Modeling and Analysis Guidelines for Slow Transients—Part III: The Study of Ferroresonance

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***Abstract*—The ability to predict or confirm ferroresonance depends primarily on the correctness of the transformer model used by the computer simulation. Ferroresonance is introduced and a general modeling approach is given. An overview of available literature and contributors to this area is provided. A simple case of ferroresonance in a single phase transformer is used to illustrate this "phenomenon." Three phase transformer core structures are discussed. Ferroresonance in three phase grounded-wye distribution systems is described and illustrated with waveform data obtained from laboratory simulations. Representation of the study zone is discussed, modeling techniques are presented, and implementation suggestions are made. Three case studies are presented. Transformer representation is critical to performing a valid simulation. The direction of ongoing research is discussed, and the reader is advised to monitor the literature for ongoing rapid improvements in transformer modeling techniques.**

***Index Terms*—Bifurcation, chaos, duality, ferroresonance, nonlinear dynamics, overvoltages, transformer models.**

I. Introduction

A. Background and Available Literature

RESEARCH involving ferroresonance in transformers has been conducted over the last 80 yrs. The word ferroresonance first appears in the literature in 1920 [7], although papers on resonance in transformers appeared as early as 1907 [4]. Practical interest was generated in the 1930's when it was shown that use of series capacitors for voltage regulation caused ferroresonance in distribution systems [9], resulting in damaging overvoltages.

The first analytical work was done by Rudenberg in the 1940's [36]. More exacting and detailed work was done later by Hayashi in the 1950's [17]. Subsequent research has been divided into two main areas: improving the transformer models and studying ferroresonance at the system level.

An understanding of the nonlinear parameters describing a transformer core is prerequisite to dealing with ferroresonance. Swift [47] and Jiles [20] have provided insight into transformer core behavior and the separation of hysteresis and eddy current losses. Frame [15] and others have developed piecewise-linear methods of modeling the nonlinearities in saturable inductances.

Hopkinson [19] performed system tests and simulations on the effect of different switching strategies on the initiation of ferroresonance in three phase systems. Smith [38] categorized the modes of ferroresonance in one type of three phase distribution transformer based on the magnitude and appearance of the voltage waveforms. Arturi [2] and Mork [29] have demonstrated the use of duality transformations to obtain transformer equivalent circuits. Kieny [21] and Mork [27] have shown that the theories and experimental techniques of nonlinear dynamics and chaotic systems can be applied to better understand ferroresonance and limitations inherent in modeling a nonlinear system. Developments in the near future are expected to be in the areas of developing improved transformer models and applying nonlinear dynamics to the simulation of ferroresonance.

B. Ferroresonance in a Single Phase Transformer

In simple terms, ferroresonance is a series "resonance" involving nonlinear inductance and capacitances. It typically involves the saturable magnetizing inductance of a transformer and a capacitive distribution cable or transmission line connected to the transformer. Its occurrence is more likely in the absence of adequate damping. A simple illustrative case follows:

When rated voltage is applied to an unloaded single phase transformer, only a very small excitation current flows (Fig. 1). In this case, the 120-V winding of a 120–240 V 1.5 kVA dry-type transformer is energized, resulting in an exciting current, whose peak amplitude is 0.05 per unit. Referring to the equivalent circuit shown, it is seen that this current consists of two components: the magnetizing current and the core loss current. The magnetizing current, which flows through the nonlinear magnetizing inductance L_m, is required to induce a voltage in the secondary winding of the transformer. The core loss current, flowing through R_C, makes up the eddy current losses and hysteresis losses in the transformer's steel core.

Although usually assumed linear, R_C, is dependent on voltage and frequency. The excitation current contains high order odd harmonics, due to transformer core saturation. R_W and L_L, are the winding resistance and winding leakage inductance, respectively. They are assumed to be linear parameters. Their magnitudes are relatively small compared to L_M and R_C, and so are usually ignored in no-load situations [3], [24].

If a capacitor is placed between the voltage source and the unloaded transformer, ferroresonance may occur (Fig. 2). An extremely large exciting current (1.92 per unit peak) is drawn and the voltage induced on the secondary may be much larger than rated (1.44 per unit peak). The high current here is due to

Manuscript received August 21, 1997; revised May 28, 1998.
Publisher Item Identifier S 0885-8977(00)00565-3.

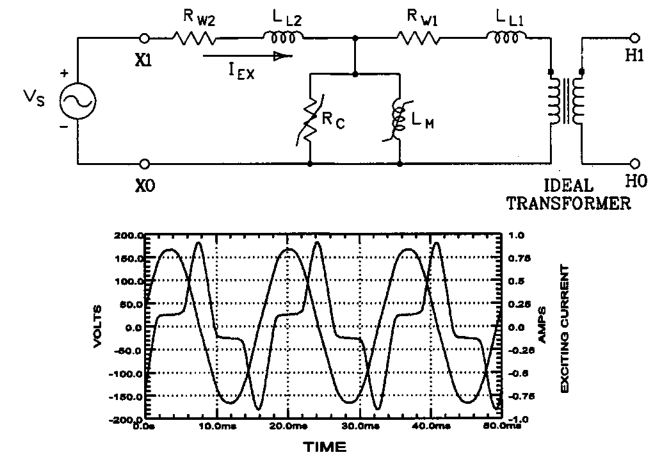

Fig. 1. Unloaded single phase transformer with rated voltage applied. Sine wave is applied voltage; spiked waveform is exciting current.

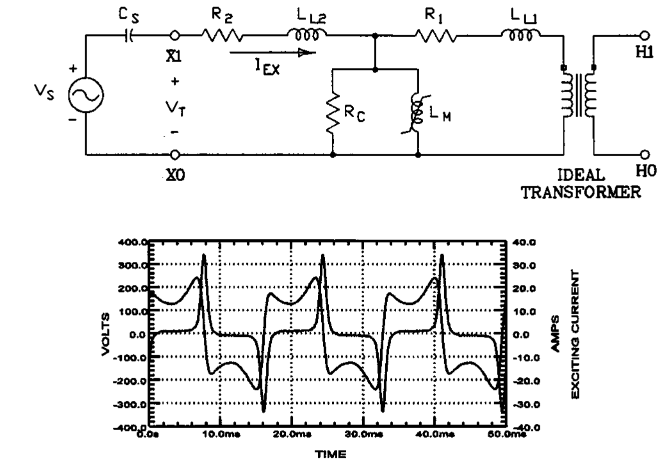

Fig. 2. Same transformer as in Fig. 1, fed through a 75 μF capacitance, operating in ferroresonance. Square waveform is terminal voltage of transformer; spiked waveform is the current.

resonance between C_S and L_M; ferroresonance in most practical situations results in smaller exciting currents. Any operating "modes" which result in a significantly distorted transformer (inductor) voltage waveform are typically referred to as ferroresonance, although the implication of resonance in a classical sense is arguably a misnomer. Even though the "resonance" occurring does involve a capacitance and an inductance, there is no definite resonant frequency, more than one response is possible for the same set of parameters, and gradual drifts or transients may cause the response to jump from one steady-state response to another.

High-order odd harmonics are characteristic of the waveforms, whose shapes might be conceptually explained in terms of the effective natural frequency $1\sqrt{L_M C_S}$ as L_M, goes in and out of saturation. Steep slopes (fast changes) occur when L_M is saturated, and flat slopes occur when L_M, is operating in its linear unsaturated region.

Due to nonlinearity, two other ferroresonant operating modes are possible, depending on the magnitudes of source voltage and series capacitance. In this case, all modes are seen to produce periodic voltage waveforms on the transformer secondary [26], [29]. In general, gradual changes in source voltage or capacitance will cause state transitions. A reversal to conditions that caused a transition will not reverse the transition, due to nonlinearity of L_M [36]. Transients can also trigger transition from mode to mode.

In modem terms, these jumps are referred to as bifurcations [16], [27], [29], [45], and may be better understood by applying the theory of nonlinear dynamics and chaos. A long-used intuitive explanation of these jumps, based on a graphical method, is given by Rudenberg [36]. However, this method is not a good analytical tool since it is based only on the fundamental frequency and neglects harmonics.

Damping added to the circuit will attenuate the ferroresonant voltage and current. Some damping is always present in the form of resistive source impedance, transformer losses, and also corona losses in high voltage systems, but most damping is due to the load applied to the secondary of the transformer. Therefore, a lightly-loaded or unloaded transformer fed through a capacitive source impedance is a prime candidate for ferroresonance.

This elementary type of ferroresonance is similar to that which occurred in the series capacitor compensated distribution systems of the 1930's. It can also occur, from different sources of capacitance, in today's single phase distribution transformers and voltage instrument transformers [1], [18]. It can also occur in series-compensated transmission lines.

Ferroresonance can lead to heating of transformer, due to high peak currents and high core fluxes. High temperatures inside the transformer may weaken the insulation and cause a failure under electrical stresses. In EHV systems, ferroresonance may result in high overvoltages during the first few cycles, resulting in an insulation coordination problem involving frequencies higher than the operating frequency of the system.

Because of nonlinearities, analytical solution of the ferroresonant circuit must be done using time domain methods. Typically, a computer-based numerical integration method is applied using time domain simulation programs such as the EMTP.

C. *Magnetic Behavior of Three Phase Transformers*

It is incorrect to assume that a three phase transformer core is magnetically equivalent to three single phase transformers, i.e., that the three phases have no direct magnetic coupling. Such an assumption can lead to serious errors, especially if one is investigating a transformer's behavior under transient or unbalanced conditions. Fig. 3 shows common core types.

The only type of core that displays magnetic characteristics similar to three single phase transformers is the triplex core. Although the cores share the same tank, they are magnetically isolated (except for leakage fluxes).

Core laminations can be stacked or wound. Zero sequence fluxes will circulate individually in each core, and tank heating is not a problem. Under normal balanced operation, exciting currents in each phase are identical, except for their 120° shift in phase angle.

All of the other core configurations provide direct flux linkages between phases via the magnetic core. Simply stated, applying a voltage to any one phase will result in voltages being

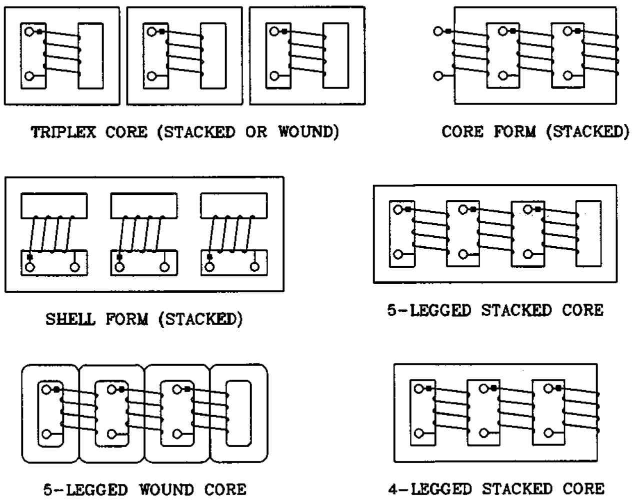

Fig. 3. Core configurations commonly used in three phase transformers. Only one set of windings is shown.

induced in the other phases [only in the adjacent phase(s) in the case of the five-legged wound core]. Further, the degree of saturation in each limb of the core affects the way flux flows divide. The apparent reluctance seen by each of the windings changes depending on the degree of saturation in each limb of the transformer core. Therefore, exciting currents vary from phase to phase, even under balanced operation. A brief discussion of each of these core types follows:

Core-form transformers require the least amount of core material to manufacture. Laminations are stacked. Their worst problem is that unbalanced operation results in zero sequence fluxes which cannot circulate in the core. These zero sequence fluxes are forced through the insulation surrounding the core and through the transformer tank. Tank steel is not laminated like the core is, so eddy currents can heat the tank and cause damage. Therefore, this type of core should only be used where load currents are balanced.

The shell-form core provides a magnetic path for zero sequence flux, and is much better-suited for unbalanced operation. Laminations are stacked. There is a large base of transformers with this type of core (about half of the installed three phase power transformers in the U.S.).

The four-legged core also provides a magnetic path for zero sequence flux. This type of core design is not very common. It is the only type of core whose outer phases do not exhibit like behavior.

The five-legged stacked core also provides a magnetic path for zero sequence flux, but has a more symmetric core. This type of core is often specified where a low-profile is desirable for shipping or for visual appearance in urban substations.

The five-legged wound core is made up of four concentrically-laminated cores. The unique feature of this core is that only adjacent phases are directly linked via a magnetic path. Assuming no flux leakage between cores, the two outer winding assemblies are not magnetically coupled. Tank heating is minimized, since there are zero sequence flux paths in the core. Because of its low cost, this type of transformer core is widely used in distribution systems.

The winding configuration used does not have any effect on the transformer core model. Delta, wye, or zig-zag winding connections are made outside of the model of the core equivalent. However, behavior of the transformer is strongly dependent on the winding configuration.

D. Ferroresonance in Three Phase Systems

Ferroresonance in three phase systems can involve large power transformers, distribution transformers, or instrument transformers (VT's or CVT's). The general requirements for ferroresonance are an applied (or induced) source voltage, a saturable magnetizing inductance of a transformer, a capacitance, and little damping. The capacitance can be in the form of capacitance of underground cables or long transmission lines, capacitor banks, coupling capacitances between double circuit lines or in a temporarily-ungrounded system, and voltage grading capacitors in HV circuit breakers. Other possibilities are generator surge capacitors and SVC's in long transmission lines. Due to the multitude of transformer winding and core configurations, system connections, various sources of capacitance, and the nonlinearities involved, the scenarios under which ferroresonance can occur are seemingly endless [5].

System events that may initiate ferroresonance include single phase switching or fusing, or loss of system grounding. The ferroresonant circuit in all cases is an applied (or induced) voltage connected to a capacitance in series with a transformer's magnetizing reactance.

Fig. 4 gives three examples of ferroresonance occurring in a network where single phase switching is used. A wye-connected capacitance is paralleled with an unloaded wye-connected transformer. The capacitance could be a capacitor bank or the shunt capacitance of the lines or cables connecting the transformer to the source. Each phase of the transformer is represented by jX_m, since ferroresonance involves only the magnetizing reactance.

If one or two poles of the switch are open and if either the capacitor bank or the transformer have grounded neutrals, then a series path through capacitance(s) and magnetizing reactance(s) exists and ferroresonance is possible. If both neutrals are grounded or both are ungrounded, then no series path exists and there is no clear possibility of ferroresonance. In all of these cases, the voltage source is the applied system voltage. Ferroresonance is possible for any of the core configurations of Fig. 3 (even for triplexed or a bank of single phase transformers).

Depending on the type of transformer core, ferroresonance may be possible even when there is no obvious series path from the applied voltage through a capacitance and a magnetizing reactance. This is possible with three phase core types which provide direct magnetic coupling between phases, where voltages can be *induced* in the open phase(s) of the transformer. To illustrate, a grounded-wye to grounded-wye transformer typical of modem distribution systems is considered. A recent survey in the US showed that 79% of underground rural distribution systems use this configuration.

Ferroresonance problems in this type of installation are of special interest [23], [25], [40], [41]. A simplified schematic

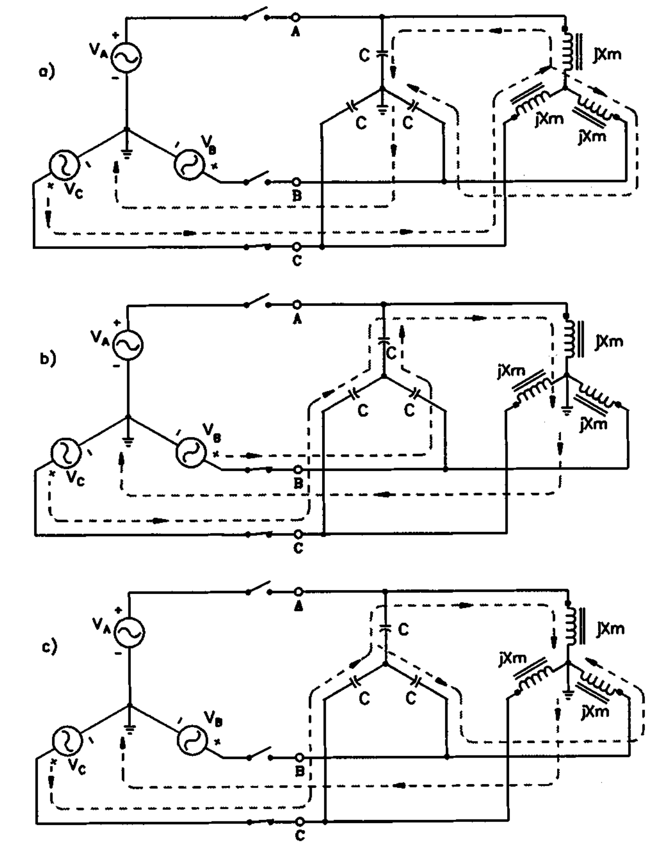

Fig. 4. Three examples of ferroresonance in three phase systems.

of such a system is shown in Fig. 5. The distribution line is represented by its RLC pi equivalent, with no interphase coupling. Three phase circuit breakers and gang-operated switches are used at the substation where distribution lines originate, but single phase switching and interrupting devices are used outside of the substation.

Either overhead lines or underground cables connect transformers to the system. Cables have a relatively large shunt capacitance compared to overhead lines, so this type of ferroresonance most often involves underground cables, but is also possible due solely to transformer winding capacitance.

Three phase or single phase transformers can appear at the end of a distribution line or at any point along the line. Three phase transformers may have any one of the several core types discussed in the previous section.

Whether ferroresonance occurs depends on the type of switching and interrupting devices, type of transformer, the load on the secondary of the transformer, and the length and type of distribution line. However, due to nonlinearities, increased capacitance does not necessarily mean an increased likelihood of ferroresonance. Operating guidelines based on linear extrapolations of capacitance may not be valid. Also, as mentioned previously, the smaller the load on the transformer's secondary, the less the system damping is and the more likely ferroresonance will be. Therefore, a highly capacitive line and little or no load on the transformer are prerequisites for ferroresonance. Binary loads (either full load or no load) such as irrigation, are essentially zero most of the time and cannot be relied upon to damp ferroresonance.

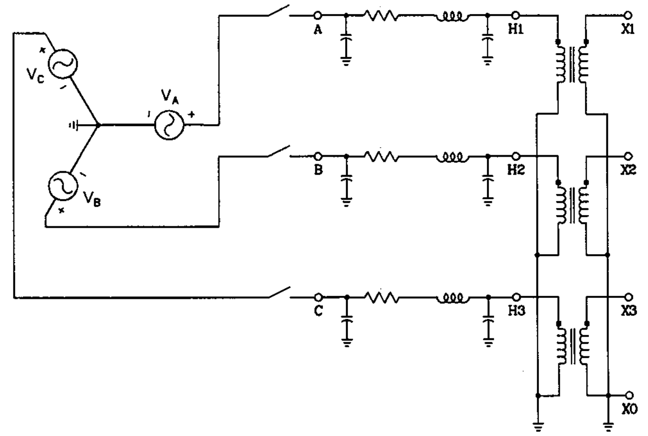

Fig. 5. Typical distribution system supplying a three phase load through a grounded-wye to grounded-wye transformer.

Ferroresonance is rarely seen provided all three source phases are energized, but may occur when one or two of the source phases are lost while the transformer is unloaded or lightly loaded. The loss of one or two phases can easily happen due to clearing of single phase fusing, operation of single phase reclosers or sectionalizers, or when energizing or deenergizing using single phase switching procedures.

If one of the three switches of Fig. 4 were open, only two phases of the transformer would be energized. If the transformer is of the triplex design or is a bank of single phase transformers, the open phase is simply deenergized and the energized phases draw normal exciting current. (Existence of capacitor banks or significant phase to phase capacitive coupling could still result in ferroresonance, but that possibility is not addressed here.)

However, if the transformer is of the three-, four-, or five-legged core type, a voltage is *induced* in the "open" phase. This induced voltage will "backfeed" the distribution line back to the open switch. If the shunt capacitance is significant, ferroresonance may occur. The ferroresonance that occurs involves the nonlinear magnetizing reactance of the transformer's open phase and the shunt capacitance of the distribution line and/or transformer winding capacitance. It has been shown that the ferroresonant circuit is a series combination of the shunt cable capacitance and the magnetizing inductance of one of the transformer's wound cores [23]. The equivalent circuit for this transformer is derived later in this paper.

An example of ferroresonant voltage and current waveforms occurring under this scenario is shown in Fig. 6. In this case, rated voltage was applied to $X2$ and $X3$, while $X1$ was unenergized and had 9 μF attached to simulate a length of underground distribution cable.

Whether in ferroresonance or not, this backfeed situation can be dangerous, as operating personnel may assume that the load side of the open switch is deenergized and safe to work on, when in fact a high voltage is present. Also, it can be seen that single phase loads connected along this backfed phase will continue to be supplied, although with dangerously high or low voltage levels and with poor power quality.

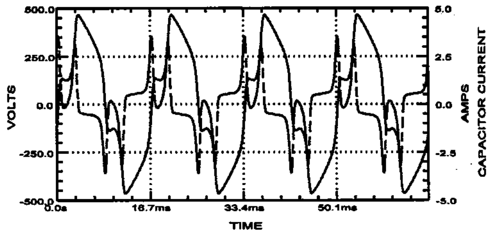

Fig. 6. Measurement of ferroresonance in a three phase grounded-wye to grounded-wye five-legged core transformer. Voltage waveform is solid; current waveform is dashed.

Therefore, use of single phase interruption and switching practices in systems containing the five-legged core transformers is the main operating tactic responsible for initiating ferroresonance. Replacement of all single phase switching and interrupting devices with three phase devices would eliminate this problem, although economics discourages such large scale upgrades. An alternate solution would be to replace all five-legged core transformers with single phase banks or triplex designs wherever there is a small load factor. System wide operation and design implications of this problem have been more fully addressed in prior work [25].

E. Nonlinear Dynamics and Chaos Applied to Ferroresonance

Ferroresonant circuits can be analyzed as damped nonlinear systems driven by sinusoidal forcing function(s) [27]. The nonlinear behavior of ferroresonance falls into two main categories. In the first, the response is a distorted periodic waveform, containing the fundamental and higher-order odd harmonics of the fundamental frequency. The second type is characterized by a nonperiodic, or chaotic, response. In both cases the response's power spectrum contains fundamental and odd harmonic frequency components. In the chaotic response, however, there are also distributed frequency harmonics and subharmonics. A good conceptual introduction to chaos and nonlinear dynamics is given by [16], and a good theoretical introduction can be found in [45].

At least two different periodic responses are possible for a single phase transformer [26], similar to that of Fig. 1. Ferroresonance in the above three phase five-legged core distribution transformer can be periodic or nonperiodic. "Lower energy modes" [1] (involving relatively low energy oscillations between the inductance and capacitance, similar to the waveforms shown in Fig. 5) produce periodic voltages on the secondary. Some of the periodic modes of ferroresonance may contain subharmonics, but still have strong power frequency components, but take longer than one fundamental cycle to repeat. This occurs more typically for very large values of C.

The "higher energy modes" [1] of ferroresonance involving relatively large capacitances and little damping can produce a nonperiodic voltage on the open phase(s). These voltage waveforms can be quite similar to those of Duffing's equation [45], which describes a nonlinear forced oscillator commonly used to illustrate the behaviors of a nonlinear dynamical system. Transitions between periodic and nonperiodic modes occur due to gradual changes in circuit parameters or to transients. And as with Duffing's equation, initial conditions determine the mode that operation stabilizes in after the transients die down.

The recognition that ferroresonance is a nonlinear and sometimes chaotic process opens up many possibilities. The newly-developed techniques for analysis of nonlinear dynamical systems and chaos are being evaluated for use with ferroresonance [21], [27]. Use of geometric graphical methods like phase plane projections and Poincaré sections can be applied to obtain a better understanding of ferroresonance.

II. Modeling and Analysis of Ferroresonance

A. Misinformation in Literature, Overview of Modeling

Ferroresonance has never been well-understood. Therefore, there is a great deal of misinformation on ferroresonance in the literature. A good example of this concerns the application of grounded-wye to grounded-wye five-legged core distribution transformers. As recently as 1989, specification of this type of transformer was recommended to eliminate or minimize the possibility of ferroresonance [14], [35]. This misinformation is gradually being corrected [25], [32], but engineers must be cautious and continue to update themselves.

Efforts in past years seem focused on refining equivalent circuit models for transformers and performing simulations using a transient circuit analysis program such as EMTP. Although these programs use fairly robust methods of numerical integration, such as the trapezoidal rule, results are only as good as the models used (and the initial conditions if the onset of ferroresonance is a concern). Simulation results have a great sensitivity to the model used and errors in nonlinear model parameters. Unfortunately, determining the model's nonlinear parameters is probably the biggest modeling difficulty. Three phase transformer modeling has not progressed as far as single phase modeling. A different model is required for each type of core, and a different means of determining the model parameters.

Ideally, use of a correct transformer model would allow an engineer to simulate situations where ferroresonance is likely. Simulation results could then be used to avoid this problem when designing a distribution system. Difficulties in determining an adequate model and in simulating every possible combination of initial condition and transient make prediction less than certain.

B. The Study Zone

Parts of the system that must be simulated are the source impedance, the transmission or distribution line(s), the transformer, and any capacitance not already included. Source representation is not generally critical. Unless the source contains nonlinearities, it is sufficient to use the steady-state thevenin impedance and open-circuit voltage. The distribution line or transmission line can be assumed to be an RLC coupled pi-equivalent, cascaded for longer lines. Shunt or series capacitors may be represented as a standard capacitance, paralleled with the appropriate dissipation resistance. Stray capacitance may also be incorporated either at the comers of an open-circuited delta transformer winding or midway along each winding. Other sources of capacitance are transformer bushings and interwinding capacitances, and possibly busbar capacitances.

One of the most critical parts of any ferroresonance study is the transformer model. The transformer contains the nonlinearities, and modeling results are most sensitive to correct representation of magnetic saturation and core loss. The rest of this discussion focuses mainly on how the transformer should be modeled. Many are dissatisfied with the transformer modeling capabilities in today's modeling packages. There has been much discussion recently as to what improvements can be made in modeling techniques [6], [13], [46].

C. More on Single Phase Transformer Models and Parameters

Single phase transformers are typically modeled as shown in Fig. 2. This model is topologically correct only for the case where the primary and secondary windings are not concentrically wound. L_{L2} is essentially zero for concentric coils. Errors in leakage representation are not significant, however, unless the core saturates. Obtaining the linear parameters for this 2-winding transformer may be difficult. Short circuit tests give total impedance $(R_1+R_2)+j(X_1+X_2)$. A judgment must be made as to how it is divided between the primary and secondary windings.

For transformers of three or more windings, a star-connected short-circuit equivalent may be obtained from binary short-circuit tests (shorting two windings at a time while leaving all others open). Although the terminal-to-terminal transfer impedances are always positive, one of the X's in this mathematical representation may be negative. This will not usually cause a problem in the time-domain transient simulation.

Such short-circuit models do not correctly account for mutual PA inductive coupling between all windings. To solve this problem, a coupled-L representation for the short-circuit inductances is recommended [11]. Parameters are obtained from binary short-circuit tests for all possible combinations of windings. A weakness of this short-circuit representation is that the core equivalent cannot be correctly incorporated (the only place it can be attached is to one of the external transformer terminals).

Model performance depends mainly on the representation of the nonlinear elements R_C and L_M. R_C has traditionally been modeled as a linear resistance. Such a core loss representation, if it represents the average losses at the level of excitation being simulated, may in fact yield reasonable results. Due to eddy current losses and hysteresis losses being nonlinear, calculation of a linear core loss resistance R_C gives different values for each level of excitation. Using the value of R_C closest to rated voltage may be a good enough estimate. Past research has shown low sensitivities to fairly large changes in R_C [29] for single phase transformers, but a high sensitivity for three-phase cores.

L_M is typically represented as a piecewise linear λ–i characteristic [22], or perhaps as a hysteric inductance [15], [20], [33]. The linear value of L_M (below the knee of the curve) does not much affect the simulation results [8], although great sensitivities are seen for the shape of the knee and the final slope in saturation.

Factory test data provided by the transformer manufacturer is often insufficient to obtain the core parameters. Open circuit tests should be made for 0.2 to 1.3 pu (or higher) instead of the typical 0.8 to 1.14 pu range. It is important that open circuit tests be performed for voltages as high as the conditions being simulated, or the final λ–i slope of L_M must be guessed. Some thought should be given to the requirements of test reports when specifying new transformers.

A method proposed by Dommel [11], [22] is often used to convert the RMS V–I open circuit characteristic to the λ–i characteristic of L_M. To successfully use this method, the first (lowest) level of excitation must result in sinusoidal current, or errors will result in the form of an S-shaped λ–i curve. Also, the V–I characteristic must extend as high as the highest voltage that will be encountered in the simulation. An extension on this method has been proposed to obtain a nonlinear v–i representation of R_C [31], but the resulting flux-linked versus I_{EX} loop does not seem to correctly represent the core losses.

Modem low-loss transformers have comparatively large interwinding capacitances which can affect the shape of the excitation curve [47]. This can cause significant errors when the above method is being used to obtain core parameters. In these cases, factory tests must be performed to get the λ–i curve before the coils are placed on the core. A means of removing the capacitive component of the exciting current has also been developed [29].

D. Three Phase Transformer Models and Model Parameters

For three phase transformers, it is possible to make a simplified model by connecting together three of the above single phase models. If this is done, a triplex core configuration is assumed (see Fig. 3). A delta-wye transformer of this type is shown in Fig. 7. It is postulated that zero sequence (homopolar) effects are included almost entirely by the leakage inductance of the delta windings [11], [22].

If the transformer does not have any delta windings, zero sequence effects may be included by adding a set of delta windings to the model whose total leakage impedance is equal to the transformer's zero sequence inductance. This may work for a three-legged core transformer that has an air path for zero sequence flux, but is highly questionable in the case of transformers having a saturable zero sequence flux path.

Factory three-phase excitation test reports will not provide the information needed to get the magnetizing inductances for this model. Note that standards require the exciting current to be stated as the "average" value of the RMS exciting currents of the three phases. Unless it is a triplexed core, this is meaningless, since the currents are not sinusoidal and they are not the same in every phase. Therefore, the waveforms of the applied voltage and exciting currents in all three phase should be given by the manufacturer for all levels of applied voltage.

The model might be improved by using a coupled inductance matrix to model the short circuit characteristics of three phase transformers. Binary short circuit tests involving all windings of all phases must be performed. Problems can arise for RMS short circuit data involving windings on different phases, since the current may be nonsinusoidal. A problem also exists with connecting the core equivalent. Three single phase core equivalents are often attached to the windings closest to the core, and may provide acceptable results in some cases, especially in

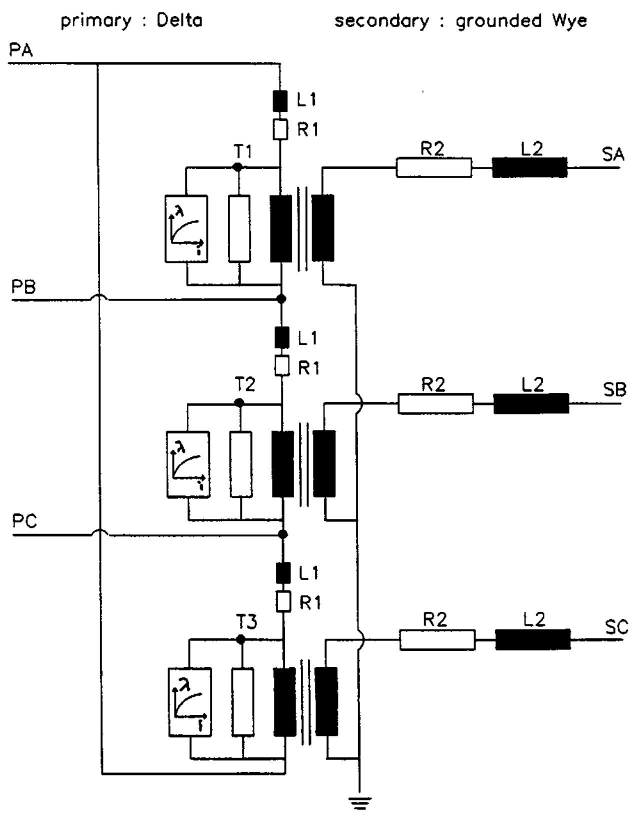

Fig. 7. Model of a delta to wye transformer bank made up of three single phase transformer models [22].

the case of the three-legged stacked core. Questions exist as to this method's validity, especially depending on the type of core being analyzed. The most important question is, however, what is the topology of the core equivalent? A method of obtaining topologically correct models is presented in the next section.

E. Use of Duality Transformations to Obtain Equivalent Circuits

This method is based on the duality between magnetic and electrical circuits. It was originally developed by Cherry [10] in 1949 and Slemon [37] in 1953. Using duality transformations, equivalent circuit derivations reduce to exercises in topology.

These methods did not receive much attention at first, presumably since computers were not available. Researchers have recently begun to use duality to provide equivalent circuit models which are more topologically correct [2], [29], [30], [34], [39], [42], [44]. This approach results in models that include the effects of saturation in each individual leg of the core, interphase magnetic coupling, and leakage effects. Results are promising, and ongoing work seems most focused on developing and improving duality-based models.

To illustrate the method, a duality derivation used to obtain the model for the five-legged wound core transformer [28] is done here and a case study is presented later in this paper. A section view of this type of transformer is shown in Fig. 8. The magnetic flux paths and assumed leakage flux paths are labeled. In the equivalent magnetic circuit, windings appear as MMF sources, leakage paths appear as linear reluctances, and magnetic cores appear as saturable reluctances.

The next step is the duality transformation itself. Using the symbol $\Leftrightarrow$ to denote the transformation between electrical and magnetic circuit elements, $\mathrm{MMF} \Leftrightarrow I(\mathrm{MMF} = NI)$, $d\lambda/dt \Leftrightarrow V$, and $\Re \Leftrightarrow L(L = N^2/\Re)$. In terms of topology, meshes, and nodes in the magnetic circuit transform into nodes and meshes, respectively, in the electrical circuit. Fig. 9 gives the resulting equivalent circuit.

To make the model practically useful, each current source resulting from the transformation has been replaced with an ideal transformer to provide primary-to-secondary isolation and coupling to the core, while preserving the overall primary to secondary turns ratio. Turns ratios are chosen so that core parameters are referenced to the low voltage windings. The portion of the model inside the coupling transformers represents the core and leakages.

Winding resistance and interconnection of the windings appears external to the coupling transformers. The advantage to this is that the derived core equivalent can be used independently of winding configuration (delta, wye, zig-zag, etc.). Winding resistance, core losses, and capacitive coupling effects are not obtained directly, but can be added to this topologically-correct equivalent electrical circuit.

Tests have been developed to determine the parameters for this model [28].

III. CASE STUDIES

A. Case Study #1: VT Ferroresonance on Floating Systems

It is possible that parts of a power system can be operated for short times without system grounding. One common example is the no-load energization of the wye side of a wye to delta power transformer. The delta side will "float" with respect to earth, until some load or other source of grounding is connected. If there is a voltage transformer (VT) connected to the delta side of the power transformer, ferroresonance can occur. The capacitance in this case comes from whatever "stray" coupling capacitance exists between the delta windings and earth. Adding a resistive burden to the VT can eliminate the problem.

A recent problem occurring in a 50-kV network in the Hafslund area near Moss, Norway, serves as an excellent example [18]. The clearing of a short circuit removed the only remaining source of grounding on the system. After the fault was cleared, the only remaining zero sequence impedance was due to capacitive coupling to earth. After operating in this way for only 3 min, ferroresonance had destroyed 72 of the VT's used for measurement and protective relaying. All 72 of the damaged VT's were from the same manufacturer. The VT's of two other manufacturers that were also in service during this time were not damaged.

Fig. 10 shows the typical VT arrangement used in this system. The VT's have two low voltage windings. The secondary is used for measurement and protective relaying purposes. The burden on that winding has a very high impedance and its effects can be ignored when considering ferroresonance. It is the tertiary windings which are shown in Fig. 10. These windings are connected in open delta and loaded with a damping resistance R_O.

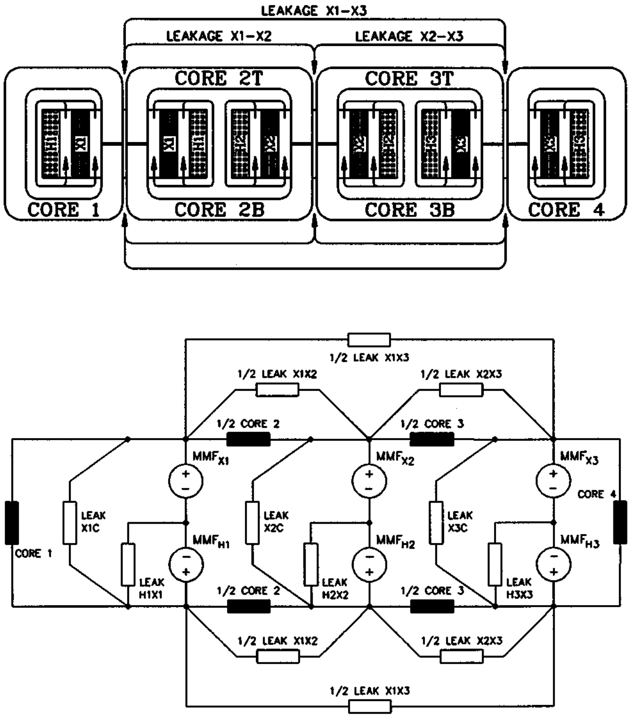

Fig. 8. Development of magnetic circuit for grounded-wye to grounded-wye five-legged wound core transformer. At top, transformer core sectional view used as a basis for duality derivation. Leakage flux paths are labeled. Bold dividing lines mark division in core reluctances. Equivalent magnetic circuit is shown at bottom.

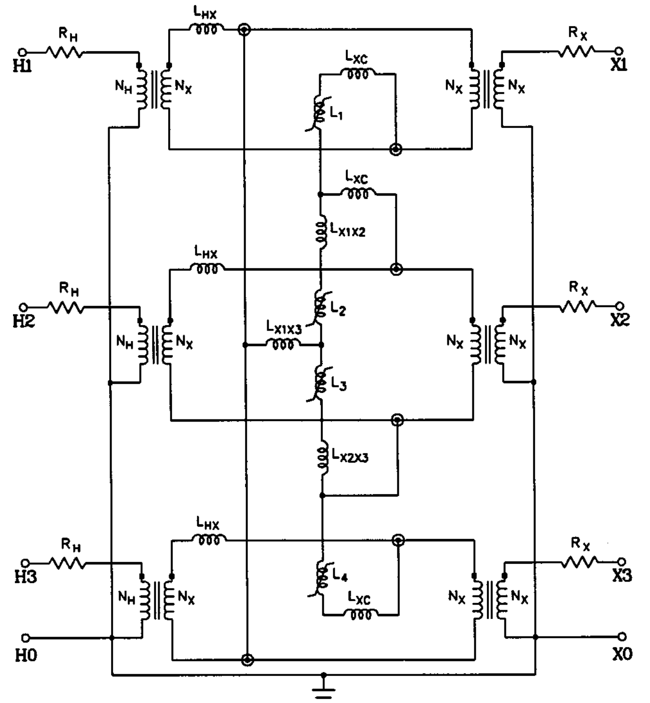

Fig. 9. Duality derived equivalent circuit with current sources replaced by ideal coupling transformers. Winding resistances have also been added.

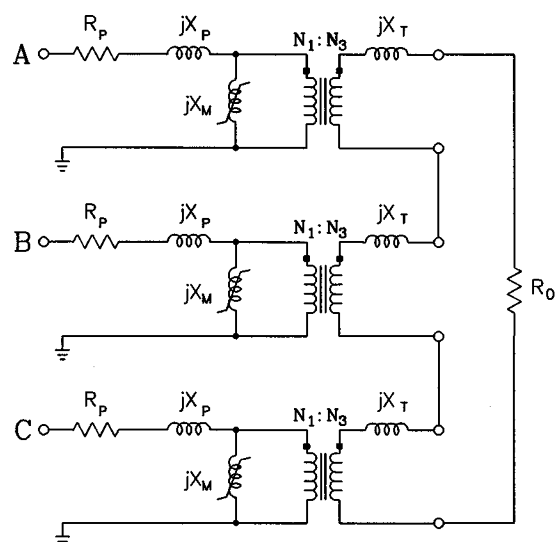

Fig. 10. Typical VT connection in 50-kV Norwegian subtransmission system.

The purpose of this damping resistance is to damp out ferroresonance, and this design has been commonly used for many years.

Since some of the VT's were damaged and the others weren't, the VT's of different manufacturers obviously must have different characteristics. The problem at Hafslund therefore forced a reevaluation of the specification and application of voltage transformers. EMTP was used to simulate the system conditions that caused the VT failures. VT model parameters were obtained from the manufacturers. Parameters are shown in Table I. Saturation characteristics were calculated based on core material B-H data, core dimensions, and number of primary turns. Data for the damaged VT's are listed as VT #1.

The designed flux densities B_M at rated voltage vary. As a more uniform basis of comparison, the flux densities were converted to flux-linked values (Fig. 11). Note that VT #1 will saturate out at lower levels than the other VT's, and one might guess this to be one of the reasons these failed and the others didn't. But this can only be confirmed from simulation results.

Fig. 12 shows the reduced equivalent used in the EMTP model. System positive and negative sequence impedances were found to be very small compared to the primary impedances of the VT's, and could be neglected. The zero sequence impedance Z_O consists almost entirely of the stray capacitance of the floating system, and is therefore very important. Values of Z_O varied from $0.6 - j219\ \Omega$ to $0.2 - j221\ \Omega$, depending where in the system. Z_O therefore becomes the only system impedance needed in the model, and the positive sequence voltage sources can be modeled as stiff sources. The core losses of the VT's were also neglected, their values being much higher than the damping resistance R_O.

Many simulations were run, with various combinations of VT's and values of R_O. It was found that ferroresonance occurred in most cases where R_O was set to the 60 Ω value typically used in system design. It was also seen that the high magnetizing currents drawn by VT #1 while in ferroresonance caused high I^2R losses in the windings, which thermally destroyed those VT's. If all of the VT's from manufacturer #1 were replaced with different VT's and if R_O was reduced to 10 Ω, ferroresonance would not occur. It was therefore recommended that the failed VT's be replaced with those of either VT

TABLE I
LINEAR PARAMETERS USED TO MODEL THE VT'S AT HAFSLUND

	R_P	X_P	X_T	$N_1{:}N_3$	B_{MAX}
VT #1	3250Ω	2500Ω	0.01Ω	20k:23	1.05T
VT #2	3218Ω	3094Ω	0.01Ω	~36k:42	0.77T
VT #3	7588Ω	4833Ω	0.01Ω	25k:29	0.83T

#2 or VT #3. A decrease in the value of R_O standardly being used was also recommended.

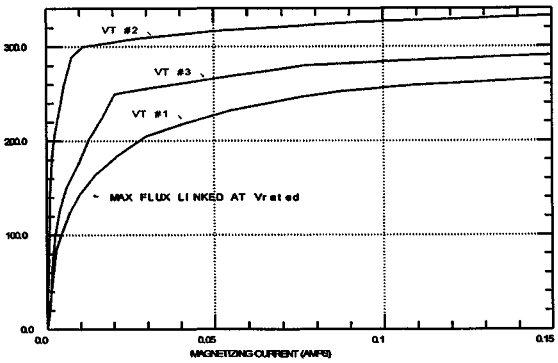

Fig. 11. Comparison of the saturation characteristics of the three VT's. Note the much lower saturation level of VT #1, the ones that were damaged.

B. Case Study #2: Ferroresonance in Distribution Systems

This case involves the verification of the 75-kVA five-legged wound-core distribution transformer model developed earlier. Ferroresonance was staged on the secondary windings in the laboratory. Balanced 3-phase voltage was applied to the secondary windings, and then one or two phases of the supply were removed and replaced with various values of shunt capacitance. Scenarios investigated were loss of one source phase to the center or an outer winding, and loss of two source phases to either the two outer windings or to the center winding and one outer winding [27].

Measured waveforms were then compared to EMTP simulations. The transformer equivalent circuit used was essentially that of Fig. 9. Details of model development, parameter values, and benchmarking are given in [28], [29].

Since many ferroresonant modes are possible, bifurcation simulations were first run. A bifurcation is essentially a jump from one mode of ferroresonance to another. A simulation technique was developed to very slowly ramp the capacitance [12], [28] and record jumps from one mode to another. Fig. 13 gives one bifurcation diagram for the case where a ramped capacitance is connected to unenergized winding $X1$ and rated positive sequence voltage is applied to $X2$ and $X3$. Due to nonlinearities, it is important to ramp the capacitance both upward and downward, to ensure that as many ferroresonant modes are discovered as possible.

Using the bifurcation diagram as a road map, ferroresonance for capacitances of 5, 10, 22.5, 14.6, and 18 μF was simulated. This corresponds to waveforms of periods 1, 2, 3, 5, and chaotic (nonperiodic). "Period 3" simply means that the waveform takes three periods of the forcing function to repeat—it contains 1/3 harmonics.

Fig. 14 shows the result of one of the EMTP simulations and compares it to the actual measurements. The model correctly predicts the existence of all modes of ferroresonance at the correct values of capacitance. The actual waveforms simulated are very close for the periods one, two, and three. Period five is generally correct, with slightly lower than actual peak amplitudes predicted. The chaotic response predicted is slightly higher than actual. The model used a simplistic linear resistance to represent the core losses of each core. The model's accuracy could be improved by implementing a more correct (complex) core loss representation.

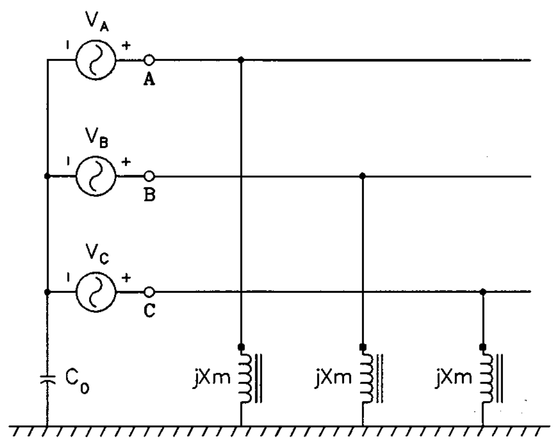

Fig. 12. Reduced system equivalent, neglecting line impedances and lumping all VT's in each phase into an aggregate jX_M.

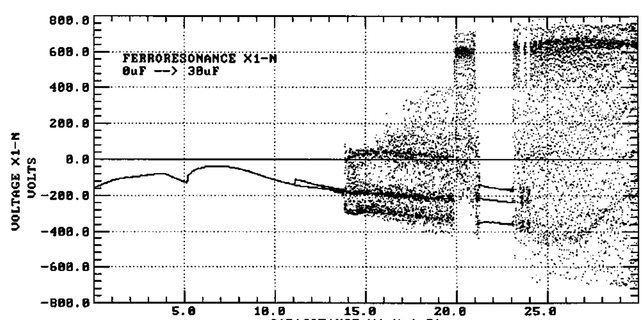

Fig. 13. Sample bifurcation diagram. Shunt capacitance on $X1$ is ramped from 0 to 30 μF. Blurred areas correspond to chaos.

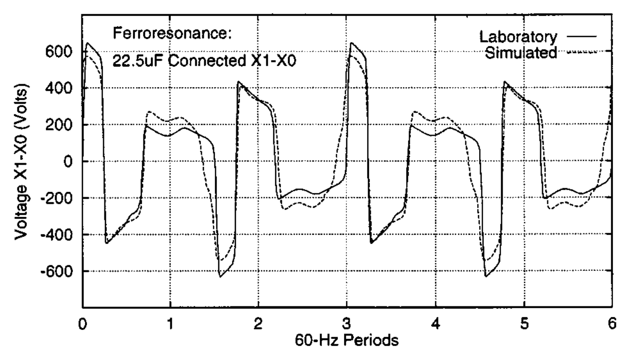

Fig. 14. Period 3 ferroresonance, 22.5 μF connected $X1$–$X0$.

C. Case Study #3: Ferroresonance of Autotransformer

This case is taken from the Ontario Hydro system where the Cataraqui 230/115-kV autotransformer $T2$, fed by line X3H, was experiencing ferroresonance upon deenergization of line X3H and the 115-kV bus (Fig. 16). The deenergizing circuit breaker was also experiencing a high recovery voltage. It was deduced that capacitive coupling between line X3H and the still-energized lines X4H and X522A was driving the autotransformer into ferroresonance. Damping resistors were added to the tertiary of $T2$, but it was not certain whether the resulting damping was sufficient to limit the duration of ferroresonance and the related recovery voltage.

Several EMTP simulations were run, with Y-connected resistive loads of zero, 133 kW/phase, and 266 kW/phase attached to the Δ tertiary of $T2$. In each case, the 115-kV breaker of $T2$ was assumed to open last. Two double-circuit 230-kV lines, an existing 500-kV line, and a future 500-kV line were included in the corridor, resulting in an 18-phase coupled-circuit transmission equivalent (Fig. 17).

Fig. 18 shows the circuit breaker recovery voltage for one of the cases.

It is interesting to note that a 133-kW/phase load did an effective job of damping ferroresonance in $T2$, but resulted in a higher recovery voltage than no damping at all. The circuit breaker was marginally able to handle the recovery voltage when the load was doubled to 267 kW/phase. Simulations were also performed for deenergization of $T1$, with similar but less severe behaviors noted. Recommendations were made to add 267 kW/phase loads to both transformers, and add surge arresters to the high and low voltage terminals of both transformers.

IV. Recommendations

Many different types of ferroresonance can and do occur. Because of the nonlinear nature of ferroresonance, it is difficult to predict if and where it might next occur. The power system engineer should be aware that lightly-loaded transformers operating in the presence of source or shunt capacitance may experience ferroresonance. Capacitance exists in the form of cables, series or shunt capacitor banks, or stray capacitances in inadequately-grounded portions of the system.

Transient simulations are helpful in confirming or predicting the likelihood of ferroresonance, but only if a correct model is used. Per phase simulations of three phase systems will not give correct results, due to various possible transformer core configurations and winding connections. A complete three phase model must be used. Therefore, the key to transient modeling is use of the proper transformer model. Development and use of acceptable transformer models should be a priority task.

The development of improved topologically correct models is a significant advancement, but model performance still depends on improving the way in which the cores are represented. Transformer core configuration must be considered and saturation characteristics must be accurately known to operating levels well above rated voltage.

At this time, it is seen that modeling of ferroresonance is as much an art as a science. As such, it is important if possible to verify the results by checking the simulations against system measurements. It is highly recommended that anyone active in

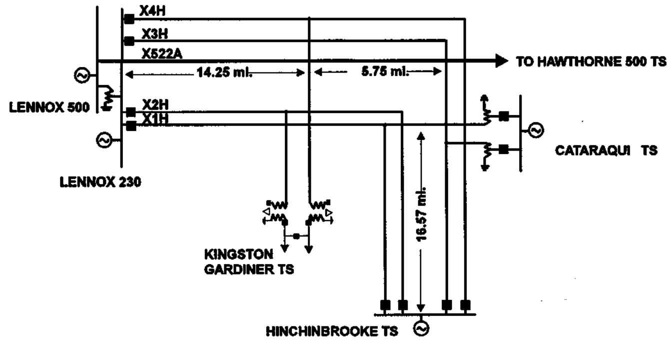

Fig. 16. Ontario Hydro 230-kV system. Ferroresonance involving line X3H and connected transformer at Cataraqui Transformer Station.

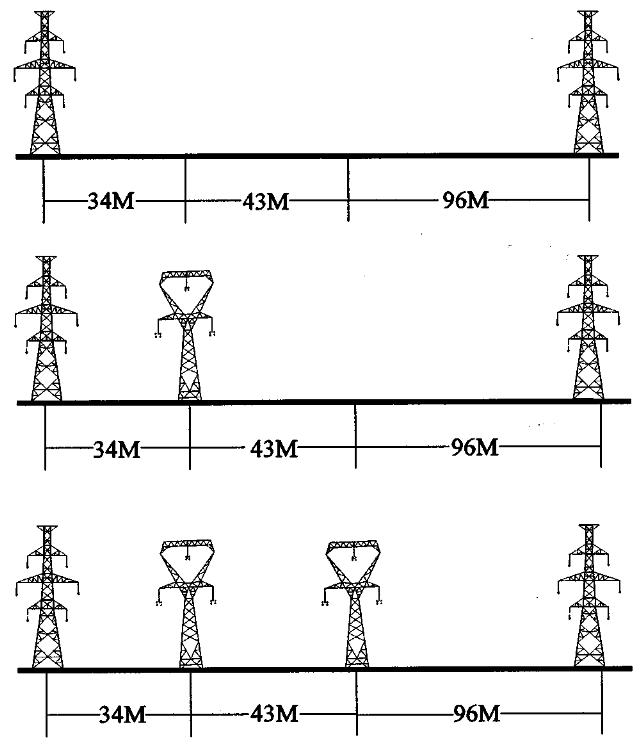

Fig. 17. Sequence of development of the transmission right-of-way.

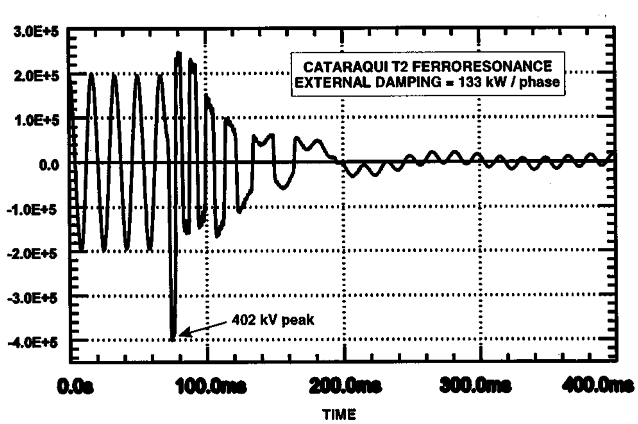

Fig. 18. Cataraqui ($T2$) autotransformer ferroresonance. HV terminal voltage on phase C is 2.0 per unit, with 133 kW/phase of damping.

this area must continually monitor the literature for improvements in modeling techniques.

ACKNOWLEDGMENT

This guideline summarizes many years of research experience. The lead author and contributor is B. Mork of Michigan Technological University. A. Morched of Ontario Hydro contributed Case Study #3 and added many valuable comments and insights based on his years of experience. Useful editorial comments on transformer design and ferroresonant behaviors were made by R. Walling of General Electric.

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