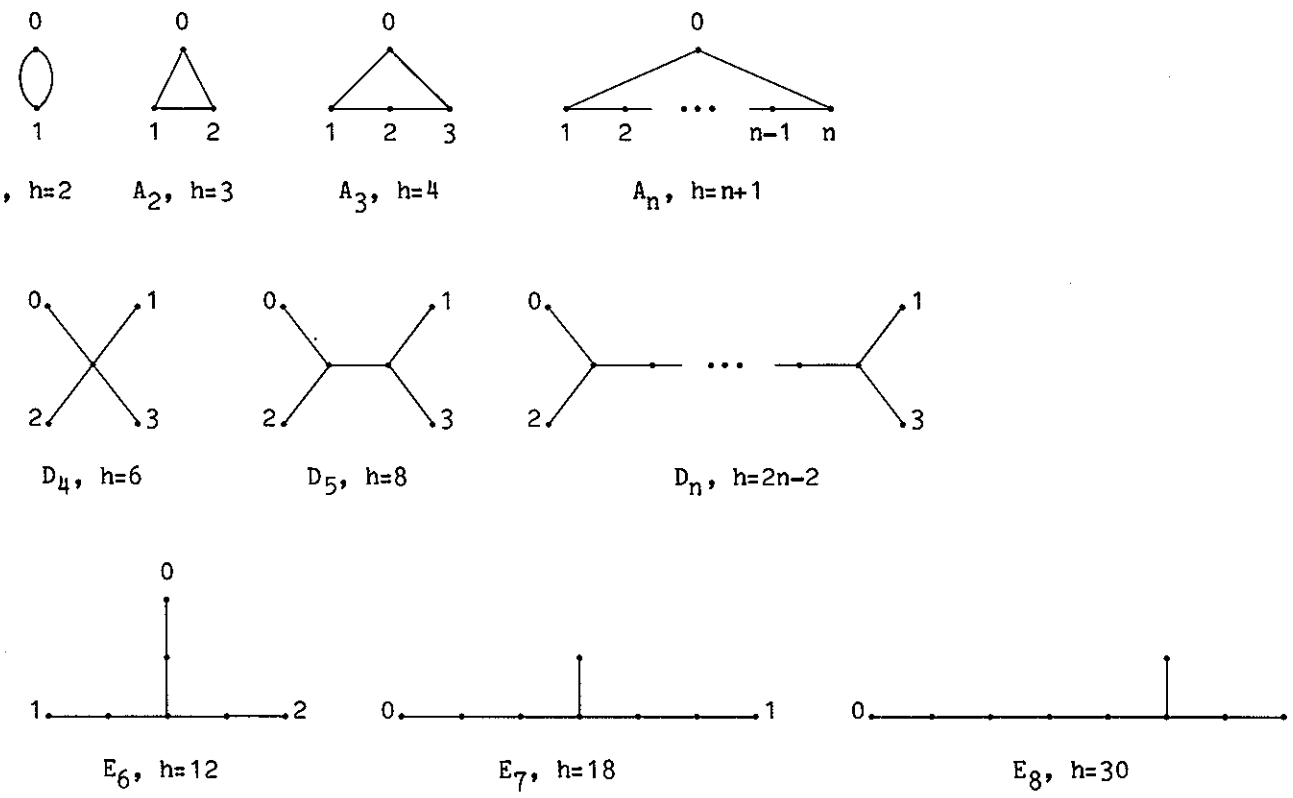


Similarly, the left and right multiplications by ±1, ±i, ±j, ±k define a group 2_+^{1+4} acting on the real 4-space of quaternions. Writing $i^0=1$, $i^1=1$, $i^w=j$, $i^{\bar{w}}=k$ we obtain from each word (abcdef) of the hexacode (see A_6) a word $(i^a,\ldots,i^f)$ of the quaternionic hexacode, and the left and right multiplications by such words (acting on the columns of the MOG, see $G_2(4)$) generate a group 2^{5+12} whose normalizer is a maximal subgroup $2^{5+12}{\cdot}(S_3 \times 3S_6)$ of $2Co_1$.

$Co_1 = F_{2-}$

Holes

The points of 24-space at maximal distance from the points of the lattice have a uniform description. The nearest lattice points to such a deep hole form a disjoint union of extended (affine) Dynkin diagrams (see below) when we join two points whose difference is a type 3 vector (and twice-join points whose difference is a type 4 vector). Each set of lattice points whose differences are at most type 4 and which forms an extended Dynkin diagram under these conventions can be found among the neighbours of a unique deep hole.



The 23 cases are just those with total subscript 24 and constant Coxeter number h. Each of the next nearest lattice points is joined (type 3 difference) to just one point in each component, which is a numbered point in the Figure above. The corresponding sequences of numbers form an additive group (the holy code) in which addition of digits is cyclic except for a component D_{2n+1}, for which it is a fourgroup. In the Code column of the table below, we use parentheses to indicate that the corresponding digits may be cyclically permuted.

Hole	Group	Code	Hole	Group	Code
D_{24}	(2)	1	$D_{16}E_8$	(2)	10
$E_8{}^3$	$(1).S_3$	000	A_{24}	(5).2	5
$D_{12}{}^2$	$(2^2).2$	(12)	$A_{17}E_7$	(6).2	31
$D_{10}E_7{}^2$	$(2^2).2$	110,301	$A_{15}D_9$	(8).2	21
$D_8{}^3$	$(2^3).S_3$	(122)	$A_{12}{}^2$	(13).4	15
$A_{11}D_7E_6$	(12).2	111	$E_6{}^4$	$(3^2).2.S_4$	1(012)
$A_9{}^2D_6$	$(5 \times 2^2).4$	240,501,053	$D_6{}^4$	$(2^4).S_4$	(012)3,1111
$A_8{}^3$	$(3^3).2.S_3$	(114)	$A_7{}^2D_5{}^2$	$(2^5).D_8$	1112,1721
$A_6{}^4$	$(7^2).2.A_4$	1(216)	$A_5{}^4D_4$	$(3^2.2^3).2S_4$	2(024)0,33001,30302,30033
$D_4{}^6$	$(2^6).3S_6$	111111,0(02332)	$A_4{}^6$	$(5^3).2.S_5$	1(01441)
$A_3{}^8$	$(2^8).2.2^3L_2(7)$	3(2001011)	$A_2{}^{12}$	$(3^6).2M_{12}$	2(11211122212)
$A_1{}^{24}$	$(2^{12}).M_{24}$	1(000001010011001101011 11)			

The portion of the group inside the initial parenthesis is the additive group of the code. In each case, the nearest and next-nearest neighbours to the hole suffice to determine the lattice.

The vertices of a hole of type D_{24} can be taken, using the shuffle numbering for M_{24}, to be the vectors

$(-3,1^{23})$, with the -3 on one of $0^-,1^-,\ldots,11^-,11^+$,

$(5,1^{23})$, with the 5 on 0^+,

$(2^8,0^{16})$, 2s on one of the 11 octads found under M_{24}, section "Mathieu 12"

The two next nearest points are (0^{24}) and $(0^{12},2^{12})$, with the 2s on the dodecad $0^+,1^+,\ldots,11^+$. The corresponding Leech roots (see "hyperbolic", below) are

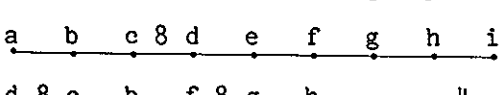
$(0^n,1,-1,0^{23-n}|0)$ $(n = 0,\ldots,22)$, $(-1,-1,0^{23}|0)$, $(0^{22},1,1,1|1)$, and $(0^{23},1,-1|0)$, $(-\frac{1}{2},\frac{1}{2}^{23}\frac{3}{2}|\frac{5}{2})$.

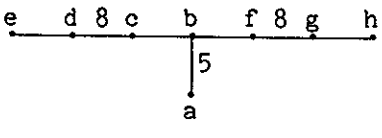
$F_2 = Co_1$

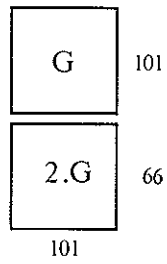
Hyperbolic

The Lorentzian lattice $II_{25,1}$ consists of the vectors $(x_0,\ldots,x_{24}|t)$ in which the coordinates are either all in $\mathbb{Z}$ or all in $\mathbb{Z}+\frac{1}{2}$, and have $\Sigma x_i + t$ in $2\mathbb{Z}$, in a space equipped with the Lorentzian norm $\Sigma x_i^2 - t^2$. A vector r of the lattice is called a Leech root if it satisfies $r.r = 2$, $r.w = -1$, where w is the particular norm 0 vector $(0,1,2,\ldots,24|70)$. Under the metric defined by $d(r,s)^2 = \mathrm{Norm}(r-s)$, the Leech roots form a (non-linear) set isometric to the Leech lattice. The reflection subgroup of $\mathrm{Aut}(II_{25,1})$ is generated by the reflections in the Leech roots, and the full group is an extension of this by the negative of the identity operation and the (infinite) affine automorphism group $\mathbb{Z}^{24}{:}2Co_1$ of the Leech lattice.

"Presentations"

$2 \times G \cong \langle$  $\mid a=(cd)^4,\ (bcde)^8=1$, more relations ? $\rangle$

$G \cong \langle$  $\mid a=(cd)^4=(fg)^4,\ 1=(abcf)^5=(abcdecbdc)^7=(abfghfbgf)^7$, more relations ? $\rangle$



Maximal subgroups

Order	Index	Structure	Specifications: Abstract	Specifications: Leech	Suzuki chain
42305421312000	98280	Co_2		minimal (type 2) vector	
2690072985600	1545600	$3{\cdot}Suz{:}2$	N(3A)	complex structure	NA_3
501397585920	8282375	$2^{11}{:}M_{24}$	$N(2^{11}) = N(2A_{759}C_{1288})$	type 4 base, A_1^{24}-hole, $\mathrm{code}_{24}(2)$	
495766656000	8386560	Co_3		type 3 vector	
89181388800	46621575	$2_+^{1+8}{\cdot}O_8^+(2)$	N(2A)	octad space	
55180984320	75348000	$U_6(2){:}S_3$		222-S-lattice	
6038323200	688564800	$(A_4 \times G_2(4)){:}2$	$N(2B^2)$	quaternionic structure	NA_4
1981808640	2097970875	$2^{2+12}{:}(A_8 \times S_3)$	$N(2A^2)$	"code" in $(2_+^{1+6})^3$	
849346560	4895265375	$2^{4+12}{\cdot}(S_3 \times 3S_6)$	$N(2A^4)$	"code" in $(2_+^{1+4})^6$, 3 bases	
235146240	17681664000	$3^{2}{\cdot}U_4(3).D_8$	$N(3^2) = N(3A_2B_2)$		NA_3^2
138568320	30005248000	$3^6{:}2M_{12}$	$N(3^6) = N(3A_{12}B_{132}C_{220})$	complex base, A_2^{12}-hole, $\mathrm{code}_{12}(3)$	
72576000	57288591360	$(A_5 \times J_2){:}2$	N(2B,3A,5A)	icosian structure	NA_5
25194240	165028864000	$3_+^{1+4}{:}2U_4(2){:}2$	N(3C)	$2^{27}3^{36}$-S-lattice	
4354560	954809856000	$(A_6 \times U_3(3)){:}2$	N(2B,3A,3B,4E,5A)	"code" in $(3_+^{1+2})^4$	NA_6
2519424	1650288640000	$3^{3+4}{:}2(S_4 \times S_4)$	$N(3^3) = N(3A_3B_6C_4)$		NA_3^3
1088640	3819239424000	$A_9 \times S_3$	N(3D)		NA_9
846720	4910450688000	$(A_7 \times L_2(7)){:}2$	N(2A,3D,4A,7A)		NA_7
314928	13202309120000	$3^2.[2.3^6].2A_4$	$N(3C^2)$		
144000	28873450045440	$(D_{10} \times (A_5 \times A_5).2).2$	N(5B), $N(2B,3A,5A)^2$		NA_5^2
139968	29705195520000	$3^2.[2^3.3^4].2A_4$	$N(3C^2)$		
60000	69296280109056	$5_+^{1+2}{:}GL_2(5)$	N(5C)	$2^5 3^{10}$-S-lattice	
60000	69296280109056	$5^3{:}(4 \times A_5).2$	$N(5^3) = N(5A_{10}B_{15}C_6)$	icosian base, A_4^6-hole	
6000	692962801090560	$5^2{:}4A_5$	$N(5C^2)$		
3528	1178508165120000	$7^2{:}(3 \times 2A_4)$	$N(7^2) = N(7A_4B_4)$	A_6^4-hole	

$Co_1 = F_{2-}$

	i	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	
		415777	8918	20127	3892	134503	587	125	544	3715	11	737	294	48	15	30	36	15	99	483	155	155	20	15	3	1		17	1	92	6	6	2			1				9	1	1								
		6806543360000	1388800	74400	83840	6492800	86560	97120	320	89120	79648	280	912	384	360	24000	000	000	53280	840	520	520	736	552	456	440	576	640	176	160	144	144	048	768	768	944	486	486	600	200	200	600	600	160	66						
		p power	A	A	A	A	A	A	A	A	A	A	A	B	C	A	A	A	AA	AB	BA	CA	BA	CA	BC	DB	DC	A	A	A	A	B	B	C	D	C	A	C	A	AA	AB	BB	BA	CA	BC	A						
		p' part	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AB	BA	CA	BA	CA	BC	DB	DC	A	A	A	A	A	A	A	A	A	A	A	A	AB	BB	BA	CA	BC	A						
	ind	1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	4E	4F	5A	5B	5C	6A	6B	6C	6D	6E	6F	6G	6H	6I	7A	7B	8A	8B	8C	8D	8E	8F	9A	9B	9C	10A	10B	10C	10D	10E	10F	11A						
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1						
x_2	+	276	20	12	-12	78	15	6	0	36	4	4	-4	0	0	21	6	1	14	-6	5	14	-1	2	-3	0	0	10	3	4	-4	8	0	0	0	6	0	3	5	-3	2	0	5	-2	1	x_2						
x_3	+	299	43	-13	11	65	20	2	-1	27	-5	11	3	-1	-1	14	9	-1	1	5	10	10	4	-2	2	-1	-1	5	5	-5	3	7	-1	3	-1	2	-1	5	-2	2	-3	3	3	1	2	x_3						
x_4	+	1771	-21	51	11	286	16	-11	7	91	-5	-5	3	7	-1	41	1	-4	30	6	-6	21	0	-3	2	3	-1	14	0	11	3	7	-1	-1	-1	7	-2	-2	9	1	1	-1	4	1	0	x_4						
x_5	+	8855	-105	15	55	935	35	-1	-7	231	7	-9	-1	-1	-5	85	0	5	39	15	-15	39	3	3	1	-3	1	21	0	-1	7	11	3	-1	-1	8	2	-4	5	5	0	0	5	0	0	x_5						
x_6	+	17250	610	90	66	285	141	-30	0	50	18	50	10	-6	6	0	25	0	-35	9	19	10	13	-2	3	0	0	-5	9	10	2	6	-2	6	2	-3	0	3	0	0	5	5	0	1	2	x_6						
x_7	+	27300	420	156	132	1794	48	30	0	308	20	20	12	0	0	90	10	0	66	30	30	30	0	6	6	0	0	14	0	20	12	8	0	0	0	3	0	6	10	6	6	0	0	2	-2	x_7						
x_8	+	37674	42	-78	-54	2925	90	9	0	602	-6	-6	2	-6	6	189	9	-1	45	-15	0	81	-6	-3	0	0	0	35	0	-14	-6	22	-2	-2	2	9	0	0	-3	-3	-3	-3	7	1	-1	x_8						
x_9	+	44275	755	-77	-77	2002	175	-5	1	307	-13	35	-13	7	-5	70	30	0	-46	-14	35	35	-1	-1	-5	1	1	0	7	-5	-5	19	3	-1	-1	-5	1	7	-10	-2	-2	0	0	-2	0	x_9						
x_{10}	+	80730	1626	-78	186	-351	216	54	0	42	10	74	34	6	6	-15	25	5	33	-15	6	6	24	6	6	0	0	-1	6	-6	2	-2	6	6	-2	0	0	0	1	-3	-3	1	1	1	1	x_{10}						
x_{11}	+	94875	155	235	-165	4785	15	-30	21	699	-5	-5	11	-1	-5	175	0	0	113	-35	5	50	-1	2	-3	1	-3	25	-3	19	-13	3	3	3	-1	6	3	0	15	-5	0	0	0	0	0	x_{11}						
x_{12}	+	313950	350	-26	318	8931	-33	75	21	910	-18	30	-10	14	10	160	-10	0	35	55	35	35	-1	-1	3	1	-3	14	0	-10	14	-6	2	-2	-2	3	3	3	0	4	-6	0	0	-2	-1	x_{12}						
x_{13}	+	345345	-511	377	33	13299	69	-39	-21	1617	-15	1	-23	1	5	350	-10	-5	179	-1	-19	89	5	-7	-3	-1	3	35	0	25	1	5	-3	1	1	6	-3	-3	14	2	2	4	-1	-2	0	x_{13}						
x_{14}	+	376740	3492	-364	100	-1638	441	9	36	132	-60	100	-12	0	-12	-35	40	-10	90	14	9	9	9	9	1	-4	4	0	7	-4	-4	-4	-4	4	0	0	0	0	-3	1	-4	2	2	0	1	x_{14}						
x_{15}	+	483000	-840	-600	120	14730	195	-30	0	1624	24	-40	8	0	0	300	0	0	-54	30	-45	90	3	6	3	0	0	14	0	-40	8	16	0	0	0	-3	0	-6	-20	0	0	0	0	0	1	x_{15}						
x_{16}	+	644644	3108	1092	484	5005	217	100	49	-28	36	20	68	28	4	14	-1	-6	141	21	15	-12	9	12	7	9	1	0	0	20	4	4	4	0	4	1	4	-2	-2	2	7	3	-2	-1	0	x_{16}						
x_{17}	o	673750	1750	350	-330	-1925	-35	235	-35	70	38	-10	-18	-14	10	0	0	0	-5	35	-5	-5	13	-5	3	5	-3	0	0	-10	-2	2	-6	-2	2	1	1	1	0	0	0	0	0	0	0	x_{17}						
x_{18}	o	673750	1750	350	-330	-1925	-35	235	-35	70	38	-10	-18	-14	10	0	0	0	-5	35	-5	-5	13	-5	3	5	-3	0	0	-10	-2	2	-6	-2	2	1	1	1	0	0	0	0	0	0	0	x_{18}						
x_{19}	+	822250	1770	-910	330	17875	-125	-35	43	1562	-6	10	2	-14	-10	225	0	0	-45	35	45	45	3	-3	-3	-1	3	16	-5	-46	10	-10	-2	2	2	1	-2	4	-15	5	0	0	0	0	0	x_{19}						
x_{20}	+	871884	972	1612	396	15795	0	0	27	972	12	-36	12	28	-4	134	9	9	243	55	0	0	0	0	0	7	3	6	-1	60	12	-4	-4	0	4	0	0	0	22	2	-3	-3	-3	1	2	x_{20}						
x_{21}	+	1434510	-882	246	-594	32967	0	81	0	2814	30	30	6	6	-6	435	-15	10	135	-69	0	81	0	9	0	0	0	21	0	6	-18	-6	2	2	-2	0	0	0	3	-9	1	3	-2	1	0	x_{21}						
x_{22}	+	1450449	4305	1001	-847	18018	252	144	63	609	65	1	-39	-7	5	84	14	-1	354	-70	30	-24	12	12	-10	-1	-1	0	0	25	-15	5	-3	-3	1	0	0	0	20	-4	6	0	-5	-2	0	x_{22}						
x_{23}	+	1771000	-3080	40	440	28600	295	70	28	2072	-40	-40	8	0	0	225	0	0	-200	40	-65	70	7	-2	-1	4	-4	-14	0	24	8	16	0	0	0	-11	1	-5	-15	5	0	0	0	0	0	x_{23}						
x_{24}	+	1821600	9120	352	-352	-792	1098	-90	-36	288	32	160	-32	0	0	-70	70	0	-24	-8	30	30	-6	-6	-10	4	-4	-10	11	0	0	0	0	0	0	0	0	0	10	2	2	0	0	-2	0	x_{24}						
x_{25}	+	2055625	7625	1625	-55	-3575	160	-290	85	105	105	25	57	13	25	0	0	0	-55	5	-10	-10	32	2	-10	5	5	5	-2	-15	1	1	1	-3	5	-2	-5	1	0	0	0	0	0	0	0	x_{25}						
x_{26}	+	2417415	5383	-1001	583	3003	231	-165	84	-217	-25	71	-73	-21	-9	35	-5	15	-5	7	-41	-5	7	19	7	4	4	0	0	-1	-1	-1	-1	7	-5	-3	3	0	3	-1	-1	-7	3	3	0	x_{26}						
x_{27}	o	2464749	-2835	-987	-243	-5103	0	0	0	189	93	45	21	21	-15	-21	-6	24	81	21	0	0	0	0	0	0	0	0	0	21	-3	1	-7	-3	5	0	0	0	-5	3	-2	0	0	2	1	x_{27}						
x_{28}	o	2464749	-2835	-987	-243	-5103	0	0	0	189	93	45	21	21	-15	-21	-6	24	81	21	0	0	0	0	0	0	0	0	0	21	-3	1	-7	-3	5	0	0	0	-5	3	-2	0	0	2	1	x_{28}						
x_{29}	+	2816856	10584	-1080	792	18954	729	0	0	504	-72	120	40	0	0	36	26	6	-54	54	81	0	9	0	9	0	0	-14	0	-40	8	16	0	0	0	0	0	0	4	0	-10	4	-6	2	-2	x_{29}						
x_{30}	+	2877875	1715	715	-429	20735	701	-55	35	1155	35	-45	-5	-1	-1	0	25	0	-385	-41	11	65	-19	5	3	-5	3	-35	0	35	-5	23	-1	-5	-1	-10	-1	-1	0	0	5	-5	0	1	0	x_{30}						
x_{31}	+	4100096	4096	0	0	64064	56	-16	-64	4096	0	0	0	0	0	476	16	-4	64	0	64	64	-8	-8	0	0	0	0	-7	0	0	0	0	0	0	-7	-1	5	-4	0	0	-4	-4	0	0	x_{31}						
x_{32}	+	5494125	19565	1365	1101	-1365	966	165	0	-35	61	205	37	21	9	0	25	0	-85	-21	-4	5	38	-19	12	0	0	0	0	5	-3	-7	1	5	5	3	0	-3	0	0	5	-5	0	1	-1	x_{32}						
x_{33}	+	5801796	-4284	-364	836	-9009	441	90	63	420	36	-76	-12	-28	-4	-84	1	21	207	-49	-9	18	9	-18	-1	-1	-1	0	0	-28	4	-4	12	0	-4	0	0	0	-4	-4	1	1	1	1	0	x_{33}						
x_{34}	+	7628985	-1863	1001	441	88452	0	0	-27	4185	-39	9	9	-7	1	385	-15	-15	324	56	0	0	0	0	0	-7	-3	0	7	9	9	-15	1	-3	1	0	0	0	17	1	1	-3	-3	1	1	x_{34}						
x_{35}	+	9221850	-5670	-1598	-198	82863	0	0	27	3402	42	90	18	-14	-10	230	-30	0	-81	-41	0	0	0	0	0	7	3	-6	8	-30	-6	-10	-2	-6	2	0	0	0	-10	2	2	0	0	2	0	x_{35}						
x_{36}	+	9669660	28	364	924	93093	-210	-174	21	3836	-4	-20	-52	-28	4	280	15	10	229	49	-50	-14	-2	10	6	1	-3	0	0	12	12	-12	4	0	-4	-3	3	0	8	4	-1	3	-2	-1	0	x_{36}						
x_{37}	+	12432420	8484	2028	2244	61776	54	54	0	1428	52	20	-4	0	24	15	-25	20	-48	120	114	6	6	6	-6	0	0	-14	0	36	12	0	8	0	0	0	0	0	-1	3	3	-1	4	-1	0	x_{37}						
x_{38}	+	16347825	35505	-351	945	-18954	1458	0	0	657	-111	225	65	-27	-15	-90	40	0	54	-54	0	0	18	0	0	0	0	11	-3	25	9	-7	-7	-3	-3	0	0	0	-10	-6	4	0	0	0	-1	x_{38}						
x_{39}	+	20083140	19908	-924	-1980	-24948	1701	405	0	1092	68	36	-92	0	12	-210	40	15	396	84	-9	45	-27	9	-9	0	0	0	0	-20	-4	-12	4	-4	0	0	0	0	-2	6	-4	-2	3	0	0	x_{39}						
x_{40}	+	21049875	2835	-2325	-1485	160380	0	0	0	5859	3	-45	27	15	15	300	0	0	-324	-120	0	0	0	0	0	0	0	-21	0	-45	-21	-9	-1	3	-1	0	0	0	-20	0	0	0	0	0	0	x_{40}						
x_{41}	+	21528000	-6720	4160	-320	138060	90	-180	36	3264	-64	0	64	0	0	175	0	0	588	20	-60	-60	-6	-24	4	-4	4	-10	4	64	0	0	0	0	0	0	0	0	15	-5	0	0	0	0	-1	x_{41}						
x_{42}	+	21579129	-7047	-2223	1881	0	0	0	162	-567	297	9	-63	-27	-11	99	9	-21	0	0	0	0	0	0	0	6	-6	-9	-2	9	-15	-3	5	-3	-3	0	0	0	3	7	-3	3	3	1	0	x_{42}						
x_{43}	+	23244375	-6825	2975	55	-23100	-105	345	105	455	167	55	47	35	-5	0	0	0	-60	140	-15	-15	-9	-3	1	5	1	0	0	-25	-17	3	-5	-1	3	3	6	0	0	0	0	0	0	0	0	x_{43}						
x_{44}	+	24174150	9030	5278	1254	57057	798	-30	21	-938	118	70	-50	14	10	70	-20	0	609	49	-30	-30	-18	-6	-6	1	-3	0	0	30	6	10	2	6	-2	-3	-6	0	-10	-2	8	0	0	4	0	x_{44}						
x_{45}	+	24667500	27500	-1300	2860	-10725	-15	300	105	-20	44	60	44	-28	-20	0	0	0	155	-85	-25	20	17	32	7	5	1	-10	-3	-20	-4	-4	-4	0	-4	3	-3	0	0	0	0	0	0	0	0	x_{45}						
x_{46}	+	24794000	28560	560	3344	40040	854	440	-70	-560	80	80	112	0	16	0	-25	0	360	56	54	0	6	12	14	-10	2	0	0	0	0	0	0	0	0	-1	2	-4	0	0	-5	5	0	-1	0	x_{46}						
x_{47}	+	25900875	-11445	3835	715	32175	1260	315	-63	1323	-85	-165	27	-41	-5	-225	0	0	-465	55	-60	75	12	3	4	1	1	-21	0	51	3	3	3	-1	-1	0	0	0	15	-5	0	0	0	0	0	x_{47}						
x_{48}	+	31574400	-5760	3328	1408	140712	-300	60	84	2176	128	0	0	0	0	20	30	0	552	16	-60	-60	-12	0	4	4	4	4	-3	0	0	0	0	0	0	-3	-6	0	20	8	-2	0	0	-2	0	x_{48}						
x_{49}	+	40166280	7560	2520	-1848	187110	-567	324	0	4200	-88	-24	-8	0	0	0	10	5	-90	-126	45	-36	9	0	-3	0	0	0	0	40	-8	-16	0	0	0	0	0	0	0	0	-10	0	5	2	0	x_{49}						
x_{50}	+	40370176	0	8192	0	22528	-512	640	112	0	0	0	0	0	0	-64	16	-24	0	128	0	0	0	0	0	8	0	14	0	0	0	0	0	0	0	-8	4	-2	0	-8	-8	0	0	0	0	x_{50}						
x_{51}	+	44013375	-16065	-1665	639	189540	729	0	0	4095	63	15	-65	27	-9	0	-25	0	-540	36	-81	0	9	0	-9	0	0	-35	0	-25	7	15	-1	3	3	0	0	0	0	0	-5	5	0	-1	-1	x_{51}						
x_{52}	+	46621575	11655	-6201	-825	138996	-567	-405	0	2247	7	-105	7	27	15	-90	40	0	-396	36	-45	-45	9	-9	3	0	0	21	0	-65	-1	-9	7	-5	3	0	0	0	-10	-6	4	0	0	0	0	x_{52}						
x_{53}	+	51571520	33600	0	-3520	40040	1610	656	56	-1344	-64	64	-128	0	0	140	0	-5	-24	0	30	-24	-6	12	-10	0	8	0	0	0	0	0	0	0	0	8	2	-4	-20	0	0	0	-5	0	0	x_{53}						
x_{54}	+	55255200	61600	-4576	-352	27456	2022	-570	-84	-1120	-96	160	32	0	0	160	-10	0	-320	-40	-50	-50	22	-14	-10	-4	-4	14	0	0	0	0	0	0	0	-3	-3	-3	0	4	-6	0	0	-2	0	x_{54}						
x_{55}	+	57544344	8856	-936	792	208494	-729	0	0	4248	24	120	-40	0	-24	-126	-11	-6	-594	90	81	0	-9	0	9	0	0	-10	-3	40	8	-8	-8	0	0	0	0	0	-14	-6	9	1	6	-3	0	x_{55}						
x_{56}	+	60435375	-8785	-3185	3311	210210	-84	330	-105	2415	47	-65	15	7	-1	0	25	0	290	154	-34	-70	-4	26	-10	-5	-1	0	0	-65	15	-1	-1	3	-1	6	3	0	0	0	5	-5	0	1	0	x_{56}						
x_{57}	+	65270205	189	-3003	-99	-81081	1701	-324	0	2541	-179	-195	85	21	9	-315	25	-20	711	-105	9	36	-27	0	9	0	0	0	0	-11	13	-7	1	-3	5	0	0	0	-11	-3	-3	-1	4	1	0	x_{57}						
x_{58}	+	66602250	51210	4290	1386	-57915	1701	-405	0	1050	250	90	-14	6	6	0	25	0	-315	69	9	-45	-27	-9	9	0	0	15	-6	10	-14	-2	6	-2	-2	0	0	0	0	0	5	5	0	1	0	x_{58}						
x_{59}	+	77702625	-34335	-975	2145	-19305	1755	-270	0	1281	129	-175	17	21	-15	-300	0	0	279	15	-45	90	27	-18	3	0	0	-14	0	-55	-7	-7	-7	-3	-3	0	0	0	20	0	0	0	0	0	0	x_{59}						
x_{60}	+	83720000	59200	0	320	-63700	560	20	20	960	-64	0	128	-56	0	0	0	0	-20	0	-20	-20	16	16	-4	0	-4	0	-7	0	0	0	0	0	8	2	2	2	0	0	0	0	0	0	1	x_{60}						
x_{61}	+	85250880	39744	7488	2112	138996	-1134	-324	0	1344	64	0	64	0	0	-225	10	-20	-396	36	36	36	18	0	-12	0	0	-6	-6	64	0	0	0	0	0	0	0	0	-1	3	-2	4	4	2	0	x_{61}						
x_{62}	+	91547820	53676	4212	-1332	18954	729	0	0	-1764	60	60	4	0	0	225	-10	-30	-54	54	-81	0	9	0	-9	0	0	-14	0	-4	4	-8	0	0	0	0	0	0	1	-3	2	-4	6	-2	1	x_{62}						
x_{63}	+	100725625	15225	6825	-2695	275275	595	-170	-140	665	25	-55	-55	-7	-15	0	0	0	1035	-105	45	-90	3	6	5	0	-4	0	0	25	-7	17	1	-3	1	1	4	-2	0	0	0	0	0	0	0	x_{63}						
x_{64}	+	106142400	-25920	11648	1728	88452	0	0	216	-1728	-192	0	0	56	0	70	30	0	324	56	0	0	0	0	0	-4	0	0	-7	0	0	0	0	0	-8	0	0	0	-10	-2	-2	0	0	-2	1	x_{64}						
x_{65}	+	109882500	32900	-9100	1540	238875	735	330	105	420	-92	-140	20	-28	-20	0	0	0	155	35	65	-70	-1	-22	1	5	1	0	0	-60	4	12	-4	0	-4	6	-3	-3	0	0	0	0	0	0	2	x_{65}						
x_{66}	+	150732800	0	4096	0	226304	512	-640	104	0	0	0	0	64	0	-160	-40	0	0	64	0	0	0	0	0	4	0	1	8	0	0	0	0	0	0	8	-4	2	0	-4	-4	0	0	0	-2	x_{66}						
x_{67}	+	163478250	-25110	-1950	-3510	331695	0	0	0	2106	-102	90	18	-6	30	-300	0	0	-81	-105	0	0	0	0	0	0	0	-9	-9	-30	-6	14	6	6	2	0	0	0	20	0	0	0	0	0	1	x_{67}						
x_{68}	+	184184000	38080	4160	3520	14300	-1090	-280	-28	448	192	0	64	0	0	-225	0	0	220	20	40	40	-2	-8	-8	-4	4	14	0	-64	0	0	0	0	0	-10	-1	-1	15	-5	0	0	0	0	0	x_{68}						
x_{69}	+	185912496	34992	11088	1584	0	0	0	162	-1296	-144	-144	144	0	16	126	-9	21	0	0	0	0	0	0	0	6	-6	0	7	0	0	0	0	0	0	0	0	0	-18	-2	3	-3	-3	-1	0	x_{69}						
x_{70}	+	185955000	35000	4200	3960	-150150	-525	1410	0	2520	152	-40	72	0	0	0	0	0	-70	-210	-25	-70	-13	-10	-9	0	0	0	0	-40	8	16	0	0	0	6	0	3	0	0	0	0	0	0	0	x_{70}						
x_{71}	+	191102976	0	-4096	0	248832	0	0	216	0	0	0	0	-64	0	-224	-24	-24	0	-64	0	0	0	0	0	-4	0	15	8	0	0	0	0	0	0	0	0	0	0	4	4	0	0	0	-2	x_{71}						
x_{72}	+	201451250	75250	-4550	-3630	-125125	-700	470	35	1890	-190	-110	-54	14	-10	0	0	0	-5	175	-50	-50	52	10	6	-5	3	0	0	50	-6	14	6	-6	-2	2	-1	5	0	0	0	0	0	0	0

$Co_1 = F_{2-}$

	e 138 240 AA 12A	e 9 216 AB 12B	e 4 608 AD 12C	e 2 592 FA 12D	e 1 728 EA 12E	e 864 BE AE 12F	e 576 EB BB 12G	e 288 DC CC 12H	e 288 EC BC 12I	e 288 ED BD 12J	e 288 FB CB 12K	e 72 HE DE 12L	e 48 IF DF 12M	e 156 A A 13A	e 168 AB AB 14A	e 56 BA BA 14B	e 5 400 AA AA 15A	e 900 BA BA 15B	e 180 AD AD 15C	e 90 BB BB 15D	e 30 CC CC 15E	e 64 C A 16A	e 64 D A 16B	e 216 AF AA 18A	e 54 BF BA 18B	e 54 CF CA 18C	e 480 AA AA 20A	e 40 FF BF 20B	e 20 EC CC 20C	e 252 AA AA 21A	e 63 AB AB 21B	e 21 BD BD 21C	e 22 AC AC 22A	e 23 A A 23A	e 23 A A B**	e 576 AA AA 24A	e 192 BC AC 24B	e 144 EA BA 24C	e 96 CF AF 24D	e 48 EB BB 24E	e 24 HE CE 24F	e 52 AB AB 26A	e 28 BA BA 28A	e 28 AE AE 28B	e 120 AAA AAA 30A	e 60 BCB BCB 30B	e 60 CBH CBH 30C	e 30 DDC DDC 30D	e 30 EED EED 30E	e 33 AA AA 33A	e 35 AA AA 35A	e 36 AD AD 36A	e 39 AA AA 39A	e 39 AA AA B**	e 40 AA AA 40A	e 84 AAB AAB 42A	e 60 AAA AAA 60A	
X_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	X_1
X_2	6	-2	2	0	3	0	1	-2	1	-1	4	0	0	3	-2	-1	3	3	0	0	1	-2	2	2	2	-1	1	0	-1	1	1	0	-1	0	0	-2	2	1	0	-1	0	-1	1	0	-1	-1	0	0	-1	1	0	0	0	0	-1	1	1	X_2
X_3	9	1	-3	0	0	-1	-2	2	2	0	4	-1	-1	0	1	1	5	0	-1	0	2	1	1	-2	1	1	2	-1	1	2	-1	-1	0	0	0	1	1	-2	-1	0	0	0	-1	-1	1	0	-1	0	0	-1	0	0	0	0	0	-2	-1	X_3
X_4	-2	-2	6	1	4	-2	-2	1	-2	0	1	1	-1	3	2	0	-4	1	2	1	-1	1	1	3	0	0	1	-1	0	-1	2	0	0	0	0	2	-2	2	2	0	-1	-1	0	0	0	1	-2	-1	1	0	-1	1	0	0	1	-1	-2	X_4
	-9	7	-1	-3	3	-1	1	3	-3	-1	1	-1	1	2	1	0	-5	0	-2	0	-1	1	1	0	0	0	1	0	1	-3	0	0	0	0	0	-1	-1	-1	-1	1	-1	2	0	-1	-1	0	2	0	-1	0	1	0	-1	-1	-1	1	1	X_5
X_6	5	-3	1	-4	5	3	3	2	-1	1	0	0	0	-1	-1	1	0	-5	0	1	0	0	0	1	-2	1	0	1	0	-2	1	0	0	0	0	1	-3	1	-1	-1	0	-1	1	1	0	-1	0	-1	0	-1	0	-1	-1	-1	0	2	0	X_6
X_7	26	2	6	2	-4	0	2	2	2	0	2	0	0	0	2	0	9	4	0	-2	0	2	-2	3	0	0	-2	0	0	2	-1	0	0	-1	-1	2	2	2	0	0	0	0	0	0	1	0	0	0	0	1	-1	-1	0	0	0	2	1	X_7
X_8	5	-3	-7	-1	2	3	0	-3	0	2	3	0	0	0	-1	0	0	0	0	0	-1	0	0	-3	0	0	-3	1	-1	-1	-1	0	1	0	0	1	1	-2	-1	0	1	0	0	1	0	0	0	0	1	-1	0	-1	0	0	1	-1	0	X_8
X_9	34	2	2	1	-5	-2	-1	-1	-1	-1	5	1	1	-3	0	-1	7	-3	1	0	0	-1	-1	-1	-1	-1	2	0	0	0	0	1	0	0	0	-2	-2	1	2	1	-1	1	-1	0	-1	1	1	0	0	0	0	1	0	0	0	0	-1	X_9
X_{10}	-15	1	1	6	0	-3	-2	2	2	4	-2	0	0	0	-1	2	-6	4	0	1	-1	0	0	0	0	0	-3	1	-1	-1	-1	0	-1	0	0	-3	1	0	1	2	0	0	0	-1	-2	0	0	1	1	1	-1	0	0	0	-1	-1	0	X_{10}
X_{11}	9	1	5	6	3	-1	1	-2	1	-1	-2	-1	1	1	-3	1	-5	0	1	0	0	-1	-1	2	-1	2	-1	0	0	4	1	0	0	0	0	1	-3	1	-1	-1	0	1	-1	-1	3	0	1	0	0	0	0	0	1	1	-1	0	-1	X_{11}
X_{12}	-5	3	-1	1	7	5	3	3	3	-1	-3	-1	1	0	2	0	16	-4	1	2	0	0	0	-1	-1	-1	0	0	0	-1	2	0	-1	0	0	-1	3	-1	1	-1	1	0	0	0	0	0	1	0	0	-1	-1	1	0	0	0	-1	0	X_{12}
X_{13}	-21	3	7	-3	-3	1	-3	1	1	1	-3	1	-1	0	-1	0	-1	-1	-1	-1	1	-1	-1	2	-1	-1	2	0	1	-1	-1	0	0	0	0	-5	-1	1	1	1	1	0	0	1	-1	-1	-1	1	-1	0	0	0	0	0	0	-1	-1	X_{13}
X_{14}	-30	-6	6	-3	-3	0	-3	1	1	-3	-3	0	0	0	0	-1	-8	7	1	1	-1	0	0	0	0	0	-3	-2	0	0	0	1	1	0	0	2	2	-1	0	-1	1	0	-1	0	0	-1	1	-1	-1	1	0	0	0	0	1	0	0	X_{14}
X_{15}	-14	-6	-10	4	-5	0	3	2	-1	-1	0	0	0	-2	2	0	-15	0	0	0	0	0	0	-3	0	0	4	0	0	2	-1	0	-1	0	0	2	-2	-1	0	-1	0	-2	0	0	1	0	0	0	0	1	-1	1	1	1	0	2	1	X_{15}
X_{16}	29	-3	5	8	5	1	3	-4	-1	5	0	1	1	0	0	0	5	5	-1	2	0	0	0	-3	0	0	2	-1	0	0	0	0	0	0	0	5	1	-1	1	1	0	0	0	0	1	1	-1	0	-2	0	0	-1	0	0	0	0	-1	X_{16}
X_{17}	-5	11	3	7	1	-5	-1	-1	-1	-3	-1	1	1	-1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	b23	**	-1	-1	-1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	X_{17}
X_{18}	-5	11	3	7	1	-5	-1	-1	-1	-3	-1	1	1	-1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	**	b23	-1	-1	-1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	X_{18}
X_{19}	35	3	-13	5	11	-5	-3	1	1	-1	-3	1	-1	0	0	-1	0	0	3	0	0	0	0	-3	0	0	-3	0	0	4	1	1	0	0	0	-1	-1	-1	-1	1	-1	0	1	0	0	0	-1	0	0	0	1	-1	0	0	-1	0	0	X_{19}
X_{20}	27	3	15	0	0	1	0	0	0	0	0	1	-1	0	2	-1	-10	0	2	0	0	0	0	0	0	0	2	1	-1	3	0	-1	0	0	0	3	-1	0	1	0	0	0	-1	0	-2	0	2	0	0	-1	1	0	0	0	0	-1	2	X_{20}
X_{21}	-33	-9	3	-3	0	-3	0	-3	0	0	-3	0	0	-1	1	0	12	-3	0	0	1	0	0	0	0	0	-1	-1	0	-3	0	0	0	0	0	3	3	0	1	0	-1	-1	0	-1	0	1	0	0	1	0	1	0	-1	-1	1	1	2	X_{21}
X_{22}	66	2	-6	6	0	2	2	4	-2	0	2	-1	-1	0	0	0	3	8	3	2	-1	-1	-1	0	0	0	4	0	1	0	0	0	0	0	0	-2	2	-2	-2	0	0	0	0	0	-1	0	-1	0	1	0	0	0	0	0	0	0	1	X_{22}
X_{23}	-10	8	8	2	-1	0	-1	2	-1	-1	2	0	0	-3	-2	0	0	0	3	0	0	0	0	1	1	1	-3	0	0	-2	1	0	0	0	0	0	4	3	0	-1	0	1	0	0	0	0	-1	0	0	0	1	-1	0	0	-1	-2	0	X_{23}
X_{24}	-24	8	-8	0	6	0	2	-2	-2	-2	-4	0	0	1	2	-1	-7	-2	-1	-2	0	0	0	0	0	0	-2	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	0	1	2	-1	0	0	0	0	0	1	1	0	-1	1	X_{24}
X_{25}	-15	9	-3	-12	0	-5	6	-2	-2	0	0	1	1	0	1	2	0	0	0	0	0	1	1	2	-1	-1	0	0	0	2	-1	1	0	0	0	-3	1	0	-1	-2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	X_{25}
X_{26}	35	11	-1	-1	-1	-3	-1	-1	-1	-1	-1	0	0	0	0	0	8	-2	-1	1	0	-1	-1	1	1	-2	3	1	1	0	0	0	0	0	0	-1	-1	-1	1	-1	1	0	0	0	0	2	-1	-1	0	0	0	-1	0	0	-1	0	0	X_{26}
X_{27}	9	-15	-3	0	0	3	0	0	0	0	0	0	0	1	0	0	-3	-3	0	0	0	-1	-1	0	0	0	-1	0	0	0	0	0	-1	0	0	-3	1	0	-1	0	0	1	0	0	1	1	0	0	0	1	0	0	b39	**	1	0	-1	X_{27}
X_{28}	9	-15	-3	0	0	3	0	0	0	0	0	0	0	1	0	0	-3	-3	0	0	0	-1	-1	0	0	0	-1	0	0	0	0	0	-1	0	0	-3	1	0	-1	0	0	1	0	0	1	1	0	0	0	1	0	0	**	b39	1	0	-1	X_{28}
X_{29}	66	-6	-2	0	-3	0	-3	0	-3	1	0	0	0	3	-2	0	9	-1	0	-1	0	0	0	0	0	0	4	0	0	-2	1	0	0	0	0	2	-2	-1	0	-1	0	-1	0	0	1	-1	0	1	0	1	1	0	0	0	0	-2	1	X_{29}
X_{30}	15	-1	7	3	9	-1	-1	-3	3	1	-1	-1	-1	0	1	0	0	-5	0	1	0	1	1	2	-1	-1	0	-1	0	1	1	0	0	0	0	-1	-1	-1	-1	1	1	0	0	-1	0	-1	0	1	0	0	0	0	0	0	0	1	0	X_{30}
X_{31}	64	0	0	-8	-8	0	0	0	0	0	0	0	0	0	0	1	-1	4	-4	1	-1	0	0	1	1	1	-4	0	0	0	0	-1	0	1	1	0	0	0	0	0	0	0	1	0	-1	0	0	-1	-1	0	0	1	0	0	0	0	-1	X_{31}
X_{32}	-5	-5	-5	1	-2	3	4	1	4	-2	1	0	0	0	0	0	0	-5	0	1	0	-1	-1	-1	2	-1	0	-1	0	0	0	0	1	0	0	-1	-1	2	-1	0	-1	0	0	0	0	-1	0	1	0	-1	0	1	0	0	0	0	0	X_{32}
X_{33}	15	15	15	-12	-3	-1	3	2	-1	-3	0	-1	-1	0	0	0	6	1	3	1	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	-1	-1	-1	-1	1	0	0	0	0	2	1	-1	1	-2	0	0	0	0	0	2	0	0	X_{33}
X_{34}	-36	-12	0	0	0	2	0	0	0	0	0	-1	1	0	0	-1	7	-3	-2	0	0	1	1	0	0	0	5	1	-1	0	0	1	1	0	0	0	0	0	-2	0	0	0	-1	0	-1	1	-2	0	0	1	0	0	0	0	-1	0	-1	X_{34}
X_{35}	-81	15	-9	0	0	-5	0	0	0	0	0	1	-1	1	-2	0	23	3	2	0	0	0	0	0	0	0	2	0	0	-3	0	-1	0	0	0	3	-1	0	-1	0	0	1	0	0	-1	-1	2	0	0	0	-1	0	1	1	0	1	-1	X_{35}
X_{36}	29	5	-7	2	2	-1	2	-2	-2	2	2	-1	1	0	0	0	-17	3	1	0	1	0	0	1	1	-2	-4	-1	0	0	0	0	0	0	0	-3	-3	0	-1	0	0	0	0	0	-1	-1	1	0	1	0	0	-1	0	0	2	0	-1	X_{36}
X_{37}	0	16	8	6	-6	0	-2	2	2	2	-2	0	0	0	-2	0	6	-4	0	-1	-1	-2	2	0	0	0	3	-1	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	-1	1	0	1	0	0	0	1	1	0	X_{37}
X_{38}	-66	6	2	0	-6	0	0	0	0	2	0	0	0	0	-1	1	-9	1	0	-2	0	1	1	0	0	0	2	0	0	2	2	0	-1	0	0	-2	2	-2	0	0	0	0	-1	1	-1	1	0	0	0	-1	1	0	0	0	0	2	-1	X_{38}
X_{39}	-36	-4	4	3	-3	0	-1	-3	3	1	-1	0	0	-1	0	0	-3	7	0	1	0	0	0	0	0	0	2	2	1	0	0	0	0	0	0	4	0	1	0	-1	-1	-1	0	0	1	-1	0	1	0	0	0	0	-1	-1	0	0	-1	X_{39}
X_{40}	36	12	0	0	0	6	0	0	0	0	0	0	0	2	-1	0	-15	0	0	0	0	1	1	0	0	0	4	0	0	3	0	0	0	-1	-1	0	0	0	2	0	0	2	0	1	1	0	0	0	0	0	-1	0	-1	-1	0	-1	1	X_{40}
X_{41}	-36	-4	4	6	-6	0	-4	0	0	-2	2	0	0	0	2	0	-5	0	1	0	0	0	0	0	0	0	-1	0	0	-1	-1	1	-1	0	0	4	0	-2	0	0	0	0	2	0	3	0	1	0	0	-1	0	0	0	0	-1	-1	-1	X_{41}
X_{42}	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	3	2	0	0	-3	0	0	-1	-1	0	0	0	3	-1	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	-1	0	0	X_{42}
X_{43}	20	20	-4	-13	-1	8	-7	1	1	-1	-1	-1	1	-2	0	0	0	0	0	0	0	1	1	3	0	0	0	0	0	0	0	0	0	0	0	-4	0	-1	0	1	-1	-2	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	X_{43}
X_{44}	49	1	1	-2	10	5	-2	-2	-2	-2	-2	-1	1	0	0	0	7	7	1	-2	0	0	0	-3	0	0	2	0	0	0	0	0	0	0	0	-3	1	0	1	0	0	0	0	0	-1	-1	1	0	0	0	0	1	0	0	0	0	-1	X_{44}
X_{45}	-5	11	11	-2	1	-1	-1	0	3	5	2	-1	1	0	2	-3	0	0	0	0	0	0	0	-1	-1	2	0	0	0	-1	-1	0	0	0	0	-5	-1	1	-1	-1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	X_{45}
X_{46}	40	8	-8	-2	10	0	2	-4	2	-2	2	0	-2	-3	0	0	0	5	0	-1	0	0	0	3	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	-1	0	0	0	1	0	0	0	0	0	X_{46}
X_{47}	15	-1	-9	-9	12	-5	-4	3	0	0	-1	1	1	0	-1	0	0	0	-3	0	0	-1	-1	0	0	0	3	0	0	3	0	0	0	0	0	3	3	0	-1	0	-1	0	0	1	0	0	1	0	0	0	-1	0	0	0	1	-1	0	X_{47}
X_{48}	-8	8	0	-2	4	0	-4	0	0	0	2	0	0	0	-4	1	2	-3	-1	0	0	0	0	-3	0	0	-4	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	-1	0	2	1	-1	0	0	0	-1	1	0	0	0	2	2	X_{48}
X_{49}	30	-10	10	6	9	0	5	0	-3	1	2	0	0	-2	0	0	0	-5	0	-2	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-2	2	1	0	1	0	-2	0	0	0	-1	0	0	-1	0	0	0	1	1	0	0	0	X_{49}
X_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	8	-2	2	-2	0	0	0	0	0	0	0	0	0	2	-1	0	0	1	1	0	0	0	0	0	0	2	0	0	0	-2	-2	0	0	0	-1	0	-1	-1	0	2	0	X_{50}
X_{51}	-60	-12	4	0	-3	0	3	0	3	1	0	0	0	3	1	0	0	5	0	-1	0	-1	-1	0	0	0	0	1	0	1	1	0	1	0	0	-4	0	-1	0	1	0	-1	0	-1	0	1	0	-1	0	-1	0	0	0	0	0	1	0	X_{51}
X_{52}	84	4	4	-3	9	0	-5	3	3	1	1	0	0	0	1	0	-9	1	0	-2	0	-1	-1	0	0	0	2	0	0	-3	0	0	0	0	0	4	0	1	0	-1	1	0	0	-1	-1	1	0	0	0	0	1	0	0	0	0	1	-1	X_{52}
X_{53}	24	8	16	6	6	0	2	4	-2	-2	2	0	0	0	0	0	5	0	-4	0	1	0	0	0	0	0	-4	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	-1	X_{53}
X_{54}	80	0	8	-4	2	0	-6	-2	-2	2	0	0	0	0	2	0	16	-4	1	2	0	0	0	1	1	1	0	0	0	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	-1	0	0	0	2	0	X_{54}
X_{55}	6	-18	2	0	3	0	-3	0	-3	-1	0	0	0	0	2	1	9	4	0	1	0	0	0	0	0	0	-2	1	0	-1	-1	0	0	0	0	-2	-2	1	0	-1	0	0	-1	0	1	0	0	1	0	0	0	0	0	0	-1	1	X_{55}
X_{56}	-30	2	-6	-6	0	-2	2	-2	-2	0	2	1	-1	0	0	0	0	-5	0	1	0	-1	-1	2	-1	2	0	-1	0	0	0	0	0	0	0	-2	2	-2	2	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	X
X_{57}	-9	7	7	-6	-3	3	1	0	-3	1	-2	0	0	0	0	0	9	4	0	1	1	-1	-1	0	0	0	1	-1	0	0	0	0	0	0	0	-5	-1	1	-1	1	0	0	0	0	1	0	0	-1	1	0	0	0	0	0	-1	0	1	X_{57}
X_{58}	-75	-11	-11	-3	-3	-3	1	3	-3	-1	1	0	0	0	-1	-2	0	-5	0	1	0	0	0	0	0	0	0	0	1	0	3	0	0	0	0	1	1	1	1	1	1	0	0	-1	0	-1	0	-1	0	0	0	0	0	0	0	-1	0	X_{58}
X_{59}	39	-9	-1	12	3	3	3	2	-1	-1	0	0	0	0	-2	0	15	0	0	0	0	1	1	0	0	0	-4	0	0	1	-2	0	0	0	0	-1	-1	-1	3	-1	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	1	-1	X_{59}
X_{60}	-60	-4	8	6	0	-2	-4	0	0	-4	2	-2	0	0	0	1	0	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	-1	1	0	0	0	0	0	2	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	X_{60}
X_{61}	84	4	4	-6	-6	0	4	0	0	-2	-2	0	0	0	-2	-2	-9	1	0	1	1	0	0	0	0	0	-1	0	0	-3	0	0	0	0	0	4	0	-2	0	0	0	0	0	0	-1	1	0	1	1	0	-1	0	0	0	-1	1	-1	X_{61}
X_{62}	66	-6	-2	0	-3	0	3	0	3	1	0	0	0	0	-2	0	9	-1	0	-1	0	2

$Co_1 = F_{2-}$

		1A	2A	2B	2C	3A	3B	3C	3D	4A	4B	4C	4D	4E	4F	5A	5B	5C	6A	6B	6C	6D	6E	6F	6G	6H	6I	7A	7B	8A	8B	8C	8D	8E	8F	9A	9B	9C	10A	10B	10C	10D	10E	10F	11A	
χ_{86}	+	299710125	-28755	6565	1485	347490	0	0	-216	-1539	-99	45	21	13	-15	-175	0	0	162	-50	0	0	0	0	0	4	0	15	1	21	-3	1	-7	-3	-3	0	0	0	-15	5	0	0	0	0	0	χ_{86}
χ_{87}	+	302176875	875	2275	4235	150150	-105	300	-105	-1925	-101	75	-45	7	-25	0	0	0	-250	70	-115	20	-25	-16	5	-5	-1	0	0	-5	-13	-1	7	3	7	3	3	3	0	0	0	0	0	0	0	χ_{87}
χ_{88}	+	309429120	-27776	-11648	1408	24024	1344	-276	84	-2688	-128	0	128	0	0	350	-10	20	-296	-56	-44	-44	-32	16	4	4	4	0	0	0	0	0	0	0	0	-6	3	3	14	2	2	4	4	-2	0	χ_{88}
χ_{89}	+	326956500	-15660	-2340	-396	189540	729	0	0	-1980	228	-60	-20	0	24	0	-25	0	-540	36	81	0	9	0	9	0	0	10	3	20	-4	0	8	0	0	0	0	0	0	0	-5	-5	0	-1	2	χ_{89}
χ_{90}	+	360062976	-57344	0	0	-139776	336	912	0	0	0	0	0	0	0	336	16	26	-512	0	64	-80	16	-32	0	0	0	0	0	0	0	0	0	0	0	-6	0	-3	16	0	0	-4	6	0	-2	χ_{90}
χ_{91}	+	387317700	3780	-4356	1188	-208494	-1458	0	0	2772	180	180	-20	0	0	-90	-10	0	594	-90	0	0	-18	0	0	0	0	-14	0	20	12	8	0	0	0	0	0	0	-10	-6	-6	0	0	-2	0	χ_{91}
χ_{92}	+	402902500	60900	-9100	3300	-25025	-770	670	-35	-700	-316	20	-44	28	-20	0	0	0	255	35	30	30	-18	6	6	5	-3	0	0	20	-12	-4	12	0	4	-5	-8	-2	0	0	0	0	0	0	0	χ_{92}
χ_{93}	+	464257024	0	4096	0	-146432	-1280	-1280	-56	0	0	0	0	-64	0	224	24	24	0	64	0	0	0	0	0	4	0	-35	0	0	0	0	0	0	0	16	-2	-2	0	-4	-4	0	0	0	0	χ_{93}
χ_{94}	+	469945476	-56700	4004	-1980	-243243	0	0	-189	3780	-252	180	36	-28	20	-224	-24	-24	405	161	0	0	0	0	0	11	3	0	0	-60	-12	4	4	0	-4	0	0	0	0	4	4	0	0	0	0	χ_{94}
χ_{95}	+	469945476	40068	9828	-8316	0	0	729	0	-252	132	-60	36	0	-36	-189	-9	26	0	0	0	81	0	-27	0	0	0	0	0	-36	12	-12	4	4	0	0	0	0	3	3	3	3	-2	-1	0	χ_{95}
χ_{96}	+	483483000	-16520	-3640	440	60060	-105	210	-84	-2856	24	-40	-24	0	0	175	0	0	28	140	55	10	7	22	-1	-4	-4	0	0	-56	-8	0	-16	0	0	-6	-3	0	15	-5	0	0	0	0	0	χ_{96}
χ_{97}	+	502078500	13860	-1260	-924	187110	-1134	405	0	-2940	68	-60	-140	0	36	0	25	0	-90	-126	-36	45	18	9	12	0	0	0	0	-20	12	-4	-4	-4	0	0	0	0	0	0	5	5	0	1	0	χ_{97}
χ_{98}	+	503513010	-19278	3354	594	-104247	0	0	0	1890	-126	-126	42	6	6	-195	15	-15	-567	-111	0	0	0	0	0	0	0	21	0	66	-6	-2	-10	6	-2	0	0	0	-3	9	-1	-3	-3	-1	0	χ_{98}
χ_{99}	+	504627200	0	-4096	0	-123904	-1792	-640	56	0	0	0	0	64	0	160	40	0	0	-64	0	0	0	0	0	-4	0	-21	0	0	0	0	0	0	0	8	2	-4	0	4	4	0	0	0	0	χ_{99}
χ_{100}	+	522161640	1512	2184	-792	-162162	1701	324	0	168	-88	-120	-56	0	24	210	-5	-10	-306	42	-9	-36	-27	0	-9	0	0	0	0	40	8	-8	-8	0	0	0	0	0	2	-6	-1	7	2	3	0	χ_{100}
χ_{101}	+	551675124	-47628	-9828	-3564	0	0	0	0	-2268	324	36	108	0	-24	189	9	-51	0	0	0	0	0	0	0	0	0	0	0	36	12	0	8	0	0	0	0	0	-3	-3	-3	-3	-3	1	0	χ_{101}
	ind	1 2	2 2	4	2	3 6	3 6	3 6	3 6	4 4	4	4 4	4	8	4	5 10	5 10	5 10	6 6	12	6 6	6 6	6 6	6 6	6	12	6	7 14	7 14	8	8	8 8	8	8 8	8	9 18	9 18	9 18	10 10	20	20	10 10	10 10	10	11 22	
χ_{102}	+	24	8	0	0	-12	6	-3	0	8	0	4	0	0	0	-6	4	-1	-4	0	-4	5	2	-1	0	0	0	-4	3	0	0	4	0	2	0	-3	0	3	-2	0	0	-2	3	0	2	χ_{102}
χ_{103}	+	2024	-8	0	0	-352	26	-1	8	120	0	-4	0	0	0	-56	4	-1	-32	0	4	31	-2	1	0	0	0	-20	1	0	0	12	0	-2	0	-10	-1	-1	-8	0	0	2	7	0	0	χ_{103}
χ_{104}	+	2576	176	0	0	-196	56	11	8	48	0	24	0	0	0	-14	16	1	20	0	-16	11	8	5	0	0	0	0	7	0	0	8	0	4	0	2	-1	5	6	0	0	-4	1	0	2	χ_{104}
χ_{105}	+	4576	160	0	0	-572	58	-14	-8	160	0	16	0	0	0	-64	16	1	-20	0	-20	34	-2	-2	0	0	0	-16	5	0	0	16	0	0	0	-5	1	7	0	0	0	0	5	0	0	χ_{105}
χ_{106}	+	40480	-160	0	0	-3080	70	34	-8	608	0	-16	0	0	0	-190	0	5	-88	0	20	74	2	2	0	0	0	-36	-1	0	0	16	0	0	0	-11	1	-5	-10	0	0	0	5	0	0	χ_{106}
χ_{107}	+	95680	1856	0	0	208	316	19	-8	64	0	96	0	0	0	20	40	5	-16	0	-16	11	20	-7	0	0	0	4	11	0	0	0	0	8	0	1	1	1	-4	0	0	-4	1	0	2	χ_{107}
χ_{108}	+	170016	-672	0	0	-7788	114	-30	0	1120	0	-16	0	0	0	-264	-4	-9	-36	0	36	90	6	-6	0	0	0	-28	0	0	0	16	0	0	0	-3	0	-6	8	0	0	+2	3	0	0	χ_{108}
χ_{109}	+	299000	680	0	0	-12220	20	-25	8	1576	0	20	0	0	0	-350	0	0	-148	0	-40	95	-4	5	0	0	0	-40	-5	0	0	4	0	2	0	-7	-1	5	-10	0	0	0	0	0	-2	χ_{109}
χ_{110}	+	315744	2848	0	0	-3432	474	60	0	288	0	80	0	0	0	-6	44	-6	136	0	-44	28	-2	4	0	0	0	16	9	0	0	16	0	0	0	6	0	3	-2	0	0	-2	-2	0	0	χ_{110}
χ_{111}	+	351624	2520	0	0	-8736	210	-51	0	728	0	44	0	0	0	-126	24	-1	-96	0	-60	21	6	-9	0	0	0	0	0	0	0	12	0	-2	0	3	0	6	-10	0	0	0	-5	0	-2	χ_{111}
χ_{112}	+	388080	336	0	0	-11088	252	9	0	1232	0	-24	0	0	0	-210	20	5	144	0	0	81	-12	3	0	0	0	0	0	0	0	24	0	-4	0	9	0	0	26	0	0	6	1	0	0	χ_{112}
χ_{113}	+	789360	-1072	0	0	-18876	6	-39	48	1616	0	40	0	0	0	-260	-20	10	-148	0	-4	41	2	-1	0	0	0	-16	5	0	0	-8	0	-4	0	-3	3	0	-12	0	0	-2	-2	0	0	χ_{113}
χ_{114}	+	1841840	11152	0	0	4004	980	-181	8	272	0	200	0	0	0	70	60	-10	-116	0	-8	19	28	1	0	0	0	0	7	0	0	-8	0	4	0	-1	-1	-1	2	0	0	2	2	0	0	χ_{114}
χ_{115}	+	1937520	720	0	0	-42120	0	81	0	3408	0	-24	0	0	0	-510	0	-5	-216	0	0	81	0	-9	0	0	0	-24	-3	0	0	-8	0	4	0	0	0	0	-10	0	0	0	-5	0	2	χ_{115}
χ_{116}	+	4004000	-3360	0	0	-37180	530	-100	56	2016	0	-80	0	0	0	-100	0	0	492	0	60	60	6	-12	0	0	0	28	0	0	0	16	0	0	0	8	2	-4	20	0	0	0	0	0	0	χ_{116}
χ_{117}	+	5051904	-3584	0	0	-75504	204	96	-48	4608	0	0	0	0	0	-526	-16	4	-80	0	64	64	4	16	0	0	0	4	4	0	0	0	0	0	0	6	-3	-3	6	0	0	-4	-4	0	0	χ_{117}
χ_{118}	+	6446440	17528	0	0	-4004	658	190	112	-392	0	124	0	0	0	-70	0	15	116	0	44	-10	38	14	0	0	0	0	0	0	0	-4	0	6	0	1	4	-2	-2	0	0	8	3	0	0	χ_{118}
χ_{119}	+	6446440	3192	0	0	-76076	-224	-161	-56	4088	0	60	0	0	0	-350	-20	-10	156	0	-96	39	0	-3	0	0	0	0	0	0	0	-20	0	-2	0	1	-2	4	22	0	0	2	2	0	0	χ_{119}
χ_{120}	+	7104240	14928	0	0	15444	1350	135	0	720	0	104	0	0	0	120	60	15	-228	0	-12	15	-30	-3	0	0	0	-4	3	0	0	-8	0	-4	0	0	0	0	8	0	0	-2	3	0	0	χ_{120}
χ_{121}	o	9152000	-7680	0	0	-11440	260	80	-112	-512	0	0	0	0	0	-50	0	0	-144	0	0	0	12	24	0	0	0	4	-3	0	0	0	0	0	0	-1	-4	2	10	0	0	0	0	0	0	χ_{121}
χ_{122}	o	9152000	-7680	0	0	-11440	260	80	-112	-512	0	0	0	0	0	-50	0	0	-144	0	0	0	12	24	0	0	0	4	-3	0	0	0	0	0	0	-1	-4	2	10	0	0	0	0	0	0	χ_{122}
χ_{123}	+	11051040	9568	0	0	-84084	-546	210	48	3424	0	-16	0	0	0	-140	20	15	388	0	-44	10	10	10	0	0	0	0	-7	0	0	-16	0	0	0	3	3	3	28	0	0	-2	3	0	0	χ_{123}
χ_{124}	+	13156000	-11040	0	0	-48620	790	-20	-56	1504	0	-80	0	0	0	100	0	0	348	0	60	60	18	12	0	0	0	32	-3	0	0	16	0	0	0	7	-2	-2	-20	0	0	0	0	0	0	χ_{124}
χ_{125}	o	15002624	-7168	0	0	11648	56	-304	-112	0	0	0	0	0	0	-56	4	-26	128	0	-16	-16	8	32	0	0	0	0	0	0	0	0	0	0	0	2	5	-1	-8	0	0	2	2	0	-1	χ_{125}
χ_{126}	o	15002624	-7168	0	0	11648	56	-304	-112	0	0	0	0	0	0	-56	4	-26	128	0	-16	-16	8	32	0	0	0	0	0	0	0	0	0	0	0	2	5	-1	-8	0	0	2	2	0	-1	χ_{126}
χ_{127}	+	15471456	24864	0	0	-60060	1302	-300	0	-224	0	80	0	0	0	-84	-4	6	-564	0	-60	-60	18	-12	0	0	0	0	0	0	0	16	0	0	0	-3	0	-6	4	0	0	-6	-6	0	0	χ_{127}
χ_{128}	o	16170000	14000	0	0	23100	-210	-705	0	560	0	-40	0	0	0	0	0	0	20	0	20	-25	26	5	0	0	0	0	0	0	0	8	0	-4	0	-3	0	3	0	0	0	0	0	0	0	χ_{128}
χ_{129}	o	16170000	14000	0	0	23100	-210	-705	0	560	0	-40	0	0	0	0	0	0	20	0	20	-25	26	5	0	0	0	0	0	0	0	8	0	-4	0	-3	0	3	0	0	0	0	0	0	0	χ_{129}
χ_{130}	+	17050176	6336	0	0	0	1620	81	0	960	0	-96	0	0	0	216	36	1	0	0	0	81	-36	-9	0	0	0	24	3	0	0	0	0	-8	0	0	0	0	-24	0	0	6	1	0	0	χ_{130}
χ_{131}	+	17310720	35840	0	0	-6720	1848	156	0	0	0	256	0	0	0	0	40	-5	320	0	-40	-4	8	20	0	0	0	0	0	0	0	0	0	0	0	-6	0	-3	0	0	0	0	-5	0	-2	χ_{131}
χ_{132}	+	18987696	7056	0	0	-147420	0	-81	0	4368	0	-120	0	0	0	-294	36	-4	-756	0	0	-81	0	9	0	0	0	0	0	0	0	-8	0	-4	0	0	0	0	-34	0	0	6	-4	0	2	χ_{132}
χ_{133}	+	19734000	-6320	0	0	-163020	-120	-165	48	6096	0	40	0	0	0	-350	0	0	-68	0	40	-5	-8	-11	0	0	0	20	-1	0	0	-8	0	4	0	6	3	0	-10	0	0	0	0	0	0	χ_{133}
χ_{134}	+	34155000	18600	0	0	-148500	540	135	0	3240	0	20	0	0	0	0	0	0	420	0	-120	15	-12	-3	0	0	0	40	-9	0	0	20	0	2	0	0	0	0	0	0	0	0	0	0	0	χ_{134}
χ_{135}	o	40370176	0	0	0	22528	-512	640	112	0	0	0	0	0	0	-64	16	-24	0	0	0	0	0	0	0	0	0	14	0	0	0	0	0	0	0	-8	4	-2	0	0	0	0	0	0	0	χ_{135}
χ_{136}	o	40370176	0	0	0	22528	-512	640	112	0	0	0	0	0	0	-64	16	-24	0	0	0	0	0	0	0	0	0	14	0	0	0	0	0	0	0	-8	4	-2	0	0	0	0	0	0	0	χ_{136}
χ_{137}	+	44204160	-2688	0	0	-6864	-96	570	168	896	0	-64	0	0	0	100	20	-15	528	0	24	-30	0	-6	0	0	0	-28	0	0	0	0	0	0	0	3	6	0	12	0	0	2	-3	0	0	χ_{137}
χ_{138}	+	49335000	61000	0	0	42900	960	870	0	840	0	100	0	0	0	0	0	0	220	0	40	-50	64	-2	0	0	0	-20	-6	0	0	4	0	-6	0	6	0	3	0	0	0	0	0	0	0	χ_{138}
χ_{139}	+	50519040	13312	0	0	27456	1248	744	120	0	0	0	0	0	0	-120	0	-10	448	0	16	-56	-32	-8	0	0	0	-16	-2	0	0	0	0	0	0	6	3	0	-8	0	0	-8	2	0	0	χ_{139}
χ_{140}	+	51571520	25536	0	0	-32032	728	305	-112	-1344	0	160	0	0	0	-140	-20	-30	-96	0	120	-15	-24	-33	0	0	0	0	0	0	0	0	0	8	0	8	-4	2	-4	0	0	6	6	0	0	χ_{140}
χ_{141}	o	59153976	-22680	0	0	-122472	0	0	0	1512	0	180	0	0	0	126	-24	-24	648	0	0	0	0	0	0	0	0	0	0	0	0	4	0	-6	0	0	0	0	10	0	0	0	0	0	2	χ_{141}
χ_{142}	o	59153976	-22680	0	0	61236	0	0	0	1512	0	180	0	0	0	126	-24	-24	-324	0	0	0	0	0	0	0	0	0	0	0	0	4	0	-6	0	0	0	0	10	0	0	0	0	0	2	χ_{142}
χ_{143}	+	59153976	-22680	0	0	61236	0	0	0	1512	0	180	0	0	0	126	-24	-24	-324	0	0	0	0	0	0	0	0	0	0	0	0	4	0	-6	0	0	0	0	10	0	0	0	0	0	2	χ_{143}
χ_{144}	+	62790000	-10800	0	0	-273000	-210	285	120	5200	0	40	0	0	0	0	0	0	-120	0	60	-75	-6	-9	0	0	0	0	7	0	0	-8	0	-4	0	-3	-6	0	0	0	0	0	0	0	-2	χ_{144}
χ_{145}	+	67358720	-4096	0	0	-265408	344	-16	-112	4096	0	0	0	0	0	-20	-40	20	-64	0	-64	-64	8	8	0	0	0	16	9	0	0	0	0	0	0	-7	5	-1	4	0	0	4	4	0	0	χ_{145}
χ_{146}	+	106260000	92000	0	0	23100	2310	-660	-120	-160	0	240	0	0	0	0	0	0	20	0	20	20	2	-16	0	0	0	0	-7	0	0	-16	0	0	0	-3	-3	-3	0	0	0	0	0	0	0	χ_{146}
χ_{147}	+	112519680	3584	0	0	-331968	-588	96	-168	3584	0	0	0	0	0	140	20	-20	-64	0	80	-64	-4	-16	0	0	0	0	0	0	0	0	0	0	0	-3	3	0	4	0	0	-6	4	0	-2	χ_{147}
χ_{148}	+	139243104	-34272	0	0	108108	2646	-270	0	3360	0	-304	0	0	0	504	4	-21	-828	0	36	90	18	18	0	0	0	0	0	0	0	-16	0	0	0	0	0	0	8	0	0	-2	3	0	0	χ_{148}
χ_{149}	+	161161000	8120	0	0	-280280	70	-380	112	2744	0	-100	0	0	0	350	0	0	1208	0	20	20	2	20	0	0	0	0	0	0	0	-4	0	6	0	-11	-5	1	10	0	0	0	0	0	0	χ_{149}
χ_{150}	+	190417920	50176	0	0	-251328	-336	-120	168	0	0	0	0	0	0	280	-40	-30	-320	0	-176	40	16	-8	0	0	0	0	0	0	0	0	0	0	0	6	-3	-3	-24	0	0	-4	6	0	0	χ_{150}
χ_{151}	+	210496000	69120	0	0	148720	1300	-320	112	3584	0	0	0	0	0	350	0	0	-432	0	0	0	-36	0	0	0	0	-20	-6	0	0	0	0	0	0	4	4	4	10	0	0	0	0	0	0	χ_{151}
χ_{152}	+	215547904	-65536	0	0	73216	1504	-224	64	0	0	0	0	0	0	-176	-16	4	512	0	-64	-64	32	-16	0	0	0	-16	-2	0	0	0	0	0	0	10	1	1	-16	0	0	4	4	0	0	χ_{152}
χ_{153}	+	230230000	-39600	0	0	-400400	910	325	160	-1840	0	40	0	0	0	0	0	0	-1200	0	60	-75	-6	27	0	0	0	0	-7	0	0	24	0	4	0	1	-2	4	0	0	0	0	0	0	0	χ_{153}
χ_{154}	+	247543296	-60928	0	0	192192	924	816	168	3584	0	0	0	0	0	196	-4	46	-448	0	-16	-16	20	-16	0	0	0	0	0	0	0	0	0	0	0	-3	-3	-3	12	0	0	2	2	0	0	χ_{154}
χ_{155}	+	282906624	61440	0	0	-192192	1848	-240	-48	-4096	0	0	0	0	0	-196	-56	24	-576	0	0	0	-24	24	0	0	0	0	7	0	0	0	0	0	0	3	6	0	20	0	0	0	0	0	0	χ_{155}
χ_{156}	+	287006720	-57344	0	0	-128128	1904	-256	-112	0	0	0	0	0	0	280	-40	20	640	0	64	64	16	-32	0	0	0	0	0	0	0	0	0	0	0	-4	5	5	-24	0	0	-4	-4	0	0	χ_{156}
χ_{157}	+	313524224	0	0	0	-372736	-1792	-640	-160	0	0	0	0	0	0	384	64	24	0	0	0	0	0	0	0	0	0	-36	-8	0	0	0	0	0	0	8	2	-4	0	0	0	0	0	0	2	χ_{157}
χ_{158}	+	342752256	21504	0	0	-133056	-1512	756	0	0	0	256	0	0	0	336	-24	31	192	0	-24	84	-24	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	0	0	4	-1	0	0	χ_{158}
χ_{159}	+	351624000	82880	0	0	43680	1470	-510	168	-1344	0	-160	0	0	0	-350	0	0	992	0	20	-70	-22	26	0	0	0	0	0	0	0	0	0	0	0	-6	-3	0	-10	0	0	0	0	0	2	χ_{159}
χ_{160}	+	394680000	78400	0	0	-171600	-240	1155	-120	-2240	0	-160	0	0	0	0	0	0	400	0	40	-5	16	1	0	0	0	-

$Co_1 = F_{2-}$

	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	13A	14A	14B	15A	15B	15C	15D	15E	16A	16B	18A	18B	18C	20A	20B	20C	21A	21B	21C	22A	23A	B**	24A	24B	24C	24D	24E	24F	26A	28A	28B	30A	30B	30C	30D	30E	33A	35A	36A	39A	B**	40A	42A	60A	
x_{86}	-54	18	6	0	0	4	0	0	0	0	0	-2	0	0	-1	1	5	0	-1	0	0	-1	-1	0	0	0	1	0	0	3	0	1	0	0	0	6	-2	0	0	0	0	0	1	-1	-3	0	-1	0	0	0	0	0	0	0	1	-1	1	x_{86}
x_{87}	70	22	6	10	-5	-2	1	0	-3	3	-2	1	-1	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	-2	2	1	-2	-1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	x_{87}
x_{88}	24	-8	8	-6	0	0	4	0	0	-4	-2	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	-2	1	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	0	0	0	0	0	0	0	-1	x_{88}
x_{89}	-60	-12	4	0	-3	0	-3	0	-3	1	0	0	0	0	-2	-1	0	5	0	-1	0	2	-2	0	0	0	0	-1	0	1	1	0	0	0	0	-4	0	-1	0	-1	0	0	1	0	0	1	0	1	0	-1	0	0	0	0	0	1	0	x_{89}
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	4	0	1	2	0	0	-2	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-1	0	1	0	0	0	0	0	0	0	x_{90}
x_{91}	-6	18	-2	0	6	0	0	0	0	-2	0	0	0	3	-2	0	-9	-4	0	2	0	2	-2	0	0	0	2	0	0	1	-2	0	0	0	0	2	2	2	0	0	0	-1	0	0	-1	0	0	0	0	0	1	0	0	0	0	1	-1	x_{91}
x_{92}	-25	-1	-5	2	2	1	2	2	2	-2	2	1	1	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	x_{92}
x_{93}	0	0	0	0	0	8	0	0	0	0	0	2	0	-3	1	0	8	3	-1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	-1	-1	0	0	0	0	0	0	0	1	0	x_{93}
x_{94}	45	-27	9	0	0	1	0	0	0	0	0	-1	-1	0	0	0	-8	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	-1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	x_{94}
x_{95}	0	0	0	-9	0	0	0	-3	0	0	3	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	x_{95}
x_{96}	-36	12	12	-12	3	0	3	2	-1	3	0	0	0	0	0	0	-5	0	1	0	0	0	0	-2	1	-2	-1	0	0	0	0	0	0	0	0	4	0	1	0	1	0	0	0	0	3	0	1	0	0	0	0	0	0	0	-1	0	-1	x_{96}
x_{97}	30	-10	10	3	-6	0	-4	-3	0	-2	-1	0	0	1	0	0	0	-5	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-2	2	-2	0	0	-1	1	0	0	0	-1	0	-1	0	0	0	0	1	1	0	0	0	x_{97}
x_{98}	9	9	-15	0	0	-3	0	0	0	0	0	0	0	0	1	0	3	3	0	0	0	0	0	0	0	0	5	1	-1	-3	0	0	0	0	0	-3	1	0	1	0	0	0	0	-1	3	-1	0	0	0	0	1	0	0	0	1	1	-1	x_{98}
x_{99}	0	0	0	0	0	-8	0	0	0	0	0	-2	0	-1	-1	0	16	1	1	-2	0	0	0	0	0	0	0	0	0	3	0	0	0	1	1	0	0	0	0	0	0	-1	0	1	0	1	1	0	0	0	-1	0	-1	-1	0	-1	0	x_{99}
x_{100}	6	14	-14	6	-3	0	-1	0	3	1	2	0	0	0	0	0	3	-2	0	1	-1	0	0	0	0	0	-2	-1	0	0	0	0	0	0	0	-2	-2	1	0	-1	0	0	0	0	-1	2	0	1	-1	0	0	0	0	0	0	0	1	x_{100}
x_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	-3	1	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	x_{101}
	12 12	12	12	12 12	12 12	24	12	12 12	12 12	12	12 12	24	12	13 26	28	14 14	15 30	15 30	15 30	15 30	15 30	16	16 16	18 18	18 18	18 18	20 20	20	20 20	21 42	21 42	21 42	22 22	23 46	23 46	24	24 24	24	24	24	24 24	52	28 28	56 56	30 30	60	60	30 30	30 30	33 66	35 70	36 36	39 78	39 78	40 40	84	60 60	
x_{102}	-4	0	0	-1	2	0	0	1	-2	0	3	0	0	-2	0	1	3	-2	0	1	2	0	2	-1	2	-1	-2	0	-1	2	-1	0	0	1	1	0	-2	0	0	0	-1	0	1	0	1	0	0	1	0	-1	1	-1	1	1	0	0	1	x_{102}
x_{103}	0	0	0	3	6	0	0	-1	2	0	3	0	0	-4	0	-1	-2	-2	-2	1	-1	0	2	-2	1	1	0	0	1	-2	-2	1	0	0	0	0	0	0	0	0	1	0	1	0	-2	0	0	-1	1	0	0	0	-1	-1	0	0	0	x_{103}
x_{104}	-12	0	0	3	0	0	0	3	0	0	3	0	0	2	0	1	4	4	-2	1	1	0	0	2	-1	-1	-2	0	-1	0	0	1	0	0	0	0	2	0	0	0	1	0	-1	0	0	0	0	-1	1	2	0	0	-1	-1	0	0	-2	x_{104}
x_{105}	-20	0	0	-2	-2	0	0	-2	-2	0	6	0	0	0	0	-1	8	-2	2	-2	1	0	0	1	1	1	0	0	1	2	2	-1	0	-1	-1	0	-2	0	0	0	0	0	-1	0	0	0	0	0	-1	0	-1	1	0	0	0	0	0	x_{105}
x_{106}	8	0	0	-4	2	0	0	2	2	0	0	0	0	-2	0	1	-10	0	2	0	-1	0	0	-1	-1	-1	-2	0	-1	0	0	-1	0	0	0	0	4	0	0	0	0	0	-1	0	2	0	0	0	-1	0	-1	-1	1	1	0	0	-2	x_{106}
x_{107}	16	0	0	1	4	0	0	3	0	0	-3	0	0	0	0	1	-7	-2	2	1	-1	0	0	-1	-1	-1	4	0	1	-2	1	-1	0	0	0	0	0	0	0	0	-1	0	1	0	-1	0	0	-1	1	-1	-1	1	0	0	0	0	1	x_{107}
x_{108}	28	0	0	4	-2	0	0	2	2	0	0	0	0	2	0	0	-3	2	0	-1	0	0	0	3	0	0	0	0	-1	-4	2	0	0	0	0	0	-2	0	0	0	0	0	0	0	-1	0	0	1	0	0	2	1	-1	-1	0	0	3	x_{108}
x_{109}	-20	0	0	1	4	0	0	-1	-4	0	-3	0	0	0	0	1	10	0	-2	0	0	0	-2	-1	-1	-1	6	0	0	2	-1	1	0	0	0	0	-2	0	0	0	-1	0	1	0	2	0	0	0	0	1	0	1	0	0	0	0	0	x_{109}
x_{110}	-24	0	0	0	6	0	0	-4	2	0	0	0	0	0	0	-1	3	8	0	-1	0	0	0	-2	-2	1	-2	0	0	-2	-2	0	0	0	0	0	4	0	0	0	0	0	1	0	1	0	0	1	-2	0	1	0	0	0	0	0	1	x_{110}
x_{111}	-64	0	0	-1	-10	0	0	5	2	0	3	0	0	0	0	0	9	-6	0	0	-1	0	-2	-3	0	0	-2	0	-1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	1	-2	0	-1	0	0	0	0	1	x_{111}
x_{112}	-16	0	0	-1	-4	0	0	-3	0	0	3	0	0	4	0	0	-3	2	0	2	-1	0	0	3	0	0	2	0	1	0	0	0	0	1	1	0	0	0	0	0	-1	0	0	0	-1	0	0	0	1	0	0	-1	1	1	0	0	-1	x_{112}
x_{113}	44	0	0	5	2	0	0	1	-2	0	-3	0	0	0	0	-1	19	4	-2	1	1	0	0	-1	-1	2	-4	0	0	-4	-1	-1	0	0	0	0	-2	0	0	0	-1	0	-1	0	-3	0	0	1	1	0	-1	-1	0	0	0	0	-1	x_{113}
x_{114}	44	0	0	-7	-4	0	0	-1	-4	0	-3	0	0	0	0	1	-11	-6	-2	0	-1	0	0	1	1	1	2	0	0	0	0	1	0	0	0	0	-2	0	0	0	1	0	-1	0	-1	0	0	2	-1	0	0	-1	0	0	0	0	-1	x_{114}
x_{115}	-24	0	0	-3	0	0	0	-3	0	0	-3	0	0	0	0	-1	-15	0	0	0	1	0	0	0	0	0	-2	0	1	6	0	0	0	0	0	0	4	0	0	0	1	0	-1	0	-1	0	0	0	1	-1	1	0	0	0	0	0	1	x_{115}
x_{116}	12	0	0	0	6	0	0	4	-2	0	0	0	0	0	0	0	-10	0	-4	0	0	0	0	0	0	0	-4	0	0	4	-2	0	0	-1	-1	0	-2	0	0	0	0	0	0	0	2	0	0	0	0	0	-2	0	0	0	0	0	2	x_{116}
x_{117}	48	0	0	0	-12	0	0	0	0	0	0	0	0	0	0	0	-4	-4	2	-1	1	0	0	-2	1	1	-2	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	-1	-1	0	-1	0	0	0	0	0	-2	x_{117}
x_{118}	-44	0	0	4	-2	0	0	-2	-2	0	0	0	0	0	0	0	11	6	2	3	0	0	-2	-1	2	2	-2	0	-1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	1	0	0	0	0	1	x_{118}
x_{119}	-4	0	0	-7	8	0	0	3	0	0	-3	0	0	0	0	0	19	4	4	1	-1	0	2	3	0	0	-2	0	0	0	0	0	0	0	0	0	-2	0	0	0	1	0	0	0	1	0	0	-1	-1	0	0	-1	0	0	0	0	1	x_{119}
x_{120}	60	0	0	9	-6	0	0	-1	2	0	-3	0	0	0	0	-3	-6	-6	0	0	0	0	0	0	0	0	0	0	-1	2	-1	0	0	0	0	0	-2	0	0	0	-1	0	-1	0	2	0	0	-2	0	0	1	0	0	0	0	0	0	x_{120}
x_{121}	16	0	0	-8	4	0	0	0	0	0	0	0	0	0	0	-1	-5	0	-2	0	0	0	0	-3	0	0	-2	0	0	-2	1	0	0	1	1	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	-1	1	0	0	215	0	1	x_{121}
x_{122}	16	0	0	-8	4	0	0	0	0	0	0	0	0	0	0	-1	-5	0	-2	0	0	0	0	-3	0	0	-2	0	0	-2	1	0	0	1	1	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	-1	1	0	0	-215	0	1	x_{122}
x_{123}	-92	0	0	4	10	0	0	2	2	0	0	0	0	0	0	-1	-14	-4	-2	-1	0	0	0	1	1	1	4	0	-1	0	0	-1	0	0	0	0	2	0	0	0	0	0	1	0	-2	0	0	1	0	0	0	1	0	0	0	0	-2	x_{123}
x_{124}	28	0	0	-8	10	0	0	4	-2	0	0	0	0	0	0	-1	10	0	4	0	0	0	0	-3	0	0	4	0	0	2	-1	0	0	0	0	0	-2	0	0	0	0	0	-1	0	-2	0	0	0	0	0	2	1	0	0	0	0	-2	x_{124}
x_{125}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	1	1	0	0	2	-1	-1	0	0	0	0	0	0	i11	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-1	-1	-1	0	0	0	0	0	0	0	x_{125}
x_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	1	1	0	0	2	-1	-1	0	0	0	0	0	0	0-i11	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-1	-1	-1	0	0	0	0	0	0	0	x_{126}
x_{127}	-116	0	0	-8	10	0	0	-4	2	0	0	0	0	0	0	0	15	-10	0	2	0	0	0	3	0	0	-4	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	-1	x_{127}
x_{128}	20	0	0	-7	2	0	0	-1	2	0	-3	0	0	2	0	0	0	0	0	0	0	0	0	-1	2	-1	0	0	0	0	0	0	0	b23	**	0	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	x_{128}
x_{129}	20	0	0	-7	2	0	0	-1	2	0	-3	0	0	2	0	0	0	0	0	0	0	0	0	-1	2	-1	0	0	0	0	0	0	0	**	b23	0	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	x_{129}
x_{130}	0	0	0	-3	12	0	0	-3	0	0	-3	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	3	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	x_{130}
x_{131}	0	0	0	0	0	0	0	4	4	0	0	0	0	-2	0	0	0	10	0	-2	1	0	0	2	2	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	x_{131}
x_{132}	-84	0	0	3	0	0	0	3	0	0	3	0	0	0	0	0	-15	0	0	0	-1	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	-2	0	0	0	-1	0	0	0	-1	0	0	0	-1	2	0	0	0	0	0	0	1	x_{132}
x_{133}	60	0	0	3	0	0	0	-5	4	0	3	0	0	0	0	1	10	0	-2	0	0	0	0	-2	1	-2	6	0	0	-4	-1	-1	0	0	0	0	-2	0	0	0	1	0	-1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	x_{133}
x_{134}	-60	0	0	9	-12	0	0	-1	-4	0	-3	0	0	-4	0	1	0	0	0	0	0	0	2	0	0	0	0	0	0	-2	1	0	0	0	0	0	2	0	0	0	-1	0	-1	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	x_{134}
x_{135}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	8	-2	2	-2	0	0	0	0	0	0	0	0	0	2	-1	0	0	1	1	0	0	0	0	0	0	0	0	i14	0	0	0	0	0	0	-1	0	-1	-1	0	0	0	x_{135}
x_{136}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	8	-2	2	-2	0	0	0	0	0	0	0	0	0	2	-1	0	0	1	1	0	0	0	0	0	0	0	0-i14	0	0	0	0	0	0	-1	0	-1	-1	0	0	0	x_{136}
x_{137}	-16	0	0	-4	8	0	0	2	-4	0	0	0	0	0	0	0	1	-4	-2	-1	0	0	0	-3	0	0	-4	0	1	-4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	-1	0	0	2	-1	0	0	0	0	-1	x_{137}
x_{138}	60	0	0	12	0	0	0	-2	4	0	0	0	0	0	0	2	0	0	0	0	0	0	2	-2	-2	1	0	0	0	4	1	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{138}
x_{139}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	6	6	0	3	-1	0	0	-2	1	-2	0	0	0	2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	1	-1	0	-1	0	0	0	0	0	0	x_{139}
x_{140}	-96	0	0	-3	0	0	0	1	4	0	-3	0	0	0	0	0	13	-2	-2	-2	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	x_{140}
x_{141}	72	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	18	-12	0	0	0	0	-2	0	0	0	2	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	-2	0	0	0	0	2	0	0	1	1	0	0	2	x_{141}
x_{142}	-36	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-9	6	0	0	0	0	-2	0	0	0	2	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0 b39	**	0	0	-1	x_{142}
x_{143}	-36	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-9	6	0	0	0	0	-2	0	0	0	2	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	**	b39	0	0	-1	x_{143}
x_{144}	40	0	0	7	10	0	0	1	-2	0	3	0	0	0	0	1	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	0	4	0	0	0	-1	0	-1	0	0	0	0	0	0	-2	0	1	0	0	0	0	0	x_{144}
x_{145}	64	0	0	-8	-8	0	0	0	0	0	0	0	0	0	0	-1	7	2	-2	-1	-1	0	0	-1	-1	-1	-4	0	0	4	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1	1	0	1	1	0	0	0	0	-1	x_{145}
x_{146}	20	0	0	2	2	0	0	0	-6	0	6	0	0	2	0	-1	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	-1	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	x_{146}
x_{147}	-64	0	0	-16	-4	0	0	0	0	0	0	0	0	0	0	0	-13	2	2	2	1	0	0	-1	-1	2	4	0	0	0

J_4

	;	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		86775	2179	18166	26	54	98	43	6	26	2	2			1					31									
		571046077562880	9895040	57920	61120	06720	304	008	720	61120	304	304	840	840	280	768	512	960	80	944	242	192	192	48	84	84	56	56	
		p power	A	A	A	A	A	B	A	AA	AA	AB	A	A	A	B	B	AA	AB	A	A	BA	BB	CC	AA	BA	AB	BB	
		p' part	A	A	A	A	A	A	A	AA	AA	AB	A	A	A	A	A	AA	AB	A	A	AA	AB	AC	AA	BA	AB	BB	
	ind	1A	2A	2B	3A	4A	4B	4C	5A	6A	6B	6C	7A	B**	8A	8B	8C	10A	10B	11A	11B	12A	12B	12C	14A	B**	14C	D**	
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	o	1333	53	-11	10	-11	5	-3	3	-10	2	-2	b7	**	1	-3	1	3	-1	2	2	-2	2	0	-b7	**	b7	**	x_2
x_3	o	1333	53	-11	10	-11	5	-3	3	-10	2	-2	**	b7	1	-3	1	3	-1	2	2	-2	2	0	**	-b7	**	b7	x_3
x_4	o	299367	-153	231	45	-89	7	7	-3	45	-3	-3	4b7	**	-1	-1	-1	-3	1	13	2	1	1	1	-2b7	**	0	0	x_4
x_5	o	299367	-153	231	45	-89	7	7	-3	45	-3	-3	**	4b7	-1	-1	-1	-3	1	13	2	1	1	1	**	-2b7	0	0	x_5
x_6	o	887778	738	-606	45	34	-14	10	3	45	-3	-3	b7	**	6	2	-2	3	-1	1	1	1	1	1	b7	**	b7	**	x_6
x_7	o	887778	738	-606	45	34	-14	10	3	45	-3	-3	**	b7	6	2	-2	3	-1	1	1	1	1	1	**	b7	**	b7	x_7
x_8	+	889111	2071	727	55	87	39	-1	6	55	7	7	-1	-1	-5	7	3	6	2	3	3	3	3	-1	-1	-1	-1	-1	x_8
x_9	o	1187145	585	-375	-45	-55	-7	17	0	-45	3	3	5b7	**	5	1	-3	0	0	14	3	-1	-1	-1	-b7	**	b7	**	x_9
x_{10}	o	1187145	585	-375	-45	-55	-7	17	0	-45	3	3	**	5b7	5	1	-3	0	0	14	3	-1	-1	-1	**	-b7	**	b7	x_{10}
x_{11}	+	1776888	2808	120	99	120	24	8	8	99	3	3	1	1	0	8	0	8	0	3	3	3	3	-1	1	1	1	1	x_{11}
x_{12}	o	3403149	-627	-627	-66	77	13	-3	14	66	6	-6	5b7	**	5	-3	-3	-2	-2	24	2	2	-2	0	b7	**	b7	**	x_{12}
x_{13}	o	3403149	-627	-627	-66	77	13	-3	14	66	6	-6	**	5b7	5	-3	-3	-2	-2	24	2	2	-2	0	**	b7	**	b7	x_{13}
x_{14}	+	4290927	1647	175	141	175	31	7	-8	-99	-3	13	4	4	-5	-1	3	-8	0	25	3	1	1	1	2	2	0	0	x_{14}
x_{15}	o	32307363	-6237	99	0	99	51	-21	3	0	0	0	6b7	**	-9	3	-1	3	-1	0	0	0	0	0	0	0	-2b7	**	x_{15}
x_{16}	o	32307363	-6237	99	0	99	51	-21	3	0	0	0	**	6b7	-9	3	-1	3	-1	0	0	0	0	0	0	0	**	-2b7	x_{16}
x_{17}	o	32897107	-5293	-749	10	211	35	-21	-3	-10	2	-2	4b7	**	-1	3	-1	-3	1	-10	1	-2	2	0	2b7	**	0	0	x_{17}
x_{18}	o	32897107	-5293	-749	10	211	35	-21	-3	-10	2	-2	**	4b7	-1	3	-1	-3	1	-10	1	-2	2	0	**	2b7	0	0	x_{18}
x_{19}	+	35411145	10185	3465	105	265	25	49	0	105	9	9	0	0	5	1	5	0	0	11	0	1	1	1	0	0	0	0	x_{19}
x_{20}	+	35411145	10185	3465	105	265	25	49	0	105	9	9	0	0	5	1	5	0	0	11	0	1	1	1	0	0	0	0	x_{20}
x_{21}	+	95288172	25452	364	231	44	108	28	7	-189	15	-5	0	0	4	-4	-4	7	-1	-54	1	-1	3	1	0	0	0	0	x_{21}
x_{22}	+	230279749	11333	6853	-308	197	37	21	14	308	-4	4	0	0	5	-3	-3	-2	-2	51	-4	-4	4	0	0	0	0	0	x_{22}
x_{23}	+	259775040	6720	6720	210	320	64	0	0	-210	-6	6	0	0	0	0	0	0	0	19	-3	2	-2	0	0	0	0	0	x_{23}
x_{24}	+	259775040	6720	6720	210	320	64	0	0	-210	-6	6	0	0	0	0	0	0	0	19	-3	2	-2	0	0	0	0	0	x_{24}
x_{25}	+	300364890	34650	-7910	420	-550	42	-14	0	0	12	-8	0	0	-10	-6	6	0	0	56	1	-4	0	-2	0	0	0	0	x_{25}
x_{26}	+	366159104	-2816	-1792	440	768	0	0	-6	-440	-8	8	-4	-4	0	0	0	-6	-2	36	3	0	0	0	-2	-2	0	0	x_{26}
x_{27}	+	393877506	-10494	7106	561	66	-110	-22	1	-99	9	5	1	1	6	2	-2	1	1	0	0	-3	1	-1	-1	-1	1	1	x_{27}
x_{28}	+	394765284	-9756	-2716	309	804	4	-28	-1	-351	-3	-7	4	4	4	4	-4	-1	-1	1	1	-3	1	-1	2	2	0	0	x_{28}
x_{29}	+	460559498	24458	6986	329	-54	90	-14	-7	329	-7	-7	0	0	6	2	6	-7	1	-19	3	-3	-3	1	0	0	0	0	x_{29}
x_{30}	+	493456605	16605	7645	-120	285	29	13	0	540	0	4	5	5	5	-3	-3	0	0	-29	4	0	-4	-2	1	1	1	1	x_{30}
x_{31}	+	690839247	23247	-10801	21	79	15	-49	7	441	-3	17	0	0	-1	-1	-1	7	-1	32	-1	1	-3	-1	0	0	0	0	x_{31}
x_{32}	+	786127419	16443	3003	252	187	-37	-21	14	-252	12	-12	0	0	-5	3	3	-2	-2	-22	0	4	-4	0	0	0	0	0	x_{32}
x_{33}	+	786127419	16443	3003	-126	187	-37	-21	14	126	-6	6	0	0	-5	3	3	-2	-2	-22	0	-2	2	0	0	0	0	0	x_{33}
x_{34}	+	786127419	16443	3003	-126	187	-37	-21	14	126	-6	6	0	0	-5	3	3	-2	-2	-22	0	-2	2	0	0	0	0	0	x_{34}
x_{35}	+	789530568	49608	-440	-111	-440	40	-8	8	-351	-15	1	1	1	0	-8	0	8	0	2	2	1	1	1	-1	-1	1	1	x_{35}
x_{36}	+	885257856	24192	-8064	0	384	0	0	21	0	0	0	0	0	0	0	0	-3	1	-45	-1	0	0	0	0	0	0	0	x_{36}
x_{37}	+	885257856	24192	-8064	0	384	0	0	21	0	0	0	0	0	0	0	0	-3	1	-45	-1	0	0	0	0	0	0	0	x_{37}
x_{38}	+	1016407168	27776	9856	-56	384	0	0	-7	56	8	-8	0	0	0	0	0	1	1	29	-4	0	0	0	0	0	0	0	x_{38}
x_{39}	+	1016407168	27776	9856	-56	384	0	0	-7	56	8	-8	0	0	0	0	0	1	1	29	-4	0	0	0	0	0	0	0	x_{39}
x_{40}	+	1085604531	-17229	-9933	330	307	19	35	6	330	-6	-6	4	4	-5	3	-5	6	2	33	0	-2	-2	2	-2	-2	0	0	x_{40}
x_{41}	+	1089007680	14400	-3520	-330	320	-64	-64	0	-90	6	-10	5	5	0	0	0	0	0	57	2	2	2	2	1	1	1	1	x_{41}
x_{42}	+	1182518964	-32076	10164	99	-396	-12	-28	-1	99	3	3	4	4	4	-4	4	-1	-1	0	0	3	3	-1	-2	-2	0	0	x_{42}
x_{43}	+	1183406741	-31339	341	-154	341	101	-35	1	-154	-10	-10	6	6	1	-3	1	1	1	0	0	2	2	-2	0	0	-2	-2	x_{43}
x_{44}	+	1183406741	39061	6677	440	-363	-75	-27	6	-440	-8	8	-1	-1	5	-3	-3	6	2	0	0	0	0	0	1	1	-1	-1	x_{44}
x_{45}	+	1184295852	-29268	10284	-99	-276	12	-20	7	-99	-3	-3	5	5	4	4	4	7	-1	3	3	-3	-3	1	-1	-1	1	1	x_{45}
x_{46}	+	1445942610	-13230	6930	0	210	-30	42	0	0	0	0	0	0	-10	-6	-2	0	0	-9	2	0	0	0	0	0	0	0	x_{46}
x_{47}	+	1445942610	-13230	6930	0	210	-30	42	0	0	0	0	0	0	-10	-6	-2	0	0	-9	2	0	0	0	0	0	0	0	x_{47}
x_{48}	+	1445942610	-13230	6930	0	210	-30	42	0	0	0	0	0	0	-10	-6	-2	0	0	-9	2	0	0	0	0	0	0	0	x_{48}
x_{49}	+	1509863773	-5027	-7139	-176	-99	45	-27	-7	-176	16	16	5	5	1	-3	-7	-7	1	0	0	0	0	0	-1	-1	1	1	x_{49}
x_{50}	+	1579061136	31632	4752	-384	528	-48	-48	-14	384	0	0	-5	-5	0	0	0	2	2	4	4	0	0	0	-1	-1	-1	-1	x_{50}
x_{51}	+	1842237992	1064	3752	-385	-472	8	56	42	-385	-1	-1	0	0	0	8	0	-6	2	45	1	-1	-1	-1	0	0	0	0	x_{51}
x_{52}	+	1903741279	26719	-737	385	-737	-17	7	-6	385	1	1	6	6	-5	7	-5	-6	-2	0	0	1	1	1	0	0	-2	-2	x_{52}
x_{53}	+	1981808640	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	-2	0	0	0	0	0	0	0	x_{53}
x_{54}	+	1981808640	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	-2	0	0	0	0	0	0	0	x_{54}
x_{55}	+	1981808640	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-24	-2	0	0	0	0	0	0	0	x_{55}
x_{56}	+	2001151845	-10395	-10395	0	165	-27	21	0	0	0	0	0	0	5	-3	5	0	0	0	0	0	0	0	0	0	0	0	x_{56}
x_{57}	+	2001151845	-10395	-10395	0	165	-27	21	0	0	0	0	0	0	5	-3	5	0	0	0	0	0	0	0	0	0	0	0	x_{57}
x_{58}	+	2001151845	-10395	-10395	0	165	-27	21	0	0	0	0	0	0	5	-3	5	0	0	0	0	0	0	0	0	0	0	0	x_{58}
x_{59}	+	2267824128	-49152	0	384	0	0	0	8	384	0	0	-4	-4	0	0	0	8	0	20	-2	0	0	0	2	2	0	0	x_{59}
x_{60}	+	2692972480	-34880	3520	-230	-320	64	-64	0	-230	10	10	-5	-5	0	0	0	0	0	-11	0	-2	-2	2	1	1	-1	-1	x_{60}
x_{61}	+	2727495848	25256	-4312	-385	-88	8	56	-42	-385	-1	-1	0	0	0	8	0	6	-2	0	0	-1	-1	-1	0	0	0	0	x_{61}
x_{62}	+	3054840657	1617	-495	231	-495	33	-7	-8	231	-9	-9	-6	-6	5	1	-3	-8	0	0	0	3	3	-1	0	0	2	2	x_{62}

	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	
	30	32	160	160	42	42	264	22	23	48	48	28	28	29	30	31	31	31	66	66	35	35	37	37	37	40	40	42	42	43	43	43	44	66	66	
	AA	C	AA	AA	BA	AA	AA	BB	A	BB	BB	CC	DC	A	AAA	A	A	A	AA	AA	BA	AA	A	A	A	BA	AA	ABA	BAA	A	A	A	AA	AAA	BAA	
	AA	A	AA	AA	AA	BA	AA	BB	A	AB	AB	AC	BC	A	AAA	A	A	A	AA	AA	AA	BA	A	A	A	AA	AA	AAA	BBA	A	A	A	AA	AAA	BAA	
	15A	16A	20A	B*	21A	B**	22A	22B	23A	24A	B*	28A	B**	29A	30A	31A	B*5	C*6	33A	B*	35A	B**	37A	B*2	C*4	40A	B*	42A	B**	43A	B*6	C*7	44A	66A	B*	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	0	-1	-1	-1	b7	**	-2	0	-1	0	0	-b7	**	-1	0	0	0	0	-1	-1	b7	**	1	1	1	1	1	-b7	**	0	0	0	0	1	1	x_2
x_3	0	-1	-1	-1	**	b7	-2	0	-1	0	0	**	-b7	-1	0	0	0	0	-1	-1	**	b7	1	1	1	1	1	**	-b7	0	0	0	0	1	1	x_3
x_4	0	-1	1	1	b7	**	1	0	-1	-1	-1	0	0	0	0	0	0	0	1	1	-b7	**	0	0	0	-1	-1	b7	**	1	1	1	-1	1	1	x_4
x_5	0	-1	1	1	**	b7	1	0	-1	-1	-1	0	0	0	0	0	0	0	1	1	**	-b7	0	0	0	-1	-1	**	b7	1	1	1	-1	1	1	x_5
x_6	0	0	-1	-1	b7	**	1	-1	1	-1	-1	b7	**	1	0	0	0	0	1	1	b7	**	0	0	0	1	1	b7	**	0	0	0	1	1	1	x_6
x_7	0	0	-1	-1	**	b7	1	-1	1	-1	-1	**	b7	1	0	0	0	0	1	1	**	b7	0	0	0	1	1	**	b7	0	0	0	1	1	1	x_7
x_8	0	1	2	2	-1	-1	3	1	0	1	1	-1	-1	0	0	0	0	0	0	0	-1	-1	1	1	1	0	0	-1	-1	0	0	0	-1	0	0	x_8
x_9	0	-1	0	0	-b7	**	2	-1	0	1	1	b7	**	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-b7	**	1	1	1	0	-1	-1	x_9
x_{10}	0	-1	0	0	**	-b7	2	-1	0	1	1	**	b7	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	**	-b7	1	1	1	0	-1	-1	x_{10}
x_{11}	-1	0	0	0	1	1	3	-1	0	-1	-1	1	1	0	-1	-1	-1	-1	0	0	1	1	0	0	0	0	0	1	1	-1	-1	-1	-1	0	0	x_{11}
x_{12}	-1	1	2	2	-b7	**	0	0	0	0	0	-b7	**	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	b7	**	0	0	0	0	0	0	x_{12}
x_{13}	-1	1	2	2	**	-b7	0	0	0	0	0	**	-b7	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	**	b7	0	0	0	0	0	0	x_{13}
x_{14}	1	1	0	0	1	1	-3	-1	1	-1	-1	0	0	0	1	0	0	0	-2	-2	-1	-1	0	0	0	0	0	-1	-1	0	0	0	-1	0	0	x_{14}
x_{15}	0	1	-1	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	b7	**	-1	-1	-1	1	1	0	0	1	1	1	0	0	0	x_{15}
x_{16}	0	1	-1	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	**	b7	-1	-1	-1	1	1	0	0	1	1	1	0	0	0	x_{16}
x_{17}	0	1	1	1	b7	**	-2	-1	0	0	0	0	0	0	0	0	0	0	-1	-1	-b7	**	0	0	0	-1	-1	-b7	**	0	0	0	2	1	1	x_{17}
x_{18}	0	1	1	1	**	b7	-2	-1	0	0	0	0	0	0	0	0	0	0	-1	-1	**	-b7	0	0	0	-1	-1	**	-b7	0	0	0	2	1	1	x_{18}
x_{19}	0	-1	0	0	0	0	-1	0	0	1	1	0	0	-1	0	0	0	0	-b33	*	0	0	-1	-1	-1	0	0	0	0	0	0	0	1	-b33	*	x_{19}
x_{20}	0	-1	0	0	0	0	-1	0	0	1	1	0	0	-1	0	0	0	0	*	-b33	0	0	-1	-1	-1	0	0	0	0	0	0	0	1	*	-b33	x_{20}
x_{21}	1	0	-1	-1	0	0	-2	1	0	-1	-1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	-2	-2	x_{21}
x_{22}	2	1	2	2	0	0	3	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	x_{22}
x_{23}	0	0	0	0	0	0	-1	-1	-1	2r3	-2r3	0	0	0	0	0	0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0	1	-1	-1	x_{23}
x_{24}	0	0	0	0	0	0	-1	-1	-1	-2r3	2r3	0	0	0	0	0	0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0	1	-1	-1	x_{24}
x_{25}	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{25}
x_{26}	0	0	-2	-2	-1	-1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	-2	0	0	x_{26}
x_{27}	1	0	1	1	1	1	0	0	-1	-1	-1	-1	-1	-1	1	0	0	0	0	0	1	1	0	0	0	1	1	-1	-1	0	0	0	0	0	0	x_{27}
x_{28}	-1	0	-1	-1	1	1	1	1	0	1	1	0	0	0	-1	0	0	0	1	1	-1	-1	0	0	0	-1	-1	-1	-1	0	0	0	1	1	1	x_{28}
x_{29}	-1	0	1	1	0	0	5	1	0	-1	-1	0	0	0	-1	0	0	0	-1	-1	0	0	0	0	0	1	1	0	0	0	0	0	1	-1	-1	x_{29}
x_{30}	0	1	0	0	-1	-1	-5	0	0	0	0	-1	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	-1	1	1	x_{30}
x_{31}	1	-1	-1	-1	0	0	4	1	0	-1	-1	0	0	0	1	0	0	0	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	0	2	1	1	x_{31}
x_{32}	2	-1	2	2	0	0	-2	0	0	0	0	0	0	1	-2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	x_{32}
x_{33}	-1	-1	2	2	0	0	-2	0	0	0	0	0	0	1	1	0	0	0	-b33	*	0	0	0	0	0	0	0	0	0	0	0	0	0	b33	*	x_{33}
x_{34}	-1	-1	2	2	0	0	-2	0	0	0	0	0	0	1	1	0	0	0	*	-b33	0	0	0	0	0	0	0	0	0	0	0	0	0	*	b33	x_{34}
x_{35}	-1	0	0	0	1	1	-2	0	0	1	1	-1	-1	0	-1	0	0	0	-1	-1	1	1	0	0	0	0	0	-1	-1	0	0	0	0	1	1	x_{35}
x_{36}	0	0	2r5-1	-2r5-1	0	0	3	-1	0	0	0	0	0	-1	0	1	1	1	0	0	0	0	0	0	0	r5	-r5	0	0	0	0	0	-1	0	0	x_{36}
x_{37}	0	0	-2r5-1	2r5-1	0	0	3	-1	0	0	0	0	0	-1	0	1	1	1	0	0	0	0	0	0	0	-r5	r5	0	0	0	0	0	-1	0	0	x_{37}
x_{38}	-1	0	2r5-1	-2r5-1	0	0	1	0	0	0	0	0	0	1	1	0	0	0	-1	-1	0	0	0	0	0	-r5	r5	0	0	0	0	0	-1	1	1	x_{38}
x_{39}	-1	0	-2r5-1	2r5-1	0	0	1	0	0	0	0	0	0	1	1	0	0	0	-1	-1	0	0	0	0	0	r5	-r5	0	0	0	0	0	-1	1	1	x_{39}
x_{40}	0	-1	2	2	1	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	1	1	0	0	0	-1	0	0	x_{40}
x_{41}	0	0	0	0	-1	-1	1	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	-2	-2	x_{41}
x_{42}	-1	0	-1	-1	1	1	0	0	0	-1	-1	0	0	0	-1	1	1	1	0	0	-1	-1	0	0	0	-1	-1	1	1	1	1	1	0	0	0	x_{42}
x_{43}	1	-1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	x_{43}
x_{44}	0	1	2	2	-1	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	1	1	0	0	0	0	0	0	x_{44}
x_{45}	1	0	-1	-1	-1	-1	3	-1	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	-1	0	0	x_{45}
x_{46}	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	c43	*6	*7	1	0	0	x_{46}
x_{47}	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*7	c43	*6	1	0	0	x_{47}
x_{48}	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*6	*7	c43	1	0	0	x_{48}
x_{49}	-1	-1	1	1	-1	-1	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	x_{49}
x_{50}	1	0	-2	-2	1	1	-4	0	0	0	0	1	1	0	-1	0	0	0	1	1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	-1	x_{50}
x_{51}	0	0	-2	-2	0	0	-3	1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	x_{51}
x_{52}	0	1	-2	-2	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{52}
x_{53}	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	-1	-1	-1	0	0	0	0	-c37	*2	*4	0	0	0	0	1	1	1	0	0	0	x_{53}
x_{54}	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	-1	-1	-1	0	0	0	0	*4	-c37	*2	0	0	0	0	1	1	1	0	0	0	x_{54}
x_{55}	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	-1	-1	-1	0	0	0	0	*2	*4	-c37	0	0	0	0	1	1	1	0	0	0	x_{55}
x_{56}	0	1	0	0	0	0	0	0	-1	0	0	0	0	1	0	c31	*5	*6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{56}
x_{57}	0	1	0	0	0	0	0	0	-1	0	0	0	0	1	0	*6	c31	*5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{57}
x_{58}	0	1	0	0	0	0	0	0	-1	0	0	0	0	1	0	*5	*6	c31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{58}
x_{59}	-1	0	0	0	-1	-1	-4	0	1	0	0	0	0	0	-1	1	1	1	-1	-1	1	1	0	0	0	0	0	-1	-1	0	0	0	0	-1	-1	x_{59}
x_{60}	0	0	0	0	1	1	1	0	0	0	0	-1	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	1	1	x_{60}
x_{61}	0	0	2	2	0	0	0	0	0	-1	-1	0	0	-1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{61}
x_{62}	1	-1	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	x_{62}

J_4

Sporadic Janko group J_4

Order = 86,775,571,046,077,562,880 = $2^{21}.3^3.5.7.11^3.23.29.31.37.43$ Mult = 1 Out = 1

Constructions

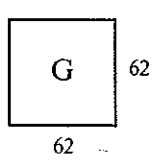
Modulo 2 G has a 112-dimensional irreducible representation over $\mathbb{F}_2$, in which there is an orbit of 173,067,389 special vectors, each fixed by a group $2^{11}:M_{24}$. Explicit matrices have been computed.

The group can be defined as the stabilizer of a certain 4995-dimensional subspace of the exterior square of this representation, and two vectors in the 112-space are called neighbours if their exterior product lies in this 4995-space. The neighbourhood of a special vector is a 12-space, and consists of the zero vector, the special vector, 2x759 octad vectors whose neighbourhood is 6-dimensional, and 2x1288 dodecad vectors whose neighbourhood is 2-dimensional. The stabilizer of an octad vector is $2^{10}:L_5(2)$. There are also pentads of special vectors, stabilized by $2^{3+12}.(S_5 x L_3(2)) \cong (2^{3+12}:S_5):L_3(2) \cong (2^{3+12}:L_3(2)):S_5$. The 112-space reduces under the four large maximal subgroups as follows:

$2^{11}:M_{24}$	1.11.44.44.11.1 (uniserial)
$2^{10}:L_5(2)$	$(1 \oplus 5).10.40.40.10.(1 \oplus 5)$
$2^{1+12}.3M_{22}.2$	$10.30.(10.10 \oplus 12).30.10$
$2^{3+12}.(S_5 x L_3(2))$	$4.12.(3.3.3.3 \oplus 12).32.(3.3.3.3 \oplus 12).12.4$

Some maximal subgroups (including all local subgroups)

			Specifications	
Order	Index	Structure	Abstract	Modulo 2
501397585920	173067389	$2^{11}:M_{24}$	$N(2^{11}) = N(2A_{1771}B_{276})$	special vector
10239344640	8474719242	$2^{10}:L_5(2)$	$N(2^{10}) = N(2A_{155}B_{868})$	octad vector
21799895040	3980549947	$2_+^{1+12}.3M_{22}:2$	N(2A)	10-space
660602880	131358148251	$2^{3+12}.(S_5 \times L_3(2))$	$N(2A^3)$	pentad
141831360	611822174208	$U_3(11):2$		vector
319440	271649045348352	$11_+^{1+2}:(5 \times 2S_4)$	N(11A)	2-space
163680	530153782050816	$L_2(32):5$		
12144	7145550975467520	$L_2(23):2$		vector
812	106866466805514240	29:28	N(29A)	
602	144145466853949440	43:14	N(43ABC)	
444	195440475329003520	37:12	N(37ABC)	vector



$^2E_6(2)$ and $E_6(2)$

Twisted group $^2E_6(2)$

Order = 76,532,479,683,774,853,939,200 = $2^{36}.3^9.5^2.7^2.11.13.17.19$ Mult = $2^2 \times 3$ Out = S_3

Constructions

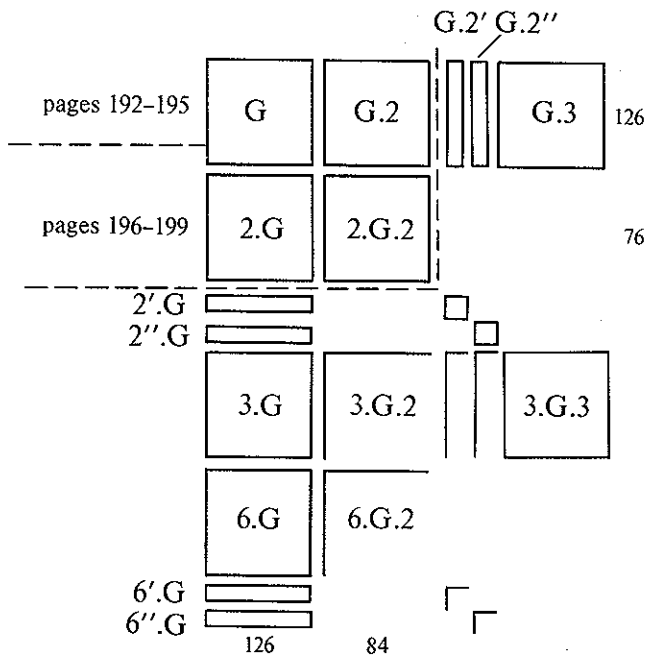
Steinberg G : the centralizer in $E_6(4)$ of an outer automorphism of order 2.

3.G.3 has a 27-dimensional representation over $\mathbb{F}_4$, which remains irreducible on restriction to $3{\cdot}Fi_{22}$. There is a cubic form which is invariant up to scalar multiplication.

Remark : 2.G.2 is the involution centralizer in the Baby Monster; $2^2.G.S_3$ is the normalizer of a certain 4-group in the Monster. For a possible presentation, see page 232.

Some maximal subgroups (including all local subgroups)

Structure	G.2	G.3	G.S_3	Specification
$2^{1+20}:U_6(2)$	: H.2	: H.3	: H.S_3	N(2A)
$2^{8+16}:O_8^-(2)$	: H.2	: H.3	: H.S_3	N(2^8) = N($2A_{119}B_{136}$)
$F_4(2)$	: $F_4(2) \times 2$	$^3D_4(2):3 \times 3$	$^3D_4(2):3 \times S_3$	C(2D)
$F_4(2)$				C(2)
$F_4(2)$				C(2)
$2^2.2^9.2^{18}:(L_3(4)xS_3)$	: H.2	: H.3	: H.S_3	N($2A^2$)
Fi_{22}	: $Fi_{22}:2$	$3^6:U_4(2):2$	$3^6:(2xU_4(2):2)$	
Fi_{22}	$O_7(3):2$			
Fi_{22}				
$O_{10}^-(2)$	: H:2	: H x 3	: (H x 3):2	C(3D)
$2^3.2^{12}.2^{15}:(S_5xL_3(2))$	: H.2	: H.3	: H.S_3	N($2A^3$)
$S_3 \times U_6(2)$	: $S_3 \times U_6(2):2$	: $S_3 \times U_6(2):3$	: $S_3 \times U_6(2):S_3$	N(3A)
$(3 \times O_8^+(2):3):2$	: $S_3 \times O_8^+(2):S_3$	: H.3	: H.S_3	N(3B)
$U_3(8):3$	: $U_3(8):6$	: H x 3	: (H x 3):2	C(3G)
$(L_3(2) \times L_3(4)):2$	: H:2	: $(L_3(2) \times L_3(4):3):2$	: H:S_3	
$3^{1+6}.2^{3+6}.(S_3x3)$	: H.2	: H.3	: H.S_3	N(3C)
$3^2Q_8 \times U_3(3):2$	: H.2	: $3^2:2A_4 \times U_3(3):2$	: $3^2:2S_4 \times U_3(3):2$	N($3B^2$)



Note: Only the characters of 2.G.2 are printed

Chevalley group $E_6(2)$

Note: The characters of $E_6(2)$ are not printed

Order = 214,841,575,522,005,575,270,400 = $2^{36}.3^6.5^2.7^3.13.17.31.73$

Mult = 1 Out = 2

Constructions

Chevalley G : the adjoint Chevalley group of type E_6 over $\mathbb{F}_2$; G has a 27-dimensional representation over $\mathbb{F}_2$, which supports an invariant cubic form.

Maximal 2-local subgroups

$2^{16}:O_{10}^+(2)$

$2^{16}:O_{10}^+(2)$

$[2^{25}]:(S_3xL_5(2))$

$[2^{25}]:(S_3xL_5(2))$

$[2^{29}]:(S_3xL_3(2)xL_3(2))$

$2^{1+20}:L_6(2)$

Maximal subgroups of G.2 include $[2^{24}]:O_8^+(2).2$ and $[2^{30}]:(S_3xS_3xL_3(2)).2$

Some other subgroups

$F_4(2)$

$S_3 \times L_6(2)$

$3{\cdot}(3^2:Q_8 \times L_3(4)).S_3$

$L_3(8):3$

$(L_3(2) \times L_3(2) \times L_3(2)):S_3$

$L_3(2) \times U_3(3):2$

$^2E_6(2)$

| | | |

$^2E_6(2)$

	8 192 IA 8L	8 192 JA 8M	8 192 KA 8N	1 024 LA 80	1 024 OA 8P	324 CA 9A	162 CA 9B	960 AA 10A	480 AB 10B	66 A 11A	66 A B**	165 888 AB 12A	62 208 DB 12B	41 472 BB 12C	18 432 AC 12D	6912 DC CC 12E	6912 DC CC F**	4 608 BA 12G	4 608 CD AD 12H	4 608 CE AE 12I	4 608 CF AF 12J	3 456 GD CD 12K	3 456 GE CE 12L	3 456 GF CF 12M	2 304 CH AH 12N	768 CM AM 12O	768 DG CG 12P	576 EH BH 12Q	576 HL BL 12R	576 IL CL 12S	576 IL CL 12T	576 IL CL 12U	384 GI CI 12V	384 GJ CJ 12W	384 GK CK 12X	192 IN CN 12Y	192 IN CN Z**	13 A AA 13A	448 AB AB 14A	56 BA BA 14B	900 AB AB 15A	90 AA AA 15B	256 D A 16A	256 D A 16B	256 D A 16C	128 IA A 16D	128 JA A 16E	128 KA A 16F	128 LA A 16G	128 MA A 16H	128 NA A 16I	17 AA A 17A	17 AA B* 18A	108 AD AA B**	108 AD AA B**	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	-2	-2	-2	2	2	3	3	5	1	2	2	5	-4	-4	-3	0	0	4	1	1	1	-2	-2	-2	-3	1	4	0	0	0	0	0	2	2	2	0	0	1	-1	2	-2	1	2	2	2	0	0	0	0	0	0	0	0	1	1			x_2
x_3	4	4	4	4	4	2	2	4	4	0	0	1	10	19	1	-2	-2	3	1	1	1	4	4	4	9	1	2	3	-1	2	2	2	0	0	0	2	2	0	3	1	5	-1	0	0	0	0	0	0	0	0	0	0	0	0	0			x_3
x_4	0	0	0	-4	4	3	3	3	-1	0	0	-5	-5	-14	11	-1	-1	2	3	3	3	-3	-3	-3	-5	-5	3	-2	2	-1	-1	-1	1	1	1	-1	-1	0	1	0	-2	-2	0	0	0	-2	-2	-2	2	2	2	0	0	-1	-1			x_4
x_5	8	8	8	4	-4	3	0	0	0	0	0	25	7	-11	17	11	11	5	5	5	5	5	5	5	1	-3	-1	1	5	-1	-1	-1	1	1	1	-1	-1	0	4	-1	0	0	0	0	0	2	2	2	-2	-2	-2	1	1	1	1			x_5
x_6	1	1	1	-3	-3	0	0	3	-5	1	1	23	14	14	-1	2	2	-2	-5	-5	-5	4	4	4	7	3	6	-2	-2	-2	-2	-2	0	0	0	-2	-2	0	0	-1	5	2	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0			x_6
x_7	0	0	0	4	-4	0	0	9	1	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-3	1	-3	0	0	0	0	2	2	2	-2	-2	-2	-1	-1	0	0			x_7
x_8	8	8	8	-4	4	0	-3	6	2	0	0	30	12	3	6	12	12	11	2	2	2	8	8	8	6	2	-4	3	3	0	0	0	0	0	0	0	0	-1	0	0	-1	2	0	0	0	-2	-2	-2	2	2	2	0	0	2	2			x_8
x_9	11	-5	-5	3	3	0	0	-1	3	0	0	10	-8	19	10	4	4	-5	10	-6	-6	-2	-6	-6	2	2	0	-1	-1	8	-4	-4	2	-2	-2	0	0	0	3	0	4	1	-1	-1	-1	3	-1	-1	3	-1	-1	0	0	0	0			x_9
x_{10}	-5	11	-5	3	3	0	0	-1	3	0	0	10	-8	19	10	4	4	-5	-6	10	-6	-6	-2	-6	2	2	0	-1	-1	-4	8	-4	-2	2	-2	0	0	0	3	0	4	1	-1	-1	-1	-1	3	-1	-1	3	-1	0	0	0	0			x_{10}
x_{11}	-5	-5	11	3	3	0	0	-1	3	0	0	10	-8	19	10	4	4	-5	-6	-6	10	-6	-6	-2	2	2	0	-1	-1	-4	-4	8	-2	-2	2	0	0	0	3	0	4	1	-1	-1	-1	-1	-1	3	-1	-1	3	0	0	0	0			x_{11}
x_{12}	0	0	0	-8	8	1	1	-3	1	0	0	-12	-3	42	-12	-3	-3	2	4	4	4	1	1	1	4	-4	-3	-2	2	5	5	5	1	1	1	-3	-3	0	-3	-1	6	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1			x_{12}
x_{13}	0	0	0	0	0	-4	2	0	0	-2	-2	52	16	-2	4	-8	-8	-2	-4	-4	-4	-4	-4	-4	4	-4	0	-2	6	0	0	0	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2			x_{13}
x_{14}	0	0	0	0	0	3	0	0	0	-1	-1	-12	69	-3	-12	-3	-3	13	4	4	4	1	1	1	-4	4	5	5	1	1	1	1	1	1	1	1	1	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1			x_{14}
x_{15}	0	0	0	0	0	0	-3	4	4	0	0	22	-14	4	-18	6	6	4	2	2	2	-4	-4	-4	-10	-6	-6	-4	4	-2	-2	-2	0	0	0	-2	-2	0	3	2	5	-1	0	0	0	0	0	0	0	0	0	0	0	-2	-2			x_{15}
x_{16}	-6	-6	-6	6	6	0	0	-2	2	0	0	23	5	-40	-17	1	1	-8	3	3	3	3	3	3	-9	3	-3	0	4	1	1	1	-1	-1	-1	1	1	0	0	-1	-5	1	-2	-2	-2	0	0	0	0	0	0	0	0	0	0			x_{16}
x_{17}	-11	5	5	-3	-3	1	1	0	0	0	0	-12	15	6	12	-9	-9	-2	16	0	0	-5	3	3	-4	0	-1	2	2	-1	-1	-1	3	-5	-5	-1	-1	0	0	0	0	0	5	-3	-3	-3	1	1	-3	1	1	0	0	1	1			x_{17}
x_{18}	5	-11	5	-3	-3	1	1	0	0	0	0	-12	15	6	12	-9	-9	-2	0	16	0	3	-5	3	-4	0	-1	2	2	-1	-1	-1	-5	3	-5	-1	-1	0	0	0	0	0	-3	5	-3	1	-3	1	1	-3	1	0	0	1	1			x_{18}
x_{19}	5	5	-11	-3	-3	1	1	0	0	0	0	-12	15	6	12	-9	-9	-2	0	0	16	3	3	-5	-4	0	-1	2	2	-1	-1	-1	-5	-5	3	-1	-1	0	0	0	0	0	-3	-3	5	1	1	-3	1	1	-3	0	0	1	1			x_{19}
x_{20}	0	0	0	0	0	0	0	5	1	-2	-2	-18	-36	-18	6	0	0	6	-6	-6	-6	-6	-6	-6	-10	2	4	2	-6	0	0	0	-2	-2	-2	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	0	1	1	0	0			x_{20}
x_{21}	-8	8	8	-4	4	0	0	-3	1	0	0	-23	4	-23	9	0	0	1	9	-7	-7	-6	2	2	1	1	-4	1	-3	0	0	0	6	-2	-2	0	0	0	0	0	-3	0	0	0	0	-2	2	2	2	-2	-2	0	0	0	0			x_{21}
x_{22}	8	-8	8	-4	4	0	0	-3	1	0	0	-23	4	-23	9	0	0	1	-7	9	-7	2	-6	2	1	1	-4	1	-3	0	0	0	-2	6	-2	0	0	0	0	0	-3	0	0	0	0	2	-2	2	-2	2	-2	0	0	0	0			x_{22}
x_{23}	8	8	-8	-4	4	0	0	-3	1	0	0	-23	4	-23	9	0	0	1	-7	-7	9	2	2	-6	1	1	-4	1	-3	0	0	0	-2	-2	6	0	0	0	0	0	-3	0	0	0	0	2	2	-2	-2	-2	2	0	0	0	0			x_{23}
x_{24}	0	0	0	0	0	0	0	-1	-1	0	0	-60	3	48	-12	-9	-9	0	-4	-4	-4	5	5	5	12	-4	-5	0	4	-5	-5	-5	1	1	1	3	3	0	0	0	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{24}
x_{25}	0	0	0	0	0	0	0	0	0	0	0	0	27	0	0	-9	-9	0	0	0	0	-3	-3	-3	0	0	3	0	0	3	3	3	1	1	1	413	-1	-413	-1	-1	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{25}
x_{26}	0	0	0	0	0	0	0	0	0	0	0	0	27	0	0	-9	-9	0	0	0	0	-3	-3	-3	0	0	3	0	0	3	3	3	1	1	1	-413	-1	413	-1	-1	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{26}
x_{27}	10	10	10	-2	-2	0	0	0	0	0	0	2	-25	-25	-14	-5	-5	-9	-6	10	10	21	-11	-11	2	2	-1	-1	-1	-1	-1	-1	1	1	1	-1	-1	0	3	0	0	0	6	-2	-2	0	0	0	0	0	0	0	0	0	0			x_{27}
x_{28}	10	10	10	-2	-2	0	0	0	0	0	0	2	-25	-25	-14	-5	-5	-9	10	-6	10	-11	21	-11	2	2	-1	-1	-1	-1	-1	-1	1	1	1	-1	-1	0	3	0	0	0	-2	6	-2	0	0	0	0	0	0	0	0	0	0			x_{28}
x_{29}	10	10	10	-2	-2	0	0	0	0	0	0	2	-25	-25	-14	-5	-5	-9	10	10	-6	-11	-11	21	2	2	-1	-1	-1	-1	-1	-1	1	1	1	-1	-1	0	3	0	0	0	-2	-2	6	0	0	0	0	0	0	0	0	0	0			x_{29}
x_{30}	0	0	0	8	-8	6	0	0	0	0	0	28	-8	28	-20	4	4	12	4	4	4	-2	-2	-2	-4	-4	0	-4	0	0	0	0	2	2	2	0	0	0	-4	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{30}
x_{31}	0	0	0	8	-8	-5	1	0	0	0	0	-40	5	14	8	-7	-7	6	0	0	0	3	3	3	8	0	-3	2	2	5	5	5	-1	-1	-1	-3	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1			x_{31}
x_{32}	0	0	0	0	0	-4	2	-2	2	0	0	8	26	8	8	-10	-10	8	8	8	8	-4	-4	-4	0	0	2	0	0	-6	-6	-6	0	0	0	2	2	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	-2	-2			x_{32}
x_{33}	4	4	4	-4	-4	0	0	-2	2	-1	-1	18	36	18	-6	12	12	-6	6	6	6	0	0	0	10	-2	4	-2	-6	0	0	0	0	0	0	0	0	0	0	0	-5	1	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{33}
x_{34}	2	2	2	-2	-2	-3	-3	0	0	0	0	13	-50	13	21	-6	-6	-11	1	1	1	4	4	4	-11	1	6	1	5	2	2	2	0	0	0	2	2	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	-1	-1			x_{34}
x_{35}	24	-8	-8	0	0	-1	-1	0	0	0	0	37	-8	37	13	4	4	5	-7	9	9	14	-6	-6	-3	1	0	-3	1	-8	4	4	2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1			x_{35}
x_{36}	-8	24	-8	0	0	-1	-1	0	0	0	0	37	-8	37	13	4	4	5	9	-7	9	-6	14	-6	-3	1	0	-3	1	4	-8	4	-2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1			x_{36}
x_{37}	-8	-8	24	0	0	-1	-1	0	0	0	0	37	-8	37	13	4	4	5	9	9	-7	-6	-6	14	-3	1	0	-3	1	4	4	-8	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1			x_{37}
x_{38}	0	0	0	-8	8	0	0	3	-1	0	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{38}
x_{39}	-1	-1	-1	-5	-5	0	0	-6	-6	0	0	0	0	-9	0	0	0	-1	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	2	-1	0	0	-1	-1	-1	1	1	1	1	1	1	0	0	0	0			x_{39}
x_{40}	-16	-16	-16	0	0	0	0	0	0	0	0	37	55	-26	21	3	3	-10	-3	-3	-3	-3	-3	-3	13	5	-1	-2	2	-1	-1	-1	1	1	1	-1	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{40}
x_{41}	12	12	12	-4	-4	3	3	0	0	0	0	133	-2	25	5	-10	-10	1	-11	-11	-11	-2	-2	-2	-3	5	-2	-3	5	2	2	2	-2	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1			x_{41}
x_{42}	-3	-3	-3	1	1	0	0	1	1	0	0	-9	0	45	-9	0	0	-3	-9	-9	-9	0	0	0	-1	-1	0	5	-3	0	0	0	0	0	0	0	0	0	-2	0	1	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0			x_{42}
x_{43}	-4	-4	-4	4	4	0	0	-2	-2	0	0	90	-36	36	18	0	0	-12	6	6	6	-6	-6	-6	2	-2	4	-4	0	0	0	0	-2	-2	-2	0	0	0	0	0	-7	-1	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{43}
x_{44}	0	0	0	0	0	0	0	0	0	-1	-1	40	-50	22	8	-10	-10	6	-8	-8	-8	10	10	10	-8	8	-2	-2	-2	-2	-2	-2	2	2	2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{44}
x_{45}	-8	-8	-8	0	0	0	0	3	-5	0	0	-37	26	-1	27	-6	-6	-9	11	11	11	2	2	2	-5	3	-6	-5	-1	2	2	2	2	2	2	2	2	0	0	0	5	2	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{45}
x_{46}	-8	8	8	4	-4	0	0	0	0	0	0	0	-9	-9	0	3	3	-9	0	0	0	-3	-3	-3	0	0	-1	3	-1	-1	-1	-1	1	1	1	-1	-1	0	-3	0	0	0	0	0	0	2	-2	-2	-2	2	2	0	0	0	0			x_{46}
x_{47}	8	-8	8	4	-4	0	0	0	0	0	0	0	-9	-9	0	3	3	-9	0	0	0	-3	-3	-3	0	0	-1	3	-1	-1	-1	-1	1	1	1	-1	-1	0	-3	0	0	0	0	0	0	-2	2	-2	2	-2	2	0	0	0	0			x_{47}
x_{48}	8	8	-8	4	-4	0	0	0	0	0	0	0	-9	-9	0	3	3	-9	0	0	0	-3	-3	-3	0	0	-1	3	-1	-1	-1	-1	1	1	1	-1	-1	0	-3	0	0	0	0	0	0	-2	-2	2	2	2	-2	0	0	0	0			x_{48}
x_{49}	0	0	0	0	0	0	0	-9	-1	1	1	72	36	-36	24	12	12	12	0	0	0	0	0	0	-8	0	4	4	0	0	0	0	0	0	0	0	0	0	0	0	7	-2	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0			x_{49}
x_{50}	0	0	0	0	0	-3	-3	0	0	2	2	56	-16	-16	-8	-8	-8	0	-8	-8	-8	-8	-8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	1			x_{50}
x_{51}	-16	-16	-16	0	0	3	0	0	0	0	0	97	-20	16	-31	-4	-4	8	1	1	1	-8	-8	-8	1	1	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1			x_{51}
x_{52}	0	0	0	0	0	-3	0	-4	-4	0	0	32	-4	32	-32	4	4	16	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	4	1	-5	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1			x_{52}
x_{53}	32	-16	-16	4	-4	0	0	0	0	0	0	-23	13	-14	9	-3	-3	2	9	-7	-7	-3	5	5	1	1	-3	-2	-2	1	1	1	5	-3	-3	1	1	0	0	0	0	0	0	0	0	2	-2	-2	-2	2	2	0	0	0	0			x_{53}
x_{54}	-16	32	-16	4	-4	0	0	0	0	0	0	-23	13	-14	9	-3	-3	2	-7	9	-7	5	-3	5	1	1	-3	-2	-2	1	1	1	-3	5	-3	1	1	0	0	0	0	0	0	0	0	-2	2	-2	2	-2	2	0	0	0	0			x_{54}
x_{55}	-16	-16	32	4	-4	0	0	0	0	0	0	-23	13	-14	9	-3	-3	2	-7	-7	9	5	5	-3	1	1	-3	-2	-2	1	1	1	-3	-3	5	1	1	0	0	0	0	0	0	0	0	-2	-2	2	2	2	-2	0	0	0	0			x_{55}
x_{56}	0	0	0	0	0	-2	4	0	0	-1	-1	112	-32	-32	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			x_{56}
x_{57}	0	0	0	0	0	-3	0	2	-2	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-7	-1	0	0	0	0	0	0	0	0	0	0	1	1			x_{57}
x_{58}	0	0	0	0	0	-3	0	0	0	0	0	28	1	10	-20	1	1	-6	4	4	4	1	1	1	-4	-4	1	2	-2	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	313	-313		x_{58}
x_{59}	0	0	0	0	0	-3	0	0	0	0	0	28	1	10	-20	1	1	-6	4	4	4	1	1	1	-4	-4	1	2	-2	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-313	313		x_{59}
x_{60}	32	-16	-16	-4	4	0	0	3	-1	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	3	0	0	0	0	-2	2	2	2	-2	-2	0	0	0	0			x_{60}
x_{61}	-16	32	-16	-4	4	0	0	3	-1	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	3	0	0	0	0	2	-2	2	-2	2	-2	0	0	0	0			x_{61}
x_{62}	-16	-16	32	-4	4	0	0	3	-1	0	0	0	0	0	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	3	0	0	0	0	2	2	-2	-2	-2	2	0	0	0	0			x_{62}
x_{63}	0	0	0	0	0	0	3	0	0	0	0	-64	-64	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	2	2			x_{63}
x_{64}	0	0	0	0	0	0	0	0	0	0	0	50	32	-4

$^2E_6(2)$

| | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | ; | ; | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @ | @

$^2E_6(2)$

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$^2E_6(2)$

		1A	2A	2B	2C	3A	3B	3C	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	4K	4L	4M	4N	4O	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	7A	7B	8A	8B	8C	8D	8E	8F	8G	8H	8I	8J	8K	
χ_{100}	+	28969740800	4139520	698880	-5632	-15400	-2800	-730	-17920	-9728	2560	10752	-3584	-3584	-1536	0	-1536	512	512	0	0	0	0	0	600	-48	24	42	0	56	30	-16	2	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{100}
χ_{101}	+	28969740800	4139520	698880	-5632	-15400	-2800	-730	-17920	-9728	2560	-3584	10752	-3584	-1536	0	512	-1536	512	0	0	0	0	0	600	-48	24	42	0	56	30	-16	2	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{101}
χ_{102}	+	28969740800	4139520	698880	-5632	-15400	-2800	-730	-17920	-9728	2560	-3584	-3584	10752	-1536	0	512	512	-1536	0	0	0	0	0	600	-48	24	42	0	56	30	-16	2	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{102}
χ_{103}	o	31819833045	-24193323	-450603	9045	0	0	0	57141	-2187	2133	-27	-27	-27	-1995	-99	69	69	69	-171	45	-99	5	45	0	0	0	0	0	0	0	0	0	-63	0	9	9	-3	21	-3	-3	-3	-7	-7	-7	-7	χ_{103}
χ_{104}	o	31819833045	-24193323	-450603	9045	0	0	0	57141	-2187	2133	-27	-27	-27	-1995	-99	69	69	69	-171	45	-99	5	45	0	0	0	0	0	0	0	0	0	-63	0	9	9	-3	21	-3	-3	-3	-7	-7	-7	-7	χ_{104}
χ_{105}	+	32324909760	-24577344	-247104	-1344	0	6075	0	52928	1728	1728	1728	1728	1728	-1344	-192	192	192	192	192	-192	-192	-64	-90	0	27	0	0	-45	0	0	-21	0	-109	3	64	64	0	0	0	0	0	0	0	0	0	χ_{105}
χ_{106}	+	32514244608	33554432	0	0	21504	-4992	-384	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	-1024	128	0	128	0	0	0	0	0	-64	6	0	0	0	0	0	0	0	0	0	0	0	χ_{106}
χ_{107}	o	33748307775	26595135	-499905	-2241	25515	0	0	-16065	11583	-11457	63	63	63	-705	-705	63	63	63	63	63	63	63	0	-405	0	-117	0	0	27	0	0	0	0	0	63	63	-1	-1	-1	-1	-1	-1	-1	-1	-1	χ_{107}
χ_{108}	o	33748307775	26595135	-499905	-2241	25515	0	0	-16065	11583	-11457	63	63	63	-705	-705	63	63	63	63	63	63	63	0	-405	0	-117	0	0	27	0	0	0	0	0	63	63	-1	-1	-1	-1	-1	-1	-1	-1	-1	χ_{108}
χ_{109}	o	33748307775	-25659585	225855	1215	25515	0	0	14175	-11745	5823	-2961	-2961	-2961	351	375	15	15	15	63	39	-9	15	0	-405	0	-117	0	0	27	0	0	0	0	0	-45	-45	71	-17	-1	-1	-1	-13	-5	-5	-5	χ_{109}
χ_{110}	o	33748307775	-25659585	225855	1215	25515	0	0	14175	-11745	5823	-2961	-2961	-2961	351	375	15	15	15	63	39	-9	15	0	-405	0	-117	0	0	27	0	0	0	0	0	-45	-45	71	-17	-1	-1	-1	-13	-5	-5	-5	χ_{110}
χ_{111}	+	39598014456	26095608	516600	71928	-18711	-6804	0	-840	21816	2808	-1512	-1512	-1512	696	520	216	216	216	-72	-344	72	-104	56	-1431	108	-87	0	60	9	0	-36	0	0	0	0	0	-56	-8	-8	-8	-8	0	-16	-16	-16	χ_{111}
χ_{112}	+	43341742080	-25264128	688128	32768	42624	-2520	-360	-32768	-32768	0	0	0	0	0	0	0	0	0	0	0	0	0	-70	384	168	0	-120	-24	-64	24	8	8	0	7	0	0	0	0	0	0	0	0	0	0	0	χ_{112}
χ_{113}	+	45254873664	-13970880	313920	28224	-37422	6804	0	33600	-1728	576	-576	-576	-576	-192	-640	192	192	192	-576	0	0	64	64	-270	-108	-30	0	-60	-54	0	36	0	126	0	0	0	-64	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	+	45456904350	47552670	585630	15390	0	0	0	22590	-28674	-5346	-3186	-3186	-3186	1854	-810	-210	-210	-210	-162	198	150	-210	0	0	0	0	0	0	0	0	0	0	-45	-3	-54	-54	54	30	6	6	6	-6	10	10	10	χ_{114}
χ_{115}	+	49497518070	-24860682	669942	14454	18711	-8505	0	-13482	-12906	-1674	-378	-378	-378	1110	-1090	486	486	486	-330	-178	126	102	70	1431	135	87	0	39	-9	0	-9	0	0	0	-6	-6	-34	-10	14	14	14	10	-6	-6	-6	χ_{115}
χ_{116}	+	53033055075	-12950685	-1131165	55395	40095	0	0	-28125	-26973	2403	3843	3843	3843	1827	-45	-189	-189	-189	99	-141	-45	3	0	1215	0	-81	0	0	-9	0	0	0	0	-7	-45	-45	3	-45	-5	-5	-5	3	11	11	11	χ_{116}
χ_{117}	+	54997242300	-13430340	1407420	15804	0	0	972	-44100	-16452	1212	-3780	-3780	-3780	-1348	540	-324	-324	-324	252	-228	-36	188	0	0	0	0	108	0	0	-36	0	-36	0	0	36	36	12	44	-12	-12	-12	20	12	12	12	χ_{117}
χ_{118}	o	58615481925	-14313915	-1250235	14661	0	0	0	19845	9477	-6075	1701	1701	1701	-891	405	-411	-411	-411	261	117	213	-27	0	0	0	0	0	0	0	0	0	0	0	0	-27	-27	-27	21	-3	-3	-3	21	-3	-3	-3	χ_{118}
χ_{119}	o	58615481925	-14313915	-1250235	14661	0	0	0	19845	9477	-6075	1701	1701	1701	-891	405	-411	-411	-411	261	117	213	-27	0	0	0	0	0	0	0	0	0	0	0	0	-27	-27	-27	21	-3	-3	-3	21	-3	-3	-3	χ_{119}
χ_{120}	+	60609205800	8660520	-670680	50472	-40095	0	0	23400	-33048	-216	1800	1800	1800	360	-360	72	72	72	360	-264	-168	-120	0	-1215	0	81	0	0	9	0	0	0	45	-11	0	0	24	-24	8	8	8	0	16	16	16	χ_{120}
χ_{121}	+	65511420975	-15997905	-31185	-51921	0	0	0	-11025	36207	3375	-945	-945	-945	1263	1215	15	15	15	-81	-33	63	-113	0	0	0	0	0	0	0	0	0	0	0	0	-9	-9	-33	15	15	15	15	-9	7	7	7	χ_{121}
χ_{122}	+	65511420975	-15997905	-31185	-51921	0	0	0	-11025	36207	3375	-945	-945	-945	1263	1215	15	15	15	-81	-33	63	-113	0	0	0	0	0	0	0	0	0	0	0	0	-9	-9	-33	15	15	15	15	-9	7	7	7	χ_{122}
χ_{123}	o	67496615550	935550	-274050	-1026	-25515	0	0	-1890	-162	-5634	-2898	-2898	-2898	-354	-330	78	78	78	126	102	54	78	0	405	0	117	0	0	-27	0	0	0	0	0	18	18	70	-18	-2	-2	-2	-14	-6	-6	-6	χ_{123}
χ_{124}	o	67496615550	935550	-274050	-1026	-25515	0	0	-1890	-162	-5634	-2898	-2898	-2898	-354	-330	78	78	78	126	102	54	78	0	405	0	117	0	0	-27	0	0	0	0	0	18	18	70	-18	-2	-2	-2	-14	-6	-6	-6	χ_{124}
χ_{125}	+	68719476736	0	0	0	-32768	4096	-512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-64	0	0	0	0	0	0	0	0	0	64	8	0	0	0	0	0	0	0	0	0	0	0	χ_{125}
χ_{126}	+	85668781275	23295195	-654885	-70821	0	0	0	-40005	-9477	-1701	1323	1323	1323	1467	675	651	651	651	27	243	-285	75	0	0	0	0	0	0	0	0	0	0	0	0	-9	-9	-45	-21	-21	-21	-21	-9	-1	-1	-1	χ_{126}

	ind	1 2	2 2	2 2	2	3 6	3 6	3 6	4 4	4	4	4 4	4	4	4	4 4	4 4	4	4	4 4	4	4	4	5 10	6 6	6 6	6 6	6 6	6 6	6	6 6	6 6	6 14	7 14	7	8	8 8	8	8	8 8	8	8	8	8 8	8	8		
χ_{127}	+	2432	384	128	0	56	29	2	64	0	0	16	0	0	0	16	16	0	0	16	0	0	0	7	24	-3	8	6	5	0	2	-3	6	10	3	0	8	0	0	4	0	0	0	4	0	0	χ_{127}
χ_{128}	+	45696	-1408	896	0	120	147	12	-64	0	0	112	0	0	0	-16	-16	0	0	48	0	0	0	21	-110	-13	8	-4	11	0	-4	3	12	0	7	0	-8	0	0	-4	0	0	0	12	0	0	χ_{128}
χ_{129}	+	537472	19328	3712	0	496	307	-44	832	0	0	208	0	0	0	80	-48	0	0	80	0	0	0	22	176	-13	16	-4	19	0	4	3	12	26	5	0	8	0	0	-12	0	0	0	4	0	0	χ_{129}
χ_{130}	+	4667520	-40832	17792	0	1056	1245	-24	-832	0	0	432	0	0	0	-80	48	0	0	240	0	0	0	70	-224	-35	32	-8	29	0	8	-3	24	-10	11	0	-8	0	0	12	0	0	0	12	0	0	χ_{130}
χ_{131}	+	5374720	127744	12544	0	2584	505	46	3200	0	0	-224	0	0	0	160	32	0	0	160	0	0	0	20	376	25	40	34	1	0	-2	9	18	64	1	0	48	0	0	8	0	0	0	-8	0	0	χ_{131}
χ_{132}	+	17736576	277376	20096	0	3696	-174	168	4816	0	0	464	0	0	0	336	80	0	0	16	0	0	0	1	176	50	80	32	26	0	8	18	0	46	-3	0	40	0	0	-12	0	0	0	4	0	0	χ_{132}
χ_{133}	+	21909888	215424	34944	0	792	1953	36	2624	0	0	784	0	0	0	144	144	0	0	208	0	0	0	63	504	-63	-24	-36	9	0	-12	-15	12	0	7	0	8	0	0	4	0	0	0	20	0	0	χ_{133}
χ_{134}	+	61395840	-38016	73344	0	0	3375	-54	-1728	0	0	2256	0	0	0	-48	-176	0	0	272	0	0	0	90	0	-81	0	54	15	0	-6	15	6	-26	9	0	8	0	0	-12	0	0	0	20	0	0	χ_{134}
χ_{135}	+	90419328	1005696	77184	0	7776	1701	0	12480	0	0	432	0	0	0	560	48	0	0	240	0	0	0	28	864	-27	96	0	21	0	0	-27	0	90	-1	0	24	0	0	12	0	0	0	-4	0	0	χ_{135}
χ_{136}	+	106419456	-498432	96000	0	5544	2871	36	-3200	0	0	-544	0	0	0	-160	-32	0	0	480	0	0	0	56	-696	87	24	-12	15	0	-12	-9	36	10	3	0	-48	0	0	-8	0	0	0	-24	0	0	χ_{136}
χ_{137}	+	289697408	325248	51072	0	-4312	728	260	4032	0	0	1008	0	0	0	-400	752	0	0	112	0	0	0	-42	456	24	24	60	24	0	-12	-24	12	0	0	0	-40	0	0	-4	0	0	0	12	0	0	χ_{137}
χ_{138}	+	4434144400	3329920	133760	0	21120	-300	-174	23360	0	0	336	0	0	0	720	-304	0	0	16	0	0	0	0	640	-44	128	118	20	0	2	-12	6	100	-5	0	104	0	0	-12	0	0	0	4	0	0	χ_{138}
χ_{139}	+	526722560	2426368	197120	0	3752	4130	134	12544	0	0	-448	0	0	0	64	-448	0	0	320	0	0	0	35	1480	130	-40	-14	26	0	38	18	18	0	0	0	-32	0	0	-16	0	0	0	-16	0	0	χ_{139}
χ_{140}	+	662165504	3627008	149504	0	12320	278	-262	21504	0	0	-768	0	0	0	256	256	0	0	256	0	0	0	-21	800	-10	32	-10	14	0	-22	6	6	70	0	0	0	0	0	0	0	0	0	0	0	0	χ_{140}
χ_{141}	+	662165504	3627008	149504	0	12320	278	-262	21504	0	0	-768	0	0	0	256	256	0	0	256	0	0	0	-21	800	-10	32	-10	14	0	-22	6	6	70	0	0	0	0	0	0	0	0	0	0	0	0	χ_{141}
χ_{142}	+	869092224	-1547392	353920	0	3696	7098	294	-5824	0	0	4816	0	0	0	-176	80	0	0	400	0	0	0	49	-976	-166	16	-22	82	0	22	-6	-6	0	0	0	40	0	0	-12	0	0	0	4	0	0	χ_{142}
χ_{143}	+	886828800	3055360	226560	0	2640	1875	-24	10880	0	0	5024	0	0	0	800	-96	0	0	-96	0	0	0	0	400	-77	48	40	75	0	-24	3	-24	-10	4	0	-80	0	0	-24	0	0	0	-8	0	0	χ_{143}
χ_{144}	+	1175451264	-2851200	339840	0	13608	6804	0	-6720	0	0	1008	0	0	0	-400	-144	0	0	432	0	0	0	64	-1080	-108	24	0	-60	0	0	36	0	36	-6	0	-72	0	0	-4	0	0	0	12	0	0	χ_{144}
χ_{145}	+	1361048832	-228096	366336	0	0	7047	0	-9344	0	0	-2592	0	0	0	96	480	0	0	480	0	0	0	57	0	135	0	0	-9	0	0	-9	0	-74	3	0	80	0	0	24	0	0	0	-24	0	0	χ_{145}
χ_{146}	+	1521172224	6614784	140544	0	21384	-1701	0	27264	0	0	1440	0	0	0	544	-96	0	0	-96	0	0	0	-1	-216	27	120	0	-21	0	0	27	0	54	5	0	48	0	0	8	0	0	0	-8	0	0	χ_{146}
χ_{147}	+	1862340480	6416256	-15744	0	22176	-180	630	15680	0	0	336	0	0	0	-560	80	0	0	208	0	0	0	-20	-96	12	-96	138	-36	0	6	12	-6	56	0	0	72	0	0	-12	0	0	0	4	0	0	χ_{147}
χ_{148}	+	1975209600	7952000	309120	0	21000	525	300	29120	0	0	-784	0	0	0	880	624	0	0	-80	0	0	0	0	680	77	120	68	45	0	60	-3	-12	0	0	0	-8	0	0	28	0	0	0	12	0	0	χ_{148}
χ_{149}	+	2128389120	3007488	494592	0	-5544	7470	-252	-512	0	0	2944	0	0	0	640	384	0	0	128	0	0	0	70	696	-114	-24	12	54	0	-12	-18	-36	-80	-3	0	0	0	0	32	0	0	0	0	0	0	χ_{149}
χ_{150}	+	2606204160	2757888	403200	0	-13608	8505	0	8064	0	0	2016	0	0	0	-416	-288	0	0	96	0	0	0	35	1080	-135	-24	0	-39	0	0	9	0	0	0	0	-48	0	0	-8	0	0	0	8	0	0	χ_{150}
χ_{151}	+	3342662400	-2069760	170240	0	18480	-2625	-564	-22400	0	0	5152	0	0	0	160	800	0	0	-96	0	0	0	0	240	159	80	60	-25	0	-4	-33	12	0	0	0	-80	0	0	-24	0	0	0	-8	0	0	χ_{151}
χ_{152}	+	3352212864	10251648	271488	0	21384	2268	-324	25152	0	0	1680	0	0	0	-112	656	0	0	144	0	0	0	14	936	-36	-72	-36	-84	0	12	12	12	0	-7	0	72	0	0	4	0	0	0	4	0	0	χ_{152}
χ_{153}	+	3476368896	-6189568	498176	0	9240	1806	-120	5376	0	0	8512	0	0	0	-1216	-192	0	0	-192	0	0	0	21	-520	110	104	-88	38	0	8	30	-24	0	0	0	32	0	0	16	0	0	0	-16	0	0	χ_{153}
χ_{154}	o	3792402432	-6752256	43008	0	7056	3654	90	21504	0	0	-1792	0	0	0	256	256	0	0	-256	0	0	0	7	-816	102	-144	-42	30	0	42	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{154}
χ_{155}	o	3792402432	-6752256	43008	0	7056	3654	90	21504	0	0	-1792	0	0	0	256	256	0	0	-256	0	0	0	7	-816	102	-144	-42	30	0	42	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{155}
χ_{156}	o	5267225600	5913600	-322560	0	27440	5075	44	-17920	0	0	-896	0	0	0	-640	128	0	0	-384	0	0	0	0	240	51	-48	60	75	0	-36	3	12	0	0	0	64	0	0	0	0	0	0	0	0	0	χ_{156}
χ_{157}	o	5267225600	5913600	-322560	0	27440	5075	44	-17920	0	0	-896	0	0	0	-640	128	0	0	-384	0	0	0	0	240	51	-48	60	75	0	-36	3	12	0	0	0	64	0	0	0	0	0	0	0	0	0	χ_{157}
χ_{158}	+	5348259840	16873472	501760	0	29568	1470	264	50176	0	0	-1792	0	0	0	256	-768	0	0	256	0	0	0	-35	1280	-34	64	-16	-26	0	-8	-18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{158}
χ_{159}	+	5877256320	-6293376	261504	0	-7776	8505	0	20160	0	0	432	0	0	0	-80	432	0	0	-336	0	0	0	20	-864	-135	-96	0	-39	0	0	9	0	54	-2	0	24	0	0	12	0	0	0	12	0	0	χ_{159}
χ_{160}	+	6962288256	-10150272	524160	0	21384	-6804	0	960	0	0	1008	0	0	0	-1040	240	0	0	-144	0	0	0	56	-216	108	120	0	60	0	0	-36	0	0	-7	0	-72	0	0	-4	0	0	0	28	0	0	χ_{160}
χ_{161}	+	7662200832	-9934848	915456	0	28512	3402	-324	-21504	0	0	-5376	0	0	0	-256	-256	0	0	768	0	0	0	-43	-1440	234	96	-36	18	0	12	-6	12	-36	-8	0	0	0	0	0	0	0	0	0	0	0	χ_{161}
χ_{162}	+	8690922240	14803712	672000	0	-7392	5145	-138	27776	0	0	3360	0	0	0	416	32	0	0	160	0	0	0	-35	800	89	-96	-118	9	0	6	57	-6	0	0	0	-80	0	0	-24	0	0	0	-8	0	0	χ_{162}
χ_{163}	+	10906255360	26492928	573440	0	32032	-6020	-8	57344	0	0	0	0	0	0	1024	0	0	0	0	0	0	0	35	-480	60	32	-120	-4	0	-40	12	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{163}
χ_{164}	+	12929963904	-2166912	825984	0	0	-4131	0	-24256	0	0	6480	0	0	0	336	-432	0	0	-48	0	0	0	-21	0	189	0	0	117	0	0	45	0	-10	-3	0	40	0	0	-12	0	0	0	20	0	0	χ_{164}
χ_{165}	+	12929963904	-2166912	825984	0	0	11178	0	-24256	0	0	6480	0	0	0	336	-432	0	0	-48	0	0	0	-21	0	-54	0	0	-126	0	0	-54	0	-10	-3	0	40	0	0	-12	0	0	0	20	0	0	χ_{165}
χ_{166}	+	13208911872	-11534336	1048576	0	384	9024	-480	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-28	-1664	64	-128	64	64	0	-32	0	0	100	2	0	0	0	0	0	0	0	0	0	0	0	χ_{166}
χ_{167}	+	15135211520	13697024	-65536	0	5984	-3520	1232	0	0	0	8192	0	0	0	0	0	0	0	0	0	0	0	20	-544	320	32	32	-64	0	-16	0	0	-26	2	0	0	0	0	0	0	0	0	0	0	0	χ_{167}
χ_{168}	+	17381844480	-670208	-835072	0	24024	5880	48	-16128	0	0	3136	0	0	0	320	64	0	0	320	0	0	0	-70	184	184	-88	40	56	0	-16	-24	-24	0	0	0	-32	0	0	16	0	0	0	16	0	0	χ_{168}
χ_{169}	+	19155502080	1115136	224256	0	-21384	5670	648	7680	0	0	8064	0	0	0	640	384	0	0	-384	0	0	0	-20	-936	198	72	-72	-66	0	-24	6	24	36	1	0	0	0	0	32	0	0	0	0	0	0	χ_{169}
χ_{170}	+	21727305600	-837760	1249920	0	-18480	5775	384	-20160	0	0	3920	0	0	0	-560	-688	0	0	-176	0	0	0	0	1040	-49	48	104	15	0	0	-33	-24	0	0	0	8	0	0	-12	0	0

$^2E_6(2)$

	8L	8M	8N	8O	8P	9A	9B	10A	10B	11A	B**	12A	12B	12C	12D	12E	F**	12G	12H	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	12S	12T	12U	12V	12W	12X	12Y	Z**	13A	14A	14B	15A	15B	16A	16B	16C	16D	16E	16F	16G	16H	16I	17A	B*	18A	B**	
χ_{100}	0	0	0	0	0	-4	2	0	0	0	0	-8	-8	-8	-8	4	4	8	24	-8	-8	6	-2	-2	0	0	0	0	0	0	0	0	-6	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{100}
χ_{101}	0	0	0	0	0	-4	2	0	0	0	0	-8	-8	-8	-8	4	4	8	-8	24	-8	-2	6	-2	0	0	0	0	0	0	0	0	2	-6	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{101}
χ_{102}	0	0	0	0	0	-4	2	0	0	0	0	-8	-8	-8	-8	4	4	8	-8	-8	24	-2	-2	6	0	0	0	0	0	0	0	0	2	2	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{102}
χ_{103}	1	1	1	5	5	0	0	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	χ_{103}
χ_{104}	1	1	1	5	5	0	0	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	χ_{104}
χ_{105}	0	0	0	0	0	0	0	6	6	0	0	0	0	-9	0	0	0	-1	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{105}
χ_{106}	0	0	0	0	0	0	-3	-8	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	8	-1	0	0	0	0	0	0	0	0	0	0	0	2	2	χ_{106}
χ_{107}	-1	-1	-1	-1	-1	0	0	0	0	-b11	**	27	0	0	-21	0	0	0	3	3	3	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	χ_{107}
χ_{108}	-1	-1	-1	-1	-1	0	0	0	0	**	-b11	27	0	0	-21	0	0	0	3	3	3	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	χ_{108}
χ_{109}	3	3	3	-1	-1	0	0	0	0	-b11	**	27	0	0	-21	0	0	0	3	3	3	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	1	1	1	1	0	0	0	0	χ_{109}
χ_{110}	3	3	3	-1	-1	0	0	0	0	**	-b11	27	0	0	-21	0	0	0	3	3	3	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	1	1	1	1	1	0	0	0	0	χ_{110}
χ_{111}	0	0	0	0	0	0	0	8	0	0	0	9	0	36	9	0	0	-12	9	9	9	0	0	0	1	1	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{111}
χ_{112}	0	0	0	0	0	0	0	2	-2	-1	-1	64	64	-8	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	5	-1	0	0	0	0	0	0	0	0	0	0	0	0	χ_{112}
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	18	0	-36	-6	0	0	12	6	6	6	0	0	0	2	6	0	-4	0	0	0	0	0	0	0	0	0	0	0	-2	0	4	-2	0	0	0	0	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	2	2	2	6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	-3	0	0	2	2	2	0	0	0	0	0	0	0	0	0	χ_{114}
χ_{115}	2	2	2	-2	-2	0	0	-2	2	0	0	-9	0	-9	-9	0	0	-9	-9	-9	-9	0	0	0	-1	-1	0	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	-5	1	2	2	2	0	0	0	0	0	0	0	0	0	χ_{115}
χ_{116}	11	11	11	3	3	0	0	0	0	0	0	-81	0	0	-9	0	0	0	3	3	3	0	0	0	-9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	χ_{116}
χ_{117}	-4	-4	-4	4	4	0	0	0	0	0	0	0	-36	0	0	-12	-12	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{117}
χ_{118}	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	χ_{118}
χ_{119}	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	χ_{119}
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	81	0	0	9	0	0	0	-3	-3	-3	0	0	0	9	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	χ_{120}
χ_{121}	-9	-9	-9	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-b17	*	0	0	χ_{121}
χ_{122}	-9	-9	-9	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	*-b17	0	0	0	χ_{122}
χ_{123}	2	2	2	-2	-2	0	0	0	0	-2b11	**	-27	0	0	21	0	0	0	-3	-3	-3	0	0	0	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	χ_{123}
χ_{124}	2	2	2	-2	-2	0	0	0	0	**	-2b11	-27	0	0	21	0	0	0	-3	-3	-3	0	0	0	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	χ_{124}
χ_{125}	0	0	0	0	0	4	-2	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-4	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	χ_{125}
χ_{126}	-9	-9	-9	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	3	3	3	1	1	1	1	1	1	0	0	0	0	χ_{126}

	8 8	8 8	8 8	8 8	8 8	9 18	9 18	10 10	10 10	11 22	11 22	12	12	12 12	12	12	12	12 12	12 12	12	12	12 12	12	12	12 12	12	12	12 12	12 12	12 12	12 12	12 12	12	12	12	12	13 26	14 14	14 14	15 30	15 30	16	16	16	16 16	16	16	16 16	16	16	17 34	17 34	18 18	18 18	
χ_{127}	4	0	0	0	4	2	2	-1	3	1	1	0	0	9	0	0	0	1	4	0	0	-2	0	0	4	0	0	1	1	4	-2	-2	-2	0	0	0	0	1	2	-1	4	1	0	0	0	2	0	0	2	0	0	1	1	0	0	χ_{127}
χ_{128}	-4	0	0	0	4	3	3	-3	1	2	2	0	0	-9	0	0	0	-1	4	0	0	4	0	0	-4	0	0	-1	3	0	0	0	-4	0	0	0	0	1	0	-1	-3	0	0	0	0	-2	0	0	2	0	0	0	0	-1	-1	χ_{128}
χ_{129}	4	0	0	0	4	1	1	-2	2	1	1	0	0	27	0	0	0	-5	-8	0	0	-8	0	0	8	0	0	-1	-1	-4	-4	-4	0	0	0	0	0	2	1	7	1	0	0	0	0	-2	0	0	-2	0	0	0	0	-1	-1	χ_{129}
χ_{130}	-4	0	0	0	4	3	3	-2	2	0	0	0	0	-27	0	0	0	5	0	0	0	0	0	0	-8	0	0	1	-3	0	0	0	0	0	0	0	0	0	-2	-1	-5	1	0	0	0	2	0	0	-2	0	0	0	0	1	1	χ_{130}
χ_{131}	-8	0	0	0	8	1	1	4	4	-1	-1	0	0	45	0	0	0	5	4	0	0	10	0	0	4	0	0	1	1	4	-2	-2	2	0	0	0	0	0	0	1	5	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{131}
χ_{132}	20	0	0	0	-4	3	0	1	1	0	0	0	0	-18	0	0	0	6	8	0	0	-4	0	0	0	0	0	6	-2	4	4	4	-4	0	0	0	0	0	6	1	1	1	0	0	0	2	0	0	-2	0	0	0	0	-1	-1	χ_{132}
χ_{133}	4	0	0	0	-4	0	0	-1	-1	0	0	0	0	45	0	0	0	5	4	0	0	-8	0	0	12	0	0	-3	1	4	4	4	0	0	0	0	0	0	0	-1	3	-3	0	0	0	2	0	0	-2	0	0	-1	-1	0	0	χ_{133}
χ_{134}	4	0	0	0	-4	0	0	-6	-6	0	0	0	0	-9	0	0	0	-9	0	0	0	6	0	0	0	0	0	3	-1	-4	2	2	-2	0	0	0	0	-1	-2	1	0	0	0	0	0	-2	0	0	2	0	0	0	0	0	0	χ_{134}
χ_{135}	12	0	0	0	4	0	0	-4	4	-1	-1	0	0	45	0	0	0	-3	0	0	0	0	0	0	8	0	0	5	-3	0	0	0	0	0	0	0	0	-1	2	-1	1	1	0	0	0	-2	0	0	2	0	0	0	0	0	0	χ_{135}
χ_{136}	8	0	0	0	8	0	0	8	0	0	0	0	0	-45	0	0	0	-5	-4	0	0	-4	0	0	-4	0	0	-1	3	0	0	0	4	0	0	0	0	0	2	3	-4	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{136}
χ_{137}	12	0	0	0	-4	-4	2	-2	2	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	-4	0	0	-4	4	4	4	4	8	0	0	0	0	0	0	0	3	3	0	0	0	2	0	0	2	0	0	0	0	0	0	χ_{137}
χ_{138}	-12	0	0	0	-4	3	0	0	0	0	0	0	0	-36	0	0	0	-4	0	0	0	-6	0	0	0	0	0	0	4	-8	-2	-2	2	0	0	0	0	4	-1	0	0	0	0	0	0	2	0	0	-2	0	0	0	0	1	1	χ_{138}
χ_{139}	-16	0	0	0	0	-1	-1	3	-5	1	1	0	0	54	0	0	0	-2	-4	0	0	2	0	0	4	0	0	-2	2	-4	2	2	2	0	0	0	0	0	0	0	5	2	0	0	0	0	0	0	0	0	0	0	0	1	1	χ_{139}
χ_{140}	0	0	0	0	0	-1	-1	3	-1	0	0	0	0	18	0	0	0	-6	0	0	0	6	0	0	-8	0	0	-2	-2	4	-14	10	-2	0	0	0	0	0	-2	0	3	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{140}
χ_{141}	0	0	0	0	0	-1	-1	3	-1	0	0	0	0	18	0	0	0	-6	0	0	0	6	0	0	-8	0	0	-2	-2	4	10	-14	-2	0	0	0	0	0	-2	0	3	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{141}
χ_{142}	-12	0	0	0	-4	-3	-3	-7	5	0	0	0	0	-18	0	0	0	-10	-8	0	0	10	0	0	-8	0	0	-2	-2	4	-2	-2	2	0	0	0	0	0	0	0	-2	1	0	0	0	2	0	0	-2	0	0	0	0	-1	-1	χ_{142}
χ_{143}	24	0	0	0	8	0	-3	0	0	0	0	0	0	-9	0	0	0	-1	8	0	0	-16	0	0	8	0	0	-1	3	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	χ_{143}
χ_{144}	-4	0	0	0	4	0	0	0	0	-2	-2	0	0	-36	0	0	0	12	12	0	0	0	0	0	-4	0	0	-4	0	0	0	0	0	0	0	0	0	0	4	-2	4	-2	0	0	0	-2	0	0	2	0	0	0	0	0	0	χ_{144}
χ_{145}	8	0	0	0	-8	0	0	9	1	0	0	0	0	-9	0	0	0	7	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0	0	0	0	0	-2	-1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{145}
χ_{146}	24	0	0	0	-8	0	0	-1	-1	0	0	0	0	-45	0	0	0	3	12	0	0	0	0	0	4	0	0	-5	3	0	0	0	0	0	0	0	0	0	-2	1	-1	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	χ_{146}
χ_{147}	4	0	0	0	4	0	0	-4	-4	0	0	0	0	36	0	0	0	20	0	0	0	-6	0	0	-8	0	0	4	4	4	-2	-2	2	0	0	0	0	0	-8	0	-5	1	0	0	0	-2	0	0	-2	0	0	0	0	0	0	χ_{147}
χ_{148}	-4	0	0	0	4	0	-3	0	0	1	1	0	0	-27	0	0	0	5	-4	0	0	8	0	0	4	0	0	1	1	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	0	0	0	0	2	2	χ_{148}
χ_{149}	0	0	0	0	0	0	0	-2	2	0	0	0	0	18	0	0	0	10	4	0	0	-8	0	0	4	0	0	-2	2	-4	-4	-4	0	0	0	0	0	0	0	1	-5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{149}
χ_{150}	8	0	0	0	-8	0	0	3	-5	-1	-1	0	0	9	0	0	0	9	-12	0	0	0	0	0	4	0	0	1	-3	0	0	0	0	0	0	0	0	1	0	0	5	2	0	0	0	0	0	0	0	0	1	1	0	0	0	χ_{150}
χ_{151}	-8	0	0	0	-8	6	0	0	0	0	0	0	0	27	0	0	0	19	-8	0	0	4	0	0	-8	0	0	-5	3	0	0	0	-4	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{151}
χ_{152}	4	0	0	0	4	0	0	-2	-2	0	0	0	0	36	0	0	0	-12	12	0	0	-12	0	0	-4	0	0	-4	0	0	0	0	-4	0	0	0	0	0	0	1	-7	-1	0	0	0	2	0	0	2	0	0	0	0	0	0	χ_{152}
χ_{153}	-16	0	0	0	0	3	0	-3	1	0	0	0	0	18	0	0	0	-6	4	0	0	16	0	0	-4	0	0	2	6	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{153}
χ_{154}	0	0	0	0	0	0	0	-1	3	b11	**	0	0	18	0	0	0	-6	8	0	0	-10	0	0	-8	0	0	-2	2	-4	2	2	-2	0	0	0	0	0	0	0	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{154}
χ_{155}	0	0	0	0	0	0	0	-1	3	**	b11	0	0	18	0	0	0	-6	8	0	0	-10	0	0	-8	0	0	-2	2	-4	2	2	-2	0	0	0	0	0	0	0	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{155}
χ_{156}	0	0	0	0	0	-4	2	0	0	2b11	**	0	0	-9	0	0	0	-1	-8	0	0	4	0	0	8	0	0	-1	3	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{156}
χ_{157}	0	0	0	0	0	-4	2	0	0	**	2b11	0	0	-9	0	0	0	-1	-8	0	0	4	0	0	8	0	0	-1	3	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{157}
χ_{158}	0	0	0	0	0	-3	0	-3	5	0	0	0	0	18	0	0	0	10	-16	0	0	8	0	0	-8	0	0	-2	-2	-8	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{158}
χ_{159}	12	0	0	0	-4	0	0	4	4	1	1	0	0	9	0	0	0	9	0	0	0	0	0	0	-8	0	0	1	-3	0	0	0	0	0	0	0	0	1	0	0	-5	-2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	χ_{159}
χ_{160}	-4	0	0	0	-4	0	0	8	0	0	0	0	0	36	0	0	0	-12	12	0	0	0	0	0	4	0	0	4	0	0	0	0	0	0	0	0	0	0	-2	2	5	-1	0	0	0	-2	0	0	-2	0	0	0	0	0	0	χ_{160}
χ_{161}	0	0	0	0	0	0	0	-3	1	0	0	0	0	-18	0	0	0	6	0	0	0	-12	0	0	8	0	0	2	-6	0	0	0	-4	0	0	0	0	1	0	1	-4	-1	0	0	0	-2	0	0	-2	0	0	0	0	0	0	χ_{161}
χ_{162}	-8	0	0	0	-8	0	3	-3	5	0	0	0	0	9	0	0	0	-7	0	0	0	-6	0	0	8	0	0	5

$^2E_6(2)$

	18C	18D	19A	B**	20A	20B	21A	21B	22A	B**	24A	24B	24C	24D	24E	24F	28A	28B	28C	28D	30A	30B	33A	B**	35A	B**			2D	2E	4P	4Q	4R	4S	4T	4U	6J	6K	6L	6M	6N	6O	8Q	8R	8S	8T	8U	8V	8W	8X	8Y	8Z	8A1	8B1	8C1	
X_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	17326400	-20160	2880	-2240	-448	320	-448	64	140	140	60	68	12	0	-448	0	0	0	64	0	0	0	0	0	0	0	0	X_{100}
X_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{101}
X_{102}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																														X_{102}
X_{103}	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	b35	**		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{103}
X_{104}	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	**	b35																														X_{104}
X_{105}	0	0	0	0	-2	-2	-1	0	0	0	1	1	0	0	0	0	-1	-1	-1	1	0	0	0	0	1	1	:	++	10740600	-42120	-7560	120	2808	120	248	-72	0	-405	0	0	27	0	216	40	24	-24	24	24	-24	-24	-8	-8	-24	8	8	X_{105}
X_{106}	-1	0	0	0	0	0	-1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	1	:	++	1114112	65536	0	0	0	0	0	0	-448	-160	64	-16	-32	16	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{106}
X_{107}	0	0	0	0	0	0	0	0	0	b11	**	0	0	-1	-1	-1	-1	0	0	0	0	0	0	b11	**	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{107}
X_{108}	0	0	0	0	0	0	0	0	**	b11	0	0	-1	-1	-1	-1	0	0	0	0	0	0	**	b11	0	0																														X_{108}
X_{109}	0	0	0	0	0	0	0	0	b11	**	0	0	-1	-1	-1	-1	0	0	0	0	0	0	b11	**	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{109}
X_{110}	0	0	0	0	0	0	0	0	**	b11	0	0	-1	-1	-1	-1	0	0	0	0	0	0	**	b11	0	0																														X_{110}
X_{111}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	-1	0	0	0	0	:	++	3508596	-39564	3060	4340	-3276	372	52	-12	189	-378	-51	0	6	0	-252	100	-12	4	-60	-12	4	4	-12	-12	-12	12	-4	X_{111}
X_{112}	0	0	0	0	2	0	0	1	1	1	0	0	0	0	0	0	0	0	0	-1	1	-1	-1	-1	0	0	:	++	11880960	35328	-7680	-3584	-512	512	-512	0	456	-84	-24	-12	12	12	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{112}
X_{113}	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	-2	-2	-2	0	0	0	0	0	1	1	:	++	14034384	35280	-5040	3920	-1008	-816	16	-48	-378	-378	30	0	6	0	0	-80	0	16	0	64	32	0	32	0	0	0	0	X_{113}
X_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	0	0	0	0	0	0	:	++	8055450	72090	7290	-1350	1674	-486	138	42	0	0	0	0	0	0	-162	30	6	-90	-18	-10	-30	42	6	14	18	-6	-6	X_{114}
X_{115}	0	0	0	0	-2	0	0	0	0	0	3	3	-1	-1	-1	-1	0	0	0	0	-1	1	0	0	0	0	:	++	8771490	46242	-5310	-4606	882	-222	-142	18	189	189	-51	0	-3	0	126	-50	-42	-26	-18	38	10	34	-10	14	-6	2	2	X_{115}
X_{116}	0	0	0	0	0	0	0	-1	0	0	0	0	3	1	1	1	0	0	0	1	0	0	0	0	0	0	:	++	9398025	-2295	-2295	-9975	-2295	393	9	105	405	0	45	0	0	0	441	-15	57	-15	-39	-7	17	-15	9	-23	-15	-15	1	X_{116}
X_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	14619150	-57330	-2610	3150	3822	-114	-18	14	0	0	0	-54	0	18	-378	30	6	-18	6	-26	54	6	-10	6	6	-18	-2	X_{117}
X_{118}	0	0	b19	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{118}
X_{119}	0	0	**	b19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																														X_{119}
X_{120}	0	0	0	0	0	0	0	1	0	0	0	0	-3	-1	-1	-1	1	1	1	-1	0	0	0	0	0	0	:	++	10740600	27000	1080	-8520	216	120	216	24	-405	0	-45	0	0	0	288	0	96	-48	0	0	-8	-24	0	-16	-24	0	0	X_{120}
X_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	11609325	-2835	-2835	-2835	-2835	-531	237	45	0	0	0	0	0	0	189	45	-3	45	-3	-3	-3	-3	-35	29	-3	-3	-3	X_{121}
X_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	11609325	-2835	-2835	-2835	-2835	-531	237	45	0	0	0	0	0	0	189	45	-3	45	-3	-3	-3	-3	-35	29	-3	-3	-3	X_{122}
X_{123}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	b11	**	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{123}
X_{124}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	**	b11	0	0																														X_{124}
X_{125}	0	0	1	1	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	:	++	16777216	0	0	0	0	0	0	0	-512	-512	0	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{125}
X_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	15181825	-59535	5265	-9135	3969	369	129	-63	0	0	0	0	0	0	189	-75	21	45	-51	-27	-15	9	37	-3	33	9	-7	X_{126}

	18 18	18 18	19 38	19 38	20 20	20 20	21 42	21 42	22 22	22 22	24 24	24 24	24	24 24	24	24	28 28	28	28	28 28	30 30	30 30	33 66	33 66	35 70	35 70	fus	ind		2 2	2 2	4 4	4 4	4 4	4 4	4 4	6 6	6 6	6 6	6 6	6 6	6 6	8 8	8 8	8 8	8 8	8 8	8 8	8 8	8 8	8 8	8 8	8 8		
X_{127}	0	2	0	0	-1	1	1	0	-1	-1	3	-1	0	-2	0	0	2	0	0	1	0	-1	1	1	0	0	:	++	220	36	36	28	20	4	12	4	-14	13	6	4	3	0	0	4	8	4	0	8	0	0	0	0	0	4	0	X_{127}
X_{128}	-1	-1	1	1	1	-1	0	1	0	0	-3	1	0	2	0	0	0	0	0	-1	1	0	-1	-1	0	0	:	++	2380	-76	-76	140	36	20	-4	-12	22	49	2	4	-1	-4	0	4	24	4	0	-8	0	0	0	0	0	-4	0	X_{128}
X_{129}	-1	1	0	0	2	0	-1	-1	1	1	3	-1	0	0	0	0	-2	0	0	-1	-1	1	1	1	1	1	:	++	11492	-484	412	292	-20	60	-12	-4	-64	71	-16	-10	-7	2	0	-20	24	12	0	24	0	0	0	0	0	-4	0	X_{129}
X_{130}	1	-1	-1	-1	-2	0	-1	-1	0	0	-3	1	0	0	0	0	-2	0	0	1	-1	1	0	0	0	0	:	++	57460	652	-500	1204	-36	44	4	12	76	211	4	4	13	4	0	-20	72	12	0	-24	0	0	0	0	0	4	0	X_{130}
X_{131}	1	1	0	0	0	0	1	1	1	1	3	3	0	2	0	0	0	0	0	1	1	1	-1	-1	-1	-1	:	++	8840	1912	248	392	344	-72	104	-8	-106	29	34	-16	19	4	0	40	16	8	0	16	0	0	0	0	0	8	0	X_{131}
X_{132}	2	-1	0	0	1	1	1	0	0	0	0	4	0	0	0	0	2	0	0	-1	1	1	0	0	1	1	:	++	78676	-3156	1196	-236	-708	76	-92	-20	196	106	-36	-2	6	-6	0	-36	-24	-36	0	8	0	0	0	0	0	-12	0	X_{132}
X_{133}	0	0	0	0	-1	-1	0	1	0	0	3	-1	0	-2	0	0	0	0	0	-1	-1	-1	0	0	0	0	:	++	162708	876	1644	1876	-132	12	100	-20	-174	231	6	6	9	-6	0	-4	72	-4	0	40	0	0	0	0	0	4	0	X_{133}
X_{134}	0	0	0	0	2	2	1	0	0	0	3	-1	0	0	0	0	2	0	0	1	0	0	0	0	-1	-1	:	++	413100	-2220	-300	3180	132	180	-100	20	0	405	0	0	-21	12	0	20	88	-12	0	-8	0	0	0	0	0	-4	0	X_{134}
X_{135}	0	0	0	0	0	0	0	-1	-1	-1	-3	-3	0	0	0	0	-2	0	0	-1	1	-1	-1	-1	0	0	:	++	302940	3492	3492	1948	468	132	204	36	-216	189	24	0	3	0	0	20	24	20	0	56	0	0	0	0	0	-4	0	X_{135}
X_{136}	0	0	0	0	0	0	1	0	0	0	-3	-3	0	-2	0	0	2	0	0	-1	0	-1	0	0	0	0	:	++	270504	-1704	-1320	1960	568	152	-120	24	126	153	-6	-36	-9	-12	0	40	48	8	0	-16	0	0	0	0	0	-8	0	X_{136}
X_{137}	0	0	0	0	2	0	0	0	0	0	0	-4	0	2	0	0	0	0	0	0	-1	1	0	0	0	0	:	++	563108	4956	732	-28	812	380	180	-68	14	14	-6	50	18	6	0	-20	24	-52	0	24	0	0	0	0	0	12	0	X_{137}
X_{138}	-2	-1	0	0	0	0	1	1	0	0	0	-4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	:	++	508300	-10380	3700	460	-540	-44	-196	-12	-20	160	-60	-2	0	6	0	-60	-8	4	0	24	0	0	0	0	0	12	0	X_{138}
X_{139}	1	-1	0	0	-1	-1	0	0	-1	-1	0	4	0	2	0	0	0	0	0	0	1	0	1	1	0	0	:	++	866320	-13328	3824	2576	-336	112	-304	-16	-182	196	-50	52	-20	-8	0	-16	32	-16	0	32	0	0	0	0	0	0	0	X_{139}
X_{140}	-1	-1	0	0	-1	1	-2	0	0	0	0	0	0	0	0	0	2	0	0	0	-1	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{140}
X_{141}	-1	-1	0	0	-1	1	-2	0	0	0	0	0	0	0	0	0	2	0	0	0	-1	0	0	0	0	0																														X_{141}
X_{142}	-1	1	0	0	1	-1	0	0	0	0	0	4	0	0	0	0	0	0	0	0	2	-1	0	0	0	0	:	++	2382380	-4396	-4396	7532	-700	564	220	-44	224	602	-16	8	-10	-4	0	36	56	36	0	-40	0	0	0	0	0	12	0	X_{142}
X_{143}	1	0	0	0	0	0	-1	1	0	0	-3	1	0	0	0	0	-2	0	0	2	0	0	0	0	0	0	:	++	2077400	-11480	7080	2520	72	488	-8	-24	20	335	-20	20	1	4	0	-40	-48	56	0	16	0	0	0	0	0	-8	0	X_{143}
X_{144}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	1	1	1	1	:	++	2076516	-5220	-5220	7460	1260	-324	-140	-36	378	378	30	0	6	0	0	-20	72	-20	0	-24	0	0	0	0	0	-12	0	X_{144}
X_{145}	0	0	1	1	1	1	-2	0	0	0	3	-1	0	0	0	0	-2	0	0	1	1	0	0	0	1	1	:	++	1288872	24408	216	5544	-1224	-360	136	24	0	243	0	0	45	0	0	-56	48	-24	0	-16	0	0	0	0	0	8	0	X_{145}
X_{146}	0	0	0	0	-1	-1	0	-1	0	0	3	3	0	2	0	0	-2	0	0	-1	-1	-1	0	0	-1	-1	:	++	1272024	-4824	5544	-1064	-2232	-24	-264	-24	594	189	6	0	3	0	0	56	-48	-40	0	16	0	0	0	0	0	8	0	X_{146}
X_{147}	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	1	1	:	++	649740	-20748	1908	-308	-1820	-428	-68	20	84	84	-36	12	12	24	0	-60	8	36	0	8	0	0	0	0	0	-12	0	X_{147}
X_{148}	-1	0	0	0	0	0	0	0	1	1	-3	1	0	-2	0	0	0	0	0	0	0	0	1	1	0	0	:	++	1082900	37100	5100	980	2940	-116	356	44	-70	245	110	38	11	2	0	60	-24	-4	0	8	0	0	0	0	0	4	0	X_{148}
X_{149}	0	0	0	0	-2	0	1	0	0	0	0	0	0	2	0	0	4	0	0	-1	-1	1	0	0	0	0	:	++	3500640	13728	5280	9056	160	160	96	32	-126	414	6	-18	18	-6	0	0	0	64	0	0	0	0	0	0	0	0	0	X_{149}
X_{150}	0	0	0	0	-1	-1	0	0	1	1	-3	-3	0	-2	0	0	0	0	0	0	1	0	-1	-1	0	0	:	++	3651480	-6552	3816	8344	-504	-216	-200	-24	-378	189	-30	0	3	0	0	56	48	-40	0	48	0	0	0	0	0	-8	0	X_{150}
X_{151}	0	2	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	:	++	3498600	-4200	-2280	-4760	-1288	-296	456	24	-420	525	-60	-60	3	12	0	40	-48	-56	0	-48	0	0	0	0	0	-8	0	X_{151}
X_{152}	0	0	0	0	2	-2	0	-1	0	0	0	0	0	-2	0	0	0	0	0	1	1	1	0	0	0	0	:	++	3063060	9708	9708	2772	1980	-116	164	12	-414	126	6	-36	-30	-12	0	-20	24	-20	0	24	0	0	0	0	0	12	0	X_{152}
X_{153}	2	-1	0	0	1	-1	0	0	0	0	0	-4	0	-2	0	0	0	0	0	0	-2	0	0	0	0	0	:	++	5371184	-2352	-9776	2352	784	592	-144	-48	-70	560	30	20	0	-12	0	-16	-96	-16	0	32	0	0	0	0	0	0	0	X_{153}
X_{154}	0	0	0	0	-1	1	0	0	b11	**	0	0	0	0	0	0	0	0	0	0	0	-1	b11	**	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{154}
X_{155}	0	0	0	0	-1	1	0	0	**	b11	0	0	0	0	0	0	0	0	0	0	0	-1	**	b11	0	0																														X_{155}
X_{156}	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	b11	**	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{156}
X_{157}	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	**	b11	0	0																														X_{157}
X_{158}	2	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	:	++	2132480	-10752	6656	3584	0	512	0	0	-448	-196	0	20	-12	12	0	-64	0	64	0	0	0	0	0	0	0	0	0	X_{158}
X_{159}	0	0	0	0	0	0	0	1	-1	-1	-3	-3	0	0	0	0	-2	0	0	0	1	1	1	1	-1	-1	:	++	4654260	15948	-4788	7028	-1764	-276	68	108	216	-189	-24	0	-3	0	0	-20	24	-20	0	24	0	0	0	0	0	-4	0	X_{159}
X_{160}	0	0	0	0	0	0	0	-1	0	0	0	0	0	2	0	0	0	0	0	1	0	-1	0	0	0	0	:	++	2274804	17676	-3060	-2380	-3492	-84	4	108	-594	378	-6	0	6	0	0	-20	-72	-20	0	-8	0	0	0	0	0	12	0	X_{160}
X_{161}	0	0	0	0	1	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-1	-1	:	++	1018368	-10752	-1536	-4608	0	512	0	0	-288	252	0	36	-12	12	0	64	0	-64	0	0	0	0	0	0	0	0	0	X_{161}
X_{162}	-1	0	0	0	1	1	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	:	++	7363720	-41608	12024	3976	-1960	440	-152	-8	28	91	20	-44	5	8	0	-56	16	-24	0	16	0	0	0	0	0	-8	0	X_{162}
X_{163}	0	2	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	:	++	2446080	-37632	5376	-5376	-1792	256	-256	0	672	294	0	-12	-6	-12	0	-64	0	-64	0	0	0	0	0	0	0	0	0	X_{163}
X_{164}	0	0	0	0	-1	1	-1	0	0	0	3	-5	0	0	0	0	-2	0	0	-1	2	0	0	0	0	0	:	++	9236916	-50868	-2484	-1932	3996	492	-316	-84	0	405	0	0	27	0	0	76	-72	12	0	-8	0	0	0	0	0	-4	0	X_{164}
X_{165}	0	0	0	0	-1	1	-1	0	0	0	0	4	0	0	0	0	-2	0	0	-1	-1	0	0	0	0	0	:	++	11814660	-2052	-2052	9156	1548	-228	-44	-36	0	162	0	0	-18	0	0	-36	24	-36	0	-40	0	0	0	0	0	12	0	X_{165}
X_{166}	1	1	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	-1	0	0

$^2E_6(2)$

	10C	10D	12A	112B	112C	112D	112E	112F	112G	112H	112I	112J	112K	112L	1	14C	14D	14E	16J	K**	16L	16M	16N	18E	18F	18G	20C	20D	24G	24H	24I	24J	24K	24L	24M	24N	24O	24P	Q**	24R	26A	28E	28F	30C	30D	30E	32A	B**	34A	B*	36A	40A	42A	42B	48A	B**	56A	60A	
χ_{100}	0	0	-12	-20	12	0	4	-4	-4	-4	2	-4	-4	-2	0	0	0	0	0	0	0	0	0	2	2	0	0	0	2	0	0	-4	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{100}
χ_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{101}
χ_{102}																																																											χ_{102}
χ_{103}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{103}
χ_{104}																																																											χ_{104}
χ_{105}	0	0	0	15	-9	0	0	-9	0	3	0	0	-1	0	-11	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	1	0	0	0	0	-1	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	χ_{105}
χ_{106}	12	-4	0	0	0	0	0	0	0	0	0	0	0	0	-8	6	2	0	0	0	0	0	0	2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-3	-1	0	0	0	0	0	0	1	0	0	0	0	0	χ_{106}
χ_{107}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{107}
χ_{108}																																																											χ_{108}
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{109}
χ_{110}																																																											χ_{110}
χ_{111}	-4	-4	21	-10	6	0	5	6	-3	6	0	0	-2	0	0	0	0	0	4	4	4	0	0	0	0	0	0	0	0	1	-2	-3	1	1	-2	0	0	0	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	1	1	0	0	χ_{111}
χ_{112}	10	-2	24	4	-12	-12	-8	4	8	-4	-8	-4	4	0	0	7	-1	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	-1	χ_{112}
χ_{113}	-16	0	-6	-10	6	0	2	6	6	6	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	-4	0	-2	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{113}
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	-3	6	6	2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	χ_{114}
χ_{115}	-10	2	21	-19	-3	0	5	-3	-3	-3	0	0	5	0	0	0	0	-2	-2	2	-4	0	0	0	0	4	0	0	1	1	-3	1	1	1	0	0	0	0	-1	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	1	1	0	1	χ_{115}
χ_{116}	0	0	9	0	0	0	-15	0	-3	0	0	0	0	0	0	-7	1	1	1	1	1	1	0	0	0	0	0	0	0	-3	0	3	-1	3	0	0	0	0	0	0	0	0	1	0	0	0	-1	-1	0	0	0	0	0	-1	1	1	0	0	χ_{116}
χ_{117}	0	0	0	0	0	-18	0	0	0	0	6	6	0	2	0	0	0	6	6	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{117}
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{118}
χ_{119}																																																											χ_{119}
χ_{120}	0	0	-9	0	0	0	15	0	3	0	0	0	0	0	3	-11	1	8	8	0	0	0	0	0	0	0	0	0	0	3	0	-3	1	-3	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	0	χ_{120}
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	b17	*	0	0	0	0	0	0	0	0	0	χ_{121}
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	*	b17	0	0	0	0	0	0	0	0	0	χ_{122}
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{123}
χ_{124}																																																											χ_{124}
χ_{125}	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	0	0	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-2	-2	0	0	0	1	1	0	0	-1	-1	0	0	0	χ_{125}
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	-3	1	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	χ_{126}
	10 10	10 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	14 14	14 14	14 14	16	16	16	16 16	16 16	18 18	18 18	18 18	20 20	20 20	24	24 24	24 24	24	24	24	24 24	24	24 24	24	24	24 24	26 26	28 28	28 28	30 30	30 30	30 30	32	32	34 34	34 34	36 36	40 40	42 42	42 42	48	48	56 56	60 60	
χ_{127}	-5	1	-6	-5	-3	0	-2	-1	-2	1	2	4	3	-2	-4	3	1	0	0	0	0	2	2	-2	-2	0	3	1	0	-2	1	0	0	0	1	0	2	0	0	1	-1	0	-1	-2	1	1	0	0	-1	-1	0	-1	-1	0	0	0	0	0	χ_{127}
χ_{128}	5	-1	-10	-1	-7	-4	2	3	2	5	0	-4	-1	0	0	7	1	0	0	0	2	-2	1	1	-1	5	-1	0	-2	1	0	0	0	1	0	0	0	0	0	-1	1	0	1	-1	2	2	0	0	0	0	-1	-1	0	1	0	0	0	1	χ_{128}
χ_{129}	-8	-4	-8	-11	-5	-2	-8	1	0	3	-2	6	-3	2	-2	5	-1	0	0	0	2	2	-1	-1	-1	2	2	0	0	-3	0	0	0	1	0	0	0	0	0	-1	0	-2	1	1	1	-1	0	0	0	0	0	1	0	1	-1	0	0	1	χ_{129}
χ_{130}	10	2	-20	1	-17	4	4	-3	-4	-1	0	-4	1	0	4	11	1	0	0	0	2	-2	1	1	1	4	0	0	0	-3	0	0	0	1	0	0	0	0	0	1	0	0	-1	1	1	-1	0	0	0	0	1	0	1	-1	0	0	0	-1	χ_{130}
χ_{131}	-10	2	-10	-1	-7	-4	2	-1	-6	-3	2	0	-1	-2	-8	-1	1	0	0	0	0	0	-1	-1	1	2	-2	0	2	5	0	0	0	1	0	-2	0	0	-1	0	0	1	-1	-1	-1	0	0	0	0	-1	0	1	-1	0	0	0	1	χ_{131}
χ_{132}	1	-1	-4	-26	2	-10	4	-6	4	-2	-6	-2	-2	-2	10	3	1	0	0	0	-2	2	1	-2	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	1	1	-1	0	0	0	0	-1	-1	1	0	0	0	0	1	χ_{132}
χ_{133}	-17	1	-6	1	-9	6	-2	-3	6	3	-6	6	1	-2	0	7	1	0	0	0	-2	2	0	0	0	1	-1	0	2	-1	0	0	0	-1	0	0	0	0	1	0	0	1	1	1	1	0	0	1	1	0	1	0	1	0	0	0	-1	χ_{133}
χ_{134}	0	0	0	15	-15	-12	0	3	0	9	6	0	-1	2	2	9	-1	0	0	0	-2	2	0	0	0	0	0	0	0	3	0	0	0	-1	0	-2	0	0	-1	-1	2	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	χ_{134}
χ_{135}	-10	2	-24	-29	-3	0	-8	3	0	-3	0	0	3	0	-6	1	-1	0	0	0	2	-2	0	0	0	-2	2	0	-4	-1	0	0	0	-1	0	0	0	0	-1	1	2	-1	-1	-1	-1	0	0	0	0	0	0	0	1	0	0	0	-1	χ_{135}
χ_{136}	4	-4	-18	-5	-3	12	10	-5	2	-7	-8	-4	3	0	-4	3	-3	0	0	0	0	0	0	0	0	0	0	0	2	5	0	0	0	1	0	0	0	0	1	0	0	1	-2	1	-1	0	0	0	0	0	-1	0	0	0	0	0	0	χ_{136}
χ_{137}	8	-4	6	14	18	-6	2	2	2	2	2	2	6	-2	0	0	0	0	0	0	2	2	2	2	0	2	2	0	2	2	0	0	0	-2	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	χ_{137}
χ_{138}	0	0	-20	-20	4	10	4	0	4	4	0	-2	-4	0	2	-5	1	0	0	0	2	-2	1	-2	0	0	0	0	4	4	0	0	0	0	0	-2	0	0	0	0	-2	-1	0	0	0	0	0	0	0	1	0	-1	1	0	0	0	0	χ_{138}
χ_{139}	-5	-3	2	-4	-4	8	-10	0	-2	-8	6	4	-4	2	0	0	0	0	0	0	0	0	1	1	1	1	-1	0	2	2	0	0	0	2	0	2	0	0	0	0	0	0	1	-2	0	0	0	0	0	-1	-1	0	0	0	0	0	-1	χ_{139}
χ_{140}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{140}
χ_{141}																																																											χ_{141}
χ_{142}	5	-1	8	-10	2	-4	8	2	0	6	2	0	-2	-2	0	0	0	0	0	0	2	-2	-1	-1	-1	-3	-1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2	-1	-1	0	0	0	0	-1	1	0	0	0	0	0	0	χ_{142}
χ_{143}	0	0	12	-15	-9	-12	-12	-3	-4	11	0	-4	1	0	-4	-4	0	0	0	0	0	0	2	-1	1	0	0	0	-4	-1	0	0	0	-1	0	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	χ_{143}
χ_{144}	16	0	-30	-10	-6	0	-10	6	6	-6	0	0	-2	0	-6	-6	2	0	0	0	-2	2	0	0	0	0	0	0	-2	-2	0	0	0	-2	0	0	0	0	0	0	-2	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{144}
χ_{145}	-3	3	0	9	-9	0	0	-3	0	-9	0	0	1	0	4	-3	-1	0	0	0	0	0	0	0	0	-1	1	0	0	-3	0	0	0	1	0	0	0	0	-1	0	0	1	3	0	0	0	0	0	0	-1	-2	0	0	0	0	1	χ_{145}
χ_{146}	-1	1	-6	-29	-3	0	-2	3	6	-3	0	0	3	0	12	5	-1	0	0	0	0	0	0	0	1	-1	0	2	-1	0	0	0	-1	0	0	0	0	0	-1	0	0	1	-1	-1	1	0	0	-1	-1	0	1	0	-1	0	0	0	-1	χ_{146}
χ_{147}	-10	2	-12	4	12	0	28	4	4	4	-2	4	4	2	0	0	0	0	0	0	-2	-2	0	0	0	2	-2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	χ_{147}
χ_{148}	0	0	-6	-25	9	6	14	3	-2	1	6	-2	-1	2	0	0	0	0	0	0	-2	2	2	-1	-1	0	0	0	2	-1	0	0	0	3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{148}
χ_{149}	-10	-2	18	14	-6	6	-10	-2	-2	-2	-2	-2	6	2	10	3	1	0	0	0	0	0	0	0	0	-4	0	0	-2	4	0	0	0	0	0	0	0	0	0	0	-2	-1	-1	-1	1	0	0	0	0	0	0	1	0	0	0	0	1	χ_{149}
χ_{150}	5	3	30	19	-3	0	10	3	-6	-3	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	2	-1	0	0	0	-1	0	0	0	0	1	1	0	0	-1	2	0	0	0	-1	-1	0	1	0	0	0	0	0	1	χ_{150}
χ_{151}	0	0	-12	-5	-3	-12	-20	-1	4	1	8	4	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	1	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{151}
χ_{152}	10	-2	-30	-6	6	-12	6	-6	-2	-2	0	4	2	0	0	-7	-1	0	0	0	-2	-2	0	0	0	2	-2	0	-2	-2	0	0	0	-2	0	0	0	0	0	0	0	-1	1	1	1	0	0	0	0	0	0	-1	0	0	0	0	1	χ_{152}
χ_{153}	9	3	-14	0	-8	-20	6	4	-2	4	-8	4	0	0	0	0	0	0	0	0	0	0	-1	2	0	-3	-1	0	2	2	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	χ_{153}
χ_{154}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{154}
χ_{155}																																																											χ_{155}
χ_{156}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{156}
χ_{157}																																																											χ_{157}
χ_{158}	5	3	-16	-16	8	-4	-16	0	8	-4	0	-4	0	0	0	0	0	0	0	0	0	0	0	-1	2	0	-1	1	0	4	-2	0	0	0	2	0	0	0	0	0	-1	0	0	-1	2	2	0	0	0	0	-1	1	0	0	0	0	0	1	χ_{158}
χ_{159}	10	-2	24	-19	3	0	8	-3	0	3	0	0	5	0	-12	2	2	0	0	0	-2	-2	0	0	0	-2	2	0	4	1	0	0	0	1	0	0	0	0	-1	0	0	0	1	1	1	0	0	0	0	0	0	0	-1	0	0	0	-1	χ_{159}
χ_{160}	4	-4	6	-10	-6	0	2	6	-6	-6	0	0	-2	0	0	7	1	0	0	0	-2	-2	0	0	0	0	0	0	-2	-2	0	0	0	-2	0	0	0	0	0	-1	0	1	-2	1	-1	0	0	0	0	0	1	0	0	0	0	0	0	χ_{160}
χ_{161}	-7	3	0	0	24	12	0	0	8	-4	0	-4	0	0	-6	8	0	0	0	0	0	0	0	0	0	-3	-1	0	-4	2	0	0	0	-2	0	0	0	0	0	0	-2	0	2	2	0	0	0	0	0	-1	0	-1	0	0	0	0	0	χ_{161}
χ_{162}	-5	-3	12	1	15	0	4	5	-4	-1	2	-4	1	-2	0	0	0	0	0	0	0	0	-2	1	-1	1	-1	0	0	-3	0	0	0	1	0	-2	0	0	1	0	0	0	1	-2	0	0	0	0	0	-1	0	0	0	0	0	-1	χ_{162}
χ_{163}	5	3	0	-6	6	12	0	2	-8	-2	8	4	2	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	-4	2	0	0	0	2	0	0	0	0	0	0	0	0	-1	2	0	0	0	1	1	0	1	0	0	0	0	0	1	χ_{163}
χ_{164}	-9	-3	0	15	9	0	0	-9	0	-3	0	0	-1	0	-4	3	1	0	0	0	-2	-2	0	0	0	3	1	0	0	-3	0	0	0	1	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	χ_{164}
χ_{165}	-15	3	0	6	18	0	0	-6	0	6	0	0	-2	0	4	-3	-1	0	0	0	-2	-2	0	0	0	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-3	0	0	0	0	0	0	-1	1	0	0	0	0	-1	χ_{165}
χ_{166}	10	-2	16	-16	16	16	-16	0	0	0	0	0	0	0	-2	-2	-2	0	0	0	0	0	-1	-1	1	2	-2

$Fi'_{24} = F_{3+}$

		1A	2A	2B	3A	3B	3C	3D	3E	4A	4B	4C	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	7B	8A	8B	8C	9A	9B	9C	9D	9E	9F	10A	10B	11A	12A	12B	12C	12D	12E	
		@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	
		12552057	258247	16052	4456961	242439	1428	1530	382	398	58	294	907	783	195	100	29	22	559	69	69	62	23	5	17	2	2	1		472	262	209	59	13		2			124	31	13	8	5	
		091906617212928 00	006617600	6499840	8329600	1326720	5134080	55008	11264	13120	06080	912	200	82080	95520	77696	85984	39488	872	984	984	208	328	184	640	058	304	536	768	392	440	952	049	122	972	400	960	132	416	104	824	640	184	
		p power	A	A	A	A	A	A	A	B	A	B	A	AA	CB	BA	AB	BB	CA	DA	DB	CB	DB	EB	A	A	A	C	A	B	B	B	B	A	D	AA	AB	A	DA	EA	DA	AB	IA	
		p' part	A	A	A	A	A	A	A	A	A	A	A	AA	CB	BA	AB	BB	CA	DA	DB	CB	DB	EB	A	A	A	A	A	A	A	A	A	A	A	AA	AB	A	AA	BA	AA	AB	CA	
	1nd	1A	2A	2B	3A	3B	3C	3D	3E	4A	4B	4C	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	7B	8A	8B	8C	9A	9B	9C	9D	9E	9F	10A	10B	11A	12A	12B	12C	12D	12E	
X_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	X_1
X_2	+	8671	351	-33	247	-77	85	4	13	31	-1	-1	21	-9	-15	27	-9	3	9	0	12	-3	0	-3	5	-2	-1	7	-1	4	-5	13	4	-5	4	1	-3	3	7	-5	7	-1	1	X_2
X_3	+	57477	1157	133	534	615	210	48	-15	69	21	5	27	86	70	23	22	7	14	-4	16	10	4	1	14	0	5	5	-3	-6	30	12	-6	3	3	7	3	2	6	15	6	6	6	X_3
X_4	+	249458	2354	370	2705	869	167	113	14	50	42	2	8	41	55	5	49	37	23	5	1	7	13	-2	6	-1	6	-2	2	32	14	23	5	5	2	4	0	0	29	5	5	9	-1	X_4
X_5	+	555611	5083	91	-535	2300	518	-49	14	155	27	-5	36	169	154	-20	-23	28	34	7	19	-2	-5	-2	21	0	3	3	3	5	50	-13	5	-4	5	8	-4	1	-7	20	-7	9	2	X_5
X_6	o	1603525	1925	-315	6475	-275	-140	76	-77	5	-35	21	0	35	0	-19	-21	45	8	8	0	12	-12	3	0	0	1	1	-3	49	-5	-5	-5	4	-2	0	0	0	23	5	-1	-5	-4	X_6
X_7	o	1603525	1925	-315	6475	-275	-140	76	-77	5	-35	21	0	35	0	-19	-21	45	8	8	0	12	-12	3	0	0	1	1	-3	49	-5	-5	-5	4	-2	0	0	0	23	5	-1	-5	-4	X_7
X_8	+	1666833	11153	273	3093	-1848	987	-66	51	209	49	-15	83	-43	-105	56	21	-24	47	2	30	3	-6	3	14	0	1	9	1	-12	-21	24	-12	6	6	3	3	3	5	-16	5	-11	-1	X_8
X_9	+	4864431	13871	-1105	4836	4917	867	-24	156	175	-97	15	56	164	155	101	-28	-43	11	20	20	-13	8	-4	5	5	-9	7	-1	3	39	21	3	12	3	-4	0	0	4	13	4	-4	-5	X_9
X_{10}	+	32715683	43043	-2653	25298	-1837	2618	26	-91	291	-77	3	133	-238	-70	275	-46	83	-22	-22	38	-22	-10	5	0	0	-5	11	3	-1	-19	44	-1	8	-1	-7	-3	0	18	3	18	10	6	X_{10}
X_{11}	+	35873145	53625	4473	27000	4158	2457	27	54	185	105	-7	120	120	189	246	120	-18	21	3	27	33	3	6	-14	0	9	9	1	27	0	54	27	0	0	0	8	0	8	14	8	0	-7	X_{11}
X_{12}	+	40536925	39325	3165	49075	880	70	-11	169	-35	85	13	0	-5	-30	40	147	96	58	13	-3	-18	21	9	-5	9	1	1	-3	70	25	34	-11	-2	-2	0	0	0	55	-8	-17	-5	10	X_{12}
X_{13}	+	48893768	95304	840	-3340	-14113	4760	62	-64	840	8	8	168	-204	-420	-33	-12	-33	84	-6	66	0	6	0	35	0	0	8	0	8	-55	-37	8	-1	8	4	0	0	-12	-33	-12	-4	0	X_{13}
X_{14}	o	74837400	37400	-4200	-4575	20130	2310	285	105	280	-120	-8	0	545	210	-22	33	-78	-22	5	21	-18	-3	9	15	-6	-8	0	0	15	60	-3	15	6	-3	0	0	0	1	10	1	-15	10	X_{14}
X_{15}	o	74837400	37400	-4200	-4575	20130	2310	285	105	280	-120	-8	0	545	210	-22	33	-78	-22	5	21	-18	-3	9	15	-6	-8	0	0	15	60	-3	15	6	-3	0	0	0	1	10	1	-15	10	X_{15}
X_{16}	+	79452373	104533	5845	-2444	24178	3010	148	247	405	245	-11	48	412	490	-38	-44	130	106	16	4	34	-8	7	14	0	13	-3	-3	-14	85	-5	13	13	-2	8	0	0	-24	18	0	20	6	X_{16}
X_{17}	+	112168056	19448	6776	42510	14322	2415	228	156	-8	168	24	6	566	35	170	110	98	-37	8	8	23	20	-4	14	0	0	0	8	66	30	21	-15	3	-6	-2	6	0	-14	10	10	6	7	X_{17}
X_{18}	+	159402880	158080	896	-260	49312	5572	-260	-260	896	64	0	105	1084	896	-32	-4	32	40	-32	32	-4	-4	-4	25	0	0	0	0	-17	145	-17	-17	-17	1	5	1	-1	-4	32	-4	4	-4	X_{18}
X_{19}	+	281380736	11648	8064	132860	10064	-952	668	182	128	0	0	-14	308	0	-16	156	144	-16	-16	0	24	12	6	0	0	0	0	0	155	-34	-34	-7	20	2	-2	-6	-1	32	-16	8	0	8	X_{19}
X_{20}	+	415098112	256256	-2816	106600	-29480	9400	166	-104	768	0	0	162	-280	-440	296	-152	-8	8	26	46	-8	10	-8	16	2	0	0	0	-23	-50	112	-23	-23	-5	6	-6	0	24	-24	24	0	0	X_{20}
X_{21}	+	635618984	323752	3752	146120	-39127	4046	-652	104	552	168	40	34	184	-406	-167	8	-55	130	-32	-28	-10	8	8	14	0	0	-8	0	-58	-4	104	23	-4	5	2	2	0	48	-15	0	-24	-6	X_{21}
X_{22}	+	1069551175	175175	13895	47425	-23045	-3605	769	427	135	-105	71	0	665	35	-109	-31	107	35	-55	-19	11	5	11	0	0	-1	7	7	-122	-50	-32	40	4	-5	0	0	0	45	27	21	-15	-9	X_{22}
X_{23}	+	1112333222	246246	-13146	415415	1694	749	938	182	230	-210	-42	-28	-273	21	6	-297	174	69	6	-6	-3	-18	6	0	0	-6	-6	-2	209	-16	83	20	2	8	-4	4	0	131	14	11	15	5	X_{23}
X_{24}	+	1264015025	352625	18865	72800	93050	7595	872	-91	625	105	65	0	1400	595	194	256	10	-13	32	28	43	4	-11	0	0	5	-3	1	-73	80	-10	8	-1	-1	0	0	-2	28	-14	4	0	7	X_{24}
X_{25}	+	1337276304	388752	-11376	147420	92016	10368	891	0	528	-112	-48	154	924	720	-48	-36	-144	-48	-21	-9	0	-9	0	0	7	0	0	0	0	81	81	0	0	0	2	-6	1	12	24	12	-4	0	X_{25}
X_{26}	+	1540153692	695772	4956	-16758	-47619	15561	9	441	1436	28	60	217	-630	-399	-99	138	-3	81	9	33	9	-15	9	0	0	-4	4	-4	9	-99	-45	9	-18	0	-3	1	0	-22	5	-22	10	5	X_{26}
X_{27}	+	2346900864	750464	-2688	-40116	92400	13020	222	-300	896	-64	0	189	-244	840	80	-180	48	80	26	30	-12	18	-12	-35	0	0	0	0	-21	15	-57	-21	15	6	-11	-3	0	-4	32	-4	-4	-4	X_{27}
X_{28}	+	3178094920	886600	-9912	-111800	-99407	17962	-263	-182	1480	-120	-56	120	-440	-966	-431	72	177	10	1	33	-6	9	-6	21	0	0	-8	0	7	-20	-74	7	34	-2	0	8	0	-8	-23	-8	0	-2	X_{28}
X_{29}	+	3208653525	648725	32725	425775	51150	3360	1011	-105	85	525	69	0	455	280	158	463	46	104	23	-17	16	-5	7	0	0	1	-7	-3	-15	-60	84	-15	-6	3	0	0	0	43	22	19	15	-8	X_{29}
X_{30}	+	3283490925	439725	25965	421200	71280	5670	1296	0	45	405	-35	0	-960	450	-24	144	288	-6	-24	-36	-18	36	0	15	-6	9	1	-3	0	0	81	0	0	0	0	0	0	24	0	-24	0	-6	X_{30}
X_{31}	+	5005499499	627627	-24213	376740	151965	-882	1305	0	747	-413	-37	-126	84	294	-123	228	-147	102	-15	-3	-18	21	0	0	0	-9	-1	3	-99	45	27	-18	18	0	2	2	0	60	-27	12	4	-6	X_{31}
X_{32}	+	5775278080	0	49152	457600	58432	1408	760	832	0	0	0	-120	0	-384	0	384	192	0	0	48	0	24	0	-16	-2	0	0	0	58	40	-32	58	13	4	0	-8	0	0	0	0	0	0	X_{32}
X_{33}	+	6471756928	267904	-43904	697060	39016	9856	1594	910	-640	0	0	28	28	-56	280	-380	-56	-8	10	-2	16	-2	-2	0	0	0	0	0	190	-44	46	28	10	1	4	-4	-1	8	8	-16	0	8	X_{33}
X_{34}	+	7150713570	484770	-16926	-46215	227403	12348	-288	-351	354	106	-14	-105	2001	84	147	-231	219	-60	12	-24	12	12	-15	35	0	-10	-2	-6	-72	45	0	9	18	0	-5	-1	0	21	3	-3	1	12	X_{34}
X_{35}	+	8529641472	1441792	0	509952	-8448	17472	-1320	768	0	0	0	272	-512	0	640	0	0	64	-8	0	0	0	0	-26	-12	0	0	0	-24	-6	120	-24	-6	-6	-8	0	0	0	0	0	0	0	X_{35}
X_{36}	+	9100908180	616980	17556	745290	-71433	8757	-477	-819	-44	84	-76	105	714	-399	327	330	87	-87	3	-21	-3	-21	-3	0	0	-4	-4	-4	117	0	45	36	27	0	5	1	1	10	-17	10	-6	1	X_{36}
X_{37}	+	9441555200	1478400	8960	-204400	305360	14840	-1900	350	1280	0	0	0	1680	1400	-336	-112	-112	24	-12	-4	-40	-4	14	0	0	0	0	0	98	80	-46	-10	-10	-4	0	0	0	-16	-16	-16	0	-16	X_{37}
X_{38}	+	10169903744	1299584	-11648	-151372	-48664	3500	2204	728	-384	64	0	69	1028	-560	344	-140	-56	200	20	-20	-44	4	-8	-21	0	0	0	0	44	35	-55	44	8	-1	9	-3	0	-36	-24	12	4	-12	X_{38}
X_{39}	+	10776585600	-105600	40320	-27000	11880	-7560	2160	27	640	0	0	0	-120	0	24	-120	72	-48	24	0	-24	24	3	-10	18	0	0	0	-27	0	27	-27	0	0	0	0	0	-8	-8	-8	0	-8	X_{39}
X_{40}	+	10776585600	-105600	40320	-27000	11880	-7560	2160	27	640	0	0	0	-120	0	24	-120	72	-48	24	0	-24	24	3	-10	18	0	0	0	-27	0	27	-27	0	0	0	0	0	-8	-8	-8	0	-8	X_{40}
X_{41}	+	13904165275	2277275	37275	-120575	-179300	18340	682	-182	1435	315	27	0	-175	-840	-364	-63	-372	176	14	-30	12	-18	-6	0	0	3	-5	3	61	-20	-47	-20	7	-2	0	0	0	-23	4	25	-15	4	X_{41}
X_{42}	+	13904165275	675675	10395	1599325	21175	-11900	304	-1001	155	35	-21	0	-315	0	-513	189	135	72	0	0	-36	0	-9	0	0	-1	7	3	196	25	-65	-20	-20	-8	0	0	0	161	-25	-7	5	-4	X_{42}
X_{43}	+	14507059905	1605825	-11583	0	-216513	19683	0	0	1665	-15	-63	-120	0	-729	567	0	-81	-81	0	0	27	0	0	9	2	9	1	1	0	0	0	0	0	0	0	-8	-3	0	-9	0	0	-9	X_{43}
X_{44}	+	17068369920	665600	43008	542400	-62832	672	996	960	0	0	0	120	320	-336	752	192	-48	-112	-4	-12	0	12	0	-14	0	0	0	0	24	60	-12	24	-21	6	0	8	-3	0	0	0	0	0	X_{44}
X_{45}	+	17161712568	1438008	47544	-218322	170478	8883	1107	-702	-200	168	-40	-132	-426	231	582	-114	222	159	15	15	27	3	18	-14	0	0	0	8	-27	0	-54	-27	0	0	8	4	0	-14	-2	10	6	19	X_{45}
X_{46}	+	18481844304	1548624	37968	-245322	150282	-3780	-1107	378	464	336	16	-21	-546	420	66	-234	186	84	-15	-39	12	-27	-6	0	0	0	0	0	27	0	-27	27	0	0	-1	3	0	-22	-22	2	6	-4	X_{46}
X_{47}	+	18481844304	1548624	37968	-245322	150282	-3780	-1107	378	464	336	16	-21	-546	420	66	-234	186	84	-15	-39	12	-27	-6	0	0	0	0	0	27	0	-27	27	0	0	-1	3	0	-22	-22	2	6	-4	X_{47}
X_{48}	+	25027497495	1216215	18711	114660	254628	10899	207	819	23	63	-89	70	252	315	324	132	36	-117	27	-9	3	-21	3	0	0	-5	3	-1	-63	-54	-45	18	0	0	-10	6	0	32	-4	8	-12	-13	X_{48}
X_{49}	+	27808330550	-250250	-6090	1171625	-77935	4550	-40	455	310	-210	166	0	-455	210	-527	105	273	-86	40	-60	-18	-12	-9	0	0	10	10	6	122	50	5	14	14	2	0	0	0	-47	-23	1	-15	10	X_{49}
X_{50}	+	29444114700	300300	-22260	655200	150975	-16695	-3087	819	1100	-420	140	0	-840	105	-129	0	-129	-39	33	-3	57	45	3	0	0	4	-4	-4	-36	0	-45	45	-27	0	0	0	0	44	-25	20	0	5	X_{50}
X_{51}	+	35594663104	192192	23744	25480	342892	154	856	-728	960	-448	64	-196	1176	-574	60	-184	-52	-30	-48	20	26	-4	8	0	0	0	0	0	46	19	100	-8	1	4	-8	4	0	-48	-12	0	-16	6	X_{51}
X_{52}	+	36858678129	-1825551	37233	132678	115182	25515	1458	0	-591	-399	17	-21	1134	-945	-378	-90	-162	135	-54	-54	-45	18	0	0	0	9	9	-7	0	0	0	0	0	0	-1	3	-2	-6	30	18	6	3	X_{52}
X_{53}	+	37337059200	560000	-49280	1470000	114060	7140	-1122	-924	-640	0	128	0	560	-140	1100	-464	76	20	-34	22	4	22	4	0	0	0	0	0	93	30	-6	12	3	-6	0	0	1	-16	-28	-16	0	-4	X_{53}
X_{54}	+	38641860608	2555904	0	366080	-224896	31712	-1336	-832	0	512	0	8	-384	0	192	0	0	-96	-24	0	0	0	0	1	8	0	0	0	-40	-31	32	41	-4	-4	4	0	-1	0	0	0	-16	0	X_{54}
X_{55}	+	40043995992	-744744	53592	1023750	-83358	8253	1692	0	-168	-280	-8	42	-210	609	618	102	114	-39	24	-12	-27	-24	0	0	0	0	0	-8	18	-45	-108	-63	-18	0	6	2	0	-6	-6	18	-10	21	X_{55}
X_{56}	+	44493328880	880880	-45584	1309490	51722	-13888	1664	728	-144	-336	48	105	1274	-224	-238	-494	154	32	32	-8	64	-8	-8	0	0	0	0	0	-172	-100	-28	-10	8	-1	5	1	0	54	18	-18	-6	0	X_{56}
X_{57}	+	45049495491	2573571	40131	1002456	68607	-7938	-1377	0	195	315	-77	-84	-168	-126	39	216	-369	138	39	-45	54	27	0	0	0	-9	7	3	0	0	81	0	0	0	-4	-4	0	-24	15	-24	0	18	X_{57}
X_{58}	+	46602926370	1171170	-10206	2100735	-197802	-11907	1215	0	354	-294	-14	-105	1071	189	-90	63	54	45	-9	27	45	-45	0	0	0	6	-2	2	0	0	0	0	0	0	-5	-1	0	39	30	-9	-9	-15	X_{58}
X_{59}	+	54234085491	1384371	93555	531441	0	0	0	729	-1485	27	99	216	729	0	0	81	0	0	0	0	0	0	9	19	12	-1	-9	3	0	0	0	0	0	0	-4	0	3	-27	0	-3	9	0	X_{59}
X_{60}	+	54481627200	1601600	34880	-382200	-260700	1740	525	-546	320	0	64	0	1400	-220	68	-184	-220	-76	41	-31	-4	5	14	0	-7	0	0	0	39	30	-33	-42	3	3	0	0	0	32	-4	-16	0	-4	X_{60}
X_{61}	+	54481627200	1601600	34880	-382200	-260700	1740	525	-546	320	0	64	0	1400	-220	68	-184	-220	-76	41	-31	-4	5	14	0	-7	0	0	0	39	30	-33	-42	3	3	0	0	0	32	-4	-16	0	-4	X_{61}
X_{62}	+	55616661100	2702700	-101780	-236600	445555	11935	-242	91	300	-420	44	0	0	595	-189	232	19	27	54	-26	-41	-2	-5	0	0	-4	-4	4	82	-80	28	28	-17	4	0	0	0	-60	3	12	0	3	X_{62}
X_{63}	+	63831063582	393822	36126	767637	216513	19683	0	0	-1314	294	78	57	405	-729	-567	405	81	-81	0	0	27	0	0	6	-1	-6	2	-2	0	0	0	0	0	0	-3	1	0	-27	9	-27	-3	-9	X_{63}
X_{64}	+	65393917952	1441792	0	-403456	-64768	-11200	248	896	0	0	0	-48	-512	0	640	0	0	64	-8	0	0	0	0	6	6	0	0	0	32	50	-40	5	14	2	-8	0	0	0	0	0	0	0	X_{64}
X_{65}	+	65393917952	1441792	0	-403456	-64768	-11200	248	896	0	0	0	-48	-512	0	640	0	0	64	-8	0	0	0	0	6	6	0	0	0	32	50	-40	5	14	2	-8	0	0	0	0	0	0	0	X_{65}
X_{66}	+	67331776512	2342912	57344	1440192	-109824	336	-2256	624	0	0	0	-188	320	896	-256	-64	-256	32	-40	32	-16	8	-16	-14	0	0	0	0	-42	-114	84	-42	21	3	12	4	0	0	0	0	0	0	X_{66}
X_{67}	+	71189326208	2946944	-24192	591500	-148192	-448	-1906	182	128	0	0	-42	-700	-504	-160	300	288	56	2	-18	-48	-6	6	0	0	0	0	0	-205	110	-34	-43	2	2	-6	-2	0	32	-16	8	0	8	X_{67}
X_{68}	+	74887473024	3978624	8064	-728676	-43416	29484	1782	0	384	64	0	-1	-1716	504	-744	-36	72	-96	-42	18	36	18	0	-21	0	0	0	0	0	-81	-81	0	0	0	-1	-1	1	36	24	-12	4	12	X_{68}
X_{69}	+	77007684600	3003000	-21000	-625275	-215820	-4410	207	189	120	-280	-40	0	1365	-210	-156	69	276	78	-21	-21	6	15	-3	0	0	-8	0	0	72	-45	0	-9	-18	0	0	0	0	-51	12	-3	5	-6	X_{69}
X_{70}	+	77007684600	3003000	-21000	-625275	-215820	-4410	207	189	120	-280	-40	0	1365	-210	-156	69	276	78	-21	-21	6	15	-3	0	0	-8	0	0	72	-45	0	-9	-18	0	0	0	0	-51	12	-3	5	-6	X_{70}
X_{71}	+	77108871168	2555904	0	437760	220032	10080	360	-1728	0	-512	0	168	-384	0	192	0	0	-96	-24	0	0	0	0	-35	0	0	0	0	-72	45	0	9	18	0	4	0	-1	0	0	0	16	0	X_{71}
X_{72}	+	77379702400	3203200	-76160	1155700	-119240	10360	-980	910	-640	0	0	0	-980	-560	136	-236	88	64	28	-20	-56	-20	-2	0	0	0	0	0	-170	100	46	-8	-8	1	0	0	0	8	8	-16	0	8	X_{72}
X_{73}	+	102385217025	-2013375	76545	921375	155925	2835	-3726	0	705	105	-143	0	735	-945	285	-225	-27	-57	42	-54	27	18	0	0	0	-3	5	1	0	0	-81	0	0	0	0	0	3	15	21	15	15	-9	X_{73}
X_{74}	+	111233322200	-1001000	-24360	2720900	129305	-700	-457	182	-40	280	-104	0	-1820	-420	-407	-252	57	-20	-29	39	-12	27	6	0	0	0	-8	0	137	65	-61	-25	-16	2	0	0	0	-76	41	20	-20	-4	X_{74}
X_{75}	+	111233322200	2202200	29400	-104650	-71170	-29995	-565	182	-40	-280	-8	0	350	105	134	150	78	161	-1	51	-3	15	6	0	0	0	0	-8	-25	-25	-25	29	20	-10	0	0	0	-22	14	2	-10	5	X_{75}

$F_{3+} = Fi'_{24}$

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$Fi'_{24} = F_{3+}$

| | | |

$F_{3+} = Fi'_{24}$

	864 IF CF 12Y	216 HG DG 12Z	216 JF DF 12A1	72 KG EG 12B1	840 AC AC 14C	168 AD AD 14D	32 B A 16B	17 496 AM AC 18I	5 832 BM BC 18J	5 832 CM CC 18K	1 944 AQ AD 18L	1 458 EM EC 18M	648 BQ BD 18N	648 CQ CD 18O	243 DQ DD 18P	162 EQ ED 18Q	162 FR FC 18R	1 200 AD AD 20C	80 AE AE 20D	132 AD AD 22B	44 AC AC 22C	288 AE AE 24H	96 CD AD 24I	72 EE CE 24J	48 HF BF 24K	78 AC AC 26B	78 AC AC C#	84 AD AD 28B	14 BG BG 28C	14 BG BG D#	360 ACL ACL 30C	360 ADP ADP 30D	90 CCO CCO 30E	90 CDS CDS 30F	45 BDN BDN 30G	17 AC AC 34A	108 BP CD 36E	108 DP BD 36F	36 CQ AF 36G	18 FW FE 36H	20 BD AD 40A	42 ACL ACL 42D	21 BDV BDV 42E	23 AC AC 46A	23 BC BC B##	27 AI AC 54A	60 ACN ACN 60B	33 ABN ABN 66A	33 BBN BBN B#	35 ACC ACC 70A	39 BBA ABA 78A	39 ACA BCA B#	42 ABO ABO 84A	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	3	-2	0	1	-3	1	-1	10	1	1	-2	1	1	1	-2	1	-2	5	1	3	-1	-1	-1	-1	-1	0	0	-1	0	0	1	5	-2	2	-1	-1	1	1	0	0	-1	0	1	0	0	1	-1	0	0	2	0	0	2	x_2
x_3	0	2	0	-1	6	2	1	6	6	6	2	-3	2	2	2	-1	3	5	1	2	2	0	0	0	1	2	2	0	0	0	3	3	0	0	0	0	0	0	0	-1	-1	0	-1	0	0	0	-1	-1	-1	1	-1	-1	0	x_3
x_4	3	-3	3	0	4	0	0	-6	12	3	6	3	0	3	-3	3	0	4	0	0	0	5	1	-1	-1	1	1	2	-1	-1	3	-1	0	2	-1	0	-1	2	0	0	0	1	0	0	0	0	1	0	0	-1	1	1	-1	x_4
x_5	-2	-3	1	0	1	-3	-1	7	-2	7	3	-2	-6	3	3	0	1	2	-2	1	1	1	1	-2	0	0	0	-1	0	0	-1	-1	-1	-1	2	0	-3	0	1	-1	0	1	0	0	0	1	-1	1	1	1	0	0	-1	x_5
x_6	-2	-2	4	1	0	0	1	3	3	3	-1	-6	-1	-1	-1	2	0	0	0	0	0	-3	1	0	1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	b23	**	0	0	0	0	0	-1	-1	0	x_6
x_7	-2	-2	4	1	0	0	1	3	3	3	-1	-6	-1	-1	-1	2	0	0	0	0	0	-3	1	0	1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	**	b23	0	0	0	0	0	-1	-1	0	x_7
x_8	-3	2	0	-1	-4	0	1	18	-9	0	-2	0	-5	4	-2	-2	0	7	-1	3	-1	1	1	1	-2	-1	-1	2	0	0	-1	3	2	0	0	0	-2	1	0	0	1	-1	0	0	0	0	1	0	0	1	-1	-1	-1	x_8
x_9	1	0	-2	0	5	-3	-1	15	-3	-3	3	6	-3	-3	3	0	3	4	0	0	0	-2	-2	1	1	0	0	1	-1	-1	-1	3	2	0	0	0	1	1	1	1	0	-1	0	0	0	0	1	0	0	0	0	0	1	x_9
x_{10}	2	-2	2	1	0	0	-1	23	5	-4	-1	-4	5	-4	-1	2	-1	3	-1	0	0	0	0	0	1	0	0	0	0	0	0	2	6	2	0	0	1	0	-3	-1	-1	1	0	0	0	0	-1	0	0	0	0	0	0	x_{10}
x_{11}	5	3	-1	0	4	0	1	27	0	0	3	0	0	0	3	0	0	0	0	0	0	2	2	-1	0	-2	-2	-2	0	0	-4	4	-1	1	1	0	0	0	-1	0	0	1	0	-1	-1	0	0	0	0	-1	1	1	1	x_{11}
x_{12}	0	-1	3	-1	-5	-1	-1	8	-1	-10	8	8	-1	2	-1	2	2	0	0	0	0	3	-1	0	0	0	0	-3	1	1	0	0	0	0	0	0	0	3	0	0	0	-2	-1	0	0	-1	0	0	0	0	0	0	0	x_{12}
x_{13}	-6	-2	0	-2	-5	-1	0	8	-1	-1	0	-1	3	3	0	-3	-4	4	0	0	0	0	0	0	1	1	1	1	0	0	1	1	-2	-2	1	0	1	1	0	0	0	1	-1	0	0	-1	1	0	0	0	1	1	1	x_{13}
x_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{14}
x_{15}																																																											x_{15}
x_{16}	0	-2	0	1	4	0	-1	-6	3	21	-10	3	-1	5	-1	-1	0	0	0	0	0	-4	0	2	-2	0	0	-2	0	0	2	-2	2	-2	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	1	x_{16}
x_{17}	3	2	0	2	-4	0	0	0	0	9	-8	9	4	1	1	1	0	-2	-2	0	0	4	0	1	0	0	0	2	0	0	1	1	-2	-2	1	-1	1	-2	0	0	0	-1	0	0	0	0	1	0	0	1	0	0	-1	x_{17}
x_{18}	-4	0	2	0	5	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	5	1	1	1	-2	-2	-2	0	0	0	-1	0	0	-1	-1	-1	-1	-1	0	-1	-1	-1	1	1	-1	1	0	0	-1	-1	1	1	0	0	0	-1	x_{18}
x_{19}	-4	0	2	0	0	0	0	-5	4	4	3	4	0	0	3	0	-2	-2	2	-1	-1	0	0	0	0	0	0	0	0	0	-4	0	2	0	0	0	-2	-2	-1	0	0	0	0	1	1	1	-2	-1	-1	0	0	0	0	x_{19}
x_{20}	4	0	-2	0	-6	2	0	1	10	10	1	1	-2	-2	1	1	1	6	2	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	1	1	0	0	-1	0	0	1	0	0	0	-1	0	0	-2	x_{20}
x_{21}	0	2	0	2	-6	-2	0	-34	-16	2	-6	2	0	6	3	0	-1	8	0	0	0	0	0	0	1	0	0	0	0	0	1	-3	1	3	0	0	0	0	0	-1	0	0	1	0	0	-1	-1	0	0	-1	0	0	0	x_{21}
x_{22}	3	-1	3	-1	0	0	1	10	10	10	-2	10	-2	-2	-2	-2	1	0	0	0	0	1	-3	1	1	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	1	1	0	x_{22}
x_{23}	5	4	2	-2	0	0	0	-27	18	-9	-7	0	2	-1	2	2	0	6	-2	0	0	-5	-1	1	0	0	0	0	0	0	2	-2	-1	1	1	0	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{23}
x_{24}	-5	2	-2	-1	0	0	-1	-5	4	-14	7	-5	4	-2	-2	1	1	0	0	-2	-2	-2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	1	0	1	1	0	0	0	x_{24}
x_{25}	0	-3	-3	0	0	0	0	0	9	9	0	0	-3	-3	0	0	0	-2	-2	-1	-1	0	0	0	0	0	0	0	1	1	3	-1	0	2	-1	0	-1	-1	0	0	0	0	0	0	0	0	1	0	1	-1	-1	0	0	x_{25}
x_{26}	-5	1	1	1	0	0	0	27	-9	-9	-1	0	-1	-1	-1	2	0	3	-1	0	0	-2	-2	1	1	1	1	0	0	0	-4	-4	-1	-1	-1	0	-1	-1	1	0	-1	0	0	0	0	0	0	0	0	0	1	1	0	x_{26}
x_{27}	-4	0	2	0	5	1	0	15	-3	-3	-1	-3	5	5	-1	-1	0	1	1	0	0	2	2	2	0	0	0	-1	0	0	-3	-3	0	0	0	0	1	1	-1	0	-1	-1	1	0	0	0	1	0	0	0	0	0	-1	x_{27}
x_{28}	-2	-3	1	0	-1	3	0	-25	-16	-16	3	2	0	0	3	0	2	0	0	0	0	0	0	0	-1	0	0	1	0	0	-4	4	-1	1	1	0	0	0	1	0	0	-1	0	0	0	-1	0	0	0	-1	0	0	1	x_{28}
x_{29}	4	-1	1	-1	0	0	-1	-15	12	12	1	-6	4	4	1	-2	-3	0	0	0	0	3	-1	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	-1	-1	0	x_{29}
x_{30}	0	0	0	0	5	-3	1	0	18	-9	0	0	-6	3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	-2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	1	x_{30}
x_{31}	2	-1	-1	2	0	0	1	-9	9	-9	7	0	1	-5	-2	-2	0	8	0	0	0	0	0	0	-1	0	0	0	0	0	-1	-1	-1	-1	2	0	1	1	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	x_{31}
x_{32}	0	0	0	0	-4	4	0	14	-4	-4	2	5	-4	-4	2	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	1	-1	-1	0	0	0	0	0	2	1	-1	-1	-1	0	0	0	1	0	0	0	0	x_{32}
x_{33}	0	0	0	0	0	0	0	8	8	8	0	8	0	0	0	0	-1	-6	2	1	1	0	0	0	0	0	0	0	0	0	-2	2	1	-1	-1	0	0	0	0	-1	0	0	0	0	0	-1	0	1	1	0	0	0	0	x_{33}
x_{34}	0	2	0	-1	-5	-1	0	0	-9	18	8	0	-1	2	-1	2	0	5	1	0	0	-3	1	0	-1	0	0	1	0	0	-1	-1	-1	-1	-1	0	2	-1	0	0	1	1	-1	0	0	0	-1	0	0	0	0	0	1	x_{34}
x_{35}	0	0	0	0	4	0	0	24	-12	-12	0	6	0	0	0	0	0	-8	0	0	0	0	0	0	0	-1	-1	2	0	0	4	0	-2	0	0	0	2	2	0	0	0	1	0	1	1	0	-2	0	0	-1	-1	-1	-1	x_{35}
x_{36}	5	-1	-1	-1	0	0	0	-9	-18	9	7	-9	-2	1	-2	1	0	-5	-1	1	1	-2	-2	1	-1	0	0	0	0	0	1	1	1	1	1	0	1	-2	-1	0	-1	0	0	0	0	0	1	1	1	0	0	0	0	x_{36}
x_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r13	*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	r13	*	0	x_{37}
x_{38}	0	0	0	0	-1	3	0	16	7	7	0	-2	3	3	0	0	1	-5	-1	0	0	-2	-2	-2	0	0	0	1	0	0	-3	-3	0	0	0	0	1	1	0	-1	1	-1	0	0	0	1	1	0	0	-1	0	0	1	x_{38}
x_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{39}
x_{40}																																																											x_{40}
x_{41}	-4	0	2	0	0	0	-1	-17	-8	1	-9	1	0	-3	0	-3	4	0	0	0	0	1	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	3	0	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	x_{41}
x_{42}	-6	4	0	1	0	0	-1	30	3	3	2	-6	-1	-1	2	2	0	0	0	0	0	3	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{42}
x_{43}	3	0	0	0	1	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	2	2	-1	-1	0	0	-1	0	0	4	-4	1	-1	-1	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	0	0	2	x_{43}
x_{44}	0	0	0	0	-6	-2	0	12	12	12	-4	3	-4	-4	-4	-1	0	0	0	-3	1	0	0	0	0	2	2	0	0	0	-4	4	-1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-1	-1	-1	0	x_{44}
x_{45}	5	-3	-1	0	-4	0	0	27	0	0	3	0	0	0	3	0	0	-2	2	0	0	-4	0	-1	0	0	0	2	0	0	-1	-5	-1	1	1	0	0	0	-1	0	0	-1	0	0	0	0	1	0	0	1	0	0	-1	x_{45}
x_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{46}
x_{47}																																																											x_{47}
x_{48}	-5	-1	1	-1	0	0	1	-9	0	9	-5	0	4	1	4	4	0	0	0	0	0	2	-2	-1	-2	0	0	0	0	0	0	0	0	0	0	0	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{48}
x_{49}	2	2	2	-1	0	0	0	-18	0	-9	-2	0	4	-5	-2	-2	0	0	0	0	0	1	1	-2	-1	0	0	0	0	0	0	0	0	0	0	0	1	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{49}
x_{50}	3	-1	3	-1	0	0	0	-18	-18	9	10	9	-2	1	1	1	0	0	0	0	0	-2	2	1	1	0	0	0	0	0	0	0	0	0	0	-1	1	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{50}
x_{51}	6	0	0	0	0	0	0	-30	-3	-12	-18	-3	-3	0	0	3	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	-1	3	-1	-3	0	0	-2	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	x_{51}
x_{52}	3	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	-5	-1	2	2	-2	2	1	0	0	0	0	0	0	-5	-1	-2	2	-1	0	0	0	0	0	1	0	0	0	0	0	1	-1	-1	0	0	0	0	x_{52}
x_{53}	4	0	4	0	0	0	0	27	-18	0	-9	-9	-6	0	0	-3	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	-1	-1	0	1	1	0	x_{53}
x_{54}	0	0	0	0	1	1	0	-16	11	2	8	2	-1	2	-1	2	2	-4	0	-1	-1	0	0	0	0	0	0	1	0	0	-3	1	0	-2	1	0	-2	1	0	0	0	1	1	0	0	-1	-1	-1	-1	1	0	0	1	x_{54}
x_{55}	-3	2	0	2	0	0	0	-18	9	0	10	0	1	4	1	-2	2	-4	0	0	0	-4	0	-1	0	0	0	0	0	0	2	2	-1	-1	-1	0	-2	1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	x_{55}
x_{56}	-6	0	0	0	0	0	0	8	8	8	0	8	0	0	0	0	-1	-5	-1	0	0	-2	2	-2	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	1	-1	0	0	0	0	-1	1	0	0	0	0	0	0	x_{56}
x_{57}	0	-3	-3	0	0	0	1	0	-18	9	0	0	6	-3	0	0	0	6	-2	0	0	0	0	0	1	0	0	0	0	0	-2	2	1	-1	-1	0	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{57}
x_{58}	3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1	0	0	1	1	1	0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	x_{58}
x_{59}	0	0	0	3	-1	-1	1	0	0	0	0	0	0	0	0	0	0	-4	0	3	-1	-3	1	0	0	1	1	-1	0	0	3	3	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	-1	0	0	-1	1	1	-1	x_{59}
x_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{60}
x_{61}																																																											x_{61}
x_{62}	3	4	0	1	0	0	0	-30	-12	6	14	-3	-4	2	-4	-1	0	0	0	0	0	2	-2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{62}
x_{63}	-3	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	1	1	1	1	0	0	-2	-1	-1	2	-2	-1	1	1	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	1	0	0	1	x_{63}
x_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{64}
x_{65}																																																											x_{65}
x_{66}	0	0	0	0	6	2	0	-6	-6	-6	-2	3	-2	-2	-2	1	-3	2	2	0	0	0	0	0	0	0	0	0	0	0	-2	-2	1	1	1	0	0	0	0	-1	0	0	-1	0	0	0	2	0	0	1	0	0	0	x_{66}
x_{67}	-4	0	2	0	0	0	0	-29	-2	-2	3	-2	-6	-6	3	0	-2	-4	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-1	-1	-1	0	0	0	-1	0	0	0	0	0	0	1	2	0	0	0	0	0	0	x_{67}
x_{68}	0	0	0	0	-1	3	0	0	9	9	0	0	-3	-3	0	0	0	-1	-1	1	1	2	2	2	0	0	0	1	0	0	-1	-1	-1	-1	-1	0	-1	-1	0	0	-1	-1	0	0	0	0	-1	1	1	-1	0	0	1	x_{68}
x_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{69}
x_{70}														

$Fi'_{24} = F_{3+}$

		1A	2A	2B	3A	3B	3C	3D	3E	4A	4B	4C	5A	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	7B	8A	8B	8C	9A	9B	9C	9D	9E	9F	10A	10B	11A	12A	12B	12C	12D	12E	
χ_{91}	+	164572397352	-577368	13608	-531441	0	0	0	-729	-216	216	72	27	-729	0	0	-81	0	0	0	0	0	0	-9	-1	6	-8	0	0	0	0	0	0	0	0	7	3	0	27	0	3	-9	0	χ_{91}
χ_{92}	+	164572397352	-577368	13608	-531441	0	0	0	-729	-216	216	72	27	-729	0	0	-81	0	0	0	0	0	0	-9	-1	6	-8	0	0	0	0	0	0	0	0	7	3	0	27	0	3	-9	0	χ_{92}
χ_{93}	+	169598100672	192192	52416	1120392	3564	-32886	648	0	960	448	64	-28	1176	-126	60	72	-180	-30	-48	36	-54	-36	0	0	0	0	0	0	0	-81	0	0	0	0	-8	-4	0	-48	-12	0	16	6	χ_{93}
χ_{94}	+	178514751987	-2814669	124659	0	0	0	0	0	243	-405	99	162	0	0	0	0	0	0	0	0	0	0	0	-9	-9	-9	-9	3	0	0	0	0	0	0	6	-6	0	0	0	0	0	0	χ_{94}
χ_{95}	+	184117100544	2555904	0	-539136	114048	-2592	3240	0	0	512	0	-56	-384	0	192	0	0	-96	-24	0	0	0	0	-15	-8	0	0	0	0	81	0	0	0	0	4	0	3	0	0	0	-16	0	χ_{95}
χ_{96}	+	190685695200	114400	-92960	978900	49830	420	1068	-624	480	-160	96	0	-2780	280	190	148	-170	-80	28	-44	-20	4	16	35	0	0	0	0	-12	-75	-3	-12	33	-3	0	0	0	-36	30	12	20	-12	χ_{96}
χ_{97}	+	197813862400	0	0	716800	-112640	-8960	1408	-896	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-104	40	-32	58	-14	4	0	0	0	0	0	0	0	0	χ_{97}
χ_{98}	+	197813862400	0	0	716800	-112640	-8960	1408	-896	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-104	40	-32	58	-14	4	0	0	0	0	0	0	0	0	χ_{98}
χ_{99}	+	200219979960	120120	-11592	-741195	-128106	-14868	-1260	-819	184	-56	24	35	861	0	78	21	198	24	24	0	-12	12	-3	0	0	8	0	0	63	-27	45	-18	0	0	-5	3	0	-23	22	1	-11	4	χ_{99}
χ_{100}	+	200219979960	120120	-11592	-741195	-128106	-14868	-1260	-819	184	-56	24	35	861	0	78	21	198	24	24	0	-12	12	-3	0	0	8	0	0	63	-27	45	-18	0	0	-5	3	0	-23	22	1	-11	4	χ_{100}
χ_{101}	+	205353825600	123200	-56000	-760200	-94380	5250	-2040	-168	-960	0	64	0	-280	70	404	184	-236	62	-28	16	10	4	-8	0	0	0	0	0	66	30	-60	-15	3	3	0	0	0	48	12	0	0	-6	χ_{101}
χ_{102}	+	205353825600	123200	-56000	-760200	-94380	5250	-2040	-168	-960	0	64	0	-280	70	404	184	-236	62	-28	16	10	4	-8	0	0	0	0	0	66	30	-60	-15	3	3	0	0	0	48	12	0	0	-6	χ_{102}
χ_{103}	+	205940550816	562848	54432	-610740	-119394	4536	-2754	0	672	-160	32	-134	-372	-756	-426	-180	270	-156	6	54	0	-18	0	-21	0	0	0	0	0	81	81	0	0	0	-2	2	0	12	6	12	20	0	χ_{103}
χ_{104}	+	222758961152	-5111808	0	-173056	-110848	29120	1904	-832	0	0	0	-48	768	0	-384	0	0	192	48	0	0	0	0	-14	0	0	0	0	-40	50	32	41	-4	-4	-8	0	2	0	0	0	0	0	χ_{104}
χ_{105}	+	225247477455	1336335	7119	-626535	126522	-3969	-1053	0	207	7	-65	-70	1113	315	402	153	234	-93	51	-9	-9	-9	0	0	0	3	3	-1	0	-81	0	0	0	0	10	-6	0	9	18	9	-23	-9	χ_{105}
χ_{106}	+	250274974950	-2252250	-54810	-511875	381150	6930	-846	819	230	350	-42	0	2205	-630	198	-3	-18	-54	-18	18	-78	6	3	0	0	10	-6	6	-63	-135	-45	18	0	0	0	0	0	77	14	-19	5	14	χ_{106}
χ_{107}	+	282049015248	-432432	-93744	132678	-304722	-20412	1458	0	336	336	-112	273	1134	756	-378	-90	-162	-108	-54	-54	36	18	0	0	0	0	0	0	0	0	0	0	0	0	-7	1	0	-6	-42	18	6	12	χ_{107}
χ_{108}	+	336033532800	-873600	-13440	-378000	-23220	-3780	-378	-756	-640	0	-128	0	-1680	420	-420	-144	-84	84	66	42	12	-18	12	0	0	0	0	0	27	0	54	27	0	0	0	0	-2	-16	-28	-16	0	-4	χ_{108}
	ind	1	2	2	3	3	3	3	3	4	4	4	5	6	6	6	6	6	6	6	6	6	6	6	7	7	8	8	8	9	9	9	9	9	9	10	10	11	12	12	12	12	12	
		3	6	6		3	3			12	12	12	15	6	6	6		6	6	6	6	6			21	21	24	24	24		9				9	30	30	33		12		12	12	
		3	6	6		3	3			12	12	12	15	6	6	6		6	6	6	6	6			21	21	24	24	24		9				9	30	30	33		12		12	12	
χ_{109}	o2	783	79	15	0	54	27	0	0	15	7	-1	8	16	15	-2	0	6	7	-2	6	3	0	0	6	-1	3	3	-1	0	9	0	0	0	3	4	0	2	0	6	0	4	3	χ_{109}
χ_{110}	o2	64584	1352	72	0	-297	216	0	0	72	8	8	34	-16	-36	-25	0	-9	20	2	18	0	0	0	9	2	0	8	0	0	-9	0	0	0	6	2	2	3	0	-9	0	-4	0	χ_{110}
χ_{111}	o2	306153	3433	489	0	1431	351	0	0	105	57	9	28	112	111	31	0	39	31	4	12	15	0	0	15	1	9	1	1	0	36	0	0	0	3	8	4	1	0	15	0	12	3	χ_{111}
χ_{112}	o2	306153	2729	-279	0	1431	351	0	0	105	-15	-7	28	128	99	-25	0	-9	11	2	18	-9	0	0	15	1	-3	5	1	0	36	0	0	0	3	4	-4	1	0	15	0	0	3	χ_{112}
χ_{113}	o2	6724809	26377	-567	0	-3861	2079	0	0	393	-15	-7	134	-128	-189	-29	0	27	43	-2	54	-9	0	0	21	0	-3	13	1	0	-36	0	0	0	6	2	-2	3	0	-21	0	0	3	χ_{113}
χ_{114}	o2	19034730	44330	1386	0	13662	2457	0	0	426	34	-6	105	464	441	-58	0	-18	41	-4	36	9	0	0	15	-6	6	6	2	0	90	0	0	0	3	5	1	0	0	30	0	4	-3	χ_{114}
χ_{115}	o2	25356672	40832	384	0	16632	2484	0	0	384	64	0	47	656	384	8	0	24	8	8	24	12	0	0	33	5	0	0	0	0	99	0	0	0	-3	7	-1	0	0	24	0	4	12	χ_{115}
χ_{116}	o2	43779879	79079	1575	0	-11880	3537	0	0	615	63	-9	104	-112	-315	104	0	-72	77	-4	36	9	0	0	24	-4	3	3	-1	0	-36	0	0	0	6	4	0	0	0	-24	0	-12	-3	χ_{116}
χ_{117}	o2	195019461	182533	5061	0	45495	4158	0	0	645	237	-11	36	544	714	-41	0	183	130	-14	-6	6	0	0	21	0	9	-7	-3	0	90	0	0	0	3	8	-4	-1	0	15	0	24	-6	χ_{117}
χ_{118}	o2	195019461	127621	-5691	0	45495	4158	0	0	645	-195	21	36	784	546	127	0	-57	10	-8	24	-42	0	0	21	0	-15	1	-3	0	90	0	0	0	3	-4	4	-1	0	15	0	-12	-6	χ_{118}
χ_{119}	o2	203843871	194271	2079	0	-15147	7371	0	0	735	119	-17	196	-336	-189	-291	0	-27	51	6	54	27	0	0	0	0	3	11	-1	0	-54	0	0	0	0	-4	4	0	0	-3	0	-4	3	χ_{119}
χ_{120}	o2	216154575	-61425	4815	0	2430	1215	0	0	15	-105	-1	0	0	-45	54	0	-18	27	54	-18	-9	0	0	0	-7	3	3	7	0	0	0	0	0	0	0	0	-1	0	6	0	0	3	χ_{120}
χ_{121}	o2	216154575	-61425	4815	0	2430	1215	0	0	15	-105	-1	0	0	-45	54	0	-18	27	54	-18	-9	0	0	0	-7	3	3	7	0	0	0	0	0	0	0	0	-1	0	6	0	0	3	χ_{121}
χ_{122}	o2	330032934	205414	-5082	0	27027	5859	0	0	486	-146	22	209	16	399	-245	0	3	7	-2	30	-21	0	0	-21	0	-6	10	-2	0	9	0	0	0	3	-11	-7	0	0	27	0	4	-9	χ_{122}
χ_{123}	o2	1050717096	580008	936	0	-68607	13446	0	0	1320	168	40	196	-96	-846	129	0	-63	138	-6	18	-18	0	0	30	2	0	-8	0	0	-54	0	0	0	0	8	-4	0	0	-39	0	-24	-6	χ_{123}
χ_{124}	o2	1152161010	-30030	-7182	0	-891	-2268	0	0	114	154	66	35	336	0	21	0	-27	-24	48	0	-36	0	0	0	0	6	-2	-6	0	-27	0	0	0	0	-5	3	0	0	-3	0	4	12	χ_{124}
χ_{125}	o2	1152161010	-30030	-7182	0	-891	-2268	0	0	114	154	66	35	336	0	21	0	-27	-24	48	0	-36	0	0	0	0	6	-2	-6	0	-27	0	0	0	0	-5	3	0	0	-3	0	4	12	χ_{125}
χ_{126}	o2	1255560075	152075	-4725	0	-14850	-3780	0	0	75	-245	-21	0	560	0	38	0	270	56	-16	0	36	0	0	0	0	3	3	3	0	-45	0	0	0	-6	0	0	0	0	30	0	-20	-12	χ_{126}
χ_{127}	o2	1255560075	152075	-4725	0	-14850	-3780	0	0	75	-245	-21	0	560	0	38	0	270	56	-16	0	36	0	0	0	0	3	3	3	0	-45	0	0	0	-6	0	0	0	0	30	0	-20	-12	χ_{127}
χ_{128}	o2	1349587008	-133056	-4032	0	-10692	3402	0	0	-192	0	64	-42	0	126	108	0	-36	54	-54	18	18	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-2	0	0	-12	0	0	-6	χ_{128}
χ_{129}	o2	1349587008	-133056	-4032	0	-10692	3402	0	0	-192	0	64	-42	0	126	108	0	-36	54	-54	18	18	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-2	0	0	-12	0	0	-6	χ_{129}
χ_{130}	o2	1553430879	519519	-18081	0	-25839	10773	0	0	927	-273	-33	154	-672	-315	633	0	225	21	12	36	-27	0	0	0	0	-9	-1	-1	0	-54	0	0	0	0	-6	-6	0	0	9	0	24	9	χ_{130}
χ_{131}	o2	1818548820	623700	29268	0	56133	9477	0	0	660	420	84	120	0	729	405	0	-27	81	0	0	45	0	0	-24	11	12	4	4	0	0	0	0	0	0	0	8	0	0	21	0	0	-3	χ_{131}
χ_{132}	o2	2400239520	656800	6048	0	-79002	16632	0	0	1440	-160	32	120	-560	-756	-434	0	54	-92	-2	54	0	0	0	21	0	0	0	0	0	-45	0	0	0	-6	0	8	-3	0	-18	0	20	0	χ_{132}
χ_{133}	o2	2801912256	448448	33216	0	105732	6210	0	0	192	448	64	6	896	330	356	0	132	-22	-22	-30	66	0	0	24	-11	0	0	0	0	45	0	0	0	-3	-2	6	0	0	12	0	16	6	χ_{133}
χ_{134}	o2	3264456195	715715	-6909	0	177903	10584	0	0	963	-357	-45	70	1232	840	-1	0	-177	8	-28	12	24	0	0	0	0	-9	-1	3	0	126	0	0	0	-3	-10	6	0	0	-9	0	-12	0	χ_{134}
χ_{135}	o2	4290428142	107822	-23058	0	134838	9828	0	0	366	6	30	-83	2000	0	-394	0	54	-88	-16	0	-36	0	0	21	0	-6	2	6	0	36	0	0	0	-6	-3	-3	0	0	6	0	-12	12	χ_{135}
χ_{136}	o2	4416617205	685685	-14091	0	177012	8316	0	0	1077	-203	21	-70	1568	840	20	0	-204	-16	20	12	-12	0	0	0	0	-3	-3	-3	0	99	0	0	0	-3	10	-6	0	0	-12	0	-8	12	χ_{136}
χ_{137}	o2	4620461076	825748	14868	0	161865	15687	0	0	468	308	-76	126	1456	1071	-47	0	153	-65	34	18	-9	0	0	0	0	12	-4	-4	0	45	0	0	0	-3	-2	-2	0	0	9	0	-4	-9	χ_{137}
χ_{138}	o2	8250621525	-492075	-25515	0	0	0	0	0	405	405	117	0	0	0	0	0	0	0	0	0	0	0	0	-15	6	-3	-3	-3	0	0	0	0	0	0	0	0	3	0	0	0	0	0	χ_{138}
χ_{139}	o2	8250621525	-492075	-25515	0	0	0	0	0	405	405	117	0	0	0	0	0	0	0	0	0	0	0	0	-15	6	-3	-3	-3	0	0	0	0	0	0	0	0	3	0	0	0	0	0	χ_{139}
χ_{140}	o2	8993909925	-675675	15525	0	-40095	7290	0	0	165	-315	-91	0	0	270	-135	0	81	54	-54	-54	18	0	0	0	7	-3	5	-3	0	0	0	0	0	0	0	0	0	0	-15	0	0	6	χ_{140}
χ_{141}	o2	8993909925	-675675	15525	0	-40095	7290	0	0	165	-315	-91	0	0	270	-135	0	81	54	-54	-54	18	0	0	0	7	-3	5	-3	0	0	0	0	0	0	0	0	0	0	-15	0	0	6	χ_{141}
χ_{142}	o2	9456453864	-216216	49896	0	32076	5103	0	0	-24	-168	-56	-111	0	-189	540	0	108	27	54	54	-9	0	0	-24	18	0	0	8	0	0	0	0	0	0	9	1	0	0	12	0	0	15	χ_{142}
χ_{143}	o2	10726070355	1390675	47187	0	120582	4158	0	0	-45	435	83	-120	-320	126	-338	0	342	238	-14	18	-18	0	0	-21	0	3	3	3	0	90	0	0	0	-6	0	-8	0	0	-18	0	0	18	χ_{143}
χ_{144}	o2	12258366120	-280280	40104	0	137808	11313	0	0	-216	56	-120	-105	1456	-531	304	0	144	-11	-20	36	9	0	0	0	7	0	0	-8	0	45	0	0	0	-3	-5	-1	0	0	0	0	-4	9	χ_{144}
χ_{145}	o2	14507059905	1605825	-11583	0	-216513	19683	0	0	1665	-15	-63	-120	0	-729	567	0	-81	-81	0	0	27	0	0	9	2	9	1	1	0	0	0	0	0	0	0	-8	-3	0	-9	0	0	-9	χ_{145}
χ_{146}	o2	15016498497	2523521	24129	0	-177606	19089	0	0	1665	105	-15	-28	-448	-819	-358	0	-198	245	20	-36	9	0	0	0	0	-3	-11	1	0	-36	0	0	0	6	-4	4	0	0	18	0	0	9	χ_{146}
χ_{147}	o2	21014341920	1716000	288	0	-73062	-2268	0	0	-480	160	-96	120	240	-576	258	0	234	168	-12	-36	-36	0	0	-9	-9	0	0	0	0	135	0	0	0	0	0	8	0	0	-30	0	-20	12	χ_{147}
χ_{148}	o2	21096751104	2342912	24576	0	-19008	25488	0	0	0	0	0	204	320	384	-256	0	-192	32	-40	24	48	0	0	-54	-5	0	0	0	0	-90	0	0	0	-3	12	-4	0	0	0	0	0	0	χ_{148}
χ_{149}	o2	21122107776	1798016	-24192	0	-2376	27972	0	0	384	-64	0	251	-2128	0	-184	0	216	-184	32	0	-36	0	0	-21	0	0	0	0	0	9	0	0	0	-6	-9	3	0	0	24	0	-4	12	χ_{149}
χ_{150}	o2	21842179632	1224496	72240	0	292788	-3780	0	0	1200	112	-16	-168	784	420	100	0	276	-44	46	6	12	0	0	0	0	0	0	0	0	-45	0	0	0	3	-4	0	-2	0	-60	0	4	-12	χ_{150}
χ_{151}	o2	34128405675	3114475	-28245	0	383670	13230	0	0	555	35	-101	0	-560	1050	-218	0	150	178	-2	-66	-42	0	0	0	0	-9	-1	3	0	-45	0	0	0	3	0	0	1	0	6	0	20	-6	χ_{151}
χ_{152}	o2	34128405675	1512875	-55125	0	383670	13230	0	0	555	-245	107	0	560	630	38	0	-234	-106	38	-18	-18	0	0	0	0	3	-5	3	0	-45	0	0	0	3	0	0	1	0	6	0	-20	-6	χ_{152}
χ_{153}	o2	41785039296	448448	-28224	0	371844	10962	0	0	192	-448	64	-154	896	-630	356	0	-252	-22	-22	18	18	0	0	0	0	0	0	0	0	9	0	0	0	3	-2	6	0	0	12	0	-16	6	χ_{153}
χ_{154}	o2	63831063582	393822	36126	0	216513	19683	0	0	-1314	294	78	57	2592	-729	-567	0	81	-81	0	0	27	0	0	6	-1	-6	2	-2	0	0	0	0	0	0	-3	1	0	0	9	0	24	-9	χ_{154}
χ_{155}	o2	78346948680	-120120	59976	0	-60588	-16443	0	0	456	-616	40	105	1344	441	84	0	468	-15	30	-18	45	0	0	0	0	0	0	-8	0	-135	0	0	0	0	5	1	0	0	-12	0	-16	21	χ_{155}
χ_{156}	o2	80256172032	1114112	0	0	-266112	30240	0	0	0	-512	0	-168	128	0	-448	0	0	-160	-16	0	0	0	0	15	-6	0	0	0	0	-45	0	0	0	-6	12	0	3	0	0	0	16	0	χ_{156}
χ_{157}	o2	80256172032	3997696	0	0	-266112	30240	0	0	0	512	0	-168	-896	0	832	0	0	-32	-32	0	0	0	0	15	-6	0	0	0	0	-45	0	0	0	-6	-4	0	3	0	0	0	-16	0	χ_{157}
χ_{158}	o2	84057367680	3569280	-54144	0	-292248	17172	0	0	-384	64	0	105	1104	-1224	-552	0	72	-48	-12	-36	-36	0	0	-15	13	0	0	0	0	135	0	0	0	0	5	1	0	0	-24	0	4	-12	χ_{158}
χ_{159}	o2	103562058600	2602600	-22680	0	81675	-30240	0	0	360	-280	168	0	-560	0	475	0	-405	160	16	0	0	0	0	0	0	0	8	0	0	45	0	0	0	-12	0	0	0	0	-45	0	20	0	χ_{159}
χ_{160}	o2	115511526900	4627700	102900	0	-283635	-16065	0	0	180	420	-76	0	2240	-105	413	0	-339	71	62	-42	15	0	0	0	0	-12	4	-4	0	-90	0	0	0	-3	0	0	0	0	45	0	0	-9	χ_{160}
χ_{161}	o2	120131987976	-184184	-11256	0	311256	-5481	0	0	264	504	-24	-49	2800	-861	-152	0	120	-53	-44	12	15	0	0	0	0	0	0	8	0	-126	0	0	0	3	-9	-1	0	0	-24	0	12	15	χ_{161}
χ_{162}	o2	135605256192	3997696	0	0	245376	6048	0	0	0	-512	0	-8	-896	0	832	0	0	-32	-32	0	0	0	0	-21	0	0	0	0	0	-9	0	0	0	6	-4	0	-1	0	0	0	16	0	χ_{162}
χ_{163}	o2	135605256192	1114112	0	0	245376	6048	0	0	0	512	0	-8	128	0	-448	0	0	-160	-16	0	0	0	0	-21	0	0	0	0	0	-9	0	0	0	6	12	0	-1	0	0	0	-16	0	χ_{163}
χ_{164}	o2	142169187069	2393469	60669	0	216513	0	0	0	-963	573	93	-6	-2592	0	-567	0	81	0	0	0	0	0	0	21	0	-3	5	-3	0	0	0	0	0	0	-6	-6	-3	0	9	0	-24	0	χ_{164}
χ_{165}	o2	150164984970	3773770	-32886	0	-476982	-8127	0	0	714	-574	154	-105	1456	189	-830	0	378	-11	34	-54	9	0	0	0	0	6	6	2	0	45	0	0	0	6	-5	-1	0	0	-6	0	-4	-3	χ_{165}
χ_{166}	o2	151397207325	2764125	42525	0	295245	0	0	0

$F_{3+} = Fi'_{24}$

	12F	12G	12H	12I	12J	12K	12L	12M	13A	14A	14B	15A	15B	15C	16A	17A	18A	18B	18C	18D	18E	18F	18G	H**	20A	20B	21A	21B	21C	D*	22A	23A	B**	24A	24B	24C	24D	24E	24F	G*	26A	27A	27B	27C	28A	29A	B*	30A	30B	33A	B*	35A	36A	36B	36C	36D	
x_{91}	0	0	0	3	3	0	0	0	-1	-1	0	9	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	-1	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	0	-1-b29	*	1	0	0	0	-1	0	0	0	0	x_{91}
x_{92}	0	0	0	3	3	0	0	0	-1	-1	0	9	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	-1	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	0	-1	*-b29	1	0	0	0	-1	0	0	0	0	x_{92}
x_{93}	6	16	4	0	0	-2	-2	-2	0	0	0	17	-1	-1	0	1	0	0	0	3	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	1	x_{93}
x_{94}	0	0	0	0	0	0	0	0	0	3	3	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	x_{94}
x_{95}	0	8	0	0	0	0	-4	0	0	1	0	-11	-2	-2	0	1	0	0	0	-3	0	0	0	0	2	0	-3	0	-2	-2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	-1	x_{95}
x_{96}	0	2	-6	0	0	0	2	0	0	-1	0	0	0	0	0	0	-11	1	4	1	1	1	1	1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-3	-3	0	-1	x_{96}
x_{97}	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	-1	-1	0	0	0	0	0	0	0	0	x_{97}
x_{98}	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	-1	-1	0	0	0	0	0	0	0	0	x_{98}
x_{99}	4	-2	-6	1	1	0	-2	0	0	0	0	5	-1	2	0	0	-3	-3	3	-3	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	x_{99}
x_{100}	4	-2	-6	1	1	0	-2	0	0	0	0	5	-1	2	0	0	-3	-3	3	-3	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	1	x_{100}
x_{101}	-6	0	4	0	0	-2	0	-2	1	0	0	0	0	0	0	0	4	-4	-2	2	1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r3	-2r3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{101}
x_{102}	-6	0	4	0	0	-2	0	-2	1	0	0	0	0	0	0	0	4	-4	-2	2	1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2r3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{102}
x_{103}	-6	2	2	0	0	-4	2	2	0	-1	0	10	1	1	0	0	9	-3	0	-3	0	0	0	0	0	0	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-2	-1	0	0	-1	-3	-3	0	-1	x_{103}
x_{104}	0	0	0	0	0	0	0	0	0	-2	0	-6	-3	0	0	1	0	0	0	6	0	0	0	0	0	0	-2	1	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	-2	0	-1	-1	1	0	0	0	0	x_{104}
x_{105}	9	-2	-2	0	0	-5	1	1	0	0	0	-10	2	-4	1	0	0	0	0	3	0	0	0	0	2	2	0	0	0	0	0	0	0	-3	-3	0	0	0	-1	-1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	1	x_{105}
x_{106}	-4	-10	-6	-1	-1	6	2	0	0	0	0	0	0	0	0	0	3	3	-3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	x_{106}
x_{107}	-6	-6	2	0	0	-4	0	2	0	0	0	3	3	3	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	x_{107}
x_{108}	2	0	4	-4	-4	4	0	-2	4	0	0	0	0	0	0	0	6	6	3	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	2	2	-1	0	x_{108}
	12	12	12	12	12	12	12	12	13	14	14	15	15	15	16	17	18	18	18	18	18	18	18	18	20	20	21	21	21	21	22	23	23	24	24	24	24	24	24	24	26	27	27	27	28	29	29	30	30	33	33	35	36	36	36	36	
		12	12			12	12	12	39	42	42	15	15	15	48	51		18		18		18	18	18	60	60			21	21	66	69	69					24	24	24	78				84	87	87	30	30	33	33	105				36	
		12	12			12	12	12	39	42	42	15	15	15	48	51		18		18		18	18	18	60	60			21	21	66	69	69					24	24	24	78				84	87	87	30	30	33	33	105				36	
x_{109}	0	-2	2	0	0	-1	1	2	3	2	1	5	-1	2	1	1	0	4	0	1	0	1	-13	13	2	0	0	0	-1	-1	2	1	1	0	0	0	0	0	-1	-1	1	0	0	0	0	0	0	1	0	-1	-1	1	0	0	0	1	x_{109}
x_{110}	0	-1	-1	0	0	-4	2	2	0	1	2	-5	-2	1	0	1	0	-4	0	-1	0	2	0	0	-2	2	0	0	-1	-1	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	1	1	-1	-1	0	0	-1	0	0	0	-1	x_{110}
x_{111}	0	3	3	0	0	3	3	0	3	3	-1	10	1	1	1	0	0	4	0	4	0	1	13	-13	2	0	0	0	1	1	1	0	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	2	1	1	1	0	0	0	0	0	x_{111}
x_{112}	0	3	-1	0	0	-1	-3	2	3	-1	1	10	1	1	-1	0	0	8	0	-4	0	-1	-3	-3	0	0	0	0	1	1	1	0	0	0	0	0	0	-1	1	1	-1	0	0	0	-1	0	0	-2	-1	1	1	0	0	0	0	0	x_{112}
x_{113}	0	3	-1	0	0	-1	-3	2	0	1	0	-10	-1	-1	-1	0	0	-8	0	4	0	-2	0	0	0	-2	0	0	0	0	-1	0	0	0	0	0	0	1	1	1	0	0	0	0	-1	-1	-1	2	1	0	0	1	0	0	0	0	x_{113}
x_{114}	0	-2	-6	0	0	-3	1	0	0	-1	0	15	-3	-3	0	0	0	8	0	2	0	-1	3	3	-1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	-1	0	0	-1	1	0	0	0	0	0	0	-2	x_{114}
x_{115}	0	-8	0	0	0	0	-2	0	3	1	-1	5	2	-1	0	-1	0	8	0	-1	0	-1	-13	13	-1	-1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	1	-1	0	0	-2	0	0	0	1	x_{115}
x_{116}	0	0	0	0	0	9	3	0	0	0	0	-10	5	2	1	0	0	-4	0	-4	0	2	0	0	-2	0	0	0	2	2	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	-2	0	0	0	-1	0	0	0	0	x_{116}
x_{117}	0	3	7	0	0	-2	0	-2	0	1	0	15	0	3	-1	0	0	-8	0	-2	0	1	13	-13	2	0	0	0	0	0	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	-1	0	0	-1	-1	-1	-1	1	0	0	0	0	x_{117}
x_{118}	0	3	3	0	0	6	0	0	0	-3	0	15	0	3	-1	0	0	4	0	-2	0	1	-13	13	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	-1	1	-1	-1	1	0	0	0	0	x_{118}
x_{119}	0	-7	1	0	0	-5	-1	-2	-3	0	0	-5	-2	1	1	-1	0	0	0	-6	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	2	x_{119}
x_{120}	0	-6	2	0	0	-1	-3	2	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0-b21	*	-1	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0	-1	-1	0	0	0	0	0	x_{120}
x_{121}	0	-6	2	0	0	-1	-3	2	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*-b21	-1	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0	-1	-1	0	0	0	0	0	x_{121}
x_{122}	0	7	7	0	0	-5	1	-2	-3	-1	0	5	2	-1	0	0	0	4	0	1	0	1	13	-13	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	1	-1	0	0	-1	0	0	0	1	x_{122}
x_{123}	0	-3	1	0	0	-2	0	-2	0	2	-2	-5	-2	1	0	0	0	0	0	6	0	0	0	0	-2	0	0	0	-1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	x_{123}
x_{124}	0	1	-3	0	0	0	-2	0	0	0	0	5	-1	2	0	1	0	0	0	-3	0	0	0	0	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	x_{124}
x_{125}	0	1	-3	0	0	0	-2	0	0	0	0	5	-1	2	0	1	0	0	0	-3	0	0	0	0	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	x_{125}
x_{126}	0	-2	-6	0	0	0	-2	0	3	0	0	0	0	0	-1	0	0	-4	0	-1	0	2	0	0	0	0	0	0	0	0	0	b23	**	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{126}
x_{127}	0	-2	-6	0	0	0	-2	0	3	0	0	0	0	0	-1	0	0	-4	0	-1	0	2	0	0	0	0	0	0	0	0	0	**	b23	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{127}
x_{128}	0	0	4	0	0	-2	0	-2	3	0	0	0	3	-3	0	1	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	2r3	-2r3	-1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	x_{128}
x_{129}	0	0	4	0	0	-2	0	-2	3	0	0	0	3	-3	0	1	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0-2r3	2r3	-1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	x_{129}
x_{130}	0	-3	-3	0	0	9	-3	0	0	0	0	-5	1	-2	-1	0	0	0	0	6	0	0	0	0	2	2	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	1	1	3	0	0	0	0	0	0	0	0	0	0	x_{130}
x_{131}	0	-3	-3	0	0	9	3	0	-3	0	1	0	3	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	1	1	1	-1	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	x_{131}
x_{132}	0	2	2	0	0	-4	2	2	3	-3	0	0	3	-3	0	0	0	-8	0	-5	0	-2	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	-1	0	0	1	0	0	0	-1	x_{132}
x_{133}	0	16	4	0	0	-2	-2	-2	0	0	1	-15	-3	0	0	0	0	-4	0	5	0	-1	-13	13	-2	2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	1	x_{133}
x_{134}	0	3	-9	0	0	0	0	0	0	0	0	10	-2	4	1	0	0	-4	0	2	0	-1	13	-13	-2	-2	0	0	0	0	0	1	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	x_{134}
x_{135}	0	6	6	0	0	0	6	0	0	1	0	-20	-2	-2	0	0	0	8	0	-4	0	2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	x_{135}
x_{136}	0	4	-12	0	0	0	-2	0	0	0	0	-10	2	-4	1	1	0	-4	0	-1	0	-1	13	-13	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	1	0	x_{136}
x_{137}	0	-7	5	0	0	-1	-1	2	-3	0	0	-15	0	-3	0	0	0	4	0	1	0	1	-3	-3	-2	-2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	x_{137}
x_{138}	0	0	0	0	0	0	0	0	3	-3	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1-b29	*	0	0	0	0	0	0	0	0	0	x_{138}
x_{139}	0	0	0	0	0	0	0	0	3	-3	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	*-b29	0	0	0	0	0	0	0	0	0	0	x_{139}
x_{140}	0	9	5	0	0	2	0	2	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b21	*	0	-1	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{140}
x_{141}	0	9	5	0	0	2	0	2	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	*	b21	0	-1	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{141}
x_{142}	0	12	4	0	0	-5	-3	-2	0	0	0	0	6	3	0	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	x_{142}
x_{143}	0	-6	2	0	0	2	6	2	0	-1	0	0	-3	3	-1	0	0	-8	0	-2	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	-1	0	0	0	0	x_{143}
x_{144}	0	-16	0	0	0	-3	-1	0	0	0	1	-15	3	3	0	0	0	4	0	1	0	1	3	3	1	-1	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	-1	x_{144}
x_{145}	0	3	3	0	0	3	-3	0	0	-3	2	0	-3	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	-1	0	0	0	1	0	0	-1	0	0	0	0	x_{145}
x_{146}	0	6	-6	0	0	-3	3	0	0	0	0	-10	-1	-1	-1	0	0	8	0	-4	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	1	0	0	0	0	0	0	0	2	1	0	0	0	0	0	0	0	x_{146}
x_{147}	0	-2	6	0	0	0	-2	0	0	-1	1	0	3	-3	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	0	0	1	0	0	0	1	x_{147}
x_{148}	0	0	0	0	0	0	0	0	0	-2	-1	0	-3	3	0	1	0	8	0	2	0	-1	-13	13	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	x_{148}
x_{149}	0	8	0	0	0	0	2	0	3	3	0	5	-1	2	0	0	0	-4	0	5	0	2	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	-1	0	0	-3	0	0	0	-1	0	0	0	-1	x_{149}
x_{150}	0	4	-4	0	0	-4	4	2	0	0	0	15	3	0	0	0	0	4	0	7	0	1	-13	13	2	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	0	0	0	0	1	x_{150}
x_{151}	0	-10	-2	0	0	-2	-4	-2	0	0	0	0	0	0	1	0	0	-8	0	-5	0	1	-13	13	0	0	0	0	0	0	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	-1	x_{151}
x_{152}	0	-2	14	0	0	2	-2	2	0	0	0	0	0	0	-1	0	0	-4	0	-1	0	-1	3	3	0	0	0	0	0	0	1	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0

$Fi'_{24} = F_{3+}$

	39A	B*	39C	D*	42A	42B	C*	45A	B*	60A			2C	2D	4D	4E	4F	4G	6L	6M	6N	6O	6P	6Q	6R	6S	6T	6U	6V	8D	8E	8F	10C	10D	12N	12O	12P	12Q	12R	12S	12T	12U	12V	12W	12X	
x_{91}	-1	-1	-1	-1	-1	0	0	0	0	1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{91}
x_{92}	-1	-1	-1	-1	-1	0	0	0	0	1																																				x_{92}
x_{93}	0	0	0	0	0	0	0	-1	-1	1	:	++	517188672	103488	-448	-192	192	0	-19656	-1836	-396	594	24	132	-54	-126	-30	-18	0	0	0	0	-28	8	-16	56	-16	12	2	0	0	12	2	6	-6	x_{93}
x_{94}	0	0	0	0	0	0	0	0	0	0	:	++	227988189	120285	-8019	-243	-243	-27	0	0	0	0	0	0	0	0	0	0	0	-15	9	9	-36	0	0	0	0	0	0	0	0	0	0	0	0	x_{94}
x_{95}	0	0	0	0	1	0	0	1	1	-1	:	++	264536064	98304	-4096	0	0	0	22464	-3456	0	432	-192	-192	-216	0	24	0	0	0	0	0	14	-6	-64	-64	8	0	8	0	0	0	8	0	0	x_{95}
x_{96}	0	0	0	0	-1	0	0	0	0	0	:	++	536764800	-77440	-3200	-640	128	0	1560	-2274	110	210	-40	-58	-60	-70	14	20	8	0	0	0	0	0	40	-8	-14	2	4	8	8	-10	-8	8	8	x_{96}
x_{97}	*	b117	1	1	0	0	0	0	0	0	:	++	504627200	0	0	0	0	0	-17920	-4096	0	-640	0	0	224	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{97}
x_{98}	b117	*	1	1	0	0	0	0	0	0	:	++	504627200	0	0	0	0	0	-17920	-4096	0	-640	0	0	224	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{98}
x_{99}	0	0	0	0	0	0	0	-b45	*	-1		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{99}
x_{100}	0	0	0	0	0	0	0	*	-b45	-1																																				x_{100}
x_{101}	1	1	1	1	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{101}
x_{102}	1	1	1	1	0	0	0	0	0	0																																				x_{102}
x_{103}	0	0	0	0	-1	0	0	1	1	0	:	++	193235328	-54912	512	0	384	0	-2808	-3618	-594	594	168	6	-216	-54	6	0	0	0	0	0	-22	-2	80	8	26	-30	-28	0	0	6	8	0	0	x_{103}
x_{104}	0	0	0	0	-2	0	0	0	0	0	:	++	97976320	-65536	0	0	0	0	-11648	-1280	-1024	1312	128	128	16	128	-16	-16	8	0	0	0	20	4	0	0	0	0	0	0	0	0	0	0	0	x_{104}
x_{105}	0	0	0	0	0	0	0	-1	-1	2	:	++	264188925	146685	-5075	-435	141	21	12285	-2862	-990	1755	-195	-6	-27	-45	-33	9	0	25	1	-15	0	0	-35	-35	-26	6	1	-3	-3	-6	1	-3	-3	x_{105}
x_{106}	0	0	0	0	0	0	0	0	0	0	:	++	176125950	-770	-2450	270	-178	182	-4095	-1908	220	360	-95	148	-126	40	-32	22	7	10	-14	2	0	0	25	49	-20	-16	16	9	-7	-4	4	0	0	x_{106}
x_{107}	0	0	0	0	0	0	0	0	0	1	:	++	529296768	-88704	-4032	576	0	0	0	-5346	-594	-486	-144	-90	0	-54	18	0	0	16	-16	0	-7	1	-36	0	18	18	18	-12	12	6	0	0	-6	x_{107}
x_{108}	1	1	-2	-2	0	0	0	0	0	0	:	++	472953600	-134400	0	0	-256	0	-15120	-7020	-780	1080	240	-156	-54	120	24	-6	0	0	0	0	0	0	0	0	0	-4	0	0	-16	-4	0	0	0	x_{108}
	39	39	39	39	42	42	42	45	45	60	fus	ind	2	2	4	4	4	4	6	6	6	6	6	6	6	6	6	6	6	8	8	8	10	10	12	12	12	12	12	12	12	12	12	12	12	x_{109}
					42	42	42	45	45	60																																				
					42	42	42	45	45	60																																				
x_{109}	0	0	0	0	2	1	1	-1	-1	-1	*	+																																		
x_{110}	0	0	0	0	-2	-1	-1	1	1	1	*	+																																		
x_{111}	0	0	0	0	0	-1	-1	1	1	2	*	+																																		
x_{112}	0	0	0	0	2	1	1	1	1	0	*	+																																		
x_{113}	0	0	0	0	-2	0	0	-1	-1	0	*	+																																		
x_{114}	0	0	0	0	2	0	0	0	0	-1	*	+																																		
x_{115}	0	0	0	0	-2	-1	-1	-1	-1	-1	*	+																																		
x_{116}	0	0	0	0	0	0	0	-1	-1	-2	*	+																																		
x_{117}	0	0	0	0	-2	0	0	0	0	-1	*	+																																		
x_{118}	0	0	0	0	0	0	0	0	0	3	*	+																																		
x_{119}	0	0	0	0	0	0	0	1	1	1	*	+																																		
x_{120}	0	0	0	0	0	-b21	*	0	0	0		o2																																		
x_{121}	0	0	0	0	0	*	-b21	0	0	0	*																																			
x_{122}	0	0	0	0	2	0	0	-1	-1	-1	*	+																																		
x_{123}	0	0	0	0	2	1	1	1	1	1	*	+																																		
x_{124}	0	0	0	0	0	0	0	-b45	*	-1		o2																																		
x_{125}	0	0	0	0	0	0	0	*	-b45	-1	*																																			
x_{126}	0	0	0	0	0	0	0	0	0	0	*	o																																		
x_{127}	0	0	0	0	0	0	0	0	0	0	*	o																																		
x_{128}	0	0	0	0	0	0	0	0	0	0	.	o2																																		
x_{129}	0	0	0	0	0	0	0	0	0	0	*																																			
x_{130}	0	0	0	0	0	0	0	1	1	-1	*	+																																		
x_{131}	0	0	0	0	0	1	1	0	0	0	*	+																																		
x_{132}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{133}	0	0	0	0	0	1	1	0	0	1	*	+																																		
x_{134}	0	0	0	0	0	0	0	1	1	-2	*	+																																		
x_{135}	0	0	0	0	-2	0	0	1	1	-2	*	+																																		
x_{136}	0	0	0	0	0	0	0	-1	-1	2	*	+																																		
x_{137}	0	0	0	0	0	0	0	0	0	1	*	+																																		
x_{138}	0	0	0	0	0	0	0	0	0	0		o2																																		
x_{139}	0	0	0	0	0	0	0	0	0	0	*																																			
x_{140}	0	0	0	0	0	-b21	*	0	0	0		o2																																		
x_{141}	0	0	0	0	0	*	-b21	0	0	0	*																																			
x_{142}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{143}	0	0	0	0	2	0	0	0	0	0	*	+																																		
x_{144}	0	0	0	0	0	1	1	0	0	1	*	+																																		
x_{145}	0	0	0	0	0	-1	-1	0	0	0	*	+																																		
x_{146}	0	0	0	0	0	0	0	-1	-1	0	*	+																																		
x_{147}	0	0	0	0	2	-2	-2	0	0	0	*	+																																		
x_{148}	0	0	0	0	-2	-1	-1	0	0	0	*	+																																		
x_{149}	0	0	0	0	0	0	0	-1	-1	1	*	+																																		
x_{150}	0	0	0	0	0	0	0	0	0	-1	*	+																																		
x_{151}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{152}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{153}	0	0	0	0	0	0	0	-1	-1	-1	*	+																																		
x_{154}	0	0	0	0	2	-1	-1	0	0	-1	*	+																																		
x_{155}	0	0	0	0	0	0	0	0	0	-1	*	+																																		
x_{156}	0	0	0	0	2	0	0	0	0	1	*	+																																		
x_{157}	0	0	0	0	0	0	0	0	0	-1	*	+																																		
x_{158}	0	0	0	0	-2	1	1	0	0	-1	*	+																																		
x_{159}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{160}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{161}	0	0	0	0	0	0	0	-1	-1	2	*	+																																		
x_{162}	0	0	0	0	0	0	0	1	1	1	*	+																																		
x_{163}	0	0	0	0	2	0	0	1	1	-1	*	+																																		
x_{164}	0	0	0	0	-2	0	0	0	0	1	*	+																																		
x_{165}	0	0	0	0	0	0	0	0	0	1	*	+																																		
x_{166}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{167}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{168}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{169}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{170}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{171}	0	0	0	0	0	1	1	0	0	0	*	+																																		
x_{172}	0	0	0	0	-2	1	1	0	0	0	*	+																																		
x_{173}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{174}	0	0	0	0	-2	0	0	1	1	0	*	+																																		
x_{175}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{176}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{177}	0	0	0	0	0	0	0	0	0	-1	*	+																																		
x_{178}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{179}	0	0	0	0	0	0	0	-1	-1	0	*	+																																		
x_{180}	0	0	0	0	2	-1	-1	0	0	0	*	+																																		
x_{181}	0	0	0	0	0	0	0	0	0	0	*	+																																		
x_{182}	0	0	0	0	0	0	0	0	0	0	*	+																																		

Sporadic Fischer group Fi'_{24}

Order = 1,255,205,709,190,661,721,292,800 = $2^{21}.3^{16}.5^{2}.7^{3}.11.13.17.23.29$ Mult = 3 Out = 2

Constructions

Fischer G.2 is generated by a conjugacy class of 306936 involutions, which are 3-transpositions, i.e. any non-commuting pair has product of order 3. Joining transpositions when they commute we obtain a rank 3 graph of valence 31671, whose automorphism group is again G.2. The centralizer of a transposition is 2 x Fi_{23}, and the centralizer of the product of two distinct transpositions is $2^2Fi_{22}.2 \equiv (2 \times 2{\cdot}Fi_{22}){:}2$ or $3 \times O^{+}_{8}(3){:}S_3$ according as they commute or not. Products of 2, 3, or 4 mutually commuting transpositions are called bi-, tri- or tetra-transpositions respectively. All maximal sets of mutually commuting transpositions (bases) are conjugate and contain 24 transpositions, which generate an elementary Abelian group 2^{12} whose normalizer is a non-split extension $2^{12}{\cdot}M_{24}$. The sets of transpositions in a base whose product is 1 are the C-sets of a binary Golay code (see M_{24}), and so the elementary Abelian 2^{12}-group has the structure of the Golay cocode. There are three orbits of $2^{12}M_{24}$ on transpositions: a transposition is either in the base, or commutes with an octad or with a duad of transpositions in the base. There are 24 basic transpositions, $2^5.759$ octadic, and $2^{10}.276$ duadic. Any 8 mutually commuting transpositions with trivial product (forming an octad) lie in just three bases B_1, B_2 and B_3. With respect to B_1 the remaining 32 elements of B_2 and B_3 are the octadic transpositions for the corresponding octad.

"Orthogonal" G has a subgroup $H \equiv O^{-}_{10}(2)$ which has three orbits on transpositions:

(i) the 528 reflections of H

(ii) 104448 transpositions centralized by A_{12} in H

(iii) 201960 transpositions centralized by $2^{6+8}(A_7 \times 3)$ in H

The transpositions of the third orbit fall naturally into octads, and the stabilizer $2^{6+8}(A_8 \times 3)$ of such an octad has index 2 in its stabilizer in G. An explicit form for the graph, and the action of a generating set of transpositions, have been found using the orbit decomposition under the subgroup S_{12} of H.2.

Complex 3.G has a 783-dimensional complex representation over $\mathbb{Q}(w)$, where $w = z3$. The 783 coordinate 1-spaces are indexed by the points i of a 24-set [U] and the octads of a Steiner system S(5,8,24) on [U] (see M_{24}). There is an invariant algebra with multiplication $x * y$, antilinear in each variable, and 3.G.2 is generated by the antilinear maps ("reflections") $x \longrightarrow x^r = x * r - \langle r,x\rangle r$, for 3 x 306936 root vectors r. These are the scalar multiples, by powers of w, of certain vectors:

$(-7,1^{23}|0^{759})$ (3x24) (basic)

$\frac{1}{2}(-1^8,1^{16}|\pm\theta^1,0^{280},0^{448},\pm1^{30})$ $(3\times2^5.759)$ (octadic)

$\frac{1}{8}(-7^2,1^{22}|\pm\theta^{77},0^{352},\pm1^{330})$ $(3\times2^{10}.276)$ (duadic)

where the inner product $\langle x,y\rangle$ of vectors $x = (\ldots x_i\ldots|\ldots x_A\ldots)$ and $y = (\ldots y_i\ldots|\ldots y_A\ldots)$ is $\frac{1}{8}\Sigma x_i\bar{y}_i + \Sigma x_A\bar{y}_A$. The reflection in $(-7,1^{23}|0^{759})$ (with the -7 in the i'th coordinate) negates the coordinates corresponding to octads containing i, in addition to complex conjugating all coordinates. For more details see the next section.

Loop The sign problem in the above construction is rather subtle, and is best understood as follows. There is a loop P of order 2^{13}, which has a homomorphism $A \longrightarrow [A]$ (with kernel $\{\pm1\}$) onto the additive group of the binary Golay code. The loop is defined up to isomorphism by the equations

$$A^2 = (-1)^{\frac{1}{4}|[A]|}$$

$$BA = (-1)^{\frac{1}{2}|[A]\cap[B]|}AB$$

$$A(BC) = (-1)^{|[A]\cap[B]\cap[C]|}(AB)C$$

and satisfies the Moufang identity $A((BC)A) = (AB)(CA)$. If $A_1, \ldots, A_{12}$ are elements of P such that $[A_1], \ldots, [A_{12}]$ generate the C-sets, then there are 2^{12} standard automorphisms S to each permutation [S] of M_{24}, such that $[A_1^S], \ldots, [A_{12}^S]$ are the images under [S] of $[A_1], \ldots, [A_{12}]$. The standard automorphisms form the group $2^{12}{\cdot}M_{24}$ and are called even or odd according as $U^S = U$ or $-U$, where U is a particular one of the two loop elements mapping to the universal C-set [U]. The standard automorphisms corresponding to the identity element of M_{24} are named by C^*-sets d and take A to A or -A according as d meets [A] evenly or oddly.

There are 24 coordinate vectors e_i (i in [U]) of norm $\frac{1}{8}$, and 2 x 759 coordinate vectors e_A of norm 1 which are indexed by the loop elements A for which [A] is an octad, and satisfy $e_{-A} = -e_A$. We can now define the 32 octadic root vectors corresponding to a given octad a as follows: There are 32 C-sets {}, $b_1,\ldots,b_{30}$, a' disjoint from a, where $b_1,\ldots,b_{30}$ are octads and a' is the complement of a. Their inverse images in P form an elementary Abelian group of order 2^6. There are therefore 32 ways to select pre-images 1, $B_1,\ldots,B_{30}$, UA forming a group. Each such way determines the signs in one of the octadic root vectors $v = \frac{1}{2}(-1^8,1^{16}|\pm\theta^1,0^{280},0^{448},\pm1^{30})$ corresponding to the octad a by:

$$2v = \Sigma e_j - \Sigma e_i + \theta e_A + \Sigma B_b \text{ (summed over i in [A], j not in [A]).}$$

If two octads a_1, a_2 meet in a duad d then the typical duadic root vector for d has the form r^s where r and s are octadic root vectors for a_1 and a_2.

$F_{3+} = Fi'_{24}$

	12Y	12Z	12A1	12B1	14C	14D	16B	18I	18J	18K	18L	18M	18N	18O	18P	18Q	18R	20C	20D	22B	22C	24H	24I	24J	24K	26B	C*	28B	28C	D*	30C	30D	30E	30F	30G	34A	36E	36F	36G	36H	40A	42D	42E	46A	B**	54A	60B	66A	B*	70A	78A	B*	84A	
x_{91}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{91}
x_{92}																																																							x_{92}
x_{93}	6	0	0	0	0	0	0	0	9	-18	0	0	-3	6	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	1	2	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	x_{93}
x_{94}	0	0	0	0	9	-3	-1	0	0	0	0	0	0	0	0	0	0	6	2	0	0	0	0	0	0	0	0	3	1	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	0	0	0	x_{94}
x_{95}	0	0	0	0	-5	3	0	0	9	-18	0	0	-3	6	0	0	0	4	0	-3	1	0	0	0	0	0	0	-1	0	0	-1	3	2	0	0	-1	2	-1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	-1	x_{95}
x_{96}	2	0	-4	0	5	1	0	-6	3	3	2	3	-1	-1	2	-1	-3	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	1	2	-1	0	-1	1	0	0	0	0	0	0	0	0	0	-1	x_{96}
x_{97}	0	0	0	0	0	0	0	8	8	8	0	-10	0	0	0	0	2	0	0	0	0	0	0	0	0	2b13	*	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1	-1	0	0	0	0-b13		*	0	x_{97}
x_{98}	0	0	0	0	0	0	0	8	8	8	0	-10	0	0	0	0	2	0	0	0	0	0	0	0	0	*	2b13	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	1	-1	0	0	0	0	*-b13		0	x_{98}
x_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{99}
x_{100}																																																							x_{100}
x_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{101}
x_{102}																																																							x_{102}
x_{103}	-6	0	0	0	-1	3	0	0	9	9	0	0	-3	-3	0	0	0	12	0	0	0	0	0	0	0	0	0	1	0	0	2	-2	-1	1	1	0	-1	-1	0	0	0	-1	0	0	0	0	0	0	0	-1	0	0	1	x_{103}
x_{104}	0	0	0	0	-6	-2	0	16	-2	-20	8	-2	2	-4	-1	2	-2	0	0	2	2	0	0	0	0	0	0	0	0	0	2	-2	2	-2	1	-1	0	0	0	0	0	0	1	0	0	1	0	-1	-1	-1	0	0	0	x_{104}
x_{105}	3	3	3	0	0	0	-1	0	-9	18	0	0	3	-6	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{105}
x_{106}	-4	2	-4	-1	0	0	0	9	-9	9	-11	0	-5	1	-2	4	0	0	0	0	0	1	1	-2	2	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{106}
x_{107}	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-7	1	0	0	2	-2	2	0	0	0	0	0	0	5	1	-1	1	1	0	0	0	0	0	1	0	0	-1	-1	0	-1	0	0	0	0	0	0	x_{107}
x_{108}	8	0	2	0	0	0	0	27	0	0	3	0	0	0	3	0	0	0	0	-2	-2	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	1	0	-1	-1	0	x_{108}
x_{109}	12	12	12	12	14	14	16	18	18	18	18	18	18	18	18	18	18	20	20	22	22	24	24	24	24	26	26	28	28	28	30	30	30	30	30	34	36	36	36	36	40	42	42	46	46	54	60	66	66	70	78	78	84	x_{109}

Algebra — The algebra is antilinear in each variable, so should properly be regarded as defined on the 2x783-dimensional real form of the representation. In terms of the "reflection" $x \to x^r$ in a root vector r, we have $x * r = x^r + \langle r,x\rangle r$. We define $E_i = 8e_i$, so that $\{\ldots,E_i,\ldots,e_A,\ldots\}$ is the dual base to $\{\ldots,e_i,\ldots,e_A,\ldots\}$. Then the non-zero products of basis elements are

$$2E_i*E_i = -81e_i+15\Sigma e_j,\quad 2E_i*E_j = 15e_i+15e_j-\Sigma e_k \ (j \neq i)\ (k \neq i,j)$$

$$2E_i*e_A = 2e_A*E_i = 3e_A,\quad 2E_j*e_A = 2e_A*E_j = -e_A,\quad 2e_A*e_A = -2e_A*e_{-A} = 3\Sigma e_i - \Sigma e_j \ (i \text{ in } A,\ j \text{ not in } A)$$

$$2e_A*e_B = e_{AB} \text{ or } \theta e_{ABU} \text{ if } [AB] \text{ or } [ABU] \text{ is an octad.}$$

Modulo 3 — G.2 has a 783-dimensional representation over $\mathbb{F}_3$ obtained from the above by reducing modulo θ. This has a fixed vector $I = \Sigma e_i$ (i in [U]), and on the 782-space modulo I there is an invariant algebra (obtained from the negative of the above algebra) for which the only non-zero products of basis vectors are

$$e_i*e_j = e_i+e_j \ (i \neq j) \qquad e_j*e_A = e_A*e_j = e_A \ (j \text{ not in } [A])$$

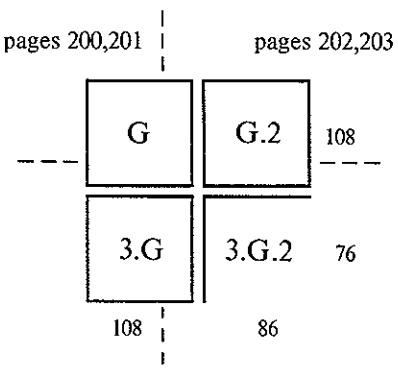
$$e_A*e_A = -e_A*e_{-A} = \Sigma e_i \ (i \text{ in } [A]) \qquad e_A*e_B = e_{AB} \ ([AB] \text{ an octad})$$

G acts irreducibly on the 781-dimensional space orthogonal to I.

"Presentation" 3.G.2 ≅ < ——————————— | more relations > ;

omitting the lowest node leaves $O^-_{10}(2).2$; omitting the lowest pair of nodes leaves S_{12}.

The leftmost and rightmost nodes are redundant. By adjoining more redundant nodes the diagram may be enlarged to the incidence graph of the 9 points and 9 lines of the Pappus configuration. See page 232.



Some maximal subgroups (including all p-local subgroups for p ≠ 2)

Structure	G.2	Character	Specifications: Abstract	Specifications: Fischer
Fi_{23}	$: Fi_{23} \times 2$	1a+57477a+249458a	C(2C)	transposition
$2{\cdot}Fi_{22}{:}2$	$: (2 \times 2{\cdot}Fi_{22}){:}2$		N(2A)	bi-transposition
$(3 \times O^+_8(3){:}3){:}2$	$: S_3 \times O^+_8(3){:}S_3$		N(3A)	triad
$O^-_{10}(2)$	$: O^-_{10}(2){:}2$			
$3^7{\cdot}O_7(3)$	$: 3^7{\cdot}O_7(3){:}2$		$N(3^7) = N(3A_{378}B_{364}C_{351})$	
$3^{1+10}_+{:}U_5(2){:}2$	$: 3^{1+10}_+{:}(2 \times U_5(2){:}2)$		N(3B)	
$2^{11}{\cdot}M_{24}$	$: 2^{12}{\cdot}M_{24}$		$N(2^{11}) = N(2A_{276}B_{1771})$	base
$2^2{\cdot}U_6(2){:}S_3$	$: (2 \times 2^2{\cdot}U_6(2)){:}S_3$		$N(2A^2)$, C(2D)	tri-transposition
$2^{1+12}_+{\cdot}3U_4(3).2_2$	$: 2^{1+12}{\cdot}3U_4(3).(2^2)_{122}$		N(2B)	tetra-transposition
$3^2.3^4.3^8.(A_5 \times 2A_4).2$	$: 3^2.3^4.3^8.(S_5 \times 2S_4)$		$N(3B^2)$	
$(A_4 \times O^+_8(2){:}3){:}2$	$: S_4 \times O^+_8(2){:}S_3$		$N(2A^2)$	
He:2	$7^{1+2}_+{:}(6 \times S_3).2$			
He:2				
$2^{3+12}.(L_3(2) \times A_6)$	$: 2^{3+12}.(L_3(2) \times S_6)$		$N(2B^3)$	
$2^{6+8}.(S_3 \times A_8)$	$: 2^{7+8}.(S_3 \times A_8)$		$N(2^6) = N(2A_{28}B_{35})$	octad
$(3^2{:}2 \times G_2(3))'2$	$: (S_3 \times S_3 \times G_2(3)){:}2$		$N(3^2) = N(3A_2E_2)$	
$(A_5 \times A_9){:}2$	$: S_5 \times S_9$		N(2A,3A,5A)	
$7{:}6 \times A_7$	$: 7{:}6 \times S_7$		N(7A)	
29:14	: 29:28		N(29AB)	
$3^3.[3^{10}].GL_3(3)$	$: 3^3.[3^{10}].(L_3(3) \times 2^2)$		$N(3B^3)$	
$A_6 \times L_2(8){:}3$	$: S_6 \times L_2(8){:}3$			

$B = F_{2+}$

		e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	
		4154781481226	30612991873	3548835956	26489012	116891823873705065303437205154109586 09	20293	190259512682415 91	60397	3019	1437	1258	44356000	000	11036	62705	3185	403	199	174	149	1069953	497	373	564									
		42619117758054400000	5099415756800	61213696000	826931200	992793600	9926400	70080	2016000	0557440	7204736	3629440	14720	91040	97760	89888	69600	29120	20000	000	1968640	66400	04960	10784	06560	18240	29920	16832	280	664	248	480			
		p power	A	A	A	A	A	A	B	B	B	B	D	B	B	D	C	D	A	A	AA	AA	AB	BA	AD	AC	BB	AD	BD	BD	BC	A			
		p' part	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	AA	AA	AB	BA	AD	AC	BB	AD	BD	BD	BC	A			
	ind	1A	2A	2B	2C	2D	3A	3B	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	5A	5B	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A		
x_1	+	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		x_1
x_2	+	4371	-493	275	-53	19	78	-3	-77	51	19	-21	35	-13	11	3	-1	-5	21	-4	-34	20	14	-7	4	-8	5	-2	13	1	1	10		x_2
x_3	+	96255	4863	2047	103	255	351	27	351	223	-1	71	111	31	39	15	-1	-9	55	5	111	57	31	3	9	13	-5	15	27	3	-5	19		x_3
x_4	+	1139374	-26962	8878	782	-338	1000	-53	-978	558	-82	-114	238	46	14	-18	-26	14	99	-1	-232	92	40	11	-20	8	-5	-8	43	-5	-1	26		x_4
x_5	+	9458750	118846	37950	3486	2110	2729	-25	2750	1214	318	286	510	190	158	-2	-26	30	175	0	505	181	105	19	37	57	15	25	55	-5	3	35		x_5
x_6	+	9550635	119339	35627	-781	-2005	3003	6	2827	1163	43	83	603	-53	-77	-5	27	3	210	10	539	161	59	26	-31	-7	14	-37	86	2	2	45		x_6
x_7	+	63532485	-468027	134597	-4811	1989	8073	135	-7547	2437	197	-267	1317	-251	181	-27	1	21	385	-15	-1143	315	137	-9	27	-77	-25	9	135	-9	-5	65		x_7
x_8	+	347643114	1511146	392426	1274	8938	16224	-219	14378	4394	-534	826	2186	-86	378	-54	78	26	539	-11	1792	334	224	-71	46	14	-19	64	181	1	5	70		x_8
x_9	+	356054375	-1901977	419175	-1377	3175	18227	-160	-18425	8327	-153	-1089	2295	-185	223	87	27	15	650	0	-2365	713	243	-16	25	-27	0	-29	160	16	0	89		x_9
x_{10}	+	1407126890	-3199638	789866	25194	-12950	25103	155	-18326	3178	-406	-406	3178	874	-406	-22	26	-86	615	15	-2001	-57	143	-57	-89	111	35	-17	235	-17	3	64		x_{10}
x_{11}	+	3214743741	3059133	1214653	22933	-32835	32670	270	17501	-3875	1469	-587	4173	157	-555	45	-27	69	616	-9	1518	-372	94	6	-36	64	-50	-114	270	6	10	55		x_{11}
x_{12}	+	4221380670	-999362	1615934	-44642	36926	37401	465	-8130	-8130	-706	414	4542	-706	414	62	-78	-34	595	45	-551	-551	281	97	89	-119	65	89	305	17	-11	45		x_{12}
x_{13}	+	4275362520	10237656	1710808	48568	69336	42471	594	45144	20056	1240	2360	3480	1368	1208	280	0	120	770	20	3927	1227	295	66	123	175	34	183	114	18	22	65		x_{13}
x_{14}	+	4622913750	11656918	2011350	-42042	-13610	58422	345	57750	22678	-1066	1414	5430	-490	-186	-10	-78	-90	1275	0	5446	1288	246	-35	-104	-84	-15	-26	265	-11	-3	126		x_{14}
x_{15}	+	9287037474	16720418	1896994	87074	90658	19734	1023	24354	31522	2850	4642	1058	1570	1058	418	26	98	99	-26	2486	470	406	155	118	134	55	118	-17	19	-1	1		x_{15}
x_{16}	+	12501781215	-15693601	2075359	65807	-27425	52404	1293	-47201	11935	-1057	1743	3647	543	-881	-1	-53	111	715	-35	-3388	446	-140	-175	-50	98	85	100	157	25	-19	56		x_{16}
x_{17}	+	13508418144	-18677152	3512928	-134368	37472	57135	1488	-60576	18272	3680	-3040	3808	-1696	1312	-32	0	-96	694	19	-4465	881	687	-172	113	-295	120	-49	32	-4	-16	46		x_{17}
x_{18}	+	27416186875	-37543429	6369275	-44149	-2565	129349	-1385	-125125	32827	2299	-1589	10395	-325	11	-37	-1	-85	1925	0	-9163	1691	389	71	-21	-49	-25	21	375	-9	23	140		x_{18}
x_{19}	+	75844139371	-56581525	3402091	162435	-58261	-1001	2887	4235	49931	875	-3549	315	203	-573	27	-273	435	-154	-4	-2233	-847	343	-109	-31	21	-17	167	23	59	3	0		x_{19}
x_{20}	+	80426400000	-87975680	10451200	104960	-75520	169170	-120	-172800	77056	-1792	-4608	8960	1280	512	-256	0	0	1750	0	-10814	2470	274	40	-106	146	40	-94	200	8	-16	100		x_{20}
x_{21}	o	90807234375	55087175	2498375	-118625	-42425	-10725	-1815	-9625	42215	-3001	1855	-105	-1625	-545	503	-325	15	0	0	1595	-475	155	-115	-155	85	65	-5	25	37	13	-10		x_{21}
x_{22}	o	90807234375	55087175	2498375	-118625	-42425	-10725	-1815	-9625	42215	-3001	1855	-105	-1625	-545	503	-325	15	0	0	1595	-475	155	-115	-155	85	65	-5	25	37	13	-10		x_{22}
x_{23}	+	108348770530	84710626	15223010	108290	77026	230230	-1187	195426	25954	-30	1666	16546	866	514	-94	26	-254	2255	-95	12166	826	662	97	106	182	5	-26	493	-23	-7	126		x_{23}
x_{24}	+	156871952640	119318784	16386304	113152	-71424	243594	-216	244992	52480	2304	-512	13056	1280	512	-256	0	0	2190	140	12858	2598	202	-48	-42	202	-104	-102	264	0	-32	109		x_{24}
x_{25}	o	252984703125	56581525	895125	-229075	-135275	-23100	795	-28875	20405	6293	-1715	805	2165	365	-123	325	-355	0	0	1540	-350	-60	-53	130	-70	75	4	-5	-5	47	0		x_{25}
x_{26}	o	252984703125	56581525	895125	-229075	-135275	-23100	795	-28875	20405	6293	-1715	805	2165	365	-123	325	-355	0	0	1540	-350	-60	-53	130	-70	75	4	-5	-5	47	0		x_{26}
x_{27}	+	309720864375	-124219017	27321975	165087	8055	316953	2025	-202025	-16809	-4233	-1857	21735	1175	159	7	-273	175	2100	0	-10647	-1197	729	153	99	63	-135	9	585	9	9	90		x_{27}
x_{28}	+	536105794455	267461271	22612887	713727	469143	208494	2187	224631	108279	6807	5727	5319	3127	2239	-153	651	15	1155	80	8910	2106	-18	-81	378	90	-45	270	27	-9	-45	35		x_{28}
x_{29}	+	551549171250	67178034	36697650	-189294	-74190	336699	-3420	89650	-74190	2866	1170	25010	-1230	-878	50	326	210	1925	0	4059	-3501	699	-288	-141	-141	60	27	660	24	-24	35		x_{29}
x_{30}	+	635966233056	350269920	41884128	127008	467424	360855	0	401632	142560	-2592	14112	14688	2784	3616	96	0	-96	2156	-69	18711	3645	1431	0	189	189	0	279	0	0	0	70		x_{30}
x_{31}	+	641238778125	348069645	35578125	-244035	-464115	405405	810	395725	131277	-1523	1149	18285	-2995	-3523	301	25	285	2750	0	18909	2655	-99	90	-129	-441	90	-291	330	42	18	99		x_{31}
x_{32}	+	1095935366250	-488660886	60806250	-268566	211050	560196	-405	-553750	124650	-5270	-7830	23850	-790	3050	42	246	-150	3200	0	-24012	3204	900	207	180	-168	-45	-108	315	-9	3	81		x_{32}
x_{33}	+	1917448516791	-707107401	68232375	373167	4023	577368	0	-628265	169047	-5193	-3249	17415	-1257	2287	-153	351	-225	2441	-84	-21384	5832	216	0	-216	-216	0	-72	0	0	0	73		x_{33}
x_{34}	+	2624476798125	-749628243	86858925	-682227	320685	725517	1620	-654675	76461	10157	5901	30765	-5715	-243	-83	-247	205	3025	0	-24003	1377	-435	-216	177	-315	60	285	420	-48	36	56		x_{34}
x_{35}	o	4097337118875	-42567525	-21278565	-1289925	10395	0	-6561	10395	10395	5019	315	2331	5019	315	-357	975	251	0	125	0	0	0	243	0	0	135	0	-81	27	-81	0		x_{35}
x_{36}	o	4097337118875	-42567525	-21278565	-1289925	10395	0	-6561	10395	10395	5019	315	2331	5019	315	-357	975	251	0	125	0	0	0	243	0	0	135	0	-81	27	-81	0		x_{36}
x_{37}	+	4375623425250	1192564450	73514210	1374450	1637090	98175	-159	210210	344610	11746	28210	-1022	7842	2930	578	350	402	0	250	11935	1225	1535	-251	425	105	41	95	-79	-19	69	0		x_{37}
x_{38}	+	4836998684375	1062116055	124444375	-946985	217815	837980	2465	645975	62807	-8233	2391	34455	-6825	-553	-361	-325	215	2750	0	24156	-1980	-164	477	-156	-356	25	348	465	-75	13	39		x_{38}
x_{39}	+	6143375289051	631449819	79710939	-1137045	759003	-81081	-2349	-68453	69531	27867	12075	5691	2587	-21	-1157	-325	-629	-924	301	7623	-1827	1863	-261	-3	-147	-189	-57	51	-21	-21	0		x_{39}
x_{40}	+	6145833622500	1307779044	127638500	2169284	1804260	467181	945	601700	188516	6628	26692	6180	6500	7876	1188	676	260	825	0	14685	3669	2093	429	357	545	185	189	-255	21	-31	19		x_{40}
x_{41}	+	6619124890560	961268672	159742912	693056	1043392	959322	3036	667072	-92736	-3136	1344	38080	2496	5952	-320	0	64	2310	-65	18074	-826	346	-448	358	182	4	154	556	40	20	0		x_{41}
x_{42}	+	6697382793750	-1667387370	112521750	123590	211990	803880	-2475	-804650	292822	-4842	3206	18550	2390	-4538	310	-350	-90	2750	0	-24024	3570	-984	141	274	434	-75	40	85	13	-43	54		x_{42}
x_{43}	+	7331420799495	-1735627257	159891975	2311335	-825849	852930	0	-896633	255879	-10233	-12249	26055	7815	-3161	327	351	615	2795	120	-29646	2430	1026	0	-594	594	0	18	0	0	0	37		x_{43}
x_{44}	+	8165038927500	828158604	116354700	3075436	268940	340704	-5895	300300	-18676	28812	-4116	6860	7180	2668	-692	676	940	175	0	11760	-2856	1440	429	8	532	225	-112	-55	-43	1	0		x_{44}
x_{45}	o	8379477898506	861288714	-17337078	322218	-1162998	0	0	-141174	112266	-8694	810	3402	-2934	1962	1098	-702	426	-594	381	0	0	0	0	0	0	0	0	0	0	0	36		x_{45}
x_{46}	o	8379477898506	861288714	-17337078	322218	-1162998	0	0	-141174	112266	-8694	810	3402	-2934	1962	1098	-702	426	-594	381	0	0	0	0	0	0	0	0	0	0	0	36		x_{46}
x_{47}	+	9958526703125	-978052075	58095125	1071525	661525	-114400	5075	103125	117205	-15851	-2715	-1035	10325	-1755	821	325	965	0	0	-8800	-2050	800	83	-290	-330	35	160	115	-29	111	-35		x_{47}
x_{48}	+	10238958815085	1052416365	16787309	530621	1666413	432432	-6102	234157	31661	-1939	-18179	525	4653	317	-691	-351	-1059	385	85	3696	-1974	-976	618	426	98	410	240	138	90	-10	0		x_{48}
x_{49}	+	10238958815085	-2423952531	147359597	-335699	-150675	432432	-6102	-569107	552685	-1939	-37331	1197	1261	5037	-83	-247	301	385	85	-22176	3780	1904	-234	-108	-224	50	-192	-198	30	46	0		x_{49}
x_{50}	+	10529575087500	-662332020	173026700	-463540	-868980	936495	-675	-438900	-209524	16268	9548	44940	3980	-4788	140	300	-820	1750	0	-9681	-5145	-721	201	-249	-49	-235	-81	525	33	-31	0		x_{50}
x_{51}	+	10853720317440	-496624128	201383424	1584128	-352768	962676	-2520	-129536	-252416	-8704	-11264	43520	5632	-1024	512	0	0	1540	-60	-5676	-5676	756	-168	-76	596	360	-364	680	-40	-16	-35		x_{51}
x_{52}	o	11890281046875	-683330725	2646875	1062075	-937125	-150150	4560	182875	75355	7259	13755	3675	-7845	-325	987	-625	-645	0	0	-8470	350	-70	260	270	210	200	-54	0	-36	-24	0		x_{52}
x_{53}	o	11890281046875	-683330725	2646875	1062075	-937125	-150150	4560	182875	75355	7259	13755	3675	-7845	-325	987	-625	-645	0	0	-8470	350	-70	260	270	210	200	-54	0	-36	-24	0		x_{53}
x_{54}	+	12927978301875	-2955201613	233687475	-1916733	817075	1067703	-2955	-1254925	513779	19635	-33789	-22995	-8845	5955	1299	-377	-285	2100	0	-36169	8057	1719	-367	313	-399	45	-329	-155	73	-3	0		x_{54}
x_{55}	+	17413195263750	-2845334778	137113350	-2228122	1018630	-18018	4500	9350	522886	32774	-23322	9030	-11130	-666	70	598	-410	-1100	0	-21714	-3318	1374	-312	154	-154	60	334	100	-80	-28	-36		x_{55}
x_{56}	+	19246163186250	3964722762	326079050	-300118	472650	1613898	-9990	1781450	532682	4938	22826	32010	-2870	4778	-246	-702	-470	3850	0	48378	8742	1418	-114	-330	50	50	90	-150	6	14	35		x_{56}
x_{57}	+	19946023665750	2050161750	100532310	696150	3011670	447525	12312	621270	122070	2902	3030	3606	13014	6230	278	650	-554	0	125	-1035	6525	-795	396	45	225	240	405	72	36	36	-35		x_{57}
x_{58}	o	21346507000000	-726689600	-83720000	1019200	145600	336700	4600	56000	-15680	2240	-2240	-6720	-320	320	960	0	-960	0	0	4060	-3500	700	-44	-140	-140	-560	28	40	-20	112	0		x_{58}
x_{59}	o	21346507000000	-726689600	-83720000	1019200	145600	336700	4600	56000	-15680	2240	-2240	-6720	-320	320	960	0	-960	0	0	4060	-3500	700	-44	-140	-140	-560	28	40	-20	112	0		x_{59}
x_{60}	+	22348485900000	3108998880	219199200	3612950	2653920	787995	-540	1029600	270816	25312	4128	19040	8160	3872	352	0	160	1750	0	18459	4905	-741	-468	201	441	-300	411	-60	-36	36	-20		x_{60}
x_{61}	+	24573501156204	2525799276	318843756	1689324	-2469012	1135134	10287	1066604	-44436	25452	-4116	38892	3180	-7956	-788	-676	-148	2079	-46	26334	-882	1566	171	-114	294	135	-546	-33	3	15	0		x_{61}
x_{62}	+	35301999416320	632067072	-1046886640	0	2659328	123760	13276	21504	21504	21504	0	7168	5120	0	-1024	1248	2048	-980	70	-4368	3696	368	420	-400	0	-244	-16	-4	116	0	0		x_{62}
x_{63}	+	38348970335820	-1121764788	448945740	-3993300	2789964	1213785	0	-511028	-372276	-16308	5292	47628	-12084	6188	396	324	-276	595	195	-10935	-6075	2457	0	405	-783	0	297	0	0	0	-10		x_{63}
x_{64}	+	38658236225625	5058537561	360257625	-3005991	-1931175	1876446	0	1857625	280665	-9639	-24615	38745	-3495	-6695	1113	729	345	3200	0	37422	5346	-1890	0	-270	-378	0	-306	0	0	0	16		x_{64}
x_{65}	+	40955835260340	4209665460	67149236	43316	1160628	27027	-17847	-246092	492212	20916	9268	-3276	11444	1844	52	-676	-76	-385	-285	8547	-7329	83	285	495	35	209	3	153	69	-19	0		x_{65}
x_{66}	+	47097393520320	5507206848	193076928	2164032	2895552	208494	8748	310464	660672	26304	33600	-576	16576	8512	960	0	64	-330	-305	8910	2106	-18	-324	378	90	-180	270	108	-36	36	-46		x_{66}
x_{67}	+	78359378686875	9011981979	383845275	-1937949	-3875685	1482624	20250	1511675	804987	-3685	6339	1035	-11845	-4189	-1877	351	115	1925	0	26928	6084	-576	630	-204	-384	450	-240	-150	30	30	-15		x_{67}
x_{68}	+	89626740328125	8725441725	522856125	1417325	6670525	997425	23805	1280125	623485	-11331	21805	13725	8445	8685	1117	-975	-435	0	0	31185	5175	945	549	55	455	45	529	-35	85	-67	-70		x_{68}
x_{69}	+	94981065075000	-9392827080	700787000	-256360	-1815240	2371005	8640	-2115400	368312	6968	-5096	44280	10680	-3688	-1416	0	600	2750	0	-48531	-3495	-259	-336	-279	149	-160	-51	0	-48	32	-36		x_{69}
x_{70}	+	118479380574555	-7935316389	800481627	-2436525	1028187	2606175	10935	-2318085	-197253	19803	12915	45675	-13253	3731	459	-273	195	1980	180	-31185	2025	-225	243	-135	-315	-225	207	135	27	-45	-56		x_{70}
x_{71}	+	119483690332160	-9270317056	645607424	-6823936	1060864	1566488	-6208	-2207744	315392	28672	0	0	-12288	8192	0	0	0	2310	-340	-22792	7448	920	752	-8	-784	-160	184	-512	16	-64	0		x_{71}
x_{72}	+	123485536419840	-12201197568	596148224	3411968	-2064384	1493856	-1728	-1835008	917504	0	-28672	0	0	4096	0	-1728	0	1365	-160	-28896	7392	608	-384	-480	392	320	-96	-192	0	32	0		x_{72}
x_{73}	+	140763279350304	-8764026336	322820640	3740256	-1490400	1791153	0	-1759968	147744	-6624	42336	6048	1824	-6048	672	0	-1056	1254	-321	-5103	5103	-2511	0	-81	567	0	81	0	0	0	-54		x_{73}
x_{74}	+	162746401888125	-12226310019	868960125	773109	-488835	2397681	-10935	-2635875	282141	-26499	16533	41805	6365	-1739	-1875	-51	1125	825	0	-40095	4131	-207	-243	243	-225	225	-63	-135	-27	45	-1		x_{74}
x_{75}	+	181844090400000	17795697920	1020243200	-304640	2100480	2781570	-8880	3027200	1147136	-21248	36352	34560	6400	512	768	0	0	1750	0	57266	4670	194	-784	174	-134	-400	18	-240	-48	-80	-35		x_{75}
x_{76}	+	210919336847010	5765644962	958293666	2955042	-3442014	2453814	0	1381666	-694494	-15966	-20574	64098	1314	-2014	354	-378	-606	385	135	16038	-11178	-1674	0	486	594	0	-378	0	0	0	-55		x_{76}
x_{77}	+	211069033500000	4639055200	1096732000	1944800	5045600	2501850	2190	604000	-819360	-2720	12000	54880	4960	4320	96	0	480	0	0	13210	-10550	730	34	170	-190	-170	410	350	74	98	-80		x_{77}
x_{78}	+	226187908369605	10449476037	-3064635	10233405	8305605	0	26244	-542619	546021	42693	6237	-1323	8613	-387	309	-351	1837	-945	230	0	0	0	972	0	0	324	0	324	108	108	0		x_{78}
x_{79}	+	260522896303125	19387175445	785531925	9700965	9936405	360855	0	877525	1353429	10773	85797	4725	16725	10213	-75	-675	645	-1750	0	18711	3645	1431	0	189	189	0	279	0	0	0	-20		x_{79}
x_{80}	+	284415522641250	23053576546	788714850	-7515326	-3026590	-24024	11535	-11550	1984738	-32158	53186	8610	-16670	-8126	1186	1274	-1470	-1925	0	59752	-7784	2856	739	-616	-56	-105	-280	95	11	43	0		x_{80}
x_{81}	+	284415522641250	4512942434	788714850	6345794	9875810	-24024	11535	974050	-226590	21602	29890	8610	14050	4418	1954	26	450	-1925	0	-19096	8624	2856	299	464	56	-105	-280	95	83	-97	0		x_{81}
x_{82}	+	349055414150625	-23852507679	1264723425	8024289	-2077215	1297296	18630	-1638175	1207521	-27167	-55839	5985	7905	737	-1951	-351	-415	-175	0	-55440	-5544	3600	-198	-168	168	-450	48	-330	-78	-30	0		x_{82}
x_{83}	+	364635285437500	-13527336900	1290173500	-6747300	7639100	1424500	26845	-2194500	-169540	-9156	-40740	-1540	-19780	9820	1660	300	-100	0	0	-4620	4200	3700	141	680	0	-35	-268	-355	-19	81	0		x_{83}
x_{84}	+	386959894750000	-26179925200	628958000	5215600	-1467600	-722150	370	638000	2035760	-41168	-36240	-2640	10800	-4240	688	0	-1040	0	0	-45430	-7450	1370	722	-330	-350	290	-150	210	66	-98	-65		x_{84}
x_{85}	+	422966584586250	13540709898	507760650	8809722	6156810	1791153	0	1672650	-89910	44298	-35910	9450	11850	2682	-1110	-378	2010	-1275	0	-5103	5103	-2511	0	-81	567	0	81	0	0	0	54		x_{85}
x_{86}	+	563729863936554	4776683562	830581290	12549978	4666410	-1791153	0	-87318	57834	37674	6426	15498	13674	-3366	-438	-378	954	-21	-321	5103	-5103	2511	0	81	-567	0	-81	0	0	0	0		x_{86}
x_{87}	+	581759023584375	18712024695	1811117175	-8284185	-3659145	3046680	-1755	2798775	-593481	-28553	-14553	49175	-14025	-2457	-1065	975	-185	-1750	0	24696	-630	1176	-279	-246	-126	-435	-72	-75	-15	9	0		x_{87}
x_{88}	+	595404085985280	-27243380736	738918400	9052160	-7667712	1235520	1728	-1441792	917504	0	8192	0	0	8192	0	0	0	-770	280	-6336	10368	-1472	1152	-384	176	-320	-192	192	0	32	-100		x_{88}
x_{89}	+	646398915093750	-29247504650	638485750	3790150	-3677450	2340975	27210	-1538250	818230	5110	60550	-9450	4790	-13370	1622	650	-1690	0	0	-18305	-5075	-1745	-350	-35	385	10	799	10	130	106	0		x_{89}
x_{90}	+	652994822390625	19229635425	737526625	7055425	9974625	-965250	-39015	-1321375	557665	5217	42305	-1695	3425	2625	1761	325	1025	0	0	27390	-2850	1150	-447	30	190	25	126	345	33	1	-125		x_{90}
x_{91}	+	662947625486250	30727030698	917620650	4332042	5236650	85293	0	315050	1317546	36522	25866	-16470	-1110	10	42	702	330	1100	0	26973	-7047	621	0	-567	621	0	189	0	0

$B = F_{2+}$

	@ 3963 61728 C A 8A	@ 117 96480 B A 8B	@ 1176 96480 B A 8C	@ 29 456 C A 8D	@ 15 240 B A 8E	@ 89 864 C A 8F	@ 157 560 E A 8G	@ 214 288 C A 8H	@ 74 608 F A 8I	@ 524 304 E A 8J	@ 196 304 H A 8K	@ 98 920 G A 8L	@ 98 768 H A 8M	@ 81 288 J A 8N	@ 32 496 B A 9A	@ 12 664 B A 9B	@ 17 200 AA AA 10A	@ 11 800 AB AB 10B	@ 403 800 AC AC 10C	@ 76 000 BB BB 10D	@ 28 200 AD AD 10E	@ 24 600 BD BD 10F	@ 19 320 A A 11A	@ 9 13981 AB CB 12A	@ 746 496 BB GB 12B	@ 737 280 AA CA 12C	@ 663 552 AB CB 12D	@ 276 480 AD CD 12E	@ 138 240 BE IE 12F	@ 73 728 AC CC 12G	@ 55 296 AD CD 12H	@ 36 864 AF CF 12I	@ 27 648 AE HE 12J	@ 20 736 BD GD 12K	@ 18 432 AG CG 12L	@ 13 824 BC GC 12M	@ 10 368 BE JE 12N	@ 9 216 BF GF 120	@ 9 216 AH HH 12P	@ 9 216 BH IH 12Q	@ 1 536 BJ IJ 12R	@ 1 152 BH JH 12S	@ 864 BI KI 12T	@ 312 A A 13A	@ 564 480 AA AA 14A	@ 4 704 AA AA 14B	@ 4 704 AC AC 14C	@ 1 792 AB AB 14D	@ 1 792 AD AD 14E	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	-21	7	-9	11	-1	-5	-5	3	-1	3	7	-1	-1	-1	-3	6	7	5	-3	0	-1	4	4	-12	-3	4	6	-6	5	-2	0	-4	2	-3	2	1	-1	5	0	-3	1	3	-1	3	-10	4	-4	2	-2	x_2
x_3	55	3	19	23	11	7	-1	-1	-5	7	11	-1	3	-1	0	9	13	7	3	-3	5	5	5	25	7	9	7	11	3	-1	5	1	3	-1	3	-1	3	7	-3	3	3	3	-1	3	19	5	5	3	3	x_3
x_4	-98	-18	-18	30	14	-2	14	-2	-2	-2	14	6	-2	-2	1	10	13	3	7	3	-3	7	5	-36	-9	12	0	-12	-5	8	0	4	4	3	-4	-1	-5	7	0	3	-1	3	1	2	-26	2	-2	2	-2	x_4
x_5	174	46	46	46	14	14	30	14	-2	14	14	6	-2	-2	2	11	21	15	11	0	5	0	4	53	-1	5	17	13	15	9	1	13	-3	7	5	3	-3	7	1	7	3	1	1	2	35	7	7	3	3	x_5
x_6	211	-1	15	51	-25	3	-5	-5	7	-13	23	-5	-1	3	6	15	14	2	-6	2	-10	10	6	65	2	1	11	11	6	-5	-7	1	3	2	-5	-2	0	10	1	-2	-6	4	0	3	45	3	3	-3	-3	x_6
x_7	-379	37	-59	69	-11	5	-19	-11	5	13	29	1	-3	1	0	18	23	17	-11	-3	-1	9	5	-101	7	-5	25	-15	15	-7	3	-5	9	3	1	-1	3	7	3	-9	3	3	1	3	-63	7	-9	1	1	x_7
x_8	490	-22	74	42	26	-22	-14	-6	10	18	34	14	2	-2	-3	15	21	11	-1	1	13	13	5	110	-7	14	-16	16	-19	0	-2	-2	8	-11	0	-3	-1	1	6	-3	5	3	-3	0	70	0	0	6	6	x_8
x_9	-657	-5	-85	143	-13	-33	15	7	3	7	27	-5	3	3	2	20	48	10	-2	0	0	0	6	-151	-16	25	11	-33	0	3	-3	1	-9	0	7	0	0	16	-3	0	0	0	0	1	-91	7	-5	1	-3	x_9
x_{10}	-518	-38	26	-6	26	26	26	-6	10	-22	42	-6	-6	2	-7	11	-13	-9	19	-9	-5	15	4	-89	19	7	-17	-1	-5	-1	-1	7	7	-1	-1	-1	1	-5	-1	11	-5	5	-1	0	-64	-8	8	0	0	x_{10}
x_{11}	405	57	-23	-75	-15	-27	29	13	-15	-27	49	5	9	-3	0	9	-42	8	8	3	-10	15	0	76	-14	-4	22	-14	30	14	-8	4	18	-2	-6	2	6	-14	0	6	-6	6	0	1	35	-7	1	-1	-5	x_{11}
x_{12}	-146	-26	-26	-146	-26	14	-26	14	-10	22	46	-14	-2	2	6	6	-37	19	-17	9	11	21	1	-39	-3	-39	-39	9	-15	-7	9	-7	9	9	9	5	-3	-19	-7	-7	5	5	3	0	-7	-7	-3	5	1	x_{12}
x_{13}	792	88	152	152	120	24	40	-8	24	40	8	0	8	0	0	9	56	18	18	8	16	-4	0	139	22	27	31	35	18	7	23	3	3	2	11	-2	6	6	7	10	6	-2	0	0	65	9	9	1	1	x_{13}
x_{14}	1302	-106	54	214	-26	22	-50	6	-10	-18	30	-14	-2	2	-6	21	43	-5	-17	0	-5	0	4	232	25	24	-2	34	-15	-10	4	-16	-6	1	-6	5	3	17	-4	-7	-3	-1	3	0	126	0	0	-2	-2	x_{14}
x_{15}	50	146	82	50	146	82	50	50	2	2	2	-6	18	2	-3	-3	43	19	-1	-6	3	-2	0	22	67	54	22	46	23	6	22	22	14	7	14	15	5	-5	-2	7	-1	1	-1	0	1	1	1	1	1	x_{15}
x_{16}	-833	-81	79	127	-1	-1	39	-17	-17	-25	7	11	7	-5	-3	6	-1	-21	7	9	15	5	0	-98	1	-50	28	0	5	20	-6	6	8	-3	-8	5	11	9	2	5	-3	-1	1	1	-56	0	0	-8	8	x_{16}
x_{17}	-672	192	-192	96	0	-32	-96	32	0	32	0	0	0	0	3	3	48	38	-18	3	-8	-13	1	-79	20	-15	47	-25	40	-1	-7	-31	7	-16	7	8	10	-4	1	-8	0	-2	0	0	-46	10	-10	6	-6	x_{17}
x_{18}	-1925	91	-69	187	11	-5	-45	11	-5	-13	35	-1	3	-1	-8	19	21	5	1	0	5	0	0	-293	-5	11	13	1	15	-11	7	-13	-15	-5	-7	-5	-9	11	-1	-1	-1	-1	-1	-1	-140	0	0	-4	4	x_{18}
x_{19}	147	-145	95	51	7	-61	35	11	-41	-5	-1	15	7	7	-2	-2	0	6	10	16	-16	4	0	17	-73	-31	-1	-21	15	-25	-3	-7	3	15	3	11	9	-1	-3	-9	3	-3	-3	0	0	0	0	0	0	x_{19}
x_{20}	-1792	-128	-128	256	128	0	0	0	0	0	0	0	0	0	15	6	70	-10	10	0	-10	0	1	-218	-56	54	-38	-66	-40	2	-6	14	14	0	-10	8	-4	8	2	8	0	-4	0	-2	-100	-2	2	4	-4	x_{20}
x_{21}	-161	-5	-85	-1	-45	111	15	23	-13	23	-5	-5	-13	11	-6	3	0	0	0	0	0	0	0	5	41	5	59	-5	-15	11	-11	-11	3	1	-5	5	3	1	5	-7	-3	-1	-1	0	10	-4	4	-2	2	x_{21}
x_{22}	-161	-5	-85	-1	-45	111	15	23	-13	23	-5	-5	-13	11	-6	3	0	0	0	0	0	0	0	5	41	5	59	-5	-15	11	-11	-11	3	1	-5	5	3	1	5	-7	-3	-1	-1	0	10	-4	4	-2	2	x_{22}
x_{23}	2226	66	66	50	34	-46	66	-46	-14	18	34	-6	-14	2	1	10	1	15	15	-15	1	1	0	250	-47	-54	-2	-14	13	6	-2	2	-14	1	10	-3	-5	-7	-10	5	1	-1	-1	0	126	0	0	-2	-2	x_{23}
x_{24}	2304	128	128	256	-128	0	0	0	0	0	0	0	0	0	0	9	34	14	2	4	-14	-4	-4	262	-8	-42	82	-2	24	-6	-14	14	6	-8	-10	0	-12	8	2	8	0	-4	0	0	109	11	11	-3	-3	x_{24}
x_{25}	-35	-175	65	-3	-55	13	5	21	25	-3	-7	5	1	-11	-15	3	0	0	0	0	0	0	0	2	47	-30	20	-20	-5	20	10	2	4	-5	-4	-25	-5	-1	-6	3	-1	3	1	-2	0	0	0	0	0	x_{25}
x_{26}	-35	-175	65	-3	-55	13	5	21	25	-3	-7	5	1	-11	-15	3	0	0	0	0	0	0	0	2	47	-30	20	-20	-5	20	10	2	4	-5	-4	-25	-5	-1	-6	3	-1	3	1	-2	0	0	0	0	0	x_{26}
x_{27}	-1729	-117	-37	-225	-45	47	55	7	-13	15	51	15	-5	7	0	0	-42	20	12	0	-10	0	-1	-141	93	-29	-15	21	-15	9	-9	11	-3	-3	-3	-3	9	-19	7	1	1	1	-3	0	-90	-6	6	2	-2	x_{27}
x_{28}	1023	259	83	95	171	-17	95	7	11	23	-13	11	-5	11	0	0	21	-13	27	12	13	8	0	90	63	90	-18	30	3	30	-6	10	6	3	-2	3	9	7	-6	3	3	-3	3	-3	35	-7	-7	3	3	x_{28}
x_{29}	610	26	26	-414	26	2	-70	2	42	-22	66	6	18	-10	9	0	-91	5	-19	0	5	0	0	51	-48	19	-21	-21	20	-5	3	3	-13	0	-5	4	-10	-24	11	-4	0	2	2	-1	35	-7	-7	3	3	x_{29}
x_{30}	2016	-64	320	224	128	96	-96	32	0	32	0	0	0	0	0	0	70	28	8	3	14	-21	0	189	0	61	-9	39	0	-9	-3	-3	-9	0	7	0	0	0	-3	0	0	0	0	-2	70	0	0	6	6	x_{30}
x_{31}	2925	-67	-163	301	-115	45	5	29	-3	-59	5	25	5	-7	0	0	20	-50	-10	0	-20	0	0	255	30	31	-51	21	-30	13	-9	-1	-3	6	5	-2	6	14	7	-14	-6	-2	-2	-3	119	-7	-15	-5	-1	x_{31}
x_{32}	-3046	50	-270	154	-110	58	50	-38	2	34	-6	-10	10	-10	0	0	64	0	-16	0	0	0	-1	-252	-9	20	-36	0	-45	4	0	-4	0	27	8	-5	9	-1	0	3	3	-3	3	-3	-119	7	3	1	-7	x_{32}
x_{33}	-2673	-157	-301	303	59	-33	135	39	27	15	-21	-1	3	-9	0	0	49	25	17	0	-7	-12	-6	-216	0	-56	72	-24	0	24	24	24	0	0	-8	0	0	0	0	0	0	0	0	2	-35	7	-17	1	5	x_{33}
x_{34}	-3395	181	181	189	-139	29	-75	-35	-27	5	-3	9	-19	9	0	0	-43	-15	-27	0	5	0	-5	-255	24	-15	69	33	60	-19	-3	9	9	-12	9	-4	-6	0	-11	-20	4	-2	-4	0	-56	0	0	-8	8	x_{34}
x_{35}	-21	-45	-45	-21	-45	-53	51	-53	-61	3	11	15	-5	-1	0	0	0	0	0	-15	0	5	0	0	27	0	0	0	-9	0	0	0	0	-9	0	15	9	3	0	-9	-1	-3	3	0	0	0	0	0	0	x_{35}
x_{36}	-21	-45	-45	-21	-45	-53	51	-53	-61	3	11	15	-5	-1	0	0	0	0	0	-15	0	5	0	0	27	0	0	0	-9	0	0	0	0	-9	0	15	9	3	0	-9	-1	-3	3	0	0	0	0	0	0	x_{36}
x_{37}	-798	210	50	34	450	98	10	-14	-14	-22	10	30	42	14	3	-6	0	0	0	10	0	10	0	9	117	195	63	55	1	-17	25	9	7	13	23	1	-5	-3	17	-7	-3	-1	-1	-1	0	0	0	0	0	x_{37}
x_{38}	2855	-193	-257	-89	95	-121	-65	-25	-17	15	7	-5	-9	11	8	-10	-20	-50	-10	0	20	0	0	68	5	36	-4	-36	-15	-4	12	-12	12	-3	-4	-7	3	-3	-4	-7	5	-1	-1	0	91	-7	-11	-1	3	x_{38}
x_{39}	-469	-285	-125	235	115	11	-101	123	-13	11	-13	-5	3	11	0	0	-56	4	-20	-11	8	13	0	-3	-21	-35	-57	15	51	39	-3	13	-9	-21	15	3	3	-5	-11	19	-5	-5	-1	0	0	0	0	0	0	x_{39}
x_{40}	356	276	596	-28	116	100	20	68	20	20	4	36	4	4	0	-9	69	25	9	0	5	0	0	-43	-79	5	101	25	-15	13	25	29	9	-11	1	13	15	-7	9	-15	-7	3	1	0	19	5	5	3	3	x_{40}
x_{41}	2240	-128	128	-320	0	-64	64	64	0	64	0	0	0	0	-15	-6	-28	22	6	-13	12	7	-5	198	-72	-26	-54	-54	-44	-22	6	6	10	12	-6	-4	10	0	-2	4	4	-2	0	0	0	0	0	0	0	x_{41}
x_{42}	-3178	-58	358	342	-170	-106	-50	70	38	-18	-18	-30	14	-14	9	-9	-20	-50	-10	0	20	0	0	-110	-47	-14	-56	-16	-35	-24	2	2	-8	-7	16	9	1	17	10	13	-3	1	1	-1	-54	-12	12	-2	2	x_{42}
x_{43}	-2745	-457	119	199	151	-57	135	-25	-9	-57	-9	31	-9	15	0	0	-7	-5	35	0	1	-24	1	-162	0	46	-54	6	0	18	-6	6	-18	0	-2	0	0	0	-6	0	0	0	0	0	-35	-7	5	5	-3	x_{43}
x_{44}	588	284	-36	204	-196	-52	220	44	28	28	-20	36	-20	4	-9	0	-21	15	11	0	-5	0	4	-136	-19	-120	8	12	65	0	-36	16	20	33	4	-15	-25	-11	4	1	1	-5	1	0	0	0	0	0	0	x_{44}
x_{45}	378	90	90	-6	-198	90	-54	26	42	-6	-6	-30	-22	-6	0	0	-36	-18	18	-3	12	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-36	6	-6	4	-4	x_{45}
x_{46}	378	90	90	-6	-198	90	-54	26	42	-6	-6	-30	-22	-6	0	0	-36	-18	18	-3	12	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-36	6	-6	4	-4	x_{46}
x_{47}	645	45	205	69	-195	-219	5	85	-35	21	-19	5	29	-11	26	8	0	0	0	0	0	0	0	-2	79	30	160	0	-45	-32	6	14	0	3	0	7	-9	-1	-10	11	-1	-1	1	0	-35	-7	-7	-3	-3	x_{47}
x_{48}	1197	-99	-259	237	141	45	-11	-3	-3	21	-11	-31	21	-15	0	-9	-35	-31	21	9	13	13	0	-118	-10	-86	80	16	42	-16	-14	-6	0	10	-16	14	-6	6	2	2	6	2	0	0	0	0	0	0	0	x_{48}
x_{49}	637	117	-459	-3	149	-163	101	125	-27	-11	13	9	-3	9	0	-9	119	17	1	-3	15	5	0	4	-86	116	40	-116	-30	-16	-8	-20	12	10	-12	14	0	-14	-8	10	-2	4	-4	0	0	0	0	0	0	x_{49}
x_{50}	-1428	188	188	-404	188	44	-20	44	28	-52	28	-20	-4	-4	0	-9	-70	-10	10	0	10	0	-4	-121	-31	39	95	71	45	-1	-1	-25	15	17	-9	5	3	-7	-1	5	-7	-1	3	1	0	0	0	0	0	x_{50}
x_{51}	-512	0	0	-512	0	0	0	0	0	0	0	0	0	0	-9	-9	-28	4	28	24	-28	12	0	52	88	-44	-92	4	-40	20	4	4	-4	-32	20	8	-4	-8	-4	8	0	-4	0	0	-35	7	7	-3	-3	x_{51}
x_{52}	203	-205	-205	-53	115	-21	-45	-21	67	3	-21	15	-5	15	24	6	0	0	0	0	0	0	0	-2	-56	-50	34	30	0	-22	-6	6	6	12	-10	-4	-6	0	-6	0	0	6	-4	-3	0	0	0	0	0	x_{52}
x_{53}	203	-205	-205	-53	115	-21	-45	-21	67	3	-21	15	-5	15	24	6	0	0	0	0	0	0	0	-2	-56	-50	34	30	0	-22	-6	6	6	12	-10	-4	-6	0	-6	0	0	6	-4	-3	0	0	0	0	0	x_{53}
x_{54}	-2093	355	-445	275	-45	-109	-165	-29	-29	27	-5	7	-5	7	-12	-12	112	20	-8	0	0	0	-1	-103	-31	89	23	-81	45	-9	-15	-7	-9	-3	15	-3	-9	-7	9	-3	-3	3	1	0	0	0	0	0	0	x_{54}
x_{55}	1078	358	358	182	-250	-106	-90	22	54	-42	6	-10	-10	-18	18	0	22	20	28	0	-10	0	0	-86	-104	26	22	-54	60	14	30	-30	18	-12	18	-4	-6	0	-2	-20	4	-2	4	0	36	-6	6	-4	4	x_{55}
x_{56}	3450	-70	570	250	-38	-38	10	-38	-54	-6	-38	-30	10	-6	0	-9	112	10	-18	0	0	0	0	134	26	-10	-46	62	-30	-6	-22	-2	6	2	-10	6	-12	-14	-6	-6	-2	0	0	0	35	7	7	3	3	x_{56}
x_{57}	454	-130	190	70	30	102	-34	6	-50	14	6	10	22	-22	0	0	0	0	0	-15	0	5	-5	93	-24	45	93	-15	48	-11	9	-27	9	24	-7	4	6	0	-7	8	4	2	2	0	-35	7	7	-3	-3	x_{57}
x_{58}	-448	0	0	64	0	64	0	-64	0	0	0	0	0	0	10	1	0	0	0	0	0	0	-5	52	-20	20	-20	-20	0	-4	4	4	-12	-8	-4	8	-6	4	12	0	0	6	0	0	0	0	0	0	0	x_{58}
x_{59}	-448	0	0	64	0	64	0	-64	0	0	0	0	0	0	10	1	0	0	0	0	0	0	-5	52	-20	20	-20	-20	0	-4	4	4	-12	-8	-4	8	-6	4	12	0	0	6	0	0	0	0	0	0	0	x_{59}
x_{60}	2016	448	-192	224	128	96	160	32	0	32	0	0	0	0	27	0	-70	-10	10	0	10	0	-4	105	60	9	-21	-69	20	43	9	9	-37	-12	11	4	-4	12	1	4	4	4	0	0	-20	-6	-6	-4	-4	x_{60}
x_{61}	2156	444	-196	-148	-196	-20	140	-20	-4	-52	-4	-36	-4	-4	0	0	-49	31	-1	6	-17	-22	0	78	-21	14	78	-66	87	-18	6	30	-18	15	-18	-9	21	3	-2	7	-1	1	-1	0	0	0	0	0	0	x_{61}
x_{62}	0	0	0	0	0	0	0	0	0	0	0	-32	0	32	19	10	-28	-20	0	10	-12	-2	-1	48	12	48	48	0	-44	48	0	-16	16	0	0	12	4	-4	-16	-4	8	-4	6	0	0	0	0	0	0	x_{62}
x_{63}	-756	-116	-52	-372	-84	12	-84	-20	12	44	12	4	12	-12	0	0	-63	35	-25	15	9	-21	1	-27	0	-107	-63	45	0	9	9	-3	-27	0	5	0	0	0	9	0	0	0	0	1	10	-4	4	-2	2	x_{63}
x_{64}	4473	-295	-295	377	25	57	-135	-7	57	-39	-39	25	-7	9	0	0	-64	0	-16	0	0	0	-6	162	0	-110	54	-30	0	-18	30	-6	18	0	10	0	0	0	6	0	0	0	0	3	-56	0	12	0	-8	x_{64}
x_{65}	-140	-316	-316	116	4	116	-44	-12	4	-44	4	-36	4	-4	0	-9	35	31	-9	11	3	3	0	-97	29	-113	11	55	9	3	-5	23	-9	-11	-1	9	-9	5	19	1	5	-5	-1	0	0	0	0	0	0	x_{65}
x_{66}	-576	-128	128	-64	0	192	-64	64	0	-64	0	0	0	0	0	0	48	38	-18	3	-8	7	0	90	36	90	-18	30	12	30	-6	10	6	12	-2	-12	0	4	-6	12	4	0	0	0	-46	-4	-4	2	2	x_{66}
x_{67}	1555	-121	-41	243	-369	35	115	-5	63	-5	-25	-1	-1	-9	0	0	-21	5	1	0	-5	0	0	36	90	116	-72	84	-30	32	-24	-4	12	-6	-4	14	0	2	16	10	-2	4	0	0	-35	-7	1	9	5	x_{67}
x_{68}	-35	-35	125	157	365	-227	-155	-115	-35	37	-11	-15	-11	1	18	9	0	0	0	0	0	0	0	55	109	55	1	25	45	-15	7	-9	9	-11	9	-27	9	-3	7	13	-3	1	-3	0	-70	0	0	-6	-6	x_{68}
x_{69}	-2952	248	312	-8	280	120	-120	-40	-72	8	-24	0	-24	0	0	-9	20	-50	-10	0	-20	0	5	185	32	41	5	1	0	-19	13	-15	9	16	-7	-16	0	0	-3	0	0	0	0	0	-56	0	8	4	0	x_{69}
x_{70}	-2877	143	-193	-285	-57	-13	35	27	55	-5	-49	15	23	7	0	0	36	12	0	-48	12	12	0	9	63	-135	-9	75	15	15	-3	1	3	-9	-13	3	9	7	-3	-9	3	-3	-3	0	56	0	0	8	-8	x_{70}
x_{71}	0	512	-512	0	0	0	0	0	0

$B = F_{2+}$

	@ 3 600 AA AA 15A	@ 120 BB BB 15B	@ 12 288 D A 16A	@ 12 288 D A 16B	@ 2 048 H A 16C	@ 2 048 H A 16D	@ 2 048 D A 16E	@ 1 024 H A 16F	@ 256 K A 16G	@ 256 M A 16H	@ 68 A A 17A	@ 1 296 BD BA 18A	@ 648 BD BA 18B	@ 432 BG BB 18C	@ 432 BJ BD 18D	@ 216 AJ AD 18E	@ 216 BK BC 18F	@ 38 A A 19A	@ 38 400 BA AA 20A	@ 4 000 DA BA 20B	@ 3 200 BA AA 20C	@ 2 560 BB AB 20D	@ 960 BD AD 20E	@ 640 BG AG 20F	@ 480 FE BE 20G	@ 400 CI AI 20H	@ 160 DF BF 20I	@ 80 FJ BJ 20J	@ 252 AA AA 21A	@ 132 AA AA 22A	@ 88 AB AB 22B	@ 46 A A 23A	@ 46 A A B**	@ 2 304 DB AB 24A	@ 2 304 DC AC 24B	@ 2 304 DE AE 24C	@ 2 304 DE AE 24D	@ 2 304 GA AA 24E	@ 2 728 MA BA 24F	@ 1 768 GD AD 24G	@ 576 FG AG 24H	@ 576 JG AG 24I	@ 192 JJ AJ 24J	@ 192 MF BF 24K	@ 192 QK AK 24L	@ 96 OI BI 24M	@ 96 RN BN 24N	@ 25 B A 25A	@ 104 AC AC 26A	@ 52 AA AA 26B	@ 27 A A 27A	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	3	2	3	-5	-1	3	-1	3	1	1	2	2	2	2	-2	1	-2	1	-7	-2	3	1	-1	1	0	-1	2	0	1	2	0	1	1	-2	0	2	-4	0	-3	2	1	-2	0	1	1	-1	-1	1	-1	1	0	x_2
x_3	1	2	-1	7	-1	3	3	3	1	1	1	3	3	1	3	0	1	1	11	1	1	3	1	-1	1	-1	1	1	1	1	1	0	0	3	1	-1	5	1	1	-1	-1	-1	1	1	-1	1	-1	0	-1	1	0	x_3
x_4	0	2	-6	-6	2	2	2	2	-2	2	0	2	2	-2	-2	1	2	1	-13	-3	-3	3	1	-1	3	-1	1	-1	-1	-1	1	0	0	0	0	-4	-4	4	1	0	-1	2	-2	1	-1	1	1	-1	2	0	1	x_4
x_5	4	0	10	10	2	2	2	2	2	-2	1	1	1	3	1	-2	3	-1	15	0	5	-1	1	3	0	-1	0	0	-1	2	0	0	0	1	1	5	5	-3	3	1	3	3	-1	-1	-1	1	1	0	2	0	-1	x_5
x_6	3	1	-1	7	-1	3	-5	3	-3	1	1	-1	-1	-1	-1	2	-1	0	22	2	2	-2	-2	-2	-2	2	2	-2	0	0	-2	0	0	-1	-3	-1	5	1	4	3	-2	1	-1	0	2	-2	0	0	-1	-1	0	x_6
x_7	-2	0	5	-11	1	1	-3	1	1	1	0	0	0	2	0	0	-2	0	-27	3	3	-3	3	1	-3	1	-1	1	2	1	1	-1	-1	1	-5	1	-5	-1	-1	-3	-1	-1	1	-1	-1	-1	1	0	-1	-1	0	x_7
x_8	-1	1	-6	10	-2	-2	2	-2	2	2	-1	1	1	-1	1	1	-1	0	23	3	-7	-1	1	3	1	3	-1	1	-2	-1	1	0	0	-4	2	-4	2	-2	-5	0	1	-2	0	-1	1	1	1	-1	0	0	0	x_8
x_9	2	0	-5	-13	-1	3	-1	3	1	-3	0	2	2	0	-2	-2	0	-1	-30	0	0	2	-4	-2	0	2	0	0	-1	0	-2	0	0	-5	5	-1	-7	-3	0	-1	0	3	1	0	0	0	0	0	1	1	-1	x_9
x_{10}	3	0	-2	-2	-2	-2	6	-2	-2	2	0	-3	-3	-1	1	1	3	0	-21	-1	-1	3	-1	-1	3	1	-1	-1	1	-2	0	0	0	7	-1	-1	-1	-5	-5	3	-1	-1	-1	-1	3	1	-1	0	0	0	-1	x_{10}
x_{11}	-5	0	9	1	1	-3	-3	-3	-1	3	0	-3	-3	1	-3	0	1	0	16	1	6	0	-2	0	3	-2	-3	-1	1	0	0	0	0	-6	4	6	0	0	0	-6	2	2	0	0	-2	0	0	1	1	-1	0	x_{11}
x_{12}	1	0	-2	-2	2	-6	-2	-6	2	2	0	-2	-2	2	2	2	-2	0	-5	-5	-5	-5	-1	-1	-3	-3	-1	1	0	-1	1	0	0	1	1	1	1	1	7	1	1	1	1	-1	1	-1	-1	0	0	0	0	x_{12}
x_{13}	-4	-1	8	8	0	0	8	0	0	0	-1	3	3	1	3	0	1	0	14	-6	4	6	0	-2	0	0	-2	0	2	0	0	-1	-1	7	-1	3	3	3	0	-1	-2	1	1	0	2	0	0	0	0	0	0	x_{13}
x_{14}	-3	0	-10	6	2	2	-2	2	-2	-2	-1	1	1	-3	1	-2	-3	0	35	0	-5	3	-1	-1	0	-3	0	0	0	-2	0	0	0	2	0	-2	4	0	-3	-2	1	-2	0	1	-3	-1	-1	0	0	0	0	x_{14}
x_{15}	9	-2	6	6	6	6	-2	-2	-2	2	0	5	-1	1	1	1	-1	0	-1	4	-1	7	7	3	-2	1	0	-2	1	0	0	0	0	2	10	2	2	2	5	2	-1	2	2	1	-1	-1	-1	-1	0	0	0	x_{15}
x_{16}	4	-2	-9	7	3	3	-1	3	-1	-1	0	-4	-4	-2	4	1	2	0	-21	-1	-1	-5	3	-1	-3	-3	3	1	2	0	0	0	0	0	0	-2	-4	2	-2	-5	4	-3	0	2	-1	1	1	1	0	1	-1	x_{16}
x_{17}	10	-2	16	-16	0	0	0	0	0	0	0	-1	-1	3	-1	-1	-1	0	-6	-1	4	2	0	2	3	0	-1	-1	1	1	1	0	0	3	-3	3	-3	-3	-6	3	0	-3	-1	-2	0	0	0	-1	0	0	0	x_{17}
x_{18}	-1	0	11	-5	-1	-1	3	-1	-1	-1	0	-1	-1	-1	3	0	-1	-1	-35	0	5	-3	1	1	0	-1	0	0	-4	0	0	0	0	1	3	5	-1	7	1	1	3	-3	-1	1	-1	1	-1	0	-1	-1	1	x_{18}
x_{19}	-1	2	7	-1	3	7	-5	-1	1	-3	0	2	-4	-2	2	2	0	0	-10	10	0	6	-4	2	0	2	-2	0	0	0	0	0	0	-1	5	-5	1	-3	3	3	-1	-1	1	-1	-1	1	1	1	0	0	1	x_{19}
x_{20}	-5	0	0	0	0	0	0	0	0	0	-2	4	4	-2	-4	-1	2	1	-10	0	-10	6	2	2	0	0	0	0	1	-1	1	0	0	-2	-2	2	2	2	8	-2	0	0	0	0	0	0	0	0	-2	0	0	x_{20}
x_{21}	0	0	7	-1	-5	-1	3	-1	1	1	1	5	-1	-1	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-5	5	3	-3	1	1	-1	-3	-3	-1	-3	1	-1	-1	0	0	0	0	x_{21}
x_{22}	0	0	7	-1	-5	-1	3	-1	1	1	1	5	-1	-1	1	-2	1	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-5	5	3	-3	1	1	-1	-3	-3	-1	-3	1	-1	-1	0	0	0	0	x_{22}
x_{23}	5	-2	6	6	-2	-2	-2	-2	2	-2	0	-2	-2	2	-2	1	2	0	31	1	1	-1	1	-1	1	1	1	1	0	0	0	0	0	-6	-6	-2	-2	6	3	2	-3	0	0	-1	1	1	-1	0	0	0	1	x_{23}
x_{24}	-6	-1	0	0	0	0	0	0	0	0	0	-3	-3	1	-3	0	1	0	22	-8	2	-10	-2	2	-4	0	0	0	1	2	0	0	0	2	2	-2	-2	-6	0	-2	0	0	0	0	0	0	0	0	0	0	0	x_{24}
x_{25}	0	0	5	-3	5	1	1	1	-1	-1	0	1	1	3	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b23	**	-4	2	-4	2	-2	1	0	-1	2	0	1	-1	1	1	0	-2	0	0	x_{25}
x_{26}	0	0	5	-3	5	1	1	1	-1	-1	0	1	1	3	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b23	-4	2	-4	2	-2	1	0	-1	2	0	1	-1	1	1	0	-2	0	0	x_{26}
x_{27}	3	0	-13	-5	-5	-1	-1	-1	1	-3	0	0	0	0	0	0	0	0	-20	0	-10	-4	-2	4	0	2	0	0	0	1	-1	1	1	9	-1	-3	3	-1	-1	-3	1	1	3	-1	-3	-1	1	0	0	0	0	x_{27}
x_{28}	-6	2	15	-9	3	-1	3	-1	1	1	-1	0	0	0	0	0	0	0	-9	6	1	-1	-3	-1	4	1	2	0	-1	0	0	0	0	-2	2	6	-6	6	-3	2	-1	2	2	1	-1	-1	-1	0	1	-1	0	x_{28}
x_{29}	-1	0	2	2	6	-2	2	-2	-2	-2	1	0	0	0	0	-3	0	0	5	0	5	5	5	-3	0	1	0	0	-1	0	0	0	0	-1	-1	-1	-1	-5	-2	3	-4	-1	-1	2	0	0	2	0	-1	1	0	x_{29}
x_{30}	5	0	-16	16	0	0	0	0	0	0	1	0	0	0	0	0	0	-1	-8	7	2	0	2	-4	3	0	-1	-1	-2	0	0	1	1	-1	5	-1	5	-3	0	-1	0	-3	-1	0	0	0	0	1	-2	0	0	x_{30}
x_{31}	5	0	5	-11	1	1	-3	1	1	1	0	0	0	0	0	0	0	0	10	0	0	2	4	2	0	0	0	0	0	0	0	0	0	5	-1	5	-1	3	0	1	2	-1	1	0	2	0	2	0	1	1	0	x_{31}
x_{32}	-4	0	-6	-6	6	-2	-6	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	1	-1	0	0	-4	0	-8	4	-4	5	4	-1	2	-2	1	3	-1	-1	0	1	1	0	x_{32}
x_{33}	-7	0	-21	3	3	-1	-1	-1	-3	1	0	0	0	0	0	0	0	0	5	10	5	-3	1	-3	0	1	-2	0	1	0	2	0	0	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	1	2	0	0	x_{33}
x_{34}	-8	0	13	13	-7	1	-3	1	1	1	0	0	0	0	0	0	0	1	-15	0	5	1	1	-3	0	3	0	0	2	-1	-1	0	0	1	1	5	5	1	-2	-3	0	3	-1	2	0	0	0	0	0	0	0	x_{34}
x_{35}	0	-1	3	3	7	-1	3	-1	-1	-1	1	0	0	0	0	0	0	1	0	-5	0	0	0	0	1	0	-1	1	0	0	0	0	0	0	0	0	0	0	-3	0	3	0	0	1	-1	-1	-1	0	0	0	0	x_{35}
x_{36}	0	-1	3	3	7	-1	3	-1	-1	-1	1	0	0	0	0	0	0	1	0	-5	0	0	0	0	1	0	-1	1	0	0	0	0	0	0	0	0	0	0	-3	0	3	0	0	1	-1	-1	-1	0	0	0	0	x_{36}
x_{37}	0	1	-14	2	6	6	-6	-2	-2	-2	0	4	-2	2	-4	-1	0	0	0	10	0	0	0	0	-2	0	2	2	0	0	0	0	0	3	5	3	-3	-3	3	-5	1	1	-1	-1	1	1	-1	0	-1	-1	0	x_{37}
x_{38}	5	0	-9	-9	-5	3	7	3	-1	-1	0	0	0	-2	0	0	-2	0	10	0	0	2	-4	2	0	0	0	0	3	0	0	0	0	-4	4	-4	-4	-4	11	4	1	4	0	-1	1	1	-1	0	0	0	-1	x_{38}
x_{39}	-6	1	11	-5	3	11	3	3	-1	-1	0	0	0	0	0	0	0	0	12	-3	-8	-4	0	4	1	0	-3	1	0	0	0	0	0	3	1	-5	1	5	-1	7	-5	1	-1	-1	-1	-1	-1	1	0	0	0	x_{39}
x_{40}	6	0	4	4	-4	-4	4	4	0	0	-1	-3	-3	-1	-3	0	-1	0	-15	0	5	1	-3	1	0	1	0	0	1	0	0	0	0	-3	-7	-7	5	5	5	5	5	-1	-1	1	1	-1	1	0	0	0	0	x_{40}
x_{41}	-3	1	0	0	0	0	0	0	0	0	0	2	2	-2	-2	1	2	0	2	-3	-8	-6	4	2	-5	0	1	-1	0	1	-1	0	0	-2	2	6	-6	2	-10	-2	4	-2	-2	2	0	0	0	0	0	0	0	x_{41}
x_{42}	5	0	6	22	2	2	-2	2	2	2	0	-3	-3	3	1	1	-1	0	10	0	0	2	-4	2	0	0	0	0	0	0	0	0	0	-4	-2	4	-2	2	-1	0	1	-2	0	-1	-3	-1	1	0	-1	1	0	x_{42}
x_{43}	5	0	-9	-9	-1	-1	7	-1	-1	-1	2	0	0	0	0	0	0	0	7	-8	-3	-1	1	-1	0	1	0	0	1	-1	1	0	0	2	2	-2	-2	-6	0	-2	0	0	0	0	0	0	0	0	0	0	0	x_{43}
x_{44}	4	0	12	12	4	4	-4	4	0	0	0	-6	0	0	2	-1	-2	1	15	0	-5	-1	-1	3	0	1	0	0	0	-2	0	0	0	-4	0	8	-4	0	3	0	1	-2	-2	-1	1	1	1	0	0	0	0	x_{44}
x_{45}	0	0	6	6	-2	-2	-2	-2	-2	2	0	0	0	0	0	0	0	-1	6	1	-4	6	0	2	-3	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{45}
x_{46}	0	0	6	6	-2	-2	-2	-2	-2	2	0	0	0	0	0	0	0	-1	6	1	-4	6	0	2	-3	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{46}
x_{47}	0	0	21	5	5	-3	-3	-3	1	1	2	2	2	-4	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-2	0	6	6	-3	0	-1	2	0	-3	-1	1	1	0	0	0	-1	x_{47}
x_{48}	7	-2	-3	-19	1	1	5	1	1	1	0	-3	-3	-1	-3	0	-1	0	17	7	-3	1	1	-3	5	-1	3	1	0	0	0	0	0	0	2	0	-6	-6	0	0	-2	-2	0	0	-2	0	0	0	0	0	0	x_{48}
x_{49}	7	-2	-3	-3	1	9	-3	1	1	1	0	3	-3	-1	3	0	1	0	13	-7	3	5	-1	-3	-3	3	1	1	0	0	0	0	0	0	0	-4	-4	4	-2	0	2	2	-2	2	-2	0	0	0	0	0	0	x_{49}
x_{50}	-5	0	4	4	4	4	4	4	0	0	-1	3	3	-1	3	0	-1	0	-10	0	10	6	-2	2	0	0	0	0	0	2	0	0	0	-1	-1	-1	-1	3	3	-5	1	-5	-1	-1	1	1	-1	0	1	1	0	x_{50}
x_{51}	1	0	0	0	0	0	0	0	0	0	0	3	3	3	-1	-1	-1	0	4	14	4	4	-4	-4	0	0	2	0	1	0	0	-1	-1	0	0	0	0	4	-8	4	0	0	0	0	0	0	0	0	0	0	0	x_{51}
x_{52}	0	0	3	3	-1	7	3	-1	-1	-1	0	-4	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b23	**	2	2	-2	-2	2	-4	-2	0	0	0	0	0	-2	0	0	1	1	0	x_{52}
x_{53}	0	0	3	3	-1	7	3	-1	-1	-1	0	-4	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b23	2	2	-2	-2	2	-4	-2	0	0	0	0	-2	0	0	1	1	0	x_{53}
x_{54}	3	0	11	-5	-1	-1	3	-1	-1	-1	0	2	2	0	-2	4	0	0	20	0	0	4	-4	0	0	-2	0	0	0	-1	-1	0	0	-5	5	3	-3	1	-5	-1	-3	-3	3	-1	1	1	1	0	0	0	0	x_{54}
x_{55}	7	0	-6	-6	-6	10	2	-6	-2	2	0	6	0	0	-2	-2	2	0	-20	0	-10	-4	-2	4	0	-2	0	0	0	0	0	0	0	-2	-2	2	2	-2	-2	2	0	0	0	2	0	0	0	0	0	0	0	x_{55}
x_{56}	-2	0	6	6	-2	-2	-2	-2	-2	2	0	3	3	-1	3	0	-1	1	-30	0	0	2	-4	-2	0	-2	0	0	-1	0	0	0	0	2	-6	-2	-2	-6	-6	-2	-2	4	0	-2	-2	0	0	0	0	0	0	x_{56}
x_{57}	0	2	-10	-10	2	-6	6	2	-2	-2	0	0	0	0	0	0	0	0	0	-5	0	0	0	0	1	0	-1	1	1	-1	-1	0	0	5	1	9	-3	1	4	1	-4	-1	-1	0	0	-2	2	0	0	0	0	x_{57}
x_{58}	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	1	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	-4	2	4	0	0	0	-2	0	0	0	0	0	0	1	x_{58}
x_{59}	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	1	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	-4	2	4	0	0	0	-2	0	0	0	0	0	0	1	x_{59}
x_{60}	-5	0	16	-16	0	0	0	0	0	0	1	0	0	0	0	3	0	0	-10	0	10	6	-2	2	0	0	0	0	-2	-2	0	0	0	7	-3	-1	5	-3	0	-1	4	1	-1	0	0	0	0	0	0	0	0	x_{60}
x_{61}	9	2	12	12	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	-1	4	-1	-1	-1	-1	2	-1	0	2	0	0	0	0	0	-6	2	2	2	2	5	2	-1	2	2	1	-1	-1	-1	-1	0	0	0	x_{61}
x_{62}	10	1	0	0	0	0	0	0	0	0	0	-6	0	2	2	-1	0	0	4	4	4	4	0	0	-2	-2	0	-2	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	1	x_{62}
x_{63}	10	0	-4	-4	4	4	-4	4	0	0	0	0	0	0	0	0	0	0	7	-3	-3	-1	-3	3	3	-1	1	-1	-1	-1	1	0	0	1	-7	-3	-3	9	0	-3	0	3	-1	0	0	0	0	0	1	-1	0	x_{63}
x_{64}	-4	0	-15	-15	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	-2	0	2	0	0	2	2	-2	-2	6	0	2	0	0	0	0	0	0	0	-1	-1	0	x_{64}
x_{65}	2	3	4	4	4	4	4	-4	0	0	0	-3	-3	-1	-3	0	-1	0	23	-17	3	7	3	-1	-1	-1	-1	-1	0	0	0	0	0	-1	-1	-5	-5	-5	-5	-1	1	1	1	-1	1	1	-1	0	0	0	0	x_{65}
x_{66}	-6	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-11	4	2	0	2	-1	0	1	-1	-1	0	0	1	1	-2	2	6	-6	6	0	2	-4	2	2	0	0	0	0	0	0	0	0	x_{66}
x_{67}	-1	0	15	-9	3	-1	-5	-1	1	-3	0	0	0	0	0	0	0	0	-35	0	-5	-3	-1	1	0	1	0	0	3	0	0	0	0	-4	4	0	0	4	-2	0	-2	-2	-2	2	2	0	0	0	0	0	0	x_{67}
x_{68}	0	0	-11	5	1	1	-3	-7	1	1	1	-3	3	-3	1	-2	-1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	1	-1	-7	-1	7	1	1	1	-5	1	1	1	1	1	0	0	0	0	x_{68}
x_{69}	5	0	8	8	0

$B = F_{2+}$

| | e | e | e

$B = F_{2+}$

		1A	2A	2B	2C	2D	3A	3B	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	5A	5B	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	
x_{92}	+	665029816320000	-40081850368	1526169600	-5640192	3112960	2400672	-12480	-3276800	1835008	0	-36864	0	0	-4096	0	1728	0	1275	0	-32416	12512	-864	416	544	216	0	-32	-320	-32	0	-44	x_{92}
x_{93}	+	665029816320000	24946638848	1526169600	12464128	3112960	2400672	-12480	3276800	0	0	-20480	0	0	12288	0	-1728	0	1275	0	10592	2528	-864	-352	544	568	0	-32	-320	-32	64	-44	x_{93}
x_{94}	+	677025031854600	-26189163000	1936087560	6762600	-1069560	2606175	-21870	-2918520	-84600	-26616	-9240	51912	4232	-2200	72	0	-984	0	225	-31185	-2025	-225	162	135	315	450	207	-270	18	-90	-110	x_{94}
x_{95}	+	714865488180480	-2370251520	-30788352	-16633344	-1370880	1046682	12960	944384	-231168	62720	-10752	-26880	-3840	14848	768	0	0	1330	480	-12726	-11970	-1062	-360	-594	42	720	-342	0	72	-48	0	x_{95}
x_{96}	+	718889622405120	25583910912	1335066624	-5640192	-9732096	2729376	0	3276800	0	0	-36864	0	0	-4096	0	1728	0	595	120	7776	7776	-864	0	-864	216	0	-288	0	0	0	-100	x_{96}
x_{97}	+	775438738408125	25760740797	930894525	13837797	13775805	577368	0	1412125	488349	67005	50373	-28755	12765	8101	3789	-27	-75	1100	0	-21384	5832	216	0	-216	-216	0	-72	0	0	0	1	x_{97}
x_{98}	+	809403858900000	35652533280	2204423200	-1732640	4492320	1683045	-540	3085600	698656	-59360	57568	-10080	-9440	8672	-1632	0	1120	1750	0	34629	3255	4069	-12	-9	-329	100	-123	-540	-12	4	0	x_{98}
x_{99}	+	820429392234375	6332778375	182634375	8642375	16336775	-848925	30330	298375	83335	25991	58695	11655	14215	-5305	5767	-25	455	0	0	-35805	-525	-285	978	-205	35	570	-349	-70	-142	26	0	x_{99}
x_{100}	+	835517991997440	37614440448	1426968576	11280384	-995328	2152008	0	2207744	995328	36864	0	0	12288	-8192	0	0	0	890	-60	29160	-1944	-1080	0	648	-432	0	-216	0	0	0	-74	x_{100}
x_{101}	+	844994666880000	-13273408512	457497600	22177792	-12723200	91728	31140	179200	-50176	7168	-14336	-35840	15360	10240	-3072	-1248	0	2100	0	3024	-4032	1104	-396	-1088	-224	420	208	100	-44	-8	0	x_{101}
x_{102}	+	968283412561920	1926299648	2337865728	0	65536	1878240	4224	0	-1310720	65536	0	65536	0	0	0	0	0	-2080	-580	5600	-12544	1248	1712	-512	0	96	-32	64	16	0	-126	x_{102}
x_{103}	+	1042755084800000	-45853716480	647680000	9052160	1167360	-165880	-39520	1126400	2142208	36864	-65536	0	-20480	-8192	0	0	0	-2750	0	-15576	-4920	904	1200	-24	176	-320	360	480	-48	32	26	x_{103}
x_{104}	+	1093905856312500	-27211361100	243512500	11078900	-1633100	-1651650	-6945	2040500	965300	112308	22260	24500	-20300	4340	-1356	300	2420	0	0	-43890	-7350	-770	111	250	350	175	46	-65	31	71	0	x_{104}
x_{105}	+	1198405322994375	-46990500921	1438655175	-23399361	6108615	2368521	0	-1977625	1165671	53703	-23841	-19305	-13785	11647	-777	351	-465	-175	0	-10935	-4131	-2295	0	1053	-27	0	-567	0	0	0	55	x_{105}
x_{106}	+	1370975689505565	-40103091171	1948003101	-16451019	-123363	-938223	0	-84931	843453	54045	-49707	-2835	-387	3445	-819	-351	-795	-385	-60	-40095	-4131	3537	0	-243	-351	0	513	0	0	0	10	x_{106}
x_{107}	+	1370975689505565	37476692253	185277213	17202861	12684573	-938223	0	-1563683	1219293	31005	32877	12285	7005	8749	2109	1053	-435	-385	-60	2673	-9477	-1647	0	27	27	0	-207	0	0	0	10	x_{107}
x_{108}	+	1391162882484375	50414138775	1216436375	-16514225	-15698025	2702700	19575	3224375	1123255	-14441	-15505	-38745	2935	-3185	-889	-325	-385	0	0	-13860	15750	-340	567	-330	-350	-25	60	-105	135	55	0	x_{108}
x_{109}	+	1458928219297500	-53463253284	1850939100	17311644	-8941860	2729376	0	-2980900	976860	-39204	-39780	-8100	32220	4508	-1188	324	540	-1925	0	7776	7776	-864	0	-864	216	0	-288	0	0	0	35	x_{109}
x_{110}	+	1523654585578125	-31693737075	727786125	-18293275	11926925	-1287000	11025	1566125	716845	2957	18565	1245	-18195	805	317	-975	85	0	0	-48840	300	360	597	380	440	585	-40	65	-19	89	35	x_{110}
x_{111}	+	1679281227336960	-53080904448	1653509376	9165312	-7803648	-1876446	0	1134848	1679616	-62208	-78336	34560	14592	512	768	0	0	-3290	-540	-37422	-5346	1890	0	270	378	0	306	0	0	0	65	x_{111}
x_{112}	+	1711786747207680	-23837958144	2124390400	-18104320	1155072	2162160	-47520	-991232	-925696	8192	0	0	-8192	16384	0	0	0	-2420	180	-11088	-2448	-272	-288	48	-352	160	48	480	-96	-64	-18	x_{112}
x_{113}	+	1807462370115584	-53936390144	42205184	0	-1835008	-267904	63872	1835008	1835008	0	0	0	0	0	0	0	0	-616	584	-11648	-11648	-640	-1280	128	0	512	128	128	128	0	0	x_{113}
x_{114}	+	1928549534519790	-20035837458	1778766318	5262894	11395566	1791153	0	-1522962	-942354	-39186	37422	25326	-10770	558	-402	702	1902	-2310	-210	-5103	-5103	-2511	0	81	-567	0	81	0	0	0	0	x_{114}
x_{115}	+	2123804897280000	56233132032	1450803200	14692352	6389760	731808	-17280	0	917504	0	45056	0	0	12288	0	1728	0	-3025	0	7392	-1248	-1120	480	-288	-40	320	96	0	-96	32	144	x_{115}
x_{116}	+	2437847336925000	-35811255480	2097117000	-8230040	3359560	2168595	39510	-3372600	-767032	-17080	12264	-1400	5320	1128	1800	0	680	-2750	0	34419	10155	-621	-582	331	91	270	307	70	-14	-62	56	x_{116}
x_{117}	+	2565217512000000	-37246809600	2452032000	26956800	-6796800	1158300	6480	-704000	-744960	-13824	-46080	7680	12800	-15360	1536	0	0	0	0	1980	-4500	540	-936	-180	540	0	252	-720	72	0	165	x_{117}
x_{118}	+	2611979289562500	20161498500	2950106500	-42144700	9178500	-386100	-24840	346500	-923260	-53884	26180	-2940	-1660	-1980	1156	-1300	-1980	0	0	1980	-10800	3340	-900	-480	-580	640	540	-120	-60	32	-10	x_{118}
x_{119}	+	2642676197359616	0	0	33554432	0	-1441792	51200	0	0	0	0	0	0	0	0	0	0	2816	-384	0	0	0	0	0	-1024	0	0	0	0	128	-128	x_{119}
x_{120}	+	2730389017356000	48886437600	2348771040	-8663200	-18532640	450450	-33606	1268960	-92960	89824	28000	-1568	-7968	-5280	-32	0	1632	0	-250	6930	-3150	1170	-918	-110	-70	-486	-110	-326	106	-34	0	x_{120}
x_{121}	+	2788834713600000	81179770880	2976972800	-9052160	9502720	3132480	-29760	3276800	917504	0	-8192	0	0	-8192	0	0	0	-1750	0	17984	1280	-1984	128	256	-176	320	64	-320	-128	-32	100	x_{121}
x_{122}	+	2908795117921875	81708865875	1059681875	-270725	-11978925	-2641275	-30645	-2722125	2712115	14931	-23205	21315	-16525	-965	483	-325	1675	0	0	70245	-11025	5	-765	-465	175	-85	165	75	3	-77	0	x_{122}
x_{123}	+	2952870715858140	-86375888676	2336120028	-2737475	721116	577368	0	-1592228	1858140	-20772	-6084	-23652	-2724	1468	-1764	-324	1404	-385	15	-21384	5832	216	0	-216	-216	0	-72	0	0	0	-35	x_{123}
x_{124}	+	2991219686193960	53556498216	1777793832	13318344	-5253336	1791153	0	2636712	-141912	59688	-66744	-8856	10920	3528	-24	0	-1272	210	210	-5103	-5103	-2511	0	81	-567	0	81	0	0	0	-45	x_{124}
x_{125}	+	3068619025500000	-58621399200	400476000	5997600	8028000	87075	46980	252000	912480	-46240	3360	8160	-16800	9760	224	0	160	0	0	7875	-4275	-3165	-1440	45	-315	-420	-765	180	-72	36	-70	x_{125}
x_{126}	+	3469068040246875	-81059700645	2507830875	1645515	4097115	360855	0	-2087525	1019547	32859	-58869	-54405	-10725	-1973	1851	675	555	2750	0	18711	3645	1431	0	189	189	0	279	0	0	0	-11	x_{126}
x_{127}	+	3555194033015625	-44982312375	3511671625	7851025	2353225	1726725	-25305	-2300375	-1256535	-35511	83825	-2695	35305	-3375	2201	-975	-1695	0	0	-28875	-9975	325	-633	-455	-35	55	-491	-185	-41	19	0	x_{127}
x_{128}	+	3555194033015625	27442039625	791415625	-37630775	-5665975	-2327325	18435	-1992375	229705	93513	-34615	32585	32585	-9015	1609	-975	1865	0	0	50435	5975	355	-433	-205	-245	-5	-61	-205	-25	151	0	x_{128}
x_{129}	+	3669877711500000	-9053044000	-1523060000	-8663200	23077600	1651650	70530	-308000	-92960	-17696	-79520	-12320	-5920	-15520	2528	0	1120	0	0	37730	2450	1090	542	-110	-70	10	34	-110	-218	-34	0	x_{129}
x_{130}	+	3839609555656875	-69439110741	2486882475	-24787581	7121835	-324324	8100	-1942325	418635	124075	-35805	-6405	-18165	-317	475	351	-845	-1925	0	19404	6174	1116	-576	-306	-294	-180	-468	180	72	12	0	x_{130}
x_{131}	+	4068660047203125	66401039925	739171125	-3744675	9782325	-2606175	21870	-3328875	1250325	6453	42045	15525	26965	-995	-2235	-675	365	0	0	31185	2025	225	-162	-135	-315	-450	-207	270	-18	-126	-25	x_{131}
x_{132}	+	4095583526034000	73329656400	-1119413680	21658000	6607440	2702700	-13230	-1250480	1007440	-22960	-70000	-34608	-18096	-7280	1488	0	16	0	-375	-13860	-12600	-340	-486	120	-140	-430	60	210	90	-50	0	x_{132}
x_{133}	+	4193877530658816	-13905475584	-3151869952	-22177792	13977600	4324320	-11448	157696	-71680	-14336	14336	-43008	10240	-10240	-2048	0	2048	616	66	7392	-7728	4064	-816	336	224	536	-288	-24	96	8	0	x_{133}
x_{134}	+	4231406449091250	113114736306	1608131250	-15701166	-10981710	-208494	10935	371250	2799666	-18510	31698	-23310	-17870	-14254	3570	-1326	-1710	825	0	-8910	2106	18	243	378	90	-225	-270	135	27	-45	-26	x_{134}
x_{135}	+	4231406449091250	-9480477006	1608131250	14531634	-659790	-208494	10935	1638450	-909774	24498	-61518	-23310	50	13106	3570	1326	-590	825	0	-8910	-2106	18	-405	-378	-90	-225	-270	135	-45	-9	-26	x_{135}
x_{136}	+	4250407956480000	79348727808	946176000	-3411968	-26378240	812448	11520	0	917504	0	28672	0	0	-4096	0	-1728	0	-4025	0	-14112	6048	672	288	352	-392	960	-416	640	-32	-32	0	x_{136}
x_{137}	+	4331775591000000	85565937600	2630232000	15163200	17496000	0	0	792000	699840	22464	25920	-25920	8640	2880	-2880	0	-960	0	0	0	0	0	0	0	0	0	0	0	0	0	45	x_{137}
x_{138}	+	4331775591000000	85565937600	2630232000	15163200	17496000	0	0	792000	699840	22464	25920	-25920	8640	2880	-2880	0	-960	0	0	0	0	0	0	0	0	0	0	0	0	0	45	x_{138}
x_{139}	+	4570963756734375	-1964187225	2831038375	-14348425	4580775	193050	-10665	-1241625	-1424665	-51033	-32585	-8505	-13465	8695	-1273	-325	-745	0	0	34650	-900	-230	567	60	140	-185	-294	375	135	23	-140	x_{139}
x_{140}	+	4619851069640625	-77275536975	242888625	30528225	1775025	3316950	-25515	-1551375	303345	-31311	26145	-62895	-9615	-1695	1809	-975	705	0	0	51030	0	-810	-243	0	0	405	342	405	45	-27	0	x_{140}
x_{141}	+	4643518737000000	83140200000	-742808000	3182400	9832000	600600	-39300	-792000	1522240	-104896	-48320	-960	-3520	-2240	-1984	0	320	0	0	-27720	0	-1640	-144	160	0	340	-8	-20	40	180	-45	x_{141}
x_{142}	+	4740258710687500	55902546700	2264223500	-18409300	11141900	275275	53740	1039500	-447860	-72436	16940	6860	-19060	940	76	1300	620	0	0	-20405	-4025	-2485	-200	-505	-385	260	971	-100	80	-52	0	x_{142}
x_{143}	+	4740258710687500	-2036934900	-1362784500	42233100	927500	275275	53740	-192500	-84980	-25844	-13300	140	-10740	780	-1652	-1300	1420	0	0	10395	1575	1035	1260	-265	-525	540	59	140	-4	-84	0	x_{143}
x_{144}	+	4919769220713750	33520660758	3556560150	18581238	6142230	-1791153	0	1113750	-1084266	20502	-29322	16470	150	4086	-426	702	630	-2100	0	5103	5103	2511	0	-81	567	0	-81	0	0	0	-45	x_{144}
x_{145}	+	5191701169700864	0	-3758096384	0	0	4100096	35840	0	0	0	0	0	0	0	0	0	0	-3136	864	0	0	4096	0	0	0	-512	0	0	0	0	0	x_{145}
x_{146}	+	5223958579125000	-55454513400	2273205000	25857000	769800	-2799225	-27810	2277000	-19320	-34552	15720	5320	-15480	-280	1096	0	680	0	0	-58905	4275	2055	-990	-45	315	-330	-345	270	-6	18	-20	x_{146}
x_{147}	+	5257393731060471	44547766263	2227974903	23683023	-24638985	0	0	639639	-852201	36855	5103	27783	-18729	-15345	2151	351	639	-2079	471	0	0	0	0	0	0	0	0	0	0	0	0	x_{147}
x_{148}	+	5257393731060471	44547766263	-1450757385	23683023	17164791	0	0	-2376297	454167	85239	5103	27783	22743	-6129	-3609	351	-4161	-2079	471	0	0	0	0	0	0	0	0	0	0	0	0	x_{148}
x_{149}	+	5485156508081250	-39603834270	3552689250	974610	-5738910	1621620	39690	111650	-1751262	90466	5586	10050	-8030	-7022	-1022	-650	-270	-2750	0	-19404	-6930	-396	522	174	126	90	-204	-150	-6	18	126	x_{149}
x_{150}	+	5992026614971875	78802563555	-1391428125	-10527165	-14419485	-852930	0	-2957725	1419363	-56349	-75069	52515	36195	8003	-93	675	1155	-2750	0	29646	-2430	-1026	0	594	-594	0	-18	0	0	0	-19	x_{150}
x_{151}	+	6414993199558125	-33733182483	-3172921875	-1717443	-8124435	938223	0	2410925	572589	-60435	-6147	53325	3885	5309	2445	1053	-675	-4025	0	-2673	9477	1647	0	-27	-27	0	207	0	0	0	35	x_{151}
x_{152}	+	6692129944500000	72104832800	2769844000	7644000	-1663200	-1876875	42420	-2156000	-256480	-93408	-16800	-10080	-33760	2400	-3168	0	-800	0	0	57365	-7525	1525	404	315	1155	-140	-171	180	-108	84	0	x_{152}
x_{153}	+	6934321313920000	-66547633152	-667110400	-22177792	9548800	32032	8380	1971200	431104	29696	14336	15360	5120	-10240	-1024	-1248	0	-1100	0	-7392	7728	-736	-156	688	224	-340	-224	220	52	8	-126	x_{153}
x_{154}	+	6979512601119240	-6685379064	-2628007416	7017192	8367624	3667599	0	-580216	-476280	-32760	25704	-28728	12936	-6680	840	0	936	440	-135	5103	-17253	783	0	891	-189	0	-81	0	0	0	-56	x_{154}
x_{155}	+	7575407965125000	77335057800	195525000	1146600	6827400	2606175	43740	693000	-343800	24456	-49560	-64440	3080	3560	456	0	680	0	0	-31185	-2025	-225	-324	135	315	-900	207	540	-36	-36	70	x_{155}
x_{156}	+	7679219111313750	-3299834538	-466747050	-15788682	23006550	-648648	16200	1166550	-349482	16982	-34314	57750	-16170	-1418	-874	-2002	630	-3850	0	-16632	5292	-2568	-972	-420	0	-480	552	120	12	0	0	x_{156}
x_{157}	+	7698465274500000	85561476000	-140668000	-16088800	-6626400	965250	6210	-1980000	344480	11168	67360	-17760	11680	3360	160	0	2080	0	0	-27390	-13350	-1150	-606	-390	50	-430	-126	-30	-78	14	35	x_{157}
x_{158}	+	7744013981383680	-80453108736	-104924160	0	-5833728	0	26244	2128896	359424	15360	0	-27648	15360	0	-3072	1248	-2048	1980	-70	0	0	0	-972	0	0	324	0	324	-108	0	-126	x_{158}
x_{159}	+	7853379563452500	29807296596	-752093100	14304276	10162260	-2237301	0	1116500	146772	-12204	30996	18900	13140	-8684	1236	324	660	1925	0	-56133	8991	459	0	-81	-189	0	27	0	0	0	-140	x_{159}
x_{160}	+	7863520369985280	-26072766720	-338671872	16633344	-15079680	-162162	-14904	-2227456	-478464	1792	96768	5376	-9472	-6656	-1792	0	0	-770	280	26334	19530	846	-72	666	-42	-504	126	72	-72	48	0	x_{160}
x_{161}	+	8351460096890625	-14930776575	-2900613375	38039625	10548225	0	-32805	1670625	236385	50433	-58455	41265	-33375	-2295	-1455	-975	665	0	0	0	0	0	-729	0	0	675	0	-405	-81	-81	-90	x_{161}
x_{162}	+	8388837260960625	-47910235023	-231058575	-13427127	11034225	-1430298	0	-1176175	-5103	4977	-18711	16065	465	2953	417	-27	-135	650	0	51030	8262	-1242	0	-810	378	0	54	0	0	0	-56	x_{162}
x_{163}	+	8602825703915520	-119801020416	1915060224	-3411968	2981888	-1574496	-27072	0	917504	0	28672	0	0	-4096	0	-1728	0	2695	520	7392	-4704	-2400	1344	-160	-392	-960	608	-448	-64	-32	0	x_{163}
x_{164}	+	8774244412835616	-132532130016	-22952160	-7446816	-6308064	-938223	0	3166240	1868832	-50400	2016	6048	5664	8416	2208	0	864	-484	-9	2673	-9477	-1647	0	27	27	0	-207	0	0	0	64	x_{164}
x_{165}	+	9211433539600384	68719476736	0	16777216	0	-1114112	-35840	0	0	0	0	0	0	0	0	4096	0	384	384	-32768	4096	0	-512	0	-512	0	0	0	0	64	64	x_{165}
x_{166}	+	9211433539600384	-68719476736	0	16777216	0	-1114112	-35840	0	0	0	0	0	0	0	0	-4096	0	384	384	32768	-4096	0	512	0	-512	0	0	0	0	64	64	x_{166}
x_{167}	+	9558000449712000	35916566400	-2950992000	30326400	-10350720	0	52488	-2471040	293760	44928	51840	40320	-31872	5760	384	0	-1920	0	-500	0	0	0	-972	0	0	0	0	-648	108	0	90	x_{167}
x_{168}	+	9569488431021750	-99418091850	2130247350	-7871850	-2519370	0	6561	-1309770	-342090	12726	111510	-19530	-14154	-15210	-3018	1650	1494	0	-125	0	0	0	-243	0	0	729	0	81	-27	-27	0	x_{168}
x_{169}	+	9573585768140625	-31562182575	1129676625	8037225	-13395375	0	0	1472625	-1148175	-41391	-36855	-14175	23985	-855	513	-675	345	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{169}
x_{170}	+	9573585768140625	-31562182575	1129676625	8037225	-13395375	0	0	1472625	-1148175	-41391	-36855	-14175	23985	-855	513	-675	345	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{170}
x_{171}	+	9596829826538955	-99702143541	-1221239349	6140043	-9316917	1876446	0	423115	422091	-13365	59787	3915	-14133	10891	843	-1053	-885	-2695	-420	37422	5346	-1890	0	-270	-378	0	-306	0	0	0	70	x_{171}
x_{172}	+	9845152706812500	-17601206700	-1435395500	8413300	9494100	-51975	-40635	-2579500	-339500	53844	-94220	-1260	-300	-140	2452	300	-1100	0	0	95865	8925	505	-579	-195	-35	1045	249	165	45	73	0	x_{172}
x_{173}	+	10134024677130240	-69442043904	671514624	5640192	-22118400	-2729376	0	1441792	0	0	36864	0	0	4096	0	1728	0	2465	240	-7776	-7776	864	0	864	-216	0	288	0	0	0	-44	x_{173}
x_{174}	+	10161349228100070	-105566975514	-2004092442	-23218650	9456102	0	0	3735270	1182438	21222	5670	21222	28134	-3546	-2970	702	-1434	-1980	-555	0	0	0	0	0	0	0	0	0	0	0	126	x_{174}
x_{175}	+	11381560819200000	48243486720	1424896000	-9052160	-17879040	-2007720	30240	-1126400	-831488	28672	-65536	0	20480	8192	0	0	0	2750	0	39864	-8520	-1064	768	1176	-176	160	120	-480	0	-32	44	x_{175}
x_{176}	+	11854109736960000	68719476736	0	-16777216	0	-2555904	15360	0	0	0	0	0	0	0	0	-4096	0	3200	0	-32768	4096	0	-512	0	512	0	0	0	0	-64	-64	x_{176}
x_{177}	+	11854109736960000	-68719476736	0	-16777216	0	-2555904	15360	0	0	0	0	0	0	0	0	4096	0	3200	0	32768	-4096	0	512	0	512	0	0	0	0	-64	-64	x_{177}
x_{178}	+	12424315904000000	27880652800	2072576000	0	14745600	-2173600	-9280	0	-1310720	-65536	0	0	0	0	0	0	0	0	0	-12320	6400	-160	496	0	0	-160	-288	0	144	0	70	x_{178}
x_{179}	+	12424315904000000	27880652800	2072576000	0	14745600	-2173600	-9280	0	-1310720	-65536	0	0	0	0	0	0	0	0	0	-12320	6400	-160	496	0	0	-160	-288	0	144	0	70	x_{179}
x_{180}	+	13046344927150080	-25954353152	-4244635648	0	16777216	1896960	-31488	0	0	0	0	0	0	0	0	0	0	2080	80	17920	-18944	512	1216	-512	0	-640	-512	-512	64	0	56	x_{180}
x_{181}	+	13940902029600000	38975489280	-4065811200	-18869760	-17038080	852930	0	-915200	186624	-6912	50688	34560	-3840	14848	768	0	0	-1750	0	-29646	2430	1026	0	-594	594	0	18	0	0	0	-35	x_{181}
x_{182}	+	14014628339712000	27880652800	-1906769920	0	-16711680	3775200	-27264	0	-1310720	65536	0	-65536	0	0	0	0	0	0	-500	-12320	6400	1760	496	0	0	-544	480	576	-48	0	-70	x_{182}
x_{183}	+	14220776132062500	51828676900	-461345500	-3248700	12996900	-1201200	-13740	654500	-617820	-9436	-36540	7140	420	-9020	868	1300	-380	0	0	-30800	13300

$B = F_{2+}$

	8A	8B	8C	8D	8E	8F	8G	8H	8I	8J	8K	8L	8M	8N	9A	9B	10A	10B	10C	10D	10E	10F	11A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	12S	12T	13A	14A	14B	14C	14D	14E	
x_{92}	0	256	256	0	-256	0	0	0	0	0	0	-64	0	0	-6	12	-43	-5	-17	0	5	0	-1	-128	-128	64	-128	-24	0	0	24	0	0	0	8	0	0	0	0	0	0	0	0	1	84	-14	2	4	4	x_{92}
x_{93}	0	256	256	0	-256	0	0	0	0	0	0	64	0	0	-6	12	-27	-5	3	0	5	0	-1	0	0	-64	0	-56	0	0	-8	0	0	-32	-24	0	0	0	0	0	0	0	0	1	-44	-2	-2	4	4	x_{93}
x_{94}	-504	-280	40	-120	-120	-120	184	104	-24	-8	56	0	-8	0	0	0	0	0	0	-15	0	-15	0	-9	126	135	-9	75	-30	15	3	-1	3	-6	-13	6	-18	14	3	-6	6	6	0	0	70	0	12	-6	-2	x_{94}
x_{95}	-1792	-128	-128	256	128	0	0	0	0	0	0	0	0	0	0	0	-70	18	6	48	10	0	-1	510	-48	14	-30	6	0	-22	18	6	6	-24	-2	8	-24	0	-6	0	0	0	0	0	0	0	0	0	0	x_{95}
x_{96}	0	256	256	0	-256	0	0	0	0	0	0	-64	0	0	0	0	37	19	-17	24	-11	24	1	0	0	-64	0	-24	0	0	24	0	0	0	8	0	0	0	0	0	0	0	0	-3	-28	14	2	-4	4	x_{96}
x_{97}	-27	113	833	-315	185	21	45	109	57	-27	9	5	-15	-3	0	0	22	-20	-28	0	-10	0	0	-216	0	-56	72	-24	0	24	24	24	0	0	-8	0	0	0	0	0	0	0	0	-1	1	1	1	1	1	x_{97}
x_{98}	-2016	-192	448	-224	128	-96	-160	-32	0	-32	0	0	0	0	0	9	-70	-10	10	0	10	0	1	-41	4	-41	-59	-59	-60	-11	-41	7	-3	4	5	4	0	4	-9	-12	4	0	0	0	0	0	0	0	0	x_{98}
x_{99}	1575	-105	-105	39	-105	-153	-265	-25	-9	23	7	-25	39	7	-18	0	0	0	0	0	0	0	0	-77	94	115	67	-45	-30	35	3	19	3	6	-13	-10	-18	-2	19	-14	2	-2	2	1	0	0	0	0	0	x_{99}
x_{100}	0	512	-512	0	0	0	0	0	0	0	0	0	0	0	0	0	48	-54	34	-24	-8	12	4	-216	0	8	-72	0	0	-24	0	24	0	0	16	0	0	0	0	0	0	0	0	-2	74	4	-4	-2	2	x_{100}
x_{101}	0	0	0	0	0	0	0	0	0	0	0	32	0	-32	9	-9	-112	20	-8	0	0	0	-1	-64	44	64	80	16	-20	16	-32	0	16	-8	16	4	16	12	0	-12	0	0	-6	0	0	0	0	0	0	x_{101}
x_{102}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-15	12	48	-32	0	28	16	-4	4	256	-32	0	-32	0	64	-32	0	0	-32	0	0	16	16	0	0	0	0	0	0	0	-126	0	0	2	2	x_{102}
x_{103}	0	-512	512	0	0	0	0	0	0	0	0	0	0	0	8	-19	20	50	10	0	-20	0	0	136	-80	104	-8	-64	0	-24	32	-8	0	-16	16	0	0	16	0	0	0	0	0	0	-26	2	-2	2	-2	x_{103}
x_{104}	532	-1020	-380	20	-60	-44	20	84	-92	-12	-28	-20	4	-4	21	12	0	0	0	0	0	0	0	-22	-85	-70	86	90	-25	-18	6	-14	2	3	2	-21	11	-5	6	-9	-1	3	3	-3	0	0	0	0	0	x_{104}
x_{105}	-225	419	19	-65	-69	111	-105	55	27	31	-21	-1	3	-9	0	0	-21	-15	-11	0	-5	0	1	189	0	125	9	-63	0	9	-27	-3	9	0	1	0	0	0	-3	0	0	0	0	2	35	-7	1	-1	-5	x_{105}
x_{106}	1701	641	-143	69	-39	21	-75	29	9	-19	-15	1	25	9	0	0	-21	-49	31	-24	-13	12	0	189	0	-115	9	45	0	33	9	-27	-27	0	-11	0	0	0	9	0	0	0	0	0	-10	4	-4	2	-2	x_{106}
x_{107}	1197	149	53	109	101	141	-147	125	-27	-35	21	29	-27	-3	0	0	-47	-17	11	-12	-7	-12	0	27	0	-5	-63	-15	0	-15	-21	-21	9	0	1	0	0	0	3	0	0	0	0	0	-98	14	-10	2	6	x_{107}
x_{108}	-1169	-405	475	-113	-605	127	15	135	-29	23	59	-5	-13	11	0	-9	0	0	0	0	0	0	0	-26	-53	-10	100	80	15	-20	-22	-2	-24	11	-8	-5	-9	-5	2	-1	-1	-1	-1	0	0	0	0	0	0	x_{108}
x_{109}	540	-356	-356	-228	284	156	-180	-36	-36	12	12	4	12	-12	0	0	91	-5	19	0	-5	0	-5	0	0	-64	0	-24	0	0	24	0	0	0	8	0	0	0	0	0	0	0	0	3	35	-7	-7	3	3	x_{109}
x_{110}	1765	225	145	-123	-455	117	85	-99	-103	-35	1	-15	-7	1	-18	-9	0	0	0	0	0	0	0	-212	49	-100	112	-20	-15	8	16	-12	12	-11	4	5	3	9	-16	17	1	-1	-3	0	35	-7	-7	3	3	x_{110}
x_{111}	2304	-128	-128	256	128	0	0	0	0	0	0	0	0	0	0	0	-98	6	-38	-24	-18	12	-6	-162	0	110	-54	30	0	18	-30	6	-18	0	-10	0	0	0	-6	0	0	0	0	0	65	9	9	1	1	x_{111}
x_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-18	56	-20	-20	0	-8	12	0	368	80	-80	80	0	0	-16	0	16	0	0	-32	-16	0	16	0	0	0	0	0	0	-70	0	4	6	2	x_{112}
x_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10	8	56	24	0	-16	-8	-8	-5	-128	-128	-128	-128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{113}
x_{114}	126	-210	366	126	-210	30	30	-66	30	46	30	30	-18	6	0	0	42	-22	-6	18	-14	6	-5	81	0	81	81	9	0	33	9	33	9	0	9	0	0	0	-15	0	0	0	0	0	0	0	0	0	0	x_{114}
x_{115}	0	256	256	0	-256	0	0	0	0	0	0	-64	0	0	0	-9	-43	15	27	0	5	0	0	-64	-64	0	-64	8	0	0	-40	0	0	-16	-24	0	0	0	0	0	0	0	0	-1	-56	0	-4	0	-8	x_{115}
x_{116}	1288	8	-568	-120	-280	-248	-40	40	72	-40	-8	0	-8	0	9	0	20	50	10	0	-20	0	0	11	38	-21	-61	51	-50	-13	-21	-5	-5	6	3	2	4	-2	3	6	2	0	0	0	-56	0	0	-8	8	x_{116}
x_{117}	-512	0	0	-512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	204	24	-20	204	60	0	-36	12	-4	-12	0	12	0	-24	8	-12	0	0	0	0	0	-35	-7	-11	5	-3	x_{117}
x_{118}	196	-140	-140	196	180	68	100	4	-12	-28	-12	-20	-12	12	0	9	0	0	0	0	0	0	0	-112	-76	0	68	-40	0	44	20	-40	24	8	0	8	-6	-4	4	16	0	-2	-4	0	10	-4	4	-2	2	x_{118}
x_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	0	0	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	16	0	0	x_{119}
x_{120}	-1568	-160	-160	-32	-160	-32	-160	96	-32	-32	32	0	32	0	9	9	0	0	0	-10	0	-10	0	82	-134	50	-62	10	34	34	10	18	34	-26	-6	-14	-2	-6	10	10	6	-2	0	0	0	0	0	0	0	x_{120}
x_{121}	0	-512	-512	0	512	0	0	0	0	0	0	0	0	0	-6	3	-70	10	-10	0	10	0	-1	-64	-64	-64	-64	16	0	0	16	0	0	16	16	0	0	0	0	0	0	0	0	0	-100	-2	2	4	-4	x_{121}
x_{122}	-2933	375	135	-149	-385	91	35	19	-17	43	-49	-5	7	11	0	18	0	0	0	0	0	0	-5	271	127	15	37	-75	-45	21	15	-1	-3	15	-11	-9	3	-1	15	3	-5	3	-1	0	0	0	0	0	0	x_{122}
x_{123}	3996	284	-292	-228	-4	156	-36	-132	-36	-36	60	-4	-4	12	0	0	49	-17	-1	3	41	-9	0	-216	0	-56	72	-24	0	24	24	24	0	0	-8	0	0	0	0	0	0	0	0	-1	-35	7	7	-3	-3	x_{123}
x_{124}	-792	168	-408	360	-120	-24	120	-120	-24	-8	24	0	24	0	0	0	-84	2	-6	-18	4	-6	1	81	0	81	81	9	0	33	9	33	9	0	9	0	0	0	-15	0	0	0	0	0	-45	-3	-3	3	3	x_{124}
x_{125}	-672	-320	320	96	0	-32	160	32	0	32	0	0	0	0	0	0	0	0	0	0	0	0	5	141	-120	45	51	75	60	35	21	-3	-21	12	-5	20	6	0	5	-4	4	2	0	-2	-70	0	0	-6	-6	x_{125}
x_{126}	1611	-253	-157	-373	-205	-21	75	-5	51	-37	3	35	-13	3	0	0	-20	-50	-10	0	20	0	0	189	0	61	-9	39	0	-9	-3	-3	-9	0	7	0	0	0	-3	0	0	0	0	1	-119	21	-3	-3	1	x_{126}
x_{127}	1729	205	285	33	-395	145	-55	-23	-59	17	-43	-15	13	1	21	21	0	0	0	0	0	0	0	9	-117	25	-99	65	-25	-27	5	1	17	-1	9	3	23	-5	5	-1	3	-1	-3	0	0	0	0	0	0	x_{127}
x_{128}	-1687	-375	265	-151	265	-215	-55	-23	41	41	41	-15	9	1	-33	-15	0	0	0	0	0	0	0	115	-65	-45	-29	35	35	51	-37	-13	11	-1	3	3	5	23	-5	-5	-1	1	-3	0	0	0	0	0	0	x_{128}
x_{129}	-1568	160	160	-32	160	224	160	96	-96	32	32	0	32	0	33	-3	0	0	0	0	0	0	0	82	-134	-110	-62	10	10	-14	-38	2	10	-14	-22	-2	-8	2	2	2	-2	-4	0	0	0	0	0	0	0	x_{129}
x_{130}	1715	271	511	339	359	-93	-45	-21	-41	11	15	-1	-9	-9	0	0	-91	-5	19	0	5	0	0	-162	108	46	36	24	-60	-20	-30	6	-24	24	16	-8	-6	-12	10	4	4	-2	0	0	0	0	0	0	0	x_{130}
x_{131}	909	-215	185	-147	-15	-99	5	21	-63	-19	-31	-35	-7	-3	0	0	0	0	0	0	0	0	0	9	90	-135	9	-75	30	-15	-3	1	-3	6	13	18	-18	10	-3	6	2	6	0	-3	-25	3	3	7	7	x_{131}
x_{132}	3024	720	80	208	400	-48	16	16	-48	16	16	0	16	0	0	-9	0	0	0	45	0	-15	0	-344	70	-200	-92	80	-30	44	-4	0	24	2	-8	-10	6	6	-12	-6	-2	6	0	0	0	0	0	0	0	x_{132}
x_{133}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-9	-84	8	8	-2	-20	10	0	-112	32	16	32	-16	24	-32	32	16	0	8	-16	-8	12	16	16	-8	8	4	0	0	0	0	0	0	0	x_{133}
x_{134}	-798	-286	-286	-158	66	194	50	2	-14	34	2	-14	-14	10	0	0	-69	25	9	0	-5	0	0	90	63	90	18	-30	15	-30	-6	10	-6	-9	2	3	9	7	-6	-9	3	-3	-3	0	26	-2	2	-2	2	x_{134}
x_{135}	-1470	-214	106	-62	-54	34	10	2	26	-38	2	46	-14	14	0	0	69	25	9	0	5	0	0	-90	99	-90	18	-30	15	-30	6	-10	-6	15	2	-9	9	11	6	15	7	-3	3	0	154	-14	-2	-2	2	x_{135}
x_{136}	0	256	256	0	-256	0	0	0	0	0	0	64	0	0	18	-9	133	-25	7	0	5	0	-4	-64	-64	0	-64	40	0	0	-8	0	0	16	8	0	0	0	0	0	0	0	0	3	0	0	0	0	0	x_{136}
x_{137}	-1728	0	0	-192	0	-192	0	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	45	3	3	-3	-3	x_{137}
x_{138}	-1728	0	0	-192	0	-192	0	-64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	45	3	3	-3	-3	x_{138}
x_{139}	-665	-25	455	167	215	103	15	55	7	-17	-1	-5	31	11	0	9	0	0	0	0	0	0	0	-388	-37	60	26	10	15	-6	-20	-4	-6	-5	10	-21	-9	11	-4	-1	-1	-1	-1	0	140	0	0	4	-4	x_{139}
x_{140}	-735	-615	-135	97	-375	-63	-15	17	-55	1	-23	-15	25	1	0	0	0	0	0	0	0	0	5	0	81	0	54	-90	45	54	0	0	-42	9	6	9	-3	-15	0	-3	-3	-3	-3	0	0	0	0	0	0	x_{140}
x_{141}	832	320	320	-192	320	-192	0	64	64	0	0	0	0	0	12	21	0	0	0	0	0	0	0	160	196	0	-56	40	60	-40	16	-16	0	-8	-8	8	-6	-4	8	-4	-4	-10	0	0	-45	-3	-3	3	3	x_{141}
x_{142}	588	-100	-420	-52	60	76	60	108	-36	-4	12	20	12	-12	-17	1	0	0	0	0	0	0	0	7	16	135	-101	-85	20	-37	-1	23	11	20	-5	-4	2	8	7	4	-4	-2	4	0	0	0	0	0	0	x_{142}
x_{143}	-308	220	220	-52	-100	76	-20	76	28	-20	-36	-20	28	12	-17	1	0	0	0	0	0	0	0	7	16	55	-29	-25	20	-5	11	-33	-1	-16	15	-32	-4	0	-5	4	4	4	-4	0	0	0	0	0	0	x_{143}
x_{144}	-666	-42	-42	486	-330	6	150	-186	6	38	54	30	6	6	0	0	-42	-20	-12	0	-10	0	-4	-81	0	-81	-81	-9	0	-33	-9	-33	-9	0	-9	0	0	0	15	0	0	0	0	0	-45	-3	-3	3	3	x_{144}
x_{145}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	20	0	-64	0	16	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{145}
x_{146}	2184	-280	40	8	-120	8	-40	104	-24	24	-40	0	24	0	-27	0	0	0	0	0	0	0	0	-141	174	-45	-105	15	70	-25	-21	3	31	6	-1	26	-8	6	-5	-2	2	4	0	0	-20	-6	-6	-4	-4	x_{146}
x_{147}	-945	819	99	111	171	-225	279	-57	-21	-33	-21	-1	3	-9	0	0	63	33	-27	3	15	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{147}
x_{148}	2079	-189	-333	63	-261	15	-9	151	-21	15	27	31	3	15	0	0	63	-15	-27	15	-9	-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{148}
x_{149}	-126	-158	258	66	370	66	-70	50	-30	-6	-22	-10	-22	-10	0	0	-20	50	10	0	20	0	0	-114	174	14	-60	-36	-30	4	30	-2	-12	6	-20	-2	6	-2	-2	-14	-6	-2	-2	0	126	0	0	-2	-2	x_{149}
x_{150}	99	-173	403	227	-205	99	-45	-61	-45	-45	3	35	3	3	0	0	-20	50	10	0	20	0	5	162	0	-46	54	-6	0	-18	6	-6	18	0	2	0	0	0	6	0	0	0	0	-3	-19	-5	-5	-3	-3	x_{150}
x_{151}	-2115	229	-251	125	-11	-99	-195	77	-75	-19	-27	29	-11	-3	0	0	-133	-25	7	0	-5	0	6	-27	0	5	63	15	0	15	21	21	-9	0	-1	0	0	0	-3	0	0	0	0	0	35	7	7	3	3	x_{151}
x_{152}	-2016	320	-320	288	0	-96	-160	96	0	-32	0	0	0	0	-24	-6	0	0	0	0	0	0	0	155	-196	-5	101	45	-60	21	51	11	-3	12	-3	12	0	-4	3	-12	4	0	0	0	0	0	0	0	0	x_{152}
x_{153}	0	0	0	0	0	0	0	0	0	0	0	32	0	-32	10	19	48	20	8	0	0	0	0	112	76	-16	-32	-16	-60	32	32	-16	0	8	-16	-4	-12	-4	-16	-4	0	-4	-6	0	-126	0	0	2	2	x_{153}
x_{154}	-504	-280	40	392	-120	-120	24	-24	-24	-40	24	0	-40	0	0	0	36	24	-8	9	4	9	-5	27	0	59	63	-9	0	15	-45	-21	-9	0	7	0	0	0	3	0	0	0	0	0	56	0	0	8	-8	x_{154}
x_{155}	-504	-280	40	-120	-120	-120	-200	-152	-24	-8	-72	0	-8	0	0	0	0	0	0	0	0	0	0	-9	-36	135	-9	75	60	15	3	-1	3	12	-13	12	0	-4	3	12	-4	0	0	0	70	0	0	6	6	x_{155}
x_{156}	70	-90	-90	198	-122	38	-10	-26	54	-26	6	-50	-10	6	0	0	112	-10	18	0	0	0	0	-84	60	60	96	12	0	-40	-24	36	12	-24	4	8	6	-12	8	-16	0	2	-4	0	0	0	0	0	0	x_{156}
x_{157}	3744	-160	-160	160	-160	-96	160	32	96	32	32	0	32	0	0	-9	0	0	0	0	0	0	0	-454	14	90	50	10	-30	-46	34	-38	-30	-14	-6	-10	6	-2	-14	10	-2	-2	0	0	35	7	7	3	3	x_{157}
x_{158}	0	0	0	0	0	0	0	0	0	0	0	-32	0	32	0	0	-36	60	0	-10	12	2	0	0	108	0	0	0	-36	0	0	0	0	0	0	-12	0	12	0	-12	-8	0	6	0	-126	0	0	2	2	x_{158}
x_{159}	-1260	116	-524	20	-204	-108	-60	-44	-12	4	36	4	-28	-12	0	0	21	5	1	0	5	0	0	351	0	-49	-45	-9	0	27	27	-9	-9	0	-17	0	0	0	3	0	0	0	0	-1	140	0	0	4	-4	x_{159}
x_{160}	1792	-128	-128	-256	128	0	0	0	0	0	0	0	0	0	0	0	-70	-2	-6	28	10	0	0	-150	-24	26	102	-54	24	-2	-18	2	18	0	-14	-8	12	8	14	8	0	-4	0	0	0	0	0	0	0	x_{160}
x_{161}	-2247	-315	405	153	45	137	-255	-31	61	9	77	-15	5	1	0	0	0	0	0	0	0	0	0	0	27	0	0	0	-45	0	0	0	0	27	0	15	9	3	0	27	-1	-3	-3	0	90	6	-6	-2	2	x_{161}
x_{162}	441	245	-155	89	-243	-87	-135	-15	-3	-15	21	5	-3	-3	0	0	-48	10	-2	0	0	0	-4	-378	0	-10	-18	18	0	-42	18	30	18	0	10	0	0	0	-6	0	0	0	0	1	56	0	0	8	-8	x_{162}
x_{163}	0	256	256	0	-256	0	0	0	0	0	0	64	0	0	-18	9	-91	39	7	24	-27	8	0	-64	-64	0	-64	40	0	0	-8	0	0	16	8	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	x_{163}
x_{164}	-2016	320	-64	-224	128	-96	96	-32	0	-32	0	0	0	0	0	0	34	-20	-16	15	26	-9	0

$B = F_{2+}$

	15A	15B	16A	16B	16C	16D	16E	16F	16G	16H	17A	18A	18B	18C	18D	18E	18F	19A	20A	20B	20C	20D	20E	20F	20G	20H	20I	20J	21A	22A	22B	23A	B**	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	24K	24L	24M	24N	25A	26A	26B	27A	
x_{92}	-3	0	0	0	0	0	0	0	0	0	0	2	2	0	-2	-2	0	0	35	0	5	3	1	-1	0	3	0	0	1	1	-1	0	0	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	x_{92}
x_{93}	-3	0	0	0	0	0	0	0	0	0	0	2	-4	0	-2	-2	-2	0	-5	0	5	-5	5	3	0	-3	0	0	1	-1	-1	0	0	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	1	1	0	x_{93}
x_{94}	0	0	-8	-8	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	-3	0	-3	1	-2	0	0	0	0	-1	-5	3	-9	3	0	3	-2	1	1	0	2	0	0	0	0	0	0	x_{94}
x_{95}	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-46	-16	-6	2	-2	-2	0	0	0	0	0	1	-1	0	0	-2	-2	2	2	2	8	-2	0	0	0	0	0	0	0	0	0	0	0	x_{95}
x_{96}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-5	0	5	-5	1	-1	0	3	0	0	-1	-1	1	1	1	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	1	1	0	x_{96}
x_{97}	-7	0	9	-15	-7	-3	5	5	3	-1	1	0	0	0	0	0	0	0	20	0	-10	4	-2	-4	0	-2	0	0	1	0	0	0	0	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	x_{97}
x_{98}	-5	0	-16	16	0	0	0	0	0	0	0	-3	-3	1	-3	0	1	0	-10	0	10	6	-2	2	0	0	0	0	0	1	1	-1	-1	-3	7	5	-1	3	0	1	-4	-1	1	0	0	0	0	0	0	0	0	x_{98}
x_{99}	0	0	-1	-1	-1	-1	-1	-1	-1	-1	0	-6	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3	0	3	2	-1	-1	0	-2	0	-2	0	1	1	0	x_{99}
x_{100}	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-26	-6	4	-2	0	-2	0	0	-2	0	-2	2	0	0	0	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	x_{100}
x_{101}	3	0	0	0	0	0	0	0	0	0	0	-3	3	3	1	1	1	0	20	0	0	4	4	0	0	2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	x_{101}
x_{102}	-10	-1	0	0	0	0	0	0	0	0	0	2	2	0	-2	1	0	0	0	0	0	0	0	0	-4	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{102}
x_{103}	-5	0	0	0	0	0	0	0	0	0	0	-9	-3	1	3	0	-1	1	-10	0	0	-2	4	-2	0	0	0	0	-1	0	0	0	0	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	x_{103}
x_{104}	0	0	12	12	-4	-4	-4	4	0	0	0	6	0	4	-2	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-2	-6	-6	-2	1	2	-1	-4	0	1	-1	1	-1	0	1	-1	0	x_{104}
x_{105}	-4	0	-5	19	3	-1	-1	-1	1	-3	0	0	0	0	0	0	0	0	-15	0	-5	1	-1	-3	0	1	0	0	1	-1	1	0	0	5	1	-3	9	-3	0	1	0	-3	1	0	0	0	0	0	2	0	0	x_{105}
x_{106}	2	0	-7	17	-3	1	-3	1	3	-1	0	0	0	0	0	0	0	0	-21	-6	-1	3	3	-5	0	-1	-2	0	1	0	0	0	0	-7	1	-3	-3	9	0	-3	0	3	-1	0	0	0	0	0	0	0	0	x_{106}
x_{107}	2	0	-11	5	5	-3	-3	-3	1	1	0	0	0	0	0	0	0	0	27	-8	-3	3	-3	-1	0	3	0	0	1	0	0	0	0	5	-1	5	-1	3	0	1	0	3	1	0	0	0	0	0	0	0	0	x_{107}
x_{108}	0	0	-9	-17	3	7	3	-1	1	1	0	-3	3	-1	-3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	-2	4	10	-2	1	4	3	0	2	1	-1	1	-1	0	0	0	0	x_{108}
x_{109}	1	0	12	12	-4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	-5	0	-5	-5	-5	3	0	-1	0	0	-1	-1	-1	-1	-1	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	-1	1	0	x_{109}
x_{110}	0	0	-27	-3	1	5	1	-3	-1	-1	0	3	3	3	-1	2	-1	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-8	4	4	4	1	0	1	-2	-2	-3	1	-1	1	0	0	0	0	x_{110}
x_{111}	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-42	-2	-2	6	-6	2	0	0	2	0	2	0	2	0	0	-2	-2	2	2	-6	0	-2	0	0	0	0	0	0	0	0	0	0	0	x_{111}
x_{112}	10	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	0	-12	18	8	4	0	4	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{112}
x_{113}	-4	2	0	0	0	0	0	0	0	0	0	-2	-2	-4	2	2	0	0	8	8	8	8	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	x_{113}
x_{114}	3	0	2	2	2	2	-6	2	-2	2	0	0	0	0	0	0	0	-1	-2	-12	-2	6	2	-2	6	2	0	2	0	1	-1	0	0	-3	-3	-3	-3	-3	0	-3	0	-3	1	0	0	0	0	0	0	0	0	x_{114}
x_{115}	8	0	0	0	0	0	0	0	0	0	0	3	3	-1	3	0	-1	0	15	0	5	-1	1	3	0	3	0	0	0	0	0	0	0	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	-1	-1	0	x_{115}
x_{116}	-5	0	8	8	0	0	8	0	0	0	0	6	0	0	-2	1	-2	0	-10	0	0	-2	4	-2	0	0	0	0	2	0	0	0	0	-1	-1	-1	-1	-5	10	3	2	-1	-1	-2	-2	0	0	0	0	0	0	x_{116}
x_{117}	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	1	1	0	0	0	0	4	-8	4	0	0	0	0	0	0	0	0	0	0	0	x_{117}
x_{118}	0	0	12	12	4	-12	-4	4	0	0	0	-3	3	1	-3	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	4	4	0	0	-8	-2	4	4	-2	2	2	0	0	0	0	0	0	0	x_{118}
x_{119}	8	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	-1	x_{119}
x_{120}	0	-1	0	0	0	0	0	0	0	0	0	-3	3	-3	1	1	-1	0	0	10	0	0	0	0	2	0	2	2	0	0	0	0	0	2	2	2	2	-2	-2	-2	2	2	-2	-2	2	-2	0	0	0	0	0	x_{120}
x_{121}	5	0	0	0	0	0	0	0	0	0	0	5	-1	-1	1	-2	1	0	10	0	10	-6	-2	-2	0	0	0	0	1	-1	-1	0	0	-8	-8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{121}
x_{122}	0	0	-13	-5	3	7	7	-1	1	1	0	0	0	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-3	-9	5	-1	-5	1	1	-1	-1	1	1	-1	1	-1	0	0	0	0	x_{122}
x_{123}	-7	0	12	12	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	-13	-3	-3	-5	1	3	3	1	1	-1	1	0	0	0	0	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	x_{123}
x_{124}	3	0	8	8	0	0	8	0	0	0	0	0	0	0	0	0	0	0	22	12	-8	-2	-4	-2	-6	0	0	-2	0	1	1	0	0	-3	-3	-3	-3	-3	0	-3	0	-3	1	0	0	0	0	0	0	0	0	x_{124}
x_{125}	0	0	-16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	1	0	0	-5	5	3	-3	-3	-6	3	4	1	-1	-2	0	0	0	0	-2	0	0	x_{125}
x_{126}	5	0	11	-5	3	-5	3	-5	-1	-1	0	0	0	0	0	0	0	1	10	0	0	2	-4	2	0	0	0	0	-2	0	0	0	0	-1	5	-1	5	-3	0	-1	0	-3	-1	0	0	0	0	0	1	-1	0	x_{126}
x_{127}	0	0	1	-23	1	5	5	-3	-1	-1	0	-3	-3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-11	-3	1	1	1	1	-3	-1	-1	-1	1	-1	1	1	0	0	0	0	x_{127}
x_{128}	0	0	1	1	1	1	1	-7	1	1	0	-1	-1	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	-5	-5	-5	-1	5	-1	-1	-1	-1	1	-1	-1	1	0	0	0	0	x_{128}
x_{129}	0	0	0	0	0	0	0	0	0	0	0	5	-1	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	0	0	0	0	0	0	x_{129}
x_{130}	1	0	-1	-9	-5	-1	3	7	-3	1	0	0	0	0	0	0	0	0	-5	0	5	-5	5	3	0	1	0	0	0	0	0	0	0	-8	-2	-4	2	2	-4	0	0	0	2	0	0	-2	0	0	0	0	0	x_{130}
x_{131}	0	0	5	-19	-3	1	-7	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	1	5	-3	9	-3	0	-3	2	-1	-1	0	2	0	0	0	1	-1	0	x_{131}
x_{132}	0	0	0	0	0	0	0	0	0	0	0	-3	3	-1	-3	0	1	0	0	-5	0	0	0	0	-3	0	-1	1	0	0	0	0	0	0	8	4	4	0	0	4	-2	-2	-2	0	-2	0	0	0	0	0	0	x_{132}
x_{133}	-5	-3	0	0	0	0	0	0	0	0	0	3	3	-1	3	0	-1	0	16	-4	-4	0	-4	0	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{133}
x_{134}	6	0	6	6	6	-10	-2	6	-2	2	0	0	0	0	0	0	0	0	-15	0	-5	1	3	1	0	-1	0	0	1	0	0	0	0	2	2	-6	-6	5	3	-2	-1	2	-2	-1	-1	1	1	0	0	0	0	x_{134}
x_{135}	6	0	-6	-6	6	-2	-6	-2	2	2	0	0	0	0	0	0	0	0	-15	0	5	1	-3	1	0	1	0	0	1	0	0	0	0	2	-2	-6	5	-6	-3	-2	-5	-2	-2	1	-1	-1	-1	0	0	0	0	x_{135}
x_{136}	-2	0	0	0	0	0	0	0	0	0	0	-3	3	3	1	-2	1	0	15	0	5	-1	-3	-1	0	-3	0	0	0	-2	0	1	1	4	4	-4	-4	0	0	0	0	0	0	0	0	0	0	-1	-1	0	x_{136}
x_{137}	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{137}
x_{138}	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{138}
x_{139}	0	0	7	-9	-5	-5	-1	-5	-1	-1	0	3	-3	1	3	0	-1	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	2	-4	2	-4	4	1	2	3	0	-2	1	-1	1	-1	0	0	0	0	x_{139}
x_{140}	0	0	-7	9	-7	1	1	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	6	0	6	0	0	3	-2	-3	0	-2	3	1	-1	1	0	0	0	0	x_{140}
x_{141}	0	0	0	0	0	0	0	0	0	0	0	-3	3	1	1	4	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	-4	-4	-4	-8	4	0	0	0	0	0	0	-2	0	0	0	0	0	x_{141}
x_{142}	0	0	4	4	-12	4	4	-4	0	0	0	-5	1	5	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	3	-9	3	3	-6	-1	0	3	-1	-2	0	0	0	0	0	0	1	x_{142}
x_{143}	0	0	4	4	12	-4	4	4	0	0	0	3	-3	-3	-1	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5	-5	-1	-1	-5	-2	-1	4	1	1	-2	0	-2	0	0	0	0	1	x_{143}
x_{144}	-3	0	10	10	2	2	2	2	-2	2	0	0	0	0	0	0	0	-1	20	0	-10	4	-2	-4	0	2	0	0	0	2	0	0	0	3	3	3	3	3	0	3	0	3	-1	0	0	0	0	0	0	0	0	x_{144}
x_{145}	-4	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	x_{145}
x_{146}	0	0	8	8	0	0	8	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	-1	-5	-9	3	3	6	-1	2	-1	3	2	2	0	0	0	0	0	0	x_{146}
x_{147}	0	0	-21	3	-5	-9	-1	-1	1	-3	0	0	0	0	0	0	0	0	9	-11	-1	9	3	5	3	1	1	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{147}
x_{148}	0	0	-9	15	-1	-5	3	-5	1	1	0	0	0	0	0	0	0	0	-27	3	3	-3	3	1	3	1	3	-1	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{148}
x_{149}	-5	0	10	-6	6	6	2	-2	2	2	0	0	0	0	0	0	0	0	-10	0	0	-2	-4	-2	0	0	0	0	0	0	0	0	0	4	6	4	-2	-6	0	0	2	2	0	0	2	0	2	0	0	0	0	x_{149}
x_{150}	-5	0	3	3	-5	-5	3	3	-1	-1	0	0	0	0	0	0	0	0	-10	0	0	-2	-4	-2	0	0	0	0	-1	1	1	0	0	-2	-2	2	2	6	0	2	0	0	0	0	0	0	0	1	1	0	x_{150}
x_{151}	-2	0	5	-11	5	-3	-3	-3	1	1	-1	0	0	0	0	0	0	0	15	0	-5	-1	3	-1	0	3	0	0	-1	0	-2	0	0	-5	1	-5	1	-3	0	-1	0	-3	-1	0	0	0	0	0	0	0	0	x_{151}
x_{152}	0	0	16	-16	0	0	0	0	0	0	1	-4	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	-5	-3	3	3	0	-3	-4	-1	1	0	0	0	0	0	0	0	0	x_{152}
x_{153}	7	0	0	0	0	0	0	0	0	0	0	-3	-3	-1	1	-2	-1	0	20	0	0	4	-4	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	1	x_{153}
x_{154}	-1	0	8	8	0	0	8	0	0	0	0	0	0	0	0	0	0	0	-16	9	4	0	4	0	-3	0	1	1	-2	-1	-1	0	0	-1	-5	-9	3	3	0	-1	0	3	-1	0	0	0	0	0	0	0	0	x_{154}
x_{155}	0	0	-8	-8	0	0	-8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	-1	-1	-1	-5	3	-9	3	0	3	4	1	1	0	0	0	0	0	0	0	0	x_{155}
x_{156}	2	0	-6	-6	10	-6	2	2	2	-2	0	0	0	0	0	0	0	0	30	0	0	-2	-4	2	0	-2	0	0	0	0	0	0	0	0	0	4	4	4	-2	0	-4	2	-2	2	0	0	0	0	0	0	0	x_{156}
x_{157}	0	0	0	0	0	0	0	0	0	0	0	-3	-3	-1	-3	0	-1	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	2	2	2	2	6	0	-2	-2	-2	2	0	2	0	0	0	0	0	0	x_{157}
x_{158}	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	36	-4	-4	4	0	0	2	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	x_{158}
x_{159}	-1	0	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	-35	0	5	-3	1	1	0	-1	0	0	4	0	0	0	0	-1	7	3	3	3	0	-1	0	-3	1	0	0	0	0	0	-1	-1	0	x_{159}
x_{160}	13	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-26	-6	-6	6	-2	-6	-4	0	-2	0	0	0	0	0	0	-2	-2	2	2	-2	-8	2	0	0	0	0	0	0	0	0	0	0	0	x_{160}
x_{161}	0	0	9	-15	-7	-3	-3	-3	-1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	3	0	0	-1	-1	1	1	0	0	0	0	x_{161}
x_{162}	2

$B = F_{2+}$

	28A	28B	28C	28D	28E	30A	30B	30C	30D	30E	30F	30G	H**	31A	B**	32A	B*	32C	D**	33A	34A	34B	C*	35A	36A	36B	36C	38A	39A	40A	40B	40C	40D	40E	42A	42B	42C	44A	46A	B**	47A	B**	48A	48B	52A	55A	56A	B*	60A	60B	60C	66A	70A	
X_{92}	0	0	-2	-2	0	-13	-1	1	1	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	-2	0	0	0	1	1	1	-1	1	0	3	1	-1	1	0	0	0	0	0	0	-1	-1	0	0	-1	1	0	1	-1	X_{92}
X_{93}	0	0	2	2	0	3	-3	3	1	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	-2	0	0	1	1	1	-1	-1	0	1	1	1	-1	0	0	0	0	0	0	1	-1	0	0	1	-1	0	-1	1	X_{93}
X_{94}	2	-2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-2	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	X_{94}
X_{95}	0	0	0	0	0	5	-1	-3	3	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	2	2	-2	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	-1	1	0	1	0	X_{95}
X_{96}	0	0	2	-2	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	1	1	-1	1	0	-1	-1	-1	-1	-1	-1	0	0	0	0	-1	1	0	0	1	1	0	-1	2	X_{96}
X_{97}	1	1	1	1	1	7	1	-1	1	-1	0	0	0	0	0	0	-1	-1	1	1	0	1	-1	-1	1	0	0	0	0	-1	-2	-2	0	0	0	1	1	1	0	0	0	0	0	0	0	-1	0	1	1	-1	1	0	0	1	X_{97}
X_{98}	0	0	0	0	0	5	-1	1	-1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	-2	-2	-2	0	0	0	0	0	0	1	-1	-1	0	0	-1	1	0	1	0	0	-1	1	0	1	0	X_{98}
X_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	-2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	0	0	0	0	0	X_{99}
X_{100}	-2	2	0	0	0	6	0	-2	0	-2	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	2	-2	0	0	0	2	-2	2	0	0	0	1	1	0	0	0	-1	0	0	-2	0	0	-1	-1	X_{100}
X_{101}	0	0	0	0	0	-7	-1	1	-1	-3	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	1	1	0	0	0	0	0	2	0	0	0	0	-1	0	0	1	1	0	0	0	-1	0	0	-1	1	0	-1	0	X_{101}
X_{102}	2	2	0	0	2	6	0	0	-2	-2	-1	1	1	0	0	0	0	0	0	1	0	0	0	-1	-2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	1	-1	X_{102}
X_{103}	-2	2	2	-2	0	5	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	0	1	0	-2	2	0	0	0	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	-1	X_{103}
X_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	X_{104}
X_{105}	3	-1	1	1	1	-6	0	-2	0	-2	0	0	0	0	0	0	1	1	-1	-1	1	0	0	0	0	0	0	0	0	-1	-1	-1	1	-1	0	-1	-1	1	-1	0	0	0	0	1	1	0	1	-1	-1	0	2	0	-1	0	X_{105}
X_{106}	2	-2	0	0	0	-6	0	4	2	2	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	1	-3	1	1	0	-1	1	-1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	X_{106}
X_{107}	-2	2	-2	-2	0	-2	-2	2	-2	2	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	-1	3	1	-1	-2	1	-1	-1	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	2	X_{107}
X_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-2	0	0	0	0	0	0	0	0	0	X_{108}
X_{109}	3	3	1	1	-1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	-1	-1	-1	1	1	1	0	0	0	0	-1	0	1	1	1	1	0	-1	0	X_{109}
X_{110}	3	3	1	1	-1	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	X_{110}
X_{111}	1	1	1	1	1	4	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	2	2	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	X_{111}
X_{112}	-2	2	-4	0	0	2	2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	2	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{112}
X_{113}	0	0	0	0	0	2	2	0	0	-2	-2	2	2	0	0	0	0	0	0	1	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	X_{113}
X_{114}	0	0	0	0	0	-3	-3	3	-1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	-4	0	0	0	0	0	0	-1	0	0	0	0	-1	-1	0	0	0	0	1	-1	0	1	0	X_{114}
X_{115}	0	0	0	4	0	2	2	0	0	2	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	-1	1	1	-1	1	0	-2	0	2	0	0	0	0	0	0	0	-1	0	0	0	0	-2	0	0	-1	X_{115}
X_{116}	0	0	0	0	0	5	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	1	0	0	-2	2	0	0	0	-2	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	1	0	0	-1	X_{116}
X_{117}	1	1	-3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	0	-1	-1	0	0	0	0	0	0	-1	-1	0	0	0	0	0	X_{117}
X_{118}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{118}
X_{119}	0	0	0	0	0	0	0	-4	0	0	0	0	0	1	1	0	0	0	0	0	2	0	0	2	0	0	0	0	-1	0	0	0	0	0	0	0	-2	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	X_{119}
X_{120}	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	X_{120}
X_{121}	0	0	2	-2	0	5	-1	-1	1	1	0	0	0	1	1	0	0	0	0	-1	0	0	0	0	-1	1	0	0	0	-2	-2	2	0	0	-1	1	-1	-1	0	0	0	0	0	0	0	-1	0	0	1	1	0	-1	0	X_{121}
X_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	1	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	0	X_{122}
X_{123}	-3	-3	-1	-1	1	7	1	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	3	1	1	-1	1	1	1	0	0	0	0	0	0	0	1	0	-1	-1	-1	1	0	0	0	X_{123}
X_{124}	-1	-1	1	1	-1	-3	-3	3	-1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-2	2	0	0	0	0	0	0	1	0	0	0	0	-1	-1	0	1	-1	-1	1	-1	0	1	0	X_{124}
X_{125}	2	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	2	0	0	1	0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	0	X_{125}
X_{126}	-3	1	1	1	-1	-5	1	-1	1	-1	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	-1	0	0	0	1	1	2	-2	0	0	0	-2	0	0	0	0	0	0	0	-1	1	-1	0	1	1	1	-1	0	0	1	X_{126}
X_{127}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	3	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	X_{127}
X_{128}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	X_{128}
X_{129}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{129}
X_{130}	0	0	0	0	0	-1	-1	1	1	-1	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	-1	0	0	0	X_{130}
X_{131}	-1	-1	3	3	-1	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	-1	-1	1	0	-1	-1	0	0	0	0	0	X_{131}
X_{132}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{132}
X_{133}	0	0	0	0	0	-3	-3	-1	-1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	X_{133}
X_{134}	2	-2	-2	2	0	6	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-1	-1	1	1	0	-1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	X_{134}
X_{135}	2	-2	2	-2	0	-6	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	X_{135}
X_{136}	0	0	0	0	0	-2	-2	-2	2	2	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	1	0	0	0	1	1	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	0	1	0	X_{136}
X_{137}	1	1	-1	-1	1	0	0	0	0	0	0	0	0	-1	-1	4r2	-4r2	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	1	1	0	0	0	0	0	X_{137}
X_{138}	1	1	-1	-1	1	0	0	0	0	0	0	0	0	-1	-1	-4r2	4r2	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	1	1	0	0	0	0	0	X_{138}
X_{139}	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	X_{139}
X_{140}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	2	0	0	0	0	0	0	0	0	1	0	X_{140}
X_{141}	-1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	X_{141}
X_{142}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	X_{142}
X_{143}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	X_{143}
X_{144}	-1	-1	1	1	-1	3	3	-3	1	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	-1	-1	-1	1	0	-1	0	X_{144}
X_{145}	0	0	0	0	0	0	0	0	-4	0	0	-2	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	X_{145}
X_{146}	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	X_{146}
X_{147}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{147}
X_{148}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	1	-3	-1	1	1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{148}
X_{149}	-2	-2	0	0	-2	-5	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	-1	-1	0	0	1	X_{149}
X_{150}	1	1	-1	-1	1	-5	1	1	-1	-1	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	1	0	0	0	0	0	2	-2	0	0	0	-1	1	1	-1	0	0	0	0	0	0	-1	0	1	1	-1	-1	0	1	1	X_{150}
X_{151}	3	3	-1	-1	-1	2	2	-2	2	-2	0	0	0	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	0	0	-1	-1	-1	-1	0	-1	1	1	0	0	0	0	0	-1	1	0	1	-1	-1	0	0	0	0	X_{151}
X_{152}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	X_{152}
X_{153}	2	2	0	0	2	3	3	-1	-1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-1	X_{153}
X_{154}	0	0	0	0	0	-3	3	1	3	1	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	2	0	0	1	0	0	0	0	-1	-1	0	0	0	0	-1	1	0	-1	1	X_{154}
X_{155}	-2	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-1	-1	0	0	1	1	0	0	0	0	0	0	0	0	0	X_{155}
X_{156}	0	0	0	0	0	-8	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	X_{156}
X_{157}	3	3	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	-1	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	X_{157}
X_{158}	2	2	0	0	2	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	0	-1	0	X_{158}
X_{159}	-4	4	0	0	0	-9	-3	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$B = F_{2+}$

ind	1A	2A	2B	2C	2D	3A	3B	4A	4B	4C	4D	4E	4F	4G	4H	4I	4J	5A	5E	6A	6B	6C	6D	6E	6F	6G	6H	6I	6J	6K	7A	
	1 2	2	2 2	4	2	3 6	3 6	4	4	4	4	4 4	4	4	4	8	4	5 10	5 10	6	6	6 6	6	6	12	6 6	6	6 6	6	12	7 14	
X_{185} +	96256	0	2048	0	0	352	28	0	0	0	0	128	0	0	0	0	0	56	6	0	0	32	0	0	0	4	0	36	0	0	20	X_{1}
X_{186} +	10506240	0	45056	0	0	3456	-108	0	0	0	0	768	0	0	0	0	0	240	-10	0	0	128	0	0	0	-20	0	108	0	0	52	X_{186}
X_{187} +	410132480	0	516096	0	0	23648	-4	0	0	0	0	3584	0	0	0	0	0	880	-20	0	0	288	0	0	0	36	0	324	0	0	128	X_{187}
X_{188} +	8844386304	0	3629056	0	0	96096	840	0	0	0	0	9728	0	0	0	0	0	1904	54	0	0	544	0	0	0	-40	0	504	0	0	180	X_{188}
X_{189} +	36657653760	0	8198144	0	0	146016	-432	0	0	0	0	12672	0	0	0	0	0	1960	10	0	0	800	0	0	0	-80	0	432	0	0	140	X_{1}
X_{190} o	53936390144	0	-1835008	0	0	11648	1280	0	0	0	0	0	0	0	0	0	0	-56	144	0	0	128	0	0	0	-128	0	0	0	0	0	X_{190}
X_{191} o	53936390144	0	-1835008	0	0	11648	1280	0	0	0	0	0	0	0	0	0	0	-56	144	0	0	128	0	0	0	-128	0	0	0	0	0	X_{191}
X_{192} +	100406462464	0	13989888	0	0	228448	-2240	0	0	0	0	16256	0	0	0	0	0	2464	-36	0	0	288	0	0	0	0	0	576	0	0	140	X_{1}
X_{193} +	772566552576	0	46067712	0	0	535392	2088	0	0	0	0	30464	0	0	0	0	0	3376	76	0	0	-480	0	0	0	120	0	792	0	0	112	X_{193}
X_{194} +	864538761216	0	65333248	0	0	691392	-996	0	0	0	0	38528	0	0	0	0	0	4816	-34	0	0	1600	0	0	0	164	0	612	0	0	200	X_{194}
X_{195} +	4322693806080	0	133728256	0	0	997152	3768	0	0	0	0	43136	0	0	0	0	0	3080	-170	0	0	352	0	0	0	-280	0	648	0	0	20	X_{195}
X_{196} +	10177847623680	0	258080768	0	0	1611456	5712	0	0	0	0	57344	0	0	0	0	0	5280	-70	0	0	1088	0	0	0	-80	0	432	0	0	72	X_{1}
X_{197} +	17069098618880	0	255963136	0	0	1029248	-13384	0	0	0	0	37632	0	0	0	0	0	880	130	0	0	1408	0	0	0	104	0	648	0	0	-112	X_{197}
X_{198} +	21400636907520	0	384933888	0	0	2122848	-8748	0	0	0	0	59904	0	0	0	0	0	5920	20	0	0	288	0	0	0	-180	0	108	0	0	108	X_{198}
X_{199} o	23459577856000	0	-7208960	0	0	-228800	16144	0	0	0	0	8192	0	0	0	0	0	0	-250	0	0	-320	0	0	0	176	0	-144	0	0	20	X_{199}
X_{200} o	23459577856000	0	-7208960	0	0	-228800	16144	0	0	0	0	8192	0	0	0	0	0	0	-250	0	0	-320	0	0	0	176	0	-144	0	0	20	X_{20}
X_{201} +	31123395403776	0	36741120	0	0	123552	16632	0	0	0	0	18944	0	0	0	0	0	-1024	26	0	0	-1056	0	0	0	-600	0	72	0	0	-52	X_{201}
X_{202} +	40345142190080	0	219127808	0	0	512096	8924	0	0	0	0	31232	0	0	0	0	0	-1120	80	0	0	-736	0	0	0	196	0	612	0	0	-140	X_{202}
X_{203} +	60780833777664	0	717674496	0	0	2018016	3384	0	0	0	0	54656	0	0	0	0	0	2464	164	0	0	5280	0	0	0	-24	0	-504	0	0	0	X_{2}
X_{204} +	60863961600000	0	548761600	0	0	2077920	-540	0	0	0	0	65280	0	0	0	0	0	2000	0	0	0	-1376	0	0	0	-100	0	540	0	0	-92	X_{20}
X_{205} o	77683916800000	0	150732800	0	0	-535040	-16640	0	0	0	0	0	0	0	0	0	0	1000	0	0	0	512	0	0	0	640	0	0	0	0	-56	X_{205}
X_{206} o	77683916800000	0	150732800	0	0	-535040	-16640	0	0	0	0	0	0	0	0	0	0	1000	0	0	0	512	0	0	0	640	0	0	0	0	-56	X_{206}
X_{207} +	93980448866304	0	360628224	0	0	-41184	-32112	0	0	0	0	7168	0	0	0	0	0	-1496	54	0	0	2400	0	0	0	624	0	-144	0	0	112	$X_{}$
X_{208} +	110949141022720	0	602601472	0	0	759616	35476	0	0	0	0	28544	0	0	0	0	0	-3080	220	0	0	3520	0	0	0	-244	0	684	0	0	-140	X_{208}
X_{209} +	138547873400000	0	699494400	0	0	1542880	-17180	0	0	0	0	65280	0	0	0	0	0	-2000	0	0	0	-864	0	0	0	540	0	540	0	0	-148	X_{209}
X_{210} o	168430710030336	0	-191102976	0	0	0	0	0	0	0	0	0	0	0	0	0	0	936	336	0	0	0	0	0	0	0	0	0	0	0	-72	X_{210}
X_{211} o	168430710030336	0	-191102976	0	0	0	0	0	0	0	0	0	0	0	0	0	0	936	336	0	0	0	0	0	0	0	0	0	0	0	-72	$X_{}$
X_{212} o	235755961319424	0	-504627200	0	0	908544	-3840	0	0	0	0	0	0	0	0	0	0	-1176	-576	0	0	1792	0	0	0	-640	0	0	0	0	0	X_{212}
X_{213} o	235755961319424	0	-504627200	0	0	908544	-3840	0	0	0	0	0	0	0	0	0	0	-1176	-576	0	0	1792	0	0	0	-640	0	0	0	0	0	X_{213}
X_{214} o	261841888000000	0	19712000	0	0	-677600	-42560	0	0	0	0	13440	0	0	0	0	0	0	0	0	0	-160	0	0	0	160	0	0	0	0	0	X_{2}
X_{215} o	261841888000000	0	19712000	0	0	-677600	-42560	0	0	0	0	13440	0	0	0	0	0	0	0	0	0	-160	0	0	0	160	0	0	0	0	0	X_{2}
X_{216} +	312199319900160	0	1806249984	0	0	5787936	14256	0	0	0	0	80640	0	0	0	0	0	6160	160	0	0	-2208	0	0	0	336	0	-432	0	0	0	X_{216}
X_{217} o	447224633856000	0	-468357120	0	0	0	26244	0	0	0	0	16128	0	0	0	0	0	0	-250	0	0	0	0	0	0	540	0	-324	0	0	0	X_{217}
X_{218} o	447224633856000	0	-468357120	0	0	0	26244	0	0	0	0	16128	0	0	0	0	0	0	-250	0	0	0	0	0	0	540	0	-324	0	0	0	$X_{}$
X_{219} +	791851002292224	0	934778880	0	0	3592512	20412	0	0	0	0	18816	0	0	0	0	0	-1176	-276	0	0	-5184	0	0	0	324	0	324	0	0	0	X_{219}
X_{220} +	828829551513600	0	3033640960	0	0	3931200	21492	0	0	0	0	37632	0	0	0	0	0	0	-150	0	0	7360	0	0	0	524	0	-756	0	0	0	X_{220}
X_{221} +	1566852857856000	0	3986554880	0	0	6177600	-34128	0	0	0	0	73728	0	0	0	0	0	0	-250	0	0	-1600	0	0	0	-560	0	-432	0	0	-160	X_{221}
X_{222} +	1793872427483136	0	-265289728	0	0	-1592448	-11328	0	0	0	0	65536	0	0	0	0	0	-2464	636	0	0	-1408	0	0	0	256	0	-576	0	0	140	X
X_{223} o	2642676197359616	0	0	0	0	-1441792	51200	0	0	0	0	0	0	0	0	0	0	2816	-384	0	0	0	0	0	0	0	0	0	0	0	-128	X_{223}
X_{224} o	2642676197359616	0	0	0	0	-1441792	51200	0	0	0	0	0	0	0	0	0	0	2816	-384	0	0	0	0	0	0	0	0	0	0	0	-128	X_{224}
X_{225} +	2995626807613440	0	1414014976	0	0	-247104	48060	0	0	0	0	39552	0	0	0	0	0	-6160	-310	0	0	64	0	0	0	836	0	324	0	0	140	$X_{}$
X_{226} +	3370903132962816	0	3979345920	0	0	6196608	-39744	0	0	0	0	-55536	0	0	0	0	0	2816	316	0	0	-1920	0	0	0	-384	0	-576	0	0	52	$X_{}$
X_{227} +	3824419701473280	0	4439457792	0	0	2122848	-34992	0	0	0	0	64512	0	0	0	0	0	-8120	-470	0	0	288	0	0	0	-720	0	432	0	0	0	X_{227}
X_{228} +	4193877530658816	0	4950865920	0	0	-864864	-2700	0	0	0	0	-27776	0	0	0	0	0	616	66	0	0	7392	0	0	0	-660	0	-180	0	0	0	X_{228}
X_{229} o	4215392896000000	0	-1295360000	0	0	-572000	3100	0	0	0	0	38400	0	0	0	0	0	0	0	0	0	-800	0	0	0	-1180	0	-540	0	0	-20	X
X_{230} o	4215392896000000	0	-1295360000	0	0	-572000	3100	0	0	0	0	38400	0	0	0	0	0	0	0	0	0	-800	0	0	0	-1180	0	-540	0	0	-20	X_{2}
X_{231} +	4250243698421760	0	3586166784	0	0	0	0	0	0	0	0	20736	0	0	0	0	0	-1440	510	0	0	0	0	0	0	0	0	0	0	0	-108	X_{231}
X_{232} o	4490309824000000	0	-1379840000	0	0	800800	99340	0	0	0	0	-8960	0	0	0	0	0	0	0	0	0	1120	0	0	0	-940	0	180	0	0	0	X_{232}
X_{233} o	4490309824000000	0	-1379840000	0	0	800800	99340	0	0	0	0	-8960	0	0	0	0	0	0	0	0	0	1120	0	0	0	-940	0	180	0	0	0	X
X_{234} +	4638342016000000	0	349184000	0	0	-2736800	-10340	0	0	0	0	74240	0	0	0	0	0	0	0	0	0	-3040	0	0	0	1060	0	-540	0	0	-140	X_{234}
X_{235} o	5191647233310720	0	-3756261376	0	0	4088448	34560	0	0	0	0	0	0	0	0	0	0	-3080	720	0	0	3968	0	0	0	640	0	0	0	0	0	X_{235}
X_{236} o	5191647233310720	0	-3756261376	0	0	4088448	34560	0	0	0	0	0	0	0	0	0	0	-3080	720	0	0	3968	0	0	0	640	0	0	0	0	0	X
X_{237} +	5191701169700864	0	3758096384	0	0	4100096	35840	0	0	0	0	0	0	0	0	0	0	-3136	864	0	0	-4096	0	0	0	-512	0	0	0	0	0	X
X_{238} o	8580585368070144	0	-377726976	0	0	0	0	0	0	0	0	-51840	0	0	0	0	0	5544	144	0	0	0	0	0	0	0	0	0	0	0	-180	X_{238}
X_{239} o	8580585368070144	0	-377726976	0	0	0	0	0	0	0	0	-51840	0	0	0	0	0	5544	144	0	0	0	0	0	0	0	0	0	0	0	-180	X_{239}
X_{240} +	8740741152000000	0	5116672000	0	0	-3775200	-50820	0	0	0	0	-13440	0	0	0	0	0	0	0	0	0	4960	0	0	0	260	0	900	0	0	-200	X
X_{241} +	12604975138529280	0	7934386176	0	0	2122848	43740	0	0	0	0	-69120	0	0	0	0	0	880	-720	0	0	288	0	0	0	900	0	-540	0	0	108	X_{2}
X_{242} +	16972115104000000	0	1277696000	0	0	3253600	60580	0	0	0	0	-67200	0	0	0	0	0	0	0	0	0	-3040	0	0	0	700	0	540	0	0	0	X_{242}
X_{243} +	21065543276560384	0	0	0	0	-3670016	-20480	0	0	0	0	0	0	0	0	0	0	3584	384	0	0	0	0	0	0	0	0	0	0	0	0	X_{243}
X_{244} +	24089453696000000	0	1813504000	0	0	-7453600	64820	0	0	0	0	89600	0	0	0	0	0	0	0	0	0	-1760	0	0	0	140	0	-180	0	0	0	X
X_{245} +	24515659148820480	0	4244635648	0	0	-3995136	-34560	0	0	0	0	0	0	0	0	0	0	-1120	480	0	0	-512	0	0	0	-640	0	0	0	0	200	X_{245}
X_{246} +	26438944243712000	0	-8124497920	0	0	1601600	-36544	0	0	0	0	57344	0	0	0	0	0	0	-500	0	0	2240	0	0	0	64	0	-576	0	0	0	X_{246}
X_{247} +	29823129106907136	0	-4410441728	0	0	5957952	-65856	0	0	0	0	-57344	0	0	0	0	0	-2464	-364	0	0	-1088	0	0	0	-64	0	576	0	0	0	X

Sporadic "Baby monster" group $B \cong F_{2+}$

Order = 4,154,781,481,226,426,191,177,580,544,000,000

$= 2^{41}.3^{13}.5^{6}.7^{2}.11.13.17.19.23.31.47$

Mult = 2 Out = 1

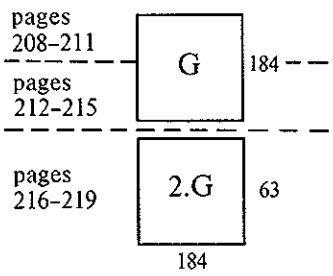
Constructions

Transposition G is generated by a conjugacy class (2A) of involutions, which are (3,4)-transpositions; that is, any non-commuting pair has product of order 3 or 4. G has been explicitly constructed as a permutation group on its 13571955000 transpositions t, which fall into orbits of sizes 1, 3968055, 23113728, 2370830336, and 11174042880 under the stabilizer $2\cdot({}^2E_6(2)).2$ of a particular transposition t_0. These orbits correspond to the conjugacy classes 1A, 2B, 2C, 3A and 4B for t_0t.

Monster 2.G is the centralizer of an involution of class 2A in the Monster group M, and is the stabilizer in M of the corresponding transposition vector (see M, section "Vectors"). The irreducible 196883-dimensional representation of M restricts to 2.G as the sum of irreducible representations of degrees 1, 4371, 96255, 96256. The first three of these yield irreducible representations of G, since the central involution acts trivially.

"Presentation" G ≅ < | more relations >

(The leftmost node is redundant. See page 232.)



$B = F_{2+}$

	8A	8B	8C	8D	8E	8F	8G	8H	8I	8J	8K	8L	8M	8N	9A	9B	10A	10B	10C	10D	10E	10F	11A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	12K	12L	12M	12N	12O	12P	12Q	12R	12S	12T	13A	14A	14B	14C	14D	14E	
	8	8	8	8	8	8	8	8	8	8	8	8	8	8	9	9	10	10	20	10	10	10	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	24	13	14	14	28	14	14	
											8				18	18		10		10		10	22						12				12				12					12		26				14		
χ_{185}	0	0	0	0	0	0	0	0	0	0	16	0	0	0	1	10	0	8	0	2	0	10	6	0	0	0	0	0	4	0	0	0	8	0	0	0	2	0	0	0	0	6	0	4	0	0	0	4	0	χ_{185}
χ_{186}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	0	18	0	16	0	-6	0	10	8	0	0	0	0	0	12	0	0	0	0	0	0	0	-6	0	0	0	0	6	0	4	0	0	0	4	0	χ_{186}
χ_{187}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	-4	32	0	16	0	4	0	20	10	0	0	0	0	0	4	0	0	0	8	0	0	0	2	0	0	0	0	6	0	4	0	0	0	0	0	χ_{187}
χ_{188}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	3	30	0	16	0	-6	0	10	6	0	0	0	0	0	-8	0	0	0	8	0	0	0	8	0	0	0	0	0	0	0	0	0	0	4	0	χ_{188}
χ_{189}	0	0	0	0	0	0	0	0	0	0	48	0	0	0	0	18	0	24	0	6	0	-10	2	0	0	0	0	0	48	0	0	0	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	χ_{189}
χ_{190}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	2	0	-8	0	8	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{190}
χ_{191}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	2	0	-8	0	8	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{191}
χ_{192}	0	0	0	0	0	0	0	0	0	0	48	0	0	0	1	10	0	-32	0	12	0	20	0	0	0	0	0	0	-32	0	0	0	8	0	0	0	-16	0	0	0	0	0	0	-4	0	0	0	-4	0	χ_{192}
χ_{193}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	9	0	0	-48	0	-12	0	20	-6	0	0	0	0	0	-8	0	0	0	8	0	0	0	8	0	0	0	0	0	0	0	0	0	0	-16	0	χ_{193}
χ_{194}	0	0	0	0	0	0	0	0	0	0	16	0	0	0	3	12	0	48	0	2	0	-30	-2	0	0	0	0	0	4	0	0	0	-16	0	0	0	-10	0	0	0	0	-6	0	0	0	0	0	8	0	χ_{194}
χ_{195}	0	0	0	0	0	0	0	0	0	0	16	0	0	0	-12	-12	0	56	0	-6	0	10	-10	0	0	0	0	0	40	0	0	0	56	0	0	0	8	0	0	0	0	0	0	0	0	0	0	4	0	χ_{195}
χ_{196}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-12	-12	0	-32	0	-18	0	-10	0	0	0	0	0	0	-80	0	0	0	-16	0	0	0	32	0	0	0	0	0	0	-4	0	0	0	8	0	χ_{196}
χ_{197}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	8	-28	0	16	0	14	0	30	0	0	0	0	0	0	-24	0	0	0	0	0	0	0	-24	0	0	0	0	0	0	-4	0	0	0	16	0	χ_{197}
χ_{198}	0	0	0	0	0	0	0	0	0	0	-64	0	0	0	0	0	0	-32	0	12	0	-20	2	0	0	0	0	0	12	0	0	0	-24	0	0	0	-18	0	0	0	0	-6	0	0	0	0	0	-4	0	χ_{198}
χ_{199}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-29	-2	0	0	0	10	0	10	0	0	0	0	0	0	16	0	0	0	-16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	4	0	χ_{199}
χ_{200}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-29	-2	0	0	0	10	0	10	0	0	0	0	0	0	16	0	0	0	-16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	4	0	χ_{200}
χ_{201}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	27	0	0	0	0	30	0	-10	-6	0	0	0	0	0	40	0	0	0	56	0	0	0	8	0	0	0	0	0	0	0	0	0	0	-4	0	χ_{201}
χ_{202}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	-13	-22	0	-32	0	-8	0	0	2	0	0	0	0	0	4	0	0	0	8	0	0	0	2	0	0	0	0	6	0	0	0	0	0	4	0	χ_{202}
χ_{203}	0	0	0	0	0	0	0	0	0	0	-16	0	0	0	9	0	0	96	0	4	0	-20	0	0	0	0	0	0	136	0	0	0	8	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	χ_{203}
χ_{204}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	0	-18	0	-80	0	0	0	0	-2	0	0	0	0	0	60	0	0	0	-24	0	0	0	-6	0	0	0	0	6	0	0	0	0	0	-12	0	χ_{204}
χ_{205}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	10	0	-40	0	0	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	8	0	χ_{205}
χ_{206}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	10	0	-40	0	0	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	8	0	χ_{206}
χ_{207}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-9	-18	0	24	0	26	0	10	0	0	0	0	0	0	-16	0	0	0	-8	0	0	0	-32	0	0	0	0	0	0	0	0	0	0	-16	0	χ_{207}
χ_{208}	0	0	0	0	0	0	0	0	0	0	-16	0	0	0	25	-2	0	72	0	28	0	20	0	0	0	0	0	0	-116	0	0	0	-16	0	0	0	14	0	0	0	0	-6	0	0	0	0	0	4	0	χ_{208}
χ_{209}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	-8	-8	0	80	0	0	0	0	-8	0	0	0	0	0	60	0	0	0	-24	0	0	0	-6	0	0	0	0	6	0	4	0	0	0	-4	0	χ_{209}
χ_{210}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	-24	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	0	χ_{210}
χ_{211}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	-24	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	0	χ_{211}
χ_{212}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	48	12	0	-40	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{212}
χ_{213}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	48	12	0	-40	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{213}
χ_{214}	0	0	0	0	0	0	0	0	0	0	-48	0	0	0	-8	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0	0	12	0	0	0	0	12	0	-4	0	0	0	0	0	χ_{214}
χ_{215}	0	0	0	0	0	0	0	0	0	0	-48	0	0	0	-8	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	0	0	0	12	0	0	0	0	12	0	-4	0	0	0	0	0	χ_{215}
χ_{216}	0	0	0	0	0	0	0	0	0	0	-96	0	0	0	0	0	0	-16	0	16	0	0	0	0	0	0	0	0	-48	0	0	0	24	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	χ_{216}
χ_{217}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	0	0	0	0	0	-30	0	10	0	0	0	0	0	0	-36	0	0	0	0	0	0	0	-18	0	0	0	0	-6	0	0	0	0	0	0	0	χ_{217}
χ_{218}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	0	0	0	0	0	-30	0	10	0	0	0	0	0	0	-36	0	0	0	0	0	0	0	-18	0	0	0	0	-6	0	0	0	0	0	0	0	χ_{218}
χ_{219}	0	0	0	0	0	0	0	0	0	0	-80	0	0	0	0	0	0	-40	0	20	0	20	6	0	0	0	0	0	36	0	0	0	48	0	0	0	6	0	0	0	0	-6	0	4	0	0	0	0	0	χ_{219}
χ_{220}	0	0	0	0	0	0	0	0	0	0	32	0	0	0	0	-18	0	0	0	-10	0	-10	10	0	0	0	0	0	-84	0	0	0	48	0	0	0	-6	0	0	0	0	6	0	0	0	0	0	0	0	χ_{220}
χ_{221}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	0	0	0	-30	0	10	0	0	0	0	0	0	-48	0	0	0	-48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{221}
χ_{222}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39	-6	0	32	0	28	0	20	0	0	0	0	0	0	-64	0	0	0	-32	0	0	0	16	0	0	0	0	0	0	0	0	0	0	-4	0	χ_{222}
χ_{223}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	χ_{223}
χ_{224}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	χ_{224}
χ_{225}	0	0	0	0	0	0	0	0	0	0	16	0	0	0	0	-18	0	16	0	-26	0	-10	0	0	0	0	0	0	36	0	0	0	48	0	0	0	6	0	0	0	0	-6	0	0	0	0	0	-4	0	χ_{225}
χ_{226}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27	0	0	0	0	-20	0	20	0	0	0	0	0	0	64	0	0	0	32	0	0	0	-16	0	0	0	0	0	0	-4	0	0	0	4	0	χ_{226}
χ_{227}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-8	0	-42	0	-10	8	0	0	0	0	0	48	0	0	0	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{227}
χ_{228}	0	0	0	0	0	0	0	0	0	0	-16	0	0	0	-27	0	0	-40	0	30	0	30	0	0	0	0	0	0	140	0	0	0	-8	0	0	0	-2	0	0	0	0	-6	0	0	0	0	0	0	0	χ_{228}
χ_{229}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	-32	4	0	0	0	0	0	0	0	0	0	0	0	0	-60	0	0	0	24	0	0	0	6	0	0	0	0	-6	0	0	0	0	0	-4	0	χ_{229}
χ_{230}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	-32	4	0	0	0	0	0	0	0	0	0	0	0	0	-60	0	0	0	24	0	0	0	6	0	0	0	0	-6	0	0	0	0	0	-4	0	χ_{230}
χ_{231}	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	-96	0	66	0	-30	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	χ_{231}
χ_{232}	0	0	0	0	0	0	0	0	0	0	-32	0	0	0	7	-2	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	0	-8	0	0	0	-14	0	0	0	0	6	0	0	0	0	0	0	0	χ_{232}
χ_{233}	0	0	0	0	0	0	0	0	0	0	-32	0	0	0	7	-2	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	0	-8	0	0	0	-14	0	0	0	0	6	0	0	0	0	0	0	0	χ_{233}
χ_{234}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	28	28	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0	8	0	0	0	-10	0	0	0	0	-6	0	-4	0	0	0	4	0	χ_{234}
χ_{235}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	0	-56	0	-24	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{235}
χ_{236}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	0	-56	0	-24	0	0	-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{236}
χ_{237}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	20	0	64	0	16	0	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{237}
χ_{238}	0	0	0	0	0	0	0	0	0	0	48	0	0	0	0	0	0	24	0	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	χ_{238}
χ_{239}	0	0	0	0	0	0	0	0	0	0	48	0	0	0	0	0	0	24	0	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	χ_{239}
χ_{240}	0	0	0	0	0	0	0	0	0	0	-80	0	0	0	-6	30	0	0	0	0	0	0	0	0	0	0	0	0	-60	0	0	0	24	0	0	0	6	0	0	0	0	-6	0	0	0	0	0	-8	0	χ_{240}
χ_{241}	0	0	0	0	0	0	0	0	0	0	64	0	0	0	0	0	0	16	0	24	0	0	-10	0	0	0	0	0	-60	0	0	0	-24	0	0	0	18	0	0	0	0	6	0	0	0	0	0	-4	0	χ_{241}
χ_{242}	0	0	0	0	0	0	0	0	0	0	-16	0	0	0	-8	-8	0	0	0	0	0	0	10	0	0	0	0	0	60	0	0	0	-24	0	0	0	30	0	0	0	0	-6	0	-4	0	0	0	0	0	χ_{242}
χ_{243}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	-32	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	χ_{243}
χ_{244}	0	0	0	0	0	0	0	0	0	0	-64	0	0	0	-7	20	0	0	0	0	0	0	0	0	0	0	0	0	-20	0	0	0	8	0	0	0	-22	0	0	0	0	6	0	-4	0	0	0	0	0	χ_{244}
χ_{245}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-18	0	-32	0	-48	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	8	0	χ_{245}
χ_{246}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-13	-4	0	0	0	20	0	20	0	0	0	0	0	0	64	0	0	0	-16	0	0	0	32	0	0	0	0	0	0	0	0	0	0	0	0	χ_{246}
χ_{247}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	6	0	32	0	28	0	-20	0	0	0	0	0	0	-64	0	0	0	16	0	0	0	-32	0	0	0	0	0	0	0	0	0	0	0	0	χ_{247}

Maximal p-local subgroups (complete for $p \neq 2$)

p	Structure	Specification	Supergroups
2	$2^{\cdot}(^2E_6(2)):2$	N(2A)	
	$(2^2 \times F_4(2)):2$	N(2C)	
	$S_4 \times {}^2F_4(2)$	$N(2C^2)$	
	$2_+^{1+22}.Co_2$	N(2B)	
	$2^2.2^{10}.2^{20}.(M_{22}:2 \times S_3)$	$N(2B^2)$	
	$2^3.[2^{32}].(S_5 \times L_3(2))$	$N(2B^3)$	
	$2^5.[2^{25}].L_5(2)$	$N(2B^5)$	
	$2^9.2^{16}.S_8(2)$	$N(2B^8)$	
3	$S_3 \times Fi_{22}:2$	N(3A)	
	$(3^2:D_8 \times U_4(3).2^2).2$	$N(3A^2)$	
	$3_+^{1+8}:2_-^{1+6}.U_4(2).2$	N(3B)	
	$3^2.3^3.3^6.(S_4 \times 2S_4)$	$N(3B^2)$	
	$3^3.[3^7].(L_3(3) \times 2)$	$N(3B^3)$	Fi_{23}
	$3^3.[3^6].(L_3(3) \times D_8)$	$N(3B^3)$	$O_8^+(3):S_4$
	$3^6:(2 \times L_4(3).2^2)$	$N(3^6) = N(3A_{234}B_{130})$	$O_8^+(3):S_4$
5	$5:4 \times HS:2$	N(5A)	
	$5_+^{1+4}:2_-^{1+4}.A_5.4$	N(5B)	
	$5^2:4S_4 \times S_5$	$N(5A^2)$	
	$5^{3\cdot}L_3(5)$	$N(5B^3)$	
7	$(7:3 \times 2^{\cdot}L_3(4).2).2$	N(7A)	$2^{\cdot}(^2E_6(2)):2$
	$(2^2 \times 7^2:(3 \times 2A_4)).2$	$N(7A^2)$	$(2^2 \times F_4(2)):2$
11	$11:10 \times S_5$	N(11A)	$S_5 \times M_{22}:2$
13	$13:12 \times S_4$	N(13A)	$S_4 \times {}^2F_4(2)$
17	$(17:8 \times 2^2)^{\cdot}2$	N(17A)	$(2^2 \times F_4(2)):2$
19	$19:18 \times 2$	N(19A)	$2^{\cdot}(^2E_6(2)):2$
23	$23:11 \times 2$	N(23AB)	$2_+^{1+22}:Co_2$
31	31:15	N(31AB)	Th
47	47:23	N(47AB)	

Some non-local maximal subgroups

Th

HN:2

Fi_{23}

$O_8^+(3):S_4$

$S_5 \times M_{22}:2$

$O_8^+(3):S_4$

$B = F_{2+}$

	15A	15B	16A	16B	16C	16D	16E	16F	16G	16H	17A	18A	18B	18C	18D	18E	18F	19A	20A	20B	20C	20D	20E	20F	20G	20H	20I	20J	21A	22A	22B	23A	B**	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	24K	24L	24M	24N	25A	26A	26B	27A	
	15 30	15 30	16	16	16	16	16	16	16	16 16	17 34	18	18	18 18	18	18 18	36	19 38	20	20	20	20	20	20	20 20	40	20	20	21 42	22 22	22 22	23 46	23 46	24	24	24	24	24	24	24	24	24	24	24	24 24	24	24	25 50	52	26	27 54	
x_{185}	2	3	0	0	0	0	0	0	0	4	2	0	0	2	0	3	0	2	0	0	0	0	0	0	2	0	0	0	2	0	2	1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	1	x_{185}
x_{186}	6	2	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	0	0	0	2	0	0	0	-2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{186}
x_{187}	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	2	0	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	-1	x_{187}
x_{188}	-4	0	0	0	0	0	0	0	0	0	-2	0	0	-2	0	-3	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	-1	0	0	0	x_{188}
x_{189}	16	-2	0	0	0	0	0	0	0	-4	2	0	0	2	0	0	0	-2	0	0	0	0	0	0	-2	0	0	0	-4	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{189}
x_{190}	-2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	111	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	x_{190}
x_{191}	-2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-111	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	x_{191}
x_{192}	-2	0	0	0	0	0	0	0	0	4	-2	0	0	-6	0	3	0	0	0	0	0	0	0	0	4	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	x_{192}
x_{193}	-8	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	-4	0	0	0	4	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	1	0	0	0	x_{193}
x_{194}	-8	-1	0	0	0	0	0	0	0	-4	0	0	0	4	0	-3	0	0	0	0	0	0	0	0	2	0	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	0	x_{194}
x_{195}	2	-2	0	0	0	0	0	0	0	4	0	0	0	4	0	0	0	0	0	0	0	0	0	0	-6	0	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	x_{195}
x_{196}	6	2	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{196}
x_{197}	-2	1	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	-1	x_{197}
x_{198}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-4	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{198}
x_{199}	0	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	-3	0	0	0	0	0	0	0	0	-2	0	0	0	2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{199}
x_{200}	0	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	-3	0	0	0	0	0	0	0	0	-2	0	0	0	2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{200}
x_{201}	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	6	0	0	0	2	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	1	0	0	0	x_{201}
x_{202}	-4	-1	0	0	0	0	0	0	0	0	0	0	0	2	0	-3	0	0	0	0	0	0	0	0	8	0	0	0	4	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	-1	x_{202}
x_{203}	16	-1	0	0	0	0	0	0	0	4	0	0	0	0	0	3	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	4	0	0	-1	0	0	0	x_{203}
x_{204}	20	0	0	0	0	0	0	0	0	0	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{204}
x_{205}	10	0	0	0	0	0	0	0	0	0	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{205}
x_{206}	10	0	0	0	0	0	0	0	0	0	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{206}
x_{207}	16	3	0	0	0	0	0	0	0	0	-2	0	0	6	0	-3	0	0	0	0	0	0	0	0	2	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	x_{207}
x_{208}	16	1	0	0	0	0	0	0	0	-4	0	0	0	-2	0	3	0	0	0	0	0	0	0	0	-4	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	1	x_{208}
x_{209}	-20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	1	x_{209}
x_{210}	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{210}
x_{211}	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{211}
x_{212}	-6	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2111	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	x_{212}
x_{213}	-6	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2111	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	x_{213}
x_{214}	0	0	0	0	0	0	0	0	0	-4	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b23	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{214}
x_{215}	0	0	0	0	0	0	0	0	0	-4	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{215}
x_{216}	-14	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{216}
x_{217}	0	-1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{217}
x_{218}	0	-1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{218}
x_{219}	12	-3	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	-1	0	0	0	x_{219}
x_{220}	0	-3	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{220}
x_{221}	0	2	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	-2	0	0	0	0	0	0	2	0	0	0	2	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{221}
x_{222}	2	-3	0	0	0	0	0	0	0	0	0	0	0	2	0	-3	0	2	0	0	0	0	0	0	4	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{222}
x_{223}	8	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	x_{223}
x_{224}	8	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	-1	x_{224}
x_{225}	-4	5	0	0	0	0	0	0	0	-4	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	x_{225}
x_{226}	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	-4	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{226}
x_{227}	-2	-2	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	2	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{227}
x_{228}	-14	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	3	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	1	0	0	0	x_{228}
x_{229}	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	x_{229}
x_{230}	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	x_{230}
x_{231}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{231}
x_{232}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b23	**	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	x_{232}
x_{233}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b23	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	x_{233}
x_{234}	0	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	x_{234}
x_{235}	-2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	111	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{235}
x_{236}	-2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-111	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{236}
x_{237}	-4	0	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	x_{237}
x_{238}	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	x_{238}
x_{239}	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	x_{239}
x_{240}	0	0	0	0	0	0	0	0	0	4	2	0	0	-2	0	-6	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	x_{240}
x_{241}	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	x_{241}
x_{242}	0	0	0	0	0	0	0	0	0	4	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	1	x_{242}
x_{243}	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	x_{243}
x_{244}	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	-1	x_{244}
x_{245}	14	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{245}
x_{246}	0	1	0	0	0	0	0	0	0	0	0	0	0	-4	0	-3	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	x_{246}
x_{247}	2	-1	0	0	0	0	0	0	0	0	0	0	0	-2	0	3	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	x_{247}

	28A	28B	28C	28D	28E	30A	30B	30C	30D	30E	30F	30G	H**	31A	B**	32A	B*	32C	D**	33A	34A	34B	C*	35A	36A	36B	36C	38A	39A	40A	40B	40C	40D	40E	42A	42B	42C	44A	46A	B**	47A	B**	48A	48B	52A	55A	56A	B*	60A	60B	60C	66A	70A	
	28	28	28	28	28 28	30	30	60	30 30	30	30 30	30 30	30 30	31 62	31 62	32	32	32	32	33 66	68	34	34	35 70	36	36	36 36	38 38	39 78	40	40	40	40	40 40	42	42	84	44 44	46 46	46 46	47 94	47 94	48	48	104 104	55 110	56	56	60	60	60 60	66 66	70 70	
χ_{185}	0	0	0	0	2	0	0	0	2	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	χ_{185}
χ_{186}	0	0	0	0	-2	0	0	0	-2	0	-2	0	0	-1	-1	0	0	0	0	2	0	0	0	2	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	-2	0	0	0	0	-2	0	0	χ_{186}
χ_{187}	0	0	0	0	0	0	0	0	-2	0	-1	-1	-1	0	0	0	0	0	0	-2	0	0	0	-2	0	0	-2	0	1	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	1	0	0	χ_{187}
χ_{188}	0	0	0	0	-2	0	0	0	4	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	-2	0	0	χ_{188}
χ_{189}	0	0	0	0	2	0	0	0	0	0	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	2	0	0	0	0	2	0	0	χ_{189}
χ_{190}	0	0	0	0	0	0	0	0	-2	0	0	-2	-2	1	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	i11	1	1	1	1	0	0	0	-1	0	0	0	0	0	i11	0	χ_{190}
χ_{191}	0	0	0	0	0	0	0	0	-2	0	0	-2	-2	1	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-i11	1	1	1	1	0	0	0	-1	0	0	0	0	0	-i11	0	χ_{191}
χ_{192}	0	0	0	0	2	0	0	0	-2	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	χ_{192}
χ_{193}	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	0	0	0	-2	0	0	χ_{193}
χ_{194}	0	0	0	0	0	0	0	0	0	0	-3	1	1	0	0	0	0	0	0	-2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	1	0	0	χ_{194}
χ_{195}	0	0	0	0	2	0	0	0	2	0	-2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{195}
χ_{196}	0	0	0	0	0	0	0	0	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	-2	0	2	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{196}
χ_{197}	0	0	0	0	0	0	0	0	-2	0	3	1	1	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{197}
χ_{198}	0	0	0	0	-2	0	0	0	-2	0	-2	0	0	1	1	0	0	0	0	2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	-2	0	0	χ_{198}
χ_{199}	0	0	0	0	2	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2i5	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	-1	0	0	χ_{199}
χ_{200}	0	0	0	0	2	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-2i5	0	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	-1	0	0	χ_{200}
χ_{201}	0	0	0	0	2	0	0	0	-6	0	2	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	χ_{201}
χ_{202}	0	0	0	0	-2	0	0	0	4	0	-3	-1	-1	0	0	0	0	0	0	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	1	0	0	χ_{202}
χ_{203}	0	0	0	0	0	0	0	0	0	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{203}
χ_{204}	0	0	0	0	-2	0	0	0	4	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	-2	0	0	0	0	0	0	0	χ_{204}
χ_{205}	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	0	0	0	0	0	135	χ_{205}
χ_{206}	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	0	0	0	0	0	-135	χ_{206}
χ_{207}	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	2	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{207}
χ_{208}	0	0	0	0	-2	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{208}
χ_{209}	0	0	0	0	-2	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	χ_{209}
χ_{210}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	i19	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	1	0	0	0	0	0	0	0	χ_{210}
χ_{211}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	-i19	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	1	0	0	0	0	0	0	0	χ_{211}
χ_{212}	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	1	0	0	0	0	0	-i11	0	χ_{212}
χ_{213}	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	1	0	0	0	0	0	i11	0	χ_{213}
χ_{214}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	b23	**	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{214}
χ_{215}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	**	b23	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{215}
χ_{216}	0	0	0	0	0	0	0	0	2	0	3	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	χ_{216}
χ_{217}	0	0	0	0	0	0	0	0	0	0	1	i15	-i15	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	1	0	0	χ_{217}
χ_{218}	0	0	0	0	0	0	0	0	0	0	1	-i15	i15	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	1	0	0	χ_{218}
χ_{219}	0	0	0	0	0	0	0	0	-4	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1	0	0	χ_{219}
χ_{220}	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{220}
χ_{221}	0	0	0	0	4	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	-2	0	0	χ_{221}
χ_{222}	0	0	0	0	2	0	0	0	2	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{222}
χ_{223}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	i26	0	0	0	0	0	0	0	0	χ_{223}
χ_{224}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	-i26	0	0	0	0	0	0	0	0	χ_{224}
χ_{225}	0	0	0	0	2	0	0	0	4	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{225}
χ_{226}	0	0	0	0	-2	0	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	χ_{226}
χ_{227}	0	0	0	0	0	0	0	0	-2	0	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	2	0	0	χ_{227}
χ_{228}	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{228}
χ_{229}	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b47	**	0	0	0	0	0	0	0	0	0	0	0	0	χ_{229}
χ_{230}	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b47	0	0	0	0	0	0	0	0	0	0	0	χ_{230}
χ_{231}	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	1	0	0	0	0	0	0	0	χ_{231}
χ_{232}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	-b23	**	1	1	0	0	0	0	0	0	0	0	0	0	0	χ_{232}
χ_{233}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	**-b23		1	1	0	0	0	0	0	0	0	0	0	0	0	χ_{233}
χ_{234}	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{234}
χ_{235}	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0-i11	0	0	-1	-1	0	0	0	0	0	0	0	0	0	i11	0	χ_{235}
χ_{236}	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 i11	0	0	-1	-1	0	0	0	0	0	0	0	0	0	-i11	0	χ_{236}
χ_{237}	0	0	0	0	0	0	0	0	4	0	0	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	-1	0	0	0	0	0	0	0	χ_{237}
χ_{238}	0	0	0	0	2	0	0	0	0	0	0	0	0	b31	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{238}
χ_{239}	0	0	0	0	2	0	0	0	0	0	0	0	0	**	b31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{239}
χ_{240}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{240}
χ_{241}	0	0	0	0	-2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	2	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{241}
χ_{242}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{242}
χ_{243}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	2	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	-2	0	0	0	0	0	0	0	χ_{243}
χ_{244}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	χ_{244}
χ_{245}	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	χ_{245}
χ_{246}	0	0	0	0	0	0	0	0	0	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	χ_{246}
χ_{247}	0	0	0	0	0	0	0	0	2	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	χ_{247}

Chevalley group $E_7(2)$

Order = 7,997,476,042,075,799,759,100,487,262,680,802,918,400

$= 2^{63}.3^{11}.5^{2}.7^{3}.11.13.17.19.31.43.73.127$

Mult = 1 Out = 1

Constructions

Chevalley G : adjoint Chevalley group of type E_7 over the field $\mathbb{F}_2$; G has a 56-dimensional representation over $\mathbb{F}_2$ which supports a non-trivial quartic invariant.

Maximal 2-local subgroups

$[2^{33}]:O^+_{12}(2)$

$[2^{47}]:(S_3 x L_6(2))$

$[2^{53}]:(S_3 x L_3(2) x A_8)$

$[2^{50}]:(L_3(2) x L_5(2))$

$[2^{42}]:(S_3 x O^+_{10}(2))$

$[2^{27}]:E_6(2)$

$[2^{42}]:L_7(2)$

Some other subgroups

$L_8(2):2$

$U_8(2):2$

$E_6(2):2$

$3^{\cdot}(^2E_6(2)):S_3$

$S_3 \times O^+_{12}(2)$

$(L_3(2) \times L

M = F1

| | | |

$M = F_1$

	9A	9B	10A	10B	10C	10D	10E	11A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	13A	13B	14A	14B	14C	15A	15B	15C	15D	16A	16B	16C	17A	18A	18B	18C	18D	18E	19A	20A	20B	20C	20D	20E	20F	21A	21B	21C	21D	22A	22B	23A	B**	
x_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	x_1
x_2	26	-1	21	5	-4	20	0	16	14	5	6	-1	-2	5	-3	13	1	-1	11	-2	10	2	9	7	-2	8	-1	3	-1	7	6	-3	6	2	-1	5	5	5	1	0	-3	2	4	5	-2	-1	4	4	0	3	3	x_2
x_3	59	5	34	-14	9	41	1	29	17	-10	-7	3	1	-6	2	22	-2	0	16	3	9	-7	14	14	5	10	-2	-4	0	8	7	3	3	-5	1	7	4	2	-2	-3	2	-3	5	0	0	3	3	1	-3	3	3	x_3
x_4	130	-5	99	19	-1	79	-1	53	40	-5	8	-8	8	15	-1	35	-1	0	26	0	26	10	17	12	-6	14	2	6	2	10	8	1	10	2	-1	5	5	3	7	3	3	3	3	3	3	-3	3	5	5	2	2	x_4
x_5	194	5	166	54	16	104	4	66	50	14	42	8	2	30	6	38	2	0	28	2	38	14	14	8	8	13	-2	12	0	8	6	5	14	10	1	1	4	6	2	-4	6	6	0	6	-1	3	2	6	2	0	0	x_5
x_6	266	-4	120	-40	-5	155	-5	91	74	20	-14	-7	-6	-20	4	60	0	1	39	0	16	-16	26	7	-2	18	3	-9	-1	15	8	2	2	-6	0	6	6	8	-8	3	0	-5	3	10	3	-2	3	-1	-5	0	0	x_6
x_7	339	-12	175	15	-25	175	-5	99	14	-31	14	20	-18	5	5	57	-3	-4	32	-7	20	4	25	35	8	13	1	2	-2	6	7	-6	3	3	0	6	0	-1	-1	-1	-9	1	3	-10	-3	5	0	-1	3	1	1	x_7
x_8	714	12	265	-55	-10	350	10	176	79	-29	-49	-20	15	-29	-13	99	3	4	52	0	9	-7	35	23	-13	17	-1	-10	2	10	6	-6	-6	-6	0	6	0	-7	1	-2	1	-4	2	-8	-1	-5	-1	-4	0	0	0	x_8
x_9	729	0	484	116	34	306	-14	145	138	66	34	0	-6	54	-2	66	-6	0	39	0	51	27	13	-1	-1	9	0	15	-5	3	0	0	9	9	0	0	0	-12	8	-2	-4	6	-2	9	-5	0	-2	5	1	-1	-1	x_9
x_{10}	923	5	715	-5	-10	350	10	173	134	-55	14	8	-10	21	5	65	5	0	43	4	81	-7	0	-21	15	10	-2	3	3	3	0	1	19	-5	1	-11	1	11	-1	6	-5	0	-6	0	0	3	0	5	-3	-2	-2	x_{10}
x_{11}	351	27	-55	201	20	156	16	55	-136	-1	56	8	-8	51	11	31	7	0	-1	12	-29	43	22	43	7	-3	0	15	3	-5	0	9	-9	15	3	9	-5	-7	1	8	1	6	4	-14	7	1	-2	-5	7	0	0	x_{11}
x_{12}	672	-3	-154	-106	46	294	-6	110	-2	70	-42	55	14	-70	-6	62	2	-1	13	0	-70	-22	20	47	20	0	3	-15	1	1	0	3	-24	0	-3	3	-4	6	-6	6	6	-4	2	-2	-2	7	-2	-10	-2	0	0	x_{12}
x_{13}	559	19	-286	-110	-11	245	5	55	269	80	-83	35	13	-56	16	40	4	-5	13	0	-54	0	0	-49	23	-2	-2	-11	1	9	-8	-3	-21	-9	3	-3	4	2	6	-3	2	-3	-3	35	0	8	0	-5	7	-3	-3	x_{13}
x_{14}	398	20	945	145	-55	65	5	61	101	101	141	56	21	69	-3	-3	9	0	12	-1	120	8	-13	49	13	3	6	14	2	-6	1	-10	26	6	0	-6	0	1	1	1	-7	3	-3	15	8	6	-1	9	-3	0	0	x_{14}
x_{15}	1325	-25	496	16	-4	476	-4	176	56	-16	56	-64	-8	0	-16	64	-8	0	26	0	9	-7	0	-63	-36	-1	-4	0	0	0	0	-6	1	1	5	-1	-7	-1	16	-8	-4	0	0	-4	0	0	-9	0	-4	0	-3	-3	x_{15}
x_{16}	-5	22	0	0	0	0	0	0	5	-40	-35	-7	21	0	-8	0	-12	-15	1	-12	0	0	0	0	0	0	0	-1	-1	-1	0	8	-1	3	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	b23	**	x_{16}
x_{17}	-5	22	0	0	0	0	0	0	5	-40	-35	-7	21	0	-8	0	-12	-15	1	-12	0	0	0	0	0	0	0	-1	-1	-1	0	8	-1	3	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	b23	x_{17}
x_{18}	1313	-10	560	-176	-40	384	4	136	-87	30	-55	-36	9	-46	26	30	6	-4	0	0	10	-38	4	70	-11	-6	-3	-8	0	-16	0	6	-3	-7	2	2	-6	-32	0	-12	8	-2	0	-15	6	-3	-3	4	-8	3	3	x_{18}
x_{19}	1085	5	505	105	80	280	-20	99	-260	-179	76	28	28	49	-23	13	-11	4	-13	0	21	21	0	48	3	-7	5	9	-3	-11	0	5	5	-3	-3	-3	-4	9	5	4	17	2	0	-21	0	0	0	-1	3	0	0	x_{19}
x_{20}	726	-30	-365	275	-115	245	-15	22	79	-110	-49	34	15	70	14	18	-6	10	-11	-11	-56	56	0	-41	22	-8	-2	14	-6	2	-6	0	-18	6	-6	0	0	-13	19	-3	3	3	1	-14	0	7	0	-2	6	0	0	x_{20}
x_{21}	521	8	1925	325	0	0	0	25	314	170	98	-34	42	134	-10	-30	6	-10	13	0	135	47	-30	-41	-5	0	0	12	8	0	-1	-6	21	1	4	-2	0	5	21	0	5	0	0	16	2	-5	-1	5	9	1	1	x_{21}
x_{22}	949	4	-1485	195	-10	350	10	-17	115	16	35	-41	-45	0	8	24	12	-9	-26	0	-196	28	0	-64	-10	-15	3	5	5	5	-7	-4	-31	13	4	4	0	3	-9	-2	-21	4	2	14	0	-7	0	-5	-1	0	0	x_{22}
x_{23}	1235	20	1365	-475	90	170	10	66	41	-31	-239	56	-39	-99	13	-39	-3	0	0	0	80	-40	-33	-27	27	-3	6	-22	-2	-10	0	6	15	-21	0	-6	0	-11	1	14	-11	-6	-2	-9	-2	6	1	6	2	2	2	x_{23}
x_{24}	792	-18	519	135	-106	70	-10	-22	-67	284	85	14	-51	56	-32	-36	-12	14	-31	-18	26	-14	0	98	-19	-9	-3	8	-8	-8	0	-6	12	0	-6	6	0	7	-9	2	7	2	2	35	0	2	0	2	-6	0	0	x_{24}
x_{25}	221	-49	-1100	-220	-100	20	0	0	-916	65	-36	71	44	-15	9	-7	5	-9	-52	0	-66	-10	39	175	22	-8	-5	-3	9	-15	6	-17	1	-11	7	7	-5	20	8	0	-12	-2	4	-15	-8	3	1	0	0	0	0	x_{25}
x_{26}	0	0	-99	45	-24	0	60	0	0	0	0	-27	0	0	0	0	0	-3	0	0	9	-7	0	0	0	0	12	-3	-7	1	0	0	0	0	0	0	-1	-3	9	12	-3	2	0	0	0	3	0	0	0	0	0	x_{26}
x_{27}	0	0	-99	45	-24	0	60	0	0	0	0	-27	0	0	0	0	0	-3	0	0	9	-7	0	0	0	0	12	-3	-7	1	0	0	0	0	0	0	-1	-3	9	12	-3	2	0	0	0	3	0	0	0	0	0	x_{27}
x_{28}	729	0	1650	50	0	0	0	-55	660	-60	76	0	36	36	4	-60	-12	0	15	15	123	-21	-35	-148	5	0	0	5	5	5	-8	0	9	9	9	0	0	6	-30	-22	0	10	0	0	9	-5	0	1	5	-7	0	x_{28}
x_{29}	476	-10	3190	230	40	-240	20	55	-288	-180	-48	-36	0	56	24	-60	24	-4	0	0	188	52	-32	34	-29	6	-3	0	0	0	6	2	20	-4	2	-10	0	38	2	-12	-2	-2	0	-21	-7	-3	2	-5	7	0	0	x_{29}
x_{30}	351	27	-1475	-275	0	0	0	-66	125	-145	77	8	29	-141	-13	-25	11	0	-13	0	-108	-44	-30	-68	-5	0	0	-11	-3	13	0	-9	-9	15	3	-9	4	45	-7	0	5	0	0	-8	6	1	3	-6	-2	0	0	x_{30}
x_{31}	-51	57	-3200	0	0	0	0	-119	155	245	-77	-118	11	-35	13	5	-19	10	-32	20	-196	28	0	104	-40	0	0	1	-3	5	7	9	-27	-3	-3	-3	1	0	16	0	0	0	0	14	0	-7	0	1	9	3	3	x_{31}
x_{32}	-265	-49	-2695	249	130	154	14	-128	1096	34	-184	91	-86	70	-58	26	14	-5	0	0	-70	42	0	93	-6	-1	2	9	5	13	-6	1	-17	-9	3	-3	5	-23	1	2	-15	0	-2	42	0	0	0	0	0	0	0	x_{32}
x_{33}	-90	72	-1000	504	150	14	34	-99	-917	-170	211	-77	27	130	-14	-18	18	-5	-39	0	-19	5	40	-69	-6	-6	6	13	1	-7	0	-12	6	6	0	12	0	-24	-8	6	0	0	2	-29	-8	-2	1	1	-3	0	0	x_{33}
x_{34}	395	-10	3565	445	190	-210	30	29	1158	285	510	-36	54	41	33	-3	21	-4	0	0	147	-5	-5	246	-15	0	-3	0	0	0	0	22	-5	11	2	2	0	13	9	18	13	-2	2	51	-12	-3	-3	1	-3	0	0	x_{34}
x_{35}	36	63	770	-30	70	-50	10	0	716	554	484	14	28	-34	-2	34	22	14	-13	0	30	22	-6	97	7	-3	-3	-7	13	5	0	-9	0	12	3	3	0	34	26	14	-14	-4	2	-1	6	2	-3	0	0	0	0	x_{35}
x_{36}	-416	-38	-1000	600	0	0	0	-69	-467	-80	245	160	-3	120	56	0	-24	0	0	0	-79	25	-10	-190	80	0	0	8	0	16	0	4	4	8	2	-4	0	40	0	0	0	0	0	-26	9	13	3	-9	-5	-1	-1	x_{36}
x_{37}	-351	-27	-1750	-550	0	0	0	-176	721	100	-167	-8	97	-84	4	-20	16	0	16	-10	-55	-7	0	-175	5	0	0	-10	2	10	-7	9	9	-15	-3	9	5	10	30	0	10	0	0	35	0	-1	0	4	0	-2	-2	x_{37}
x_{38}	233	-10	3760	-240	-15	-455	-35	22	-536	76	56	-20	-120	-56	-16	-84	-12	4	0	0	171	-21	0	91	19	9	-3	-6	-2	-10	8	4	13	1	-2	4	0	-16	-8	-11	-16	-1	1	0	0	-3	0	-2	6	0	0	x_{38}
x_{39}	-61	47	231	-185	-69	35	-25	0	71	-37	119	35	103	35	-61	-5	-5	11	0	0	0	0	0	-7	2	2	-13	-4	-4	-4	0	13	-5	3	-9	-3	1	7	-1	7	-9	1	3	0	0	0	0	0	0	0	0	x_{39}
x_{40}	-61	47	231	-185	-69	35	-25	0	71	-37	119	35	103	35	-61	-5	-5	11	0	0	0	0	0	-7	2	2	-13	-4	-4	-4	0	13	-5	3	-9	-3	1	7	-1	7	-9	1	3	0	0	0	0	0	0	0	0	x_{40}
x_{41}	34	-74	56	120	-144	0	-40	-11	-128	-128	0	-56	0	0	0	0	0	0	0	0	0	0	0	14	32	2	8	0	0	0	0	16	-2	-6	6	0	0	8	0	8	0	0	0	0	0	0	0	1	-3	-1	-1	x_{41}
x_{42}	34	-74	56	120	-144	0	-40	-11	-128	-128	0	-56	0	0	0	0	0	0	0	0	0	0	0	14	32	2	8	0	0	0	0	16	-2	-6	6	0	0	8	0	8	0	0	0	0	0	0	0	1	-3	-1	-1	x_{42}
x_{43}	-274	-58	616	-344	216	-104	-24	-110	512	-640	0	104	0	0	0	0	0	0	0	0	40	-40	16	118	10	0	-6	0	0	0	0	2	2	-10	2	14	0	-40	0	0	0	0	0	4	11	7	-4	10	2	0	0	x_{43}
x_{44}	0	0	189	-51	139	-21	-61	-15	0	81	0	0	0	21	-27	9	45	0	0	13	0	0	0	0	-27	3	0	-6	-10	-2	0	0	0	0	0	0	0	-3	9	7	13	3	-1	0	0	0	0	-3	1	0	0	x_{44}
x_{45}	0	0	189	-51	139	-21	-61	-15	0	81	0	0	0	21	-27	9	45	0	0	13	0	0	0	0	-27	3	0	-6	-10	-2	0	0	0	0	0	0	0	-3	9	7	13	3	-1	0	0	0	0	-3	1	0	0	x_{45}
x_{46}	-372	6	-680	280	120	-280	0	-22	0	0	0	216	0	0	0	0	0	0	4	4	-56	56	0	148	58	8	-10	0	0	0	8	6	-12	12	-6	-6	-1	-24	0	-24	0	0	0	-14	0	7	0	2	-6	1	1	x_{46}
x_{47}	-65	-11	0	64	25	49	69	0	151	358	-209	-35	-25	14	-2	-10	14	5	0	0	35	-21	0	-15	21	-4	2	-2	-6	2	3	-5	-5	-9	-3	3	0	16	-8	1	8	3	-3	-21	0	0	0	0	0	0	0	x_{47}
x_{48}	-65	-11	0	64	25	49	69	0	151	358	-209	-35	-25	14	-2	-10	14	5	0	0	35	-21	0	-15	21	-4	2	-2	-6	2	3	-5	-5	-9	-3	3	0	16	-8	1	8	3	-3	-21	0	0	0	0	0	0	0	x_{48}
x_{49}	-505	-46	-3500	500	0	0	0	22	262	190	78	160	54	42	-30	30	6	0	-16	-3	-145	7	0	210	30	0	0	0	0	0	0	6	12	3	-1	2	-4	-1	20	-20	0	20	0	0	35	0	5	0	-2	6	0	0	x_{49}
x_{50}	-390	150	880	-784	80	96	56	0	-512	640	0	112	0	0	0	0	0	0	-32	46	128	-64	48	42	60	-10	8	0	0	0	-6	30	30	2	2	2	0	16	0	-24	0	0	0	30	2	6	2	0	0	-3	-3	x_{50}
x_{51}	-207	-99	924	396	-176	56	76	0	587	-700	-245	98	11	-56	16	20	20	10	-16	23	0	0	0	-14	-23	2	-10	-4	8	0	0	9	9	-15	9	-3	0	-4	0	36	-4	6	0	0	0	0	0	0	0	1	1	x_{51}
x_{52}	-468	-63	715	91	-185	-329	31	0	-71	334	-351	-98	73	-70	-54	-26	10	-10	13	0	21	21	0	-85	-22	13	10	-4	8	0	7	-3	-12	12	9	-3	0	-37	-1	23	3	3	-1	-21	0	0	0	0	0	0	0	x_{52}
x_{53}	15	-174	0	0	0	0	0	0	155	65	-125	50	75	-35	37	-15	-15	10	0	0	-10	-2	5	0	0	0	0	7	11	3	1	-6	3	7	-2	-2	0	0	0	0	0	0	0	-5	-5	1	-2	0	0	0	0	x_{53}
x_{54}	15	-174	0	0	0	0	0	0	155	65	-125	50	75	-35	37	-15	-15	10	0	0	-10	-2	5	0	0	0	0	7	11	3	1	-6	3	7	-2	-2	0	0	0	0	0	0	0	-5	-5	1	-2	0	0	0	0	x_{54}
x_{55}	-130	-22	0	0	0	0	0	0	70	-200	-210	7	-42	0	24	0	-12	15	11	24	0	0	0	0	0	0	0	-3	1	-7	0	-24	-6	6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3b23	**	x_{55}
x_{56}	-130	-22	0	0	0	0	0	0	70	-200	-210	7	-42	0	24	0	-12	15	11	24	0	0	0	0	0	0	0	-3	1	-7	0	-24	-6	6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	3b23	x_{56}
x_{57}	0	0	3850	154	-400	-336	-16	110	-558	-234	378	0	-30	30	54	-18	-6	0	0	0	110	-18	10	78	-12	3	0	0	0	0	0	0	0	0	0	0	0	26	-2	16	-30	0	0	6	6	0	0	-10	-2	-1	-1	x_{57}
x_{58}	-1325	52	-4675	-275	0	0	0	-55	-940	-445	-140	-133	-12	79	31	75	-9	-5	-39	0	-29	-21	35	36	-45	0	0	3	-1	7	0	6	15	-21	0	-6	0	45	9	0	5	0	0	-14	0	-8	0	5	-7	-3	-3	x_{58}
x_{59}	-10	71	0	0	250	50	-50	0	-320	-176	0	64	-64	0	0	-16	32	0	0	0	-20	-4	-4	0	0	5	-10	0	0	0	0	29	2	-2	-5	1	0	0	0	-10	0	0	2	10	-11	-5	-2	0	0	-1	-1	x_{59}
x_{60}	-10	71	0	0	250	50	-50	0	-320	-176	0	64	-64	0	0	-16	32	0	0	0	-20	-4	-4	0	0	5	-10	0	0	0	0	29	2	-2	-5	1	0	0	0	-10	0	0	2	10	-11	-5	-2	0	0	-1	-1	x_{60}
x_{61}	660	39	6336	-256	-314	-226	34	0	1240	-416	-552	64	-40	0	-48	-16	8	0	52	0	51	51	-12	84	12	-3	6	0	0	0	-6	-3	-12	-12	3	9	0	0	16	-10	0	0	2	5	-2	-1	-2	0	0	0	0	x_{61}
x_{62}	-729	0	-595	285	105	105	-55	-129	1284	141	-124	0	-60	105	-7	21	-15	0	39	0	70	-42	0	-149	-32	3	0	3	-5	11	-8	0	-9	-9	0	0	0	-19	1	1	-3	-3	1	-42	0	0	0	-1	-1	0	0	x_{62}
x_{63}	-529	38	2100	100	0	0	0	-1	339	465	-149	-126	3	49	49	-15	-15	10	39	0	-70	42	0	-257	-5	0	0	-8	-16	0	1	-6	3	-9	-6	6	-5	20	-8	0	20	0	0	42	0	0	0	-1	-1	0	0	x_{63}
x_{64}	-675	135	-4720	240	-420	300	20	-176	1504	-224	608	64	-32	0	32	64	16	0	26	0	-82	-58	-16	147	48	3	0	0	0	0	0	-9	9	21	3	3	1	-16	-24	4	0	0	-4	15	-13	3	-4	4	0	2	2	x_{64}
x_{65}	351	189	1904	-240	154	-350	30	176	-70	-214	-238	140	106	-70	50	-14	-26	-4	0	0	0	0	0	280	10	5	-7	0	0	0	0	-27	-9	15	-3	-3	0	0	-16	10	24	4	-2	0	0	0	0	-4	0	-1	-1	x_{65}
x_{66}	0	0	230	630	30	-90	-30	99	-1485	0	27	72	99	0	0	0	0	24	-26	0	4	36	36	216	0	0	9	-8	0	-16	0	0	0	0	0	0	5	6	2	6	6	-4	-6	19	12	4	0	-1	3	0	0	x_{66}
x_{67}	-1026	-135	-3025	-225	0	0	0	55	-1120	275	336	-64	-64	-105	-33	35	35	0	28	-11	-81	7	0	-112	-40	0	0	12	8	0	0	9	18	6	-3	-3	-4	15	-13	0	15	0	0	0	0	-3	0	-5	7	0	0	x_{67}
x_{68}	0	0	6270	-130	220	-180	20	75	351	594	279	0	15	54	-18	66	18	0	0	0	8	-72	-23	-63	-9	-9	0	-2	10	18	0	0	0	0	0	0	-6	46	-10	-4	-10	0	4	15	1	0	-2	-5	-5	0	0	x_{68}
x_{69}	-221	211	1155	-45	-295	-15	45	0	773	-91	-35	98	-139	41	73	-19	-7	26	0	0	0	0	14	14	-49	1	-11	8	16	0	0	35	-1	-5	-5	-5	0	19	-9	29	-5	-5	1	0	14	0	-1	0	0	0	0	x_{69}
x_{70}	5	221	0	0	0	0	0	-55	185	-445	505	-28	25	-85	19	-5	55	20	13	0	5	45	10	0	0	0	0	4	-8	0	0	23	5	-11	1	-5	-5	0	0	0	0	0	0	5	12	8	3	5	-7	0

$M = F_1$

| | e | e | e

$M = F_1$

| | e | e | e

$M = F_1$

		1A	2A	2B	3A	3B	3C	4A	4B	4C	4D	5A	5B	6A	6B	6C	6D	6E	6F	7A	7B	8A	8B	8C	8D	8E	8F	
x_{96}	+	39100908183747737832960 0	−2044214061926 40	2781677516 80	35988211896	21594300	−2572752	−41284608	5870592	−308224	279552	−125400	32725	−289224	92092	−4424	−14148	−68	1072	−13720	0	0	0	0	1024	0	0	x_{96}
x_{97}	+	39261165197506560000000 0	2126603059200000	76775424000	209858031360	−29583360	6669000	504627200	0	0	0	2375000	0	80640	−105600	40704	28800	3840	−1080	−9520	280	0	0	0	0	0	0	x_{97}
x_{98}	+	43352869456059897852518 4	889610385291264	377461733376	49627809504	43766424	5687136	−42534912	22177792	817152	−292864	569184	6434	760032	−35880	85728	33624	3864	−672	0	686	0	0	0	0	0	0	x_{98}
x_{99}	o	59778752220731557107794 7	−226187908369605	182632149819	0	44641044	0	3064635	−7894341	−225477	282555	599697	24822	0	−61236	0	−26244	2916	0	0	0	1659	891	−1053	315	−389	−189	x_{99}
x_{100}	o	59778752220731557107794 7	−226187908369605	182632149819	0	44641044	0	3064635	−7894341	−225477	282555	599697	24822	0	−61236	0	−26244	2916	0	0	0	1659	891	−1053	315	−389	−189	x_{100}
x_{101}	+	60002077268506450239290 7	1967052076247115	328187590731	178514751987	0	0	320575563	13586859	−203445	−3861	558657	8532	−2814669	0	124659	0	0	0	−17424	1539	891	4851	351	195	27	351	x_{101}
x_{102}	o	62687740361388730404044 8	0	0	−6897532928	−65601536	26411008	0	0	0	0	360448	40448	0	0	0	0	0	0	−4096	1392	0	0	0	0	0	0	x_{102}
x_{103}	o	62687740361388730404044 8	0	0	−6897532928	−65601536	26411008	0	0	0	0	360448	40448	0	0	0	0	0	0	−4096	1392	0	0	0	0	0	0	x_{103}
x_{104}	+	65515923107370540492187 5	1365102860281875	93555193875	121406755041	44169840	−23122125	237459475	11403379	−209645	−50765	−653125	0	−4088799	68640	54753	−25920	−1200	−1485	−24479	−665	3779	3003	−1001	−245	99	−169	x_{104}
x_{105}	o	68976322274489500594924 2	−389556578898630	−206943847110	0	0	767637	50535738	8055450	707130	−263718	−1058508	30492	0	0	0	0	0	405	7497	0	−918	−990	702	−270	−246	126	x_{105}
x_{106}	o	68976322274489500594924 2	−389556578898630	−206943847110	0	0	767637	50535738	8055450	707130	−263718	−1058508	30492	0	0	0	0	0	405	7497	0	−918	−990	702	−270	−246	126	x_{106}
x_{107}	o	68976672617955508099422 3	−389645722280529	−207699116625	0	0	−767637	50720175	8216559	746415	−244881	−1066527	34848	0	0	0	0	0	−405	7650	−288	−945	−1089	351	−225	−369	63	x_{107}
x_{108}	o	68976672617955508099422 3	−389645722280529	−207699116625	0	0	−767637	50720175	8216559	746415	−244881	−1066527	34848	0	0	0	0	0	−405	7650	−288	−945	−1089	351	−225	−369	63	x_{108}
x_{109}	+	10376058869846974817553 04	2308415562554024	592924211880	58568622343	43458415	−750386	38976168	15973608	129192	−146328	−1890196	58429	4728647	39039	111111	18335	−1713	462	0	0	−56	2200	−352	−136	392	224	x_{109}
x_{110}	+	13615491261057529822728 75	4188169855744875	552058501995	289603899585	11050182	0	375040875	10347883	324459	−168597	1227875	22875	1223937	−223938	133569	−17874	2646	0	4998	294	−1813	1419	−1053	11	363	−189	x_{110}
x_{111}	+	15991103878635588828125 00	5104422882812500	−409762307500	372791032250	45459050	30875	548860500	−30304300	216660	63700	0	0	1929850	−186550	−80710	18250	−790	155	−1225	0	−3436	−6060	−1300	20	−364	140	x_{111}
x_{112}	+	16626861804838655720161 28	4588173708754944	−282732527616	335535705344	−34635328	−28861000	464257024	0	0	0	401128	16128	−1478400	−192192	−123648	−42432	192	−840	0	0	0	0	0	0	0	0	x_{112}
x_{113}	+	21816945858215056803970 72	−523196633312496	120771400464	0	75996063	0	66646800	10372752	706320	139152	−2093553	6447	0	104247	0	45927	5103	0	−4725	−1785	−48	−4752	−624	−336	−176	−48	x_{113}
x_{114}	+	22163430209133519667968 75	2750493342796875	774144076875	−3482490375	98394075	−6669000	−21939125	29873675	468555	129675	0	0	3735225	−18525	9465	−15885	−885	1080	−1700	260	2219	−1605	−325	−165	619	−357	x_{114}
x_{115}	+	24775487505552980686810 32	3115429591028040	−375445270200	−42189210063	122666544	12266514	182186312	−17911608	−522936	−190008	−5319468	15407	5692401	264264	19761	17784	384	−1518	−20825	0	2568	−2200	−1352	936	−632	56	x_{115}
x_{116}	+	32825105402836314421751 04	3374456886043776	−201641581440	255324254328	64993266	0	124822656	254592	−576384	−104832	−1192896	−21771	−5837832	18954	20088	53946	2754	0	−12648	−496	1920	0	0	256	384	0	x_{116}
x_{117}	+	35372927965387414150749 00	4545779486739540	225818500180	240777360516	−35439495	22165248	245640276	6227508	736596	−195468	−1622600	−34475	−4384380	−98735	47236	−15807	−3191	1792	0	0	2884	−3300	1006	−596	−28	168	x_{117}
x_{118}	+	36192090507743754267924 0	7409717053209576	168829098984	309045839697	103223484	0	262431912	11122216	367080	220584	−2203201	−20076	6248529	−127764	6993	−33588	2268	0	0	0	1288	−936	1728	−200	−312	0	x_{118}
x_{119}	+	40043082748232704000000 0	77683916800000	617250816000	−38207644160	6885760	53599000	150732800	0	0	0	−2375000	0	−535040	0	32256	−16640	5760	1560	13720	0	0	0	0	0	0	0	x_{119}
x_{120}	+	42393156529790097281250 00	7503509140925000	389541501000	307909104165	−8500140	0	354121800	9768200	250440	491400	−2375000	0	1907685	−148500	53541	26460	−540	0	−5355	−700	5032	2520	0	120	−536	0	x_{120}
x_{121}	+	49266701743234840696832 00	8106988248719360	−527149342720	395355008048	34538504	22253504	312958976	−10436608	−630784	0	−1306800	26950	−299728	−248248	−53200	2888	−4600	−64	13720	0	0	0	0	0	0	0	x_{121}
x_{122}	+	53340461629692083522156 25	−405229276200375	846109348425	0	0	72925515	922185	−7125975	445257	238185	0	28125	0	0	0	0	0	−2997	4725	2079	−1863	−495	−675	−351	−39	−99	x_{122}
x_{123}	+	55141324248814632084439 04	2642676197359616	0	65393917952	47151104	30507008	0	33554432	0	0	−1284096	43904	1441792	0	0	51200	0	0	8448	−960	0	0	0	0	0	0	x_{123}
x_{124}	o	55141324248814632084439 04	−2642676197359616	0	65393917952	47151104	30507008	0	0	0	0	−1284096	43904	1441792	0	0	−51200	0	0	8448	−960	0	0	0	0	0	0	x_{124}
x_{125}	o	55141324248814632084439 04	−2642676197359616	0	65393917952	47151104	30507008	0	0	0	0	−1284096	43904	1441792	0	0	−51200	0	0	8448	−960	0	0	0	0	0	0	x_{125}
x_{126}	+	71184653287617884753756 16	−1793872427483136	−72424095744	113944431744	136730880	48514752	−265289728	0	−1048576	0	1331616	−11884	1592448	192192	−29568	11328	−4224	1728	−29308	1072	0	0	0	0	0	0	x_{126}
x_{127}	+	73758925004096094082031 25	6131165888203125	470535883125	77007684600	−18018000	57605625	−68404875	29259125	−953995	407925	0	0	3003000	−46200	−21000	74040	−7680	1785	0	0	2037	1045	1325	405	−395	−371	x_{127}
x_{128}	o	75671515765424524257812 50	−3099399926218750	511362101250	89656289800	−44244200	28861000	−85450750	−16285150	−680190	122850	0	0	1925000	0	−32760	−80560	4680	840	0	0	1554	2970	−650	−630	−142	−42	x_{128}
x_{129}	o	75671515765424524257812 50	−3099399926218750	511362101250	89656289800	−44244200	28861000	−85450750	−16285150	−680190	122850	0	0	1925000	0	−32760	−80560	4680	840	0	0	1554	2970	−650	−630	−142	−42	x_{129}
x_{130}	+	78509349592079406000000 00	5443097899200000	9248237440000	52341277560	29564340	45942000	−72716800	8486400	192000	499200	2375000	0	1355640	−193700	4984	−29460	1060	−2800	6580	−770	3584	0	0	0	−512	0	x_{130}
x_{131}	+	83940370471550834876344 50	7113733997294610	−165371747310	156321775827	0	81153009	156607506	−16058574	−122094	−442638	−2808675	9450	−85293	0	39123	0	0	−783	−13720	0	5922	−374	−1026	−390	258	126	x_{131}
x_{132}	+	88742608755270179360651 00	−40955835260340	1341062582860	−556166611	204211007	37835252	−67149236	−1082900	1758540	41132	1688225	−44275	−27027	−49049	11053	17847	3439	2548	0	0	28	−4444	−728	596	252	168	x_{132}
x_{133}	+	94160318588618587515556 96	9211433539600384	0	358364217344	−97936384	−26411008	0	16777216	0	0	−1915904	−43904	−1114112	0	0	−35840	0	0	−6272	0	0	0	4096	0	0	0	x_{133}
x_{134}	+	94794957458053056531250 00	9453998491525000	−356505723000	319423783525	−127760465	28861000	148152200	−8629880	−798840	−207480	−4156250	0	−355355	21735	−26523	50455	−1665	840	0	0	2184	−1848	−1000	200	136	−168	x_{134}
x_{135}	o	95922981436508902551715 84	−1807462370115584	111379480576	6471756928	−253425536	51026248	42205184	0	0	0	−339416	−3416	267904	−11648	−43904	−63872	−7040	2632	0	0	0	0	0	0	0	0	x_{135}
x_{136}	o	95922981436508902551715 84	−1807462370115584	111379480576	6471756928	−253425536	51026248	42205184	0	0	0	−339416	−3416	267904	−11648	−43904	−63872	−7040	2632	0	0	0	0	0	0	0	0	x_{136}
x_{137}	+	95925843869185829796577 28	8575886032896000	118893838336	6190376192	−246098944	52672256	−44040192	0	0	0	−370272	7728	8456448	−11648	−51968	6528	−5888	1792	0	0	0	0	0	0	0	0	x_{137}
x_{138}	+	100238549981714890834375	−9955408040015625	472113744375	39151317200	−208435500	57605625	−60335625	18971575	861175	−225225	0	0	−327600	27300	8400	−27900	−780	1785	0	0	343	−2585	−25	135	279	7	x_{138}
x_{139}	+	126508821004661870337062 50	10034257030762250	−1019197128950	151310033875	223293070	55092115	−101146870	−31812950	305674	−550550	0	−43750	5280275	−130130	−39725	1262	4942	2707	22050	0	−1190	−990	−650	210	442	182	x_{139}
x_{140}	+	149301642835630489600000 00	11854109736960000	0	423758135296	−50785280	4096000	0	−16777216	0	0	−3200000	0	−2555904	0	0	15360	0	0	2176	−960	0	0	−4096	0	0	0	x_{140}
x_{141}	+	161094072692210325656303 70	8698184821732770	1131976353186	156321775827	0	81153009	−90471006	30235842	−27486	177282	−2185380	−32130	−85293	0	39123	0	0	−783	23324	0	−3150	2650	702	330	−366	126	x_{141}
x_{142}	+	226266213651605370999275 52	16597016119934976	−793450577920	224256581472	−59867806	−74864256	−49741824	2228224	−983040	0	−2230448	27552	6077280	−128128	−61600	−52800	2336	−3712	28322	0	0	0	0	0	0	0	x_{142}
x_{143}	+	245463847192898255981866 95	1592440993194925 5	912248087751	329144794704	0	−81153009	79410375	11510343	−77625	−81081	−4994055	−22680	−1154736	0	27216	0	0	0	783	6154	960	3591	1991	351	135	519	63	x_{143}
x_{144}	+	275019176097091022471875 00	8051368635667500	1408838931500	−88953942077	31401370	−29670875	159712300	13428012	111660	−63700	−5225000	0	1804803	250250	32963	−15510	−3910	−475	−7497	0	1324	−3124	52	460	−340	84	x_{144}
x_{145}	+	297349414199093821628744 368	11469314221670400	882884214784	289744287744	82377984	−33045504	0	0	0	0	−4197632	24368	−5892096	73216	96256	−3072	1792	1024	9520	896	0	0	0	0	0	0	x_{145}
x_{146}	+	336843888303599810445312 00	6479020806635520	−2108058501120	256457380608	199907136	6669000	−464257024	0	0	0	2466200	31200	−2306304	−27456	768	−9792	−5568	−1080	11984	2576	0	0	0	0	0	0	x_{146}
x_{147}	+	337221913270026881570473 80	8232122887328340	163655127636	−326469800657	−350991641	−4127123	577593940	−19015724	147028	−122668	−7117495	6755	3162159	183183	92463	3807	−2793	−147	0	0	−364	−3564	572	−1324	148	28	x_{147}
x_{148}	+	373107152115466240000000 0	0	2684354560000	−37500108800	−63334400	81472000	0	0	0	0	0	0	102400	−81920	0	10240	2560	−14400	2260	0	0	0	0	0	0	0	x_{148}
x_{149}	+	384717957392565650805751 80	1175432471971758 0	−2421327344436	90098990982	−40783743	−30390633	−335774516	5501132	562380	562380	1075305	−12320	2648646	81081	−64890	34857	7569	−105	0	0	−1204	−2068	1924	−660	−52	−28	x_{149}
x_{150}	+	417389411512439538046875 00	7835937868687500	1272214927500	160161390675	−11518650	−34456500	−217200500	−3204500	47500	329900	0	0	−3185325	−21450	136275	−52650	3750	−180	17850	−1260	−2436	−2860	0	100	284	0	x_{150}
x_{151}	+	417623227383858201956250 00	1828205914090500 0	1363696425000	157848705693	−58378320	0	−333015000	−5235608	304680	98280	415625	0	4113693	0	66717	30240	−2160	0	17493	0	−4024	1176	−1728	120	−504	0	x_{151}
x_{152}	+	420014540879545151675034 90	8436234038236290	1759984379010	178514751987	0	0	−321866622	12478050	750978	55458	1015740	6615	−2814669	0	124659	0	0	0	18648	−315	−3726	−4950	702	186	18	−162	x_{152}
x_{153}	+	426014748606395796698963 97	10403286114483405	2088171969741	−178514751987	0	0	−1291059	26064909	547533	51597	1574397	15147	2814669	0	−124659	0	0	0	1224	1224	−2835	−99	1053	381	45	189	x_{153}
x_{154}	+	435271309901479817556510 72	1212935168180736 0	1532481606656	−186984149352	39555516	2572752	52073472	−16307200	152576	−279552	−1335928	−58303	4993560	220220	101528	4284	−964	−1072	−23324	0	0	0	0	1024	0	0	x_{154}
x_{155}	+	505725420249495984037500 00	7126932153510000	1173798054000	57826834416	182669175	0	46450800	4069104	140400	−34320	−4809375	0	−2914704	289575	−187920	−34425	−2025	0	−12579	−1995	−848	4752	624	80	−208	48	x_{155}
x_{156}	+	513243503895580974140625 00	−5318034873937500	2756716462500	29444114700	218468250	34456500	60846500	−27959100	1284260	391300	0	0	300300	−150150	−22260	114810	−10470	180	22050	0	308	220	0	−820	404	0	x_{156}
x_{157}	+	563564332731466750054891 52	−235755961319424	2066448384000	−220039735552	388309376	−1646008	504627200	0	0	0	−4103848	89152	−908544	0	−112896	3840	5760	840	0	0	0	0	0	0	0	0	x_{157}
x_{158}	+	584373946332752618332160 0	11456032491438080	−1547242045440	2505494467 84	−150410624	74925760	−265289728	0	1048576	0	−3138400	9100	−4051840	331968	−51072	−13504	3072	1728	−23324	0	0	0	0	0	0	0	x_{158}
x_{159}	+	620380574867922491329740 80	0	−1284195221504	31396511744	60849152	50964992	0	0	0	0	−3793920	74080	0	372736	114688	0	−5120	2560	−35904	−36	0	0	0	0	0	0	x_{159}
x_{160}	+	637508128450358280790084 41	8199319123688121	1321700072121	0	0	72925515	−344343879	24005241	−1246023	118521	4723191	−4059	0	0	0	0	0	−2997	−19278	−2079	2457	−2871	−351	−279	729	225	x_{160}
x_{161}	+	643261634275226242057031 25	1347192195463125	1067684743125	118588933386	0	76271625	−260728875	1703349	−42795	45045	2784375	0	−2453814	0	−105462	0	0	−2295	−13769	0	1989	−451	−351	−435	−411	−63	x_{161}
x_{162}	+	665503395143561520000000 00	21956826816000000	−790766080000	119147944200	−7105800	−70680000	−172544000	10444800	634880	0	0	0	−113400	−28600	−169720	29160	−1960	−3520	−2975	700	4096	0	0	0	0	0	x_{162}
x_{163}	+	690848590080050364312240 66	7794089847487746	2167809420546	0	0	−72925515	−343421694	16879266	−800766	356706	4723191	24066	0	0	0	0	0	2997	−14553	0	594	−3366	−1026	−630	690	126	x_{163}
x_{164}	+	746122135297203836547793 56	−8190850502407716	1755246352860	0	0	190373976	326822364	−8377668	−785700	226044	−5019894	4356	0	0	0	0	0	−3240	23409	−1728	−2484	1188	0	180	492	0	x_{164}
x_{165}	+	774233984548530646462822 50	1747379318326785 0	−1774351528950	−232398191025	168630462	−61677135	1510410	−32421142	−901878	130858	1358500	16625	5097807	102102	74319	−2682	2958	177	−28322	0	1274	−110	702	−94	282	126	x_{165}
x_{166}	+	839747744590503356308593 75	5151403598859375	726455819375	−348658091925	−165583275	−11485500	406539375	3994575	2756975	520975	0	0	847275	−17875	−6805	−42915	9125	−2620	−25075	−820	2591	55	−325	135	63	−357	x_{166}
x_{167}	+	862066216809778349118750 00	16184160707715000	−1110415229000	31224782589	−116404365	22044750	19177400	−37858184	1178040	−280840	−5878125	0	−1200771	192115	−18179	−1485	4915	1870	23324	0	−3304	3784	−624	360	344	−112	x_{167}
x_{168}	+	889438206202883432616723 93	17936063528043465	1948844717001	−274910709213	0	0	100983753	12174057	−173367	−272727	711018	−21357	2539107	0	66339	0	0	0	−17493	0	4185	33	−675	−111	−327	189	x_{168}
x_{169}	+	103354104243912727763091 456	168430710030336	−782566686720	0	0	190373976	−191102976	0	0	0	3108456	66456	0	0	0	0	0	−3240	−8856	1728	0	0	0	0	0	0	x_{169}
x_{170}	+	115165062362004433625000000	−17824668455000000	2091409320000	370568024750	101451350	−63317500	−338008000	−27268800	1867840	−145600	0	0	4231150	150150	−94290	15910	−6090	−1020	−22050	0	2240	7040	0	−1280	448	0	x_{170}
x_{171}	+	121170799240938738783416925	7181081417421405	380574823005	−78338123487	151992126	0	110916189	30027933	430173	120861	−4971825	−4950	−1999647	208494	−24543	−48114	−5346	0	22134	1799	−6531	−1011	−1755	−275	−451	−27	x_{171}
x_{172}	+	124058385593021471188320256	9211433539600384	0	−142502789120	113032192	26411008	0	16777216	0	0	992256	−79744	−1114112	0	0	−35840	0	0	−2176	960	0	0	−4096	0	0	0	x_{172}
x_{173}	+	124982156072747647257292800	−15795123167416320	−563627151360	323920599912	−149130072	0	−115593216	12673024	−479232	0	−6437200	11550	2389608	452088	−22680	71064	4104	0	−17493	0	−4096	0	0	0	0	0	x_{173}
x_{174}	+	125517264890136048242396811	−8188002282979701	2564289531531	0	−241805655	0	4107915	−21928725	414603	−36309	4653936	81186	0	−331695	0	6561	729	0	0	0	5691	−1485	975	1155	−37	399	x_{174}
x_{175}	+	129572518017902934396764160	11854109736960000	0	−77108871168	160183296	56918016	0	−16777216	0	0	−291840	−35840	−2555904	0	0	15360	0	0	6272	0	0	0	4096	0	0	0	x_{175}
x_{176}	+	130287135266837289237316743	2675614428674163 9	−809445325689	0	0	0	−354143097	−607257	−697977	−208089	2582118	54243	0	0	0	0	0	0	0	0	567	−2673	675	−801	87	−189	x_{176}
x_{177}	+	135226984222789977095703125	8295452743703125	−4497154536875	−318660767425	37948625	46899125	335383125	13265525	467285	847925	0	0	−25025	−89375	9535	50305	−2255	2485	0	−1715	1253	1485	−975	−195	−59	433	x_{177}
x_{178}	+	136107644194473772613203125	7027988249683125	197582419125	−156321775827	0	76271625	−48249675	2270997	238005	−348075	2025000	0	85293	0	−39123	0	0	−2295	3298	−720	−2331	3389	1377	525	261	−63	x_{178}
x_{179}	+	136574874874360806036041889	8176832509726881	453678690465	−45049495491	23675652	60781266	−52756319	−51578527	−118111	126945	1101639	−17486	−2189187	−36036	121149	−27540	−4140	210	0	0	385	209	2353	−15	65	−175	x_{179}
x_{180}	+	136574874874360806036041889	−210922551590751	−21604437855	−45049495491	23675652	60781266	−52756319	7677761	−1496159	834561	1101639	−17486	−459459	36036	−56259	−22140	3780	210	0	0	−3087	473	−299	−375	465	245	x_{180}
x_{181}	+	138988549876584520148320256	−21065543276560384	0	281255346176	62246912	30507008	0	0	0	0	−2207744	−79744	3670016	0	0	20480	0	0	0	0	0	0	0	0	0	0	x_{181}
x_{182}	+	161561864971171113287540625	−12273909074571375	1405532430225	0	−116917020	0	205602705	15181425	−1367919	−394575	0	−37500	0	−160380	0	26244	2916	0	0	1029	3969	−4455	−1275	−855	−351	69	x_{182}
x_{183}	+	163216709667196367710937500	−12721880097062500	−1628116262500	318444612850	−301014350	−46899125	−331558500	11911900	1413020	555100	0	0	86450	86450	−14350	8690	−2830	−2485	0	0	700	−660	−1300	1100	508	140	x_{183}
x_{184}	+	172248852397651745653437500	−15266577308482500	1893241350 0	6832904078	−93683590	−4936750	539427900	24188892	−968900	−405860	−8662500	0	−24882	90090	−882	21450	2970	210	17493	0	−244	−2860	1352	−380	−20	−56	x_{184}
x_{185}	+	173865305251972140447265625	−25468473312734375	−2298055134375	477019824050	301901600	28744625	−345566375	17347225	522585	−102375	0	0	4577650	0	77490	−79040	4320	945	0	0	3577	−3575	625	105	−71	49	x_{185}
x_{186}	+	175867626988794162227008203	−7872254072319285	280190603979	210555861318	0	0	−586448181	−14663349	214731	−307125	7313328	30078	1299078	0	132678	0	0	0	−16149	−1596	−2421	−2805	675	75	−885	−189	x_{186}
x_{187}	+	177966317773633111417870812	−8022419792377380	972679666140	0	0	−190373976	135719388	−8377668	−785700	226044	−1911438	70812	0	0	0	0	0	3240	14553	0	−2484	1188	0	180	492	0	x_{187}
x_{188}	+	198203900044423845494482560	−6392453634103680	1830980511360	100109989980	1387386	−78009516	75163264	−29391232	−1235328	640640	−4474690	−36190	−2054052	162162	10332	−29214	−2934	−2220	7497	0	−1408	−1408	0	640	−384	0	x_{188}
x_{189}	+	200390867219082687273984375	8262689332224375	1319623680375	−54234085491	0	−76271625	−322252425	6641271	123255	−487305	4809375	0	−1384371	0	−93555	0	0	2295	−7021	−1680	−1161	3223	351	−105	−777	63	x_{189}
x_{190}	+	203314261261157852274218750	−6257595111381250	−1493402069250	−324851861425	206916710	−23807875	727389950	15485470	742910	−93730	−4156250	0	−469105	−90090	−17073	−49930	−7290	−1155	0	0	1806	−2058	650	230	238	−182	x_{190}
x_{191}	+	207467089840006711558593750	−4754485177406250	−1224482366250	−115651847025	−86229000	17228250	280513750	−16807050	182230	311350	0	0	−1683825	−257400	13455	48360	5400	2010	22575	1505	1574	5390	650	190	−762	106	x_{191}
x_{192}	+	212490247553365721772656250	1174047622256250	−1732832655750	−241832956050	84990465	0	168253050	10690330	−257670	−274470	4156250	0	−893970	−248535	49518	−26055	−7695	0	0	0	714	−462	−1650	−30	170	−210	x_{192}
x_{193}	+	241866941438795926688759808	0	0	−162357313536	−98942976	−56918016	0	0	0	0	3063808	−40192	0	0	0	0	0	0	−17408	2192	0	0

$M = F_1$

	9A	9B	10A	10B	10C	10D	10E	11A	12A	12B	12C	12D	12E	12F	12G	12H	12I	12J	13A	13B	14A	14B	14C	15A	15B	15C	15D	16A	16B	16C	17A	18A	18B	18C	18D	18E	19A	20A	20B	20C	20D	20E	20F	21A	21B	21C	21D	22A	22B	23A	B**	
x_{96}	240	-111	-440	40	485	245	5	-70	696	12	-504	-48	-40	84	36	-52	44	0	0	0	56	-56	0	321	15	10	-2	0	0	0	-1	27	0	16	1	-5	0	-8	-8	17	32	-3	1	14	0	-7	0	-10	-6	1	1	x_{96}
x_{97}	-216	108	-1000	600	0	0	0	176	-1792	-640	0	56	0	0	0	0	0	0	12	30	-144	16	-40	-190	80	0	0	0	0	0	0	18	0	-24	0	6	0	40	0	0	0	0	0	-16	-23	2	-2	-4	0	1	1	x_{97}
x_{98}	315	126	2464	96	14	6	26	-110	1248	24	-224	0	96	-40	-8	24	24	-16	0	0	0	0	14	-21	24	-1	11	0	0	0	0	-18	-9	3	-6	-6	-4	48	-8	38	-24	-4	2	0	-7	0	2	10	2	0	0	x_{98}
x_{99}	0	0	945	-111	-230	-126	94	-66	0	-324	0	0	0	0	0	36	0	0	0	-26	0	0	0	0	54	9	0	3	11	-5	0	0	0	0	0	0	0	-15	9	10	-15	0	-2	0	0	0	0	-6	-2	0	0	x_{99}
x_{100}	0	0	945	-111	-230	-126	94	-66	0	-324	0	0	0	0	0	36	0	0	0	-26	0	0	0	0	54	9	0	3	11	-5	0	0	0	0	0	0	0	-15	9	10	-15	0	-2	0	0	0	0	-6	-2	0	0	x_{100}
x_{101}	0	0	-3135	321	240	216	156	0	243	0	-405	0	99	0	0	0	0	0	0	0	-24	0	-21	162	0	0	0	-5	-1	-9	8	0	0	0	0	0	0	-47	9	-12	9	-6	0	-9	-9	0	0	0	0	-1	-1	x_{101}
x_{102}	256	-176	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-128	-56	-16	8	0	0	0	8	0	0	0	0	0	4	0	0	0	0	0	0	8	-6	8	0	0	0	0	0	x_{102}
x_{103}	256	-176	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-126	-56	-16	8	0	0	0	8	0	0	0	0	0	4	0	0	0	0	0	0	8	-6	8	0	0	0	0	0	x_{103}
x_{104}	-351	297	-1925	-325	0	0	0	0	1441	280	-503	-125	-47	76	-44	40	-8	-5	0	0	-55	17	-25	41	5	0	0	-13	7	-1	8	-27	9	-15	9	-3	-5	-5	-21	0	-5	0	0	19	19	-5	1	0	0	0	0	x_{104}
x_{105}	0	0	-1980	180	120	0	-60	0	0	0	0	-27	0	0	0	0	0	-3	0	0	81	-7	0	0	0	0	12	-6	14	6	0	0	0	0	0	0	-4	-12	0	-12	12	2	0	0	0	3	0	0	0	0	0	x_{105}
x_{106}	0	0	-1980	180	120	0	-60	0	0	0	0	-27	0	0	0	0	0	-3	0	0	81	-7	0	0	0	0	12	-6	14	6	0	0	0	0	0	0	-4	-12	0	-12	12	2	0	0	0	3	0	0	0	0	0	x_{106}
x_{107}	0	0	-2079	225	96	0	0	0	0	0	0	27	0	0	0	0	0	3	0	0	90	-14	0	0	0	0	-12	-9	7	7	0	0	0	0	0	0	-5	-15	9	0	9	4	0	0	0	-3	0	0	0	0	0	x_{107}
x_{108}	0	0	-2079	225	96	0	0	0	0	0	0	27	0	0	0	0	0	3	0	0	90	-14	0	0	0	0	-12	-9	7	7	0	0	0	0	0	0	-5	-15	9	0	9	4	0	0	0	-3	0	0	0	0	0	x_{108}
x_{109}	-218	-2	924	-20	-351	105	5	0	2631	-33	399	126	-201	-105	-105	15	-57	6	-39	0	0	0	0	-7	20	10	-11	-8	-16	0	0	2	2	-18	-6	6	0	28	8	-7	12	7	-3	0	0	0	0	0	0	0	0	x_{109}
x_{110}	729	0	275	995	-125	195	-5	0	-447	534	601	0	-15	-90	-2	-66	18	0	0	0	-98	-42	14	-265	-58	-3	0	-13	11	-5	0	0	9	9	0	0	0	-45	-17	-25	3	3	-1	21	0	0	3	0	0	0	0	x_{110}
x_{111}	1985	14	0	0	0	0	0	-55	1530	450	50	27	-54	70	14	-30	-6	-5	0	0	-145	7	0	0	0	0	0	12	4	-12	6	-2	-11	-7	2	2	0	0	0	0	0	0	0	35	0	5	0	5	-7	0	0	x_{111}
x_{112}	-214	110	5544	-536	-56	504	-16	-128	-1280	-1280	0	-56	0	0	0	0	0	0	4	30	0	0	0	-56	-38	-18	0	0	0	0	0	12	30	18	6	12	0	24	0	24	0	0	0	0	0	0	0	0	0	0	0	x_{112}
x_{113}	0	0	2079	399	-621	-21	39	0	0	-81	0	0	0	111	135	-9	27	0	0	0	-45	-69	15	0	-27	3	0	0	0	0	-6	0	0	0	0	0	0	15	27	-25	7	-3	-5	0	0	0	3	0	0	3	3	x_{113}
x_{114}	-675	-108	0	0	0	0	0	-55	-455	1075	785	-56	9	-105	119	75	15	0	0	0	60	92	-20	0	0	0	0	-13	3	3	0	18	9	21	0	6	0	0	0	0	0	0	0	-35	-7	-2	-4	5	-7	0	0	x_{114}
x_{115}	-720	198	1540	340	-85	35	-25	0	1841	464	-519	50	129	0	-24	24	-36	-30	0	0	55	7	0	-63	-36	-1	14	8	-8	-8	0	18	0	0	-6	6	0	52	-8	-13	12	-3	-1	-35	0	1	0	0	0	0	0	x_{115}
x_{116}	0	0	-1024	320	-599	225	-115	-66	1656	-126	-504	0	24	-126	90	-54	30	0	0	0	288	72	16	153	-9	-9	0	0	0	0	0	0	0	0	0	0	0	16	-8	-19	8	3	1	24	-4	0	2	-6	-2	0	0	x_{116}
x_{117}	-672	-240	3640	440	165	245	5	-66	324	81	84	0	-60	21	-123	-39	-27	0	13	0	0	0	0	91	-35	-5	-2	-12	-8	0	0	-12	24	16	-8	4	0	56	8	1	-8	-3	1	0	0	0	0	-6	-2	0	0	x_{117}
x_{118}	729	0	-2849	-321	-424	-336	-116	55	1425	300	385	0	-111	84	52	60	-12	0	-39	0	0	0	0	-28	-1	-6	0	-8	-16	0	0	0	9	9	0	0	6	-33	-9	12	-1	-6	0	0	0	0	0	-5	7	0	0	x_{118}
x_{119}	-914	4	1000	-600	0	0	0	74	512	-640	0	104	0	0	0	0	0	0	12	64	-56	56	0	190	-80	0	0	0	0	0	-6	-8	10	6	12	0	0	-40	0	0	0	0	0	-14	0	7	0	-6	-6	0	0	x_{119}
x_{120}	-729	0	1000	-600	0	0	0	99	-2523	60	365	0	117	0	32	-60	-48	0	0	-26	-51	-27	-20	190	-80	0	0	0	8	0	16	0	0	-9	-9	0	0	-40	0	0	0	0	0	-9	-16	0	2	-1	3	0	0	x_{120}
x_{121}	725	-193	-2640	80	610	-70	30	0	176	1112	-784	-64	-16	0	80	-40	8	0	0	0	-56	56	0	-27	54	4	4	0	0	0	-6	-1	17	-19	-1	-1	1	16	-8	26	0	0	6	-14	0	7	0	0	0	0	0	x_{121}
x_{122}	0	0	0	0	-375	225	-75	0	0	0	0	27	0	0	0	0	0	51	-27	51	45	5	-9	0	0	0	15	9	13	5	6	0	0	0	0	0	0	0	0	-15	0	-5	-3	0	0	-9	0	0	0	0	0	x_{122}
x_{123}	-400	-184	2816	0	-384	0	0	0	0	0	-1024	0	0	0	128	0	0	0	26	0	-128	0	0	-48	24	-16	8	0	0	0	6	8	8	0	0	0	0	0	32	0	0	0	0	6	6	0	0	0	0	0	0	x_{123}
x_{124}	-400	-184	-2816	0	384	0	0	0	0	0	0	0	0	0	0	0	0	0	26	0	128	0	0	-48	24	-16	8	0	0	0	6	-8	-8	0	0	0	0	0	0	0	0	0	0	6	6	0	0	0	0	0	0	x_{124}
x_{125}	-400	-184	-2816	0	384	0	0	0	0	0	0	0	0	0	0	0	0	0	26	0	128	0	0	-48	24	-16	8	0	0	0	6	-8	-8	0	0	0	0	0	0	0	0	0	0	6	6	0	0	0	0	0	0	x_{125}
x_{126}	450	45	2464	416	-636	196	-44	0	-1408	-256	0	-64	128	0	0	-64	32	0	0	0	-140	-28	0	-6	30	5	2	0	0	0	0	-39	6	-6	9	-3	6	32	0	-28	0	0	4	-4	10	-1	-8	0	0	0	0	x_{126}
x_{127}	-360	-198	0	0	0	0	0	0	120	-240	-280	105	-40	0	8	-40	-4	-15	11	24	0	0	0	0	0	0	0	-3	21	5	0	6	-12	0	-6	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{127}
x_{128}	-650	-110	0	0	0	0	0	0	200	200	-280	56	168	0	8	0	12	0	26	0	0	0	0	0	0	0	0	-6	-2	-10	0	8	-10	6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2b23	**	x_{128}
x_{129}	-650	-110	0	0	0	0	0	0	200	200	-280	56	168	0	8	0	12	0	26	0	0	0	0	0	0	0	0	-6	-2	-10	0	8	-10	6	-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**	2b23	x_{129}
x_{130}	-1011	-66	-1000	600	0	0	0	66	248	140	120	-112	120	-120	-24	-60	-24	0	0	0	116	-12	30	-190	80	0	0	0	0	0	-6	-30	-3	13	10	-2	0	40	0	0	0	0	0	19	-2	-8	1	6	2	0	0	x_{130}
x_{131}	0	0	-4115	285	-390	-630	90	144	-621	0	-189	225	-45	0	0	0	0	9	5	57	56	-56	0	27	0	0	9	2	-2	6	-8	0	0	0	0	0	0	-19	1	6	-3	2	6	14	0	-7	0	4	0	0	0	x_{131}
x_{132}	17	152	385	-95	285	-35	-115	0	-83	-209	133	-140	93	119	-65	39	3	20	0	0	0	0	0	14	77	-13	2	-4	8	0	0	0	9	1	-8	-8	4	-31	25	-11	17	-3	5	0	0	0	0	0	0	0	0	x_{132}
x_{133}	1256	176	384	0	384	0	0	-176	0	0	-512	0	0	0	64	0	0	0	-15	24	64	0	0	-56	16	16	-8	0	0	0	1	16	-20	0	0	0	-1	0	16	0	0	0	0	-35	0	-8	0	4	0	0	0	x_{133}
x_{134}	-569	-110	1750	550	0	0	0	66	869	-985	-203	56	-123	-21	-5	15	27	0	-43	-82	0	0	0	175	-5	0	0	-8	8	8	0	-8	19	15	-6	0	0	-10	-30	0	-10	0	0	0	0	0	0	6	2	-2	-2	x_{134}
x_{135}	136	136	616	296	-584	56	-24	-55	-640	512	0	56	0	0	0	0	0	0	0	0	0	0	0	28	-26	4	-2	0	0	0	0	10	-8	-8	4	-2	0	24	0	-16	0	0	0	0	0	0	0	5	1	0	0	x_{135}
x_{136}	136	136	616	296	-584	56	-24	-55	-640	512	0	56	0	0	0	0	0	0	0	0	0	0	0	28	-26	4	-2	0	0	0	0	10	-8	-8	4	-2	0	24	0	-16	0	0	0	0	0	0	0	5	1	0	0	x_{136}
x_{137}	170	62	-5600	416	1000	56	-64	-66	768	-384	0	0	0	0	0	0	0	0	0	0	0	0	0	42	6	6	6	0	0	0	0	-6	30	-14	10	-2	0	-32	0	8	0	0	0	0	0	0	0	-6	-2	-1	-1	x_{137}
x_{138}	-495	207	0	0	0	0	0	110	1680	60	-560	105	112	0	-56	-20	28	-15	-13	0	0	0	0	0	0	0	0	15	-1	-1	0	-9	9	-15	3	3	0	0	0	0	0	0	0	0	0	0	0	-10	-2	1	1	x_{138}
x_{139}	-845	127	0	0	-250	350	50	0	-685	1070	-245	35	-125	70	70	-50	-2	-5	0	0	90	-14	0	0	0	-10	-10	10	6	14	8	-7	-25	-5	7	1	0	0	0	30	0	0	-6	0	0	-6	0	0	0	0	0	x_{139}
x_{140}	856	-8	3200	0	0	0	0	-176	0	0	512	0	0	0	-64	0	0	0	11	24	-64	0	0	-104	40	0	0	0	0	0	7	24	-12	0	0	0	-1	0	-16	0	0	0	0	-29	6	-8	0	4	0	0	0	x_{140}
x_{141}	0	0	1420	-804	270	126	-114	-99	-621	0	-189	225	-45	0	0	0	0	9	32	-33	-196	28	0	27	0	0	9	2	14	-10	7	0	0	0	0	0	0	44	-8	-6	12	2	-6	14	0	-7	0	1	-3	0	0	x_{141}
x_{142}	-1329	156	176	80	-24	280	80	0	96	-480	-608	0	96	0	-32	0	48	0	-26	0	26	-14	0	-203	22	12	-6	0	0	0	0	12	3	23	-4	-4	0	16	24	-24	0	0	0	35	0	2	0	0	0	2	2	x_{142}
x_{143}	0	0	-2695	-519	-120	-504	-24	0	-432	0	432	-225	144	0	0	0	0	-9	52	0	-110	-70	0	54	0	0	-9	15	7	-9	0	0	0	0	0	0	0	25	-7	0	9	4	0	-2	12	7	0	0	0	0	0	x_{143}
x_{144}	-1169	154	6600	600	0	0	0	0	-701	10	-357	-155	-141	-70	-6	90	-30	5	0	0	-81	7	0	-77	40	0	0	4	12	-4	-7	-12	15	-1	2	-4	0	-40	12	0	0	0	0	0	0	-3	0	0	0	3	3	x_{144}
x_{145}	-204	66	-3200	-256	400	-176	-16	176	0	0	0	0	0	0	0	0	0	0	-52	0	144	-16	-16	-356	4	14	-4	0	0	0	0	6	-12	4	-2	10	5	0	0	0	0	0	0	16	23	-2	-4	-4	0	0	0	x_{145}
x_{146}	-1026	108	5720	-360	520	120	80	0	1280	1280	0	56	0	0	0	0	0	0	0	0	-144	16	16	308	-34	6	0	0	0	0	0	-8	-18	18	6	0	-6	0	-24	0	-24	0	0	0	-40	2	2	2	0	0	0	x_{146}
x_{147}	-1226	-11	3465	-439	215	511	11	0	751	-185	511	-35	-17	119	79	-65	-5	5	0	0	0	0	0	-7	29	14	2	-4	4	-12	0	-27	-18	6	-3	-3	0	25	1	15	17	-3	3	0	0	0	0	0	0	0	0	x_{147}
x_{148}	40	-68	0	0	0	0	0	120	0	0	0	0	0	0	0	0	0	0	-16	-68	0	64	-20	0	0	0	0	0	0	0	-8	0	0	-8	4	16	0	0	0	0	0	0	0	0	7	-6	4	0	-8	2	2	x_{148}
x_{149}	405	162	-2695	489	80	-336	64	0	1030	-383	-658	7	150	105	17	105	-3	15	0	0	0	0	0	-168	-78	-8	-8	-12	12	-4	0	0	9	-3	-6	0	0	9	-43	-16	-15	0	0	0	0	0	0	0	0	0	0	x_{149}
x_{150}	225	9	0	0	0	0	0	0	595	190	-365	28	19	50	-86	-50	-2	20	0	-39	-30	-22	20	0	0	0	0	-4	0	8	0	-27	9	-15	9	-3	0	0	0	0	0	0	0	-15	-15	6	-3	0	0	0	0	x_{150}
x_{151}	729	0	-6775	-375	0	0	0	-99	-1059	-240	349	0	-3	0	16	0	-24	0	39	0	21	21	0	293	-40	0	0	-8	-16	0	-7	0	9	9	0	0	0	25	17	0	-15	0	0	-21	0	0	0	1	-3	1	1	x_{151}
x_{152}	0	0	-660	60	-585	135	135	0	243	0	-405	0	99	0	0	0	0	0	0	0	48	-24	-3	162	0	0	0	10	-10	-2	-1	0	0	0	0	0	0	28	0	3	-12	3	3	-9	-9	0	0	0	0	-1	-1	x_{152}
x_{153}	0	0	-3795	381	-345	351	291	0	-243	0	405	0	-99	0	0	0	0	0	0	0	24	-24	-24	-162	0	0	0	5	-11	-11	7	0	0	0	0	0	0	-19	9	-9	-3	-3	3	9	9	0	0	0	0	-2	-2	x_{153}
x_{154}	-321	111	-440	136	485	301	-119	0	408	-132	-280	48	56	-84	44	-4	44	0	0	0	196	-28	0	-202	-49	-19	2	0	0	0	7	-9	-9	-1	-1	-1	0	-8	0	-3	-32	3	1	-14	0	7	0	0	0	0	0	x_{154}
x_{155}	0	0	-175	225	0	0	0	176	2160	135	672	0	-48	15	15	-105	63	0	0	0	-155	-51	5	216	0	0	0	0	0	0	1	0	0	0	0	0	0	-15	29	0	-15	0	0	-6	-6	0	3	-4	0	0	0	x_{155}
x_{156}	-900	-171	0	0	0	0	0	0	1100	-430	-420	-28	140	70	-114	50	-22	-20	0	0	90	-14	0	0	0	0	0	-12	0	-8	1	33	-12	12	9	-3	0	0	0	0	0	0	0	0	0	-6	0	0	0	0	0	x_{156}
x_{157}	884	74	1176	600	576	0	0	-176	-1792	-640	0	56	0	0	0	0	0	0	0	0	0	0	0	98	-64	16	-8	0	0	0	0	-48	-12	-12	-6	0	0	40	0	0	0	0	0	0	0	0	0	4	0	-3	-3	x_{157}
x_{158}	814	-131	2080	160	580	-140	60	-136	-1408	-256	0	-64	-128	0	0	64	-32	0	0	0	196	-28	0	134	26	1	10	0	0	0	0	23	-22	6	9	3	1	32	0	-28	0	0	-4	-14	0	7	0	-4	8	0	0	x_{158}
x_{159}	1520	116	0	-1024	0	-384	96	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	64	36	-456	12	-8	-8	0	0	0	0	0	0	16	4	-8	0	0	0	0	0	0	0	24	-18	-6	0	0	-8	0	0	x_{159}
x_{160}	0	0	2871	-9	621	-99	21	0	0	0	0	27	0	0	0	0	0	51	0	0	-54	2	9	0	0	0	15	9	-15	1	0	0	0	0	0	0	0	-9	-9	21	-9	1	-3	0	0	-9	0	0	0	0	0	x_{160}
x_{161}	0	0	3575	375	0	0	0	0	1674	0	594	153	90	0	0	0	0	-15	-26	0	-17	7	0	-189	0	0	0	-3	-7	1	8	0	0	0	0	0	1	-25	-1	0	15	0	0	-35	0	-11	0	0	0	-3	-3	x_{161}
x_{162}	-750	195	0	0	0	0	0	-70	-1400	40	840	64	8	0	-96	-40	-16	0	26	0	225	97	20	0	0	0	0	0	0	0	0	-27	-18	-22	-1	-7	0	0	0	0	0	0	0	40	-2	1	-2	-10	-6	3	3	x_{162}
x_{163}	0	0	2871	-9	246	126	-54	0	0	0	0	-27	0	0	0	0	0	-51	-27	51	-9	7	0	0	0	0	-15	18	-2	6	6	0	0	0	0	0	0	-9	-9	6	-9	-4	-6	0	0	9	0	0	0	0	0	x_{163}
x_{164}	0	0	-3366	-630	-216	0	60	0	0	0	0	-216	0	0	0	0	0	0	0	0	81	49	0	0	0	0	-24	12	0	8	0	0	0	0	0	0	-4	-6	-18	-36	-6	-6	0	0	0	9	0	0	0	0	0	x_{164}
x_{165}	261	-414	-1100	-220	-275	245	25	0	399	30	-1	-63	15	70	-82	-90	-6	25	0	0	-26	14	0	175	22	2	-10	-22	14	6	0	18

$M = F_1$

	24A	24B	24C	24D	24E	24F	24G	24H	24I	24J	25A	26A	26B	27A	27B	28A	28B	28C	28D	29A	30A	30B	30C	30D	30E	30F	30G	31A	B**	32A	32B	33A	33B	34A	35A	35B	36A	36B	36C	36D	38A	39A	39B	39C	D**	40A	40B	40C	D**	41A	42A	42B	42C	42D	44A	B**	45A	
x_{96}	0	0	0	-8	0	0	0	4	0	0	0	0	0	0	0	0	-8	0	0	0	2	1	1	7	2	2	2	0	0	0	0	2	-1	-1	5	0	0	-3	0	-1	0	0	0	0	0	0	0	-1	-1	0	2	0	1	0	-2	-2	0	x_{96}
x_{97}	0	0	0	0	0	0	0	0	0	0	0	4	2	0	0	0	0	0	0	0	0	-10	-6	0	0	0	0	1	1	0	0	-4	-1	0	5	0	-4	2	0	0	0	3	0	-1	-1	0	0	0	0	-1	0	2	-2	-1	0	0	-1	x_{97}
x_{98}	0	0	0	0	0	0	0	0	0	0	-1	0	0	-6	3	0	0	0	2	0	15	7	3	0	3	-1	-1	0	0	0	0	-2	-2	0	0	1	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-1	-2	-2	0	x_{98}
x_{99}	0	0	-6	0	0	0	0	0	-2	0	2	0	2	0	0	0	0	0	0	0	9	0	0	-6	0	1	1	0	0	-1	-1	-3	0	0	0	0	0	0	0	0	0	0	0	b39	**	-3	1	0	0	1	0	0	0	0	2	2	0	x_{99}
x_{100}	0	0	-6	0	0	0	0	0	-2	0	2	0	2	0	0	0	0	0	0	0	9	0	0	-6	0	1	1	0	0	-1	-1	-3	0	0	0	0	0	0	0	0	0	0	0	**	b39	-3	1	0	0	1	0	0	0	0	2	2	0	x_{100}
x_{101}	-9	-9	0	3	0	0	0	0	0	0	2	0	0	0	0	-8	0	-4	3	0	0	6	-6	0	0	0	0	0	0	1	1	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	3	0	0	3	0	0	0	x_{101}
x_{102}	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	x_{102}
x_{103}	0	0	0	0	0	0	0	0	0	0	-2	0	0	-2	-2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	x_{103}
x_{104}	-3	5	8	1	3	2	-2	4	0	-1	0	0	0	0	0	1	1	5	-1	0	0	1	-7	5	0	0	0	0	0	1	1	0	0	0	-4	0	1	1	1	1	-1	0	0	0	0	-1	3	0	0	0	-1	5	-1	-1	0	0	-1	x_{104}
x_{105}	0	0	0	0	-3	0	0	0	0	-3	2	0	0	0	0	-3	1	-3	0	0	0	0	0	0	0	0	0	b31	**	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	0	0	0	0	x_{105}
x_{106}	0	0	0	0	-3	0	0	0	0	-3	2	0	0	0	0	-3	1	-3	0	0	0	0	0	0	0	0	0	**	b31	0	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	0	0	0	0	x_{106}
x_{107}	0	0	0	0	3	0	0	0	0	3	3	0	0	0	0	-6	2	-2	0	0	0	0	0	0	0	0	0	2b31	**	-1	-1	0	0	0	0	2	0	0	0	0	-1	0	0	0	0	1	1	0	0	-1	0	0	1	0	0	0	0	x_{107}
x_{108}	0	0	0	0	3	0	0	0	0	3	3	0	0	0	0	-6	2	-2	0	0	0	0	0	0	0	0	0	**	2b31	-1	-1	0	0	0	0	2	0	0	0	0	-1	0	0	0	0	1	1	0	0	-1	0	0	1	0	0	0	0	x_{108}
x_{109}	-5	-5	7	-1	-2	-1	-1	-1	-1	2	-1	1	0	1	1	0	0	0	0	0	24	-3	1	4	-3	0	2	0	0	0	0	0	0	0	0	0	6	0	0	0	0	-3	0	0	0	-2	0	-1	-1		0	0	0	0	0	0	2	x_{109}
x_{110}	-3	5	-4	-7	0	0	0	2	0	0	0	0	0	0	0	14	6	2	-2	0	-3	-13	-1	2	0	1	1	0	0	-1	-1	0	0	0	-2	-1	-3	0	1	0	0	0	0	0	0	-3	-1	1	1	0	1	-1	0	2	0	0	-1	x_{110}
x_{111}	6	-10	2	2	3	-4	-4	2	2	-1	0	0	0	-1	-1	-5	-1	3	0	1	0	0	0	0	0	0	0	-1	-1	0	0	-1	-1	-2	0	0	9	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	1	1	0	x_{111}
x_{112}	0	0	0	0	0	0	0	0	0	0	3	-4	-6	2	2	0	0	0	0	0	18	0	-8	-2	0	-2	2	0	0	0	0	-2	4	0	0	0	-2	-2	0	0	0	-5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	x_{112}
x_{113}	0	0	-3	0	0	-3	-3	3	1	0	-3	0	0	0	0	-9	3	-1	-1	0	-3	0	0	-3	0	-3	3	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	1	3	-1	-1	0	0	3	0	0	0	0	0	x_{113}
x_{114}	-3	5	5	9	0	3	-1	3	1	0	0	0	0	0	0	-8	-12	-4	0	0	0	0	0	0	0	0	0	2	2	-1	-1	-1	-1	0	0	0	1	-2	-1	0	0	0	0	0	0	0	0	0	0	0	-3	4	2	1	1	1	0	x_{114}
x_{115}	5	-3	-6	9	2	2	-2	0	-2	2	2	0	0	6	-3	-1	7	-1	0	0	-1	1	1	4	2	-1	-1	0	0	0	0	0	0	0	0	0	-4	-4	0	0	0	0	0	0	0	-2	0	1	1	0	1	0	1	0	0	0	0	x_{115}
x_{116}	0	0	12	-8	0	0	0	-2	0	0	-1	0	0	0	0	-12	0	-4	0	0	9	-7	-7	-1	0	1	-1	0	0	0	0	-3	0	0	2	-1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	-2	0	-2	2	2	0	x_{116}
x_{117}	12	4	-5	4	0	-3	1	1	-1	0	0	-3	0	0	0	0	0	0	0	0	-25	-5	11	5	2	3	-1	0	0	-2	2	-3	0	0	0	0	0	0	0	0	0	4	1	0	0	0	0	-1	-1	0	0	0	0	0	2	2	-2	x_{117}
x_{118}	9	1	-8	1	0	0	0	4	0	0	-1	1	0	0	0	0	0	0	0	0	6	4	-12	-9	0	2	-2	0	0	0	0	1	1	0	0	0	-3	0	1	0	-2	-3	0	0	0	3	-1	0	0	0	0	0	-2	0	-1	-1	-1	x_{118}
x_{119}	0	0	0	0	0	0	0	0	0	0	0	4	0	1	1	0	8	0	0	2	0	10	6	0	0	0	0	0	0	0	0	2	-4	-2	-5	0	2	-4	0	0	0	3	0	-2	-2	0	0	0	0	0	-2	0	-1	0	-2	-2	1	x_{119}
x_{120}	9	1	10	-3	0	0	0	0	-2	0	0	0	2	0	0	1	-3	1	0	-2	0	10	6	0	0	0	0	0	0	0	0	0	3	0	-5	0	3	0	-1	0	0	0	0	1	1	0	0	0	0	0	3	-2	0	-2	-1	-1	1	x_{120}
x_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	8	0	0	0	2	-3	5	2	-4	-2	0	-1	-1	0	0	0	0	2	-5	0	-7	-1	-1	-1	1	0	0	0	0	0	0	0	0	0	-2	0	-1	0	0	0	0	x_{121}
x_{122}	0	0	0	0	3	0	0	0	0	3	0	-3	3	0	0	-3	5	1	3	0	0	0	0	0	3	0	0	-1	-1	-1	-1	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	-1	0	0	0	0	x_{122}
x_{123}	0	0	0	0	0	0	0	0	0	0	-1	2	0	-1	-1	16	0	0	0	0	0	8	0	0	0	0	0	1	1	0	0	0	0	-2	-2	0	0	0	-4	0	0	-1	-1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	x_{123}
x_{124}	0	0	0	0	0	0	0	0	0	0	-1	-2	0	-1	-1	0	0	0	0	0	0	-8	0	0	0	0	0	1	1	0	0	0	0	2	-2	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	x_{124}
x_{125}	0	0	0	0	0	0	0	0	0	0	-1	-2	0	-1	-1	0	0	0	0	0	0	-8	0	0	0	0	0	1	1	0	0	0	0	2	-2	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	x_{125}
x_{126}	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-4	-4	0	0	7	-2	2	2	-2	3	1	0	0	0	0	0	0	0	-8	2	2	-1	0	-1	-2	0	0	0	0	0	0	0	0	0	4	0	-1	0	0	0	0	x_{126}
x_{127}	-8	0	-6	0	1	-2	2	0	-2	1	0	3	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	6	2	2	0	2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x_{127}
x_{128}	0	0	6	0	0	-6	-2	0	2	0	0	2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	-1	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	x_{128}
x_{129}	0	0	6	0	0	-6	-2	0	2	0	0	2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	-1	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	x_{129}
x_{130}	0	8	2	0	0	0	0	0	-2	0	0	0	0	0	0	-8	12	4	2	0	0	-10	-6	0	0	0	0	0	0	0	0	3	0	-2	5	0	5	-4	-3	0	0	0	0	0	0	0	0	0	0	1	-1	-3	0	0	-2	-2	-1	x_{130}
x_{131}	-5	3	0	3	1	0	0	0	0	-3	0	-3	-3	0	0	0	-8	0	0	0	0	7	3	0	-3	0	0	0	0	0	0	0	-3	0	5	0	0	0	0	0	0	-4	2	0	0	-1	1	0	0	1	2	0	1	0	0	0	0	x_{131}
x_{132}	-7	1	1	5	-4	-3	1	-1	-3	0	0	0	0	2	2	0	0	0	0	0	1	-2	-2	1	-2	-3	-1	0	0	-2	2	0	0	0	0	0	1	4	1	0	0	0	0	0	0	-3	1	1	1	0	0	0	0	0	0	0	2	x_{132}
x_{133}	0	0	0	0	0	0	-8	0	0	0	1	1	0	-1	-1	8	0	0	0	-1	0	-12	0	0	0	0	0	0	0	0	0	4	1	1	3	0	0	0	-2	0	-1	3	0	0	0	-4	0	0	0	0	1	0	0	0	0	0	1	x_{133}
x_{134}	-3	-3	-3	5	0	3	-1	-1	1	0	0	-3	2	1	1	0	0	0	0	0	0	-5	7	-5	0	0	0	0	0	0	0	3	0	0	0	0	-1	2	1	0	0	2	-1	-1	-1	0	2	0	0	0	0	0	0	0	-2	-2	1	x_{134}
x_{135}	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	0	0	0	0	0	2	4	-4	2	2	-2	0	0	0	0	0	-1	-1	0	0	0	-4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2111	-1	-2111	-1	1	x_{135}
x_{136}	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	1	0	0	0	0	0	2	4	-4	2	2	-2	0	0	0	0	0	-1	-1	0	0	0	-4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2111	-1	2111	-1	1	x_{136}
x_{137}	0	0	0	0	0	0	0	0	0	0	-2	0	0	2	2	0	0	0	0	0	2	-2	2	2	2	-2	2	1	1	0	0	-3	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	x_{137}
x_{138}	16	-8	-8	0	1	-2	2	0	0	1	0	3	0	-6	3	0	0	0	0	-2	0	0	0	0	0	0	0	2	2	-1	-1	2	2	0	0	0	-3	-3	1	1	0	-4	-1	0	0	0	0	0	0	0	0	0	0	0	2	2	0	x_{138}
x_{139}	-9	7	-2	3	3	2	-2	-6	-2	-1	0	0	0	1	1	6	-6	-2	0	0	-10	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	-1	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	x_{139}
x_{140}	0	0	0	0	0	0	8	0	0	0	0	3	0	-2	-2	-8	0	0	0	-1	0	-4	0	0	0	0	0	1	1	0	0	4	1	-1	1	0	0	0	2	0	-1	2	-1	0	0	4	0	0	0	0	-1	0	0	0	0	0	1	x_{140}
x_{141}	-5	3	0	3	1	0	0	0	0	-3	0	0	3	0	0	0	4	-4	0	0	0	7	3	0	-3	0	0	0	0	0	0	0	-3	-1	-1	0	0	0	0	0	0	-4	2	0	0	2	0	0	0	0	2	0	1	0	1	1	0	x_{141}
x_{142}	0	0	0	0	0	0	0	0	0	0	-3	-2	0	0	0	-2	-6	-2	0	0	-8	5	5	2	-2	0	-4	0	0	0	0	0	0	0	-3	0	3	0	1	0	0	1	1	0	0	0	0	0	0	0	-1	0	-2	0	0	0	1	x_{142}
x_{143}	-16	0	0	0	-1	0	0	0	0	3	0	-4	0	0	0	-2	2	-2	0	1	0	14	6	0	3	0	0	0	0	-1	-1	0	0	0	4	0	0	0	0	0	0	-2	-2	0	0	1	1	0	0	0	-2	0	-1	0	0	0	0	x_{143}
x_{144}	-1	7	10	-5	5	-6	-2	-2	2	-3	0	0	0	-2	-2	3	-1	3	0	0	0	3	3	0	0	0	0	0	0	0	0	0	0	0	1	3	0	-5	-2	3	0	0	0	0	0	2	-4	0	0	0	0	0	1	0	0	0	1	x_{144}
x_{145}	0	0	0	0	0	0	0	0	0	0	-2	4	0	0	0	0	0	0	0	0	-14	4	-4	-4	4	-2	2	0	0	0	0	-4	-1	0	-5	1	0	0	0	0	1	2	2	0	0	0	0	0	0	0	0	-4	2	-1	0	0	1	x_{145}
x_{146}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-4	-12	-6	0	-2	2	1	1	0	0	0	0	0	9	1	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	0	0	-1	x_{146}
x_{147}	-9	-1	5	-1	-3	1	5	-1	1	1	0	0	0	-2	-2	0	0	0	0	0	-2	9	-7	-7	-2	2	2	0	0	0	0	0	0	0	0	0	-2	1	-2	1	0	0	0	0	0	-3	1	1	1	0	0	0	0	0	0	0	-1	x_{147}
x_{148}	0	0	0	0	0	0	0	0	0	0	0	0	-4	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	0	0	3	0	0	0	0	0	0	0	0	0	2	-1	1	1	0	0	0	0	0	0	4	-2	1	0	0	0	x_{148}
x_{149}	2	2	11	6	-1	-1	7	-3	-1	-1	0	0	0	0	0	0	0	0	0	0	6	-4	0	6	0	2	4	0	0	0	0	0	0	0	0	0	1	-2	-1	0	0	0	0	0	0	-1	-3	0	0	0	0	0	0	0	0	0	0	x_{149}
x_{150}	11	3	-6	-5	-4	0	0	-2	2	0	0	0	-3	6	-3	2	-6	-2	4	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	-3	5	2	-1	0	0	0	x_{150}
x_{151}	-3	5	8	-3	0	0	0	0	0	0	0	-1	0	0	0	-7	-3	5	0	0	0	-7	-3	0	0	0	0	0	0	0	0	0	-3	1	-7	0	-3	0	1	0	0	3	0	0	0	-3	1	0	0	0	3	0	0	0	1	1	-1	x_{151}
x_{152}	-9	-9	0	3	0	0	0	0	0	0	0	0	0	0	0	4	0	4	-3	0	0	6	-6	0	0	0	0	0	0	0	0	0	0	0	-1	-2	0	0	0	0	0	0	0	0	0	2	0	1	1	0	3	0	0	3	0	0	0	x_{152}
x_{153}	9	9	0	-3	0	0	0	0	0	0	2	0	0	0	0	-4	0	0	0	0	0	-6	6	0	0	0	0	0	0	1	1	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	3	1	1	1	0	-3	0	0	-3	0	0	0	x_{153}
x_{154}	0	0	0	-8	0	0	0	4	0	0	2	0	0	0	0	-4	4	0	0	0	-5	10	-2	-5	-2	-1	1	0	0	0	0	0	0	-1	1	0	3	3	-1	-1	0	0	0	0	0	0	0	-1	-1	0	-2	0	-1	0	0	0	-1	x_{154}
x_{155}	0	-8	7	8	0	3	3	-1	-1	0	0	0	0	0	0	-3	-3	1	1	-1	0	-4	0	0	0	0	0	0	0	0	0	0	-4	-1	1	-4	0	0	0	0	0	0	0	0	0	-1	-3	0	0	0	-2	-1	0	2	0	0	0	x_{155}
x_{156}	4	-4	2	-4	4	0	0	2	2	0	0	0	0	0	0	6	-6	-2	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	1	0	0	-4	-1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	0	x_{156}
x_{157}	0	0	0	0	0	0	0	0	0	0	-3	0	0	2	2	0	0	0	0	0	0	6	-6	0	0	0	0	2	2	0	0	4	1	0	0	0	-4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	x_{157}
x_{158}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	-4	4	0	0	13	10	-2	-2	-2	1	-3	0	0	0	0	-1	-1	0	1	0	2	-1	0	1	1	0	0	0	0	0	0	0	0	0	-2	0	-1	0	0	0	-1	$x_{158}

$M = F_1$

	46A	B**	46C	D**	47A	B**	48A	50A	51A	52A	52B	54A	55A	56A	56B	C**	57A	59A	B**	60A	60B	60C	60D	60E	60F	62A	B**	66A	66B	68A	69A	B**	70A	70B	71A	B**	78A	78B	C**	84A	84B	84C	87A	B**	88A	B**	92A	B**	93A	B**	94A	B**	95A	B**	104A	B**	105A	110A	119A	B**	
X_{96}	1	1	1	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	1	1	2	-2	-1	0	0	0	-1	0	-1	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	0	-1	-1	X_{96}
X_{97}	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-2	0	0	0	0	-1	-1	-1	0	0	1	1	1	0	0	0	1	-1	-1	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	-1	1	0	0	X_{97}
X_{98}	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0	1	3	-1	-1	0	-1	0	0	-2	2	0	0	0	0	-1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	X_{98}
X_{99}	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	-1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0-b39	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	X_{99}
X_{100}	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	-1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	**-b39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	X_{100}
X_{101}	-1	-1	-1	-1	0	0	1	0	-1	0	0	0	0	2	1	1	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	-1	0	0	0	1	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	1	0	1	1	X_{101}
X_{102}	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	b59	**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	-1	-1	0	0	-2	0	1	1	X_{102}
X_{103}	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	**	b59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	-1	-1	0	0	-2	0	1	1	X_{103}
X_{104}	0	0	0	0	0	0	-1	0	-1	0	0	0	0	-1	-1	-1	1	0	0	-3	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	X_{104}
X_{105}	0	0	0	0	1	1	0	0	0	0	0	0	0	-1	0	0	-1	-1	-1	0	0	0	0	0	2	b31	**	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	b31	**	-1	-1	1	1	0	0	0	0	0	0	X_{105}
X_{106}	0	0	0	0	1	1	0	0	0	0	0	0	0	-1	0	0	-1	-1	-1	0	0	0	0	0	2	**	b31	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	**	b31	-1	-1	1	1	0	0	0	0	0	0	X_{106}
X_{107}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0-b31	**	0	0	0	0	0	0	0	0	0	0	0	X_{107}
X_{108}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	**-b31	0	0	0	0	0	0	0	0	0	0	0	X_{108}
X_{109}	0	0	0	0	0	0	1	-1	0	1	0	-1	0	0	0	0	0	0	0	-1	1	2	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	X_{109}
X_{110}	0	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	3	-1	-1	0	0	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	X_{110}
X_{111}	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	2	0	0	0	0	0	0	0	0	0	1	0	-1	1	1	1	1	0	0	-1	-1	0	0	0	0	0	0	0	0	-1	-1	X_{111}
X_{112}	0	0	0	0	0	0	0	-1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{112}
X_{113}	-1	-1	1	1	0	0	0	-1	0	0	0	0	0	1	1	1	0	0	0	0	0	-1	1	1	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	X_{113}
X_{114}	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	X_{114}
X_{115}	0	0	0	0	0	0	-1	0	0	0	0	0	0	-1	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{115}
X_{116}	0	0	0	0	0	0	0	1	0	0	0	0	-1	2	0	0	0	0	0	1	1	-1	1	-1	0	0	0	0	1	0	0	0	-2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	X_{116}
X_{117}	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	0	-1	-1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	-1	0	0	X_{117}
X_{118}	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	0	X_{118}
X_{119}	0	0	0	0	1	1	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	-1	-1	-1	0	0	0	0	1	1	0	0	0	0	1	-1	1	1	0	X_{119}
X_{120}	0	0	0	0	0	0	-1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	0	0	0	0	-1	0	0	0	0	-1	-1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	X_{120}
X_{121}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	1	1	2	0	0	0	1	1	0	0	-2	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	-1	-1	0	0	1	1	0	0	1	0	1	1	X_{121}
X_{122}	0	0	0	0	0	0	0	0	0	1	-1	0	0	-1	-1	-1	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	-1	-1	0	0	0	0	1	1	0	0	-1	-1	X_{122}
X_{123}	0	0	0	0	1	1	0	1	0	2	0	-1	0	0	0	0	0	0	0	-4	0	0	0	0	0	1	1	0	0	2	0	0	2	0	-1	-1	-1	0	0	-2	0	0	0	0	0	0	0	0	1	1	-1	-1	0	0	0	0	1	0	-1	-1	X_{123}
X_{124}	0	0	0	0	1	1	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	-2	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	126	-126	1	0	-1	-1	X_{124}
X_{125}	0	0	0	0	1	1	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	-2	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	-126	126	1	0	-1	-1	X_{125}
X_{126}	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	X_{126}
X_{127}	0	0	0	0	0	0	0	0	0	-1	-2	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	-1	-1	0	0	0	0	X_{127}
X_{128}	2b23	**	0	0	0	0	0	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0-b23	**	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{128}
X_{129}	**	2b23	0	0	0	0	0	0	0	2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**-b23	0	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{129}
X_{130}	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	X_{130}
X_{131}	0	0	0	0	0	0	-1	0	1	1	-1	0	-1	0	0	0	0	0	0	1	-1	0	0	0	-1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	-1	X_{131}
X_{132}	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	1	0	0	-2	2	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	X_{132}
X_{133}	0	0	0	0	0	0	0	-1	1	1	0	1	-1	0	0	0	-1	0	0	-2	0	0	0	0	0	0	0	1	0	1	0	0	-1	0	1	1	1	0	0	-1	0	0	-1	-1	0	0	0	0	0	0	0	0	-1	-1	1	1	0	-1	1	1	X_{133}
X_{134}	-2	-2	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	-3	-1	0	0	-1	0	0	0	0	-1	0	1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	X_{134}
X_{135}	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	-1	-1	0	0	2	0	0	0	0	0	-1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0-i11	i11	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{135}
X_{136}	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	-1	-1	0	0	2	0	0	0	0	0	-1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	i11-i11	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{136}
X_{137}	-1	-1	1	1	-1	-1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	-2	-4	0	0	0	1	1	0	1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-1	-1	0	0	0	0	0	-1	0	0	X_{137}
X_{138}	1	1	-1	-1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	-1	-1	-1	-1	0	0	0	0	1	1	0	0	0	0	X_{138}
X_{139}	0	0	0	0	0	0	1	0	-1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	X_{139}
X_{140}	0	0	0	0	1	1	0	0	1	-1	0	0	-1	0	0	0	0	-1	0	0	2	0	0	0	0	0	1	1	1	0	-1	0	0	1	0	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	1	1	-1	-1	-1	-1	-1	-1	1	-1	0	X_{140}
X_{141}	0	0	0	0	-1	-1	-1	0	1	0	1	0	1	0	0	0	0	0	0	0	1	-1	0	0	0	-1	0	0	1	0	-1	0	0	-1	0	0	0	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	1	1	0	0	0	0	-1	1	0	X_{141}
X_{142}	2	2	0	0	0	0	0	1	0	-2	0	0	0	0	0	0	0	0	0	-3	1	0	0	0	0	0	0	0	0	0	-1	-1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{142}
X_{143}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-2	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	-2	0	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	-2	0	0	0	X_{143}
X_{144}	-1	-1	-1	-1	0	0	1	0	-1	0	0	0	0	1	0	0	0	0	0	3	-1	0	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	-1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	X_{144}
X_{145}	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1	-1	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	X_{145}
X_{146}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	-1	-1	0	X_{146}
X_{147}	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{147}
X_{148}	2	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	-1	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	-1	-1	0	0	X_{148}
X_{149}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{149}
X_{150}	0	0	0	0	0	0	-1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X_{150}
X_{151}	1	1	-1	-1	1	1	1	0	-1	-1	0	0	1	1	0	0	0	0	0	-1	1	0	0	0	0	0	0	1	0	1	1	1	1	0	0	0	-1	0	0	-1	0	0	0	0	-1	-1	-1	-1	0	0	-1	-1	0	0	1	1	-1	1	0	0	X_{151}
X_{152}	-1	-1	1	1	0	0	1	0	-1	0	0	0	0	-2	-1	-1	0	0	0	0	-2	0	0	0	0	0	0	0	0	-1	-1	-1	-2	2	1	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	-1	-1	X_{152}
X_{153}	-2	-2	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	-1	1	1	-1	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	X_{153}
X_{154}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-2	3	1	1	0	0	0	0	0	-1	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	X_{154}
X_{155}	0	0	0	0	0	0	0	0	1	0	0	0	1	-1	-1	-1	0	0	0	2	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	X_{155}
X_{156}	0	0	0	0	-1	-1	0	0	1	0	0	0	0	0	0	0	0	-1	-1	0	0

$M = F_1$

Sporadic Fischer-Griess "Monster" or "Friendly Giant" group $M \equiv FG \equiv F_1$

Order = 808,017,424,794,512,875,886,459,904,961,710,757,005,754,368,000,000,000

$= 2^{46}.3^{20}.5^{9}.7^{6}.11^{2}.13^{3}.17.19.23.29.31.41.47.59.71$

Mult = 1 Out = 1

Constructions

Griess G : the group of orthogonal automorphisms of a certain 196884-dimensional algebra. There is a fixed vector I, and G acts irreducibly on the 196883-space orthogonal to I. A base for the space can be defined in terms of the action of the maximal subgroup $2^{1+24}Co_1$. The algebra and an element of G outside this subgroup have been computed explicitly in this base. The multiplication used below differs slightly from that of Griess, so care is necessary in comparing results.

Algebra The inner product (a,b) and algebra product a * b defined later have the following properties:

a * b = b * a , and (a,b) is positive definite;

(a * b , c) = (a , b * c) = (a,b,c), say;

if (a * a) * c = a * (a * c), then (a * b) * c = a * (b * c) for all b (and we say that a and c alternate);

(a * a , c * c) $\geq$ (a * c , a * c) , with equality just when a and c alternate.

If a set S is closed under taking algebra squares s * s , then its annihilator { x | x * s = 0 for all s in S } and alternator { x | x alternates with all s in S } are subalgebras. If elements v_1, v_2, ... all alternate with their own squares and with each other, then the subalgebra they generate is associative.

The algebra has a unit 1, and the projection 1_S of 1 onto any subalgebra S is a unit for S. For an idempotent i, the eigenvalues of the map $x \longrightarrow x * i$ lie in [0,1], and the 0- and 1-eigenspaces $V_0(i)$ and $V_1(i)$ are subalgebras with $V_0(i) * V_1(i) = 0$. Moreover, if i and j are idempotents with (i,j) = 0, then $V_1(i) * V_1(j) = 0$.

To each element of class 2A there is a corresponding transposition vector t with t * t = 64t, and there is a lattice containing 1 and these vectors that is closed under algebra multiplication. When equipped with the inner product 2(a,b) this lattice is integral (and probably unimodular). For further details see the section "Vectors" below.

In the following sections, we shall describe a simplified construction for G and the algebra using the loop P described under Fi_{24}', section "Loop". The several sections construct:

The group N, which is a 4-fold cover of the normalizer N_0 of a certain four-group {1,x,y,z} in the Monster.

The Leech link, which is a homomorphism from a certain group Q_x onto the Leech lattice modulo 2.

Representations of various groups, which can be assembled into representations of degree 196884 for N_x, N_y, N_z, the centralizers of x, y, z in N.

A dictionary which identifies the 196884-spaces for N_x, N_y, N_z compatibly with the action of their intersection N_{xyz}.

A multiplication on the resulting 196884-space, invariant under G.

Enlargements of certain quotients N_{x0}, N_{y0}, N_{z0} of N_x, N_y, N_z to groups G_{x0}, G_{y0}, G_{z0} which preserve the algebra and generate the Monster, in which they are the centralizers of x, y, z.

Vectors corresponding to various elements of G. The action of a transposition can be defined in terms of the corresponding vector and the algebra structure.

The group N This is defined using the loop P described under Fi_{24}', section "Loop". It is the group of permutations of the triples (A,B,C) of loop elements with ABC = 1 generated by the particular maps (specified by the image of (A,B,C)):

x_D : $(\bar{D}A\bar{D}, DB, CD)$ y_D : $(AD, \bar{D}B\bar{D}, DC)$ z_D : $(DA, BD, \bar{D}C\bar{D})$ (D in P)

$x_S = y_S = z_S$: (A^S, B^S, C^S) ($U^S = U$, i.e. S even)

x_S : $(\bar{A}^S, \bar{C}^S, \bar{B}^S)$ y_S : $(\bar{C}^S, \bar{B}^S, \bar{A}^S)$ z_S : $(\bar{B}^S, \bar{A}^S, \bar{C}^S)$ ($U^S = -U$, i.e. S odd)

where S is a standard automorphism of P and $\bar{A} = A^{-1}$ is the loop inverse of A. Important particular cases of x_S, y_S, z_S are x_d, y_d, z_d for d a C^*-set. The composition factors in the structure $(2^2x2^2).2^{11}.2^{22}.(S_3xM_{24})$ are defined modulo earlier ones by

2^2 : $K = \{ 1, k_1=y_Uz_{-U}, k_2=z_Ux_{-U}, k_3=x_Uy_{-U} \}$.

2^2 : $V = \{ 1, x=x_{-1}, y=y_{-1}, z=z_{-1} \}$

2^{11} : $\langle x_d \mid$ d an even C^*-set $\rangle$

2^{22} : $\langle x_D, y_D, z_D \mid$ D in P $\rangle$, in which we have $x_Dy_Dz_D=1$

S_3 : $\langle x_i, y_i, z_i \rangle$ for some i in [U], regarded as an (odd) C^*-set

M_{24} : $\langle x_S$ or y_S or $z_S \mid$ all S $\rangle$, in which x_S, y_S, z_S all act as [S], the permutation corresponding to S.

By considering the subgroups S_2 and S_1 in S_3, we see that N has subgroups N_x, N_y, N_z of index 3 whose intersection N_{xyz} has index 6 in N. The group $N_x = (2^2x2^2).2^{11}.2^{22}.(S_2xM_{24})$ has the alternative structure $2.2^{1+24}.(2^{12}:M_{24})$ in which the normal subgroup 2.2^{1+24} is called Q_x and is generated by the x_D and x_d, while the quotient N_x/Q_x is isomorphic to the group of monomial automorphisms of the Leech lattice, and is generated by y_D (or z_D), acting as the sign-change on [D], and x_S, acting as the permutation [S].

The Leech link The typical element $x_A x_B x_C \ldots$ of Q_x is written $x_{A.B.C.\ldots}$, and we write $x_{-R} = x_{-1.R}$. The formula $x_D x_E = z^{\frac{1}{2}|d|} x_{DE} x_d$, where d is the intersection of [D] and [E], regarded as a C^*-set, shows that Q_x contains z, and so contains $k_1 = x_U z$. There is a natural homomorphism taking $x_R \to v_R$, from Q_x onto the Leech lattice modulo 2, defined by

$$x_D \to (2_{\text{on } [D]}, 0_{\text{elsewhere}}) \qquad x_i \to (-3_{\text{on } i}, 1_{\text{elsewhere}})$$

where i is one of the 24 points of [U], regarded as a C^*-set. The preimages of the 98280 type 2 vectors of the Leech lattice (modulo 2) are called the <u>short elements</u> of Q_x. We use x_R^+ and x_R^- to mean $x_{U.R}$ and $x_{-U.R}$ when these are short, and otherwise x_R. Then typical short elements, and their images in the Leech lattice, are

$$x_{\pm ij} \to (4_{\text{on } i}, -4_{\text{on } j}, 0_{\text{elsewhere}})$$
$$x^+_{\pm ij} \to (4_{\text{on } i,j}, 0_{\text{elsewhere}})$$
$$x^+_{D.i} \to (-3_{\text{on } i}, 1_{\text{elsewhere}}) \text{ sign-changed on } [D]$$
$$x^+_{D.d} \to (2_{\text{on } [D]}, 0_{\text{elsewhere}}) \text{ sign-changed on } d$$

where in the last line [D] is an octad and d an even subset of it.

Representations The group N_x has the following representations (named by their degrees subscripted by x):

24_x : kernel $\langle x_D, x_d \rangle$ of order 2^{26}, image $2^{12}{:}M_{24}$.

This acts on the space generated by 24 orthogonal vectors i_1 (i in [U]) of norm $\frac{1}{8}$. The image is the group of monomial automorphisms of a copy of the Leech lattice L (with $(\ldots, x_i, \ldots)$ interpreted as $\Sigma x_i i_1$) in which y_D and z_D act as the sign-change on [D], and x_S as the permutation [S] in M_{24} that corresponds to S.

4096_x : kernel $\langle y_U, z_{-U} \rangle$ of order 2^2, image $2^{1+24}.(2^{11}{:}M_{24})$.

This is contained in the matrix normalizer of the faithful representation of the extraspecial group 2^{1+24}. There are 2 x 4096 norm 1 basis vectors D_1^- and D_1^+, permuted like the triples (?,D,?) and (?,?,D) under conjugation (? indicates an immaterial entry) with the identifications $(-UD)_1^- = D_1^-$, $(UD)_1^+ = D_1^+$, $(-D)_1^\pm = -(D_1^\pm)$.

98280_x : kernel $\langle k_1, k_2, k_3, x \rangle$ of order 2^3.

This is a monomial representation on norm 1 vectors X_R with $X_R = -X_R$ permuted in the way that the short elements x_R of Q_x are permuted under conjugation by the quotient group $N_{x1} = N_x/\langle k_1 \rangle$ of N_x.

300_x : the symmetric tensor square of 24_x.

This acts on symmetric 24 x 24 matrices A. As basis elements, we take the matrices $(ij)_1$ (for $i \neq j$) and $(ii)_1$, whose only non-zero entries are 1 in places (i,j) and (j,i) (for $(ij)_1$) and (i,i) (for $(ii)_1$). In tensor notation we have $(ij)_1 = i_1 \otimes j_1 + j_1 \otimes i_1$ and $(ii)_1 = i_1 \otimes i_1$.

98304_x : the tensor product of 4096_x and 24_x. We take the basis elements $D_1^\pm \otimes i_1$ to have norm 1.

Dictionary We define $196884_x = 300_x + 98280_x + 98304_x$. Since K is in the kernel, this can be regarded as a representation of $N_{x0} = N_x/K$. We define similar representations 196884_y of N_y or N_{y0} and 196884_z of N_z or N_{z0} by cyclically permuting the letters (x,y,z), (X,Y,Z), (A,B,C), and the subscripts (1,2,3) in all our notations. Thus there is a representation 300_y of N_y with basis elements $(ij)_2$ and $(ii)_2$, and typical element B. The dictionary reads:

(A,X,1)-name		(B,Y,2)-name		(C,Z,3)-name	
$(ij)_1$	=	$Y_{ij} + Y^+_{ij}$	=	$Z_{ij} - Z^+_{ij}$	
$X_{ij} - X^+_{ij}$	=	$(ij)_2$	=	$Z_{ij} + Z^+_{ij}$	
$X_{ij} + X^+_{ij}$	=	$Y_{ij} - Y^+_{ij}$	=	$(ij)_3$	
$(ii)_1$	=	$(ii)_2$	=	$(ii)_3$	= (ii), say
$X_{D.i}$	=	$D_2^+ \otimes i_2$	=	$D_3^- \otimes i_3$	
$D_1^- \otimes i_1$	=	$Y_{D.i}$	=	$D_3^+ \otimes i_3$	
$D_1^+ \otimes i_1$	=	$D_2^- \otimes i_2$	=	$Z_{D.i}$	
$X^+_{D.d}$	=	$\frac{1}{8}\Sigma -_e -_d e Y^+_{D.e}$	=	$\frac{1}{8}\Sigma -_d -_d e Z^+_{D.e}$	
$\frac{1}{8}\Sigma -_d -_d e X^+_{D.e}$	=	$Y^+_{D.d}$	=	$\frac{1}{8}\Sigma -_e -_d e Z^+_{D.e}$	
$\frac{1}{8}\Sigma -_e -_d e X^+_{D.e}$	=	$\frac{1}{8}\Sigma -_d -_d e Y^+_{D.e}$	=	$Z^+_{D.d}$	

The sums in the last three rows are over all C^*-sets e which can be represented by even subsets of [D], and the symbol $-_d$ means $(-1)^{\frac{1}{2}|d|}$. In the three previous rows it is supposed that i is not in [D].

Multiplication There is a symmetric invariant multiplication * defined on 196884-space. We specify it below in the notation of 196884_x.

$$A * A' = 2(AA' + A'A)$$
$$X_R * A = A * X_R = (A, v_R \otimes v_R).X_R$$
$$X_R * X_S = \begin{cases} \pm v_R \otimes v_R & (X_{R.S} = X_{\pm 1}) \\ X_{R.S} & (X_{R.S} \text{ short}) \\ 0 & (\text{otherwise}) \end{cases}$$
$$A * (W \otimes v) = (W \otimes v) * A = W \otimes vA + \frac{1}{8} \operatorname{tr} A \,.\, W \otimes v$$
$$X_R * (W \otimes v) = (W \otimes v) * X_R = \frac{1}{8} W^R \otimes [v - 2(v, v_R).v_R]$$
$$(W \otimes v) * (W' \otimes v') = \Sigma \tfrac{1}{2}(W^R, W')[(v,v') - 2(v,v_R)(v',v_R)].X_R + (W,W').[(v,v')I + 4.v \otimes v' + 4.v' \otimes v]$$

Vectors of 300_x are represented by 24 x 24 symmetric matrices A, A', ... The general vector of 24_x is called v, and v_R denotes either of the two type 2 Leech lattice vectors corresponding to the short element x_R of Q_x. W is the typical vector in 4096_x-space. Its image under x_R is written W^R. The sum in the last line is over all short X_R (the factor $\frac{1}{2}$ may be omitted if we sum instead over just one of each pair X_R, X_{-R}). The quadratic form on 196884-space is defined by the condition that $(ij)_1$ has norm 2, and all our other basis vectors have norm 1. Our basis vectors are orthogonal except when opposite or equal.

$M = F_1$

Enlargements The images of N_x in the representations 24_x and 4096_x are matrix groups $N_x(24) = 2^{12}{:}M_{24}$ and $N_x(4096) = 2^{1+24}.(2^{11}{:}M_{24})$ which have a common quotient $2^{11}{:}M_{24}$, and can be extended to matrix groups $G_x(24) = 2{\cdot}Co_1$ and $G_x(4096) = 2^{1+24}.Co_1$ having a common quotient Co_1 (extending $2^{11}{:}M_{24}$). Since the intersection of the kernels of N_x for 24_x and 4096_x is $K_1 = \langle k_1 \rangle$, it follows that $N_{x1} = N_x/K_1$ is isomorphic to the diagonal product $N_x(24) \Delta_{2^{11}:M_{24}} N_x(4096)$ which can be extended to a larger group $G_{x1} = G_x(24) \Delta_{Co_1} G_x(4096)$ of structure $2^{1+24}.2Co_1$. (The diagonal product $A \Delta_C B$ is the set of (a,b) in the Cartesian product whose coordinates have the same image in C.) Plainly the representations 24_x and 4096_x extend to G_{x1}, as does 98280_x since $Q_{x1} = Q_x/K_1$ is still normal and G_{x1} still permutes the short elements of Q_{x1} by conjugation. So the representation 196884_x extends to G_{x1}. Since k_2 and k_3 are represented trivially, the result is actually a representation of $G_{x0} = G_{x1}/K_2$, where $K_2 = \langle k_2 \rangle$. The three such groups G_{x0}, G_{y0}, G_{z0} which can be supposed (using the dictionary) to act on a single 196884-space, generate the Monster group G.

Vectors There are certain vectors in the algebra that are permuted by G in the way that it transforms the elements of certain conjugacy classes. Thus the entire group fixes the unit 24x24 matrix I, regarded as a vector of 300_x, 300_y or 300_z, which satisfies $(I,I) = 24$, $I * x = 4x$, and so $\frac{1}{4}I$ is a unit element 1 for the algebra, corresponding to the unit element of G. Again, the centralizer (a double cover of the Baby Monster) of an element of class 2A (a <u>transposition</u>) fixes a further vector t (which we call a <u>transposition vector</u>) satisfying $(t,t) = 128$, $t * t = 64t$. The lattice generated by 1, t, $t * t'$, where t and t' range over all transposition vectors, is closed under algebra multiplication, and integral (and probably unimodular) with respect to the inner product $2(a,b)$.

There are similar vectors corresponding to (written $\longleftrightarrow$) elements of certain other conjugacy classes. We describe the structure of the algebra generated by transposition vectors t_0 and t_1 corresponding to two transpositions a and b. The conjugacy classes of ab and its powers are given in the table, followed by certain relations in the algebra. These involve the vectors t_n corresponding to transpositions $a(ab)^n$, and t, u, v, $\pm w$ corresponding to any powers of ab that lie in the respective conjugacy classes 2A, 3A, 4A, 5A.

1A	$t_0 * t_0 = 64t_0$, $(t_0,t_0) = 128$, $(t_0,1) = 2$, $(1,1) = 3/2$
2A, 1A	$t_0 * t_1 = 8(t_0+t_1-t)$, $(t_0,t_1) = 16$, $t \longleftrightarrow ab$
2B, 1A	$t_0 * t_1 = 0 = (t_0,t_1)$
3A, 1A	$t_0 * t_1 = 4t_0+4t_1+2t_2-3u$, $(t_0,t_1) = 13/2$, $u \longleftrightarrow ab$, $(u,1) = 9/2$
	$t_0 * u = 10(2t_0-t_1-t_2+u)$, $(t_0,u) = 45$, $u * u = 90u$, $(u,u) = 405$
3C, 1A	$t_0 * t_1 = t_0+t_1-t_2$, $(t_0,t_1) = 2$
4A, 2B, 1A	$t_0 * t_1 = 3t_0+3t_1+t_2+t_3-v$, $(t_0,t_1) = 4$, $v \longleftrightarrow ab$, $(v,1) = 12$
	$t_0 * v = 12(5t_0-2t_1-t_2-2t_3+v)$, $(t_0,v) = 144$, $v * v = 192v$, $(v,v) = 2304$
4B, 2A, 1A	$t_0 * t_1 = t_0+t_1-t_2-t_3+t$, $(t_0,t_1) = 2$, $t \longleftrightarrow (ab)^2$
5A, 1A	$t_0 * t_1 = \frac{1}{2}(3t_0+3t_1-t_2-t_3-t_4+w)$, $(t_0,t_1) = 3$, $w \longleftrightarrow ab$
	$t_0 * t_2 = \frac{1}{2}(3t_0+3t_2-t_4-t_1-t_3-w)$, $(t_0,t_2) = 3$, $-w \longleftrightarrow (ab)^2$
	$t_0 * w = 14(t_1-t_2-t_3+t_4+w)$, $(t_0,w) = 0$, $(w,1) = 0$
	$w * w = 350(t_0+t_1+t_2+t_3+t_4)$, $(w,w) = 3500$
6A, 3A, 2A, 1A	$t_0 * t_1 = t_0+t_1-t_2-t_3-t_4-t_5+t+u$, $(t_0,t_1) = 5/2$, $t \longleftrightarrow (ab)^3$, $u \longleftrightarrow (ab)^2$
	$t * u = 0 = (t,u)$

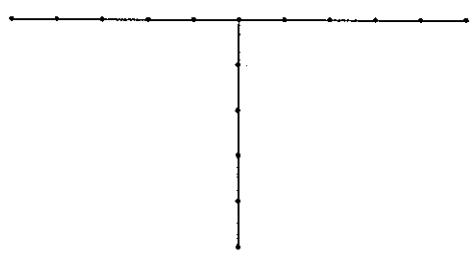
The complete structure of the algebra generated by t_0 and t_1 can in fact be read off from the table. Thus if ab is in class 6A, then $(ab)^2$ is in class 3A, and so $t_0 * t_2 = 4t_0+4t_2+2t_4-3u$.

In terms of the algebra, the action of a transposition can be recovered from the corresponding vector t by the formula

$$x \longrightarrow x - \frac{1}{14}\,[\,24(x,t)t + 16x * t - (x * t) * t\,]$$

The vector corresponding to a short element x_R of Q_x is $2A_R - 8X_R$, where the matrix $A_R = v_R \otimes v_R = X_R * X_R$. In particular, for $x_{ij} = y_{ij} = z_{ij}$ the vector is $4[(ii)+(jj)-(ij)_1-(ij)_2-(ij)_3]$.

The Monster is generated by N_0 together with any transposition outside N_0. We exhibit the vector t for such a transposition, from which the action of that transposition can be recovered from the above formula. Take an involution of $2Co_1$ outside $2^{12}M_{24}$ which fixes an 8-dimensional sublattice L_8. For each of the type 2 vectors v_R in L_8 (modulo 2) select one of the preimages x_R in Q_x, in such a way that the chosen preimages generate an elementary abelian group 2^8. Then $t = A - \Sigma X_R$ (summed over the corresponding X_R), where A is the matrix of the projection onto L_8.

"Presentation" $(G \times G).2 \cong \langle$  $\mid$ more relations $\rangle$

The three outermost dots are redundant. By omitting the lowest two dots we obtain generators for G. Further redundant generators may be added so as to complete the diagram to the 26-point graph whose nodes are the 13 points and 13 lines of the order 3 projective plane, joined by incidence. See page 232.

Moonshine There is an infinite sequence of characters H_n (the head characters) of G such that for each element g of G the function $T_g(z) = q^{-1} + H_1(g).q + H_2(g).q^2 + \ldots$ (where $q = \exp(2\pi i z)$) is fixed up to scalar factors by a certain group E(g) of transformations

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} : z \longrightarrow \frac{Az + B}{Cz + D}$$

In particular for g = 1 we find that E(1) is the group $PSL_2(\mathbb{Z})$ and $T_1(z) = j(z) - 744$, where j(z) is the classical elliptic modular function

$$q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + 333202640600q^5 + \ldots .$$

The group E(g) always has the form $\Gamma_0(n|h + e_1, e_2, \ldots)$, where the parameters are integers with h dividing n and each e_i a divisor of n/h prime to $n/(he_i)$. This group is defined to consist of all matrices

$$\begin{pmatrix} ae & b/h \\ cn & de \end{pmatrix} \quad \text{of determinant } e$$

for which a, b, c, d are integers and e is one of 1, e_1, e_2, ... We abbreviate its name by omitting "|h" when h = 1, and by writing $\Gamma_0(n|h)$ or $\Gamma_0(n|h-)$ when no e_i (save 1) is present, and $\Gamma_0(n|h+)$ when all possible e_i are present. The scalar factors for elements of E(g) are :

1	for elements of the normal subgroup $\Gamma_0(nh + e_1, e_2, \ldots)$
$\exp(-2\pi i/h)$	for the element $\begin{pmatrix} 1 & 1/h \\ 0 & 1 \end{pmatrix}$
$\exp(\pm 2\pi i/h)$	for the element $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$,

where the sign is + or − according as n/h is or is not among the e_i. These rules completely determine the subgroup F(g) of index h in $\Gamma_0(n|h+e_1, e_2, \ldots)$ that fixes $T_g(z)$ exactly. In fact $T_g(z)$ is characterized as the canonical Hauptmodul for this group F(g) – that is to say, it generates the field of all modular functions invariant under F(g) and has a series of the form $1/q + a_1 q + a_2 q^2 + \ldots$.

The symbols $n|h+e_1, e_2, \ldots$ serve as names for the conjugacy classes of G, with the slight ambiguities that g and g^{-1} have the same symbol, and that classes 27A and 27B both have symbol 27+. In this notation the "class" of g^k is $n'|h'+e_1', e_2', \ldots$ where $n' = n/(n,k)$, $h' = h/(h,k)$, and e_1', e_2', ... are the divisors of n'/h' among e_1, e_2, ...

Many properties of the elements of G and their centralizers are illuminated by this parametrization. Using $F_{n|h+e_1, e_2, \ldots}$ for the non-Abelian composition factor (when this exists) of the appropriate centralizer, we have :

F_1	F_{2+}	F_{2-}	F_{3+}	F_{3-}	$F_{3\|3}$	F_{4+}	$F_{4\|2+}$	F_{4-}	$F_{5\|2-}$	F_{5+}	F_{5-}	F_{7+}	F_{7-}
M	B	Co_1	Fi_{24}'	Suz	Th	Co_3	$F_4(2)$	$S_6(2)$	$G_2(4)$	HN	J_2	He	A_7

The names F_1, F_2, F_3, and F_5 often used for M, B, Th, and HN may be regarded as abbreviations of these F-names. The full forms can be used to exhibit the analogies between these groups.

There are replication formulae relating the various characters H_n, for instance the duplication formula

$$H_{2n}(g) = H_{n+1}(g) + \Sigma' H_i(g) H_{n-i}(g) \quad (1 \leq i \leq n/2)$$

where Σ' indicates that when n = 2m the term $H_m(g)^2$ must be replaced by $H_m^{2-}(g) = \frac{1}{2}(H_m(g)^2 - H_m(g^2))$. The duplication formula can be used to express all the H_n in terms of H_1, H_2, H_3 and H_5. The head characters H_{-1}, ... H_9 are the linear combinations of the first few irreducible characters with the coefficients

H_{-1} :	1															
H_0 :	0															
H_1 :	1	1														
H_2 :	1	1	1													
H_3 :	2	2	1	1												
H_4 :	2	3	2	1	0	1										
H_5 :	4	5	3	2	1	1	1									
H_6 :	4	7	5	3	1	3	1	1								
H_7 :	7	11	7	6	3	4	2	2	1							
H_8 :	8	15	12	8	4	8	4	4	1	1	0	1				
H_9 :	12	23	16	14	8	12	7	7	3	2	1	1	0	0	1	

Groups involved Which simple groups are involved in M ? We list all simple groups whose order divides that of M, in an abbreviated notation. The relevant parameter is followed by x when the group is not involved in M, by ? when the involvement is undecided, and by ' when the simple group intended is the derived group of the group named,

$C_{2,3,5,7,11,13,17,19,23,29,31,41,47,59,71}$ $A_{5-12,13x-32x}$

$L_2(4,5,7,8,9,11,13,16,17,19?,23,25,27?,29?,31?,32x,41?,47x,49?,59?,64x,71?,81,125x,169x,1024x)$

$L_3(2,3,4,5,7x,9x,16x,25x)$ $L_4(2,3,4x,5x,7x,9x)$ $L_5(2,3x,4x)$ $L_6(2x,3x,4x)$

$U_3(3,4,5,8)$ $U_4(2,3,4x,5x,8x)$ $U_5(2,4x)$ $U_6(2,4x)$

$S_4(2',3,4,5x,7x,8x,9x)$ $S_6(2,3x,4x,5x)$ $S_8(2,3x)$ $S_{10}(2x)$ $S_{12}(2x)$

$O_7(2,3,5x)$ $O_8^+(2,3)$ $O_8^-(2,3)$ $O_9(2,3x)$ $O_{10}^+(2,3x)$ $O_{10}^-(2)$ $O_{11}(2x)$ $O_{12}^+(2x)$ $O_{12}^-(2x)$ $O_{13}(2x)$

$Sz(8?,32x)$ $G_2(2',3,4,5x)$ ${}^3D_4(2)$ $F_4(2)$ ${}^2F_4(2')$ ${}^2E_6(2)$

$M_{11,12,22,23,24}$ $Co_{1,2,3}$ $Fi_{22,23,24'}$ J_2 Suz HS M^cL He HN Th B M J_1? J_3x O'N x Ru x

The Y-groups

We amplify the remarks in the section "Presentation", and in so doing provide presentations for a large number of groups. The wreath square of the Monster is a homomorphic image of the Coxeter group defined by Figure 1. Redundant generators can be added to form a graph of 26 nodes a, z_i, a_i, b_i, c_i, d_i, e_i, f_i, g_i, f ($i = 1,2,3$) in which the joins are from

a to b_i, b_j, b_k, f	z_i to a_i, c_j, c_k, e_i
a_i to z_i, b_i, f_j, f_k	b_i to a, a_i, c_i, g_i
c_i to z_j, z_k, b_i, d_i	d_i to c_i, e_i, g_j, g_k
e_i to z_i, d_i, f_i, f	f_i to a_j, a_k, e_i, g_i
g_i to b_i, d_j, d_k, f_i	f to a, e_i, e_j, e_k

where $\{i,j,k\} = \{1,2,3\}$. Abstractly, this is the incidence graph of the 13 points (z_i,b_i,d_i,f_i,f) and 13 lines (a,a_i,c_i,e_i,g_i) of the projective plane of order 3.

We define Y_{pqr} to be the subgroup of M wr 2 generated by a and

the first p terms of the sequence b_1, c_1, d_1, e_1, f_1

the first q terms of the sequence b_2, c_2, d_2, e_2, f_2

the first r terms of the sequence b_3, c_3, d_3, e_3, f_3 (see Figure 1).

Similarly, we define X_{pqr} to be the group generated by a and

the first p terms of the sequence b_1, c_1, d_1, e_1 ($p \leq 4$)

the first q terms of the sequence b_2, c_2, d_2, e_2 ($q \leq 4$)

the first r terms of the sequence b_3, a_3 ($r \leq 2$) (see Figure 2a),

and if p, $q \leq 3$ obtain X_{pqrs} by adjoining the first s terms of f, e_3 (see Figure 2b). Similarly, Q_{pqr} is generated by f_1, f_2, f_3 and

the first p terms of the sequence a_1, b_1, c_1, d_1

the first q terms of the sequence a_2, b_2, c_2, d_2

the first r terms of the sequence a_3, b_3, c_3, d_3 (see Figure 3).

The structures of these groups, and multiple covers of certain of them, are tabulated on the next page. The intersection of Y_{pqr}, $X_{pqr(s)}$, Q_{pqr} with M x M is a subgroup of index 2 which projects isomorphically into M in all cases of the table except

Y_{444} (image = M) $\quad X_{2222}$ (image = $2^{10+16}.O_9(2)$) $\quad X_{3222}$ (image = $2^{10+16}.O_{10}^{+}(2)$)

We have

$Y_{5qr} = Y_{4qr}$ if $q > 1$, $r > 0$ $\qquad Q_{4qr} = Q_{3qr} = Y_{3,q+1,r+1}$ if $q > 0$

$X_{pqr} = Y_{pqr}$ if $r < 2$ $\qquad X_{442} = X_{432}$ $\qquad X_{33rs} = X_{32rs}$ if r, $s > 0$

<u>Defining relations.</u> We define $f_{ij} = (ab_ib_jb_kc_ic_jd_i)^9$, whenever $\{i,j,k\} = \{1,2,3\}$. We conjecture that M wr 2 is the abstract group defined by the Coxeter relations of Y_{555} (Figure 1) together with

$f_1 = f_{12}$ or f_{13}, $\quad f_2 = f_{23}$ or f_{21}, $\quad f_3 = f_{31}$ or f_{32}.

It is known that the eight alternatives here are mutually equivalent, and define the same group as the Coxeter relations of Y_{444} together with the relation $S = 1$ of the table, when the above relations are used to define the f_i. With the further definitions

$$z_i = f_i(ab_ib_jb_kc_id_i)^5 \qquad f = f_i(c_id_ib_jc_jd_jz_k)^5$$

$$a_i = f_je_jd_jc_jb_jab_kc_kd_ke_kf_k \qquad g_i = c_i(ab_jc_jc_ke_if)^5$$

these relations imply all the Coxeter relations of the projective plane.

The "relations" column gives relations that complete (or conjecturally complete) the Coxeter relations to a presentation in the other cases. Using also the information in the "centre" column of the table one can derive presentations for many groups. Thus the Coxeter relations of Y_{332} together with

$f_{12}=f_{13}$, $f_{21}=f_{23}$;	$f_{12}=f_{13}$, $f_{21}=1$;	$f_{12}=f_{21}=1$;	$S=1$;	$f_{12}=f_{21}=S=1$

are presentations for the respective groups

$(3\times 2^2)Fi_{22}$;	$6Fi_{22}$;	$3Fi_{22}$;	2^2Fi_{22};	Fi_{22}.

$M = F_1$

Group	Structure	Relations	Centre
Y_{mn0}	S_{m+n+2}	none	
Y_{111}	2^3S_4	none	$(ab_1b_2b_3)^3$
Y_{211}	2^4S_5	none	
Y_{311}	2^5S_6	none	$(ab_1b_2b_3c_1d_1)^5$
Y_{411}	2^6S_7	none	
Y_{511}	2^6S_8	P=1	
Y_{221}	$O_6^-(2):2$	none	
Y_{321}	$O_7(2) \times 2$	none	f_{12}
Y_{421}	$O_8^+(2):2$	Q=1	
Y_{331}	$2^2.2^6.O_7(2)$	R=1	f_{12}, f_{21}
Y_{431}	$O_9(2) \times 2$	R=1	f_{21}
Y_{441}	$O_{10}^-(2):2$	R=1	
Y_{222}	$3^5:O_5(3):2$	S=1	
Y_{322}	$O_7(3) \times 2$	S=1	f_{12}
Y_{422}	$O_8^+(3):2$	Q=S=1	
Y_{332}	$2^2.Fi_{22}$	S=1	f_{12}, f_{21}
Y_{432}	$2 \times Fi_{23}$	S=1	f_{21}
Y_{442}	$3Fi_{24}$	S=1 ?	
Y_{333}	$2^3.{}^2E_6(2)$	S=1 ?	f_{12}, f_{23}, f_{31}
Y_{433}	$2^2.B$	S=1 ?	f_{21}, f_{31}
Y_{443}	$2 \times M$	S=1 ?	f_{31}
Y_{444}	M wr 2	S=1 ?	
$2Y_{511}$	2^7S_8	none	P
$2Y_{421}$	$2.O_8^+(2):2$	none	Q
$3Y_{222}$	$3^6.O_5(3):2$	T=1	S
$3Y_{322}$	$3.O_7(3) \times 2$	$U_1=1$	f_{12}, S
$2Y_{422}$	$2.O_8^+(3):2$	S=1	Q
$3Y_{332}$	$(2^2 \times 3).Fi_{22}$	$U_1=U_2=1$	f_{12}, f_{21}, S
$3Y_{333}$	$(2^3 \times 3).{}^2E_6(2)$	$U_1=U_2=U_3=1$?	f_{12}, f_{23}, f_{31}, S

Group	Structure	Relations
Q_{110}	S_6	none
Q_{210}	$2^5:S_6$	none
Q_{220}	$2^6:2^5:S_6$	X=1
Q_{111}	$3^4:S_6$	V=1
Q_{211}	$2.O_6^-(3):2$	V=1
Q_{221}	$2^3.U_6(2)$	V=1
Q_{222}	$2^4.2^{20}.U_6(2)$	V=1 ?
X_{222}	$2^6:O_6^-(2):2$	W=1
X_{322}	$O_8^-(2):2$	W=1
X_{422}	$O_9(2) \times 2$	W=1
X_{332}	$2^8:O_8^-(2):2$	W=1
X_{432}	$O_{10}^-(2):2$	W=1
X_{1111}	$2^2.2^{1+6}:S_4$	
X_{2111}	$2.2^4.2^{1+8}:S_5$	
X_{3111}	$2.2^4.[2^{11}].S_6$	
X_{2211}	$2^8.2^8.2^6.O_6^-(2):2$	
X_{3211}	$2^8.2^8.2^7.(O_7(2)\times 2)$	
X_{2221}	$2^{10}.2^{16}.O_8^-(2):2$	
X_{3221}	$2^{10}.2^{16}.(O_9(2)\times 2)$	
X_{2222}	$(2^{10+16}\times 2^{10+16}).(O_9(2)\times 2)$	
X_{3222}	$(2^{10+16}\times 2^{10+16}).O_{10}^+(2):2$	

$P = (ab_1b_2b_3c_1d_1e_1f_1)^7$

$Q = (f_{12}e_1)^3$ or $(ab_1b_2b_3c_1c_2d_1e_1)^{15}$

$R = [f_{12},d_2]$ or $[f_{21},d_1]$

$S = (ab_1c_1ab_2c_2ab_3c_3)^{10}$

$T = (ab_1c_1b_2c_2b_3c_3)^{18}$

$U_i = f_{ij}f_{ik}$ or $[f_{ij},c_k]$ or $[f_{ik},c_j]$

$V = (a_1f_1a_2f_2a_3f_3)^4$

$W = (ab_1b_2b_3c_1c_2a_3)^{12}$

$X = (a_1a_2b_1b_2f_1f_2f_3)^8$

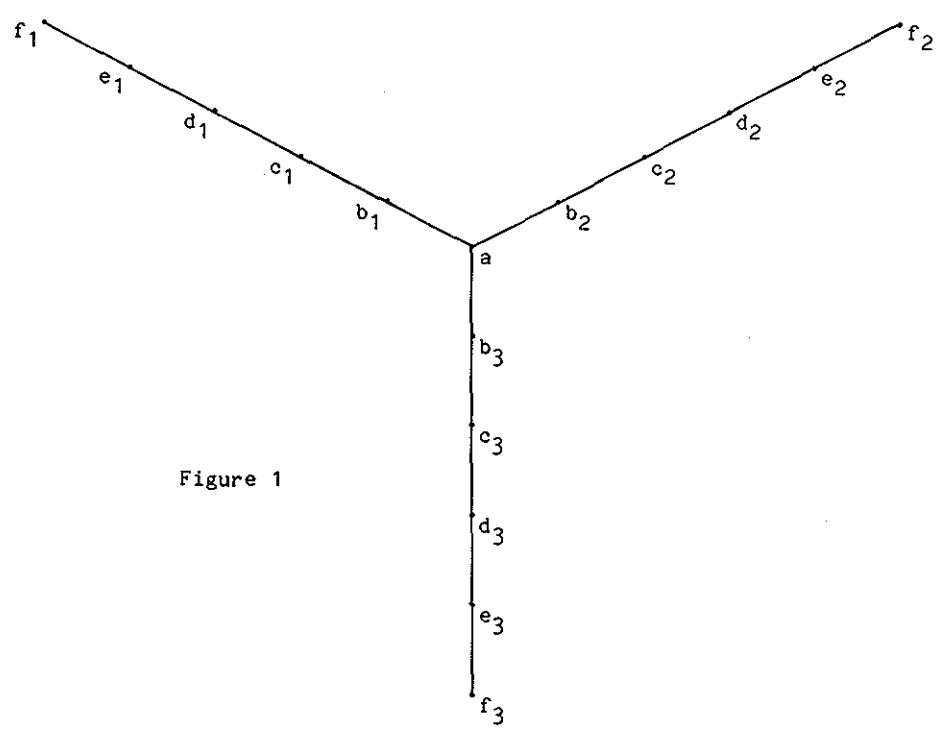


Figure 1

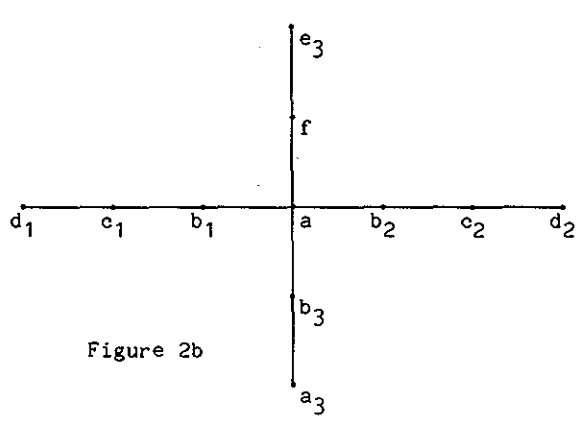


Figure 2b

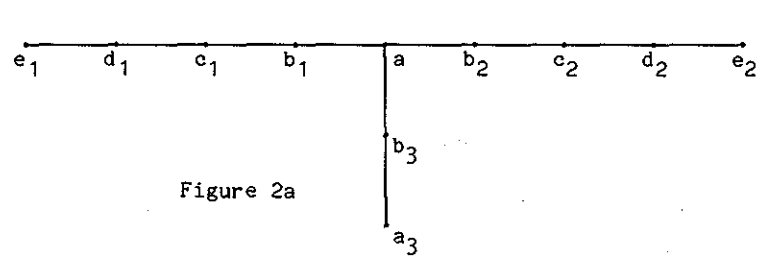


Figure 2a

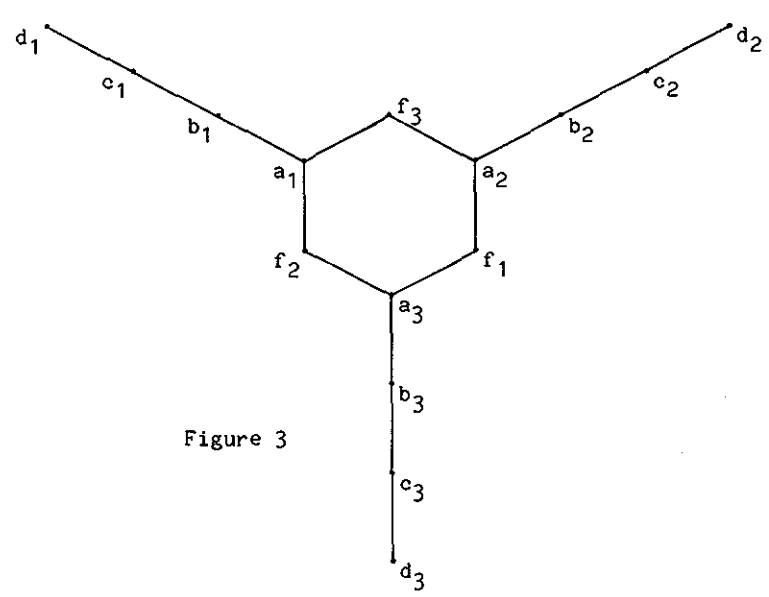


Figure 3

$M = F_1$

Maximal p-local subgroups (complete for $p \neq 2$)

p	Structure	Specification	Supergroups
2	$2^{\cdot}B$	N(2A)	
	$2^{2\cdot}({}^2E_6(2)):S_3$	$N(2A^2)$	
	$2_+^{1+24}\cdot Co_1$	N(2B)	
	$2^2.2^{11}.2^{22}.(S_3 x M_{24})$	$N(2B^2)$	
	$2^3.2^6.2^{12}.2^{18}.(L_3(2) x 3S_6)$	$N(2B^3)$	
	$2^5.2^{10}.2^{20}.(S_3 x L_5(2))$	$N(2B^5)$	
	$2^{10+16}\cdot O_{10}^+(2)$	$N(2^{10}) = N(2A_{496}B_{527})$	
3	$3^{\cdot}Fi_{24}$	N(3A)	
	$(3^2{:}2 \times O_8^+(3))^{\cdot}S_4$	$N(3A^2)$	
	$3_+^{1+12}\cdot 2Suz{:}2$	N(3B)	
	$S_3 \times Th$	N(3C)	
	$3^2.3^5.3^{10}.(M_{11} x 2S_4)$	$N(3B^2)$	
	$3^3.3^6.[3^8].(L_3(3) x [2^4])$	$N(3B^3)$	
	$3^{8\cdot}O_8^-(3)\cdot 2$	$N(3^8) = N(3A_{2214}B_{1066})$	
5	$(D_{10} \times HN)^{\cdot}2$	N(5A)	
	$5_+^{1+6}{:}4J_2{\cdot}2$	N(5B)	
	$(5^2{:}[2^4] \times U_3(5)).S_3$	$N(5A^2)$	
	$5^2.5^2.5^4.(S_3 x GL_2(5))$	$N(5B^2)$	
	$5^{3+3\cdot}(2 \times L_3(5))$	$N(5B^3)$	
	$5^4{:}(3x2L_2(25).2)$	$N(5B^4)$	
7	$(7{:}3 \times He){:}2$	N(7A)	
	$7_+^{1+4}{:}(3x2S_7)$	N(7B)	
	$(7^2{:}(3x2A_4) \times L_2(7)).2$	$N(7A^2)$	
	$7^2.7.7^2{:}GL_2(7)$	$N(7B^2)$	
11	$(11{:}5 \times M_{12}){:}2$	N(11A)	$(L_2(11) \times M_{12}){:}2$
	$11^2{:}(5x2A_5)$	$N(11A^2)$	
13	$(13{:}6 \times L_3(3))^{\cdot}2$	N(13A)	
	$13_+^{1+2}{:}(3x4S_4)$	N(13B)	
	$13^2{:}4L_2(13)^{\cdot}2$	$N(13B^2)$	
17	$(17{:}8 \times L_3(2))^{\cdot}2$	N(17A)	$(S_4(4){:}2 \times L_3(2))^{\cdot}2$
19	$(19{:}9 \times A_5){:}2$	N(19A)	$(U_3(8){:}3 \times A_5){:}2$
23	$23{:}11 \times S_4$	N(23AB)	$2^2.2^{11}.2^{22}.(M_{24} x S_3)$
29	$(29{:}14 \times 3)^{\cdot}2$	N(29A)	$3^{\cdot}Fi_{24}$
31	$31{:}15 \times S_3$	N(31AB)	$S_3 \times Th$
41	41:40	N(41A)	$L_2(41){:}2$?
47	$47{:}23 \times 2$	N(47AB)	$2^{\cdot}B$
59	59:29	N(59AB)	$L_2(59)$?
71	71:35	N(71AB)	$L_2(71)$?

Some non-local subgroups

$(A_5 \times A_{12}){:}2$	N(2A,3A,5A)
$(L_2(11) \times M_{12}){:}2$	N(2A,3A,5A,11A), N(2A,2B,3B,3A,5A)
$M_{11} \times A_6{\cdot}2^2$	N(2A,3A,5A,11A), N(2B,3B,3B,4A,5A)
$(A_6 \times A_6 \times A_6).(2 \times S_4)$	$N(2A,3A,3A,4B,5A)^3$
$(A_7 \times (A_5 \times A_5).4).2$	N(2A,3A,3A,5A,7A), $N(2A,3A,5A)^2$
$(L_3(2) \times S_4(4){:}2)^{\cdot}2$	N(2A,3A,4B,7A)
$(A_5 \times U_3(8){:}3){:}2$	N(2A,3C,5A)
$(S_5 \times S_5 \times S_5){:}S_3$	$N(2A,3A,5A)^3$
$(L_2(11) \times L_2(11)){:}4$	$N(2A,3A,5A,11A)^2$

$E_8(2)$

Chevalley group $E_8(2)$

Order = 337,804,753,143,634,806,261,388,190,614,085,595,079,991,692,242,467,651,576,160,959,909,068,800,000

$= 2^{120}.3^{13}.5^5.7^4.11^2.13^2.17^2.19.31^2.41.43.73.127.151.241.331$

Mult = 1 Out = 1

Constructions

Chevalley G : adjoint Chevalley group of type E_8 over the field $\mathbb{F}_2$

Maximal 2-local subgroups

$[2^{78}]:O^+_{14}(2)$

$[2^{98}]:(S_3 x L_7(2))$

$[2^{106}]:(S_3 x L_3(2) x L_5(2))$

$[2^{104}]:(A_8 x L_5(2))$

$[2^{97}]:(L_3(2) x O^+_{10}(2))$

$[2^{83}]:(S_3 x E_6(2))$

$[2^{57}]:E_7(2)$

$[2^{92}]:L_8(2)$

Some other subgroups

S_3 x $E_7(2)$

$(L_3(2)$ x $E_6(2)):2$

$3\cdot(3^2:Q_8$ x ${}^2E_6(2)):S_3$

$(L_5(2)$ x $L_5(2)).4$

$(U_5(2)$ x $U_5(2)).4$

$L_9(2):2$

$3\cdot U_9(2):S_3$

$O^+_{16}(2)$

$5\cdot U_5(4).4$

$U_5(4).5.4$

$(O^+_8(2)$ x $O^+_8(2)).D_{12}$

$O^+_8(4).D_{12}$

$({}^3D_4(2)$ x ${}^3D_4(2)).6$

${}^3D_4(4).6$

$3^{2+8}.2^{4+8}.3^2.2S_4$

$(L_3(2)$ x $L_3(2)$ x $L_3(2)$ x $L_3(2)).2S_4$

$(U_3(4)$ x $U_3(4)).8$

$U_3(16).8$

$(S_3)^8.2^3L_3(2)$

$U_3(3):2$ x $F_4(2)$

Partitions and the classes and characters of S_n

The table gives the usual correspondences between partitions and the classes and characters of S_n. In the table,

N is the largest part of the partition,

m+ refers to the m'th printed character of the ATLAS table,

m− to its associated character, and

m,M to the sum of the m'th and M'th ATLAS characters of A_n.

The characters of the Schur double covers are also indexed by partitions. More precisely, a partition of n into distinct parts refers either to two characters of $2A_n$ that fuse in $2S_n$, or to two characters of $2S_n$ that agree on $2A_n$. The partition indicates the S_n image of the classes on which the two characters differ.

The following provides enough information to define the Schur double covers $2A_n$, 2^+S_n and 2^-S_n. The preimages in $2^{\pm}S_n$ of the permutation

$$(abc\ldots)(ghi\ldots)(pqr\ldots)\ldots$$

are $\pm\theta$, where

$$\theta = [abc\ldots][ghi\ldots][pqr\ldots]\ldots$$

and we have

$$[a_1a_2a_3\ldots a_k] = [a_1a_2][a_1a_3]\ldots[a_1a_k].$$

If ϕ is either preimage of the permutation taking

$$a \longrightarrow A,\ b \longrightarrow B,\ c \longrightarrow C,\ \ldots$$

then we have

$$\phi^{-1}\theta\phi = \pm\ [ABC\ldots][GHI\ldots][PQR\ldots]\ldots$$

where the sign is − just when the images of θ and ϕ are both odd permutations.

We have

$$[a_1a_2\ldots a_k]^k = \begin{cases} +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & \text{in } 2^+S_n \\ +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & \text{in } 2^-S_n \end{cases}$$

$$\text{according as} \qquad k \equiv 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \pmod 8$$

form of partition	n = 13 (pp 102,103)		n = 12 (pp 92,93)		n = 11 (page 76)		n = 10 (page 49)		n = 9 (page 37)		n = 8 (page 22)		n = 7 (page 10)		n = 6 (page 5)		n = 5 (page 2)	
N	13AB	1+	12I	1+	11AB	1+	10C	1+	9AB	1+	8A	1+	7AB	1+	6B	1+	5AB	1+
N1	12M	2+	11AB	2+	10D	2+	9AB	2+	8A	2+	7AB	2+	6C	2+	5AB	2+	4A	4+
N2	22A	3+	10B	3+	18A	3+	8A	3+	14A	5+	6B	4+	10A	5+	4A	6+	6A	5+
N11	11A	4+	10F	4+	9A	4+	8B	4+	7A	6+	6E	5+	5A	7+	4B	7+	3A	2,3
N3	30F	5+	9BC	6+	24A	5+	21AB	6+	6G	10+	15AB	8+	12A	6+	3B	3-		
N21	10C	7+	18A	10+	8A	12+	14A	10+	6B	13+	10A	13+	4A	9+	6A	4,5	2A	5-
N111	10G	6+	9A	7+	8B	6+	7A	7+	6F	11+	5A	9+	4B	3,4	3A	7-	2B	4-
N4	36A	8+	8B	8+	28A	10+	12B	8+	20A	9+	4A	3+						
N31	9BC	17+	24A	15+	21AB	16+	6I	18+	15AB	15+	12A	14+	3B	8+				
N22	18A	15+	8D	14+	14A	14+	6H	14+	10A	14+	4D	12+	6A	8-	2C	3+		
N211	18B	18+	8A	16+	14B	17+	6C	19+	10B	17+	4B	10,11	6B	9-	2A	6-		
N1111	9A	12+	8C	11+	7A	11+	6E	9+	5A	7,8	4C	9-	3A	7-	2B	2-	1A	1-
N5	40A	13+	35AB	9+	30B	9+	5B	5+										
N41	8B	19+	28A	23+	12C	21+	20A	16+	4B	12+								
N32	24A	23+	42A	27+	6E	24+	30A	22+	12A	16+	6D	6,7						
N311	24B	29+	21A	31+	6L	28+	15A	24+	12B	18+	3B	12-						
N221	8D	27+	14A	29+	6K	26+	10A	23+	4D	18-	6A	14-	2C	6-				
N2111	8A	22+	14B	26+	6D	23+	10B	12,13	4A	17-	6C	13-	2A	5-				
N11111	8C	14+	7A	12+	6G	7,8	5A	9-	4C	11-	3A	5-	2B	2-	1A	1-		
N6	42B	9+	6F	5+														
N51	35AB	21+	30C	19+	5B	13+												
N42	28A	36+	12H	32+	20A	25+	4F	15+										
N411	28B	38+	12D	34+	20B	27+	4C	17+										
N33	21B	26+	6N	25+	15B	20+	12D	11+	3B	3,4								
N321	42A	49+	6G	42+	30A	31+	12A	20,21	6E	16-								
N3111	21A	40+	6O	37+	15A	30+	12C	23-	3C	14-								
N222	14C	28+	6D	28+	10C	22+	4A	17-	6D	12-	2A	3-						
N2211	14A	39+	6P	36+	10A	30-	4E	24-	6A	15-	2D	8-						
N21111	14B	30+	6E	17,18	10B	23-	4B	19-	6C	13-	2B	4-						
N111111	7A	10,11	6I	12-	5A	11-	4D	7-	3A	6-	2C	2-	1A	1-				
N61	6H	16+																
N52	30B	34+	10E	20+														
N511	30E	35+	5B	24+														
N43	12F	37+	60A	30+	12A	15+												
N421	12L	53+	20A	43+	4F	29+												
N4111	12E	46+	20B	35+	4B	22-												
N331	6S	47+	15B	40+	12F	18,19	3C	11-										
N322	6R	50+	30A	41+	12E	29-	6B	15-										
N3211	6G	54+	30B	38,39	12B	31-	6G	22-										
N31111	6P	45+	15A	36-	12D	26-	3B	14-										
N2221	6F	44+	10D	35-	4C	27-	6F	16-	2B	9-								
N22111	6Q	45-	10A	37-	4E	28-	6A	18-	2D	10-								
N211111	6C	30-	10C	26-	4A	17-	6D	10-	2A	5-								
N1111111	6J	14-	5A	11-	4D	6-	3A	4-	2C	2-	1A	1-						
N53	15C	24+																
N521	10F	43+																
N5111	5B	33+																
N44	20A	20+	4H	13+														
N431	60A	52+	12C	33+														
N422	20D	51+	4D	21,22														
N4211	20B	55+	4I	41-														
N41111	20C	44-	4B	28-														
N332	30D	48+	12B	33-	6H	15-												
N3311	15B	41,42	12F	40-	3C	20-												
N3221	30A	55-	12G	43-	6C	25-												
N32111	30C	54-	12A	42-	6J	24-												
N311111	15A	39-	12E	29-	3B	14-												
N2222	10A	33-	4F	24-	6A	13-	2D	5-										
N22211	10E	46-	4C	34-	6I	21-	2B	8-										
N221111	10B	40-	4G	31-	6B	16-	2E	6-										
N2111111	10D	22-	4A	16-	6F	12-	2A	3-										
N11111111	5A	12-	4E	7-	3A	4-	2C	2-	1A	1-								
N441	4I	25+																
N432	12K	31,32																
N4311	12C	48-																
N4221	4D	51-																
N42111	4H	50-																
N411111	4B	28-																
N333	12H	25-	3C	13-														
N3321	12C	52-	6M	30-														
N33111	12I	47-	3D	25-														
N3222	12B	43-	6L	20-														
N32211	12J	53-	6C	32-														
N321111	12A	49-	6K	27-														
N3111111	12G	27-	3B	14-														
N22221	4G	35-	6B	19-	2E	9-												
N222111	4C	38-	6J	23-	2B	10-												
N2211111	4F	29-	6A	15-	2D	5-												
N21111111	4A	18-	6H	10-	2A	3-												
N111111111	4E	6-	3A	4-	2C	2-	1A	1-										
N3331	3D	20-																
N3322	6D	24-																
N33211	6O	37-																
N331111	3C	26-																
N32221	6N	34-																
N322111	6E	36-																
N3211111	6L	23-																
N31111111	3B	15-																
N22222	6K	16-	2B	5-														
N222211	6B	21-	2F	9-														
N2221111	6M	19-	2C	8-														
N22111111	6A	17-	2E	6-														
N211111111	6I	7-	2A	3-														
N1111111111	3A	4-	2D	2-	1A	1-												
N222221	2C	9-																
N2222111	2F	13-																
N22211111	2B	8-																
N221111111	2E	5-																
N2111111111	2A	3-																
N11111111111	2D	2-	1A	1-														
N111111111111	1A	1-																

Involvement of sporadic groups in one another

For each sporadic simple group G, we ask which smaller sporadic groups are involved in G (that is, are quotients of subgroups of G)? In the table the entry in the row for G and the column for S is

- \+ if S is involved in G
- \- if S is not involved in G
- ? if we don't know ($S \cong J_1$, $G \cong M$)

If J_1 is involved in M, then it must be as a maximal subgroup, with conjugacy classes 1A, 2B, 3C, 5B, 6F, 7B, 10E, 11A, 15D, 19A.

	M_{11}	M_{12}	J_1	M_{22}	J_2	M_{23}	HS	J_3	M_{24}	M^cL	He	Ru	Suz	O'N	Co_3	Co_2	Fi_{22}	HN	Ly	Th	Fi_{23}	Co_1	J_4	Fi'_{24}	B	M
M_{11}	·																									
M_{12}	+	·																								
J_1	-	-	·																							
M_{22}	-	-	-	·																						
J_2	-	-	-	-	·																					
M_{23}	+	-	-	+	-	·																				
HS	+	-	-	+	-	-	·																			
J_3	-	-	-	-	-	-	-	·																		
M_{24}	+	+	-	+	-	+	-	-	·																	
M^cL	+	-	-	+	-	-	-	-	-	·																
He	-	-	-	-	-	-	-	-	-	-	·															
Ru	-	-	-	-	-	-	-	-	-	-	-	·														
Suz	+	+	-	-	+	-	-	-	-	-	-	-	·													
O'N	+	-	+	-	-	-	-	-	-	-	-	-	-	·												
Co_3	+	+	-	+	-	+	+	-	-	+	-	-	-	-	·											
Co_2	+	-	-	+	-	+	+	-	-	+	-	-	-	-	-	·										
Fi_{22}	+	+	-	+	-	-	-	-	-	-	-	-	-	-	-	-	·									
HN	+	+	-	+	-	-	+	-	-	-	-	-	-	-	-	-	-	·								
Ly	+	-	-	+	-	-	-	-	-	+	-	-	-	-	-	-	-	-	·							
Th	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	·						
Fi_{23}	+	+	-	+	-	+	-	-	-	-	-	-	-	-	-	-	+	-	-	-	·					
Co_1	+	+	-	+	+	+	+	-	+	+	-	-	+	-	+	+	-	-	-	-	-	·				
J_4	+	+	-	+	-	+	-	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	·			
Fi'_{24}	+	+	-	+	-	+	-	-	+	-	+	-	-	-	-	-	+	-	-	-	+	-	-	·		
B	+	+	-	+	-	+	+	-	-	+	-	-	-	-	-	+	+	+	-	+	+	-	-	-	·	
M	+	+	?	+	+	+	+	-	+	+	+	-	+	-	+	+	+	+	-	+	+	+	-	+	+	·

Order of simple groups

This list contains all the simple groups of order less than 10^{25}, except that we have stopped the series

	$L_2(q)$	$L_3(q)$	$U_3(q)$	$L_4(q)$	$U_4(q)$	$S_4(q)$	$G_2(q)$
at orders	10^6	10^{12}	10^{12}	10^{16}	10^{16}	10^{16}	10^{20}

Name(s)	Order		Mult	Out
$A_5 \cong L_2(4) \cong L_2(5)$	60	$2^2.3.5$	2	2
$L_3(2) \cong L_2(7)$	168	$2^3.3.7$	2	2
$A_6 \cong L_2(9) \cong S_4(2)'$	360	$2^3.3^2.5$	6	2^2
$L_2(8) \cong R(3)'$	504	$2^3.3^2.7$	2	3
$L_2(11)$	660	$2^2.3.5.11$	2	2
$L_2(13)$	1092	$2^2.3.7.13$	2	2
$L_2(17)$	2448	$2^4.3^2.17$	2	2
A_7	2520	$2^3.3^2.5.7$	6	2
$L_2(19)$	3420	$2^2.3^2.5.19$	2	2
$L_2(16)$	4080	$2^4.3.5.17$	1	4
$L_3(3)$	5616	$2^4.3^3.13$	1	2
$U_3(3) \cong G_2(2)'$	6048	$2^5.3^3.7$	1	2
$L_2(23)$	6072	$2^3.3.11.23$	2	2
$L_2(25)$	7800	$2^3.3.5^2.13$	2	2^2
M_{11}	7920	$2^4.3^2.5.11$	1	1
$L_2(27)$	9828	$2^2.3^3.7.13$	2	6
$L_2(29)$	12180	$2^2.3.5.7.29$	2	2
$L_2(31)$	14880	$2^5.3.5.31$	2	2
$A_8 \cong L_4(2)$	20160	$2^6.3^2.5.7$	2	2
$L_3(4)$	20160	$2^6.3^2.5.7$	3×4^2	D_{12}
$L_2(37)$	25308	$2^2.3^2.19.37$	2	2
$U_4(2) \cong S_4(3)$	25920	$2^6.3^4.5$	2	2
$Sz(8)$	29120	$2^6.5.7.13$	2^2	3
$L_2(32)$	32736	$2^5.3.11.31$	1	5
$L_2(41)$	34440	$2^3.3.5.7.41$	2	2
$L_2(43)$	39732	$2^2.3.7.11.43$	2	2
$L_2(47)$	51888	$2^4.3.23.47$	2	2
$L_2(49)$	58800	$2^4.3.5^2.7^2$	2	2^2
$U_3(4)$	62400	$2^6.3.5^2.13$	1	4
$L_2(53)$	74412	$2^2.3^3.13.53$	2	2
M_{12}	95040	$2^6.3^3.5.11$	2	2
$L_2(59)$	102660	$2^2.3.5.29.59$	2	2
$L_2(61)$	113460	$2^2.3.5.31.61$	2	2
$U_3(5)$	126000	$2^4.3^2.5^3.7$	3	S_3
$L_2(67)$	150348	$2^2.3.11.17.67$	2	2
J_1	175560	$2^3.3.5.7.11.19$	1	1
$L_2(71)$	178920	$2^3.3^2.5.7.71$	2	2
A_9	181440	$2^6.3^4.5.7$	2	2
$L_2(73)$	194472	$2^3.3^2.37.73$	2	2
$L_2(79)$	246480	$2^4.3.5.13.79$	2	2
$L_2(64)$	262080	$2^6.3^2.5.7.13$	1	6
$L_2(81)$	265680	$2^4.3^4.5.41$	2	2×4
$L_2(83)$	285852	$2^2.3.7.41.83$	2	2
$L_2(89)$	352440	$2^3.3^2.5.11.89$	2	2
$L_3(5)$	372000	$2^5.3.5^3.31$	1	2
M_{22}	443520	$2^7.3^2.5.7.11$	12	2
$L_2(97)$	456288	$2^5.3.7^2.97$	2	2
$L_2(101)$	515100	$2^2.3.5^2.17.101$	2	2
$L_2(103)$	546312	$2^3.3.13.17.103$	2	2
J_2	604800	$2^7.3^3.5^2.7$	2	2
$L_2(107)$	612468	$2^2.3^3.53.107$	2	2
$L_2(109)$	647460	$2^2.3^3.5.11.109$	2	2
$L_2(113)$	721392	$2^4.3.7.19.113$	2	2
$L_2(121)$	885720	$2^3.3.5.11^2.61$	2	2^2
$L_2(125)$	976500	$2^2.3^2.5^3.7.31$	2	6
$S_4(4)$	979200	$2^8.3^2.5^2.17$	1	4
$S_6(2)$	1451520	$2^9.3^4.5.7$	2	1
A_{10}	1814400	$2^7.3^4.5^2.7$	2	2

Name(s)	Order		Mult	Out
$L_3(7)$	1876896	$2^5.3^2.7^3.19$	3	S_3
$U_4(3)$	3265920	$2^7.3^6.5.7$	$3^2 \times 4$	D_8
$G_2(3)$	4245696	$2^6.3^6.7.13$	3	2
$S_4(5)$	4680000	$2^6.3^2.5^4.13$	2	2
$U_3(8)$	5515776	$2^9.3^4.7.19$	3	$3 \times S_3$
$U_3(7)$	5663616	$2^7.3.7^3.43$	1	2
$L_4(3)$	6065280	$2^7.3^6.5.13$	2	2^2
$L_5(2)$	9999360	$2^{10}.3^2.5.7.31$	1	2
M_{23}	10200960	$2^7.3^2.5.7.11.23$	1	1
$U_5(2)$	13685760	$2^{10}.3^5.5.11$	1	2
$L_3(8)$	16482816	$2^9.3^2.7^2.73$	1	6
${}^2F_4(2)'$	17971200	$2^{11}.3^3.5^2.13$	1	2
A_{11}	19958400	$2^7.3^4.5^2.7.11$	2	2
Sz(32)	32537600	$2^{10}.5^2.31.41$	1	5
$L_3(9)$	42456960	$2^7.3^6.5.7.13$	1	2^2
$U_3(9)$	42573600	$2^5.3^6.5^2.73$	1	4
HS	44352000	$2^9.3^2.5^3.7.11$	2	2
J_3	50232960	$2^7.3^5.5.17.19$	3	2
$U_3(11)$	70915680	$2^5.3^2.5.11^3.37$	3	S_3
$S_4(7)$	138297600	$2^8.3^2.5^2.7^4$	2	2
$O_8^+(2)$	174182400	$2^{12}.3^5.5^2.7$	2^2	S_3
$O_8^-(2)$	197406720	$2^{12}.3^4.5.7.17$	1	2
${}^3D_4(2)$	211341312	$2^{12}.3^4.7^2.13$	1	3
$L_3(11)$	212427600	$2^4.3.5^2.7.11^3.19$	1	2
A_{12}	239500800	$2^9.3^5.5^2.7.11$	2	2
M_{24}	244823040	$2^{10}.3^3.5.7.11.23$	1	1
$G_2(4)$	251596800	$2^{12}.3^3.5^2.7.13$	2	2
$L_3(13)$	270178272	$2^5.3^2.7.13^3.61$	3	S_3
$U_3(13)$	811273008	$2^4.3.7^2.13^3.157$	1	2
M^cL	898128000	$2^7.3^6.5^3.7.11$	3	2
$L_4(4)$	987033600	$2^{12}.3^4.5^2.7.17$	1	2^2
$U_4(4)$	1018368000	$2^{12}.3^2.5^3.13.17$	1	4
$S_4(8)$	1056706560	$2^{12}.3^4.5.7^2.13$	1	6
$L_3(16)$	1425715200	$2^{12}.3^2.5^2.7.13.17$	3	$4 \times S_3$
$S_4(9)$	1721606400	$2^8.3^8.5^2.41$	2	2^2
$U_3(17)$	2317678272	$2^6.3^4.7.13.17^3$	3	S_3
A_{13}	3113510400	$2^9.3^5.5^2.7.11.13$	2	2
He	4030387200	$2^{10}.3^3.5^2.7^3.17$	1	2
$U_3(16)$	4279234560	$2^{12}.3.5.17^2.241$	1	8
$S_6(3)$	4585351680	$2^9.3^9.5.7.13$	2	2
$O_7(3)$	4585351680	$2^9.3^9.5.7.13$	6	2
$L_3(19)$	5644682640	$2^4.3^4.5.19^3.127$	3	S_3
$G_2(5)$	5859000000	$2^6.3^3.5^6.7.31$	1	1
$L_3(17)$	6950204928	$2^9.3^2.17^3.307$	1	2
$L_4(5)$	7254000000	$2^7.3^2.5^6.13.31$	4	D_8
$U_6(2)$	9196830720	$2^{15}.3^6.5.7.11$	$2^2 \times 3$	S_3
R(27)	10073444472	$2^3.3^9.7.13.19.37$	1	3
$S_4(11)$	12860654400	$2^6.3^2.5^2.11^4.61$	2	2
$U_4(5)$	14742000000	$2^5.3^4.5^4.7.13$	2	2^2
$U_3(19)$	16938986400	$2^5.3^2.5^2.7^3.19^3$	1	2
$L_6(2)$	20158709760	$2^{15}.3^4.5.7^2.31$	1	2
$U_3(23)$	26056457856	$2^7.3^2.11.13^2.23^3$	3	S_3
Sz(128)	34093383680	$2^{14}.5.29.113.127$	1	7
A_{14}	43589145600	$2^{10}.3^5.5^2.7^2.11.13$	2	2
$S_8(2)$	47377612800	$2^{16}.3^5.5^2.7.17$	1	1
$L_3(25)$	50778000000	$2^7.3^2.5^6.7.13.31$	3	D_{12}
$S_4(13)$	68518981440	$2^6.3^2.5.7^2.13^4.17$	2	2
$L_3(23)$	78156525216	$2^5.3.7.11^2.23^3.79$	1	2
Ru	145926144000	$2^{14}.3^3.5^3.7.13.29$	2	1
$U_3(25)$	152353500000	$2^5.3.5^6.13^2.601$	1	4
$U_3(29)$	166557358800	$2^4.3^2.5^2.7.29^3.271$	3	S_3
$L_5(3)$	237783237120	$2^9.3^{10}.5.11^2.13$	1	2
$U_5(3)$	258190571520	$2^{11}.3^{10}.5.7.61$	1	2
$L_3(27)$	282027786768	$2^4.3^9.7.13^2.757$	1	6
$U_3(27)$	282056445216	$2^5.3^9.7^2.13.19.37$	1	6
$L_3(31)$	283991644800	$2^7.3^2.5^2.31^3.331$	3	S_3
$U_3(32)$	366157135872	$2^{15}.3^2.11^2.31.331$	3	$5 \times S_3$

Name(s)	Order		Mult	Out
Suz	448345497600	$2^{13}.3^{7}.5^{2}.7.11.13$	6	2
O'N	460815505920	$2^{9}.3^{4}.5.7^{3}.11.19.31$	3	2
Co_3	495766656000	$2^{10}.3^{7}.5^{3}.7.11.23$	1	1
$L_3(29)$	499631102880	$2^{5}.3.5.7^{2}.29^{3}.871$	1	2
A_{15}	653837184000	$2^{10}.3^{6}.5^{3}.7^{2}.11.13$	2	2
$G_2(7)$	664376138496	$2^{8}.3^{3}.7^{6}.817$	1	1
$U_3(31)$	852032133120	$2^{11}.3.5.7^{2}.19.31^{3}$	1	2
$S_4(17)$	1004497044480	$2^{10}.3^{4}.5.17^{4}.29$	2	2
$S_4(16)$	1095199948800	$2^{16}.3^{2}.5^{2}.17^{2}.257$	1	8
$U_4(7)$	1165572172800	$2^{10}.3^{2}.5^{2}.7^{6}.43$	4	D_8
$L_4(7)$	2317591180800	$2^{9}.3^{4}.5^{2}.7^{6}.19$	2	2^2
$S_4(19)$	3057017889600	$2^{6}.3^{4}.5^{2}.19^{4}.181$	2	2
$S_6(4)$	4106059776000	$2^{18}.3^{4}.5^{3}.7.13.17$	1	2
$G_2(8)$	4329310519296	$2^{18}.3^{5}.7^{2}.19.73$	1	3
$O_8^+(3)$	4952179814400	$2^{12}.3^{12}.5^{2}.7.13$	2^2	S_4
$O_8^-(3)$	10151968619520	$2^{10}.3^{12}.5.7.13.41$	2	2^2
A_{16}	10461394944000	$2^{14}.3^{6}.5^{3}.7^{2}.11.13$	2	2
${}^3D_4(3)$	20560831566912	$2^{6}.3^{12}.7^{2}.13^{2}.73$	1	3
$S_4(23)$	20674026236160	$2^{8}.3^{2}.5.11^{2}.23^{4}.53$	2	2
$G_2(9)$	22594320403200	$2^{8}.3^{12}.5^{2}.7.13.73$	1	2
$O_{10}^+(2)$	23499295948800	$2^{20}.3^{5}.5^{2}.7.17.31$	1	2
$O_{10}^-(2)$	25015379558400	$2^{20}.3^{6}.5^{2}.7.11.17$	1	2
$L_4(8)$	34558531338240	$2^{18}.3^{4}.5.7^{3}.13.73$	1	6
$U_4(8)$	34693789777920	$2^{18}.3^{7}.5.7^{2}.13.19$	1	6
Sz(512)	35115786567680	$2^{18}.5.7.13.37.73.109$	1	9
Co_2	42305421312000	$2^{18}.3^{6}.5^{3}.7.11.23$	1	1
$S_4(25)$	47607300000000	$2^{8}.3^{2}.5^{8}.13^{2}.313$	2	2^2
$L_4(9)$	50759843097600	$2^{10}.3^{12}.5^{2}.7.13.41$	4	$2 \times D_8$
$U_5(4)$	53443952640000	$2^{20}.3^{2}.5^{4}.13.17.41$	5	5:4
Fi_{22}	64561751654400	$2^{17}.3^{9}.5^{2}.7.11.13$	6	2
$U_4(9)$	101798586432000	$2^{9}.3^{12}.5^{3}.41.73$	2	2×4
$S_4(27)$	102804157834560	$2^{6}.3^{12}.5.7^{2}.13^{2}.73$	2	6
$L_7(2)$	163849992929280	$2^{21}.3^{4}.5.7^{2}.31.127$	1	2
A_{17}	177843714048000	$2^{14}.3^{6}.5^{3}.7^{2}.11.13.17$	2	2
$S_4(29)$	210103196385600	$2^{6}.3^{2}.5^{2}.7^{2}.29^{4}.421$	2	2
$U_7(2)$	227787103272960	$2^{21}.3^{8}.5.7.11.43$	1	2
$S_6(5)$	228501000000000	$2^{9}.3^{4}.5^{9}.7.13.31$	2	2
$O_7(5)$	228501000000000	$2^{9}.3^{4}.5^{9}.7.13.31$	2	2
$L_5(4)$	258492255436800	$2^{20}.3^{5}.5^{2}.7.11.17.31$	1	2^2
HN	273030912000000	$2^{14}.3^{6}.5^{6}.7.11.19$	1	2
$G_2(11)$	376611192619200	$2^{6}.3^{3}.5^{2}.7.11^{6}.19.37$	1	1
$S_4(31)$	409387254681600	$2^{12}.3^{2}.5^{2}.13.31^{4}.37$	2	2
$U_4(11)$	1036388695478400	$2^{7}.3^{4}.5^{2}.11^{6}.37.61$	4	D_8
$S_4(32)$	1124799322521600	$2^{20}.3^{2}.5^{2}.11^{2}.31^{2}.41$	1	10
$L_4(11)$	2069665112592000	$2^{7}.3^{2}.5^{3}.7.11^{6}.19.61$	2	2^2
$S_4(37)$	2402534664555840	$2^{6}.3^{4}.5.19^{2}.37^{4}.137$	2	2
A_{18}	3201186852864000	$2^{15}.3^{8}.5^{3}.7^{2}.11.13.17$	2	2
$F_4(2)$	3311126603366400	$2^{24}.3^{6}.5^{2}.7^{2}.13.17$	2	2
$G_2(13)$	3914077489672896	$2^{6}.3^{3}.7^{2}.13^{6}.61.157$	1	1
$S_4(41)$	6707334818822400	$2^{8}.3^{2}.5^{2}.7^{2}.29^{2}.41^{4}$	2	2
$L_6(3)$	21032402889738240	$2^{11}.3^{15}.5.7.11^{2}.13^{2}$	2	2^2
$U_6(3)$	22837472432087040	$2^{13}.3^{15}.5.7^{2}.13.61$	2	2^2
$S_{10}(2)$	24815256521932800	$2^{25}.3^{6}.5^{2}.7.11.17.31$	1	1
Sz(2048)	36011213418659840	$2^{22}.5.23.89.397.2113$	1	11
R(243)	49825657439340552	$2^{3}.3^{15}.7.11^{2}.31.61.271$	1	5
Ly	51765179004000000	$2^{8}.3^{7}.5^{6}.7.11.31.37.67$	1	1
$L_5(5)$	56653740000000000	$2^{11}.3^{2}.5^{10}.11.13.31.71$	1	2
$U_5(5)$	57604365000000000	$2^{9}.3^{5}.5^{10}.7.13.521$	1	2
A_{19}	60822550204416000	$2^{15}.3^{8}.5^{3}.7^{2}.11.13.17.19$	2	2
$S_8(3)$	65784756654489600	$2^{14}.3^{16}.5^{2}.7.13.41$	2	2
$O_9(3)$	65784756654489600	$2^{14}.3^{16}.5^{2}.7.13.41$	2	2
$O_8^+(4)$	67010895544320000	$2^{24}.3^{5}.5^{4}.7.13.17^{2}$	1	D_{12}
$O_8^-(4)$	67536471195648000	$2^{24}.3^{4}.5^{3}.7.13.17.257$	1	4
${}^3D_4(4)$	67802350642790400	$2^{24}.3^{4}.5^{2}.7^{2}.13^{2}.241$	1	6
$G_2(16)$	71776114783027200	$2^{24}.3^{3}.5^{2}.7.13.17^{2}.241$	1	4
Th	90745943887872000	$2^{15}.3^{10}.5^{3}.7^{2}.13.19.31$	1	1
$G_2(17)$	167795197370551296	$2^{10}.3^{5}.7.13.17^{6}.307$	1	1

Name(s)	Order		Mult	Out
$S_6(7)$	273457218604953600	$2^{12}.3^4.5^2.7^9.19.43$	2	2
$O_7(7)$	273457218604953600	$2^{12}.3^4.5^2.7^9.19.43$	2	2
$G_2(19)$	796793353927300800	$2^6.3^5.5^2.7^3.19^6.127$	1	1
A_{20}	1216451004088320000	$2^{17}.3^8.5^4.7^2.11.13.17.19$	2	2
Fi_{23}	4089470473293004800	$2^{18}.3^{13}.5^2.7.11.13.17.23$	1	1
Co_1	4157776806543360000	$2^{21}.3^9.5^4.7^2.11.13.23$	2	1
$L_8(2)$	5348063769211699200	$2^{28}.3^5.5^2.7^2.17.31.127$	1	2
$U_8(2)$	7434971050829414400	$2^{28}.3^9.5^2.7.11.17.43$	1	2
$O_8^+(5)$	8911539000000000000	$2^{12}.3^5.5^{12}.7.13^2.31$	2^2	S_4
$S_6(8)$	9077005607176765440	$2^{27}.3^7.5.7^3.13.19.73$	1	3
$G_2(23)$	11570921621943780096	$2^8.3^3.7.11^2.13^2.23^6.79$	1	1
$O_8^-(5)$	17880203250000000000	$2^{10}.3^4.5^{12}.7.13.31.313$	2	2^2
A_{21}	25545471085854720000	$2^{17}.3^9.5^4.7^3.11.13.17.19$	2	2
Sz(8192)	36888985097480437760	$2^{26}.5.53.157.1613.8191$	1	13
$G_2(25)$	37193298187500000000	$2^8.3^3.5^{12}.7.13^2.31.601$	1	2
${}^3D_4(5)$	35817806390625000000	$2^6.3^4.5^{12}.7^2.31^2.601$	1	1
$O_{12}^+(2)$	50027557148216524800	$2^{30}.3^8.5^2.7^2.11.17.31$	1	2
$O_{12}^-(2)$	51615733565620224000	$2^{30}.3^6.5^3.7.11.13.17.31$	1	2
$S_6(9)$	54025731402499584000	$2^{12}.3^{18}.5^3.7.13.41.73$	2	2^2
$O_7(9)$	54025731402499584000	$2^{12}.3^{18}.5^3.7.13.41.73$	2	2^2
J_4	86775571046077562880	$2^{21}.3^3.5.7.11^3.23.29.31.37.43$	1	1
$L_5(7)$	187035198320488089600	$2^{11}.3^5.5^2.7^{10}.19.2801$	1	2
$U_5(7)$	188151359720376729600	$2^{15}.3^2.5^2.7^{10}.11.43.191$	1	2
$L_6(4)$	361310134959341568000	$2^{30}.3^6.5^3.7^2.11.13.17.31$	3	D_{12}
A_{22}	562000363888803840000	$2^{18}.3^9.5^4.7^3.11^2.13.17.19$	2	2
$O_{10}^-(3)$	650084965259666227200	$2^{15}.3^{20}.5^2.7.13.41.61$	4	D_8
$U_6(4)$	1120527288631296000000	$2^{30}.3^4.5^6.7.13^2.17.41$	1	4
$O_{10}^+(3)$	1289512799941305139200	$2^{15}.3^{20}.5^2.7.11^2.13.41$	2	2^2
$S_6(11)$	3669292720793456064000	$2^9.3^4.5^3.7.11^9.19.37.61$	2	2
$O_7(11)$	3669292720793456064000	$2^9.3^4.5^3.7.11^9.19.37.61$	2	2
$S_8(4)$	4408780839651901440000	$2^{32}.3^5.5^4.7.13.17^2.257$	1	2
$L_5(8)$	4638226007491010887680	$2^{30}.3^4.5.7^4.13.31.73.151$	1	6
$U_5(8)$	4656663745464977326080	$2^{30}.3^9.5.7^2.11.13.19.331$	1	6
A_{23}	12926008369442488320000	$2^{18}.3^9.5^4.7^3.11^2.13.17.19.23$	2	2
$U_5(9)$	15775810414207914240000	$2^{11}.3^{20}.5^4.41.73.1181$	5	5:4
Sz(32768)	37777778976635853209600	$2^{30}.5^2.7.13.31.41.61.151.1321$	1	15
$L_7(3)$	67034222101339041669120	$2^{13}.3^{21}.5.7.11^2.13^2.1093$	1	2
$U_7(3)$	72853912155490594652160	$2^{16}.3^{21}.5.7^2.13.61.547$	1	2
${}^2E_6(2)$	76532479683774853939200	$2^{36}.3^9.5^2.7^2.11.13.17.19$	$2^2 \times 3$	S_3
$L_5(9)$	78660280796419613491200	$2^{15}.3^{20}.5^2.7.11^2.13.41.61$	1	2^2
$O_8^+(7)$	112554991177798901760000	$2^{16}.3^5.5^4.7^{12}.19.43$	2^2	S_4
$S_6(13)$	122796979335906113871360	$2^9.3^4.5.7^3.13^9.17.61.157$	2	2
$O_7(13)$	122796979335906113871360	$2^9.3^4.5.7^3.13^9.17.61.157$	2	2
$S_{12}(2)$	208114637736580743168000	$2^{36}.3^8.5^3.7^2.11.13.17.31$	1	1
$E_6(2)$	214841575522005575270400	$2^{36}.3^6.5^2.7^3.13.17.31.73$	1	2
$O_8^-(7)$	225297574007560801689600	$2^{13}.3^4.5^2.7^{12}.19.43.1201$	2	2^2
R(2187)	2391899102643523493326 32	$2^3.3^{21}.7^2.43.547.1093.2269$	1	7
${}^2F_4(8)$	2649053526995861766144 00	$2^{36}.3^5.5^2.7^2.13^2.19.37.109$	1	3
A_{24}	3102242008666197196800 00	$2^{21}.3^{10}.5^4.7^3.11^2.13.17.19.23$	2	2
$U_9(2)$	3254732927211084447744 00	$2^{36}.3^{11}.5^2.7.11.17.19.43$	3	S_3
${}^3D_4(7)$	4507829741566495552965 12	$2^8.3^4.7^{12}.13.19^2.43^2.181$	1	3
$U_6(5)$	4687555201875000000000 00	$2^{11}.3^6.5^{15}.7^2.13.31.521$	6	D_{12}
$L_9(2)$	6996123100331976425472 00	$2^{36}.3^5.5^2.7^3.17.31.73.127$	1	2
Fi_{24}^1	1255205709190661721292 800	$2^{21}.3^{16}.5^2.7^3.11.13.17.23.29$	3	2
$L_6(5)$	1383059427750000000000 00	$2^{13}.3^4.5^{15}.7.11.13.31^2.71$	2	2^2
$L_5(11)$	1952052708565059186240 000	$2^9.3^2.5^4.7.11^{10}.19.61.3221$	5	D_{10}
$F_4(3)$	5734420792816671844761 600	$2^{15}.3^{24}.5^2.7^2.13^2.41.73$	1	1
$S_8(5)$	6973279267500000000000 00	$2^{14}.3^5.5^{16}.7.13^2.31.313$	2	2
$O_9(5)$	6973279267500000000000 00	$2^{14}.3^5.5^{16}.7.13^2.31.313$	2	2
A_{25}	7755605021665492992000 000	$2^{21}.3^{10}.5^6.7^3.11^2.13.17.19.23$	2	2
$U_5(11)$	9775062020994743678515 200	$2^{11}.3^5.5^2.11^{10}.37.61.13421$	1	2
B	4154781 481226426191177580544000000	$2^{41}.3^{13}.5^6.7^2.11.13.17.19.23.31.47$	2	1
$E_7(2)$	7997476042075 799759100487262680802918400	$2^{63}.3^{11}.5^2.7^3.11.13.17.19.31.43.73.127$	1	1
M	808017424794512875886459904 961710757005754368000000000	$2^{46}.3^{20}.5^9.7^6.11^2.13^3.17.19.23.29.31.41.47.59.71$	1	1
$E_8(2)$	337804753143634806261 388190614085595079991692242 467651576160959909068800000	$2^{120}.3^{13}.5^5.7^4.11^2.13^2.17^2.19.31^2.41..43.73.127.151.241.331$	1	1

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$S_4(5)$	61	62–63
$S_6(2)$	46	47
$S_6(3)$	112–113	110–113
$S_8(2)$	123	124–125
Th	177	176
Tits	74	75
$U_2(q)$, see $L_2(q)$		
$U_3(11)$	84	84
$U_3(3)$	14	14
$U_3(4)$	30	30
$U_3(5)$	34	35
$U_3(7)$	66	67
$U_3(8)$	66	64–65
$U_3(9)$	79	79
$U_4(2)$	26	27
$U_4(3)$	52–53	54–59
$U_5(2)$	72–73	72–73
$U_6(2)$	115	116–121
$W(A_n)$, see A_{n+1}		
$W(E_6)$	26	27
$W(E_7)$	46	47
$W(E_8)$	85	86–87
${}^2A_n(q)$, see $U_{n+1}(q)$		
${}^2B_2(2^{2m+1})$, ${}^2C_2(2^{2m+1})$, see $Sz(2^{2m+1})$		
${}^2D_n(q)$, see $O^-_{2n}(q)$		
${}^2E_6(2)$	191	192–199
${}^2F_4(2)'$	74	75
${}^2G_2(27)$	123	122
${}^2G_2(3)'$	6	6
${}^3D_4(2)$	89	90
·1	180–183	184–187
·2	154	154–155
·223	100	101
·233	80	81
·3	134	135