Selection of Homework Questions

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Topic 4: Luminosity Functions

(1) The Schechter Function:

Lets evaluate some basic properties arising from the Schechter Luminosity function of galaxies. First, the function reads:

$$\Phi(L)\, dL = n_* \left(\frac{L}{L_*}\right)^{\alpha} exp\left(-\frac{L}{L_*}\right)\, d\left(\frac{L}{L_*}\right)$$

a. By approximating $\Phi(L)$ for just the low luminosity galaxies $L << L_*$, show that for $-2 < \alpha < -1$, the total number of galaxies is infinite but the total light is not.

b. Derive an expression for the "mid-rank" galaxy luminosity, L_{mid}, such that half the light comes from galaxies with $L > L_{mid}$ and half comes from galaxies with $L < L_{mid}$. What is L_{mid} / L_* for $\alpha = -1$?

c. Transform the Schechter luminosity function expressed in L to an equivalent function expressed in M, absolute magnitude (don't just copy the formula given in B&M, but show how it comes about).

d. Using whatever computing environment you prefer, generate plots of the following related LFs:
   (a) Log $\Phi(L)$ dL *vs* Log L/L_*
   (b) Log $\Phi(M)$ dM *vs* $M - M_*$
   (c) Log $N(>L)$ *vs* Log L/L_*
   (d) Log $N(< M)$ *vs* $M - M_*$
   where the second two are *cumulative* functions integrated over L or M to **brighter** galaxies. Take the normalization n_* to be unity; take the range in L/L_* to be from 10^{-2} to 10; and overplot lines with three values of α : -1.5, -1.0, -0.5 (dotted, solid, dashed). Be careful to account for the fact that graph (a) expresses Φ per unit interval of luminosity (dL), while graph (b) expresses Φ per magnitude (dM, which is an interval in Log L). Also, note that graphs (c) and (d) are **not** expressed per interval, but are integrated, and so they should look the same (excluding, possibly, the direction of the x-axis).

   Summarize, briefly, the various features you see in the plots and their differences. Why does the graph of Log Φ (M) dM immediately tell you that $\alpha = -1.0$ is the critical value separating finite from infinite numbers of galaxies?

(2) Application to the Coma Cluster :

The Coma cluster of galaxies has a luminosity function which is moderately well represented by the Schechter function, with $\alpha = -1$ and $M_{B,*} = -19.2$ (H_o = 100 km/s/Mpc). The redshift of Coma is 7000 km/s and its total luminosity is about 250 L_*.

a. How would α and $M_{B,*}$ change if H_o = 50 km/s/Mpc ?

b. What is L_* (in solar luminosity units) corresponding to $M_{B,*}$ in these two cases (use $M_{B,\odot} = 5.48$)

c. Use the total luminosity to evaluate n_* (the normalization of the Schechter function), and hence estimate how many galaxies are brighter than L_*, 0.1 L_*, and 0.01 L_*.

d. Estimate the expected luminosity of the brightest galaxy, L_1, by setting L_1 equal to the total luminosity expected from the luminosity function in galaxies brighter than L_1. Express L_1 in units of L_* and as an apparent magnitude. Compare the latter with the observed apparent magnitude for the brightest Coma galaxy, NGC 4889 ($B_T^{o,i}$ from RC3). Comment on any difference you find between the two magnitudes.

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