Spatiotemporal order and emergent edge currents in active spinner materials

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Collections of interacting, self-propelled particles have been extensively studied as minimal models of many living and synthetic systems from bird flocks to active colloids. However, the influence of active rotations in the absence of self-propulsion (i.e., spinning without walking) remains less explored. Here, we numerically and theoretically investigate the behavior of ensembles of self-spinning dimers. We find that geometric frustration of dimer rotation by interactions yields spatiotemporal order and active melting with no equilibrium counterparts. At low density, the spinning dimers self-assemble into a triangular lattice with their orientations phase-locked into spatially periodic phases. The phase-locked patterns form dynamical analogs of the ground states of various spin models, transitioning from the three-state Potts antiferromagnet at low densities to the striped herringbone phase of planar quadrupoles at higher densities. As the density is raised further, the competition between active rotations and interactions leads to melting of the active spinner crystal. Emergent edge currents, whose direction is set by the chirality of the active spinning, arise as a nonequilibrium signature of the transition to the active spinner liquid and vanish when the system eventually undergoes kinetic arrest at very high densities. Our findings may be realized in systems ranging from liquid crystal and colloidal experiments to tabletop realizations using macroscopic chiral grains.

active matter | nonequilibrium steady states | edge currents | geometrical frustration

The past two decades have seen significant progress in our understanding of active matter. Early theoretical progress (1–3) has been accompanied by the engineering of soft materials made of self-propelled polymers, colloids, emulsions, and grains (4–11), which exhibit novel nonequilibrium phenomena. Prominent examples include phase separation of repulsive spheres, giant number fluctuations away from criticality, and long-range orientational order in 2D flocks (12–14).

The systems mentioned above have in common the characteristic that constituents acquire translational momentum because of active propulsion but rotate only in response to collisions or diffusion. By contrast, insights into the consequences of active rotation without self-propulsion remain scarce, although this situation is relevant to a wide range of experimental systems (15), including spinning microorganisms (16, 17), treadmilling proteins (18), sperm cell and microtubule aggregates (19, 20), shaken chiral grains (21), light-powered chiral colloids (22), thermally and chemically powered liquid crystals (23, 24), electrorheological fluids (25), and biological and synthetic cilia driven by rotary molecular motors (26).

Until now, theoretical and numerical studies on ensembles of active spinners have separately addressed their phase dynamics and their spatial organization. The emergence and robustness of synchronized rotation in lattices of hydrodynamically coupled rotors (27, 28) have been studied as an archetype of Kuramoto dynamics in coupled oscillator systems (29). In these models, the lattice geometry is imposed, a situation relevant, for instance, to the propagation of metachronal waves at the surface of ciliated tissues (30–33). Local orientational synchronization has also been observed in self-organized disordered arrays of rotating rods (34, 35). A separate class of numerical studies has been devoted to the spatial structures of ensembles of active spinners interacting via either contact or hydrodynamic interactions (36–42). Special attention has been paid to phase separation in binary mixtures of counterrotating spinners and hydrodynamic interactions yielding spatial ordering.

Here, we bridge the gap between these two lines of research. Combining numerical simulations and analytical theory, we show the inherent interplay between the spatial structure and the phase dynamics of active spinners. We uncover a generic competition between monopole-like interactions that dominate at large separations and shorter-range multipole gear-like interactions. We find that their interplay frustrates ordered states but also, yields unique spatiotemporal order and unanticipated collective flows including edge currents.

We study a prototypical system of soft dimers interacting via repulsive interactions and undergoing unidirectional active rotation as sketched in Fig. 1. When isolated, dimers spin in response to the active torque, attaining a steady-state spinning speed caused by background friction. As they get closer, the multipole character of the pair interactions resists the rotation of adjacent dimers (Fig. 1 *B* and *C*). At very high densities, the relative motion of neighbors is completely obstructed (Fig. 1*D*). By tuning the density, we explore how the frustration

Significance

Active materials are composed of building blocks individually powered by internal energy or external fields. Here, we explore the collective behavior of interacting particles with active rotations but no self-propulsion (i.e., active spinning without walking). When many such spinners are brought together, they form unique nonequilibrium steady states reminiscent of crystals, liquids, and glasses. Unlike equilibrium phases of matter, which stem from the balance between entropy and internal energy, the spinner states arise from the competition between active torques and interactions. Active spinner crystals have distinctive features: they vary periodically in time as well as space and melt under increasing pressure. Emergent unidirectional edge currents are the nonequilibrium probes of the transition between crystalline and liquid-like states in active spinner materials.

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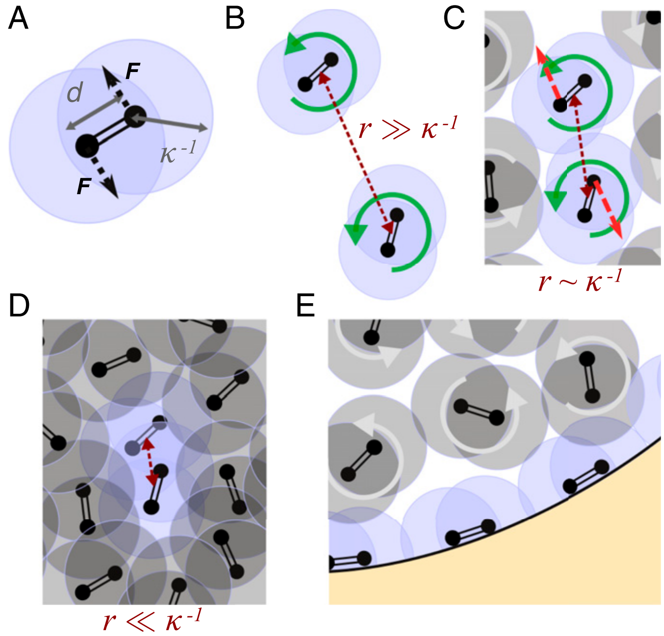

Fig. 1. Competing rotation and interactions in active spinners. (*A*) Makeup of a single self-spinning dimer, consisting of a pair of identically charged particles (black circles) connected by a rigid rod of length *d* (double line). Particles repel each other with a Yukawa interaction with screening length κ^{-1}, which determines the soft exclusion zone (light-blue disks), beyond which the repulsion falls off exponentially with distance. Each particle experiences a force of magnitude *F* and direction indicated by dotted arrows, oriented to provide zero net force and a net torque $\tau = Fd$ on the dimer at all times. (*B*–*D*) The density determines the influence of interactions on dimer dynamics. (*B*) At large separations, interactions are negligible, and dimers freely rotate at the terminal angular velocity set by the activity and the background drag. (*C*) As separations become comparable with the screening length, adjacent dimers still rotate past each other but experience interaction forces (red dashed arrows show instantaneous force caused by the interaction between two of the particles) that depend on their instantaneous orientations. (*D*) At very high densities, interactions completely obstruct dimer rotation. (*E*) Hard boundaries also obstruct dimer rotation, and their effect is transmitted into interior dimers by interactions.

between monopole and multipole interactions plays out as their relative strengths are varied (Movie S1). We observe transitions from collections of independently spinning dimers to unusual crystal states, which are ordered in particle position as well as orientation over time (Movies S2 and S3), to active spinner liquids to jammed states. Repulsive interactions with boundaries also obstruct spinning (Fig. 1*E*); to compensate, the system channels the rotational drive into linear momentum, giving rise to robust edge currents and collective motion (Movie S1).

Our model system consists of a 2D ensemble of N like-charge dimers, each consisting of two point particles of mass m connected by a stiff link of length d (Fig. 1*A*). Point particles interact only via a repulsive pair potential of the Yukawa form $be^{-\kappa r}/r$, where b sets the overall strength of the repulsion, r is the interparticle separation distance, and κ is the inverse screening length (Fig. 1). By setting $\kappa^{-1} \sim d$, we discourage dimer links from crossing each other and also, maximize the orientational dependence of the effective pair interaction between dimers.

Each dimer is actively driven, implemented by a torque $\tau = Fd$ implemented as a force dipole (Fig. 1*A*). Energy is dissipated by drag forces acting on each particle with associated drag coefficient γ. The equations of motion for the position $\mathbf{r}_i$ and orientation θ_i of the ith dimer are

$$2m\ddot{\mathbf{r}}_i = -2\gamma\dot{\mathbf{r}}_i - \partial_{\mathbf{r}_i}\sum_{j\neq i} V\left(\mathbf{r}_j - \mathbf{r}_i, \theta_i, \theta_j\right) \qquad [1]$$

and

$$I\ddot{\theta}_i = \tau - \gamma_\Omega\dot{\theta}_i - \partial_{\theta_i}\sum_{j\neq i} \mathcal{V}\left(\mathbf{r}_j - \mathbf{r}_i, \theta_i, \theta_j\right), \qquad [2]$$

where $I = md^2/2$ and $\gamma_\Omega = \gamma d^2/2$ are the moment of inertia and rotational friction coefficients, respectively, and the position- and orientation-dependent interaction potentials V and $\mathcal{V}$, respectively, are derived from the Yukawa pair interactions between the point particles. An isolated dimer attains a steady state of counterclockwise rotation about its center with a constant spinning speed $\Omega_0 = \tau/\gamma_\Omega$ (Fig. 1*B*). In contrast to systems where the dimer orientation is slaved to an external field [e.g., colloids driven by a rotating magnetic field (43, 44)], the instantaneous dimer orientation is not dictated by the internal drive in our system.

On rescaling distances by κ^{-1} and time by Ω_0^{-1}, the dynamical equations are characterized by three dimensionless quantities: κd, $\alpha \equiv I\tau/\gamma_\Omega^2$, which measures the characteristic dissipation time for angular momentum in units of the spinning period, and $\beta^{-1} \equiv \tau/\kappa b$, which quantifies the drive in units of the characteristic interaction energy scale. We focus here on the competition between rotational drive and interactions as the dimer density is varied for fixed α and β as sketched in Fig. 1 *B*–*D*. We constrain ourselves to the asymptotic limit where both $\alpha \gg 1$ and $\beta \gg 1$.

Phase Behavior

We characterized the bulk behavior of interacting spinners through simulations under periodic boundary conditions in which the dimer density was varied by changing the dimensions of the simulation box with constant screening parameter $\kappa = 0.725/d$, particle number $N = 768$, and activity parameters $\alpha = 131$ and $\beta = 133$ (*Materials and Methods*). Density is quantified by the packing fraction $\phi = A\rho$, where ρ is the number density of dimers, and $A = \pi(d + 2\kappa^{-1})^2/4$ is the soft excluded area of a spinning dimer on timescales $t \gg 1/\Omega_0$. Fig. 2 characterizes the phase behavior of our system via changes in particle ordering, orientational ordering, and dynamics in the non-equilibrium steady states reached at long times. Nearly identical behavior is observed for simulations with $N = 3{,}072$, indicating that finite size effects are negligible (Fig. S1).

Active Spinner Crystals. At low packing fractions, the dimers self-organize into a hexagonal crystalline pattern, with little or no change in position, as shown for two representative densities in columns 1 and 2 in Fig. 2*A*, *Right*. In this regime, the repulsions between dimers give rise to a Wigner-like crystal quantified by high values of the bond-orientational order parameter $|\langle\psi_6\rangle|$ (Fig. 2*D*, triangles). Although the dimers are highly restricted in their position, they continue to spin without hindrance, attaining the same angular speed as an isolated dimer ($\langle\dot{\theta}\rangle \equiv \Omega \approx \Omega_0$) (Fig. 2*E*). Apart from small fluctuations, the orientation of dimer i at time t has the form $\theta_i(t) = \Omega_0 t + \delta_i$, with the angular phase δ_i defined up to a global phase shift. This state is reminiscent of plastic crystals but with the equilibrium fluctuations of the orientational dfs replaced by active rotation: we term this state an active spinner crystal.

The crystals display ordering in not only dimer positions but also, dimer orientations, which are phase-locked into regular spatial patterns (Movies S2 and S3). The angular phases δ_i take on a few discrete values determined by the lattice position. We find evidence for two distinct configurations. At low densities, δ_i acquires one of three values $\{0, \pi/3, 2\pi/3\}$, with no two neighbors sharing the same value (column 1 in Fig. 2*C*, *Right*). This

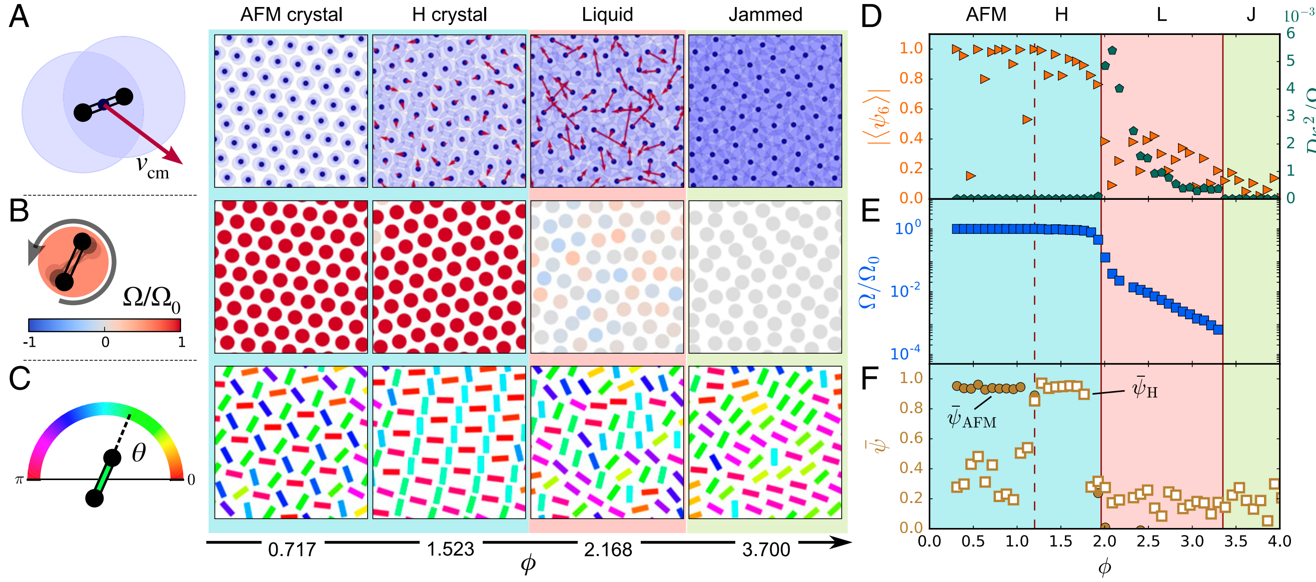

Fig. 2. Bulk phases of the active spinner system. The behavior of dimer positions and orientations is investigated as a function of packing fraction ϕ for constant activity level $\alpha = 131.026$. *A–C* highlight different physical quantities of the system, shown schematically and displayed for simulation snapshots for four representative values of ϕ in *A–C*. The snapshots cover roughly 10% of the simulation area. (*A*) Center of mass position (dark dots) and velocities (red arrows) shown along with the soft exclusion area of individual charges (translucent disks). (*B*) Angular rotation speed Ω/Ω_0. (*C*) Orientation represented by a fixed-length segment colored by the angle made by the dimer with the x axis. Segment length does not represent actual dimer size. (*D–F*) Ensemble measurements of steady-state physical quantities as a function of ϕ. (*D*) Bond-orientational order parameter and diffusivity of dimer positions. (*E*) Average angular speed. These quantities identify three distinct phases in different density ranges: crystal (blue background), liquid (red background), and jammed (green background). The rotational speed abruptly drops to zero (within numerical precision) in the jammed phase. (*F*) Order parameters quantifying Potts AFM (ψ_{AFM}) and striped H (ψ_{H}) order in the phase relationships between rotating dimers in the crystal.

pattern is identical to the equilibrium ground state of the three-state Potts antiferromagnet (3P-AFM) on the triangular lattice (45). When $\phi > \phi_{C_3-C_2} \approx 1.2$, the rotational symmetry of the pattern changes from C_3 to C_2 as stripes of alternating $\delta_i \in \{0, \pi/2\}$ form along a spontaneously chosen lattice direction (column 2 in Fig. 2*C*, *Right*). This phase is a dynamical analog of the striped herringbone (H) phase observed in lattices of elongated molecules (46). Local order parameters ψ_{AFM} and ψ_{H} (defined in *Materials and Methods*) measure the extent to which phase differences among neighboring dimers match those prescribed by the respective ordered states. As shown in Fig. 2*F*, the 3P-AFM and H states are each observed over a range of densities.

To understand the origin of the phase-locked patterns, we study a minimal model of the dimer–dimer interactions. To lowest order in dimer size d, each dimer is a superposition of a charge monopole and a charge quadrupole. The monopole repulsion arranges the dimer centers into a triangular crystal with lattice constant $a \sim 1/\sqrt{\phi}$. We assume that the dimer positions are thus fixed and focus on the orientation dynamics (Eq. **2**) caused by the quadrupolar interactions. When averaged over the common rotation period $2\pi/\Omega_0$, Eq. **2** reduces to $\partial_{\theta_i}\langle\sum_{j\neq i}\mathcal{V}(\mathbf{r}_j - \mathbf{r}_i, \theta_i, \theta_j)\rangle_t = 0$ (i.e., the nonequilibrium steady states extremize the time-averaged potential energy as a function of orientation).

On ignoring fluctuations around the constant speed evolution $\theta_i(t) = \Omega_0 t + \delta_i$ and considering only nearest neighbor interactions among dimers, the average effective energy takes the compact form

$$V_{\mathrm{eff}} \equiv \left\langle \sum_{j\neq i} \mathcal{V}(\theta_i - \theta_j) \right\rangle_t = \sum_{\langle ij\rangle} \left[A_1 + A_2 \left(\frac{d}{a}\right)^4 \cos 2(\delta_i - \delta_j) \right], \tag{3}$$

where A_1 and A_2 vary with density (details are in *SI Materials and Methods*). For an infinite lattice of dimers, V_{eff} has arbitrarily many extrema. However, the extrema can be exhaustively listed for a triangle of neighboring dimers. Up to a global phase shift and vertex permutations, the effective energy as a function of the phase shifts $\{\delta_1, \delta_2, \delta_3\}$ on the triangle vertices has three unique extrema at $\{0, \pi/3, 2\pi/3\}$, $\{0, 0, \pi/2\}$, and $\{0, 0, 0\}$. The 3P-AFM and H phases extend the first and second of these extrema, respectively, onto the infinite triangular lattice and are, thus, also extremal states of the periodic crystal. In fact, the 3P-AFM state is the global energy minimum for V_{eff}, as seen by mapping the effective energy to the antiferromagnetic (AFM) *XY* model on the triangular lattice (47).* The extremum with phase values $\delta_i = 0$, which would correspond to all dimers sharing the same orientation at all times, maximizes the frustration of spinning by interactions and is not observed in our simulations.

In summary, spinning dimers are frustrated. The spatiotemporal crystal states that are compatible with the mutual frustration of the position and orientation dfs are captured by the extrema of the effective potential V_{eff}. However, in principle, active spinner crystals could harbor a multitude of other phase-locked patterns, which cannot be reduced to repetitions of a single triangular unit but nevertheless, extremize V_{eff}. These states may be accessible by modifying the initial or boundary conditions or the dynamics of approaching the nonequilibrium steady state.

Melting and Kinetic Arrest. We now elucidate how synchronized spinning motion frustrates positional order and melts dense spinner crystals. As the packing fraction is increased, we observe

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*The effective energy inherits a discrete and a continuous ground-state degeneracy from the AFM *XY* model. An arbitrary global phase shift gives the same state, but this shift is equivalent to a choice of $t = 0$ in the description of the orientations. The discrete degeneracy is in the chirality of phase order ($0 \to \pi/3 \to 2\pi/3$ vs. $0 \to 2\pi/3 \to \pi/3$) on circling a plaquette. Adjacent plaquettes always have opposite chirality, and the two possible chirality arrangements on the triangular lattice provide two distinct ground states.

a loss of crystalline ordering, signaled by a sharp drop in $|\langle\psi_6\rangle|$ from 1 to 0.2 at $\phi = \phi_{\mathrm{melt}} \approx 1.9$. This drop coincides with the onset of diffusive dynamics of the dimer centers of mass at long times (Fig. S2). The diffusivity $D \equiv \lim_{t\to\infty}\langle|\mathbf{r}_i(t_0+t) - \mathbf{r}_i(t)|^2\rangle_i/t$ is nonzero for a range of densities above ϕ_{melt}, characteristic of a liquid phase. Melting is accompanied by a disruption of the phase-locked spinning dynamics as quantified by (*i*) a drop in the average spin velocity to below $0.1\Omega_0$ (Fig. 2*E*), (*ii*) a marked increase in spin speed fluctuations (Fig. S3), and (*iii*) a loss of H order in the orientations (Fig. 2*F*). Column 3 in Fig. 2 *A*–*C*, *Right* shows a typical liquid configuration with no discernible order in the positions, orientations, or spinning speeds.

The melting of the dimer crystal on increasing the density, at odds with the typical behavior of athermal or equilibrium repulsive particles, is a direct result of the orientational dependence of dimer–dimer interactions coupled with the active spinning. The monopole part of the pair interaction is responsible for the crystalline arrangement of dimer centers. The quadrupolar component generates a gearing effect, which hinders the activity-driven corotation of adjacent dimers as shown schematically in Fig. 1*C*. The competition between interactions and active spinning results in geometrical frustration of the crystalline order, akin to the frustration of AFM Ising spins on the triangular lattice. Increasing the density strengthens the quadrupolar component of the interactions relative to the monopole component, destabilizing the crystal at the threshold packing fraction ϕ_{melt}. In the liquid state, the frustration of in-place dimer rotation by interactions is partially relieved by dimers constantly sliding past each other at the cost of crystalline and phase-locked order.

On increasing the packing fraction beyond ϕ_{melt}, the diffusive and spinning dynamics slow down as interactions become more prominent. At $\phi = \phi_{\mathrm{J}} \approx 3.3$, the diffusivity and spinning speed of the ensemble both drop abruptly to zero, signifying a sharp transition from a liquid to a frozen solid, in which interactions completely overwhelm the external drive (48). As shown by representative snapshots (column 4 in Fig. 2 *A*–*C*, *Right*) and the bond-orientational order parameter (Fig. 2*D*), the dimer positions and orientations in the frozen state do not exhibit the ordering of the crystalline phases. However, a different form of short-range orientational order persists: dimers tend to form ribbon-like assemblies, which share a common alignment (column 4 in Fig. 2*C*, *Right* and Fig. S4). This structure, which locally resembles smectic ordering in liquid crystals, is a consequence of the constraints on tightly packing repulsive dimers. The full description of this state, reminiscent of a degenerate crystal (49), goes beyond the scope of our work.

Confinement-Induced Collective Motion

At a microscopic level, the bulk phases are distinguished by the relative importance of rotational drive and orientation-dependent interactions. For a steady state to be attained, torques must be balanced globally as well; in a confined system, the overall torque may be balanced by viscous drag as well as boundary forces. To investigate the interplay between rotational drive, interactions, and confinement, we simulated a system confined by a circular frictionless boundary as depicted in Fig. 3*A* for the same particle number ($N = 768$), activity level, and density range as in Fig. 2. Densities are changed by varying the circle radius, because $\phi = NA/\pi R^2$. Fig. 3*A* and Movie S1 show the dimer center of mass motion for three representative densities across different phases, all of which display spontaneous macroscopic flows.

Measurements of the coarse-grained azimuthal velocity $v_\theta(r)$ as a function of distance r from the disk center (*Materials and Methods*) reveal qualitative differences in the collective flows across phases. In both the crystal ($\phi = 0.827$) and frozen ($\phi = 3.750$) phases, the angular velocity about the disk center,

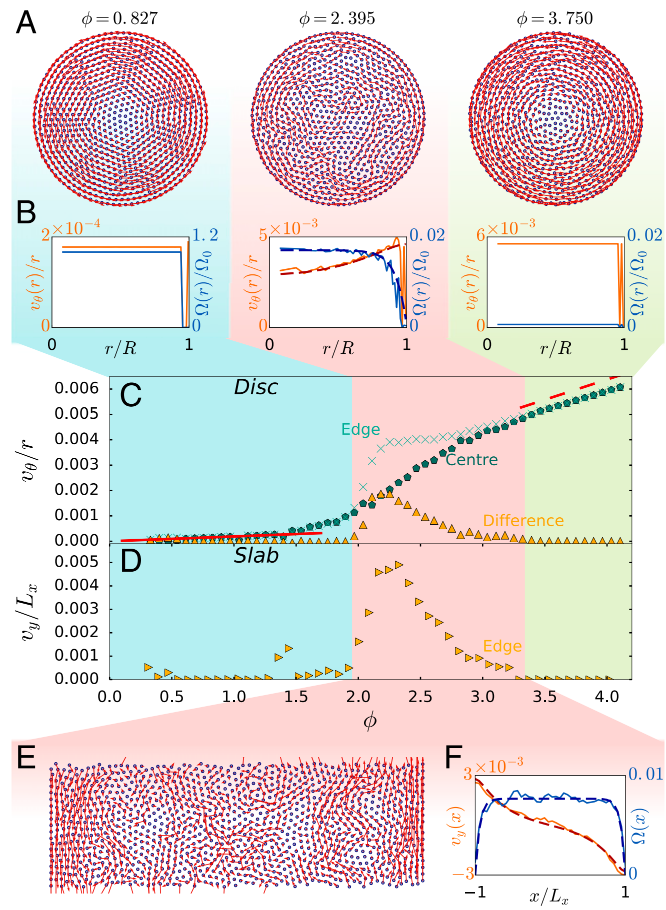

Fig. 3. Collective motion reflects phase changes. (*A*) Snapshots showing the drift of $N = 768$ dimers confined by a circular boundary. Arrows indicate the displacements after $\Delta t = 164/\Omega_0$ for $\phi = 0.827$ (crystal), $\phi = 2.395$ (liquid), and $\phi = 3.750$ (jammed). Arrows are scaled differently for visibility. (*B*) Time-averaged steady-state radial distributions of the orbital angular speed $\omega(r)$ about the disk center (orange) and the local spin speed (blue) for the simulations shown in *A*. Dashed lines are fits to the hydrodynamic theory. (*C*) Steady-state orbital angular speed $\omega(r)$ in simulation units as a function of density measured at the disk center ($r = 0$) and edge ($r = R$). Coincidence of the two values is consistent with rigid body rotation. The solid and dashed red lines show the theoretical prediction for the rigid body rotation speed in the crystal and jammed phases, respectively. (*D*) Steady-state tangential speed of dimers at the wall as a function of density for $N = 768$ dimers confined by two walls perpendicular to the x direction and periodic boundary conditions along y. Density is varied by changing the area between the slabs, while keeping the aspect ratio $L_y/L_x = 2$ unchanged. (*E*) Snapshot of dimer motion for 768 dimers confined between parallel slabs at $\phi = 2.410$, with $L_y/L_x = 1/3$. (*F*) Averaged steady-state velocity profile between the slabs (orange) and local spin speed (blue) for the simulation shown in *E*. Dashed lines are fits to the hydrodynamic theory.

$\omega(r) = v_\theta(r)/r$, is constant throughout the disk (Fig. 3*B*), showing that the ensemble rotates around the center in unison as a rigid body. By contrast, the angular velocity profile is nonuniform for the liquid ($\phi = 2.395$), growing monotonically with distance from the disk center. These distinct behaviors persist over the entire phase diagram, as shown in Fig. 3*C*, which compares the steady-state values of the flow angular velocity at the center [$\omega(0)$] and edge [$\omega(R)$] of the disk as a function of density. The center and edge values coincide in the solid phases, consistent with rigid body rotation, whereas the liquid phase shows a persistent enhancement of flow at the edge. Collective vortical motion and boundary flows were previously shown in suspensions of swimming cells (50, 51). However, their spatial structure and physical origin are profoundly different from the confinement-induced flows reported here, which depend on the chiral activity of the spinners as we now elucidate.

Spontaneous Collective Rotation of Rigid Phases. The rigid body rotation in the two solid phases, ordered and jammed, can be understood by balancing torques about the center of the circular boundary to obtain an acceleration-free steady state. The forces exerted by the boundary, being radially oriented, do not exert torque. Thus, the driving torques acting on the dimers must be balanced by drag forces. In the crystal interior, dimers homogeneously and steadily spin about their individual centers at a rate Ω_0 (Fig. 3*A*), and the resulting friction balances the driving torques at all times. However, the spinning of the outermost layer of N_e dimers is obstructed by the hard boundary as shown schematically in Fig. 1*E*, which implies that the driving torques on these dimers are not balanced by spinning. Rather, these torques drive an overall rotation of the crystal. The corresponding rigid body rotation speed, ω_rb, is obtained by balancing the net drive $N_\mathrm{e}\tau$ against the net drag torque caused by the rigid body rotation, which scales as $N\gamma\Omega R^2$, thereby leading to $\omega_\mathrm{rb}\sim(N_\mathrm{e}/N)\tau/\gamma R^2\propto\phi$.

In the frozen phase, local spinning of dimers relative to their neighbors is completely frustrated by interactions. Therefore, the entire external torque $N\tau$ is balanced solely by the drag caused by orbital motion, giving rise to $\omega_\mathrm{rb}\sim\tau/\gamma R^2\propto\phi$. The measured rotation speeds quantitatively match the predictions because of overall torque balance (solid and dashed lines in Fig. 3*C*).

Emergent Edge Current in Active Spinner Liquids. The rigid body motion of the two solid phases relies on the transmission of torque via shear stresses throughout the sample. If the disk is partitioned into circular annuli, the net external drive acting on each annulus differs from the net drag torque; neighboring annuli must exert shear forces on each other to balance the total torque. Unlike the solid phases, the liquid cannot support a shear stress through elastic deformations, which qualitatively explains the absence of pure rigid body rotation (Fig. 3 *A* and *B*). For a quantitative description of the emergent flow, we use a continuum theory of an active chiral liquid coupled to a solid substrate. This phenomenological model, introduced in refs. 21 and 52, generalizes the so-called micropolar fluid hydrodynamics (53, 54) by including couplings to a frictional substrate.

Assuming incompressibility (as justified by the lack of significant spatial variations in dimer density), the hydrodynamic description relies solely on the conservation of momentum and angular momentum and therefore, involves two coarse-grained fields: the flow velocity $\mathbf{v}(\mathbf{r})$ and the internal angular rotation, or spin, field $\Omega(\mathbf{r})$. The hydrodynamic equations take on a compact form when written in terms of $\Omega(\mathbf{r})$ and the scalar vorticity $\zeta(\mathbf{r})=1/2\hat{\mathbf{z}}\cdot\nabla\times\mathbf{v}(\mathbf{r})$. In the viscous steady-state limit, these equations, which amount to local torque and force balance, respectively, are (21, 52)

$$D_\Omega\nabla^2\Omega-\Gamma^\Omega\Omega-\Gamma(\Omega-\zeta)+\rho\tau=0 \qquad [4]$$

and

$$(4\eta+\Gamma)\nabla^2\zeta-4\Gamma^v\zeta-\Gamma\nabla^2\Omega=0, \qquad [5]$$

where ρ is the active spinner fluid density, η is the shear viscosity, and D_Ω is a spin viscosity controlling the diffusive transport of angular momentum. The coefficients Γ^Ω and Γ^v quantify the dissipation of angular and linear momentum, respectively, caused by substrate friction. The crucial spin vorticity coupling is embodied in the rolling friction Γ, which coarse grains the frustration between rotations and interactions outlined in Fig. 1*C*. Orientation-dependent interactions hinder the free spinning of adjacent fluid elements, causing shear stresses proportional to Γ, unless the elements flow past each other in such a way that the vorticity cancels the local spin.

Analysis of the hydrodynamic equations reveals that spatial variations in the local spin field induce persistent flows. In the absence of boundaries, the equations admit the flow-free solution $\Omega=\rho\tau/(\Gamma^\Omega+\Gamma)=\Omega$, $\zeta=0$. If a hard boundary hinders spinning, however, $\Omega(\mathbf{r})$ varies from its value imposed by the boundary to the constant interior value Ω over a length scale $\lambda_\Omega=[D_\Omega/(\Gamma+\Gamma_\Omega)]^{1/2}$ set by the competition between diffusion and dissipation of local spin. The spatial variations in Ω, confined to the boundary, act as a source for vorticity, which itself decays over a length scale $\lambda_\zeta=[(4\eta+\Gamma)/(4\Gamma^v)]^{1/2}$ set by drag. These predictions match the simulation results, and a fit to radially symmetric spin and flow fields (dashed lines in Fig. 3*B*, *Center*) provides quantitative agreement with four fitting parameters (more details are in *SI Materials and Methods*).

The spontaneous liquid flow only requires the obstruction of spinning by the boundary, independent of its geometry. To highlight the robustness of this emergent flow, we also study active spinner liquids in a slab geometry with two edges aligned perpendicular to the x axis and periodic boundary conditions along y, as shown in Fig. 3*E*. This geometry suppresses rigid body rotation in all phases; excess driving torques are balanced by normal boundary forces. Accordingly, no dimer motion is measured in the crystal and jammed phases (Fig. 3*D*). However, a persistent flow parallel to the slab edges arises in the liquid phase, showing that the emergence of localized shear flows at edges is a robust feature of geometrically confined active spinner liquids. The mechanism for the edge current is the exchange between local spin and vorticity described above, which hinges on the orientation dependence of dimer–dimer interactions. The hydrodynamic description quantitatively reproduces the flow velocity profile $v_y(x)$ and spin field $\Omega(x)$ (Fig. 3*F*, dashed lines).

Conclusion

Combining numerical simulations and analytical theory, we have elucidated the phase behavior of interacting active spinners. The mutual frustration of positional and time-periodic orientational order has been shown to yield a variety of crystal and disordered phases. Although we have focused on the density dependence of the bulk and edge phenomena, the phases and their associated emergent flows persist over a broad range of activity strengths (Fig. S5), which makes experimental realizations feasible. Colloidal dumbbells (55, 56) spun by phoretic stresses (57) or Quincke rotation (6) would provide a near-literal realization of our model. More broadly, the essential ingredients of active spinners with orientation-dependent repulsive interactions are present in a wide variety of experimental systems, including chiral liquid crystals confined to a monolayer and driven via the Lehmann effect (58), rotating nanorods propelled by biomolecular motors (59), and light-driven micromotors (22). We also envision macroscopic realizations using chiral particles driven by airflow (60) or vibrations (21), with soft interactions provided by electrostatic or magnetic repulsion. Other than opening up avenues to explore nonequilibrium physics in simple settings, the phases arising from the interplay between interactions and spinning may be exploited for tunable torque transmission (61) or self-assembly of anisotropic particles into ordered patterns.

Materials and Methods

Details of the molecular dynamics simulations, including implementation of dimers and boundaries, ensemble averaging and spatial coarse graining of relevant physical quantities, and order parameters used to distinguish various phases, are provided in *SI Materials and Methods*.

APPLIED PHYSICAL SCIENCES

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