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Some comments on the two prism tunnelling experiment

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Abstract

We show that the two prism tunnelling experiment of Mizobuchi and Ohtake [Phys. Lett. A 168 (1992) 1] does not verify the quantum optical prediction due to insufficient statistical precision in the anticoincidence measurement. We reanalyze their data and show that the observed number of coincidences is actually even larger than what is expected from a classical coherent light source.

In an experiment "to throw more light on light", Mizobuchi and Ohtake [1] considered the (anti) coincidence rate of photons in two detectors kept in the path of tunnelled and reflected beams. This experiment was originally suggested by Ghose, Home and Agarwal [2] and in a later paper [3] these authors discussed the interpretation and implications of the experiment.

The two prism arrangement for the tunnelling experiment uses two right angled prisms with their larger, hypotenuse faces kept parallel, in close proximity, with typical separation comparable to the wavelength [1]. The air gap between these planes defines the tunnelling gap. When the tunnelling gap is arranged such that there is 50% tunnelling transmission and 50% reflection approximately, there are equal average counts in the photodetectors in the two paths. The rate of coincidences in the two detectors can be estimated for a classical, thermal or quantum source and the quantum optical prediction for a single photon source is zero for these coincidences. The nature of the source can be parametrized [4] by the quantity $\alpha = N_c N / N_r N_t$, where N_c is the rate of coincidences, N_r and N_t are the rate of singles in the reflected and transmitted beams and N is the number of gates over which the photodetectors are active. If the source is classical $\alpha \geqslant 1$ and the average rate of coincidence is given by $N_c \geqslant N_r N_t / N$. A classical coherent wave description would give a coincidence rate corresponding to $\alpha = 1$. Clearly, the quantum optical prediction for the coincidences when a single photon source is used, is zero identically. Of course, in an actual experiment even when a nearly single photon source is employed, noise would have to be considered which would give some number of coincidences and α would be a number much less than 1, but greater than zero. Essentially, to test the quantum optical prediction, the experiment should have sufficient statistical accuracy to distinguish between this nonzero value of α and $\alpha = 1$.

In the experiment of Mizobuchi and Ohtake (referred to as MO in this paper), the number of *anticoincidences* for different rates of singles in each de-

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tector was counted [1]. Since a pulsed Nd:YAG laser with a pulse rate of 5 Hz is used, $N = 5$ (the photodetectors can count only once during every pulse since their pulse pair resolution time is much larger than the pulse width) and the expected coincidence rate is very small. For example, for a singles rate of 1 photon/s, the expected number of coincidences when $\alpha = 1$ is 0.2/s. So, when counting for 5 seconds, they would see one coincidence and eight anticoincidences. (There are 10 photons in in the two detectors and one pair is required for the coincidence. The other eight will give anticoincidences. If only one preselected counter is used for generating the anticoincidence, as in their experiment, then the number of anticoincidences is 4 in this case.) Then the normalized anticoincidence rate, as defined by the ratio of the number of anticoincidences to the number of singles, is 0.8 for this example, for a classical wave. Similarly the expected classical coincidences can be calculated for other singles rates. It is then possible to compare the observed anticoincidence rate to the one predicted from the classical picture. We summarize such an analysis in Fig. 1 and it is clear that the result of MO is in conflict with the quantum optical prediction. First of all their data show that the observed anticoincidence rate is linear in the rate of singles, a prediction from the classical theory. Also, all observed anticoincidence rates are smaller than predicted for a classical wave, which means that there are more coincidences than predicted for $\alpha = 1$. Actually the data seem to fit a near thermal source of light [5] [3]. We have made an approximate estimate of the value of α from their plotted data, for singles count rates larger than 0.3/s, and we get $\alpha \simeq 1.5 \pm 0.6$ (this is only indicative, and the standard deviation may vary from 0.5 to 0.7 depending on the individual data points chosen for the estimate). This may point to some chaotic character in the light which is detected at the photodetectors, possibly reflecting the large fluctuations in the laser intensity. For smaller count rates, *the error on α exceeds the thermal value of 2*, and no meaningful conclusion can be deduced. In short, the experiment of MO does not confirm the quantum optical prediction in tunnelling. On the other hand, the data show an anomalously large number of

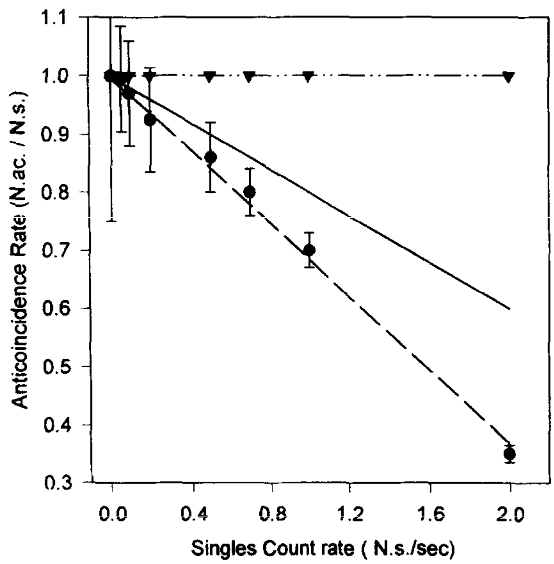

Fig. 1. Data from the experiment of Mizobuchi and Ohtake plotted on a linear scale. The data points with their error bars are from Ref. [1]. The top horizontal line and triangles are the quantum optical prediction. The unbroken line (which also approaches unity for low singles rate) is the classical prediction for coherent radiation and the broken line is a fit to the data. The insufficient statistical accuracy in the data and the role of intensity fluctuations are clear.

coincidences. Also, we note that *in the region of very small count rates, the error on the anticoincidence rates is too large to claim any meaningful test of the quantum optical prediction or to distinguish between various values of α (thermal, coherent, quantum etc.)*.

We do not believe that the data presented by MO represents any fundamentally important violation of the quantum optical prediction. In fact, recently MO have started another experiment in which the pump laser is a CW Ar ion laser, since they have realized that their data fits the anticoincidence rates predicted for light with thermal character and this may be overcome due to the stability of the Ar ion laser [5,6]. This expectation is certainly true, but even with the Ar ion laser the statistical accuracy from their preliminary experiment seems to be at about the 1σ level for the verification of the quantum optical prediction. We want to point out that this is due to the mode in which the experiment is done and *there is a basic inadequacy in using the anticoincidence mode in this experiment*; to count the number of anticoincidences rather than the much smaller number of coincidences. *If there is no*

[3] The anticoincidence mode is, however, retained in this modified experiment as well. The preliminary results are published in Ref. [6].

noise at all, these two measurements represent identical information for the same amount of counting time. But, in the presence of random noise, the situations are vastly different and it is much more advantageous to look at the coincidence rate in this particular case. Since it is worth discussing this point in some more detail we now consider the signal to noise ratio in the two situations, in the presence of random noise.

If N_s is the average singles rate at each detector and N_c the coincidence rate, then the anticoincidence rate is $N_{ac} = N_s - N_c$, since there will be an anticoincidence for counts at either detector which are not coincident. For the individual count rates the mean error on the count rates is given by the square root of the number of counts. (When the photon statistics itself is sub-Poissonian as in the case of quantum light, there would be a correction to this, and the mean error is typically smaller than the square root of count rates. But in actual cases, in the raw data, this correction factor, $\delta \equiv 1 - (\langle n \rangle - \langle \Delta n^2 \rangle)/\langle n \rangle$, is of order unity and may be ignored in the present discussion. Strictly speaking, it is necessary to determine the statistics of photon arrival in individual experiments to account for this correction factor if a high precision is required, and if the anticoincidence mode is used to resolve a small signal of coincidences.) In an experiment of duration T, the counts would be $N_s T \pm \sqrt{N_s T}$ for singles. In the ideal case, the error on coincidence or anticoincidence counts can be estimated using the error propagation formula for the functional dependence of these counts on the singles, given by the relation $N_c = \alpha N_r N_t / N \simeq \alpha N_s^2 / N$, assuming approximately equal singles rate at the two counters. The anticoincidence rate then is $N_{ac} = 1 - N_c/N_s$. The quantities of interest are N_c/N_s and N_{ac}/N_s (these are the same as n_c/n_s and n_{ac}/n_s, where n_c, n_{ac}, and n_s are total counts obtained by multiplying the rates by the counting time T). For low singles count rates as in these experiments, the number of coincidences are very small compared to singles counts and $n_{ac} \simeq n_s$ since $n_c \ll n_s$. Since $N_c \ll N_{ac}$ in the situation under consideration, $N_{ac} \simeq N_s$ and $N_{ac} T \pm \sqrt{N_{ac} T} \simeq N_{ac} T \pm \sqrt{N_s T}$. (In the experiment of MO, $(N_s - N_{ac})/N_s$ is less than 15% for $N_s \simeq 0.5/\mathrm{s}$, and less than 3% for $N_s \simeq 0.1/\mathrm{s}$.) This is the root of the problem, since the error in the anticoincidence count rate is decided by the singles rate which is numerically large compared to the coincidence rate, whereas the *signal* (departure of the anticoincidence rate from unity) is provided by the small coincidence rate. Since both N_c and N_{ac} are normalized to the singles rate, the errors on both the quantities also are normalized to the singles and consequently what is important for a comparison of accuracies obtainable is the absolute error on these two quantities. With $n_{ac} \simeq n_s$, the random error on the anticoincidence counts is approximately $\sqrt{n_s}$. The noise on the coincidence counts can be estimated as $\alpha N_s \sqrt{2 n_s}/N$. Clearly the absolute error on coincidence counts is smaller by a factor of $\alpha N_s \sqrt{2}/N$ compared to that in anticoincidence counts. This shows that the coincidence mode is much better than the anticoincidence mode in this experiment, since this factor can easily be 10 or more.

In reality, due to randomness arising from limited efficiency of detectors, other random noises etc., the error on the coincidence counts is seen to be more like $\sqrt{n_c}$ itself, which can be larger than the error we estimated, but it is still much smaller than $\sqrt{n_s}$, which is the error on the anticoincidences. Therefore, the conclusion that coincidence mode is much more advantageous statistically remains valid. It is as if the error on the coincidence rate is amplified by a factor $\sqrt{N_s/N_c}$, and the required counting time increased by the factor N_s/N_c. If the experiment is done in the coincidence mode, then the time needed for acquiring a statistically significant amount of data is estimated from the condition that $\sqrt{n_c} \ll n_c$. The criterion in the anticoincidence mode is different, since the expected number of coincidences is a small fraction of the singles, and therefore the departure of the ratio of the expected number of anticoincidences to the total number of singles from 1 is much smaller than what is given by the fractional statistical error on the anticoincidence rate. In other words, to compare the experimental result with the classical prediction, *the required statistical accuracy in the anticoincidence mode is much more stringent than in the coincidence mode.* Quantitatively, the requirement can be stated as the minimum counting time required for a signal to noise ratio of 1 and this is given approximately by the inequality $N_c T > \sqrt{N_{ac} T}$, since we *require that the statistical error on the anticoincidence count is smaller than the expected coincidence counts.* Therefore we get for the minimum counting time, $T > N_{ac}/N_c^2 \simeq N_s/N_c^2$. Of course, the required counting time for a reasonable statistical significance could be about 10 times this minimum requirement in practice.

From the discussion above, it is clear why the experiment of MO was not sensitive enough to distinguish between the classical and quantum predictions, for low count rates, contrary to general belief. For example, for singles count rates of the order of 0.1 counts/s, the expected classical coincidence rate is 2×10^{-3}/s in their experiment. Then the minimum counting time required in the anticoincidence mode would be larger than $N_s/N_c^2 = 3\times10^4$ s, whereas their typical counting times are an order of magnitude smaller. For a statistical significance of 3σ, this would increase to around 3×10^5 s. For their lowest count rate point, which is around 1.5×10^{-2} counts/s, the coincidence rate is smaller than 5×10^{-5}/s. Then the minimum counting time required in the anticoincidence mode is much larger than 10 million seconds! In the coincidence mode, the same statistical accuracy would be achieved for a counting time of about 10^5 seconds, which itself is considered difficult, though manageable. In the region where the singles rate is larger in their experiment, say 1 count/s, the expected coincidence rate is 0.2/s, and the required counting time is only a few hundred seconds (the actual counting times in the experiment of MO are around 2×10^3 to 4×10^3 s [5]). The observed coincidence rate, as deduced from their plot, is 0.3/s, a factor of 1.5 larger than the classical wave prediction (corresponding to $\alpha = 1.5$). So, in this region *they do have enough sensitivity and their result does not verify the quantum optical prediction.* As stated earlier, their observed anticoincidence rate is smaller than the classical prediction, implying a larger coincidence rate than the classical prediction.

It is clear that the ideal way to do such an experiment is to use a relatively stable CW laser for the down conversion since the intensity fluctuations would be smaller and the count rates would be orders of magnitude higher. It is possible to do the experiment even with a pulsed laser if counts are taken for sufficiently long times in the coincidence mode. In fact we have recently completed one such experiment [7] [4] in which we employed a birefringent crystal to split the single photon beam into two beams which show smaller coincidences than classically expected. A similar issue has been addressed earlier by the experiment in Ref. [4], in which photons from atomic cascade were split into two beams at a multilayer dielectric coated beam splitter. We are testing the quantum optical prediction for a variety of situations involving birefringence, refraction and tunnelling all of which have some importance to the issue of interpretation [2,8] regarding wave particle duality and complementarity.

[4] These experiments use a 10 Hz, Nd:YAG laser. The typical counting time needed for each data point with statistical error smaller than 30% exceeds 10^4 s.

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