Appl. Phys. 17, 99—103 (1978)

Applied Physics

Trapping and Storage of Atoms in a Laser Field

V. S. Letokhov and *V. G. Minogin*

Institute of Spectroscopy, Academy of Sciences USSR, SU-142092 Troizk, Moscow, USSR

Received 20 January 1978/Accepted 15 February 1978

Abstract. Analysis of the possibility of spatial trapping of cold atoms in a standing-wave laser field is presented. It is shown that cold atoms can be trapped for a long time in region $\sim\lambda$ in a nonresonant standing-wave field. In a resonant standing-wave field cold atoms can be stored for a long time in the region determined by the cross-section of the laser beam. Trapping and storage together with cooling of atoms that has been suggested earlier allow us in specific cases to increase the sensitivity and the resolution of spectroscopic studies.

PACS: 32

Confinement of spatial motions of atoms by light pressure is of considerable interest for solution of the two basic problems of laser spectroscopy. Firstly, the sensitivity of spectroscopic researches can be drastically increased by spatial limitation of the atomic motion in the light field. Really, if light pressure holds atoms in a restricted region for a long time so that atoms may not leave the field, it makes long spectroscopic measurement with a small number of atoms possible. In principle, such an approach permits us to investigate individual atoms, and thus to achieve the ultimate sensitivity of optical spectroscopy.

Secondly, the resolution of spectroscopic investigations can be drastically increased by limiting the atomic motion in the region with sizes $\sim\lambda$ (λ being the wavelength of light). The idea of such an approach has been proposed in [1a] and the holding of atoms in the region $\sim\lambda$ was called "the trapping of atoms". The main point of this approach is the usage of a standing light wave for holding atoms in the minima of periodical field potential and thus eliminating the Doppler broadening of spectral lines. Trapping of ions in a rf field has been analysed and achieved in the past [1b]. Detailed classical analysis showed [2] that such a method in case of a three-dimensional standing light wave allows us to observe narrow spectral lines with the natural width of the atom.

It is easy to show that a light field can hold only very slow atoms. In the light field $\mathscr{E}=\mathscr{E}(z,t)$ the force of light pressure acting on the atom has the order $d\nabla\mathscr{E}$ [3], where d is the atomic dipole moment. The work of light pressure in the order of magnitude equals the interaction energy between atoms and the field $U=d\mathscr{E}$. Therefore, both in a macroscopic region and in the region with sizes $\sim\lambda$ the light pressure can hold only atoms having the kinetic energy of $E\lesssim U$. For nonresonant light field even at a very high intensity of continuous laser radiation of $I\simeq 10^4\,\mathrm{W/cm^2}$ the maximum energy of confined atoms is as small as $U=0.25\alpha\mathscr{E}^2\simeq 10^{-10}\,\mathrm{eV}$ due to the small value of nonresonant polarizability $\alpha\simeq 310^{-24}\,\mathrm{cm^3}$. For a resonant light field at the intensity which is enough for resonant transition saturation ($I\simeq 0.1\,\mathrm{W/cm^2}$), the maximum energy of confined atoms is $U\simeq 10^{-7}\,\mathrm{eV}$ (for $d\simeq 5D$).

Thus, direct usage of light pressure for holding atoms in a low-pressure gas at room temperature is unsuccessfull. However, the method of cooling atoms [4] by resonant light pressure allows in some cases to eliminate this difficulty. The analysis showed [5, 6] that in a resonant field of a three-dimensional standing light wave of a tunable frequency at the intensity $I\simeq 0.1\,\mathrm{W/cm^2}$, atoms can be cooled by a light field down to energies of $E\simeq\hbar\Gamma\sim 10^{-8}-10^{-7}\,\mathrm{eV}$. It is possible for the two level atoms, the upper level of which has a probability 2Γ, decays only into the ground level. The possibility of a strong radiative cooling of atoms in a low-pressure gas allows us to realize the spatial trapping of cold atoms by light pressure.

0340-3793/78/0017/0099/$01.00

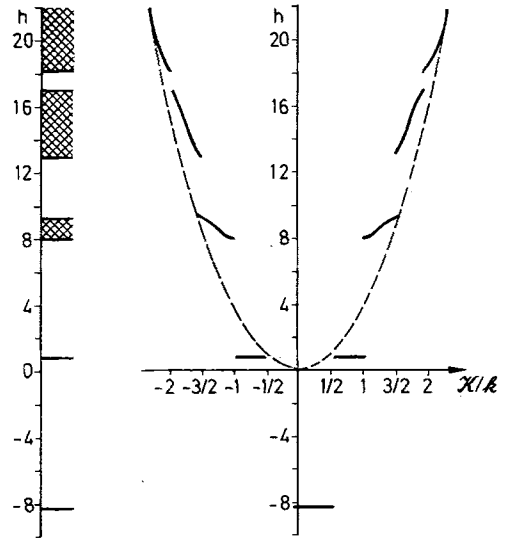

Fig. 1. Structure of allowed energy bands (on the left) and the dispersion law $h=h(\mathscr{K}/k)$, $h=(E+U_0)/R$ for an atom in nonresonant standing-wave field with $U_0=13.2R$

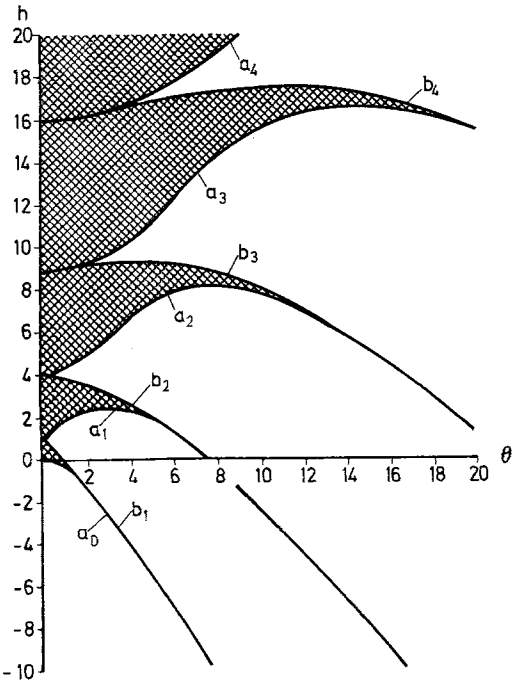

Fig. 2. Dependence of structure of allowed energy bands for an atom in a nonresonant standing-wave field on the interaction energy of an atom with the field. Energies are normalized to the recoil energy: $h=(E+U_0)/R$, $\theta=U_0/2R$

The purpose of this paper is the quantum-theoretical analysis of motions of atoms in standing light waves. This analysis shows that trapping of atoms can be realized only in a nonresonant standing light wave. It also shows that the effective storage of atoms in a light field is more easily realized in a resonant standing light wave. In conclusion, evaluations of the storage time of atoms in laser fields are presented.

1. Trapping of Atoms. Nonresonant Standing Light Wave

Let us discuss stationary atomic motion in a resonant standing light wave

$$\mathscr{E}(z,t)=2\mathscr{E}_0\cos\omega t\cos kz. \tag{1}$$

Schrödinger's equation for a spatial part of the atomic wave function in the field (1) has the form

$$\frac{d^2\psi(kz)}{d(kz)^2}+\left[\frac{E+U_0}{R}+\frac{U_0}{R}\cos 2kz\right]\psi(kz)=0, \tag{2}$$

where E is the kinetic energy, $U_0=\alpha\mathscr{E}_0^2/2$ is the average interaction energy of the atom with the field, and $R=\hbar^2k^2/2M$ is the recoil energy.

Equation (2) is the well known Mathieu equation [7]. All the basic properties of atoms in the field (1) expressed by this equation.

According to (2), the atomic eigenstates in a nonresonant standing light wave are the states with a certain wavevector $\mathscr{K}$. To each wavevector $\mathscr{K}$ corresponds an eigenfunction

$$\begin{aligned}\Psi_{\mathscr{K}}(z,t)&=\psi_{\mathscr{K}}(z)\exp[-iE(\mathscr{K})t/\hbar]\\&=w^{\mathscr{K}}(z)\exp[i\mathscr{K}z-iE(\mathscr{K})t/\hbar],\end{aligned} \tag{3}$$

where the functions $w^{\mathscr{K}}(z)$ are periodic with the spatial period $\Delta z=\pi/k=\lambda/2$, and the eigenvalue $E(\mathscr{K})$ is determined by the value of wavevector $\mathscr{K}$. Thus eigenvalues in a nonresonant standing light wave are described by the Bloch functions.

From the properties of eigenvalues of the Mathieu equation, we can find out further that the eigenvalues $E(\mathscr{K})$ always belong to certain allowed energetic bands. The multitude of eigenvalues $E(\mathscr{K})$ forms a structure of allowed energy bands, as shown in the left-hand side of Fig. 1.

At last, the dispersion law—dependence of $E=E(\mathscr{K})$—for every allowed energy band is determined by the values of characteristic indices of the Mathieu equation, and can be calculated numerically (Fig. 1).

Having analysed stationary motions of atoms in the field (1) one can clarify the condition at which atoms can be trapped in the nodes or antinodes of the standing waves. It is well known that the vibration of atoms for a long time in a periodic potential means that atoms are strongly coupled with the field [8]. At strong coupling the frequency of vibrations of atoms in the potential well considerably exceeds the diffusion probability of an atomic wave packet through the periodic potential barrier. In this case the energy spectrum consists of narrow allowed energy bands devided by wide forbidden bands. Figure 2 presents the dependence of the energy spectrum of atoms in a nonresonant standing wave field on the interaction

energy (stability regions of Mathieu's equation). Borders of allowed energy bands are marked by the corresponding eigenvalues of Mathieu's equation, a_n and b_n. On the base of the spectral properties we may conclude that the condition $U_0 \gg R$ is a criterion of strong coupling. This condition, taking the tunnelling effect into consideration, is more valid than the condition $U_0 > R$ found in [1a, 2] by a classical calculation.

When $U_0 \gg R$ the coefficient of the atomic tunneling from one potential well into the neighbouring one can be found by using a quasiclassical approximation

$$D \simeq \exp\left[-4\left(\frac{2U_0}{R}\right)^{1/2}\right]. \tag{4}$$

Then taking (4) into consideration, the probability of tunnelling from the region $\Delta z \simeq \lambda/2$ is determined by the product of frequency of vibrations of the atom in the well by the tunnelling coefficient D

$$P \simeq \frac{2}{\hbar}(2U_0R)^{1/2}\exp\left[-4\left(\frac{2U_0}{R}\right)^{1/2}\right]. \tag{5}$$

2. Trapping of Atoms. Resonant Standing Light Wave

Let us now consider the stationary motion of an atom in a resonant standing light wave of the type described by (1). It is convenient to accomplish such analysis considering first only the induced transitions and then the induced and spontaneous transitions.

The problem of the motion of the atom in a resonant field (1) with only induced transitions is similar to the one considered above.

Eigenwave functions of stationary states of the atom in a resonant field of a standing light wave by taking consideration of only induced transitions have the form [9]

$$\Psi_{\mathcal{K}}(r, z, t) = [\psi_g(r) w_g^{\mathcal{K}}(z) e^{-i(\varepsilon_g + E_g)t/\hbar} + \psi_e(r) w_e^{\mathcal{K}}(z) e^{-i(\varepsilon_e + E_e)t/\hbar}] e^{i\mathcal{K}z}, \tag{6}$$

where the products $u_e = w_e^{\mathcal{K}}(z)\exp(i\mathcal{K}z)$ and $u_g = w_g^{\mathcal{K}}(z)\exp(i\mathcal{K}z)$ satisfy a set of equations on the eigenvalues

$$\begin{aligned} &\frac{d^2u_e}{dz^2} + \frac{2ME_e}{\hbar^2}u_e + \frac{4MV_0}{\hbar}u_g\cos kz = 0 \\ &\frac{d^2u_g}{dz^2} + \frac{2ME_g}{\hbar^2}u_g + \frac{4MV_0}{\hbar}u_e\cos kz = 0. \end{aligned} \tag{7}$$

In (6) and (7) $\psi_g(r)$, $\psi_e(r)$ and ε_g, ε_e are the eigenfunctions and energies of the ground "g" and the excited "e" states; $w_g^{\mathcal{K}}(z)$, $w_e^{\mathcal{K}}(z)$ and $E_g = E_g(\mathcal{K})$, $E_e = E_e(\mathcal{K})$ are

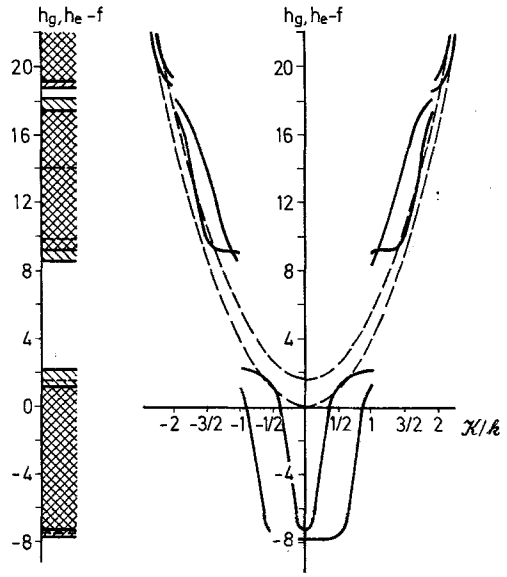

Fig. 3. Structures of allowed energy bands (on the left) and dispersion laws (on the right) for an atom in a resonant standing-wave field with $4\hbar\Omega = -1.6R$, and $4\hbar V_0 = 6.5R$. Energies are normalized to the recoil energy: $h_g = 4E_g/R$, $h_e = 4E_e/R$

spatial wavefunctions and kinetics energies of the atom in the "g" and "e" states. Functions $w_i^{\mathcal{K}}(z)$ (i = g, e) are periodic with the period $\Delta z = 2\pi/k = \lambda$, while the kinetic energies are connected by the relation

$$E_e(\mathcal{K}) - E_g(\mathcal{K}) = \hbar\Omega, \tag{8}$$

where $\Omega = \omega - \omega_0$ is the detuning of the frequency ω relative to the atomic transition frequency ω_0, $V_0 = d\mathscr{E}_0/2\hbar$ is the interaction energy of the atom with the field, and d is the dipole moment of the "g" – "e" transition.

Analysis of the set of equations (7) shows [10] that the atom with a wavevector $\mathcal{K}$ is in a superposed state in a standing light wave when $\Omega \neq 0$. Two energies $E'_g(\mathcal{K})$ and $E''_g(\mathcal{K})$ of spatial motion in the "g" state and two kinetics energies $E'_e(\mathcal{K}) = E'_g(\mathcal{K}) + \hbar\Omega$, $E''_e(\mathcal{K}) = E''_g(\mathcal{K}) + \hbar\Omega$ in the "e" state correspond to every wavevector $\mathcal{K}$. Figure 3 illustrates examples of dispersion curves $E_g = E_g(\mathcal{K})$ and $E_e = E_e(\mathcal{K})$ and the bands of allowed energies E_g and E_e. Systems of allowed energy bands E'_g, E'_e and E''_g, E''_e are shaded with different slopes.

Figure 4 presents the dependence of the energy spectrum of the atom in a resonant standing-wave field on the value of the interaction energy V_0 (for $\hbar\Omega = -0.4R$). The borders of the first system of allowed energy bands are marked by eigenvalues of the system (7) [10, 11]: $a_0^1, b_2^1, a_2^2, b_4^2, \ldots$; the borders of the other system by the eigenvalues: $a_0^2, b_2^2, a_2^1, b_4^1, \ldots$.

The analysis of the properties of energy spectrum shows that at nonzero detuning ($\Omega = 0$ is a peculiar

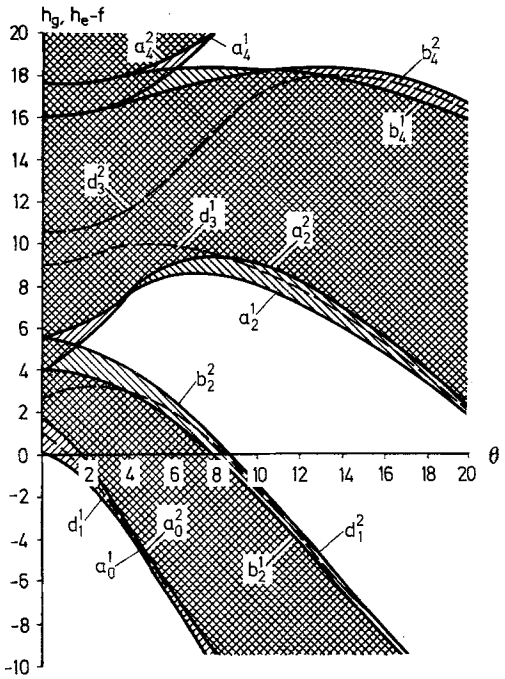

Fig. 4. Dependences of structure of allowed energy bands for an atom in a resonant standing-wave field on the interacting energy of the atom with the field

case which has no physical sense) widths of the allowed energy bands are always of the same order as the distance between the bands. Therefore, the case of strong coupling can not be realized in a resonant standing-wave field. In other words, the atom has generally equal probabilities both in the node and in the antinode of the field. Thus the atom can not be trapped into the region $\Delta z \sim \lambda$ in a resonant field of a standing light wave.

This conclusion is also valid when spontaneous emission in the field are involved. Really, the spontaneous decay leads only to transitions of atoms inside and between allowed energy bands and it can not change the type of the atom-field coupling for which induced transitions are completely responsible.

3. Storage of Atoms in a Spatially Inhomogeneous Field

An actual laser field is always limited by a finite aperture. Inhomogeneous transversal field distribution of a standing light wave causes a gradient force on the moving atom [12].

Let the one-dimensional standing light wave be formed by laser radiation of the TEM_{00q} mode. Then the field at the point r, z (r and z are cylindrical field coordinates) has the form

$$\mathscr{E}(r,z)=2\mathscr{E}_0(r)\cos\omega t\cos kz. \tag{9}$$

It is convenient to write the transversal field distribution in the form

$$\mathscr{E}_0(r)=\frac{2}{a}\left(\frac{2P_0}{c}\right)^{1/2}\exp\left(-\frac{r^2}{2a^2}\right), \tag{10}$$

where $2a$ is the effective diameter of the light beam, and P_0 is the radiation power of one travelling light wave.

Let us discuss the possibility of holding atoms for a long time (the storage of atoms) in a resonant field described by (9). In such a field the atom is acted upon by four forces. Two of them are forces of spontaneous light pressure and induced light pressure. They are determined by Ref. [5], Eqs. (2) and (3), if only E_0 is replaced by $\mathscr{E}_0(r)$. The other two forces are directed along the r vector and are caused by the interaction of the atomic dipole moment induced by the field with the field gradient in the beam cross-section

$$F_{gr}^s=\frac{1}{2}\hbar\Gamma\frac{r}{a^2}G(r)\cdot\frac{\mathscr{L}_- -\mathscr{L}_+}{1+G(r)(\mathscr{L}_-+\mathscr{L}_+)}\sin 2kz, \tag{11}$$

$$F_{gr}^i=\hbar\Omega\frac{r}{a^2}G(r)\cdot\frac{\left(1-\frac{kv}{\Omega}\right)\mathscr{L}_-+\left(1+\frac{kv}{\Omega}\right)\mathscr{L}_+}{1+G(r)(\mathscr{L}_-+\mathscr{L}_+)}\cos^2 kz, \tag{12}$$

where $G(r)=(d\mathscr{E}_0(r)/\hbar\Gamma)^2/2$, 2Γ is the probability of the spontaneous decay "e"→"g", and $\mathscr{L}_\pm=\Gamma^2/[(\Omega\pm kv)^2+\Gamma^2]$.

The structure of the forces F_{gr}^s and F_{gr}^i is similar to F_{sp} and F_{ind} [5]. A dipole moment which determines F_{gr}^i is caused by the induced transitions. A dipole moment which determines F_{gr}^s is caused by the induced and spontaneous transitions.

Dependences of F_{gr}^s and F_{gr}^i on the projection of atomic velocity on the z axis are similar to the corresponding dependences of F_{sp} and F_{ind} [5]. Magnitudes of F_{gr}^s and F_{gr}^i forces are $a/\lambda\simeq 10^4$ times smaller than the F_{sp} and F_{ind} forces.

Gradient forces, being week, are of some importance only for cold atoms. Only the average magnitude of the F_{gr}^i force affects practically the motion of cold atoms because the average magnitude of F_{gr}^s is zero. At negative detuning $\Omega=-|\Omega|$ this force is always directed inside the beam and holds cold atoms in the beam. Resonant light pressure accomplishes the strongest cooling of atoms at the detuning $\Omega=-\Gamma$ [6]. At such detuning the F_{gr}^i force holds all the atoms with energy $E\leqq E_{st}$ in the beam, where

$$E_{st}=\tfrac{1}{4}\hbar\Gamma\ln[1+2G(0)]. \tag{13}$$

This energy has the same value as the energy $E_c = \hbar\Gamma$ which cold atoms should have at the end of an optimum cooling process [6].

It is also possible to hold atoms in a nonresonant standing wave field in a macroscopic region. However, as it has been mentioned in the Introduction the holding effect in a nonresonant field is much weaker.

Conclusion

Thus spatial limitation of atomic motion in a microscopic region $\Delta z \sim \lambda$ is possible only in a nonresonant standing light wave. Holding of atoms in a macroscopic region of the light field may be of interest only for a resonant standing light wave.

Let us define the storage time of cold atoms in the region $\sim\lambda$. The main reasons for the atom to escape from the field nodes or antinodes of the nonresonant standing wave are tunnelling and collisions of the atom with other particles. Therefore, a considerable laser intensity and an extremely low pressure are required for a long storage time. For instance, with a laser radiation intensity $I = 10^4 \mathrm{W/cm^2}$ corresponding to an interaction energy of $U_0 = 20R \simeq 10^5$ Hz, the tunnelling time is $\tau = P^{-1} \simeq 1$ h. The collision time will be the same at a pressure of $p \simeq 10^{-10}$ Torr.

The storage time of atoms in a resonant light field is limited only by collisions. Therefore, the above-mentioned evaluation ($\tau \simeq 1$ h when $p \simeq 10^{-10}$ Torr) also defines the storage time of atoms in a macroscopic region. Naturally a resonant field will hold a large number of cold atoms only when the field intensity is sufficient to saturate the resonant transition: ($I \simeq 0.1 \mathrm{W/cm^2}$).

References

1a. V.S.Letokhov: Pis'ma Zh. Eksp. i Teor. Fiz. **7**, 348 (1968)
1b. E.Fischer: Z. Physik **156**, 1 (1959)
D.A.Church, H.G.Dehmelt: J. Appl. Phys. **40**, 3421 (1969)
2. V.S.Letokhov, B.D.Pavlik: Appl. Phys. **9**, 229 (1968)
3. L.D.Landau, E.M.Lifshits: *Theory of Field* (Fizmatgiz, Moscow 1960)
4. T.W.Hänsch, A. L. Schawlow: Opt. Commun. **13**, 68 (1975)
5. V.S.Letokhov, V.G.Minogin, B.D.Pavlik: Opt. Commun. **19**, 72 (1976)
6. V.S.Letokhov, V.G.Minogin, B.D.Pavlik: Zh. Esp. i Teor. Fiz. **72**, 1328 (1977)
7. N.V.Mac-Lachlan: *Theory and Applications of Mathieu Functions* (Moscow 1953)
8. *A.S.Davidov: Solid State Theory* (Nauka, Moscow 1976)
9. V.S.Letokhov, V.G.Minogin: Phys. Lett. **61**A, 370 (1977)
10. V.S.Letokhov, V.G.Minogin: Zh. Eksp. i Teor. Fiz. **74**, 1318 (1978)
11. V.A.Yakubovitch, V.M.Starzhinsky: *Linear Differential Equations with Periodical Coefficients and their Applications* (Nauka, Moscow 1972)
12. G.A.Askarjan: Zh. Eksp. i Teor. Fiz. **42**, 1567 (1962)