6. STELLAR DYNAMICS I : DISKS

(1) Introduction and Overview

- We now begin a study of **Stellar Dynamics** : the theory of gravitational many-body systems.
  This subject is mature, extensive and can be highly sophisticated
  Fortunately for us, we don't really need (and don't have the time for) a **detailed** treatment
  Instead, my aim will be to :
  - present the major themes in broad outline
  - extract some important analytic results
  - develop our intuitive understanding of each topic
  - connect each topic to our observational knowledge of galaxies

- Since the subject is so weighty, it is sensible to spread it out a bit :
  Topic 6 : considers disk dynamics and the formation of bars and spirals
  Topic 8 : covers the meat of the subject : ways to understand the general 3-D system
  Topic 12 : considers dynamical friction, encounters and mergers.
  Topic 13 : discusses how black holes can affect galaxy structure

  By the end we will have covered almost all of B&T while omitting much of the detail.

- So lets now begin with disk dynamics :
  The path for this topic takes us through a number of interesting themes :
  Circular Orbits : Simple kinematics in a differentially rotating disk
  Epicycles : perturbations of simple circular orbits
  Resonances : when epicycles synchronize with the passage of disk patterns
  Density waves : a self-consistent response to coherent epicyclic motion
  Instabilities : when disks begin to clump up under self-gravity
  Amplification : when this clumping grows to form bars and spirals.

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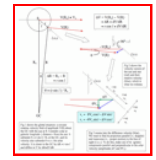
(2) Circular Rotation: Oort's Constants

In the 1920s theorist Bertil Lindblad (Swedish) and observer Jan Oort (Dutch) studied the rotation of the Milky Way disk. From the moving sun's location, nearby stars appear to move systematically as a function of galactic longitude. Two parameters, Oort's A & B, help parameterize the local velocity field.

- A simple derivation of their functional forms is given here: [image]



For the radial and tangential velocities, we find:

$$V_r = A\, r\, \sin 2l \qquad (6.1a)$$

$$V_t = B\, r + A\, r\, \cos 2l \qquad (6.1b)$$

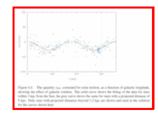
Where Oort's constants A and B are given by (R_0 = solar radius):

$$A = \frac{1}{2}\left(\frac{V_c}{R} - \frac{dV_c}{dR}\right)_{R_0} = -\frac{1}{2}R\left(\frac{d\Omega}{dR}\right)_{R_0} \qquad (6.2a)$$

$$B = -\frac{1}{2}\left(\frac{V_c}{R} + \frac{dV_c}{dR}\right)_{R_0} = -\frac{1}{2}R\left(\frac{d\Omega}{dR} + \frac{2\Omega}{R}\right)_{R_0} \qquad (6.2b)$$

Oort's A expresses **local shear**, while
Oort's B expresses **local vorticity**, ie local rotation: $\Omega_{loc} = \nabla \times \mathbf{V}$ (curl **V**)

- Here are some examples of apparent stellar motions for several different rotation curves: [image]



- Current best (Hipparcos) estimates for Oort's A and B are (B&M p642): [image]
  A = 14.8 ± 0.8 km/s/kpc and
  B = -12.4 ± 0.6 km/s/kpc

  Notice their dimensions are velocity gradients which are also frequencies
  E.g. using psm units: A = 0.0148 km/s/pc ≈ 0.014 Myr^{-1} = 14.8 Gyr^{-1}

- From their definition some interesting properties of the MW rotation can be measured:

$$A + B = \left(\frac{dV_c}{dR}\right)_{R_0} = +2.4 \text{ km/s/kpc} \qquad (6.3a)$$

$$A - B = \left(\frac{V_c}{R}\right)_{R_0} = \Omega(R_0) = 27.2 \text{ km/s/kpc} \qquad (6.3b)$$

The first of these confirms that the rotation curve is fairly flat near the sun (gently rising).

The second yields an **orbital period for the sun**:
$P(R_0) = 2\pi / \Omega(R_0) = 2\pi / 27.2\ Gyr^{-1} = 0.23$ Gyr = 230 Myr

And when coupled with an estimate for the galactocentric distance, R_0, yields an **orbital velocity**:
$V_c(R_0) = 218\ (R_0 / 8$ kpc) km/s
Which agrees fairly well with radio VLBI measurements of Ω from proper motion of Sgr A^*
$V_c(R_0) = 241\ (R_0 / 8$ kpc) km/s

- This analysis assumes the sun and stars are all on circular orbits.
  In truth, this is only approximately true: the stars are in fact perturbed from circular orbits.
  This kind of motion can be analyzed using the concept of **epicycles**.

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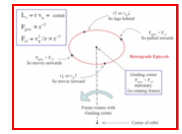
(3) Epicycles

(a) Overview

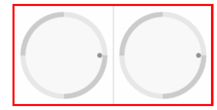
- Disk stars have approximately circular orbits with small deviations :
  As with the Ptolemaic system: star orbits can be described by superposition of:
  - Circular orbit along **guiding center** (= deferent), radius R_g, angular velocity Ω_g
  - Smaller elliptical **epicycle**, angular velocity κ, **retrograde**

- Here's an intuitive way to understand the origin of the epicyclic motion: [image]



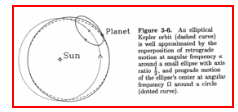
  - Consider star at guiding center (GC) and give it a kick radially **outwards**
  - Conserving AM = mrv_ϕ, since r increases, v_ϕ decreases
    → w.r.t. the guiding center, the star moves **backwards**
  - Consider the new balance between gravity and centrifugal forces:
    Under AM conservation, $F_{centrifugal} \propto v_\phi^2 / r \propto r^{-3}$ while F_{grav} falls more **slowly** than r^{-2}
    → at larger radii $F_{grav} > F_{centrifugal}$ and the star gets pulled back **inwards**
  - As it falls in, r decreases so v increases and the star moves **forward** relative to the GC
  - But now $F_{centrifugal} > F_{grav}$ and the star moves **outwards** again

    → the cycle repeats, and we have a small **retrograde epicycle**



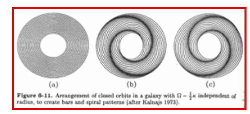
- An equivalent description considers the coriolis force in a rotating frame: [image]

- For a Keplerian potential, the orbit and epicycle frequencies are the **same**, $\kappa_g = \Omega_g$
  The full orbit is **closed**: we have an offcentered Keplerian ellipse. [image]
  However, in general Ω_g and κ are different so orbits don't close.......
  Unless they are observed from frame rotating at Ω_g - ½ κ:
  → orbits then appear **closed ellipses**, centered on galactic center.



- At this point, lets briefly anticipate the relevance of epicycles for spiral arms:

  - For epicycle phases which vary systematically with radius:
    → nested elliptical orbits may crowd in a spiral pattern
    this is called a **kinematic density wave** [image]
  - Orbits are in turn perturbed by the spiral density pattern,
    this modifies the simple epicycles
  - Self-consistent solution is a **density wave**
    gas reponse causes shocks and star formation → visible spiral arms



- Now let's return to the epicycles, and derive their characteristics :

  - Consider a smooth axisymmetric flattened mass distribution with potential Φ(R,z)
    Since we have no azimuthal forces AM is conserved, and we have :

$$\ddot{\mathbf{r}} = -\nabla\Phi(R,z)\ ; \quad L_z = R^2\dot{\phi} \;=\; const \tag{6.4}$$

  - Separating the Equations of motion into components in cylindrical coordinates (R, Φ, z):

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial\Phi}{\partial R}\ ; \quad \frac{d}{dt}(L_z) = 0\ ; \quad \ddot{z} = -\frac{\partial\Phi}{\partial z} \tag{6.5}$$

(b) Vertical z Motions

- Take first the z motion about the plane z = 0
  since the disk is symmetric about z = 0, then the z-force at z = 0 is zero :

$$\left(\frac{\partial\Phi}{\partial z}\right)_{z=0} \;=\; 0 \tag{6.6}$$

  consider small motion above and below the plane,
  we simply expand the z-force linearly for small z:

$$\ddot{z} \;=\; -\left(\frac{\partial\Phi}{\partial z}\right)_{z=0} - z\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{z=0} \;=\; -z\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{z=0} \;=\; -\nu^2 z \tag{6.7}$$

- This gives Simple Harmonic Motion (SHM), with frequency ν where $\nu^2 = (\partial^2\Phi / \partial z^2)_{z=0}$:

$$z(t) = Z\cos(\nu t + \psi_0) \qquad (6.8)$$

- For Milky Way disk near the sun, $\nu^2 = 4\pi G \rho_o$, which gives $\nu \sim 0.072$ Myr^{-1},
  → vertical oscillation period $2\pi / \nu \sim 87$ Myr ~ 1/3 circular period, Ω.

(c) Radial Motions

- First consider the circular guiding orbit (R = const) :
  it has radius R_g, circular velocity V_c and angular velocity Ω_g defined by

$$\left(\frac{\partial\Phi}{\partial R}\right)_{R_g} = \frac{V_c^2}{R_g} = R_g \Omega_g^2 \qquad (6.9)$$

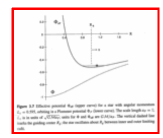
For non-circular orbits, the radial acceleration is given by (centrifugal - gravity):

$$\ddot{R} = R\dot{\phi}^2 - \frac{\partial\Phi}{\partial R} \qquad (6.10)$$

- However, since $L_z = R^2\,\phi$(dot), then this can also be written as

$$\ddot{R} = -\frac{\partial\Phi_{\rm eff}}{\partial R} \quad \text{where} \quad \Phi_{\rm eff} = \Phi(R,z) + \frac{L_z^2}{2R^2} \qquad (6.11)$$

Where the **effective potential**, Φ_{eff}, allows us to describe the radial motion in 1-D form [image]
Typically, Φ_{eff} has a minimum, rising steeply at small R and slowly at large R
This inner steep term imposes an **angular momentum (or centrifugal) barrier**
At the minimum in Φ_{eff}, we recover the circular guiding orbit of radius R_g



$$\left(\frac{\partial\Phi_{\rm eff}}{\partial R}\right)_{R_g} = 0 = \left(\frac{\partial\Phi}{\partial R}\right)_{R_g} - R_g\dot{\phi}_g^2 = \left(\frac{\partial\Phi}{\partial R}\right)_{R_g} - \frac{V_c^2}{R_g} \qquad (6.12)$$

- Other orbits will oscillate in radius, about R_g.
  Consider the potential at $R = R_g + x$

$$\ddot{R} = \ddot{x} = -\left(\frac{\partial\Phi_{\rm eff}}{\partial R}\right)_{R_g} - x\left(\frac{\partial^2\Phi_{\rm eff}}{\partial R^2}\right)_{R_g} = -x\left(\frac{\partial^2\Phi_{\rm eff}}{\partial R^2}\right)_{R_g} = -\kappa^2 x \qquad (6.13)$$

This gives SHM about the guiding radius

$$x(t) = X\cos(\kappa t + \phi_0) \qquad (6.14)$$

with frequency κ, where

$$\kappa^2 = \left(\frac{\partial^2\Phi_{\rm eff}}{\partial R^2}\right)_{R_g} = \left(\frac{\partial}{\partial R}\left(\frac{\partial\Phi}{\partial R}\right)\right)_{R_g} + \frac{3L_z^2}{R_g^4} = \left(R\frac{d\,\Omega^2}{d\,R} + 4\Omega^2\right)_{R_g} \qquad (6.15)$$

(d) Azimuthal Motion

- Since $L_z = R_g^{\,2}\,\Omega_g = R^2\Omega$ = const, changes in R yield changes in Ω (recall $\Omega = \phi$(dot))

$$\dot{\phi} = \frac{L_z}{R^2} = \frac{L_z}{(R_g + x)^2} \simeq \frac{L_z}{R_g^2}\left(1 - \frac{2x}{R_g}\right) = \Omega_g\left(1 - \frac{2x}{R_g}\right) \qquad (6.16)$$

Integration gives :

$$\phi(t) \;=\; \Omega_g t \;-\; \frac{2\Omega_g X}{\kappa R_g}\sin(\kappa t + \phi_0) \tag{6.17}$$

- Thus, ϕ(t) follows the guiding center with small amplitude SHM superposed.
  Taking the y-axis in the forward direction with origin on the guiding center, we have

$$y(t) \;=\; -\frac{2\Omega_g}{\kappa} X \sin(\kappa t + \phi_0) \tag{6.18}$$

the oscillation of frequency κ is the same as in x, but out of phase by 90°

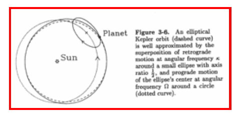
- Taken together, (and setting the initial phase $\phi_0 = 0$), we have

$$\begin{aligned} x(t) &= X\cos(\kappa t) \\ y(t) &= -\frac{2\Omega_g}{\kappa} X \sin(\kappa t) \end{aligned} \tag{6.19}$$

Some properties of this motion are:

- **elliptical** epicycle with radial/azimuthal axis ratio = κ / 2Ω
- epicyclic motion is **retrograde** w.r.t. orbit (c.f. Ptolemy's were prograde)

- For **Keplerian** potential: $\Omega \propto R^{-3/2}$ we get $\kappa = \Omega$
  → epicycle axis ratio 2:1 (cf Ptolemy's were 1:1 circles)
  → full orbit is closed ellipse, centered at the ellipse focus [image]
- For **flat rotation curve**: $\Omega \propto R^{-1}$ we get $\kappa = \Omega\sqrt{2}$
- For **solid body rotation**: Ω = const (harmonic potential [example]):
  → κ = 2 Ω giving **circular** epicycles and **closed** oval orbits



- In general, $\Omega < \kappa < 2\Omega$ so $\kappa > \Omega$
  → epicycle completed **before** rotation
  → from inertial frame, orbits don't close, but regress

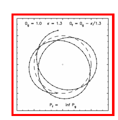
(e) Values Near the Solar Neighborhood

We can express the epicyclic and orbital frequencies at the solar radius, κ_0, Ω_0, in terms of Oort's constants:

$$\kappa_0^2 \;=\; -4B(A-B) \;=\; -4B\Omega_0\;; \quad \kappa_0 \;=\; 37\ \mathrm{km/s/kpc} \tag{6.20}$$

$$\Omega_0 \;=\; A - B \;=\; 27\ \mathrm{km/s/kpc} \tag{6.21}$$

- The ratio $\kappa_0 / \Omega_0 \approx 1.3$
  → Solar neighborhood stars make 1.3 epicyclic rotations per orbit. [image]

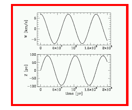


- The ratio $\kappa_0 / 2\,\Omega_0 \approx 0.7$
  → Epicycles have radial/azimuthal extent of ~0.7
  → Stars with $R_g = R_0$ have velocity dispersions $\sigma_R / \sigma_\phi = \kappa_0 / 2\Omega_0 \approx 0.7$
  → However, at R_0, velocity dispersions are in fact $\sigma_R / \sigma_\phi = 2\Omega_0 / \kappa_0 \approx 1.5$
  this is because stars found at R_0 tend to have $R_g < R_0$ (more stars at smaller radii)

- Epicycle **sizes** are $\approx \sigma / \kappa$, so for $\sigma_R \sim 30$ km/s, we find ~ 1 kpc excursions
  Similarly, for $\sigma_z \sim 30$ km/s and $\nu \approx 0.096\ \mathrm{Myr}^{-1}$ we find vertical excursions ~ 300 pc.

- For the sun, z ~ 40pc and W ~ 7 km/s, so we expect modest vertical excursions.

An amusing (speculative) theory is that disk crossings coincide with terrestrial craterings and/or extinctions
The higher stellar densities perturb the Oort comet cloud, causing more impacts (!): [image]



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(4) Resonances

(a) Rotating Patterns

- As we shall see (§ 5 below) there can be regions of **enhanced stellar density** in the disk
  These can have the shape of spiral and bar **patterns**
  These patterns are neither stationary nor move with the disk stars,
  Instead they move at some intermediate angular velocity, Ω_p, called the **pattern speed**
  The interaction of these patterns with the epicyclic motion can lead to **resonances**:

(b) Corotation Resonance (CR)

- First consider a star which orbits **at the pattern speed**
  we have $\Omega_* = \Omega_p$ or $\Omega_p - \Omega_* = 0$
  Such stars experience a **persistant** non-axisymmetric perturbation, and their response builds up

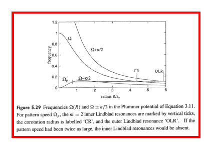
(c) Lindblad Resonances (ILR, OLR)

- Consider stars which complete exactly 1 epicycle between the passage of each arm
  → Their interaction with the spiral arm is **resonant**
  → Epicyclic amplitude is amplified and wave propagation is strongly modified

- Where do such resonances occur in the galaxy?
  The angular frequency of the star w.r.t. the pattern is $\Omega_p - \Omega_*$
  There are two cases :
  -ve if $\Omega_* > \Omega_p$; star moves past arms
  +ve if $\Omega_p > \Omega_*$; arms move past stars
  The angular frequency of encountering **each** arm in a 2 armed spiral is $2(\Omega_p - \Omega_*)$
  So the condition for resonance is:
  $2(\Omega_p - \Omega_*) = \pm\kappa$ or $\Omega_p - \Omega_* = \pm\kappa / 2$ ($\pm\kappa$ / m for an m armed spiral)

- There are two classes of resonance:
  $\Omega_p - \Omega_* = -\kappa / 2$: **Inner Lindblad Resonance** (stars move past pattern)
  $\Omega_p - \Omega_* = +\kappa / 2$: **Outer Lindblad Resonance** (pattern moves past stars)

- To establish the radii of these resonances, one needs to know: [image]
  The pattern speed: Ω_p
  The rotation curve, $V_c(R)$, which gives $\Omega(R)$ and $\kappa(R)$
  Depending on $V_c(R)$ and Ω_p there may be 0,1,2,.... resonances
  (there can also be 0,1,2 **inner** Lindblad resonances)



(d) Importance of Resonances

Resonances are important for several reasons :

- Density waves: (see below)
  Can only survive inbetween the ILR and OLR (where we find arms)
  Cannot pass an ILR (they are absorbed, like waves on a beach)
  → Important in allowing/preventing propagation across the disk through the center

- Orbit **shapes** change across the resonances:
  → Bars don't extend beyond CR, stop close to it
  → Bars probably rotate with pattern speed $\Omega_p \sim \Omega(R{=}CR)$
  → Expect stellar **rings** to form at CR and OLR (as found)

- Gas driven **inwards** to ILR and **outwards** to ILR
  → often find gas rings/disks/starformation near ILR

- For the Milky Way, estimates are (for m=2):
  ILR at ~3kpc, CR at ~14kpc, and OLR at ~20kpc, with Ω_p ~ 15 km/s/kpc

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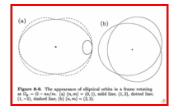
(5) Density Waves

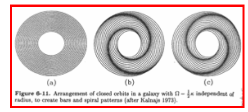
(a) Kinematic Density Waves

- In general, orbits are **not closed** (~1-2 epicycles per orbit)

- However, they **are** closed in a frame which rotates at Ω_g - ½ κ [image]
  In this frame, the orbit is a closed ellipse with the Galactic center at the center.



- Consider set of orbits whose epicyclic phases vary **monotonically** with radius
  (i.e. PA of ellipses rotates with increasing R)
  → Simple **orbit crowding** will generate a two arm spiral pattern.
  This is called a **kinematic density wave** [image]



- **If** Ω - ½ κ is roughly independent of radius (roughly true for flat rotation curve):
  → the pattern is **fixed** and rotates at $\Omega_p = \Omega_g$ - ½ κ = **pattern speed**
  In fact, for V_c = const, pattern speed varies **slowly** with R, so spiral slowly **winds up**

- However, including **self-gravity** can yield a **different** Ω_p which is almost **independent of radius**
  this is a result from "density wave theory", to which we now turn:

(b) Lin-Shu (QSSS) Density Wave Theory

(i) Sketch of Approach

- The generation of **kinematic** spirals assumes **axisymmetric** potential
  However, orbit crowding yields a **non-axisymmetric** spiral perturbation

  Star (and gas) orbits are **modified** by the spiral perturbation
  Their new orbits define a **new** surface density and associated potential.

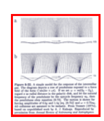
- We need to look for a **self consistent** solution: response to input potential gives **same** potential
  The analysis is difficult (Lin & Shu 1964, 1966 and much subsequent work)
  Considers **waves** propagating in a differentially rotating disk
  Derive a **dispersion relation**: $\omega = f(k)$ with phase velocity, ω/k, and group velocity, $d\omega/dk$.

  Look for **Quasi-Stationary Steady State** solution (QSSS).

(ii) Results

- Solutions are found with $\Omega_p = \Omega - (d\omega/dk) \times ½\kappa$ which is ~ independent of R
  → Pitch angles ψ(R) ~ const, yielding **logarithmic spirals**
  → Waves **survive** between ILR and OLR
  → Waves are **absorbed** at ILR.
  → Waves weaker in disks with higher velocity dispersion
  need cold component to be replenished via star-formation (c.f. S0 disks don't have arms).

- Gas response:
  → Non-linear, leading to collisions/shocks above a threshold response [image].
  → Gas runs into itself (c.f. traffic jams) creating **narrow gas features** (as observed)
  → Predict **velocity streaming** in vicinity of arms, roughly as found: [image]
  → Geometry of density wave & strength of shock depend on central concentration & V_c
  explains correlation of pitch angle with bulge/disk ratio
  explains lack of dwarf-spirals: need threshold V_c to form disk and arms.



(c) Alternative Sources of Global Density Waves

It is unclear how often the QSSS density wave theory is applicable: (see [o-link])

Some galaxies have spiral arms within the region where $V_c \propto r$ (solid body):

$\rightarrow$ arms don't wind up in this region.

Many galaxies are **flocculent**, with no clear density wave pattern (see § 6 below)

Many galaxies have an **alternative** source of density wave (see next)

There are two other obvious sources of density waves, both are m = 2.

(i) Tides from companions

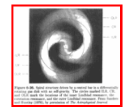
Tidal field of passing neighbor creates a strong m = 2 perturbation: [image]

This drives a strong kinematic density wave.

Self gravity enhances this perturbation.

However, these spirals are **transient**.

(ii) Bars and Oval Distortions



Bars are another source of m = 2 perturbations [images]

Sanders & Huntley (1976) find even weak bars can generates strong spiral arms

However, the mechanism needs **viscosity** (ie gas dissipation) to work.

Oval distortions may have a similar (though weaker) effect.

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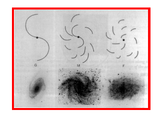
(6) Disk Instabilities and Their Amplification



For the significant number of **flocculent** spirals, a different mechanism may be at work [image]

This involves the tendency of stars and gas within a disk to clump up gravitationally and form stars

These clumps then get pulled into spiral arm fragments by differential rotation

(a) Local Disk Stability: Toomre's Q Parameter

- When are self-gravitating disks vulnerable to **local** gravitational instabilities ?

  Instabilities can arise from a competition between:

  - gravity causing overdense regions to collapse
  - stellar dispersion which inhibits the collapse
  - angular momentum which inhibits the collapse

  Toomre (1964) found the conditions for instability: Q < 1 where $Q \approx \kappa \sigma / (3 G \Sigma)$

  Where σ is the stellar velocity dispersion and Σ is the local surface density

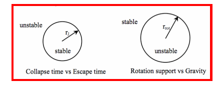
- Here's a simplified derivation based on a **modified Jeans analysis**:
  Consider overdense region radius R in a **non-rotating** disk

  - The collapse time is $t_{coll} \sim R / V$ where $V \sim$ gravitational velocity $\sim (G M / R)^{1/2}$
    So $t_{coll} \sim R / (GM / R)^{1/2} \sim (R^3 / GM)^{1/2} \sim (R / G \Sigma)^{1/2}$ (Σ is surface density)
    The time for stars to escape the region is : $t_{esc} \sim R / \sigma$ (σ is dispersion)
    So collapse occurs if $t_{coll} < t_{esc}$ ie $(R / G \Sigma)^{1/2} < R / \sigma$
    $\rightarrow$ The critical size **for stability** due to dispersion is therefore : $R_J < \sigma^2 / G \Sigma$

  - Now consider a **rotating** disk:
    The **local** angular velocity is Oort's constant B
    The region is **stable** if $F_{centrifugal} > F_{gravity}$
    In this case $R B^2 > GM / R^2 = G \Sigma$
    $\rightarrow$ The critical size **for stability** due to rotation is therefore : $R_{rot} > G \Sigma / B^2$

  - Combining these: the disk is **unstable** in the range $R_J < R < R_{rot}$

And therefore the disk is locally **stable** if $R_J > R_{rot}$ [image]



ie $\sigma^2 / G\Sigma > G\Sigma / B^2$ or $\sigma B / G\Sigma > 1$

Recall that $B = \kappa^2 / 4\Omega$ and $\kappa \sim$ 1-2 Ω so $B \sim \kappa / 3$

The final condition for disk **stability** is therefore

$$Q \equiv \frac{\sigma |B|}{G\Sigma} \cong \frac{\sigma\kappa}{3G\Sigma} > 1 \qquad (6.22)$$

[A similar relation for gravitational stability for a **gas** disk is: $Q \equiv V_s \kappa / 3 G\Sigma > 1$]

- Factors which promote gravitational **instability** (i.e. promote spiral structure) are:
  low stellar velocity dispersion σ
  high surface mass density Σ
  low epicyclic frequencies (and/or low local rotation, i.e. low $|B|$).

- Solar neighborhood : $\sigma \sim 30$ km/s ; $\Sigma \sim 50\ M_\odot\,pc^{-2}$; $\kappa \sim 36$ km/s/kpc
  these give $Q \sim 1.4$ and so the MW disk is locally stable near the sun

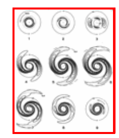
(b) Swing Amplification

Some circumstances allow a powerful amplification of spiral patterns (eg Toomre 1981)

(i) The Swing Amplifier

If we have a **leading** spiral density wave, then

- Differential rotation will gradually rotate it into a **trailing** spiral wave [image]
- The rotation of the pattern is **retrograde**
- The timescale for rotation is $\sim \kappa$

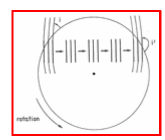


→ Epicyclic motion approximately **follows** the arm
→ **Long** perturbation duration so epicycle amplified
→ The emerging trailing pattern is strongly **amplified**

(ii) Feedback for the Amplifier

For this to work, we need a source of **leading** spiral waves
However, these are not normally generated in a rotating disk
Instead, look for **feedback**: trailing waves converted into leading waves.

- Waves **reflected** from outer edge experience 180° phase shift (trailing → leading)
  unlikely to operate in real galaxies: edges too soft
- Trailing waves passing through the central regions emerge as leading waves [image]
  This can only occur when we have no ILR (which blocks wave passage)



Swing amplification with feedback is probably very important in maintaining strong sprial structure.

(c) Bar Instability and its Suppression

- N-Body simulations of disks seem to form bars **remarkably easily** [image]
  Indeed, it is difficult to devise stable disk models **even with Q > 1**
  Reality of this **bar instability** has been verified using analytic methods

- Swing Amplification helps explain the instability:
  Recall: leading waves are strongly amplified into trailing ones
  Nothing happens unless there is a source of leading waves
  Trailing waves pass through center and emerge as leading
  Hence **feedback** keeps the amplifier going
  → bar grows quickly.

- Early work (Hohl 1971, Ostriker & Peebles 1973) noted the severity of the bar instability
  As you might expect, increasing stellar **dispersion** can calm the instability
  They found disks were stabilized against bar formation for KE(σ) / KE(rot) > 5

Note that for the MW disk near the sun, KE(σ) / KE(rot) ~ 0.15
$\rightarrow$ so our disk should be highly unstable to bar formation!

- What might suppress the bar instability in real galaxies?
  There are several possible mechanisms:
  - Put mass in a dark halo: this acts like a high dispersion component
    More of historical interest: back in 1973, any evidence for a dark halo was promoted
    However, dynamics of the inner regions are **not** influenced much by the halo
    The halo may nevertheless help stabilize disks at larger radii.
  - Achieve the same by having a high dispersion bulge or inner disk
    Ingoing waves are damped before they pass though the center: cuts feedback
  - An ILR will shield the center and cut the feedback:
    A large central bulge mass yields an Ω(R) with an ILR