$$\Phi(\mathbf{r}) = -G \int_V \frac{\rho(\mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|} \, d^3\mathbf{r}'$$

$$\mathbf{F}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = G \int_V \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|^3} \, \rho(\mathbf{r}') \, d^3\mathbf{r}'$$

$$\nabla \cdot \mathbf{F}(\mathbf{r}) = -4\pi G\rho(\mathbf{r})$$

$$\nabla^2\Phi(\mathbf{r}) = 4\pi G\rho(\mathbf{r})$$

$$\nabla^2\Phi(\mathbf{r}) = 0$$

$$4\pi GM = 4\pi G \int_V \rho(\mathbf{r}) \, d^3\mathbf{r}$$

$$= \int_V -\nabla \cdot \mathbf{F}(\mathbf{r}) \, d^3\mathbf{r} = \int_A -\mathbf{F}(\mathbf{r}) \cdot d^2\mathbf{S}$$

$$W = \frac{1}{2} \int_V \rho(\mathbf{r}) \, \Phi(\mathbf{r}) \, d^3\mathbf{r} = -\frac{1}{8\pi G} \int_V |\nabla\Phi|^2 \, d^3\mathbf{r}$$

$$\frac{V_0^2}{r} = F_r = -\frac{d\Phi}{dr} \quad ; \quad \Phi(r) = V_0^2 \ln r + const$$

$$\rho_H(r) = \frac{Ma}{2\pi r(r+a)^3} \quad ; \quad \Phi_H(r) = -\frac{GM}{(r+a)}$$

$$\rho_J(r) = \frac{Ma}{4\pi r^2(r+a)^2} \quad ; \quad \Phi_J(r) = \frac{GM}{a} \ln\left(\frac{r}{r+a}\right)$$

$$\rho_P(r) = \left(\frac{3M}{4\pi b^3}\right)\left(1 + \frac{r^2}{b^2}\right)^{-5/2} \quad ; \quad \Phi_P(r) = -\frac{GM}{\sqrt{r^2+b^2}}$$

$$-\frac{\partial\,\Phi}{\partial z} = g_z = 4\pi G\rho_0 \, z \qquad \text{(inside)}$$

$$= 2\pi G\Sigma \qquad \text{(above)}$$

$$V_c^2(R) = 4\pi G \, \Sigma_0 \, R_d \, y^2 \, [\, I_0(y)K_0(y) - I_1(y)K_1(y) \,]$$

$$\Sigma_K(R) = \frac{aM}{2\pi(R^2+a^2)^{3/2}} \quad ; \quad \Phi_K(R,z) = -\frac{GM}{\sqrt{R^2+(a+|z|)^2}}$$

$$\Sigma_{T_n}(R) = \left(\frac{d}{da^2}\right)^{n-1} \Sigma_K(R) \; ; \; \Phi_{T_n}(R) = \left(\frac{d}{da^2}\right)^{n-1} \Phi_K$$

$$\rho_M(R,z) = \left(\frac{Mb^2}{4\pi}\right) \frac{aR^2 + (a+3B)(a+B)^2}{[R^2+(a+B)^2]^{5/2} B^3}$$

$$\Phi_M(R,z) = -\frac{GM}{\sqrt{R^2+(a+B)^2}} \quad ; \; B^2 = z^2 + b^2$$

$$\rho_{S_n}(R,z) = \left(\frac{d}{db^2}\right)^n \rho_M \; ; \; \Phi_{S_n}(R,z) = \left(\frac{d}{db^2}\right)^n \Phi_M$$

$$\frac{d}{dt}(m^\alpha v_i^\alpha) \;=\; F_i^\alpha \;=\; -\,G m^\alpha \sum_{\beta\neq\alpha} m^\beta \frac{r_i^\alpha - r_i^\beta}{|\mathbf{r}^\alpha - \mathbf{r}^\beta|^3}$$

$$\frac{1}{2}\frac{d^2}{dt^2} I_{i,j} \;=\; 2\,K_{i,j} \;+\; W_{i,j} \;=\; 2\,T_{i,j} \;+\; \Pi_{i,j} \;+\; W_{i,j}$$

$$\begin{aligned}
I_{i,j} &= \int \rho\, r_i\, r_j\, d^3 r && = \text{ moment of inertia} \\
K_{i,j} &= \int \tfrac{1}{2}\,\rho\,\langle v_i v_j\rangle\, d^3 r && = \text{ total KE} \\
T_{i,j} &= \int \tfrac{1}{2}\,\rho\,\langle v_i\rangle\,\langle v_j\rangle\, d^3 r && = \text{ ordered KE} \\
\Pi_{i,j} &= \int \rho\,\sigma_{i,j}^2\, d^3 r && = \text{ dispersion KE} \\
W_{i,j} &= -\frac{1}{2} G \int\!\!\int \rho(\mathbf{r})\,\rho(\mathbf{r}')\,\frac{(r_i - r_i')\,(r_j - r_j')}{|\mathbf{r}' - \mathbf{r}|^3} d^3 r d^3 r' && = \; PE
\end{aligned}$$

$$2K_{ij} \;+\; W_{ij} \;=\; 0$$

$$2K \;+\; W \;=\; 0$$

$$E \;=\; K \;+\; W \;=\; -K \;=\; \frac{1}{2} W$$

$$M_{tot} \;\simeq\; \frac{<v^2>R_m}{0.4\,G}$$

$$\frac{V_o}{\sigma_o} \;=\; \sqrt{2\,[(1-\delta)(1-\epsilon)^{-0.9} \;-\; 1]}$$

$$\frac{V_o}{\sigma_o} \;\simeq\; \sqrt{\frac{\epsilon}{(1-\epsilon)}}$$

$$v_x dt\, dv_x\,[f(x,v_x,t) - f(x+dx,v_x,t)] \;=\; -\,v_x\,dt\,dv_x \frac{\partial f}{\partial x}\,dx$$

$$dx\,\frac{dv_x}{dt} dt\;[f(x,v_x,t) - f(x,v_x+dv_x,t)] \;=\; -\,dx\,dt\,\frac{dv_x}{dt}\,\frac{\partial f}{\partial v_x} dv_x$$

$$dx\,dv_x\,\frac{\partial f}{\partial t} dt \;=\; -\,dt\,dx\,v_x\,\frac{\partial f}{\partial x}\,dv_x \;-\; dx\,dt\,\frac{dv_x}{dt}\,\frac{\partial f}{\partial v_x}\,dv_x$$

$$\frac{\partial f}{\partial t} \;+\; v_x\,\frac{\partial f}{\partial x} \;+\; \frac{dv_x}{dt}\,\frac{\partial f}{\partial v_x} \;=\; 0$$

$$\frac{dv_x}{dt} \;=\; a_x \;=\; -\frac{\partial \Phi}{\partial x}$$

$$\frac{\partial f}{\partial t} \;+\; v_x \frac{\partial f}{\partial x} \;-\; \frac{\partial \Phi}{\partial x}\frac{\partial f}{\partial v_x} \;=\; 0$$

$$\frac{\partial f}{\partial t} \;+\; \mathbf{v} \cdot \nabla f \;-\; \nabla \phi \cdot \frac{\partial f}{\partial \mathbf{v}} \;=\; 0$$

$$\frac{d\, f}{d\, t} \;=\; \frac{\partial f}{\partial t}\frac{dt}{dt} \;+\; \frac{\partial f}{\partial x}\frac{dx}{dt} \;+\; \frac{\partial f}{\partial v_x}\frac{dv_x}{dt} \;=\; 0$$

$$\frac{\partial n}{\partial t} \;+\; \frac{\partial \big(n \langle v_x \rangle\big)}{\partial x} \;=\; 0$$

$$\frac{\partial \langle v_x \rangle}{\partial t} \;+\; \langle v_x \rangle \frac{\partial \langle v_x \rangle}{\partial x} \;=\; -\frac{\partial \Phi}{\partial x} \;-\; \frac{1}{n}\frac{\partial (n\sigma_x^2)}{\partial x}$$

$$\frac{\partial \langle v_j \rangle}{\partial t} \;+\; \langle v_i \rangle \frac{\partial \langle v_j \rangle}{\partial x_i} \;=\; -\,\frac{\partial \Phi}{\partial x_j} \;-\; \frac{1}{n}\,\frac{\partial (n\sigma_{i,j}^2)}{\partial x_i}$$

$$\frac{\partial \mathbf{v}}{\partial t} \;+\; (\mathbf{v} \cdot \nabla)\, \mathbf{v} \;=\; -\,\nabla \Phi \;-\; \frac{1}{\rho}\, \nabla p$$

$$\frac{1}{n}\,\frac{d(n\sigma_r^2)}{dr} \;+\; \frac{1}{r}\left[2\sigma_r^2 \;-\; (\,\sigma_\theta^2 \;+\; \sigma_\phi^2)\right] \;-\; \frac{\langle v_\phi \rangle^2}{r} \;=\; -\frac{d\Phi}{dr}$$

$$\frac{1}{n}\,\frac{d(n\sigma_r^2)}{dr} \;+\; 2\beta\,\frac{\sigma_r^2}{r} \;-\; \frac{V_{\rm rot}^2}{r} \;=\; -\frac{d\Phi}{dr}$$

$$V_{\rm rot}^2 \;-\; \sigma_r^2 \left(\frac{d\ln n}{d\ln r} \;+\; \frac{d\ln(\sigma_r^2)}{d\ln r} \;+\; 2\beta\right) \;=\; \frac{GM(<r)}{r} \;=\; {V_c}^2$$

$$\begin{aligned} \frac{dI}{dt} \;&=\; \sum_{i=1}^{3} \frac{\partial I}{\partial x_i}\,\frac{dx_i}{dt} \;+\; \sum_{i=1}^{3} \frac{\partial I}{\partial v_i}\,\frac{dv_i}{dt} \;=\; 0 \\ &=\; \nabla I \cdot \frac{d\mathbf{x}}{dt} \;+\; \frac{\partial I}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} \;=\; 0 \\ &=\; \mathbf{v} \cdot \nabla I \;-\; \nabla \Phi \cdot \frac{\partial I}{\partial \mathbf{v}} \;=\; 0 \end{aligned}$$

$$\begin{aligned} 4\pi G \int f(\mathbf{r}, \mathbf{v})\, d^3\mathbf{v} \;&=\; 4\pi G \rho(\mathbf{r}) \\ &=\; \nabla^2 \Phi(\mathbf{r}) \end{aligned}$$

$$\frac{1}{r^2}\,\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) \;=\; 4\pi G \int f(\,\tfrac{1}{2}v^2 + \Phi\,, |\,\mathbf{r} \times \mathbf{v}|\,)\, d^3\mathbf{v}$$

$$\frac{1}{r^2}\,\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right) \;=\; -\,16\pi^2 G\int_0^{\sqrt{2\Psi}} f(\Psi - \tfrac{1}{2}v^2)\,v^2\,dv$$

$$\;=\; -\,16\pi^2 G\int_0^{\Psi} f(E_r)\,\sqrt{2(\Psi - E_r)}\;dE_r$$

$$f(E_r) \;=\; \frac{1}{\pi^2\sqrt{8}}\left[\int_0^{E_r}\frac{d^2\rho}{d\,\Psi^2}\,\frac{d\,\Psi}{\sqrt{E_r-\Psi}} \;+\; \frac{1}{\sqrt{E_r}}\left(\frac{d\rho}{d\Psi}\right)_{\Psi=0}\right]$$

$$N(E_r)\,dE_r \;=\; 16\pi^2\,f(E_r)\int_0^{r_m(E_r)} r^2\sqrt{2(\Psi(r)-E_r)}\,dr$$

$$\rho \;=\; 4\pi\int_0^{\infty} f(E_r)\,v^2\,dv \;=\; 4\pi F\int_0^{\sqrt{2\Psi}}(\Psi - \tfrac{1}{2}v^2)^{n-\frac{3}{2}}\,v^2\,dv$$

$$\frac{1}{r^2}\,\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right) \;+\; 4\pi G c_n\Psi^n \;=\; 0$$

$$f(E_r) \;=\; \frac{\rho_1}{(2\pi\sigma^2)^{\frac{3}{2}}}\;e^{E_r/\sigma^2}$$

$$\frac{d}{dr}\left(r^2\frac{d\ln\rho}{dr}\right) \;=\; -\,\frac{4\pi G}{\sigma^2}\,r^2\rho$$

$$f(E_r) \;=\; \frac{\rho_1}{(2\pi\sigma_0^2)^{\frac{3}{2}}}\;(e^{E_r/\sigma_0^2}-1)$$

$$\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right) \;=\; -\,4\pi G\rho_1 r^2\left[e^{\Psi/\sigma_0^2}\mathrm{erf}\left(\frac{\sqrt{\Psi}}{\sigma_0}\right) - \sqrt{\frac{4\Psi}{\pi\sigma_0^2}}\left(1 \;+\; \frac{2\Psi}{3\sigma_0^2}\right)\right]$$

$$|\Delta V_{\rm tot}|^2 \;=\; \sum|\Delta V|^2 \;=\; \int_{b_{min}}^{b_{max}}\left(\frac{2Gm}{bV}\right)^2 t\;nV\,2\pi b\;db$$

$$\;=\; \frac{8\pi G^2m^2nt}{V}\;\ln\Lambda$$

$$t_{\rm relax} \;\simeq\; 0.34\,\frac{\sigma^3}{G^2m\rho\ln\Lambda}$$

$$\;\simeq\; \frac{1.8\times10^{10}\ {\rm yr}}{\ln\Lambda}\;\sigma_{10}^3\;m_\odot^{-1}\;\rho_3^{-1}$$

$$t_{\rm relax} \;\simeq\; t_{\rm cross}\,\frac{N}{6\ln N}$$

$$\frac{d\,f}{d\,t} \;=\; \Gamma(f) \;=\; \int S(\mathbf{w}-\Delta\mathbf{w}\,,\,\Delta\mathbf{w})\,f(\mathbf{w}-\Delta\mathbf{w}) \;-\; S(\mathbf{w},\Delta\mathbf{w})\,f(\mathbf{w}) \;\; d^3\mathbf{w}$$

$$\frac{d\,f}{d\,t} \;=\; \Gamma(f) \;=\; -\sum_{i=1}^{6}\frac{\partial}{\partial w_i}[f(\mathbf{w})\,D(\Delta w_i)] \;+\; \tfrac{1}{2}\sum_{i,j=1}^{6}\frac{\partial^2}{\partial w_i \partial w_j}[f(\mathbf{w})\,D(\Delta w_i \Delta w_j)]$$

$$D(\Delta v_{\parallel}) \;=\; -\frac{16\pi^2 G^2 m_a (m+m_a)\ln\Lambda}{v^2}\int_0^v v_a^2 f_a(v_a)dv_a$$

$$D(\Delta v_{\parallel}^2) \;=\; \frac{32\pi^2 G^2 m_a^2 \ln\Lambda}{3v}\left[\int_0^v \frac{v_a^4}{v^2} f_a(v_a)dv_a \;+\; v\int_v^{\infty} v_a f_a(v_a)dv_a\right]$$

$$D(\Delta v_{\perp}^2) \;=\; \frac{32\pi^2 G^2 m_a^2 \ln\Lambda}{3v}\left[\int_0^v \left(3v_a^2 - \frac{v_a^4}{v^2}\right) f_a(v_a)dv_a \;+\; 2v\int_v^{\infty} v_a f_a(v_a)dv_a\right]$$

$$D(\Delta E) \;=\; 16\pi^2 G^2 m m_a \ln\Lambda\left[m_a\int_v^{\infty} v_a f_a(v_a)dv_a \;-\; m\int_0^v \frac{v_a^2}{v} f_a(v_a)dv_a\right]$$

$$\frac{1}{n}\,\frac{d\big(n\langle v_r^2\rangle\big)}{dr} \;+\; 2\beta\,\frac{\langle v_r^2\rangle}{r} \;=\; -\,\frac{d\Phi}{dr}$$

$$-\,\langle v_r^2\rangle\left(\frac{d\,\ln n}{d\,\ln r} \;+\; \frac{d\,\ln\langle v_r^2\rangle}{d\,\ln r} \;+\; 2\beta\right) \;=\; \frac{GM(<r)}{r} \;=\; V_c^2$$