

Whittle : EXTRAGALACTIC ASTRONOMY

[Home](#)[Main](#)[Index](#)[Toolbox](#)

| | | | |
|--------------------|------------------|------------------------|-------------------------|
| 1 : Preliminaries | 6 : Dynamics I | 11 : Star Formation | 16 : Cosmology |
| 2 : Morphology | 7 : Ellipticals | 12 : Interactions | 17 : Structure Growth |
| 3 : Surveys | 8 : Dynamics II | 13 : Groups & Clusters | 18 : Galaxy Formation |
| 4 : Lum. Functions | 9 : Gas & Dust | 14 : Nuclei & BHs | 19 : Reionization & IGM |
| 5 : Spirals | 10 : Populations | 15 : AGNs & Quasars | 20 : Dark Matter |

7. ELLIPTICAL GALAXIES

[Index](#)[Questions](#)[References](#)[PDF](#)[Next](#)[Top](#)

(1) Introduction

(a) The Myths

Our view of Elliptical galaxies has changed greatly : [\[image\]](#)

In the 1970s, Ellipticals were thought to be :

- Diskless bulges with deVaucouleurs ($R^{1/4}$) profiles and constant density (King) cores.
- Oblate spheroids flattened by rotation
- Void of gas and dust
- Contain a single ancient population of stars
- Relaxed dynamically quiescent systems



To a large extent, **all** of the above are now thought to be wrong.

(b) Subdividing the Elliptical Class

In what follows, it will be useful to consider **three** classes of Ellipticals :

- **Luminous** : L greater than 1-few L_* , M_B brighter than about -20
- **Midsize** (including massive bulges) : L between 0.1 L_* & L_* , M_B in the range -18 to -20
- **Dwarfs (dE & dSph)** : L less than 0.1 L_* , or M_B fainter than -18.

Luminous and midsize have somewhat different properties, but form a single sequence in mass.

Diffuse dwarf Es & dwarf spheroidals are significantly different.

(c) Parameters

Here are a few recurring parameters we need to be familiar with :

- **Surface Brightness (SB)** : $I(R)$ (flux units), $\mu(R)$ (mag/ss units). I_B is for B band, etc
- **Total flux** : (a) within projected radius $L(<R)$, or (b) integrated: L_{tot} (equivalent to M_B)
- **Effective Radius** : R_e defined as the half light radius: $L(<R_e) = 0.5 L_{tot}$ [c.f. $I_e \equiv I(R_e)$]
- **Stellar Velocity Dispersion**: $\sigma(R)$
Assumes a Gaussian projected stellar velocity distribution
Usually single central aperture [ideally, includes light out to R_e , giving $\sigma_e = \langle \sigma(<R_e) \rangle$]

Also important, are properties of the **core** :

- **Central Surface Brightness:** $I_0 = I(0)$
- **Core Radius:** r_c or R_c , such that $I(R_c) = 0.5 I(0)$
Sometimes r_c (or r_b) refers to a **break** in the near nuclear light profile.
- **Central Stellar Velocity Dispersion:** $\sigma_0 = \sigma(0)$

Remember : $I(R)$ is **independent of distance** ! (for small redshifts).

(d) Deprojection

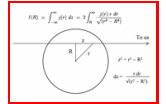
Note that all the above quantities are **projected** onto the sky.

Ultimately we want true 3D spatial information. I.e. we want to derive :

- The **luminosity density** $j(r)$ from the surface brightness $I(R)$, where
- r is true (3D) radius and R is projected radius
- For constant M/L ratio, $j(r)$ and $I(R)$ track the space and projected **mass** densities [i.e. $\rho(r)$ and $\mu(R)$]

In general, with $z^2 = r^2 - R^2$ and $dz = r dr / (r^2 - R^2)^{1/2}$, we have [\[image\]](#)

$$I(R) = \int_{-\infty}^{\infty} j(r) dz = 2 \int_R^{\infty} \frac{j(r) r dr}{\sqrt{r^2 - R^2}} \quad (7.1)$$



This is an Abel Integral equation, with solution

$$j(r) = \frac{-1}{\pi} \int_r^{\infty} \frac{dI}{dR} \frac{dR}{\sqrt{R^2 - r^2}} \quad (7.2)$$

- There are a few useful $I(R)$ & $j(r)$ pairs that can both be expressed algebraically
- For smooth (fitted) profiles, evaluate the integral directly
- For noisy data, use the Richardson-Lucy iterative inversion

Note : if the image is elliptical, a unique inversion is only possible for an axisymmetric figures viewed from the equatorial plane.

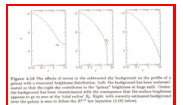
Just to orient ourselves, consider a **single power law** of index α (typically, $0.5 < \alpha < 1.5$)

- $I(R) \propto R^{-\alpha}$ Note: $I(0)$ diverges for $\alpha > 0$
- $j(r) \propto r^{-\alpha-1}$ Note: $j(0)$ diverges for $\alpha > -1$
- $\langle I(\langle R \rangle) \rangle \propto R^{2-\alpha}$
- $L(\langle r \rangle) \propto r^{2-\alpha}$ Note: L_{tot} diverges for $\alpha < 2$
- $V_c \propto r^{(1-\alpha)/2}$ Note: $V_c(0)$ diverges for $\alpha > 1$

(e) Observational Concerns

There are a number of practical difficulties facing accurate surface photometry

- **Sky Subtraction** is critical. Typically one aims for $I(R)$ about 5% to 0.5% of sky. [\[image\]](#)
Difficult especially with small CCDs which may not extend far enough
- **Seeing** affects the central regions: convolving with the PSF (e.g. Gaussian + PL wings) :
 - $I(R)$ turns over into a **flat core** for $R < 1 \sigma$
 - Ellipticity decreases significantly for $R < 4 \sigma$
 - a_4 is affected even further out



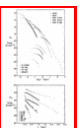
Next Prev Top

(2) Radial Light Profiles: Outside The Core

Elliptical galaxy light profiles are relatively similar with important differences: [\[image\]](#)

There is a long history of trying to fit these brightness profiles.

Reynolds (1913) and Hubble (1930) used: $I(R) = I(0) / (1 + R/R_0)^2$



A modified form has the benefit of having an analytic deprojected density:

$$I(R) = I(0) / [1 + (R/r_0)^2] \quad \text{with} \quad j(r) = I(0) / 2r_0 [1 + (r/r_0)^2]^{3/2}$$

Today's preferred fitting functions are a little different.

(a) Fitting Functions

(i) deVaucouleurs ($R^{1/4}$) and Sersic ($R^{1/n}$) Laws

deVaucouleurs noticed (1948) that for many ellipticals $\mu \propto R^{1/4}$

The fit is usually good over all but the inner and outermost regions (typically 0.03 - 20 R_e) [\[image\]](#)

The law is usually written :

$$I(R) = I_e \exp \left(-7.67 \left[(R/R_e)^{1/4} - 1 \right] \right) \quad (7.3)$$

It has the following properties :

- $L_{\text{tot}} = 7.22 \pi R_e^2 I_e$
- $I(0) = 2000 I_e$
- $\langle I(\langle R_e \rangle) \rangle = 3.61 I_e$ (which we abbreviate to $\langle I_e \rangle$ and equivalently $\langle \mu_e \rangle$)
- Asymptotically, at small R , $I(R) \propto R^{-0.8}$ while at large R , $I(R) \propto R^{-1.7}$
- In terms of surface brightness: $\mu(R) = \mu_e + 8.325 \left[(R/R_e)^{1/4} - 1 \right] = \mu(0) + 8.325 (R/R_e)^{1/4}$
- While originally purely empirical, Binney (1982) has shown that the $R^{1/4}$ law arises naturally from a reasonable distribution function.

The deVaucouleurs law is a special case of a more general, **Sersic** (1963,1968), law:

$$I(R) = I_e \exp \left(-b \left[(R/R_e)^{1/n} - 1 \right] \right) \quad (7.4)$$

Where

- $b = 1.999 n - 0.327$ ($n > 1$) ensures $0.5 L_{\text{tot}} = L(\langle R_e \rangle)$
- $n=4$ gives the deVaucouleurs $R^{1/4}$ law with $b = 7.67$
- $n=1$ gives an exponential profile with $b=1.67$
- A key feature is that the **slope** in $\mu(R)$ vs $\log R$ is a **steadily increasing** function of R : [\[image\]](#)
 $d \log I / d \log R = -(b/n) (R/R_e)^{1/n} \approx -2 (R/R_e)^{1/n}$
 → higher n means slower roll-over in the gradient.
 → inside R_e higher n means steeper profile; outside R_e higher n means shallower profile.
 → higher n is more **concentrated** (e.g. meaning higher values of $R90/R50$).
 (Note: at R_e all Sersic profiles have logarithmic slope of -2 .)
- Unfortunately, deprojection isn't straightforward, although Ciotti (1991 [o-link](#)) give approximations.
 A comprehensive analysis of the Sersic Law is given by Graham & Driver (2005 [o-link](#))

It turns out (see below) that different n 's fit the different classes of Ellipticals

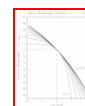
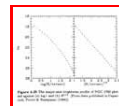
(ii) Double Power-Law (Dehnen) Laws

An alternative approach is to:

- (a) Choose a double power law to match inner and outer slopes
- (b) Specify the space luminosity density, $j(r)$, rather than the projected light $I(R)$.

One well-studied example is:

$$j(r) = \frac{j(0)}{(r/r_0)^\alpha [1 + (r/r_0)]^{\beta-\alpha}} \quad (7.5)$$



Clearly, for $r \ll r_0$ we have $j(r) \propto r^{-\alpha}$ and for $r \gg r_0$ we have $j(r) \propto r^{-\beta}$

Specific models include:

- $\beta = 4$ as a group were introduced by Dhenen (1993), with nice analytic properties.
- $\beta = 4$ $\alpha = 1$ is called the **Hernquist** law
- $\beta = 4$ $\alpha = 2$ is called the **Jaffe** law.
- $\beta = 4$ $\alpha = 3/2$ is closest to the deVaucouleurs $R^{1/4}$ law
- $\beta = 3$ $\alpha = 1$ is the **NFW profile** used for dark matter halos (where $\beta(r)$ replaces $j(r)$).

For example the Jaffe model has the following properties:

- r_0 contains half the **unprojected** light
- $R_e = 0.76 r_0$ contains half the **projected** light
- Expressions for the integrated (aperture) light, and gravitational potential are:

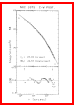
$$L(< r) = L_{tot} \frac{r}{r + r_0} \quad (7.6a)$$

$$\phi(r) = \frac{G L_{tot}}{r_0} \left(\frac{M}{L} \right) \ln \frac{r}{r + r_0} \quad (7.6b)$$

- The projected light profile has an analytic form ($a=R/r_0$):

$$I(a) = \begin{cases} (4a)^{-1} + \frac{1}{\pi}(1-a^2)^{-1} - (1-a^2)^{-3/2}(2-a^2)\text{arcosh}(1/a) & \text{for } a < 1 \\ (4a)^{-1} - \frac{1}{\pi}(a^2-1)^{-1} + (a^2-1)^{-3/2}(a^2-2)\text{arccos}(1/a) & \text{for } a \geq 1 \end{cases} \quad (7.7)$$

- An example of a Jaffe model fit to NGC 3379 is shown here [\[image\]](#)



(b) Resulting Fits

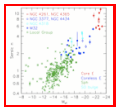
For many years, the deVaucouleurs $R^{1/4}$ was the primary fitting function
 It now seems there are systematic variation that require the other 3-parameter functions
 The nuclear regions are in any case poorly fit, and are discussed separately below.
 A recent and very thorough study is that of Kormendy et al (2009 [o-link](#))

(i) Variation with Luminosity

Galaxies of different luminosity have different concentrations (n-index) [\[image\]](#)
 The $R^{1/4}$ law fits best near $M_B \sim -21$.

Fitting Sersic profiles to find n-index gives:

- Lower luminosity Es less concentrated, $n < 4$ (Jaffe $\alpha \sim 2$)
- Higher luminosity Es more concentrated $n > 4$ (Hernquist $\alpha \sim 1$)



(ii) Variation with Environment

There is some evidence that outer light profiles can be affected by neighbors :

- Ellipticals in dense clusters have profiles that are **cutoff** at large radii likely caused by stars being **lost** due to tidal evaporation
- Ellipticals with a near neighbor can have a **raised** outer profile likely caused by **tidal heating** which puffs up the outer envelope

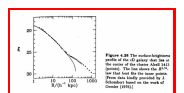
(iii) cD galaxies

cD galaxies are well fit by the $R^{1/4}$ law out to about $20R_e$

Outside this, their light profiles lie **above** the fit (eg $I(R) \propto R^{-1.6}$), in an extended halo [\[image\]](#)

This halo light may **not** come from the galaxy but from stars in the **cluster**

- the stellar velocity dispersion **increases** with radius, as expected for cluster stars (note : velocity dispersion usually **drops** with radius in normal Es)
- the isophotes can change to match the **isopleths** of the cluster galaxy distribution.



(iv) Ripples and Shells

About 10% - 20% of Ellipticals have "ripples", of amplitude 3-5% [\[image\]](#)
 These shells indicate recent accretion of disk galaxies (discussed more in Topic 12)
 Clearly, mergers play a role in the formation of at least some ellipticals.

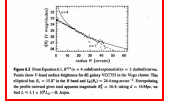


(v) Dwarf Ellipticals & Dwarf Spheroidals

There are **two** classes of Dwarf Ellipticals :

- **Compact dEs** : these are quite rare, but are clearly a continuation of their more luminous counterparts eg M32 which has a reasonable $R^{1/4}$ profile, perhaps slightly steeper, as expected.
- **Diffuse dEs** : these are common and are **quite different** from the more luminous Ellipticals
 Kormendy calls these "Spheroidals" to keep the distinction clear
 They extend in luminosity down to include the Milky Way dwarf spheroidals, like Leo I etc.

These Spheroidals have **exponential** light profiles (Sersic with $n \sim 1$) [\[image\]](#)



[Next](#) [Prev](#) [Top](#)

(3) Nuclear Regions

It transpires that the nuclear regions are very important:
 they give insight into the galaxy's formation history
 they can be influenced significantly by a central black hole

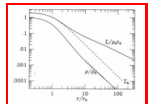
Early (pre-1975) work suggested that $I(R)$ turned over in a **flat core** of constant density
 This was naturally understood in terms of isothermal and King models (see below)
 However, the **serious** influence of seeing, especially in photographic work, had not been appreciated.
 The existence of flat cores was shown to be **incorrect** with CCD images (eg Kormendy 1977)
 Significant progress was only possible using HST.

In general, the above functions fail to match the nuclear regions very well.
 Within a "break radius" there can be deviations both below and above the best fit to the outer parts.

(a) Fitting Functions

(i) Isothermal and King Profiles

Historically, isothermal and King models were used to understand "flat cores": [\[image\]](#)
 Because they yield simple physical results, it is worth looking at them briefly.



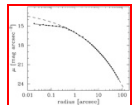
- They assume a self-gravitating spherical system with isotropic velocity dispersion.
- They have a Boltzmann distribution in star energy (potential + kinetic) (see Topic 8.8b,c)
- Their projected light, $I(R)$, turns over in a **flat core**, with known dynamical properties:
 - Within $3r_0$ it approximates the modified Hubble law: $I(R) = I(0) / [1 + (R/r_0)^2]$
 - It has a central density $\rho(0) = 9 \sigma(0)^2 / 4 \pi G r_0^2$
 - It has core M/L ratio $= \rho(0)/j(0)$ where $j(0) = 0.495 I(0)/r_0$
- At large R, isothermal models have $j(r) \propto R^{-2}$ with divergent mass.
- King models introduce a cutoff in energy that truncates the outer profile yielding finite mass.

Important Note: These models in general do **not** make good fits to Elliptical galaxies.
 They are included here (i) for historical reasons, and (ii) they yield quick estimates of $\rho(0)$ and M/L.

(ii) The Core Sersic Profile

One can combine an inner power law and a Sersic outer profile: [\[image\]](#)

An alternative approach is simply to measure the inner deviation from a best fit Sersic law.
 This was the approach of Kormendy et al (2009 [o-link](#))

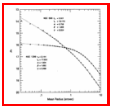


(iii) The Two-Power-Law "Nuker" Profile

Lauer et al (1995 [o-link](#)) introduced a broken power-law function to fit around the break radius.
 They call it the "Nuker Profile" (referring to their team's name):

$$I(R) = I(R_b) 2^{(\beta-\gamma)/\alpha} \left(\frac{R}{R_b}\right)^{-\gamma} \left[1 + \left(\frac{R}{R_b}\right)^\alpha\right]^{(\gamma-\beta)/\alpha} \quad (7.8)$$

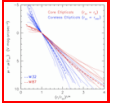
- This is a **five** parameter fit which describes two power laws: [image]
- For $R \gg R_b$ we have $I(R) \propto R^{-\beta}$ describing the outer power law
- For $R \ll R_b$ we have $I(R) \propto R^{-\gamma}$ describing the inner cusp or core
- The "break" between the two regions comes at R_b with $I(R_b)$
- α sets the sharpness of the transition near R_b .



(b) Results: Two Types, Cores & Power-laws

For significant samples, "Nuker" profiles were fitted, and showed : [image]

- There are **no** cases where $I(R)$ is flat at the center; all continue to rise down to 0.1 arcsec.
- The inner profiles divide into **two groups**:
 - **Power laws**: profile keeps rising steeply: $j(r) \propto r^{-1.9}$ with $I(R)$ diverging at $R=0$
 - **Cuspy cores**: profile breaks to shallower power-law: $j(r) \propto r^{-0.8}$ with $I(R)$ finite at $R=0$
- These two types depend on the galaxy's total **luminosity**:
 - Nuclear **power laws** are found in **Lower** Luminosity Ellipticals and Spiral bulges ($L < \sim L_*$)
 - Nuclear **cores** are found in **Higher** Luminosity Ellipticals

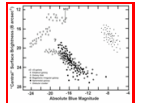


(c) Summary: Three Kinds of Ellipticals

There now seems to be **three** types of "Elliptical" galaxies:
[[image](#)] In order of increasing total luminosity:

Spheroidals: (diffuse dEs and dSph).

- Luminosity profiles more like disks ($n \sim 1$).
- They follow a sequence in M_V vs $\mu(0)$ which is similar to disks.
- In the cube R_e, M_V, μ_e , they are distinct from true ellipticals.
- Local group examples have multiple age populations (e.g. 10 Gyr + 6 Gyr).
→ They are gas stripped dIrr and dS galaxies.



Coreless Ellipticals:

- Have central profiles with no break, a steep inner power law to the smallest radii.
- They are lower luminosity Ellipticals, fainter than $M_V \sim -20.5$.
- They have **high** central brightness, **decreasing** to lower luminosity galaxies.
→ They may have formed from **wet mergers** of other ellipticals.
→ Gas moves to center to form new stars, leaving a denser core.

Core Ellipticals:

- Have central profiles with a **break to shallower slope** -- a "cuspy (not flat) core"
- They are **luminous** Ellipticals, brighter than $M_V \sim -20.5$.
- They have relatively **low** central brightness, **increasing** to lower luminosity galaxies.
→ They may have formed from **dry mergers** of other ellipticals (no gas dissipation).
→ Maybe binary black holes "scour out" central regions.

Next

Prev

Top

(4) Scaling Relations

There are many correlations between the various properties of Ellipticals.
The tightness of some are quite remarkable, and point to an underlying homogeneity of this class of galaxy.

(a) Early 2-Parameter Correlations

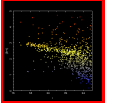
(i) Color-Magnitude and Similar Correlations

Several correlations exist between :

- Parameters tracking metallicity (and/or age): (B-V) and Mg_2 strength
- Parameters of total galaxy mass: M_B and σ_e

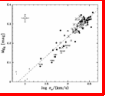
Color-Magnitude Relation

- More luminous ellipticals are slightly redder [image]



Mg_2 vs Velocity Dispersion

- Galaxies with deeper potentials have stronger Mg_2 [image].

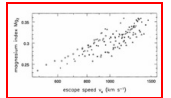


We conclude: more massive galaxies are more **metal rich** (stronger Mg_2 & UV blanketing)

The reason: deeper potentials hold ISM longer allowing metals to build up

Note: there are similar correlations **within individual galaxies**

Suggests metallicity in fact correlates with **escape velocity** [image].

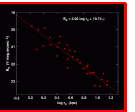


(ii) Size & Luminosity vs Surface Brightness (Kormendy) Relation

A couple of correlations suggest larger, more luminous galaxies have lower surface brightness

$\langle I_e \rangle$ correlates with R_e : Kormendy Relation [image]

- $R_e \propto \langle I_e \rangle^{-0.83 \pm 0.08}$



$\langle I_e \rangle$ correlates with L_{tot} :

- $L_{tot} \propto \langle I_e \rangle^{-2/3}$
- this follows from the above relation, given $L_e = 1/2 L_{tot} = \pi \langle I_e \rangle R_e^2$

We conclude: larger and more luminous galaxies are **fluffier** with **lower densities**

An interpretation is not yet too clear, though galaxy formation models must explain it.

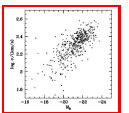
One inference: low-luminosity ellipticals formed with more gaseous dissipation than giant ellipticals.

(iii) Luminosity vs Velocity Dispersion (Faber-Jackson) Relation

Faber & Jackson (1976) first found that more luminous ellipticals and bulges have deeper potentials

σ_e correlates with L_{tot} : [image]

- $L \propto \sigma_*^n$ with $3 < n < 5$
- The scatter is ~ 0.6 mag (greater than measurement errors)
- A very rough argument shows why this might apply :
 $V^2 \propto GM/R$ and $SB \propto L/R^2$ (independent of distance)
 Squaring the first and substituting the second, we get $L \times SB \propto (M/L)^2 \propto V^4$
 If we assume M/L and SB for ellipticals doesn't vary much, then we have $L \propto V^4$
 This underlies both the Tully-Fisher and Faber-Jackson relations.



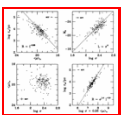
(b) The 3-Parameter Fundamental Plane

The above 2-parameter correlations have considerable **real scatter** ([image], B&M 4.43)

Furthermore, the residuals in one plot correlate with those in another.

This suggests we look for a tighter correlation among **three** parameters:

- A tilted **plane** of points in 3-D volume, which
- Projects onto 2-D planes as the (looser) correlations seen above
- One example is: $\text{Log } R_e = a \text{ Log } \sigma + b \text{ Log } I_e + c$



The choice of the 3 parameters is not unique

Three choices have been studied --- they are essentially equivalent.

(i) Log R_e , $\langle \mu_e \rangle$, Log σ_e (Djorgovski & Davis 1987)

Here, R_e is in kpc; $\langle \mu_e \rangle$ is in B mag/ss; σ_e is in km/s

[Note : we could have used μ_e rather than $\langle \mu_e \rangle$ or even $L_{tot} = 2 L_e = 2 \pi R_e^2 \langle \mu_e \rangle$]

Several statistical methods can identify/characterise correlations in n-dimensions :

- Principal Component Analysis (PCA)
- Multiple Linear Regression
- Partial Correlation Analysis

Using these, we find the equation of the **Fundamental Plane** to be :

▪ $\text{Log } R_e = 0.36 \langle \mu_e \rangle + 1.4 \text{Log } \sigma_e + \text{const}$ [normal vector (-0.65, 0.22, 0.86)]

Viewed edge on, the plane has very little scatter ~15% [image]

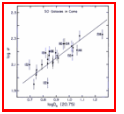
(ii) The $D_n - \sigma$ Relation (Dressler et al 1987 : Seven Samurai)

Before the F-P was found, a very tight 2-parameter correlation was identified: [image]

D_n vs σ_e , where

D_n = Diameter (in kpc) where $\langle \mu \rangle = 20.75$ B mag/ss

(the actual value of 20.75 is not important)



It turns out that this choice of parameters renders the F-P essentially edge-on
Here's why :

For $R^{1/4}$ law, integration gives $D_n \propto R_e \langle I_e \rangle^{0.8}$ or, equivalently

▪ $\text{Log } D_n = \text{Log } R_e + 0.8 \text{Log } \langle I_e \rangle = \text{Log } R_e - 0.32 \langle \mu_e \rangle$

(since $\langle \mu_e \rangle = -2.5 \text{Log } \langle I_e \rangle$)

Substituting for R_e in the F-P relation, we get :

▪ $\text{Log } D_n + 0.32 \langle \mu_e \rangle = 0.36 \langle \mu_e \rangle + 1.4 \text{Log } \sigma_e$

$\text{Log } D_n = 1.4 \text{Log } \sigma_e - 0.02 \langle \mu_e \rangle$

and we see that the dependency on $\langle \mu_e \rangle$ has essentially vanished, leaving

$D_n \propto \sigma_e^{1.4}$: a tight 2-parameter correlation

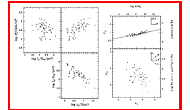
(iii) Kappa Space : $\kappa_1 \ \kappa_2 \ \kappa_3$ (Bender et al 1993)

A deliberate attempt to render the F-P "edge-on" using more physical parameters:

• $\kappa_1 = 2^{-1/2} \text{Log}(\sigma_e^2 R_e) \propto \text{Log } M$ (M = Mass)

• $\kappa_2 = 6^{-1/2} \text{Log}(\sigma_e^2 I_e^2 / R_e) \propto \text{Log} [I_e (M/L)^{1/3}]$

• $\kappa_3 = 3^{-1/2} \text{Log}(\sigma_e^2 / I_e / R_e) \propto \text{Log} (M/L)$



In this K-space we find tight projections in κ_1 vs κ_3 [image]

This suggests a narrow range of M/L which correlates weakly with total mass

(iv) The Physical Basis of the Fundamental Plane

The following gives some insight into the origin of the F-P relation :
Consider:

• $\langle I_e \rangle = \frac{1}{2} L_{tot} / \pi R_e^2$ (just a definition)

• $M/R_e = c \sigma_e^2$ (virial equilibrium, $KE \propto PE$; c = "structure parameter" containing all details)

Taken together, these give:

- $R_e = (c/2\pi) (M/L)^{-1} \sigma_e^2 <I_e>^{-1}$ or equivalently,
- $\text{Log } R_e = \text{Log} [(c/2\pi) (M/L)^{-1}] + 2 \text{Log } \sigma_e - \text{Log } <I_e>$ or
- $\text{Log } R_e = \text{Log} [(c/2\pi) (M/L)^{-1}] + 2 \text{Log } \sigma_e + 0.4 <\mu_e>$ (since $<\mu_e> = -2.5 \text{Log } <I_e>$)

So, if c and M/L are constants, then we expect

- $\text{Log } R_e = 2 \text{Log } \sigma_e + 0.4 <\mu_e> + \text{Log} [(c/2\pi) (M/L)^{-1}]$

Which is **close to**, but not quite, the F-P relation:

- $\text{Log } R_e = 1.4 \text{Log } \sigma_e + 0.36 <\mu_e> + \text{const}$

To bring these into agreement, we require:

- $(2\pi/c) (M/L) \propto M^{1/5} \propto L^{1/4}$

We conclude:

- The F-P is rooted principally in virial equilibrium
- To first order, the M/L ratios and dynamical structures of ellipticals are **very similar**
This, in turn, suggests the populations, ages & dark matter properties are highly uniform
- There is a weak trend for M/L to **increase** slightly with Mass ($\times 3$ across 5 magnitudes)
- The actual M/L values, ~ 10 -20 ($h=1$), are consistent with **no** dark matter (within R_e)
- The narrow scatter on F-P and $Mg_2 - \sigma$ relations place limits on the **ranges** of ages and metallicities:
Ages $\sim 10 - 13$ Gyr; $Z \sim 2-4 Z(\text{solar})$
- None of these relations seems to depend on **environment**: internal properties are relatively robust

(v) Use of F-P in Distance Measurement

Much motivation for the above work was to improve methods of distance measurement

In general, if a ΔV (km/s) correlates with a luminosity or size, we have a distance indicator, eg :

- Tully-Fisher : ΔV_{rot} vs M_I
- Faber-Jackson : σ_e vs M_B

Both the F-P and the $D_n - \sigma$ relations yield a **physical** length (kpc) from SB & σ , with low scatter (~ 10 -15%)

This has been used to derive **distances** :

- Used in the distance ladder to get H_0
- Used with cz to map the peculiar velocity field and large scale flows

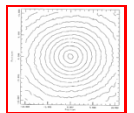


(5) The Shape of Elliptical Galaxies

To first order, isophotes are concentric, aligned, ellipses: [\[image\]](#)

Ellipticity $\epsilon = 1 - b/a$, eccentricity $e = 1 - (b/a)^2$ [$e = \epsilon(2 - \epsilon)$; $\epsilon = 1 - (1 - e)^{1/2}$]

Recall, ellipticals are classified E_n where $n = 10\epsilon$



The **apparent** ellipticity combines the **true** shape and **projection** effects

Hence, unlike other morphological designations, n (in E_n) is **not intrinsic**

(a) 3-D Shapes

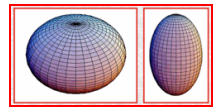
We start assuming that surfaces of constant density are **ellipsoidal**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = r^2 \quad (7.9)$$

where a, b, c may be functions of r

Basic questions :

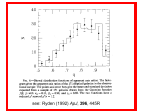
- Are ellipticals predominantly: [\[image\]](#)
 - oblate** ie $a = b > c$ (flying saucer)
 - prolate** ie $a > b = c$ (cigar)
 - triaxial** ie $a > b > c$ (smooth box)
- What is the **distribution** of true shapes?



The distribution of **observed** axial ratios, $N(b/a)$, is shown here: [\[image\]](#)

Note: it has a **rise** from E0 to E2, followed by a decline to E7

Can we reproduce this from a random orientation of oblate or prolate ellipsoids?



- Projected shape of ellipsoids is more complex than that of disk (eg B&M 4.3.3)
- However, difficult to generate **rising** distribution from E0 to E2 with just oblate or prolate
- Can** be fit by distribution of triaxial, closer to oblate than prolate:
 - $b/a \sim 0.98$ (close to oblate)
 - $c/a \sim 0.69$ (quite flattened)
 - each have Gaussian dispersion ~ 0.11

The conclusion of triaxiality is supported by the presence of **isophote twists** in many ellipticals: [\[image\]](#)

- PA of major axis **changes** with radius (so can ϵ)
- Cannot** result from projection of oblate or prolate shapes
- Intrinsically twisted galaxies are not stable
- Can occur if triaxial with axial ratios varying with radius.



(b) Isophote Shapes: Boxy & Disky

- Isophotes are not **exact** ellipses : typical deviations \sim few %
- In general, one can express the isophote as a Fourier series :

$$R(\phi) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\phi) + \sum_{n=1}^{\infty} b_n \sin(n\phi) \quad (7.10)$$

a_0 is the mean radius

a_1, b_1 define the ellipse center

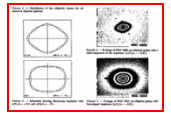
a_2, b_2 define the eccentricity and position angle

a_3, b_3 are useful diagnostics of dust (asymmetries)

Note : a_3 and all b_n are zero for 4-fold symmetry

a_4 defines the boxiness or diskiness (pointiness is better term)

- parameter a_4/a_0 typically in the range -0.02 to $+0.04$ [\[image\]](#)
 - $a_4 < 0$: boxy
 - $a_4 > 0$: disky



The a_4 parameters are very important since they correlate with many other variables (see below, § 8)



(6) Kinematics of Elliptical Galaxies

(a) Methods of Analysis

If stars produced single isolated emission lines, their (projected) velocity field would be easy to find:

The distribution of projected velocities $N(v) \propto F(\lambda)$ i.e. the emission line profile

However, Ellipticals have complex **absorption line** spectra:

Similar to a K giant, but **broadened** by Doppler motion of the stars: [\[image\]](#)

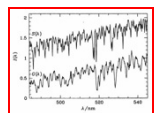
Consider :

$S(\lambda) =$ **Stellar Template** = a single star spectrum

$N(v) =$ relative (normalised) number of stars of **projected** velocity v (ie v_{los})

$N(v)$ is usually called the LOSVD (Line Of Sight Velocity Distribution)

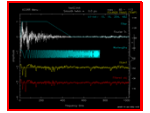
$G(\lambda) =$ observed (broadened) galaxy spectrum



Loosely speaking, $G(\lambda)$ is the same as $S(\lambda)$ convolved (smoothed) by $N(v)$
 We observe $G(\lambda)$ and $S(\lambda)$ and try to obtain $N(v)$.

Details :

- First rebin $G(\lambda)$ and $S(\lambda)$ into pixels of $\Delta u = c \text{ Log}(\Delta\lambda)$ space, i.e. km/s/pix rather than $\text{\AA}/\text{pix}$
 our convolution is, in fact: $G(u) = S(u) \otimes N(u)$ (where \otimes is convolution)
- Sum several template stars to match the overall galaxy stellar population
 template **mismatch** is a principle source of error
- Remove low frequencies (e.g. $>50\text{\AA}$ continuum variations) and high frequencies (noise)
 do this in Fourier space: apply filter to FT and transform back [\[image\]](#)



Several methods have been devised to extract $N(v)$

(i) Fourier Quotient (Sargent et al 1977)

Writing Fourier transforms (in k space) in **bold** face :
 Starting with the galaxy spectrum :

$$G(u) = S(u) \otimes N(u) \tag{7.11}$$

From the convolution theorem we have :

$$\mathbf{G(k)} = \mathbf{S(k)} \times \mathbf{N(k)} \tag{7.12}$$

giving

$$\mathbf{N(k)} = \mathbf{G(k)} / \mathbf{S(k)} \tag{7.13}$$

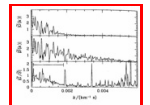
We **cannot** simply inverse transform $\mathbf{N(k)}$ because noise is introduced by the division.

Instead, **assume** $N(v)$ is Gaussian, so $\mathbf{N(k)}$ is also Gaussian

Estimate $\mathbf{N(k)}$ by fitting a Gaussian to the quotient [\[image\]](#)

from this fit, we quickly obtain $N(v)$ as a Gaussian

so the LOSVD is characterized by just cz , σ , and γ (effective line strength)



(ii) Cross Correlation (Tonry and Davis 1979)

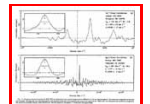
It is not difficult to show that:

$$G(u) \odot S(u) = N(u) \otimes [S(u) \odot S(u)] \tag{7.14}$$

where \odot is **cross-correlation** and \otimes is convolution
 (note $S(u) \odot S(u)$ is also called **auto-correlation**)

In English: the cross-correlation of the galaxy and template spectra is just
 the cross-correlation of the template with itself convolved by the broadening function.

In general, cross-correlating the galaxy and template produces an **offset peak** [\[image\]](#)
 Cross-correlating the template with itself produces a **narrower peak at zero offset**



- The **offset** of the peak from zero gives the redshift
- The **difference in shape** of the two peaks gives the LOSVD
- In practice, only Gaussians are used to model $N(u)$, yielding σ

(iii) Other Methods (more recent)

A number of related methods have been devised since these originals

Many try to extract more information from the LOSVD (i.e. deviations from Gaussian)

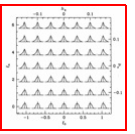
These characterisations include:

- Sums of Gaussians of different velocity, width, and strength
- Higher classical moments (ie $k > 2$), eg $\zeta_k = \mu^k / \sigma^k$ where μ is the k^{th} moment
 however, they weight the noisy wings too much, so alternatives are better

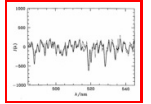
- Gauss-Hermite functions: h_k (Gaussians multiplied by a polynomial)

Ultimately, we really only deal with two extra parameters for the LOSVD: [image]

- Skewness ($k=3$), i.e. asymmetric slope to higher/lower velocities
- Kurtosis ($k=4$), i.e. stubby/peaky



In practice, these parameters are evaluated as part of an optimized χ^2 fit [image] to the observed spectrum of the template convolved by a parameterised LOSVD



(b) Amount of Rotation

(i) Expectations (pre-1975)

Ellipticals should be **rotationally flattened**

Assuming axisymmetry and isotropic velocities, stellar dynamics gives (Topic 8.5f):

$$\left(\frac{V_r}{\sigma_e} \right) = \left[\frac{1-b/a}{b/a} \right]^{\frac{1}{2}} = \left[\frac{\epsilon}{1-\epsilon} \right]^{\frac{1}{2}} \quad (7.14)$$

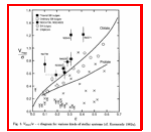
To compare with observations, let's define:

$$\left(\frac{V_r}{\sigma_e} \right)^* = \left(\frac{V_r}{\sigma_e} \right)_{obs} / \left(\frac{V_r}{\sigma_e} \right)_{expect} = \left(\frac{V_r}{\sigma_e} \right)_{obs} / \left[\frac{\epsilon}{1-\epsilon} \right]^{\frac{1}{2}} \quad (7.16)$$

(ii) Results

The rotation amplitudes results are shown in a few ways: [image]

- as V_r / σ_e vs ϵ
- as $(V_r / \sigma)^*$ vs M_B
- as $(V_r / \sigma)^*$ vs a_4



We conclude:

- Lower luminosity ellipticals and bulges have $(V_r / \sigma)^* \sim 1$
→ **Are** rotationally flattened
- For **luminous** ellipticals, $(V_r / \sigma)^* < 1$
→ **Not** rotationally flattened
→ Flattened by **velocity anisotropy** (i.e. $\sigma_x \neq \sigma_y \neq \sigma_z$ and $R_x \propto \sigma_x$ etc.)
→ Further evidence for triaxiality

Interestingly, $(V_r / \sigma)^*$ correlates **even better** with a_4/a as expected:

- Disky galaxies rotate
- Boxy galaxies don't rotate

(c) Axis of Rotation

Naively, for an axisymmetric rotating galaxy, one expects:

- Major axis slit should show **maximum** rotation
- Minor axis slit should show **no** rotation
- i.e. kinematic axis = photometric minor axis

While for a triaxial rotating galaxy, one expects:

- The projected photometric minor axis need not align with any true axis
- The rotation axis can be anywhere in the plane defined by the longest and shortest axes

What do the data suggest?

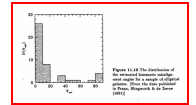
Let Ψ = the projected kinematic misalignment
= angle between kinematic and photometric minor axes

For some fiducial radius, R_f , a good estimate of this is:

$$\Psi_{est} \approx \arctan \left[\frac{V_r(R_f)_{minor\ axis}}{V_r(R_f)_{major\ axis}} \right]$$

(7.17)

A histogram of Ψ_{est} shows: [\[image\]](#)



- Most are ~ 0
- Some are 0 - 90
- Significant minor-axis rotation occurs in boxy Ellipticals, suggesting triaxiality.

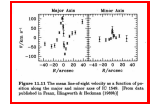
(d) Kinematically Distinct Cores and Dust Lanes

$\sim 25\%$ Ellipticals show a **separate, rotating** component in the nuclear regions ($\sim 1\text{kpc}$; $0.1-0.3 R_e$)

These are called **Kinematically Distinct Cores (KDC)**

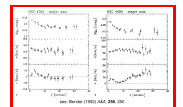
Projection effects and difficult detection suggest maybe 30%-60% Ellipticals have KDCs.

Here are some examples: [\[image\]](#)



The KDCs show the following :

- Rapid rotation ($V_r / \sigma)^* > 1$, with a range from "warm" to "cold": $V_r / \sigma = 1$ to 4.5
- Kinematic axis aligned with the photometric axis.
- Some KDCs even **counter-rotate** relative to the host
Clearly, they have a different (later?) origin than the main galaxy
- They have higher metallicity than the rest of the galaxy [\[image\]](#)
- Photometrically, they are difficult to identify (e.g. not necessarily disk isophotes) :
they don't contain much mass
kinematically prominent in LOSVD because they have **low** σ (e.g. large h_3)
- May be related to subtle **gas/dust** lanes/disks seen in **many** ($\sim 40\%$) ellipticals
Often randomly aligned at large radii but aligned with minor axis at small radii
extreme example (NGC 5128) shown here: [\[image\]](#)



KDCs (and dust lanes) are likely to be a byproduct of dissipational tidal capture

- Gas and/or star system captured
- Dissipation (loss of orbital energy) occurs: stellar system decays by dynamical friction; gas settles, losing energy by line radiation.
- Angular momentum (AM) inherited from merger (not from host): at large radii we have random orientation (of gas/dust); at center torques/precession aligns with minor axis.
- Gas disk undergoes star formation to generate a stellar disk. Stars age and disk becomes photometrically difficult to identify.

Conclusion :

Formation of ellipticals via single event is only part of story

Ongoing mergers/accretion plays at least **some** role in construction of present-day ellipticals

(7) Mass to Light Ratios

In principle :

Stellar velocities & radius give **Mass**

Photometry gives **Light**

Together, we get M/L ratios.

In practice, not so straightforward:

- Ideally, need full velocity distribution function at each location.
- Now possible to build reasonable models using LOSVD (see Topic 8).
- Usually, however, we need to assume nearly isotropic velocity field.

(a) Inner Parts

Simple estimates:

Assume approximately isothermal and fit a King profile (sec 3a above)

We find a central luminosity density, central mass density, and central M/L:

- $j(0) = I(0) / 2r_o$
- $\rho(0) = 9 \sigma(0)^2 / 4 \pi G r_o^2$
- $M/L = 9 \sigma(0)^2 / 2 \pi G I(0) r_o$

Typically, $M/L \sim 10 h M_{\odot} / L_{B,\odot}$ so dark matter **does not** dominate in the center.

(b) Outer Parts & Halo

For proper analysis, need to consider **velocity anisotropy**

- σ_r = radial velocity dispersion
extreme : radial orbits
- σ_θ = tangential velocity dispersion (assume $\sigma_\phi = \sigma_\theta$)
extreme : circular orbits

Define **anisotropy parameter** : $\beta = 1 - \langle \sigma_\theta^2 \rangle / \langle \sigma_r^2 \rangle$

We have three cases :

- $\beta = 0$: isotropic
- $\beta < 0$: tangential anisotropy
- $\beta > 0$: radial anisotropy

For Jaffe models with $\beta = \text{const}$, stellar dynamics gives (see Topic 8) :

- $M(r) = (3-2\beta)/(1-\beta) \sigma_\theta^2 r / G$

and so for three extreme cases :

- isotropic ($\beta = 0$) : $M(r) = 3 \sigma_\theta^2 r / G$
- fully tangential ($\beta = -\infty$) : $M(r) = 2 \sigma_\theta^2 r / G$
- fully radial ($\beta = 1$) : $M(r) = \infty$

Notice that $M(r)$ is sensitive to β if there is strong radial anisotropy.

Can we measure the anisotropy? Just now possible to do this:

β affects h_4 : the LOSVD kurtosis [\[image\]](#)

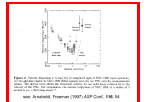
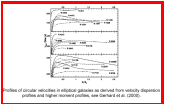
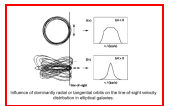
- tangential ($\beta < 0$) gives **stubby** LOSVD with $h_4 < 0$
- radial ($\beta > 0$) gives **peaky** LOSVD with $h_4 > 0$

These type of measurements can yield the mass profile or equivalently, the circular rotation curve: [\[image\]](#).

Note that if σ **increases** at large R, we **know** σ_θ is increasing in this case Dark Matter is clearly **present**.

In practice, the most distant tracers of the potential are GCs and PN.

They **do** suggest DM halos are present [\[image\]](#)



(8) Two Kinds of Ellipticals : Boxy and Disky

Summary of properties which differ for **boxy** ($a_4 < 0$) and **disky** ($a_4 > 0$) ellipticals and bulges :

| Property | Boxy ($a_4 < 0$) | Disky ($a_4 > 0$) |
|----------|--------------------|---------------------|
| | | |

| | | |
|-------------------------|--|------------------------------------|
| Luminosity | high : $M_B < -22$ | low : $M_B > -18$ |
| Rotation Rate | slow/zero : $(V_r / \sigma)^* < 1$ | faster : $(V_r / \sigma)^* \sim 1$ |
| Flattening | velocity anisotropy | rotational |
| Rotation Axis | anywhere | photometric minor axis |
| Velocity Field | anisotropic | nearly isotropic |
| Shape | moderately triaxial | almost oblate |
| Core Profile | cuspy core | steep power law |
| Core Density | low | high |
| Radio Luminosity | radio loud and quiet $10^{20} - 10^{25}$ W/Hz | radio quiet $< 10^{21}$ W/Hz |
| X-ray Luminosity | high | low |

Some of these are shown here: [\[image\]](#)

Note that to first order : Boxy and Disky galaxies have the **same** :

- color-magnitude relation
- M_{g_2} vs σ relation
- Fundamental Plane relation

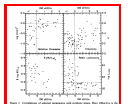
It is still unclear quite how to interpret this dichotomy :

The two types may be closely related, or may have quite different histories

Semi-empirically, Kormendy and Bender suggest a modified Hubble diagram [\[image\]](#)

- Disky Ellipticals form an extension of the S0s
- Boxy Ellipticals lie at the extreme left end
they may or may not be related to the other ellipticals and S0s

This all has important implications for Elliptical Formation



(9) Formation of Ellipticals

(Note Mergers and Galaxy Formation discussed more in Topics 12 & 19)

Still unclear -- but we have made progress

(a) Two scenarios discussed

(i) Monolithic Dissipative Collapse

- Early massive gas cloud undergoes dissipative collapse
- Huge starburst **during** collapse
Note: sub-mm detections of $\sim 10^{10} M_{\odot}$ cold gas at $z \sim 2-3$ with high SFR.
- Clumpiness during collapse \rightarrow violent relaxation \rightarrow \sim isothermal
incomplete violent relaxation \rightarrow non-isothermal & non-isotropic
- Probably rotate "rapidly" \rightarrow "Disky" Ellipticals ???

(ii) Hierarchical Mergers

- Early universe much denser: e.g. $z \sim 2$ density ~ 27 times higher than today.
 \rightarrow Mergers/interactions probably common.
- Sequence of galactic mergers, starting with pre-galactic substructures
- Galaxies continue to grow during $z \sim 1-2$
Note : HST finds **old** ellipticals at $z \sim 0.5$
- Galaxies fall into clusters and merging ceases (encounter velocities too high)
- Random accretions \rightarrow low AM & anisotropic \rightarrow "Boxy" Ellipticals ???

(b) Relevant Issues & Problems

(i) The Need for Dissipation (loss of energy)

- High stellar densities require **dissipation** during formation
 - eg collapse factors of ~ 10 needed to convert spiral disks to elliptical cores
 - eg at $M_B \sim -16$, Dwarf Spiral vs M32 we have $r_c \sim 700\text{pc}$ vs 1pc and $\mu(0) \sim 23$ vs 11
- Unlikely/impossible by mergers of **stellar systems** alone
 - such mergers cannot increase the (phase-space) density
- Dissipation requires **gaseous** collapse/merger & subsequent star formation

(ii) Observational Support for Importance of Mergers

- Ongoing mergers are seen (eg NGC 7252, NGC 6240, Arp 220)
 - They have $\sim R^{1/4}$ profiles (in K light)
 - Have **high** central (gas) densities $\sim 10^2 M_\odot \text{pc}^{-3}$ comparable to Ellipticals of similar total mass
 - Lie close to the F-P for Ellipticals (σ from CO; photometry in K)
- Brightest cluster galaxies are clearly accreting (eg multiple nuclei)
- Ongoing accretion is common in ellipticals:
 - KDCs
 - gas/dust lanes in non-equilibrium configurations
 - ripples & shells(however, don't confuse minor accretion with major merger formation).

(iii) Theoretical Support for Importance of Mergers

- Compact groups have expected lifetimes $\sim 10^{8-9}\text{yr}$ \rightarrow must merge
- N-body simulations of major mergers of two bulge/disk/halo galaxies $\rightarrow R^{1/4}$ however, highest densities inherited from pre-existing bulges
- N-body simulations of mergers **including gas** (in SPH formalism) and **star formation** ($\text{SFR} \propto \rho(\text{gas})^{1.5}$) yields:
 - rapid gas collapse to center (before merger complete) with nuclear starburst
 - final profile: dense core with "break" in power-law

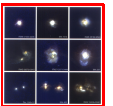
(iv) Problems to Overcome

- Dense nuclei survive mergers - so why do giant Es have larger **lower density** cores if built from denser bulges and Es?
- Mergers of Es &/or bulges should destroy the F-P
- GC frequency (per unit luminosity) is ~ 25 times higher in Es than Spirals (maybe GCs form during merger - e.g. as in NGC 1275)
- difficult to generate metallicity vs luminosity correlation by merging lower luminosity systems (unless metals made during merging starburst)

(c) Hybrid : Merger Induced Dissipational Collapse

Kormendy & Sanders (1992) combine the two formation scenarios:

Similar to Ultra-Luminous Infra-red Galaxies (ULIRGs; e.g. Arp 220 etc; [image](#))



- They are **mergers**, showing tidal debris etc
- They have **high central densities** $\sim 10^2 M_\odot \text{pc}^{-3}$, similar to the central stellar densities of ellipticals
- They have similar gas mass and stellar mass in the central regions
 - \rightarrow dissipation has occurred, sending the gas to the center
- There is a huge ongoing starburst, creating a homogeneous metal rich population

We have, in these systems, a substantial dissipative collapse which is associated with a major merger

They speculate that the ULIRGs are **ellipticals caught in formation**

Interestingly, ULIRGs are also thought to be **proto-quasars** :

- feeding (and possible creation of) nuclear black holes
- star formation luminosity comparable to quasar luminosity
- currently hidden by dust (which re-radiates in the far IR)
- ultimately becomes visible when surrounding gas swept clear

The ULIRGs are locally rare - maybe they were common at high-z
Elliptical formation may be associated directly with the quasar era ?

[Prev](#)

[Top](#)

[Home](#)

[Main](#)

[Index](#)

[Toolbox](#)
