Selection of Homework Questions

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Topic 8: Theory II : Stellar Dynamics

(1) Potential-Density Pairs

a. Use Poisson's equation in spherical polar form to show that Jaffe's (1983) spherical density distribution:

$$\rho(r) = \left(\frac{M}{4\pi r_J^3}\right) \frac{r_J^4}{r^2(r+r_J)^2} \qquad \text{(Q8.1)}$$

gives the potential:

$$\Phi(r) = \frac{GM}{r_J} \ln\left(\frac{r}{r+r_J}\right) \qquad \text{(Q8.2)}$$

Where M and r_J are constants.

b. Verify that the total mass is M.

c. Show that the circular speed is roughly constant for $r \ll r_J$ and decreases as $r^{-1/2}$ for $r \gg r_J$ (B&T-1 Q 2.3)

(2) Potential Energy

a. Show that the Gravitational potential energy of a spherical system can be written:

$$W = -\frac{G}{2}\int_0^\infty \frac{M^2(r)}{r^2}\, dr \qquad \text{(Q8.3)}$$

where M(r) is the mass interior to radius r (B&T Q2.2).

b. Evaluate this for a uniform density sphere of radius R.

c. Approximate a SN progentior star as a small (R 1.4 $M_\odot$ core of radius 10^4km plus a 20 $M_\odot$ envelope of radius 1AU, each of uniform density. Calculate the binding energy of just the envelope.

d. If the core collapses to form a neutron star of uniform density and radius 10km, and 1% of the gravitational energy released is dumped into the envelope (99% escapes as neutrinos), can the core collapse jettison the envelope?

e. If it can, what is the velocity of the ejected envelope material (assuming it all moves radially at the same velocity) ?

(3) Power Law Cores, and the Jeans Equation :

The goal of this problem is to explore the behaviour of the velocity dispersion near the center of a spherical non-rotating galaxy. At radii $r < r_o$ assume that the density has the power law form $\rho(r) = \rho_o(r/r_o)^{-\gamma}$, with $0 < \gamma < 3$. Assume that the velocity dispersion is isotropic at all radii and equal to σ_o at r_o.

a. Why is the constraint $\gamma < 3$ necessary ?

b. Use the Jeans equation in spherical form to derive an expression for the dispersion profile $\sigma^2(r)$ for $r < r_o$

c. For what range of γ does $\sigma^2(r) \to 0$ as $r \to 0$?
d. For what range of γ does $\sigma^2(r)$ diverge as $r \to 0$?
e. For what value(s) of γ is $\sigma^2(r)$ independent of r as $r \to 0$?
f. For the latter situation, what value of σ_o (expressed in terms of ρ_o, r_o, G) makes σ independent of r at **all** r ? Evaluate this for the case in which $\rho_o = 100\ M_\odot\ pc^{-3}$ and r_o=100 pc.

(4) Central Mass to Light Ratios :

Print out the pdf figure here (link) which contains light profiles for three elliptical galaxies (taken from Lauer et al). The units for μ_V are mag/ss in the V band. The central line-of-sight velocity dispersions in these galaxies are : σ(N1400) = 265 km/s; σ(N2832) = 330 km/s; σ(N3608) = 195 km/s. Assuming that the galaxies are spherical and the velocity dispersion is isotropic and the core is approximately isothermal, use "King's Method" to find the core mass-to-light ratio of each galaxy in solar units. (Note that the physical scale is plotted along the TOP axis; and think how core radius is defined in terms of central surface brightness).

(5) Relaxation Times :

Estimate the 2-body relaxation time in the following systems :

a. The galactic bulge, which we approximate as a singular isothermal sphere with circular speed V_c = 200 km/s containing stars of mass 0.6 $M_\odot$. The relaxation time should be given as a function of radius. At what radius is the relaxation time equal to 10^{10} years?
b. A typical open cluster, with median radius 2 pc, mass 250 $M_\odot$, and stellar mass 1 $M_\odot$.
c. The core of the clobular cluster M4, with core radius 0.5 pc and central surface brightness 17.88 mag/ss in V. You may assume that the typical stellar mass is 0.6 $M_\odot$ and the mass-to-light ratio is 1.6 $M_\odot/L_\odot$.

(6) Conceptual Question on DFs :

a. Systems of stars can be described by a 7-dimensional distribution function, DF or just f. What are those 7 dimensions and what, exactly, does the DF describe? What, in qualitative terms, is the form of the velocity portion of the DF for (i) stars at the galaxy center; (ii) stars in the solar neighborhood?
b. Write down the collisionless Boltzmann equation (CBE) for f, and briefly discuss each term. Why must physically plausible DFs also be solutions to the CBE? In other words, what does the CBE describe about a system of stars and the nature of the DF?
c. Imagine you are living "on" a star which is caught in a galaxy merger. Although your trajectory in physical space may hurl you through dense bulges or sparse halos, your trajectory through the 6-D position-velocity phase space keeps you moving along a path of **constant** stellar density. Why is this?
d. For a static potential, why is a distribution function with simple form f(E) automatically a solution of the CBE, where E is the energy at a particular point in position-velocity phase space? What kind of potentials would support DFs of the form f(E,|L|) and f(E,|L|,L_z)?
e. Describe, in conceptual terms, how the CBE is "processed" to yield an observationally more accessible equation: the Jeans equation? What properties of a stellar system does the Jeans equation describe?
f. Write down the Jeans equation for a spherical galaxy or star cluster. How do astronomers use the Jeans equation to derive the mass distribution in a spherical non-rotating elliptical galaxy. What basic observations and assumptions must be made, and how can higher quality observations help inform those assumptions?

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