Selection of Homework Questions

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Topic 1: History & Preliminaries

(1) Quick estimates using "psm" units (see : Topic 1.3d)

a. A globular cluster moves at 150 km/s on a circular orbit of radius 25 kpc.

(i) What's the period of the cluster?
(ii) What's the mass and mean density interior to its orbit?
(iii) If the cluster has radius of ~10pc and internal velocity dispersion ~10 km/s, what's its mass density?

b. What, roughly, are the speeds and periods of:

(i) comet nuclei in the Oort cloud, ~¼ pc from the sun.
(ii) gas in an accretion disk 1000 AU from a $10^8 M_\odot$ black hole.
(iii) stars orbiting in the galactic center cluster (distance 8.5 kpc) of radius ~30 arcsec and density ~$10^6\ M_\odot\ pc^{-3}$

c. Cosmological estimates are often easy using psm units:

(i) Hubble's original estimate was H_o = 530 km $s^{-1} Mpc^{-1}$. What cosmic age does this imply?
(ii) For H_o = 72 km s^{-1} Mpc^{-1}, what's the Universe's critical density in $M_\odot\ pc^{-3}$ (use $\rho_c = 3H_o^2/8\pi G$).
(iii) How much denser than the critical density is a typical galaxy (radius ~ 10 kpc, V_{rot} ~ 250 km/s) and a typical galaxy cluster (radius ~1 Mpc and velocity dispersion ~500 km/s).

d. Consider the formation of giant elliptical via monolithic collapse of a protogalactic gas cloud:

(i) What's the total kinetic energy (KE) of a giant E galaxy with L ~ 10^{11} $L_\odot$, σ ~ 300 km s^{-1} and M/L ~ 10?

(ii) From the virial theorem, the galaxy's gravitational binding energy (BE) is equal to its KE, which was liberated when it formed. If it collapsed on its current dynamical timescale, what was the "collapse luminosity", L_{BE}, in PLU and $L_\odot$?

(iii) Is this significant compared to the luminosity of the associated starburst (which would be classified as a "ULIRG" -- or Ultra-Luminous Infrared Galaxy)?

(2) Magnitudes and Mass to Light Ratios :

a. Apparent magnitude, m, can be defined as m = const - 2.5 log (flux), and absolute magnitude, M, can be defined as the apparent magnitude if the object is 10 pc distant. From these definitions, show that the distance modulus, m - M, is given by:

$$m - M = -5 + 5 \log_{10} d_{pc}$$

b. If I_V is the V band surface brightness measured in $L_{V,\odot}\ pc^{-2}$ and μ_V is the corresponding V band surface brightness measured in V mag $arcsec^{-2}$ (mag/ss), show that

$$\mu_V = 26.39 - 2.5 \log(I_V)$$

Derive similar relations for B, I, and bolometric. **These are potentially very useful relations.** (B&M Q2.2)

Hint: consider placing the object at a distance such that 1 pc = 1 arcsec. You will also need to use the solar absolute magnitudes given in the Lecture notes: Topic 1.3e .

c. The elliptical galaxy NGC 1399 has a central surface brightness of μ_V ~ 16.0 mag/ss. What is the corresponding surface brightness I_V, in $L_{\odot,V}\ pc^{-2}$?

d. If most of the central light comes from a core with radius ~1.0 arcsec, and velocity dispersion ~150 km/s, estimate the core's luminosity density (in $L_{\odot,V}\ pc^{-3}$) and mass density (in $M_\odot\ pc^{-3}$), and hence its M/L_V ratio. Take the redshift to be 1350 km/s and include "h" in your expressions and answers.

(3) You outshine the stars!

Putting mass-to-light ratios into physical units can be surprising:

a. What's M/L_{bol} for the sun, in units of kg/Watt? About 50% of the sun's mass is in its nuclear burning core. What's the M/L_{bol} of this "nuclear furnace"? Think about this value: does it jive with the cliche of the sun as a "roaring fusion furnace"? You should conclude that stars are **not** luminous because they're intrinsically luminous **per kg** -- indeed they are quite feeble in that sense. They are luminous only because their furnaces are so **massive**.

b. To emphasize this, estimate **your own** M/L_{bol} ratio, using the same units. Assume you weigh 100 kg and radiate like a black body of area 2 m^2 at 300K -- not unreasonable.

c. Imagine a big ball of 2×10^{28} people -- a bizarre and fairly unpleasant notion -- it has about the same mass and size as the sun. Assume that people survive until they starve (they don't eat each other!) so that the "lifetime" of this human star is about 1 week.

(i) What would its luminosity be, expressed in Watts and in $L_{\odot}$?

(ii) Compare the **total** energy liberated by the sun and the human star integrated over their respective lifetimes.

(iii) Compare this ratio to the ratio of typical outer-electron binding energies (driving chemistry) with typical nuclear binding energies (driving fusion).

Notice: solar type stars live so long for **two** reasons: (a) their fuel is indeed very energy rich; but (b) they burn it at an extremely frugal rate -- their furnaces are surprisingly feeble, per kg, well below even to your own metabolic rate.

d. Finally, use the stellar mass-luminosity relation ($L \propto M^{3.5}$) to find out what star mass and spectral type has roughly the same M/L_{bol} as you.

When your supervisor next asks you whether you have "fire in the belly" for your work, you can honestly reply, "more, even, than the sun and stars!"

(4) Alien Astronomers in Virgo study the Milky Way

Galaxy disks often have exponential surface brightness profiles: $I(R) = I(0) \exp(-R/R_d)$, where R_d is the disk (e-folding) "scale length", and $I(0)$ is the central surface brightness. Recall, the units of $I(R)$ are $L_{\odot}\ pc^{-2}$.

For example, the disk of the Milky Way has R_d = 3.5 kpc, and $I(R)$ at the solar radius (8.5 kpc) is 15 $L_{V,\odot}\ pc^{-2}$.

a. Convert the exponential form of $I(R)$ into an equivalent expression for $\mu(R)$, in mag/ss (i.e. find $\mu(R)$ in terms of $\mu(0)$, R, and R_d)? In the questions that follow, work with either $I(R)$ or $\mu(R)$, which ever your prefer.

b. What's the surface brightness, $I(0)$, at the center of the Milky Way disk, and what's the disk's total luminosity in $L_{V,\odot}$.

c. Using $M_{V,\odot}$ = 4.82, calculate the Milky Way's absolute magnitude, M_V. If viewed from Virgo (distance 15 Mpc) what would its apparent magnitude, m_V be? Would an Alien equivalent of Charles Messier have included the Milky Way in his catalogue of bright nebulae?

d. What is the apparent inclination of the Milky Way galaxy, as seen by our Virgo Alien (face-on is 0° and edge on is 90°). Hint: Virgo is at RA = $12^h\ 30^m$ Dec = +12° 30', which can be converted to galactic coordinates (l,b) using this NED tool: [o-link].

e. What is the surface brightness of the Milky Way disk, μ_V, in magnitudes per square arcsec, at the radius of the sun and at the disk's center.

f. As seen from Virgo, what is the angular **diameter** of the Milky Way, D_{25}, as defined by the 25^{th} V mag/ss isophote.

g. What's the luminosity contained within D_{25}, expressed in $L_{V,\odot}$? (Do the integral!).

h. If the rotation curve stays flat at ~220 km/s outside the solar circle, what is the value of M/L_V measured inside the D_{25} isophote?

(5) Scaled Hubble Constant: H_o :

A few examples of dealing with different values of the Hubble parameter: $h = H_o / 100$.

a. What are the h dependencies of:
(i) linear size (from angular size and distance)
(ii) luminosity (from flux)
(iii) luminosity density
(iv) mass (from dynamics)
(v) mass density
(vi) M/L.

b. An old paper by Sandage uses H_o = 55 km/s/Mpc to give the central luminosity density and mass density in a galaxy to be $10^3\ L_{B,\odot}/pc^3$ and $10^4\ M_{\odot}/pc^3$. Recast these values as new values which include h and then evaluate them for h = 0.72 (i.e. H_o = 72 km/s/Mpc).
