Laser cooling and electromagnetic trapping of neutral atoms

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Atoms in a thermal beam can be cooled, decelerated, and stopped using the radiation pressure from a nearly resonant laser beam. Several groups have already used this laser-cooling process on an atomic sodium beam. The techniques and results of the various experimental groups are reviewed, and applications of laser-cooled atoms, in particular the possibility of confining them in electromagnetic traps, are discussed.

1. INTRODUCTION

Many atomic-physics experiments are limited by the random motion of the atoms. For example, ultrahigh-resolution spectroscopy is often limited by Doppler or transit time effects. Similarly, velocity and angular resolution of collision experiments are frequently limited by the spread in velocity of the collision partners. A reasonably dense atomic sample with well-defined velocity (or zero velocity) would be highly desirable for such experiments. In addition, recent interest in the possibility of electromagnetic traps for neutral atoms has created a need for atoms at nearly zero velocity. These kinds of atomic samples can now be produced using radiation pressure from laser beams.

The use of radiation pressure to affect the velocity of atoms has a long history: Frisch first reported the deflection of an atomic beam by resonant light in 1933.[1] The velocity change occurs when an atom absorbs a photon. Its momentum changes the atomic velocity by $\Delta v = h\nu/Mc$ (=3 cm/sec for the D_2 line of Na), where M is the atomic mass. The spontaneously radiated photon also changes the atomic velocity by Δv, but the direction of the radiation is symmetrically distributed so that there is no net contribution to the atomic velocity from spontaneous emission. The average momentum transfer, therefore, is in the direction of the incident light (with some spread from the statistical nature of the emission).

If the incident light is a traveling plane wave, absorption followed by stimulated emission has no effect on the atomic velocity. If the incident light is not a plane wave, the absorbed light and the stimulated emission need not be in the same direction, and a net momentum transfer can occur. The force implied by this momentum transfer is called the "dipole" or "gradient" force. It can be understood as resulting from the interaction between the induced, oscillating dipole moment of the atom and the oscillating electric field of the light wave. It occurs only when there is a gradient in the light intensity and is the basis of various proposals for laser atom traps.[2,3] It has also been used for focusing[4] and diffracting[5] an atomic beam. There are proposals to use the dipole force to cool or decelerate atoms,[6,7] but they are outside the scope of this review; we will consider this force only in connection with traps. The dipole force has been treated theoretically and in detail from a variety of viewpoints. References 7–11 represent a sampling of such treatments.

The use of radiation pressure to cool or to reduce the relative velocity of atoms was proposed in 1975[12,13] and realized for ions stored in an ion trap in 1978.[14,15] In this Doppler cooling technique, the incident radiation is from a laser tuned slightly below resonance with an atomic transition. Atoms moving toward the laser are preferentially excited since they are Doppler shifted into resonance, whereas atoms moving away are Doppler shifted further out of resonance. The velocity change from absorption is therefore more likely to slow the atom than to speed it up, and the atomic sample cools. The process is limited by heating from the random nature of the absorption and emission processes. This type of laser cooling is reviewed in Ref. 16; heating processes are treated more fully in Refs. 9–11 and 17.

Laser cooling of neutral atoms, as opposed to trapped ions, requires somewhat different techniques. A major difficulty arises from the fact that despite many proposals (see, for example, Refs. 2, 3, and 18–27), no electromagnetic traps for neutral atoms have yet been demonstrated. As a result, the available interaction time between the atoms and the light is limited to the transit time across an apparatus rather than the potentially much longer residence time in a trap. The relatively short interaction time available for cooling of neutral atoms demands that the transfer of momentum occur at a high rate. Several processes can impede efficient momentum transfer, and the various methods for overcoming these impediments distinguish the experimental techniques being used to cool neutral atoms.

Other techniques of laser-assisted cooling use radiative processes to decelerate or cool atoms but do not rely on momentum transfer between the laser photons and the atoms. Among such techniques are cooling by anti-Stokes Raman scattering, first proposed by Kastler in 1950[28] and recently demonstrated in CO_2 (Ref. 29), laser excitation of a quasi-molecule in a binary atomic collision and subsequent energy loss as the atoms separate[30,31]; laser dissociation of a moving molecule against the direction of motion[32]; repeated Stark deceleration of repeatedly excited atoms[33]; and laser heating causing evaporation of atoms from a rapidly moving droplet or aggregate.[34] Detailed discussion of such techniques is outside the scope of this paper.

So far, all experiments on laser cooling of neutral atoms have used a sodium beam as the atomic sample. [*Note added in proof:* A group at the University of Colorado has

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recently decelerated and stopped a cesium beam by using a frequency chirp technique with diode lasers. See R. N. Watts and C. E. Wieman, in *Laser Spectroscopy VII*, T. Hänsch and Y. R. Shen, eds. (Springer-Verlag, Berlin, 1985).] Sodium is chosen because of the availability of powerful, tunable, cw lasers to excite the resonance lines, a short radiative lifetime ($\tau = 1/\gamma = 16$ nsec), allowing rapid repeated excitation, and ease of handling. An atomic beam allows a long interaction time when the laser beam is collinear with and counterpropagating to the atomic beam, and it ensures that two of the three velocity components are initially well defined.

A typical thermal ($T \cong 1000$ K) sodium beam has a velocity distribution that peaks near $v_0 = 1000$ m/sec. Since $\Delta v \cong 3$ cm/sec, on the order of $N = v_0/\Delta v \cong 3 \times 10^4$ photons must be scattered if the atoms are to be brought to rest. If the laser saturates the transition so that the atoms spends half of its time in the excited state, the scattering rate saturates at $1/2\tau$ and the maximum acceleration is

$$a_{\max} = \Delta v/2\tau = h\nu/2Mc\tau \cong 10^6 \text{m/sec}^2. \quad (1)$$

With this acceleration Na atoms with initial velocity $v_0 = 1000$ m/sec can be brought to rest in ~1 msec over a distance of ~0.5 m. During the deceleration, the transverse velocity of the atoms will execute a random walk of N steps, each of size Δv. Thus the final spread of transverse velocity is approximately $\delta v \cong \sqrt{N} \times \Delta v \cong 5$ m/sec.

The experimental problem, then, centers on making the Na atoms scatter a large number of photons in a short time. Two phenomena can interrupt this scattering process: changing Doppler shift and optical pumping.

After an atom has absorbed only 100 photons, its velocity changes by 3 m/sec, corresponding to a Doppler shift change of 5 MHz. Since the natural linewidth for absorption is 10 MHz (FWHM), the absorption rate will be reduced by a factor of 2 if the laser was initially resonant. Further absorption will Doppler shift the atom further out of resonance, so that for a laser power that does not significantly power broaden the transition, only a few hundred photons can be scattered before the changing Doppler shift puts the atom so far out of resonance that the absorption rate is small. This is only a small fraction of N, the number of scattering events required to bring the atoms to rest.

The optical pumping problem can be understood by referring to Fig. 1, which shows the relevant energy levels for Na.

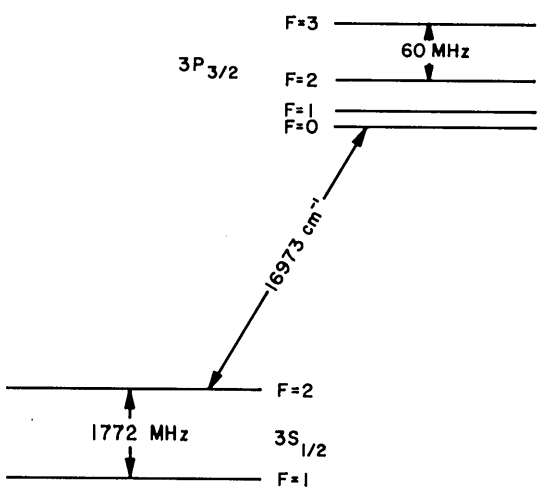

Fig. 1. Na energy-level diagram (not to scale).

Consider that a laser is tuned to be resonant with the $3S_{1/2}$ ($F = 2$) → $3P_{3/2}$ ($F = 2$) transition. Between excitations, the atoms decay with equal probability to the $3S_{1/2}$, $F = 1$ and $F = 2$ levels. If the decay is to the $F = 1$ level, the laser is out of resonance for further excitation. This optical pumping makes it impossible for the atoms to scatter a sufficient number of photons to be significantly decelerated before they traverse an apparatus of reasonable dimensions.

If the initial excitation is $3S_{1/2}$ ($F = 2$) → $3P_{3/2}$ ($F = 3$) the selection rule for the change in F ($\Delta F = \pm 1, 0$) requires that the decay be back to $F = 2$, and the atom is easily re-excited. Unfortunately, the $3P_{3/2}$, $F = 2$ state is only 60 MHz or six linewidths from the $F = 3$ state, so off-resonant excitation can occur, albeit at a reduced rate. For every few hundred excitations to the $F = 3$ level, one excitation to the $F = 2$ level is expected, and the atom may then decay to $3S_{1/2}$ ($F = 1$), effectively ending the absorption process. Thus, in the absence of some corrective action, atoms are pumped into $F = 1$ after only a few hundred absorptions. Therefore optical pumping limits deceleration to about the same small fraction of thermal velocity as does the changing Doppler shift.

2. TECHNIQUES

We now consider various methods for avoiding the limitations on cooling imposed by changing Doppler shift and optical pumping. One of the earliest suggestions for dealing with the Doppler shift problem was proposed by Letokhov *et al.*[35] By sweeping (or chirping) the laser frequency from ν to $\nu + \Delta\nu$ to keep it resonant with the Doppler-shifted decelerating atoms, all atoms with velocity slower than the initially resonant velocity v_i are swept into a narrow velocity group around v_f. Here, $v_i - v_f = \lambda\Delta\nu$. Since there is a maximum acceleration $a_{\max}$ (see above), there is a maximum frequency scan rate $\dot{\nu}_{\max} = a_{\max}/\lambda$ (=1.55 GHz/msec for Na using D_2 light). If $\dot{\nu}$ is larger than this, the atom's Doppler shift cannot change rapidly enough and the atom will go out of resonance. If the scan rate is slower than $\dot{\nu}_{\max}$, and there is sufficient laser power, the atomic velocity will adjust itself to be just far enough out of resonance that the scattering rate produces the required rate of change of Doppler shift.

A closely related approach to the Doppler-shift problem is to change the frequency of the atoms rather than that of the laser so that there is also constant deceleration. This can be done with the Zeeman frequency shift produced by an inhomogeneous magnetic field. For atoms having initial velocity v_0 and decelerating at a rate a, the atomic velocity as a function of distance z is

$$v(z) = (v_0^2 - 2az)^{1/2}. \quad (2)$$

For a linear Zeeman shift to compensate the changing Doppler shift, the magnetic field must vary as

$$B(z) = B_0(1 - x)^{1/2}, \qquad x = 2az/v_0^2, \quad (3)$$

where B_0 is the field producing a Zeeman shift equal to the Doppler shift for atoms with velocity v_0.

Other field profiles are allowed, but because of the existence of a maximum possible acceleration there is also an upper limit on the field gradient given by

$$\frac{d\nu}{dB}\frac{dB}{dz} \cdot v \leqq a_{\max}/\lambda, \quad (4)$$

where $d\nu/dB$ depends on the Zeeman effect. This restric-

tion is equivalent to the restriction on the scan rate of the laser in chirped cooling.

For both chirping and Zeeman tuning, the laser is kept nearly resonant for all atoms involved in the cooling process. This makes efficient use of the light. For very high laser intensity ($I/I_{sat} = S \gg 1$, where $I_{sat} = 6$ mW/cm^2 for the strongest transition), Na atoms with $v_0 = 1000$ m/sec can be brought to rest in about 1 msec over a distance of 50 cm with either method, and smaller velocities are also decelerated to $v = 0$ in correspondingly shorter times and smaller distances. Each of these methods compresses the velocity distribution. For chirping the compression takes place in time—all affected atoms being instantaneously in resonance with the changing laser frequency; for Zeeman tuning the compression takes place in space—all affected atoms at a point in space are resonant and therefore at the same velocity.

For both methods, another effect contributes to the velocity compression: when the laser power is high enough that the on-resonance deceleration is greater than required by the magnetic-field gradient or the chirp rate, the atomic velocity will adjust itself to be slower than the resonant velocity. Under these conditions, atoms faster than the average velocity, being closer to resonance, decelerate more, while slower atoms decelerate less. This further compresses the velocity distribution.

Deceleration and cooling need not be accomplished with a resonant compensation of the Doppler shift. Indeed, ions in traps are routinely cooled from room temperature or above to subkelvin temperatures with a fixed frequency laser. (This is possible without correction for the changing Doppler shift because of the much longer interaction time.) Minogin and others have given the problem of cooling an atomic beam in this way extensive theoretical treatment.[36–38] We shall refer to such cooling, which does not use any compensation of the changing Doppler shift, as nonresonant.

In this case one tunes the counterpropagating laser to be resonant with a particular velocity component of the atomic beam. Atoms that are initially resonant with the laser scatter photons at a high rate and are slowed down and pushed out of resonance. Once out of resonance they scatter photons at a lower rate, continuing to decelerate, but more slowly, as do the initially slower atoms. Atoms initially too fast to be in resonance are gradually decelerated into resonance, rapidly decelerated through resonance, and continue to be decelerated gradually on the slow side of resonance. Thus atoms in the vicinity of resonance with the laser are decelerated and compressed into a narrow velocity group on the slow side of resonance.

The range of velocities affected, the total velocity change, and the width of the final velocity distribution all depend on the interaction time and the laser intensity. Larger velocity changes require larger intensity or longer interaction times. Particular values of intensity and interaction time produce much smaller velocity changes than for chirped or Zeeman-tuned cooling. For example, Minogin[36] has calculated that for a regime where the velocity change is a significant fraction of the thermal velocity, increasing the laser intensity from a saturation parameter $S = 10$ to $S = 100$, or increasing the interaction time by a factor of 10, only doubles the velocity change. By contrast, $S = 10$ in chirped cooling is more than sufficient for nearly maximum deceleration, while increasing the interaction time produces a proportional increase in the velocity change. Still, quite narrow velocity distributions can be produced with modest powers and interaction times by using the nonresonant method.

Chirping, Zeeman tuning, and nonresonant deceleration have all been used successfully to decelerate and cool atomic beams (see below). Other methods are also possible: One might use a white-light source rather than a single-frequency laser. That is, the optical spectrum should cover the entire Doppler width of the atomic beam. In this way, regardless of the atom's velocity there would always be some light in resonance with it to cause deceleration. An appropriate white-light source might be a multimode laser (Moi has demonstrated such a laser and has considered its use for laser cooling[39]), a mode-locked laser, or even an atomic-resonance lamp. The total power required would be higher than for chirped or Zeeman-tuned cooling since a laser mode with saturation intensity would need to be supplied for each of a large number of frequencies separated by about a natural linewidth and extending over the Doppler width. For Na this is about 200 modes. With white-light deceleration there is no compression of the velocity distribution, in contrast to the case of chirped or Zeeman-tuned cooling. All velocity groups decelerate at the same rate at all points in space. For a fixed interaction distance, in fact, the velocity distribution will spread out since the slow atoms, interacting for the longest time, will decelerate the most. The slowest atoms will be stopped and accelerated backward.

Velocity compression can be achieved in white-light cooling by arranging for the spectrum of the light source to cut off sharply on the high-frequency side. Deceleration below the velocity that is resonant with the cutoff frequency will be nonresonant and relatively ineffective, so there will be a pileup of atoms somewhat below the lowest resonant velocity. The width of the velocity distribution will depend on the sharpness of the cutoff as well as on details such as laser power and interaction time. Zueva and Minogin[40] have done an explicit calculation for the case of five equally spaced laser frequencies and find a compression of the velocity distribution as well as greater deceleration than with a single laser frequency of the same total power. Another approach to achieving a narrow velocity distribution would be to apply a proposal by Migdall[41] for extracting atoms from the cooling process. In this scheme atoms of a given velocity are selectively excited or optically pumped into an energy level that is far out of resonance with the cooling laser.

White-light deceleration should require somewhat less power than nonresonant cooling since the velocity range that can be covered depends linearly on intensity as opposed to the square-root dependence for the fixed-frequency method. This discussion of relative intensity requirements for various deceleration methods assumes that the atomic beam is optically thin (i.e., no significant fraction of the laser beam is absorbed). When this is not the case the power requirement is determined more by the number of atoms to be decelerated than by the need to saturate the transition, and questions about which method requires more or less optical power become less distinct.

A group at the University of Aarhus[42] has proposed a Doppler-shift compensation scheme for laser cooling a fast ion beam. The idea is to compensate the laser deceleration with electrostatic acceleration of the ions. The average velocity of the beam is unchanged, but if the laser is tuned red of resonance, the velocity spread will be reduced.

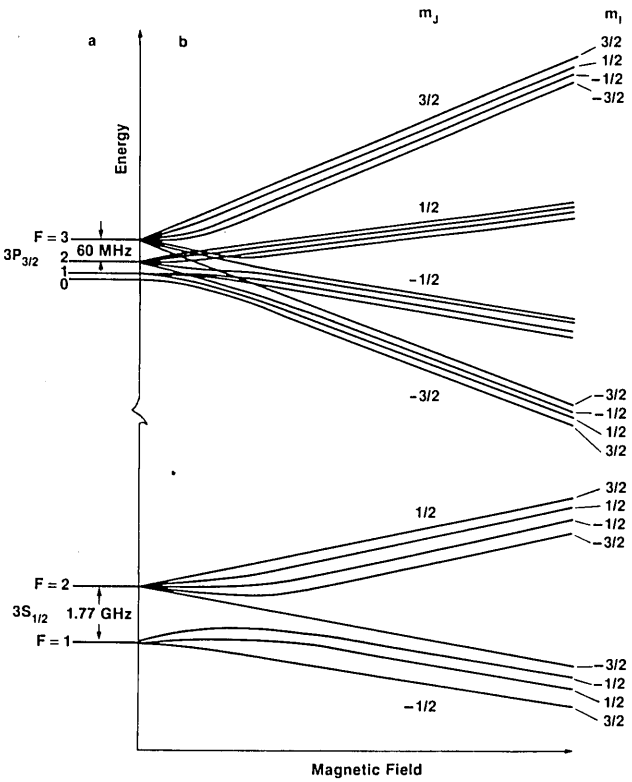

Fig. 2. Na energy levels in a magnetic field (not to scale). The nuclear quantum number $m_I = m_F - m_J$.

Another approach to compensating the changing Doppler shift is to vary the angle of the laser beam with respect to the atomic beam.[43] In this scheme a fixed-frequency laser beam would be focused onto an atomic beam symmetrically from two or more directions in such a way that the laser is more nearly counterpropagating as one moves away from the source. This is equivalent to increasing the laser frequency as the atoms decelerate. This method is less efficient than Zeeman tuning or chirping since even when the atomic transition is saturated, maximum deceleration is not achieved because the photon momentum is not opposite to the atomic momentum. This is a particular problem closest to the atomic source, where the atoms are fastest and the angle of the laser must be most nearly orthogonal to the atomic beam but the need for efficient deceleration is greatest. Also, since a greater number of photons must be scattered to accomplish the same deceleration, there is more transverse momentum spread with this method.

Another possibility for overcoming the Doppler-shift problem is a variation on Zeeman tuning: the magnetic field could be uniform in space but modulated in time. This would produce results similar to chirped cooling without scanning the laser but requires a rapid change in magnetic field.

As with changing Doppler shift, optical pumping can be dealt with in a variety of ways. Perhaps the most obvious way of avoiding optical pumping is to use two laser frequencies. (This technique is also used in ion cooling.[15]) If one frequency is resonant with the $3S_{1/2}$ $(F = 2) \rightarrow 3P_{3/2}$ $(F = 3)$ transition, and another with the $3S_{1/2}$ $(F = 1) \rightarrow 3P_{3/2}$ $(F = 2)$ transition, then any atoms pumped into the $F = 1$ state can still be excited and even pumped back into the $F = 2$ by the second laser.

Another method for avoiding optical pumping involves making use of the optical selection rules on the magnetic quantum numbers m (see Fig. 2). If we choose the axis for quantization of the projection of total angular momentum, m_F, along the incident laser beam, then a σ^+ circularly polarized laser beam will excite only $\Delta m_F = +1$ transitions. Atoms originally in $3S_{1/2}$ $(F = 2, m_F = 2)$ will be excited only to $3P_{3/2}$ $(F = 3, m_F = 3)$. The radiative decay can only go back to the original state (because of the selection rule $\Delta m_F = \pm 1, 0$), so optical pumping to $F = 1$ does not occur. Any magnetic field must be aligned with the laser beam so that the eigenstates of energy correspond to eigenstates of m_F (F, of course, will no longer be a good quantum number). This ensures that the $m_F = 2$ ground state and $m_F = 3$ excited state will not evolve into other states during the deceleration process.

Imperfect polarization or imperfect alignment of the magnetic field with the laser beam will lead to failure of the simple two-level scheme described above. Fortunately, in fields on the order of 0.05 T or more, this is not a significant problem. Laser-induced $\Delta m_F = 0$ or $\Delta m_F = -1$ transitions are suppressed for one of two reasons: Either the unwanted transitions are Zeeman shifted far out of resonance or they involve a change in the nuclear magnetic quantum number m_I, which is forbidden in a field high enough to decouple nuclear and electronic spins.

Both two-frequency excitation and circular polarization have been used successfully in laser cooling experiments (see below) to avoid optical pumping. There are other possible approaches: the ground hyperfine states could be mixed using direct rf excitation of the hyperfine transition. This is conceptually similar to the two-frequency laser excitation. Still another way is to use a very-high-power cooling laser so that the power broadening would be sufficient for significant off-resonant excitation from the $F = 1$ level. Unfortunately, if the laser is tuned to the $3S_{1/2}$ $(F = 2) \rightarrow 3P_{3/2}$ $(F = 3)$ transition, increased power broadening will increase the rate of $3S_{1/2}$ $(F = 2) \rightarrow 3P_{3/2}$ $(F = 2)$ transitions without increasing the $F = 2 \rightarrow F = 3$ rate. This will result in an increased pumping to the $3S_{1/2}$, $F = 1$ level. It would probably be necessary to have so much power broadening that both the $3S_{1/2}$ $(F = 2) \rightarrow 3P_{3/2}$ and $3S_{1/2}$ $(F = 1) \rightarrow 3P_{3/2}$ transitions were saturated in order to make power broadening an effective way of avoiding optical pumping.

Successful deceleration requires compatible methods for dealing with optical pumping and Doppler shift. For example, if Zeeman tuning is used for Doppler compensation, neither two-laser frequency nor rf hyperfine mixing is appropriate for optical pumping avoidance since the magnetic field splits and shifts the hyperfine levels. If chirping is used with two laser frequencies, both must be chirped together, while with rf mixing no chirp of the rf is needed since the Doppler shift of the hyperfine frequency is negligible.

3. EXPERIMENTS

A. Institute of Spectroscopy, Moscow

The earliest attempt at laser cooling an atomic beam was reported by Balykin *et al.* at the Institute of Spectroscopy in Moscow[44,45] in 1979. A schematic diagram of their appara-

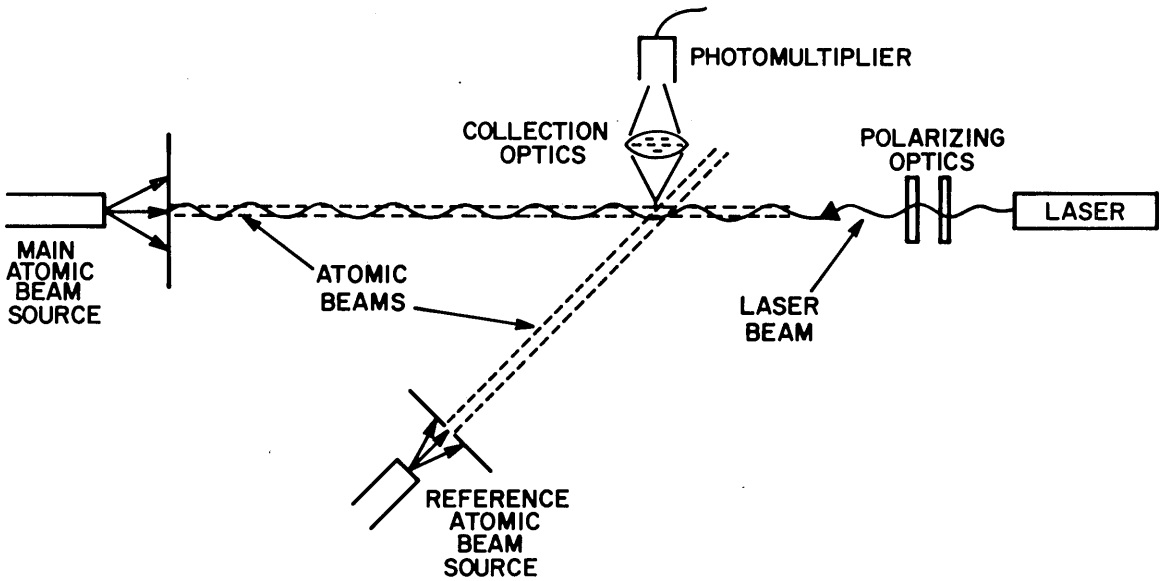

Fig. 3. Simplified schematic of Balykin *et al.* apparatus.[36] The reference beam crosses the laser at right angles and provides a calibration marker for the laser scan.

tus is shown in Fig. 3. The idea of the experiment was to combine chirping to compensate the Doppler shift with circular polarization to avoid optical pumping. As the frequency of the cooling laser was scanned, the fluorescence that it induced was observed by the photomultiplier tube and recorded. The fluorescence signal as a function of laser frequency indicated the atomic density as a function of velocity. The velocity distribution obtained for small laser scan rate and intensity was compared with that for large scan rate and intensity. Some differences suggestive of deceleration were observed. However, an analysis of the experiment by Balykin[46] showed that with the impurity of circular polarization existing in the experiment there would be so much optical pumping that little deceleration could be expected. The experiment also suffered from using the same laser both to produce cooling and to do the velocity analysis.

Accordingly, the Moscow group modified its experimental approach, using two cooling laser frequencies (from a double-mode laser) to avoid optical pumping, a separate probe laser to analyze the velocity distribution, and fixed-frequency, nonresonant deceleration. This resulted in clear deceleration and dramatic cooling of the velocity distribution, reported by Andreev *et al.* in 1981.[47] The new apparatus is shown schematically in Fig. 4. The mechanical chopper shuts off the cooling laser light, allowing the observation of fluorescence induced by the probe without interference from the fluorescence induced by the cooling laser or from effects caused by joint action of the probe and cooling lasers. The modified velocity distribution was determined by rapidly scanning the probe frequency (after the cooling laser was off) and recording the induced fluorescence oscillographically.

Figure 5 shows a typical result obtained with the nonresonant cooling method. [The data are not actually from the Moscow group but were taken in our laboratory at the National Bureau of Standards (NBS) in 1983 by the nonresonant method. As described below, the method that we used for avoiding optical pumping is not the two-frequency technique used in Moscow, but the results are quite similar.] The arrow indicates the velocity group resonant with the fixed-frequency laser. Just as predicted, the laser removes atoms from the velocity groups near resonance, depositing them in a narrow peak on the slow side of resonance. Andreev *et al.* reported the narrow peak to be shifted 15% in velocity (90 m/sec) from the resonant velocity, while the width was 41 m/sec, characteristic of a 1.5-K temperature, compared with the 573-K temperature of their Na source.

In order to increase the velocity shift and obtain slow

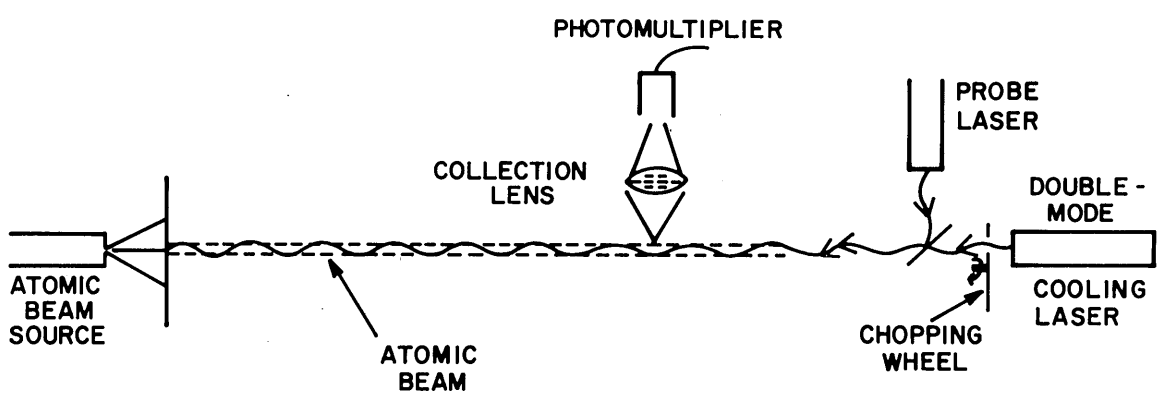

Fig. 4. Apparatus of Andreev *et al.*[38]

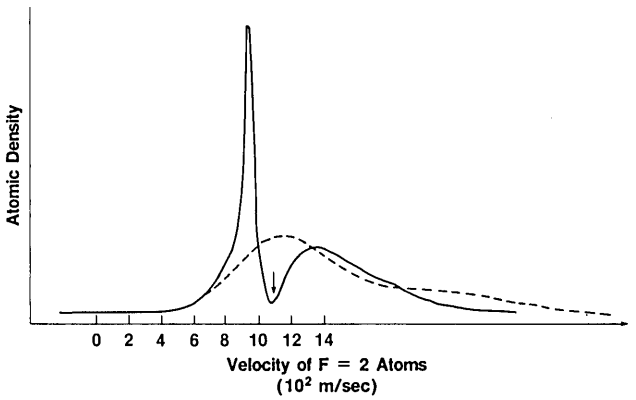

Fig. 5. Deceleration and cooling similar to that observed by Andreev *et al.* using the nonresonant technique. Dashed curve is the atomic velocity distribution before cooling; solid curve is after cooling.

atoms, the Moscow group has extended their nonresonant technique by increasing the laser power and interaction time. In addition, the laser beam focusing is arranged so that the intensity increases as the atoms propagate toward it, giving increased power broadening as the atomic velocity shifts farther from resonance. Results reported by Balykin *et al.*[48,49] in 1984 show velocity shifts of over 600 m/sec, about seven times greater than in their 1981 experiments. (See Fig. 2 of Ref. 49.) Atoms with final velocities as small as 25 m/sec were observed. As expected from theoretical calculations, the final velocity distributions are wider when higher laser intensity is used. The lowest velocities are observed in the tail of a distribution centered at several hundred meters per second. (See Fig. 5a of Ref. 49.) The distribution presumably includes atoms with zero or negative velocities, although these were not directly observed.

B. National Bureau of Standards, Gaithersburg

Meanwhile, our group at NBS had been pursuing a different approach to beam cooling, using Zeeman tuning to compensate the Doppler shift and circular polarization to avoid optical pumping. The early work of the Moscow group was limited by optical pumping problems even with circular polarization.[46] By contrast, our cooling was done in a magnetic field high enough both to decouple the electronic and nuclear angular momenta in the $3P$ level and to provide large enough Zeeman shifts that the imperfection of laser alignment and circular polarization did not disturb the process of repetitive excitation. (See above and Ref. 50.) The magnetic field use by Balykin *et al.*[45] was about 10^{-4} T, whereas our field was always several tens of millitesla in the cooling region.

Our apparatus is shown in Fig. 6. The Zeeman tuning that compensates the changing Doppler shift is provided by a solenoid wound with more turns near the entrance (close to the Na source) than near the exit. The current distribution was chosen to approximate the spatially varying field of Eq. (3), with the addition of a constant bias field to ensure that the field was always high enough to prevent optical pumping. The chopping wheel is used to turn the cooling laser light off during observation. The velocity distribution is measured using a probe laser that crosses the atomic beam at a slight angle (to avoid a long probe interaction time and the likelihood of optical pumping by the probe). Probe-laser intensity is always well below saturation. A boxcar integrator averages the probe-induced fluorescence over a short time just after the cooling laser shuts off [see Fig. 7(a)]. The probe frequency scans slowly compared to the averaging time so that it is nearly constant over many chopping cycles. This procedure provides good signal-to-noise ratio, allows us to scan the probe laser over a large frequency range, and allows us to choose the observation time independently.

In our experiments reported in 1982,[50] using a 60-cm-long solenoid, we first observed nonresonant cooling by using only the constant bias field and achieved a 15% velocity reduction as reported in Ref. 47. We then used the tapered or varying-field part of the solenoid and observed a velocity change of 40% of the initial velocity. Later[51] we used a 110-cm-long solenoid, made other improvements, such as increased laser power and focusing the laser onto the Na source, and observed velocity changes as large as about 800 m/sec (80% of the initial velocity).

A typical result of our Zeeman-tuned cooling is shown in Fig. 8. The dashed curve, taken with the cooling laser off, shows the original velocity distribution, consisting of two broad peaks. These two peaks arise from the ground-state

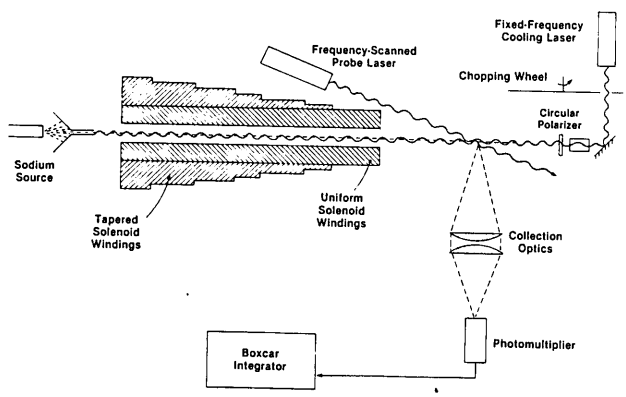

Fig. 6. The apparatus of the present authors.

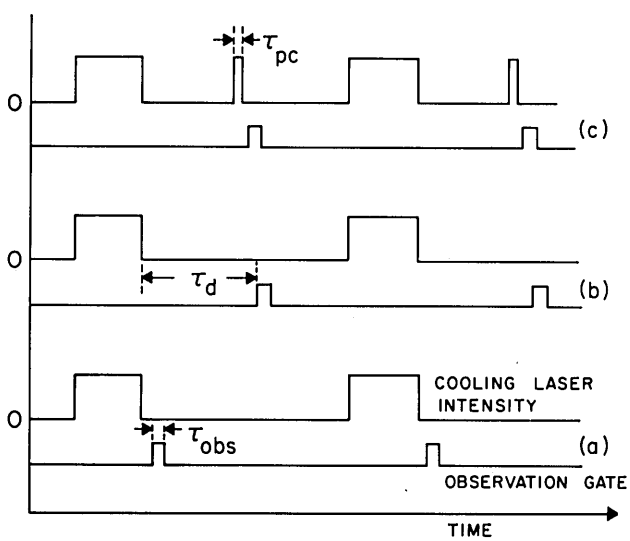

Fig. 7. Timing diagrams for prompt observation of (a) cooled atoms, (b) delayed observation, and (c) postcooling.

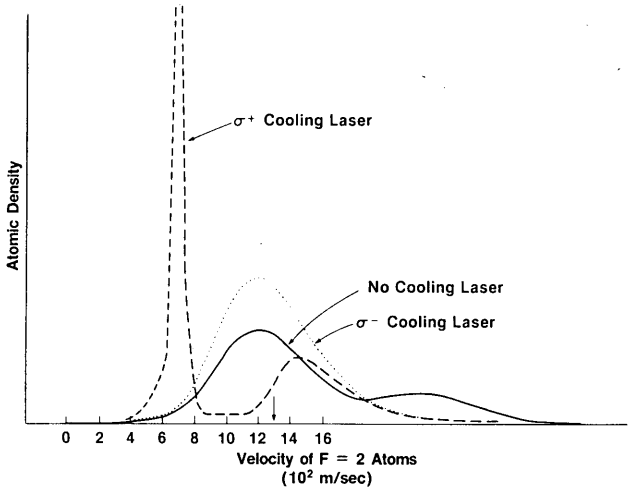

Fig. 8. Results of Zeeman-tuned cooling. Arrow indicates the highest velocity resonant with the cooling laser. The velocity scale refers only to $F = 2$ atoms.

hyperfine splitting and represent similar velocity distributions, centered near 1100 m/sec for $F = 2$ and $F = 1$ atoms. (The velocity scale refers only to $F = 2$ atoms.) The solid curve shows the velocity distribution with the cooling laser on (σ^+ so optical pumping is avoided). Atoms from the highest velocity resonant with the laser are decelerated into a narrow velocity group (whose peak density is off scale on the figure). We have observed densities per unit velocity in the cooled peak as much as 30 times higher than that in the peak of the uncooled velocity distribution and with relative velocity widths as small 5%.[52]

We have discussed the effect of optical pumping and how to avoid its interruption of the cooling process. Figure 8, however, shows some advantages of optical pumping. Note that after cooling with σ^+ light there are no atoms left in the $F = 1$ distribution. Furthermore, nearly all the atoms participate in the cooling process, whereas one might expect only 1/8 of the atoms to be in the $3S_{1/2}$ ($F = 2$, $m_F = 2$) state, which participates in cooling. The explanation is that while the atoms travel from the Na source to the solenoid they are optically pumped by the σ^+ light into the ground state with highest projection of angular momentum, i.e., $m_F = 2$. The process is aided by the fact that the laser beam is focused on the Na source. The resulting power broadening ensures a significant excitation rate even for off-resonance transitions. It has been proposed[53] that similarly advantageous optical pumping could be realized by using chirping techniques. (Figure 8 also shows that use of σ^- cooling light results in the disappearance of the $F = 1$ atoms. In this case the atoms are pumped into the $F = 2$, $m_F = -2$ state, which can be excited repeatedly to the $F = 3$, $m_F = -3$ state. This transition's frequency increases as the magnetic field decreases, so that the changing Doppler shift is not compensated and essentially no cooling occurs.)

With Zeeman-tuned cooling the total velocity change is determined by the available field change. For a given field distribution, the final velocity can be chosen by varying the cooling laser frequency and thus the initial velocity. Figure 9 shows a series of cooled velocity distributions obtained with various laser tunings. As expected, the lower the initial velocity, the lower the velocity of the peak of the cooled distribution. Note that at lower velocities the total velocity change is larger than at higher velocities because the atoms that emerge from the solenoid are those that violate the condition of expression (4) and stop cooling. For lower velocities this occurs at higher dB/dz. For the field of Eq. (3), approximated in our solenoid, higher dB/dz occurs at lower field, so that more total field change is available for deceleration.

As the final velocity approaches 200 m/sec, our observed density decreased rapidly. Useful signals cannot be obtained for final velocities less than about 200 m/sec; at lower velocities the cooled velocity distribution simply disappears, as has nearly happened in Fig. 9(f). This occurs in spite of the fact that about half of the original atoms have velocities lower than the initial velocity and can be cooled. The explanation is that in our magnet fringing fields make the field gradient small enough that the condition of expression (4) is never violated if v is less than 200 m/sec. Thus any atom slower than about 200 m/sec will not go out of resonance with the laser before it is brought to rest. Although the production of atoms at rest is one ultimate goal of our work, we cannot easily observe these atoms because they are inside the solenoid, in a high field gradient, more than 40 cm from the observation region.

Consider now the steady-state situation with the laser

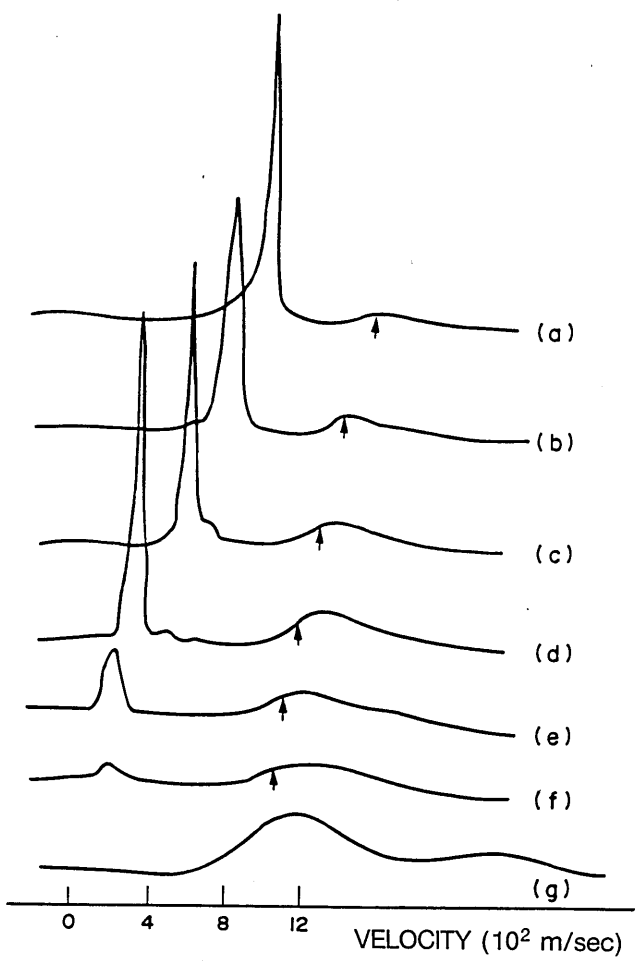

Fig. 9. Laser-cooled velocity distributions for different laser tunings. The highest velocity resonant with the laser in each case is indicated by the arrows. Trace (g) is the uncooled velocity distribution.

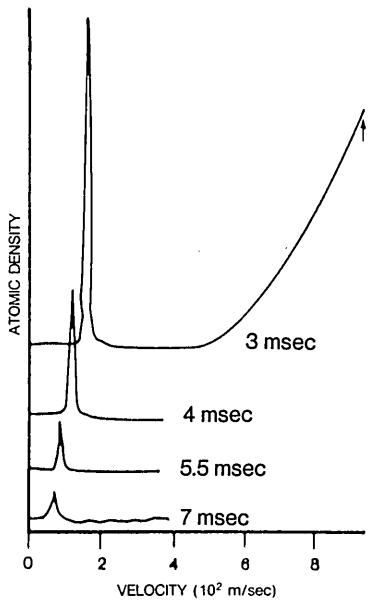

Fig. 10. Velocity distributions obtained with Zeeman-tuned cooling and delayed observation. The delay times are indicated for each curve.

tuning such that atoms are brought to rest at some point near the exit end of the solenoid. Slightly further upstream, toward the entrance end, atoms are going at low velocities determined from the resonance condition for the laser frequency and magnetic field. When the laser light shuts off, deceleration stops and the atoms propagate freely. For such a laser tuning, with prompt observation [small delay time τ_d as in Fig. 7(a)], no atoms are seen except those so fast that they had never been resonant with the laser. On the other hand, with delayed observation [significant τ_d, as in Fig. 7(b)], the very slow atoms will have time to travel from the end of the solenoid to the observation region. The starting points for velocities smaller than 200 m/sec are within a few centimeters of each other, so the observed velocity varies nearly inversely as τ_d.

Figure 10 shows a series of velocity distributions obtained with different delay times. Note that the velocities are all lower than the smallest velocities of Fig. 9. As expected, longer delays result in observation of slower velocities. In addition to the peak of slow atoms, one also sees the uncooled velocity distribution (the wing of which is visible in the figure). This arises from thermal atoms that come from the Na source to the observation region during the long delay without ever interacting with the laser. The observed width of the laser-cooled velocity distribution is determined largely by time of flight. When the delay is sufficiently long that the observation time is not a significant fraction of the delay, velocity widths as narrow as about 10 m/sec are observed. (This width comes from the true velocity width and the natural-lifetime-limited resolution of the probe.) As the velocity decreases, so does the magnitude of the observed atomic density, because of the increased spatial spreading of the atomic beam. As the longitudinal velocity decreases, the beam diverges, owing to the spread in the transverse velocity. There are two sources of transverse spread: initial finite collimation of the atomic beam and momentum transfer from the random spontaneous emission of photons in the transverse direction (heating). Focusing the laser on the Na source helps to reduce some of this transverse spread since there is a small component of the decelerating force that tends to compress the atomic beam transversely. If finite collimation of a pointlike Na source were the only reason for transverse spread, the focused laser would reduce transverse and longitudinal velocity in the same proportion, so there would be no increase in atomic-beam divergence. The contribution to the divergence from radiative heating, however, is random and cannot be completely corrected, so the beam diverges significantly as the velocity is reduced. Other factors that contribute to the decrease in observed atomic density with velocity are longitudinal spreading of the atoms, which affects observations if the observation gate is too short, and scattering from background gas when the vacuum is poor. This last factor is particularly important at low velocity because of the long time available for scattering.

The loss of density with velocity limits the lowest velocity that we can observe by this method. In 1982 we reported a velocity as low as 40 m/sec (Ref. 51), and we have since been able to observe atoms as slow as about 30 m/sec. (*Note added since preparation of manuscript*: More recently[54] we have observed atoms after delays of 65 msec, corresponding to velocities of less than 10 m/sec. At such velocities the distance the atoms fall under the influence of gravity becomes significant.)

The transverse spreading of the atomic beam can be reduced by using a magnetic hexapole lens, and we have demonstrated significant improvement using such focusing.[55] The Moscow group had used radiation pressure to reduce the transverse atomic velocity: laser light tuned slightly below resonance is directed transverse to the atomic beam, symmetrically from all directions, using a coaxial reflector called an axicone.[56,57] This technique also produced significant increases in atomic density. In our experiments at NBS refocusing and transverse cooling techniques can extend the observation of slow atoms to lower velocities, but we still cannot observe zero-velocity atoms for the simple reason that they never get to the observation region.

In order to obtain zero-velocity atoms in our observation region we introduced an additional deceleration stage. Consider the timing diagram of Fig. 7(c). After the delay appropriate to a chosen atomic velocity the desired atoms have drifted into the observation region, and the cooling laser light is again turned on for a brief time τ_{pc}. This "postcooling" pulse can be used to stop or even reverse the direction of the atoms.[58,59] The pulse is short enough (a few hundred microseconds) that the slow atoms do not travel far during the pulse. After the pulse, the observation gate is opened and the atoms are observed in the usual way.

The postcooling process is essentially nonresonant, since the laser frequency is fixed and there is no magnetic-field gradient. Nevertheless the required velocity change is small enough that the atoms can be close to resonance during this postcooling process. In order to achieve near resonance, as well as to prevent optical pumping, we add a uniform magnetic field of about 0.02 T to the observation region. In principle, we could use a different laser frequency for the postcooling and reduce or eliminate the need for so large a field in the observation region.

Figure 11 shows the velocity distributions observed following a 10-msec delay after the cooling laser shut off. The upper trace was taken without any postcooling pulse, and the lower one was taken following a 160-μsec pulse. The cooling laser was resonant with a velocity indicated by the

arrow. In this case, postcooling changes the velocity from 50 to −30 m/sec and has no significant effect on the shape of the distribution.

The velocity change produced by postcooling depends critically on the tuning of the cooling laser. Figure 12 compares the experimental results with theoretical predictions of a one-dimensional analytic solution of the postcooling problem. A three-dimensional Monte Carlo simulation of postcooling gives a similarly good fit to the data. The only adjustable parameter is the postcooling laser intensity. The data were taken with a 6-msec delay, yielding a 68-m/sec velocity before postcooling. The 250-μsec postcooling pulse produced velocity shifts as great as 130 m/sec, enough nearly to reverse the initial velocity.

More recently[54] we have used a spatially varying field in the observation region to compensate partially for the changing Doppler shift during postcooling. Atoms with velocities as high as 120 m/sec have been stopped, and the atomic density per unit velocity interval is actually higher after postcooling because of the spatial compression of the atoms. We have achieved atomic densities per unit velocity interval of nearly 10^6 Na atoms/cm^3 within 3 m/sec of zero longitudinal velocity.

In addition to our Zeeman-tuned cooling work, we have made some preliminary measurements using chirped cooling.[60,61] The procedure used is similar to that for delayed observation [Fig. 7(b)] except that the cooling laser is frequently modulated sinusoidally[62] at the chopping frequency. The phase of the modulation is arranged so that while the chopper passes laser light, the frequency of the laser is increasing nearly linearly. Only the bias part of the solenoid is used, producing a uniform field to avoid optical pumping. Observation is delayed long enough to allow the chirp-cooled atoms to travel from the solenoid to the observation region. (Figure 5 was obtained using this observation procedure but with the frequency of the cooling laser fixed.)

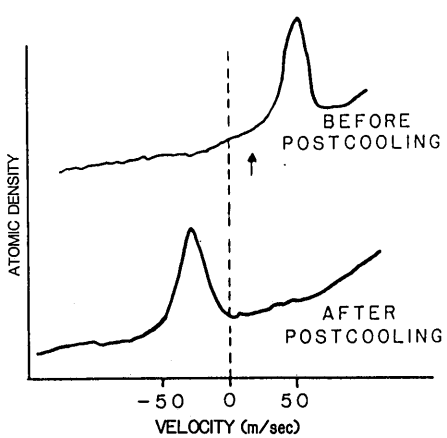

Fig. 11. Effect of postcooling after a 10-msec delay. The sloping baseline is from the uncooled thermal distribution, Zeeman shifted into resonance with the probe.

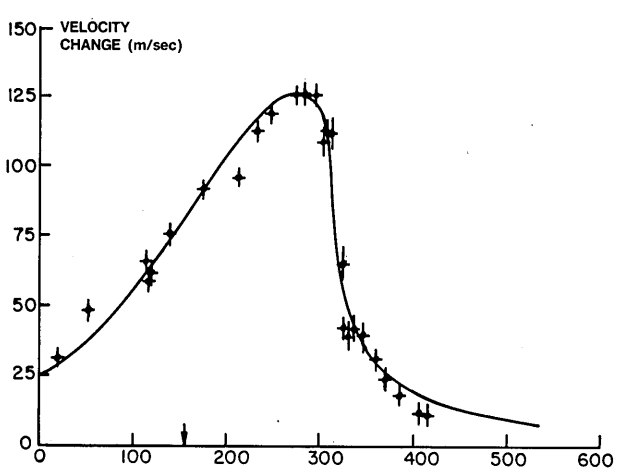

Fig. 12. Velocity changes versus laser tuning for postcooling after a 6-msec delay. The arrow indicates the frequency that is resonant with the atomic velocity before postcooling.

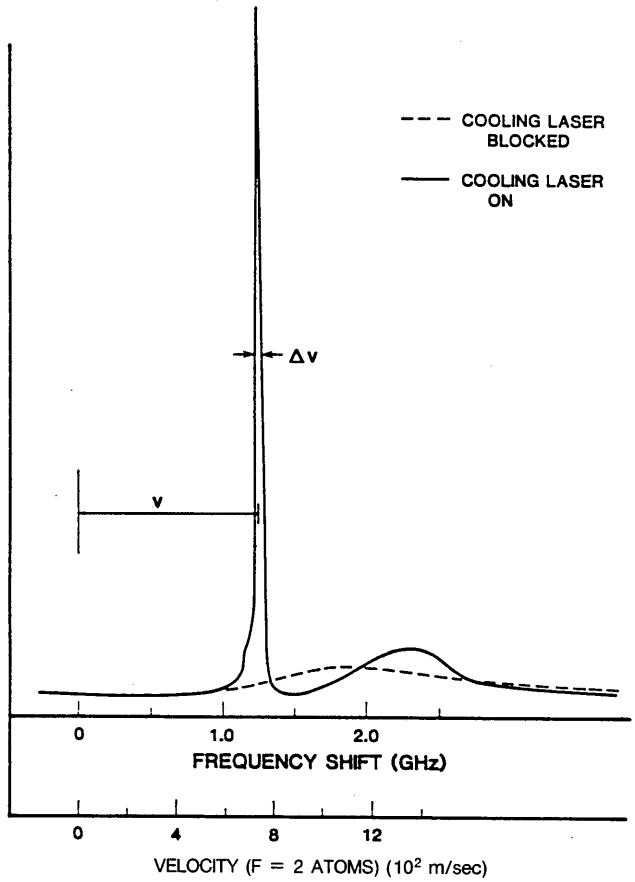

Fig. 13. Chirp-cooled velocity distribution (solid line) compared with the uncooled thermal distribution (dashed line).

Figure 13 shows a velocity distribution obtained with chirped cooling. Atomic velocities were swept from 1260 m/sec down to 720 m/sec as the laser scanned 890 MHz at a rate of 1.18 GHz/msec. The width of the final distribution is 15 m/sec or 2% of its central velocity. While we have achieved dramatic velocity compression, we have not observed deceleration by the chirping method to velocities lower than about 600 m/sec.

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A group at NBS–JILA has taken a different approach to the chirped cooling problem, using two laser frequencies to avoid optical pumping.[63,64] Figure 14 shows a simplified diagram of their apparatus. The chirp is obtained with an electro-optic phase modulator (EOM-1) driven by a variable-frequency oscillator that impresses scannable side-

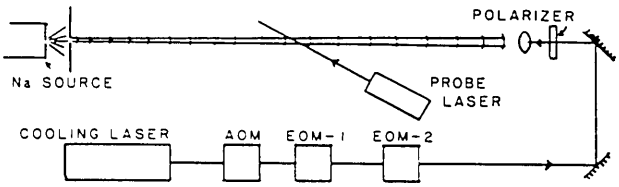

Fig. 14. Apparatus of the NBS–JILA group.[56]

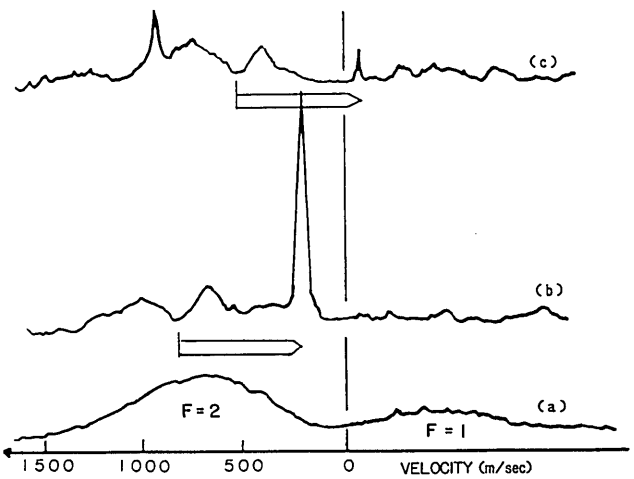

Fig. 15. Chirped-cooling results of the NBS–JILA group.[56] The velocity scale is for the $F = 2$ atoms only.

bands onto the cooling laser frequency. The cooling laser is circularly polarized, but no magnetic field is applied along the beam, so the circular polarization alone is not sufficient to prevent optical pumping. Since the rate of optical pumping to the $F = 1$ ground hyperfine level is much smaller than the overall rate of optical excitation, and the chirping rate is significantly lower than the maximum allowable rate, only a fraction of the main cooling laser power is needed at the additional frequency to pump atoms out of the $F = 1$ level. A second modulator (EOM-2) driven at 1772 MHz (the ground hyperfine frequency) provides this additional laser frequency.

While the electro-optic modulators produce the two scanned frequencies for the $3S_{1/2}$ ($F = 1$, $F = 2$) $\rightarrow$ $3P_{3/2}$ transition, other extra frequencies, both scanned and fixed, are present in the light that irradiates the atomic beam. In most cases these do little to affect the chirped cooling process, although the fixed frequencies do perform some off-resonant cooling.

The acousto-optic modulator (AOM) acts as chopper to turn the cooling light on and off: the modulator is on while the driver to EOM-1 chirps over an appropriate range and off during the observation of the fluorescence induced by the probe. The observation time occurs just after the AOM turns off, so the velocity distribution is measured before the atoms move significantly. The location of the observation region may be varied along the atomic beam axis.

Figure 15 shows some results from the NBS–JILA group. In Fig. 15(a) we see the uncooled velocity distribution. [The difference between Fig. 15(a) and Fig. 8 occurs because of the different direction of the probe.] In Fig. 15(b) the chirped sideband of the cooling laser was swept from resonance with atoms at about 800 to 200 m/sec (scan of about 1 GHz), as indicated by the arrow. The large, narrow peak at the head of the arrow is the resonantly cooled velocity distribution. The dip and peak near the tail of the arrow is the nonresonantly cooled distribution created by the fixed-frequency portion of the cooling light. When the initially resonant velocity is changed to about 500 m/sec, the resonantly cooled velocity distribution [Fig. 15(c)] is at about −100 m/sec. [The narrow peak at 900 m/sec in Fig. 15(c) arises from the action of one of the extra chirped sidebands, which sweep from about 1500 to 900 m/sec.] This negative velocity peak is smaller than the peak in Fig. 15(b) partly because a much smaller fraction of the velocity distribution participates in the cooling process when the starting velocity is 500 m/sec. Nevertheless, the atomic density per unit velocity obtainable at zero or negative velocity is a substantial fraction of that observed at the peak of the uncooled velocity distribution. The cooled velocity distributions observed by the NBS–JILA group had widths as narrow as 15 m/sec. The width comes from the longitudinal velocity spread, some of the transverse spread, and the natural-lifetime-limited resolution of the probe.

More recently[65] the NBS–JILA group has used a relay technique to extend the range of its chirp cooling: with the carrier of the cooling laser frequency resonant at about the center of the atomic velocity distribution, and EOM-1 driven at 1 GHz, the low-frequency sideband is resonant with 600 m/sec higher-velocity atoms. As the drive frequency is swept toward zero, and the higher-velocity atoms are swept toward the center of the distribution, the high-frequency sideband comes into resonance with the decelerating atoms. Now the drive frequency is swept up to 1 GHz again as the high-frequency sideband carries the atoms to zero velocity. By allowing both the high- and low-frequency sidebands to cool, one obtains a 2-GHz scan by sweeping the drive oscillator only 1 GHz.

4. DISCUSSION

Each of the methods discussed for cooling atomic beams has its own particular advantages and disadvantages. The method that one chooses will probably depend on the available equipment and the nature of the application. We will compare those methods that have already been used experimentally.

A major advantage of the nonresonant cooling technique is its simplicity. One needs only a fixed-frequency laser and a way of avoiding optical pumping. Use of a double-mode laser to do this complicates the method, but the fact that scanning is not needed makes this less difficult. The technique produces a continuous beam of slow atoms and is capable of extreme velocity compression if one does not require much deceleration. The difficulty in getting large velocity changes and the requirements for large laser power and long interaction times represent the major disadvantages.

Zeeman-tuned cooling is also relatively simple to execute. The only appropriate way to avoid optical pumping is with circular polarization, so there will necessarily be only one laser frequency. Large velocity changes along with extreme cooling are possible. The cooled atomic beam is continuous and of very high density since the velocity compression is accompanied by a spatial compression. In addition, the

circular polarization ensures that the cooled atomic beam is completely state selected. This is particularly important if magnetic focusing or confinement is to be used on the cold atoms. The major disadvantage is that the production of slow atoms is in a gradient magnetic field, which may not be desirable for some applications. If the atoms are to be transported out of the field, one may lose much of the high density inherent in the technique. For the lowest velocities, extraction of the atoms from the field has involved changing continuous production of slow atoms into pulsed observation. For many applications, of course, this may be acceptable or even desirable.

Chirped cooling has the advantage of large velocity change, extreme velocity compression, and compatibility with most methods of avoiding optical pumping. The possibility for velocity compression appears to be greater than in the other methods. Velocity compression is compromised in nonresonant cooling if large deceleration is achieved; in Zeeman-tuned cooling the compression depends on details of the magnetic-field gradient or on time-of-flight considerations. With chirping, the laser can be made to interact optimally for velocity compression as well as deceleration. In practice, both chirping and Zeeman tuning have produced similar velocity compressions. Chirping has the advantage over Zeeman tuning in that there is no need to extract the atoms from a magnetic field. On the other hand, chirping is fundamentally a pulsed production process, and it does not produce spatial compression of the velocities. That is, atoms of a particular velocity appear at a particular time but spread out in space. If the interaction distance is long enough, production can be quasi-continuous if one completes a chirping cycle in a time short compared with the total flight time in the apparatus. The technological difficulties are greater in chirping than in other methods. Standard commercially available lasers do not scan rapidly enough, nor are suitable modulators commercially available.

The hybrid technique of combined Zeeman-tuned and nonresonant cooling,[59] or two-stage Zeeman-tuned cooling,[54] used by our group at NBS looks quite promising. It can combine large velocity change, high density, zero velocity, and extreme velocity compression. A combination of Zeeman tuning with chirped cooling may prove to be even more useful.

5. APPLICATIONS

Laser-cooled atomic beams may find important applications in spectroscopy, collision experiments, and traps. At present the highest resolution optical spectroscopy (on the intercombination line in Ca) is limited in its resolution almost entirely by second-order Doppler width.[66] Even a modest reduction in the velocity spread by laser cooling would effectively eliminate the second-order Doppler width. Cooling Ca looks quite feasible—the cooling transition frequency is within the range of available dye lasers having sufficient power to saturate the transition, and the distance required to decelerate thermal velocity Ca atoms to zero under saturated conditions is less than half of that for Na. Letokhov and Minogin[67] and Ertmer *et al.*[68] have considered other candidate atoms that could be laser cooled and also used as possible frequency standards.

Another possible spectroscopic application is to measurements where motionally induced fields are a problem. This is often the case in experiments that look for small effects of parity or time-reversal asymmetries.

In order to be better understood, many collision experiments require velocity selection of the collision partners. Laser cooling can provide such selection without loss of intensity by compressing rather than selecting the velocity distribution. In addition, some laser-cooling techniques can provide completely state-selected atoms, a matter of considerable importance in the case of state-specific scattering experiments. Besides atomic scattering experiments, one can also foresee experiments involving deflection of laser-cooled atomic beams by light as a test of photon-counting statistics,[69] experiments involving collisions of laser-cooled beams with surfaces, and measurements of long atomic lifetimes.

6. ELECTROMAGNETIC TRAPS

One of the most exciting possible applications of slow or stopped atoms is to confine them in an electromagnetic trap. Specific proposals for such traps for neutral atoms are more than 20 years old, but as of this writing we know of no successful demonstration of a stable trap. This has been due in large part to a lack of suitably slow atoms to trap. Among the types of trap proposed are magnetostatic, radiative, electrostatic,[22] and hybrid magnetostatic–radiative[23] traps. At present the first two types seem most likely to be used with laser-cooled atoms, so we will concentrate our discussion on them.

Magnetostatic Traps

The principle behind magnetostatic traps is illustrated in the magnetic deflection experiments of Stern and Gerlach and Stern[70] and the magnetic focusing experiments of Friedburg and Paul[71]: Inhomogeneous magnetic fields exert forces on atoms with magnetic dipole moments. Quantum states whose energy increases with increasing magnetic field (such as the $3S_{1/2}$, $m_F = 2$ state of Na in Fig. 2) experience a force in the direction of decreasing field magnitude. This principle is used in the well-known linear hexapole focusing magnet,[71] which has zero field along its axis, the field magnitude increasing quadratically with radial distance from the axis. Heer[72] and Vladimirskii[73] suggested closed magnetic bottles. Heer suggested bending a linear six-pole field into a closed torus, producing a trap whose equilibrium points form a circle. A modified version of such a trap was used by Paul and colleagues[74] to contain ultracold neutrons.

Paul has also described a simple and elegant magnetic trap, with a single equilibrium point, that he calls a spherical hexapole.[75] By placing three coils on the surface of a sphere, one each at +45° and −45° latitude and a third coil between them on the equator, the field can be made zero at the center of the sphere when the equatorial coil carries current equal in magnitude but opposite in direction to that of each of the other two.[76] By analogy to a magnetic hexapole lens, the field increases quadratically with radial position, and the restoring force, proportional to the field gradient, is therefore harmonic. Other configurations of three loops can produce similar traps, although not necessarily having equal coefficients of their harmonic potential in all directions or zero field at their center. A particularly simple trap has

three loops (or short solenoids) wound on a single cylindrical form, appropriately spaced and energized to produce the desired field.

A similar but simpler type of trap consists of two identical separated coaxial coils carrying opposite currents. This pair of coils forms a spheroidal quadrupole trap.[77] It clearly has a single center where the field is zero and is the simplest configuration for a magnetic trap. The field magnitude varies linearly with displacement from the center, the slope being twice as great along the axis as in the plane perpendicular to the axis. As a result, the trap has equal depth in the radial and longitudinal directions when the coils are separated by about 1.25 times their radius. Its experimental simplicity makes it most attractive, because of ease of construction and of optical access to the interior. The restoring force for the quadrupole trap is constant for displacements along a given direction, so the trap is not at all harmonic. Nevertheless, the orbital calculations are straightforward.[78]

Attempts to trap neutral atoms (as opposed to neutrons) in magnetic traps have not been successful.[21] A key difficulty is in supplying sufficiently low-energy atoms to be trapped. A trap (for magnetic moments of a Bohr magneton) whose field varies from 0 to 2 T has a potential energy depth equivalent to the kinetic energy of Na atoms at 30 m/sec. This is an energy of about 1 K. Before laser cooling, isolated samples of such atoms did not exist.

The Zeeman-tuned laser-cooling technique that we have developed not only provides atoms slow enough to be contained by magnetic traps with reasonable fields but also provides the atoms in the proper magnetic orientation to be trapped. We have constructed a trap of the quadrupole design with a loop radius of 2.7 cm. The trap has a potential energy depth about equal to the kinetic energy of Na atoms with velocities of 3 m/sec. Such atoms initially at the center of the trap would be confined to a volume of about 20 cm^3.

Our proposed experiment to demonstrate magnetic trapping would proceed as follows: Atoms are cooled in the solenoid as described above. At $t = 0$, the cooling laser beam and the atomic beam are shut off. Four milliseconds later, 100-m/sec atoms arrive in the center of the trap, which is coaxial with the atomic and laser beams and located at the center of the observation region. Only the upstream (closest to the Na source) coil of the trap pair is energized at this time, and it provides a slight spatial variation in magnetic field to aid the postcooling, which begins at 4 msec and lasts several hundred microseconds. When the postcooling laser pulse ends, more than 10^5 atoms having velocities within 3/msec of zero are in a few cubic centimeters near the center of the trap. At this time, the downstream trap coil is turned on (in a few hundred microseconds), opposing the upstream coil, and the trapping field is formed. The trapped atoms can be destructively probed at any later time by reversing the downsteam coil current to produce a nearly uniform field near the trap center and irradiating the trap once again with a probe laser, observing the induced fluorescence.

By varying the time between loading the trap and probing it, we will be able to measure the lifetime of the trapped atoms. Besides collisions with background gas atoms, a mechanism that can limit the lifetime in the trap is nonadiabatic or Majorana transitions, which reorient the atomic magnetic moment. While the initial orientation of the atoms produced by laser cooling with Zeeman tuning is correct, that orientation must be preserved while the atoms move about in the trap even though the trap fields change directions in a complicated way. The atomic magnetic moments precess about the field at the Larmor frequency $\omega_L = \mu B/\hbar$, where μ is about one Bohr magneton. As long as the atoms move slowly enough, the magnetic field changes sufficiently slowly that they follow it adiabatically. This requires that $\omega_L \gg \omega_T$, the angular frequency of orbital motion in the trap. Violation of this adiabatic condition results in a large probability of a magnetic transition to a state of different orientation that may not be confined by the trap.

Qualitatively, the adiabatic condition requires that a fast atom should never be too close to the trap center. An atom near the center of our trap cannot be going faster than 3 m/sec, since it would otherwise escape. At the point of closest approach to the center the velocity is orthogonal to the radius vector, so the instantaneous frequency of orbital motion is given by $\omega_T = v_0/r_c$, where r_c is the distance of closest approach and v_0 is the atomic velocity at r_c. At r_c, the field is given by $B = r_c(dB/dr)$, where dB/dr is the field gradient and is minimum in the plane perpendicular to the trap axis. For our trap this dB/dr is 1 T/m. The condition $\omega_L \gg \omega_T$ becomes

$$\frac{\mu r_c(dB/dr)}{\hbar} \gg \frac{v_0}{r_c},$$

$$r_c^2 \gg \frac{\hbar v_0}{\mu(dB/dr)}, \tag{5}$$

which leads to $r_c \gg 5.8$ μm. If the orbit is circular the adiabatic criterion is less stringent. For orbits in the plane perpendicular to the axis we have

$$r^3 \gg \frac{\hbar^2}{\mu(dB/dr)M}, \tag{6}$$

where M is the atomic mass. This requires $r \gg 0.3$ μm, which implies $v \gg 8.5$ mm/sec.

We have done detailed numerical calculations of the orbital motion for more general cases. Many orbits are irregular and aperiodic, although there are several classes of nearly closed orbits. Since typical orbit times are a few tens of milliseconds, and since the nonadiabatic region occupies less than 10^{-10} of the trap volume, we expect even the general, unclosed orbits to last for many seconds in sufficiently good vacuum. [*Note added since preparation of manuscript*: We recently performed the experiment described above. Na atoms with velocities up to 3.5 m/sec were confined in the quadrupole trap. The time constant for decay of the trapped atom population was 0.83(7) sec and is believed to be limited mainly by collisions with background gas. Details of the experiment are given in Ref. 54.]

A particularly interesting possibility is cooling atoms in a magnetic trap using a recently proposed rf-optical pumping scheme.[24] This or a related technique may actually allow cooling below the limit associated with the usual Doppler cooling.[16]

Radiative Traps

Radiative or laser traps were first proposed by Letokhov in 1968.[2] His idea was that atoms under the influence of the oscillating electric field of a laser acquire an oscillating

electric-dipole moment that interacts with the laser field. The energy of interaction (which we might call an ac Stark shift) acts as a potential just as in the case of a static magnetic-dipole moment interacting with a static magnetic field. For the laser tuned below the atomic resonance frequency, the sign of the interaction is such that the atom is attracted to the region of maximum laser field intensity; whereas if it is above resonance, the attraction is to the region of minimum field intensity. Letokhov proposed that atoms could be captured on the nodes or antinodes of a plane standing wave[2] or of intersecting standing waves.[19]

In 1978 Ashkin[3] proposed a laser trap that combined the dipole or gradient force of Letokhov's trap with the more common radiation pressure force or scattering force that results from absorption and spontaneous reradiation of photons. Ashkin's trap design is shown schematically in Fig 16. Two focused laser beams with Gaussian transverse intensity profiles are directed coaxially and oppositely, with their foci slightly separated. The frequency is below resonance, so transverse confinement by the dipole force results from the atoms being drawn into the axis, where the intensity is maximum. Axial confinement is achieved by virtue of the scattering force: as an atom moves away from the equilibrium point midway between the two foci it sees an increased intensity in one beam and a decreased intensity in the other. The imbalance results in a net radiation pressure that forces the atom back to the equilibrium point.

Besides the introduction of radiation pressure scattering force into the laser trap, a distinguishng feature of Ashkin's trap is the use of the gradient of laser intensity resulting from the Gaussian profile of the beam rather than from a standing wave. As an alternative trap design Ashkin proposed a single, tightly focused laser beam—a configuration with an absolute maximum in intensity at the focus and thus a three-dimensionally stable dipole-force trap. Laser traps can also cool the atoms that they contain, since when the laser is tuned below resonance, the Doppler cooling[16] described above will reduce the kinetic energy of the trapped particles.

Unfortunately, it was later recognized that heating mechanisms would destabilize laser traps.[9–11] Besides the heating or diffusion of momentum arising from the statistical nature of absorption and spontaneous emission of photons (fluctuations in the scattering force) there is heating associated with fluctuations in the dipole force. Unfortunately, as a dipole-force trap is made stronger, by making the intensity gradient and thus the dipole force larger, the fluctuations also get larger. The result is that the steady-state kinetic energy of atoms in such a trap, resulting from equilibrium between the heating and cooling mechanisms, is always about equal to the trap depth [see, for example, Eq. (53) and the following discussion in Ref. 11).

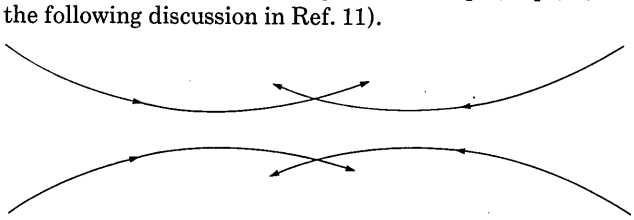

Fig. 16. Optical trap formed from two opposed, diverging laser beams with Gaussian intensity profiles. When tuned below resonance the trap provides transverse confinement by the dipole force. Longitudinal confinement is from radiation pressure.

Gordon and Ashkin[11] and Ashkin and Gordon[79] proposed the use of separate damping laser beams to provide cooling in an attempt to separate the cooling and trapping functions and optimize each. Unfortunately, trapping and cooling remain coupled through the ac Stark shift, which changes the nature of the cooling as a function of position in the trap. Gordon and Ashkin were unable to show that, without compensation of the ac Stark shifts, a laser trap could be stable.

Dalibard *et al.*[80,81] showed how to effect a complete separation of cooling and trapping functions. In their design, the trapping field is switched on and off rapidly, with cooling taking place while the trapping field is off. In this way, the cooling and trapping functions may be separately optimized, do not interact, but are effective only for half of the total time. Their design also minimizes heating mechanisms by avoiding the large field gradients of standing waves while minimizing undesirable radiation pressure forces by using opposed, focused laser beams with opposite senses of circular polarization. An expelling force resulting from imbalance between the intensities of the opposed beams can be overcome by separating the foci of the beams as in Ashkin's design (see Fig. 16), so as to provide a stronger axial restoring force.

Thus trapping laser beams configured as in Fig. 16, with opposite polarization and alternated with plane-wave damping beams, appear to present the best prospect for an optical dipole trap. An optimized design for such a trap could capture Na atoms having velocities of a few meters/second over a volume of $\sim 10^{-4}$ cm^3 and confine them to a much smaller volume as the atoms cool and the strong axial scattering trapping force compresses them into the trap center.[82] Since we can produce more than 10^5 such atoms per cubic centimeter by using our present laser-cooling techniques and switch the magnetic field to zero in a very short time, we should certainly be able to trap several atoms, which should be easily observed by their fluorescence—even single trapped ions have been observed by eye.[83]

The strong axial trapping provided by the scattering or radiation pressure force in a separated focus design such as Ashkin's[3] suggests the possibility of making a strong trap relying only on the scattering force. Some possible designs were considered,[84,85] but Ashkin and Gordon were able to show that no such trap, using time-independent laser intensities, could be stable.[86] The reason for the instability is closely analogous to Earnshaw's theorem for electrostatics: there is no point of stable equilibrium for a test charge in a charge-free region.

Of course, it is well known that charged particles can be trapped electrically in a charge-free region if the electric fields oscillate. The kinetic energy of the ionic micromotion, which is driven by the oscillating, spatially varying electric fields, is a function of the position of the ion in the trap. This variation of the micromotion energy can be thought of as a variation of a pseudopotential that can stably trap the ion. This is the basis of the rf or Paul trap for ions.[87,88] Ashkin proposed that a similar situation would hold for a light-pressure trap.[26] Consider the trap as shown in Fig. 16. An atom on the axis and centered between the two foci feels a restoring force for small displacements along the axis and an expelling force for small displacements transverse to the axis. Ashkin proposed that this configuration be alternated with its time-reversed configuration,

which produces an axial expelling force and a transverse restoring force.

Ashkin showed that such an alternation produced a situation exactly analogous to that in a quadrupole rf trap, neglecting the fluctuations in the scattering force and ignoring any dipole forces. He also showed explicitly how such a time-reversed alternation could be achieved. Because of the heating due to fluctuations in the scattering force, the trapped atoms must be cooled, either by tuning the trapping laser beams below resonance or by supplying separate, plane-wave damping beams. Damping or cooling of the atomic motion occurs because of the Doppler cooling process described in Section 1, above, and reviewed in Ref. 16.

Dalibard and Phillips[89] have considered a modification of Ashkin's idea wherein the configuration of Fig. 16 is alternated with a set of two similar pairs of opposing beams, all mutually orthogonal. In other words, a pair of opposed, diverging beams along z is alternated with two pairs along x and y. This configuration leads to exactly the same quadrupolelike force as Ashkin's design but has different damping characteristics. For example, if the trapping beams also provide the damping, an atom at the center of the trap, moving along the z axis, will be damped for both types of trap. However, if the motion is in the x–y plane, the Ashkin design will not damp it since there is no component of laser wave vector transverse to z at the center of the trap.

The analysis by Dalibard and Phillips of their trap design shows that the heating and damping actually dominate the dynamics of the trapped atom. In particular, they found no operating parameters that produce a stable trap in three dimensions when damping is provided only by the trapping beams. The reason is as follows: The strong damping from the laser beams damps not only the random motion that is due to radiative heating but also the micromotion that creates the trapping pseudopotential. Using typical parameters for a Na atom trap, the micromotion frequency will be a few kilohertz, while the damping time is of the order of 10 μsec. This strong damping effectively destroys the trapping potential, and the trap is unstable.

Such a trap can be stabilized by providing separate damping beams and tuning the frequency of the trapping beams so that their damping is not too strong. Best stability is achieved by providing strong damping in the x–y plane when trapping beams are applied along the z axis and damping along the z axis when trapping is applied in the x–y plane. Under these conditions the trapping force is not properly said to derive from a pseudopotential established by a micromotion but rather from the direct restoring forces whose accompanying expelling forces have been rendered ineffective by strong damping.

The recognition of the dominant role played by damping in a radiation pressure trap leads to another curious observation: with strong damping, even in the absence of any trapping beams, atoms remain "trapped" for relatively long periods of time. In effect, the damping laser beams act as a highly viscous fluid that inhibits the escape of the atoms. It is as if the atoms were in a pot of molasses from which they take a long time to diffuse. This "molasses trap" is not a true trap, in the sense that there is no restoring force, but the diffusion times can be remarkably long.

For a rough idea of how effective the optical molasses can be, consider the example of three mutually orthogonal pairs of opposing laser beams with a saturation parameter of about unity and tuned about half a natural linewidth below resonance. For a pair of opposed σ^+ and σ^- polarized beams, which produce no standing wave, this is the condition for optimum damping (see Ref. 81, for example). This leads to a damping rate for the kinetic energy along any axis of almost $\hbar k^2/2M$, where k is the laser wave vector and M is the atomic mass. In Na this gives a corresponding damping time $\tau_{\text{cool}} \cong 6$ μsec.

Under the combined influence of this damping and of the heating from both the spontaneous emission and the fluctuations in the rate of absorption (but ignoring fluctuations in the dipole or gradient force), the atoms will reach an energy of about $\hbar\gamma/3$ for each of the three translational degrees of freedom. This result can be obtained two ways. First, by assuming no interaction between the three pairs of beams one may apply the result given by Cook for weak opposed beams [Ref. 10, Eq. (177) using a spontaneous diffusion coefficient increased to account for all three coordinate directions]. Second, one may use the result of Dalibard et al.[81] for σ^+–σ^- beams configured for maximum damping. These give about $0.3\hbar\gamma$ and $0.4\hbar$, respectively, for the energy along one axis. Of course, there will be interaction between the pairs of beams: Interference produces spatial intensity gradients, which lead to dipole forces whose fluctuations add to the heating, while saturation effects may reduce the final energy from that obtained by simply summing the one-dimensional energy over the three degrees of freedom. Without doing the explicit calculation for a particular configuration, one can say that the energy is not likely to be less than $\hbar\gamma$, which leads to an optimistic estimate of $v_c = 0.6$ m/sec for the rms Na velocity in the cooling limit. Assuming that the direction of the velocity is randomized during the damping time, we suppose that the atom executes a random walk with a step size of 3.6 μm. In order to diffuse a distance of 1 cm requires roughly 7×10^6 steps, or about 40 sec.

[*Note added since preparation of manuscript*: A group at AT&T Bell Laboratories recently demonstrated the "optical molasses" effect.[90] After chirp cooling an atomic Na beam using the NBS–JILA technique (Subsection 3.A above) they were able to contain the atoms for times of the order of 100 msec by using intersecting laser beams a few millimeters in diameter. They find their measured temperature of 240^{+200}_{-60} μK to be consistent with the cooling limit for their conditions. Reference 90 also contains a careful treatment of the diffusion of atoms in the molasses, including the effects of boundary conditions.]

Migdall[91] has proposed an intriguing application for the molasses trap consisting of six intersecting laser beams in the center of a cell filled with Na vapor at room temperature. A small fraction of the atoms in the vapor have velocities low enough (<15 m/sec) to be captured by the molasses trap and damped to the cooling limit. The rate of capture by the trap is $nvA/4$, where n is the density of atoms with sufficiently low velocity, v is their average velocity, and A is the surface area of the trap. The rate of loss from the trap is $n'V/\tau_{\text{dif}}$, where n' is the atomic density in the trap, V is the trap volume, and τ_{dif} is the diffusion time out of the trap. This diffusion time is roughly $\tau_{\text{dif}} \cong r^2/(v_c^2\tau_{\text{cool}})$, where r is the trap radius and v_c is the velocity of the cooled atoms in the trap. Equilibrium is established when

$$\frac{nvA}{4} = \frac{n'V}{\tau_{\text{dif}}} \tag{7}$$

or

$$\frac{n'}{n} = \frac{vA\tau_{\text{dif}}}{4V} = \frac{3vr}{4v_c^2\tau_{\text{cool}}}, \tag{8}$$

which is about 5×10^4 for Na in a 1-cm-radius region. That is, the molasses trap concentrates the density of slow Na atoms by almost 5 orders of magnitude, while reducing their velocity to the cooling limit.

At room temperature, where the Na vapor density is about 10^5 cm^{-3} (Ref. 92) and the rms velocity is about 500 m/sec, the density of atoms below 15 m/sec is about 2 cm^{-3}. The molasses trap would concentrate this to 10^5 cm^3 at velocities around 0.6 m/sec. That is, the density of very cold atoms inside the trap would be about equal to the total density outside. This density is comparable with what we can achieve by laser cooling of an atomic beam but can be accomplished continuously in a cell. Such an accumulation of atoms could be used to load other traps or as a source of very cold atoms.

The above estimates do not include effects due to varying penetration of the vapor atoms into the molasses trap—the more plentiful faster atoms will penetrate farther, while the more scarce slow atoms will remain near the surface. Extra heating effects that are due to dipole forces are not included, nor is the effect of the Gaussian intensity distribution of the intersecting beams. In addition, imbalance in the intensity of the opposed beams will lead to an expelling force in the direction of the stronger beam and a finite average drift velocity for the atoms. For a small fractional imbalance of δ, half-linewidth detuning, and small saturation parameter, the drift velocity is $\delta\ \gamma/4k$. A 0.3% misbalance, which is typical of the reflection loss in a good mirror, results in a drift velocity of 0.5 cm/sec and a time to leave the trap of only 2 sec.

Another limiting factor is scattering of the cooled atoms by fast vapor atoms, which can pass through the molasses trap unimpeded. It appears that if the Na vapor density is increased to the point where scattering reduces the lifetime in the trap to a few seconds (about 10^8 cm^{-3}) the equilibrium density of very slow atoms will be about 10^6 cm^{-3}, with the lifetime limited by scattering or escape, owing to misbalance of the opposed beams. Reduction of the misbalance can increase the achievable density.

The strong damping provided by intersecting laser beams presents another interesting possibility for trapping atoms—the use of active feedback. Active feedback has long been used to trap macroscopic particles and has been suggested for single atoms (see Ref. 20, for example). But without the viscous action of the optical molasses, even cold atoms move too fast to provide sufficient signal-to-noise ratio (S/N) in the short response times required for a servo loop. Now, consider a single atom moving in a molasses trap as described above. The atom scatters about 10^7 photons per second, so a typical detection system with a combined collection and quantum efficiency of 10^{-3} can detect the presence of the atom with a S/N of 10 in 10 msec. During this time, the atom will diffuse or drift a distance considerably less than 1 mm. If multiple, position-sensitive detectors are used, one can determine whether the atom has moved less than 1 mm from the trap center with a S/N better than 10. The error signal generated could then be used to adjust the relative intensity of the intersecting laser beams so as to push the atom back toward the center. Some details of such a feedback trap have been considered by Edge.[93] The advantage of feedback is that the trap becomes stable and instabilities owing to imbalance of intensities are removed. The disadvantage is that the error signal is ambiguous if more than one atom is in the trap.

All the methods for electromagnetic trapping of neutral atoms have the difficulty that they strongly perturb the energy levels of the atoms. This presents severe difficulties if one wants to use the atoms for spectroscopy. One approach is simply to use the trap (or quasi-trap as in the molasses trap) as an accumulator and refrigerator for atoms that are later released and examined. Another approach is to turn the trapping fields on and off so as to keep the atoms trapped but provide perturbation-free times for observation. A related approach is to have a trap that has perturbation-free regions where the atoms can be observed, such as the evanescent-wave trap proposed by Cook,[94] but which has an extremely small depth. Hall and co-workers have considered schemes where atoms are reflected back and forth between "mirrors" formed by pulsed laser beams.[65]

7. CONCLUSIONS

The ability to produce significant densities of very slow neutral atoms by laser cooling, which has now been demonstrated in a number of laboratories, has opened possibilities for important applications, such as ultrahigh-resolution spectroscopy. Moreover, it now seems likely that the availability of slow atoms will allow atoms to be confined in stable or quasi-stable electromagnetic traps.

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REFERENCES

1. R. Frisch, Z. Phys. **86**, 42 (1933).
2. V. S. Letokhov, Pisma Zh. Eksp. Teor. Fiz. **7**, 348 (1968) [JETP Lett. **7**, 272 (1968)].
3. A. Ashkin, Phys. Rev. Lett. **40**, 729 (1978).
4. J. E. Bjorkholm, R. R. Freeman, A. Ashkin, and D. B. Pearson, Phys. Rev. Lett. **41**, 1361 (1978).
5. P. Moskowitz, P. L. Gould, S. R. Atlas, and D. E. Pritchard, Phys. Rev. Lett. **51**, 370 (1983).
6. A. P. Kazantsev, Usp. Fiz. Nauk **124**, 113 (1978) [Sov. Phys. Usp. **21**, 58 (1978)].
7. J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **2**, 1707 (1985).

8. S. Stenholm, Phys. Rep. **43,** 151 (1978). Section 6 contains an early review of mechanical effects of light.
9. R. J. Cook, Phys. Rev. Lett. **44,** 976 (1980).
10. R. J. Cook, Phys. Rev. A **22,** 1078 (1980).
11. J. P. Gordon and A. Ashkin, Phys. Rev. A **21,** 1606 (1980).
12. T. Hänsch and A. Schawlow, Opt. Commun. **13,** 68 (1975).
13. D. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. **20,** 637 (1975).
14. D. Wineland, R. Drullinger, and F. Walls, Phys. Rev. Lett. **40,** 1639 (1978).
15. W. Neuhauser, M. Hohenstatt, P. Toschek, and H. Dehmelt, Phys. Rev. Lett. **41,** 233 (1978).
16. D. Wineland and W. Itano, Phys. Rev. A **20,** 1521 (1979).
17. S. Stenholm, Phys. Rev. A **27,** 2513 (1983).
18. A. Ashkin, Phys. Rev. Lett. **24,** 156 (1970); **25,** 1321 (1970).
19. V. S. Letokhov and V. G. Minogin, Phys. Rep. **73,** 1 (1981). Much of the early work on laser trapping and cooling is reviewed in this paper.
20. A. Ashkin, Science **210,** 1081 (1980). This contains references to early work on both atom trapping and trapping of macroscopic particles.
21. N. Niehues, thesis, Universität Bonn (Bonn-IR-76-35) (1976), unpublished; B. Martin, thesis, Universität Bonn (Bonn-IR-75-8) (1975), unpublished. Martin saw some indications of magnetic trapping.
22. W. Wing, Phys. Rev. Lett. **45,** 631 (1980); Prog. Quantum Electron. **8,** 181 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 74 (1983).
23. W. Stwalley, Prog. Quantum Electron. **8,** 203 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 95 (1983).
24. D. E. Pritchard, Phys. Rev. Lett. **51,** 1336 (1983); Prog. Quantum Electron. **8,** 209 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 103 (1983).
25. J. Dalibard, S. Reynaud, and C. Cohen-Tannoudji, Opt. Commun. **47,** 357 (1983).
26. A. Ashkin, Opt. Lett. **9,** 454 (1984).
27. H. Metcalf, Prog. Quantum Electron. **8,** 169 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 59 (1983).
28. A. Kastler, J. Phys. Rad. **11,** 255 (1950).
29. N. Djeu and W. T. Whitney, Phys. Rev. Lett. **46,** 236 (1981).
30. P. R. Berman and S. Stenholm, Opt. Commun. **24,** 155 (1978).
31. E. Giacombino, *et al.*, Phys. Rev. A **28,** 2555 (1983); E. Giabobino and P. A. Berman, Prog. Quantum Electron. **8,** 215 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 112 (1983).
32. J. Zorn, presented at Ninth International Conference on Atomic Physics, Seattle, Wash., July 23–27, 1984 (preprint, R. S. VanDyck, Jr., and E. N. Fortson, eds., p. B22).
33. T. Breeden and H. Metcalf, Phys. Rev. Lett. **47,** 1726 (1981).
34. F. C. Hansen and K. Harvey, University of Michigan (personal communication, 1984).
35. V. S. Letokhov, V. G. Minogin, and B. D. Pavlik, Opt. Commun. **19,** 72 (1976). This proposal was made in connection with cooling of trapped atoms.
36. V. G. Minogin, Opt. Commun. **34,** 265 (1980).
37. T. V. Zueva, V. S. Ketokhov, and V. G. Minogin, Zh. Eksp. Teor. Fiz. **81,** 84 (1981) [Sov. Phys. JETP **54,** 38 (1981)].
38. I. V. Krasnov and N. Ya Shaparev, Zh. Eksp. Teor. Fiz. **77,** 899 (1970) [Sov. Phys. JETP **50,** 453 (1979)].
39. L. Moi, Opt. Commun. **50,** 349 (1984); J. Liang, L. Moi, and C. Fabre, Opt. Commun. **52,** 131 (1985).
40. T. V. Zueva and V. G. Minogin, Pis'ma Zh. Tekh. Fiz. **7,** 953 (1981) [Sov. Tech. Phys. Lett. **7,** 411 (1981)].
41. A. Migdall, National Bureau of Standards, Gaithersburg, Md. (personal communication, 1983).
42. J. Javanainen, M. Kaivola, U. Nielsen, O. Poulsen, and E. Riis, J. Opt. Soc. Am. B **2,** 1768 (1985).
43. J. J. Snyder, National Bureau of Standards, Gaithersburg, Md. (personal communication, 1981).
44. V. I. Balykin, V. S. Letokhov, and V. I. Mushin, Pis'ma Zh. Eksp. Teor. Fiz. **29,** 614 (1979) [JETP Lett. **29,** 560 (1979)].
45. V. I. Balykin, V. S. Letokhov, and V. I. Mushin, Zh. Eksp. Teor. Fiz. **78,** 1376 (1980) [Sov. Phys. JETP **51,** 692 (1980)].
46. V. I. Balykin, Opt. Commun. **33,** 31 (1980).
47. S. V. Andreev, V. I. Balykin, V. S. Letokhov, and V. G. Minogin, Pis'ma Zh. Eksp. Teor. Fiz. **34,** 463 (1981) [JETP Lett. **34,** 442 (1981)].
48. V. I. Balykin, V. S. Letokhov, and A. I. Sidorov, Opt. Commun. **49,** 248 (1984).
49. V. I. Balykin, V. S. Letokhov, and A. I. Sidorov, Zh. Eksp. Teor. Fiz. **86,** 2019 (1984).
50. W. D. Phillips and H. Metcalf, Phys. Rev. Lett. **48,** 596 (1982).
51. J. V. Prodan, W. D. Phillips, and H. Metcalf, Phys. Rev. Lett. **49,** 1149 (1982).
52. W. D. Phillips, J. V. Prodan, and H. J. Metcalf, Prog. Quantum Electron. **8,** 119 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 1 (1983).
53. M. S. Lubell and K. Rubin, Prog. Quantum Electron. **8,** 223 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed. Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 125 (1983).
54. A. Migdall, J. Prodan, W. Phillips, T. Bergeman, and H. Metcalf, Phys. Rev. Lett. **54,** 2596 (1985).
55. H. Metcalf, W. Phillips, and J. Prodan, Bull. Am. Phys. Soc. **29** (795) 1984.
56. V. I. Balykin, V. S. Letokhov, V. G. Minogin, and T. V. Zueva, Appl. Phys. B **35,** 149 (1984).
57. V. I. Balykin, V. S. Letokhov, and A-I. Sidorov, Pis'ma Zh. Eksp. Teor. Fiz. **40,** 251 (1984).
58. J. Prodan, W. D. Phillips, J. Dalibard, and H. Metcalf, Bull. Am. Phys. Soc. **29** (822) 1984.
59. J. V. Prodan, A. Migdall, W. D. Phillips, I. So, H. Metcalf, and J. Dalibard, Phys. Rev. Lett. **54,** 992 (1985).
60. J. V. Prodan and W. D. Phillips, Prog. Quantum Electron. **8,** 231 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 137 (1983).
61. W. D. Phillips and J. V. Prodan in *Coherence and Quantum Optics V*, L. Mandel and E. Wolf, eds. (Plenum, New York, 1984), p. 15; W. D. Phillips, J. V. Prodan, and H. J. Metcalf, in *Laser Spectroscopy VI*, H. P. Weber and W. Luthy, eds. (Springer-Verlag, Berlin, 1983), p. 162.
62. W. D. Phillips, Appl. Opt. **20,** 3826 (1981).
63. R. Blatt, W. Ertmer, and J. L. Hall Prog. Quantum Electron. **8,** 237 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 142 (1983).
64. W. Ertmer, R. Blatt, J. Hall, and M. Zhu, Phys. Rev. Lett. **54,** 996 (1985).
65. W. Ertmer, R. Blatt, C. G. Aminoff, M. Zhu, C. Salamon, and J. Hall, J. Opt. Soc. Am. B (to be published).
66. J. C. Berquist, R. L. Barger, and D. J. Glaze, in *Laser Spectroscopy IV*, H. Walther and K. Rothe, eds. (Springer-Verlag, Berlin, 1979), p. 120; R. L. Barger, Opt. Lett. **6,** 145 (1981).
67. V. S. Letokhov and V. G. Minogin, J. Phys. **42,** C8-374 (1981).
68. W. Ertmer, R. Blatt, and J. Hall, Prog. Quantum Electron. **8,** 249 (1984); in *Laser-Cooled and Trapped Atoms*, W. D. Phillips, ed., Nat. Bur. Stand. (U.S.) Spec. Publ. **653,** 154 (1983).
69. R. Cook, Opt. Commun. **35,** 347 (1981).
70. O. Stern, Z. Phys. **7,** 249 (1921); W. Gerlach and O. Stern, Ann. Phy. **74,** 673 (1924); **76,** 163 (1925).
71. H. Friedburg and W. Paul, Naturwissenschaften **38,** 159 (1951); Z. Phys. **130,** 493 (1951).
72. C. V. Heer, Rev. Sci. Instrum. **34,** 532 (1963).
73. V. V. Vladimirskii, Zh. Eksp. Teor. Fiz. **39,** 1062 (1960) [Sov. Phys. JETP **12,** 740 (1961)].
74. K. J. Kugler, W. Paul, and U. Trinks, Phys. Lett. **72B,** 422 (1978).
75. W. Paul and U. Trinks, Inst. Phys. Conf. Ser. No. 42, p. 18 (1978).
76. The placement of the coils at 30-deg latitudes, as indicated in some papers, does not produce zero field at the origin unless the currents are unequal. It is a simple exercise to determine the optimal placements and currents for particular application.
77. W. Paul, Universität Bonn (personal communication, 1985).
78. T. Bergeman and H. Metcalf, presented at the Ninth Interntional Conference on Atomic Physics, Seattle, Wash., July 23–27, 1984 (preprint, R. S. VanDyck, Jr., and E. N. Fortson, eds.)
79. A. Ashkin and J. P. Gordon, Opt. Lett. **4,** 161 (1979).
80. J. Dalibard, S. Reynaud, and C. Cohen-Tannoudji, Opt. Commun. **47,** 395 (1983).

81. J. Dalibard, S. Reynaud, and C. Cohen-Tannoudji, J. Phys. B **17,** 4577 (1984).
82. J. Dalibard, École Normale Supérieure, Paris (personal communication, 1984).
83. W. Neuhauser, M. Hohenstatt, P. E. Toschek, and H. Dehmelt, Phys. Rev. A **22,** 1137 (1980).
84. V. G. Minogin, Kvantovaya Electron. (Moscow) **9,** 505 (1982) [Sov. J. Quantum Electron. **12,** 299 (1982)].
85. V. G. Minogin and J. Javanainen, Opt. Commun. **43,** 119 (1982).
86. A. Ashkin and J. P. Gordon, Opt. Lett. **8,** 511 (1983).
87. E. Fischer, Z. Phys. **156,** 1 (1959); R. Wuenker, H. Shelton, and R. V. Langmuir, J. Appl. Phys. **30,** 342 (1958).
88. H. Dehmelt, Adv. At. Mol. Phys. **3,** 53 (1967).
89. J. Dalibard and W. D. Phillips, Bull. Am. Phys. Soc. **30,** 748 (1985).
90. S. Chu, L. Hollberg, J. E. Bjorkholm, A. Cable, and A. Ashkin, Phys. Rev. Lett. **55,** 48 (1985).
91. A. Migdall, National Bureau of Standards, Gaithersburg, Md. (personal communication, 1985).
92. W. Fairbank Jr., T. Hänsch, and A. Schawlow, J. Opt. Soc. Am. **65,** 199 (1975).
93. C. Edge, University of Virginia (personal communication, 1985).
94. R. Cook and R. Hill, Opt. Commun. **43,** 258 (1982).