Cosmology Lite

A brief treatment of some basic topics in cosmology

1) Overall framework: expansion and three differential relations

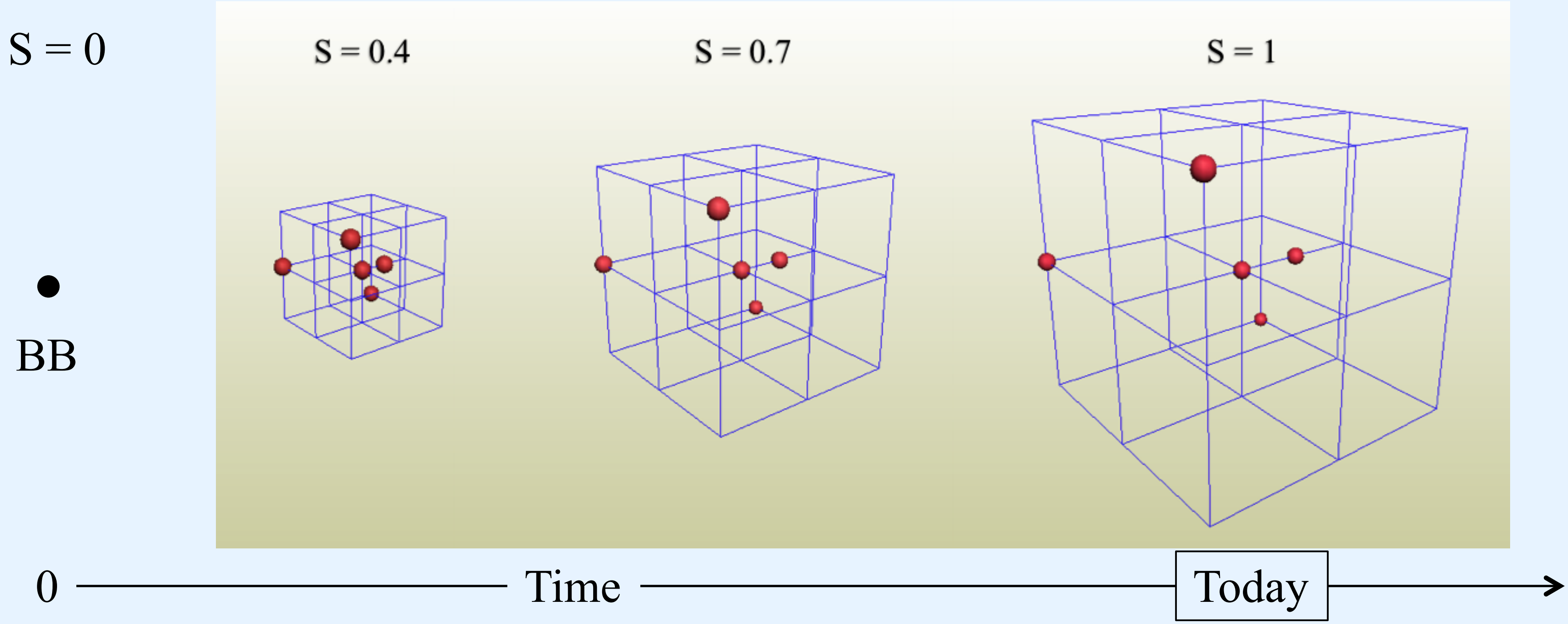
2) The Friedman equation and its solutions

3) The angular relations: curved space

4) Space time diagrams

5) Real time cosmology

Cosmic Expansion & the Scale Factor



Space (shown by the grid) expands and galaxies are "fixed" on it.
Note: The galaxies themselves do <u>not</u> expand!
Grid size specified by scale factor: *S* *S*=1 today, *S*=0 at BB (often "a" in literature)
Pick galaxy for us. Today: others galaxies at comoving distances: r_0 ("0" = today)

Other times: $r = S\, r_0$ or $S = r/r_0$ (same for all galaxies)

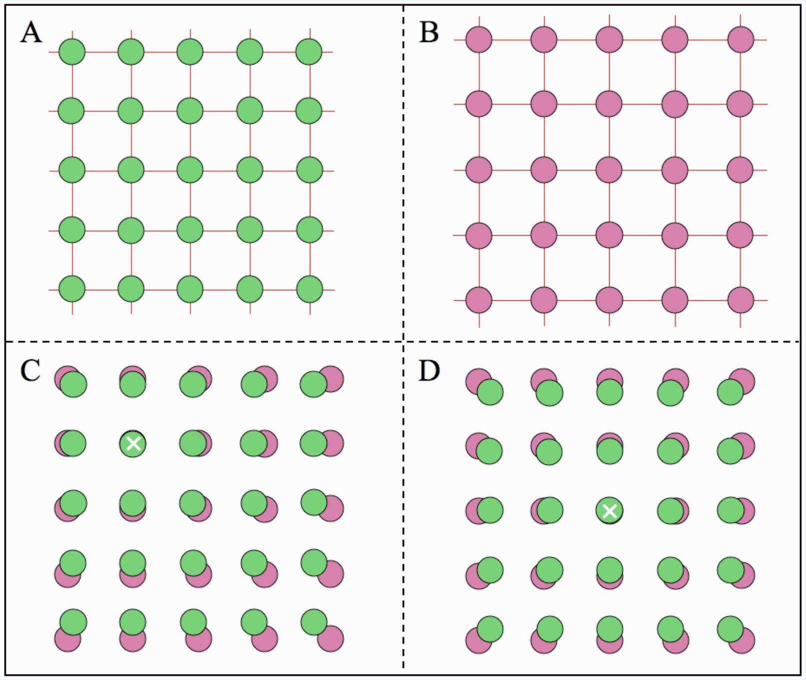
The velocity-distance law

Expanding grid: galaxy distances increase.
Cast as velocity. Take the time derivative:

$$v = \frac{dr}{dt} = \frac{r_0\, dS}{dt} = \frac{1}{S}\frac{dS}{dt} \times r \equiv H \times r$$

Thus, we have a linear velocity-distance law

$$v = H \times r \quad \text{and} \quad v_0 = H_0 \times r_0 \ \text{today.}$$



Introduces the Hubble parameter/constant: H, H_0
Units of 1/time, defining Hubble time and distance:

$$H = \frac{1}{S}\frac{dS}{dt}.$$

$$t_H = \frac{1}{H} \quad \text{and} \quad r_H = c \times t_H = \frac{c}{H}$$

Today's values:
$H_0 \approx 22$ km/s/Mly
(0.0735 Gyr^{-1})
$t_{H0} \approx 13.6$ Gyr
$r_{H0} \approx 13.6$ Gly

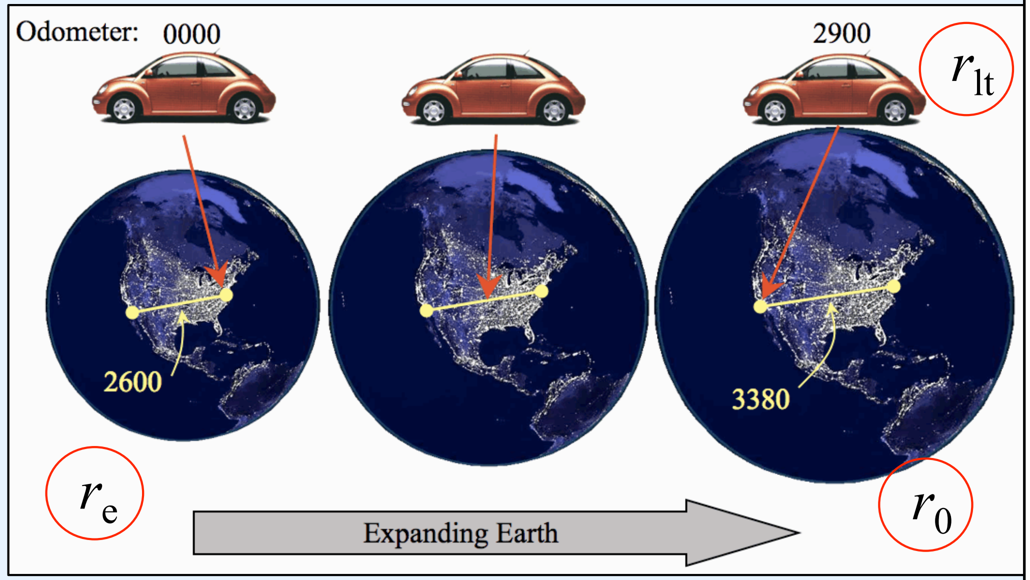
Note 1: Rigorously true to all distances (follows from uniform expansion)
e.g. At $r = r_H$, v = c; at $r = 100\, r_H$, v = $100c$. (This is OK in GR)
Note 2: Same law for all observers (galaxies) – see figure.

Light's motion across expanding space

When light moves from a distant galaxy to us, there are THREE distances to consider:

If Earth static, then all three distances are the same. But if Earth expands, they're all different.

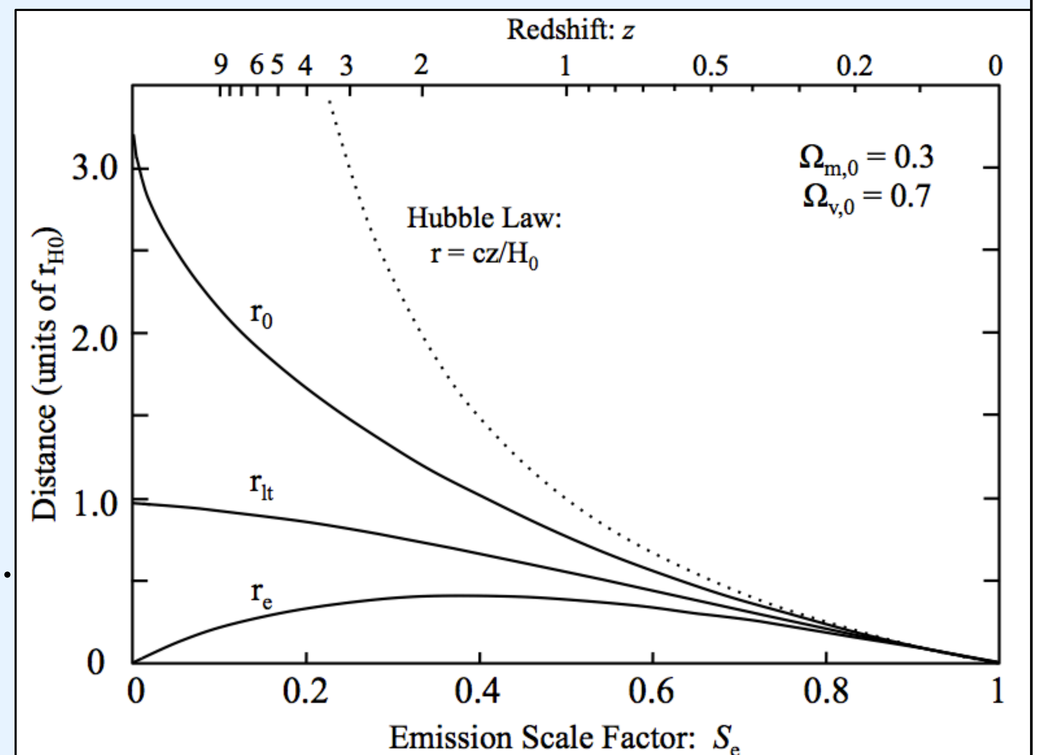
Analog: car moves at 60 mph (c) across expanding Earth



r_e emission distance = the distance when light set out.

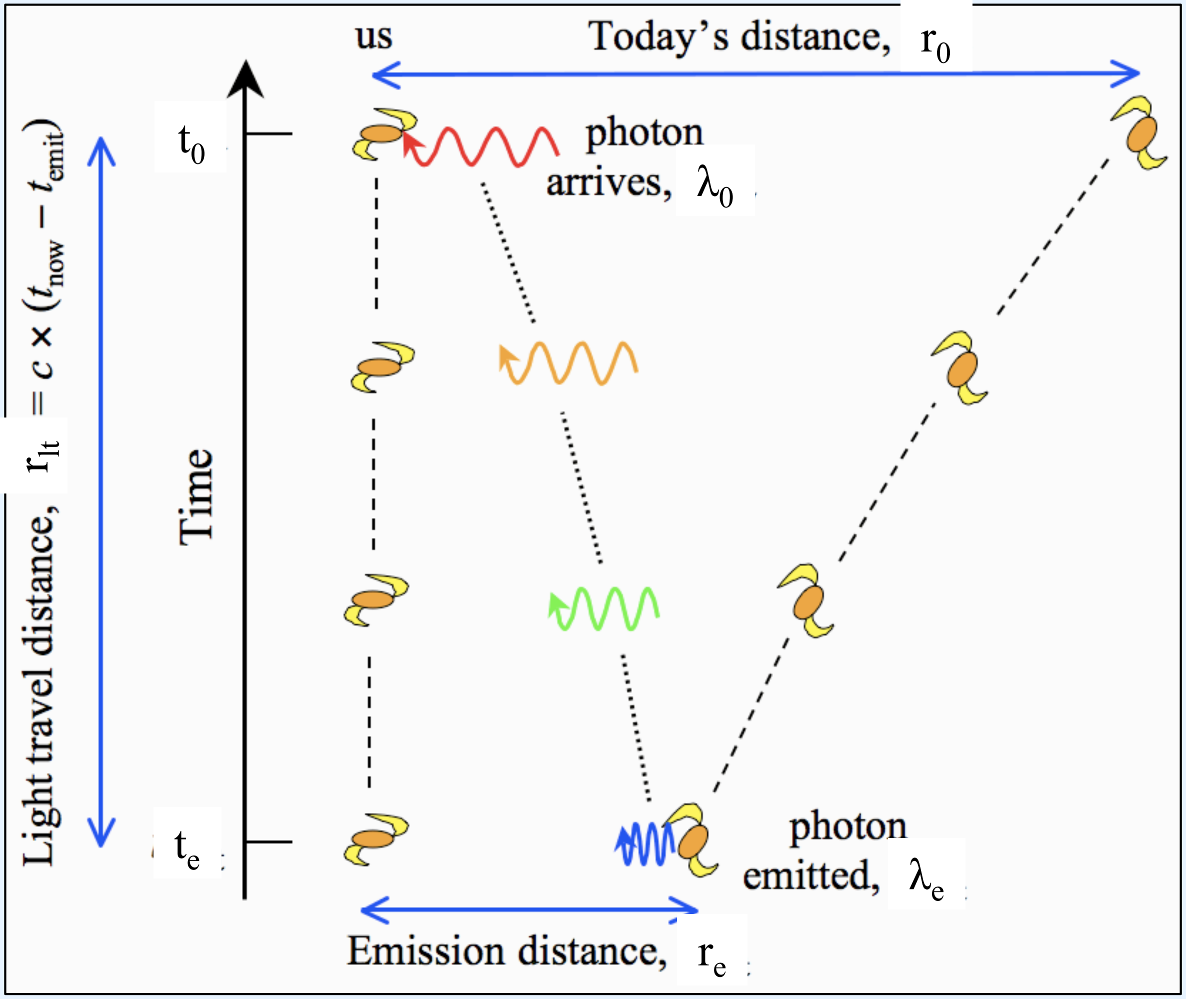
r_0 today's distance = distance when light arrives, today. (= comoving distance)

r_{lt} light travel distance (odometer reading). = journey's duration = look-back time



Redshift: S_e and z

Space-time diagram (see later)



Light waves and the Universe are stretched by the same factor

$$\frac{\lambda_e}{\lambda_0} = \frac{r_e}{r_0} = S_e$$

Reshift tells us the scale factor when the light set out!

Conventionally, use z:

$$z = \frac{\Delta\lambda}{\lambda_e} = \frac{(\lambda_0 - \lambda_e)}{\lambda_e} = \frac{\Delta r}{r_e}$$

Clearly:

$$z = \frac{1}{S_e} - 1$$ and $$S_e = \frac{1}{1+z}$$

Note: Cosmological redshift is <u>different</u> from Doppler or gravitational redshifts.

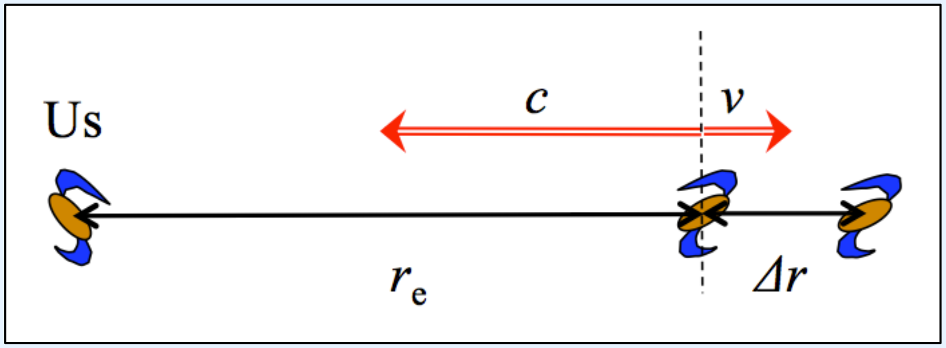
Redshift is our primary observable!!

The Hubble Law

More distant objects have longer light travel times, so expansion is greater.
➔ Expect larger redshift for more distant objects. Hubble finds this in 1926:

$$cz \approx H_0 \times D$$

This is a low redshift observational <u>approximation</u> to the velocity-distance law.
Question: is $cz = \mathrm{v}_0$ and is $D = r_0$? The answer is "<u>only approximately</u>".



$$z \approx \frac{\Delta r}{r_e} \approx \frac{v}{c} \quad \text{and} \quad D \approx r_0$$

This is an an approximate <u>Doppler</u> expression

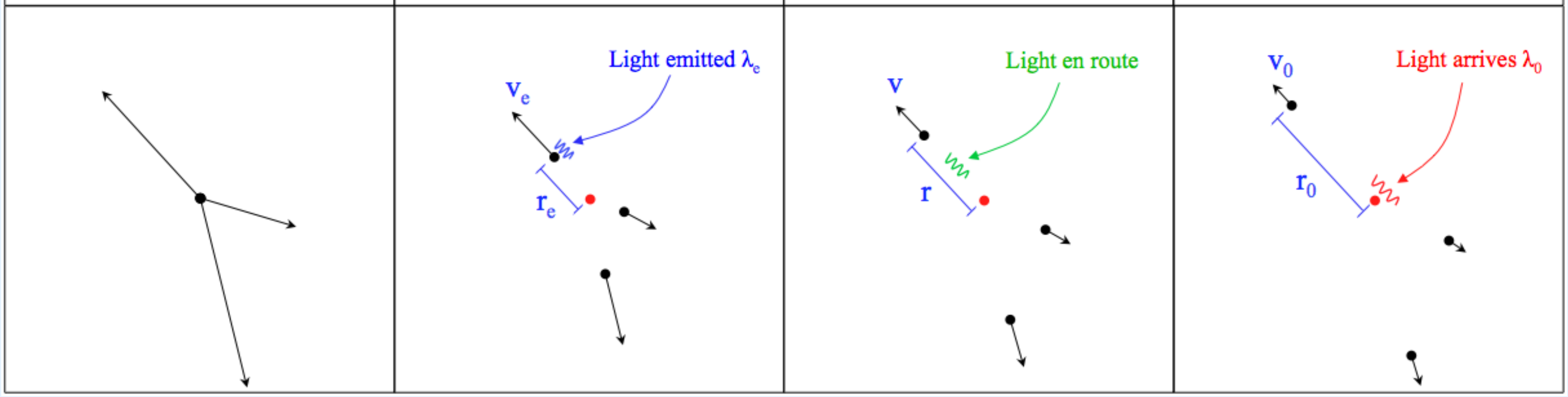
Also: $v \approx ½\,(\mathrm{v}_e + \mathrm{v}_0)$, not v_0; light travels r_{lt} not r_e; and Doppler cz isn't v_e or v_0.

The velocity-distance law: $\mathrm{v}_0 = H_0\, r_0$ is exact but unobservable.
The Hubble law: $cz \approx H_0\, D$ is an observational approximation to it.

Changing velocities; the velocity factor: *V*

How does a given galaxy's recession velocity change over time?
(Note: velocity-distance law is for a fixed time; different galaxies have $v \propto r$)

Big Bang: $t \approx 0$	Emission: $t = t_e$	Intermediate: $t = t$	Today: $t = t_0$
$S \approx 0$; $V \approx \infty$	$S_e = r_e/r_0$; $V_e = v_e/v_0$	$S = r/r_0$; $V = v/v_0$	$S = 1$; $V = 1$
$H \approx \infty$; $t_H \approx 0$		$H = v/r$; $t_H = r/v$	$H_0 = v_0/r_0$; $t_{H0} = r_0/v_0$



Analogous to scale factor is velocity factor, *V*:

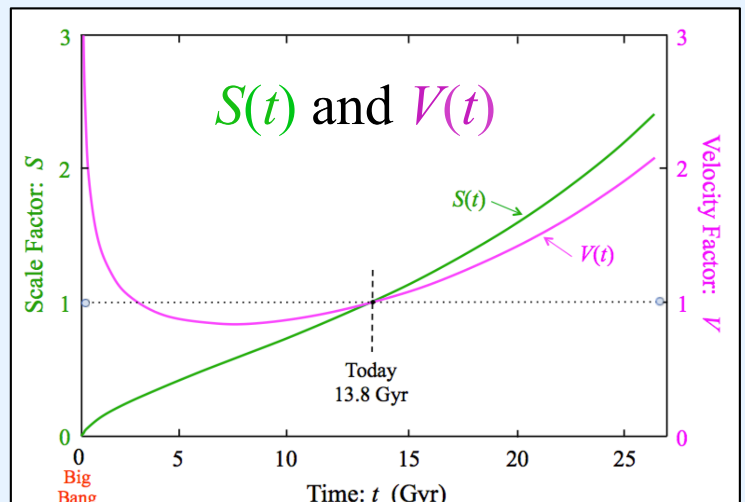
$$S = \frac{r}{r_0}\,; \qquad V = \frac{v}{v_0}.$$

S and *V* are dimensionless

Divide $v = dr/dt$ by $v_0 = H_0\, r_0$

$$V = t_{H0} \frac{dS}{dt}.$$

Normalized version of $v = dr/dt$



Three Cosmic Constituents

For our purposes, there are three kinds of cosmic constituent.

Matter: non-relativistic; energy in rest mass → no change with expansion.
Radiation ($\gamma + \nu$): relativistic; energy in motion → redshift during expansion.
Vacuum: ρ_v same for all observers; SR → ρ_v remains constant during expansion.

Their *density* changes with expansion:

Matter: $\rho_m = \rho_{m,0} / S^3$ simple dilution

Radiation: $\rho_r = \rho_{r,0} / S^4$ dilution + redshift

Vacuum: $\rho_v = \rho_{v,0}$ constant

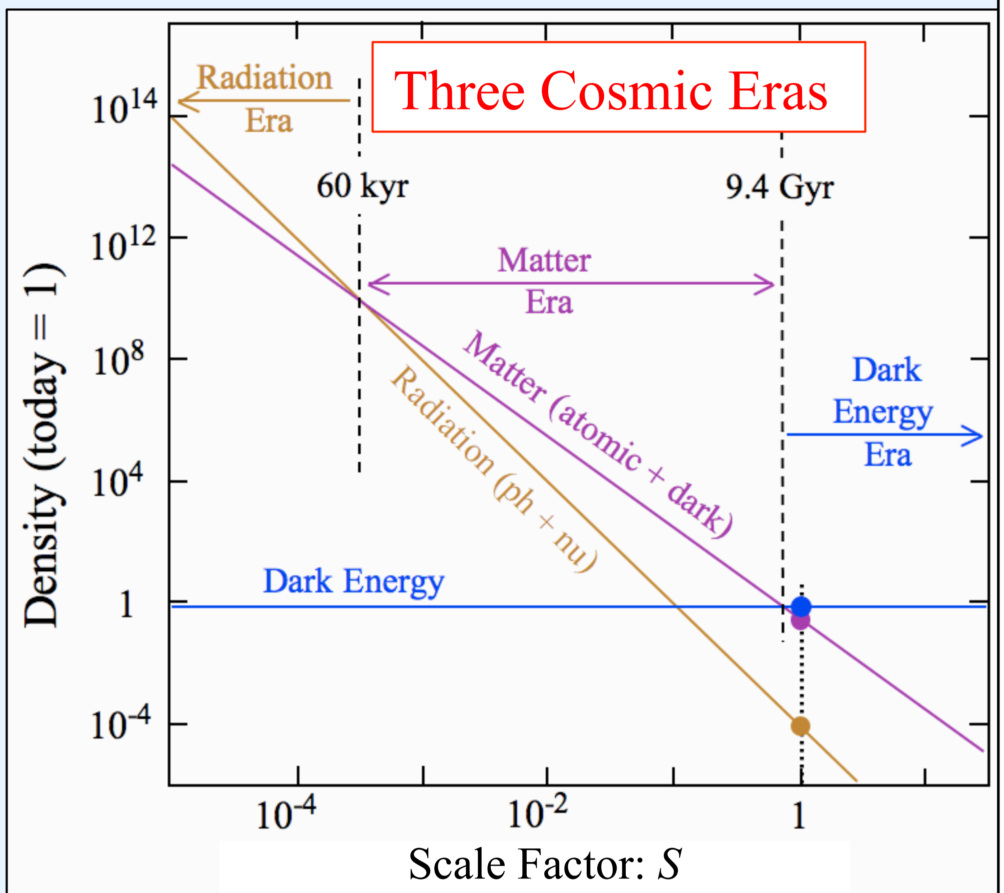
Two transitions of density equality:

1) Matter-Radiation: $\rho_m = \rho_r$

$$S_{eq} = \frac{\rho_{r,0}}{\rho_{m,0}} = \frac{\Omega_{r,0}}{\Omega_{m,0}} = 2.8 \times 10^{-4}$$

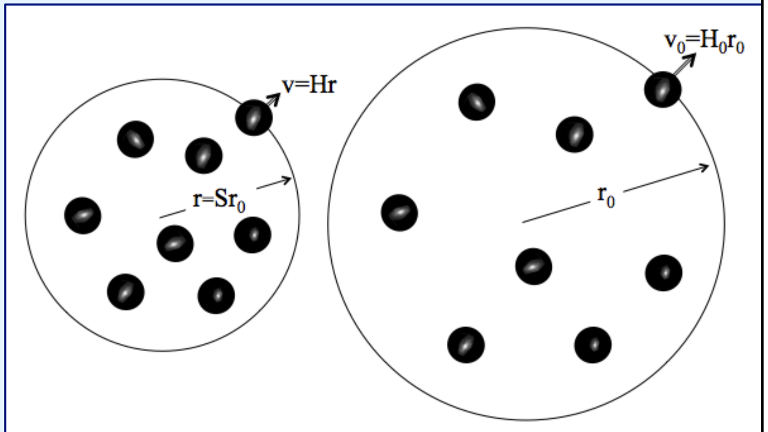
2) Matter-Vacuum: $\rho_m = \rho_v$

$$S_{eq} = \left(\frac{\rho_{m,0}}{\rho_{v,0}}\right)^{1/3} = \left(\frac{\Omega_{m,0}}{\Omega_{v,0}}\right)^{1/3} = 0.75$$



An Expanding Sphere of Matter

A Newtonian analysis of an expanding sphere contains the relevant physics.
Start with just matter, expanding with v = H × r



$$\frac{r}{r_0} = S \; ; \; \frac{v}{v_0} = V \; ; \; \frac{M}{M_0} = 1 \; ; \; \frac{\rho}{\rho_0} = S^{-3}$$

Consider the KE & GE of a galaxy mass, m, on the edge:

$$KE = \tfrac{1}{2}mv^2 \quad \text{so} \quad \frac{KE}{KE_0} = V^2$$

$$GE = -\frac{GMm}{r} \quad \text{so} \quad \frac{GE}{GE_0} = \frac{M}{M_0}\frac{1}{S} = \frac{1}{S}$$

Define the <u>ratio</u> of GE to KE, and call it Ω:
(this is independent of *r* so holds for all gals)

$$\Omega \equiv -\frac{GE}{KE} = \frac{2GM}{rv^2} = \frac{8\pi G\rho}{3H^2}$$

Consider <u>special case</u>: Ω = 1 so TE = 0

$$v^2 = v_{esc}^2 = \frac{2GM}{r} \quad \& \quad \rho = \rho_{crit} = \frac{3H^2}{8\pi G}$$

Ω can also be written:

$$\Omega = \frac{v_{esc}^2}{v^2} = \frac{\rho}{\rho_{crit}}$$

The Friedman Equation: *V*(*S*)

How does the velocity factor, *V*, change as the Universe expands, i.e. what is V(S)? A Newtonian analysis gets us the answer.

Consider a galaxy: its energy is conserved during the expansion:

$$TE = KE + GE = KE_0 + GE_0$$

Normalize (make dimensionless): divide by KE_0

Matter only

$$\frac{KE}{KE_0} + \frac{GE}{KE_0} = \frac{KE_0}{KE_0} + \frac{GE_0}{KE_0} \Rightarrow V^2 + \frac{GE}{GE_0}\frac{GE_0}{KE_0} = 1 - \Omega_0 \Rightarrow V^2 - \frac{\Omega_0}{S} = 1 - \Omega_0$$

KE + GE = TE

$$\frac{GE}{GE_0} = \frac{M}{M_0}\frac{1}{S}$$

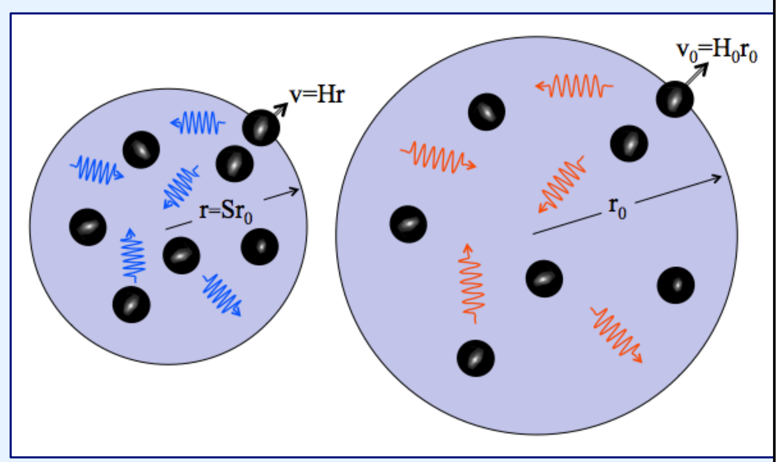
Adding the other components is straightforward:

$$V^2 - \left[\frac{\Omega_{m,0}}{S} + \frac{\Omega_{r,0}}{S^2} + \Omega_{v,0}S^2\right] = 1 - \Omega_{t,0}$$

KE + GE = TE

where $$\Omega_{m,0} = \frac{\rho_{m,0}}{\rho_{0,crit}} = \rho_{m,0} \div \frac{3H_0^2}{8\pi G}$$

and $$\Omega_{t,0} = \Omega_{m,0} + \Omega_{r,0} + \Omega_{v,0}$$

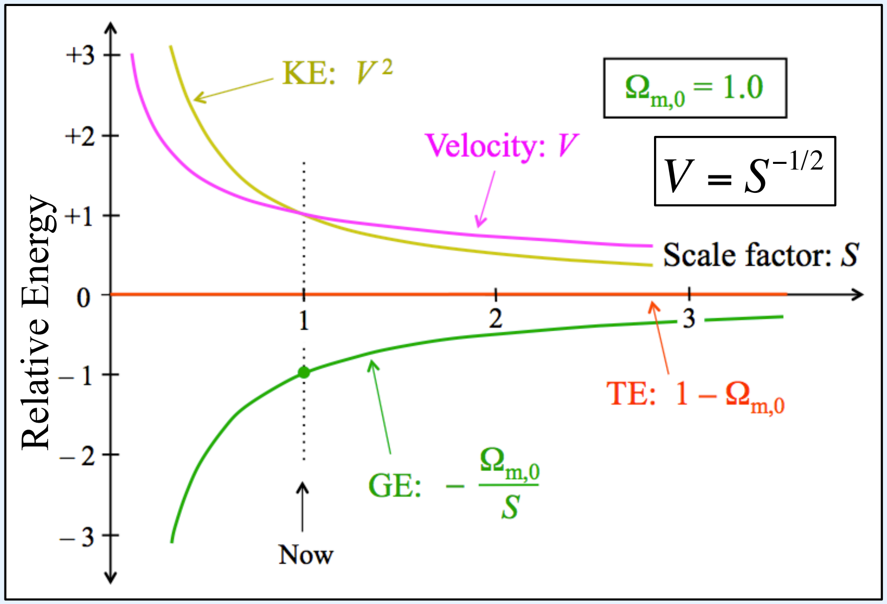


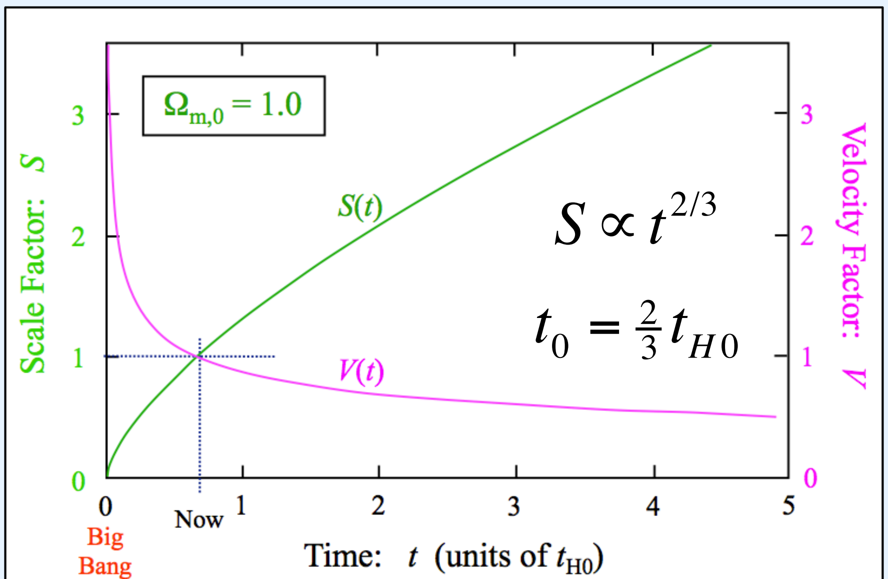
Note: reality has $\Omega_{t,0} = 1$ so RHS = 0.

Friedman Solutions: Pure Matter

Choice of Ω's define the expansion solution.
First review matter-only, then do other components.

$$V^2 - \frac{\Omega_{m,0}}{S} = 1 - \Omega_{m,0}$$





Energy diagram: Shows Friedman terms
Gives $V^2(S)$ and hence $V(S)$.

How to go to $S(t)$ and $V(t)$?
We have:

$$V = t_{H0} \frac{dS}{dt} \qquad so \qquad dt = t_{H0} \frac{dS}{V}$$

Integration gives:

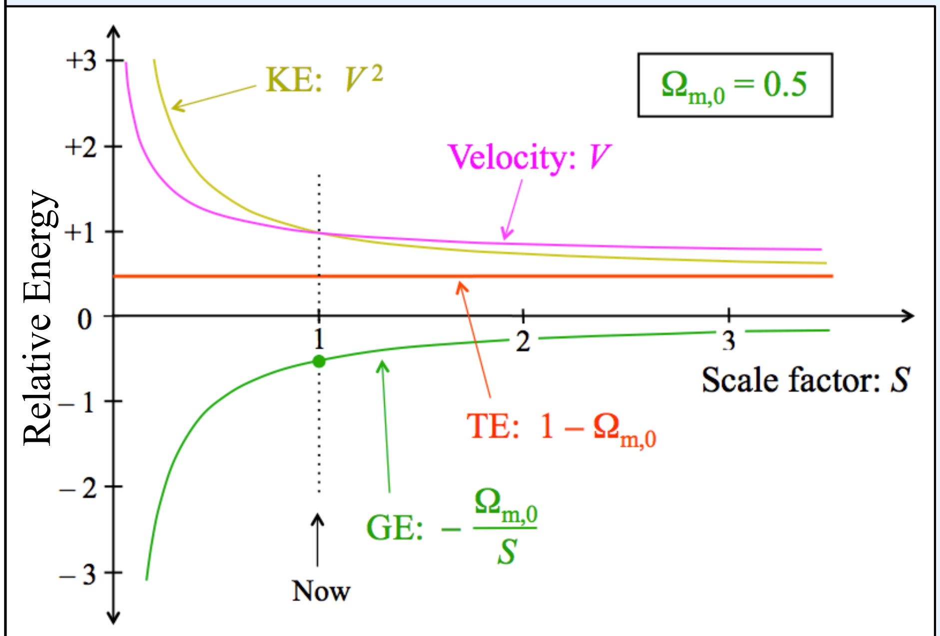
$$t = \int_0^t dt = t_{H0} \int_0^S \frac{dS}{V}$$

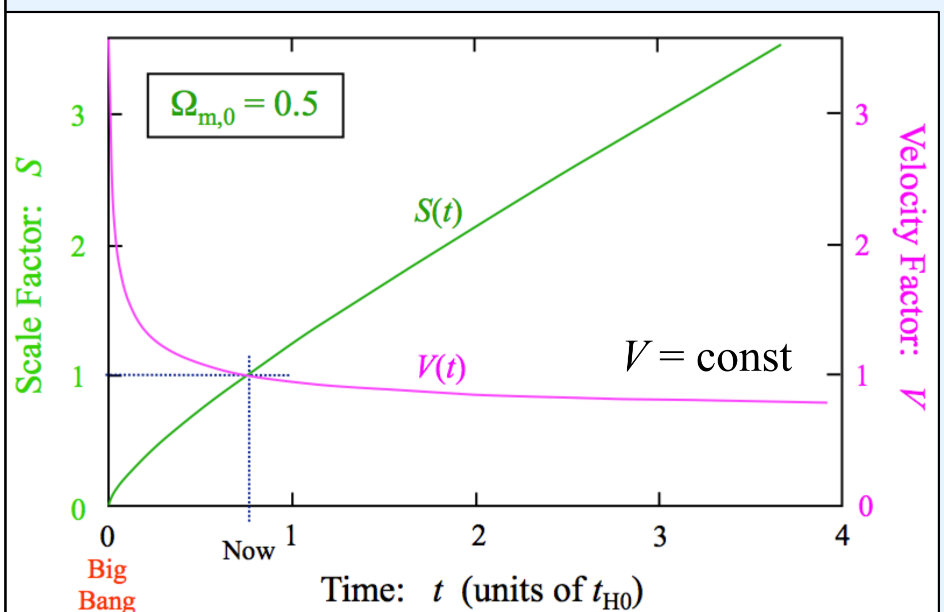
This gives $t(S)$. Invert to get $S(t)$.
Get current age from:

$$t_0 = t_{H0} \int_0^1 \frac{dS}{V}$$

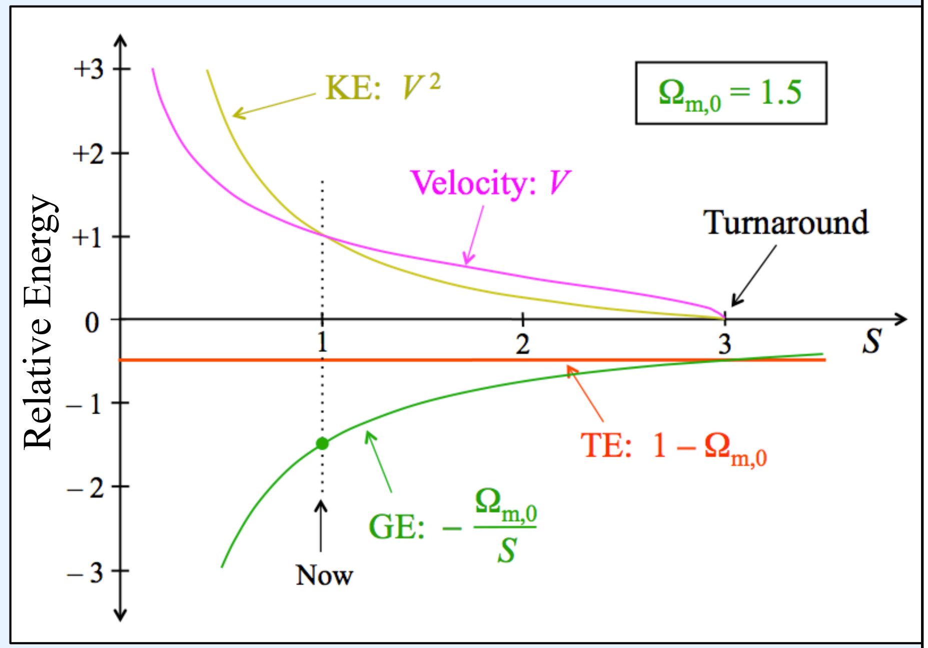
Friedman Solutions: Pure Matter

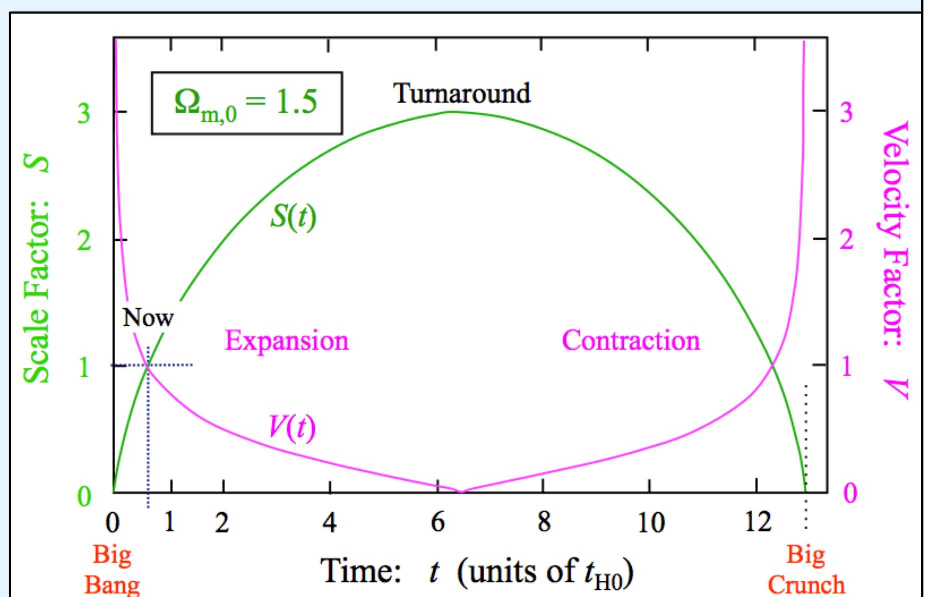
Lower matter density



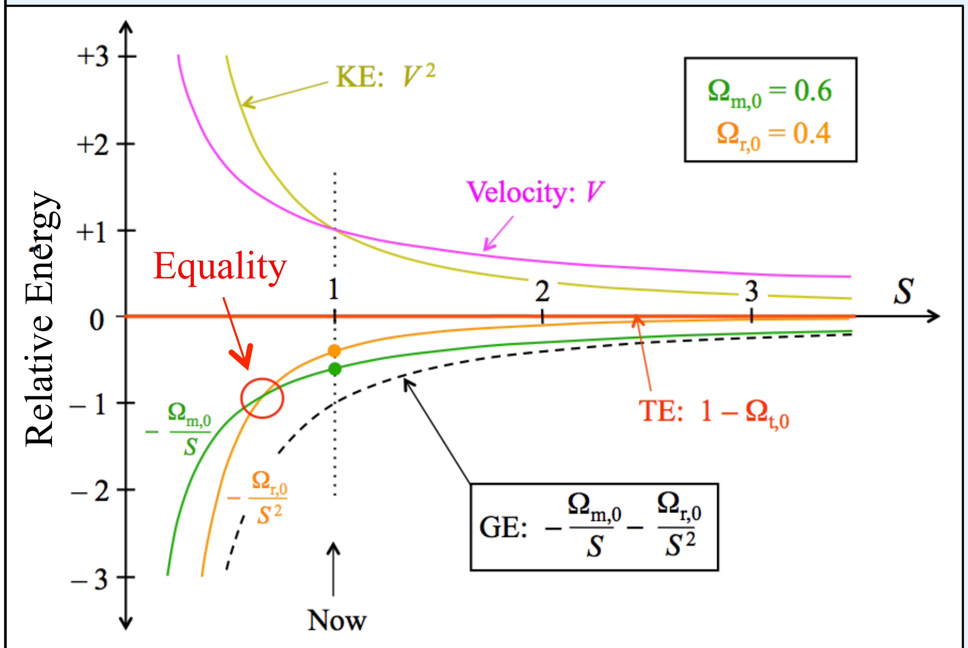


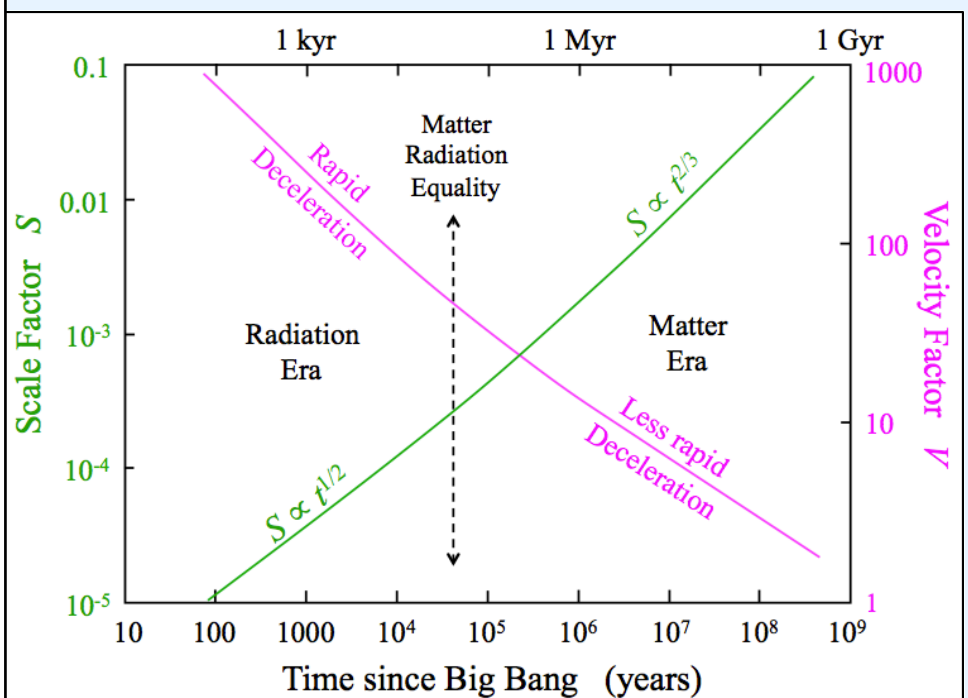
Higher matter density





Matter plus Radiation: Early Times





Radiation dominates at early times.
Equality occurs when $\rho_r = \rho_m$

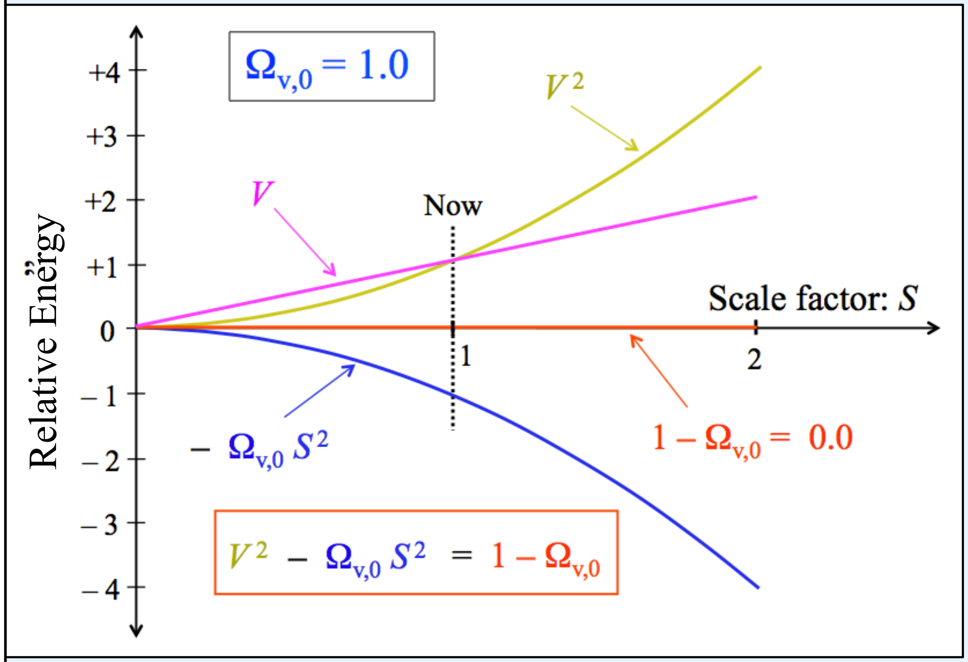
$$\frac{\Omega_{m,0}}{S_{eq}} = \frac{\Omega_{r,0}}{S_{eq}^2} \quad \rightarrow \quad \boxed{S_{eq} = \frac{\Omega_{r,0}}{\Omega_{m,0}}}$$

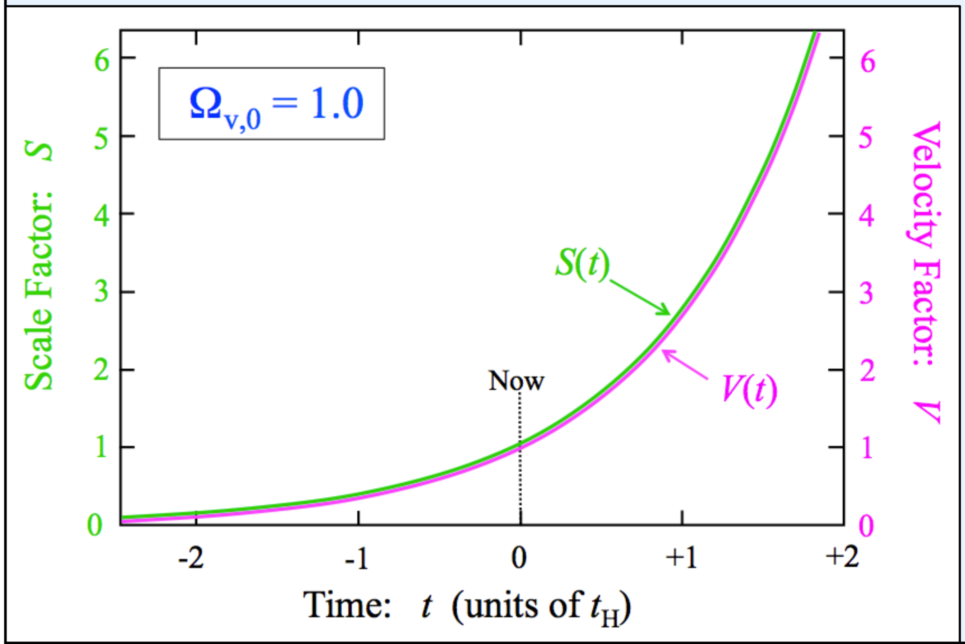
Real Universe: $\Omega_{m,0} = 0.3$ $\Omega_{r,0} = 8.4\text{E-}5$
$S_{eq} = \Omega_{r,0}/\Omega_{m,0} = 2.8\text{E-}4$ or $t_{eq} = 50$ kyr
Transistion is very important.

Radiation era: $S \sim t^{1/2}$ $V \sim t^{-1/2}$
Matter era: $S \sim t^{2/3}$ $V \sim t^{-1/3}$

Slope of GE&KE curves → deceleration
$d(V^2)/dS = 2V\, dV/dt \times dt/dS$
$= 2V\, dV/dt \times t_{H0}/V$
$= 2t_{H0}\, dV/dt = 2A$ (acceleration)
Or:
$a = -\text{grad}\,\Phi = -d(GE)/dS$

Vacuum





Because vacuum GE gets more negative with S, KE increases with S giving acceleration.

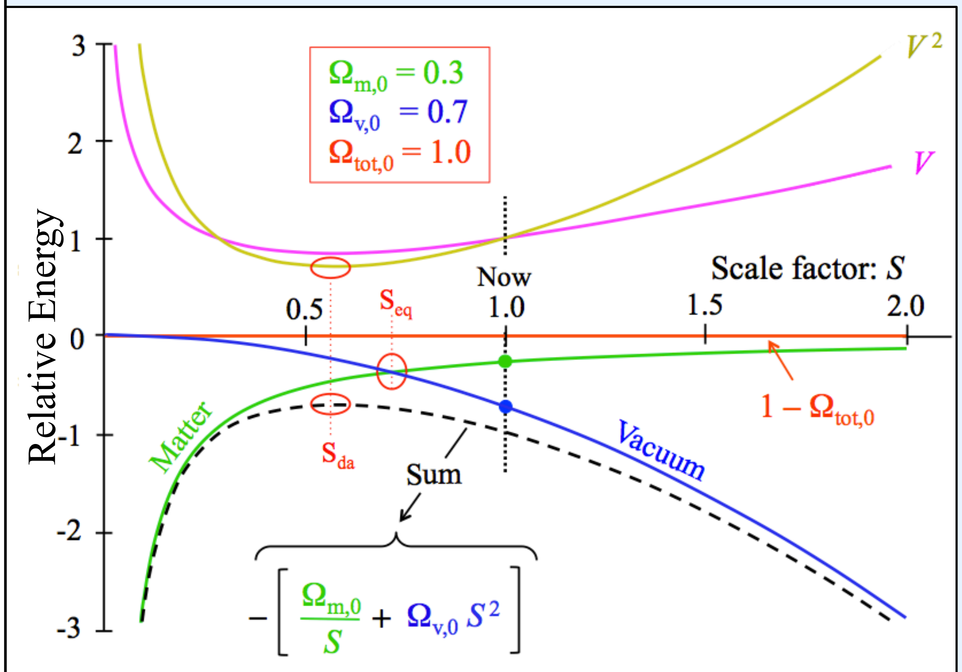
For $\Omega_{v,0} = 1$, we have $V = S$.
This gives exponential growth with e-folding time t_{H0}:

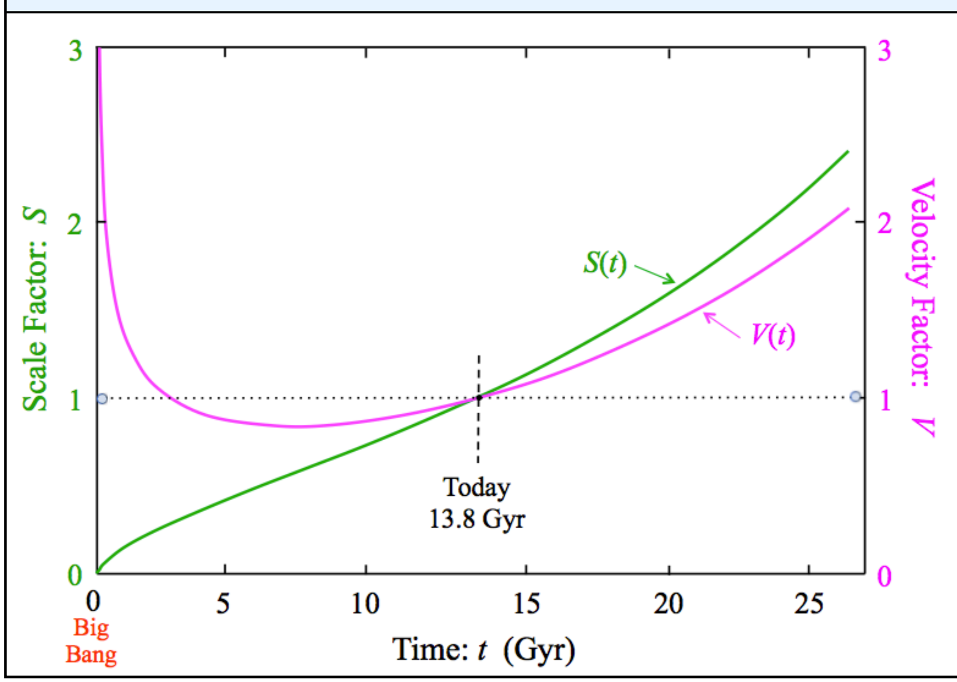
$$V = t_{H0} \frac{dS}{dt} = S \qquad \Rightarrow \qquad \frac{dS}{S} = \frac{dt}{t_{H0}}$$

$$S(t) = e^{(t-t_0)/t_{H0}} \text{ and } V(t) = e^{(t-t_0)/t_{H0}}$$

Note: this is the nature of Inflation, where the vacuum density is very high so t_{H0} is very short.

Matter plus Vacuum: Late Times





Sum of matter and vacuum gives a "hilltop" GE shape:

Early deceleration (matter domiantes)

Late acceleration (vacuum dominates)

Equality occurs when $\rho_m = \rho_v$

$$\frac{\Omega_{m,0}}{S_{eq}} = \Omega_{v,0} S_{eq}^2 \quad \rightarrow \quad S_{eq} = \left(\frac{\Omega_{m,0}}{\Omega_{v,0}}\right)^{1/3}$$

For $\Omega_{m,0} = 0.3$ and $\Omega_{v,0} = 0.7$, $S_{eq} \approx 0.6$

Coasting when slopes of GE curves for matter and vacuum equal and opposite.

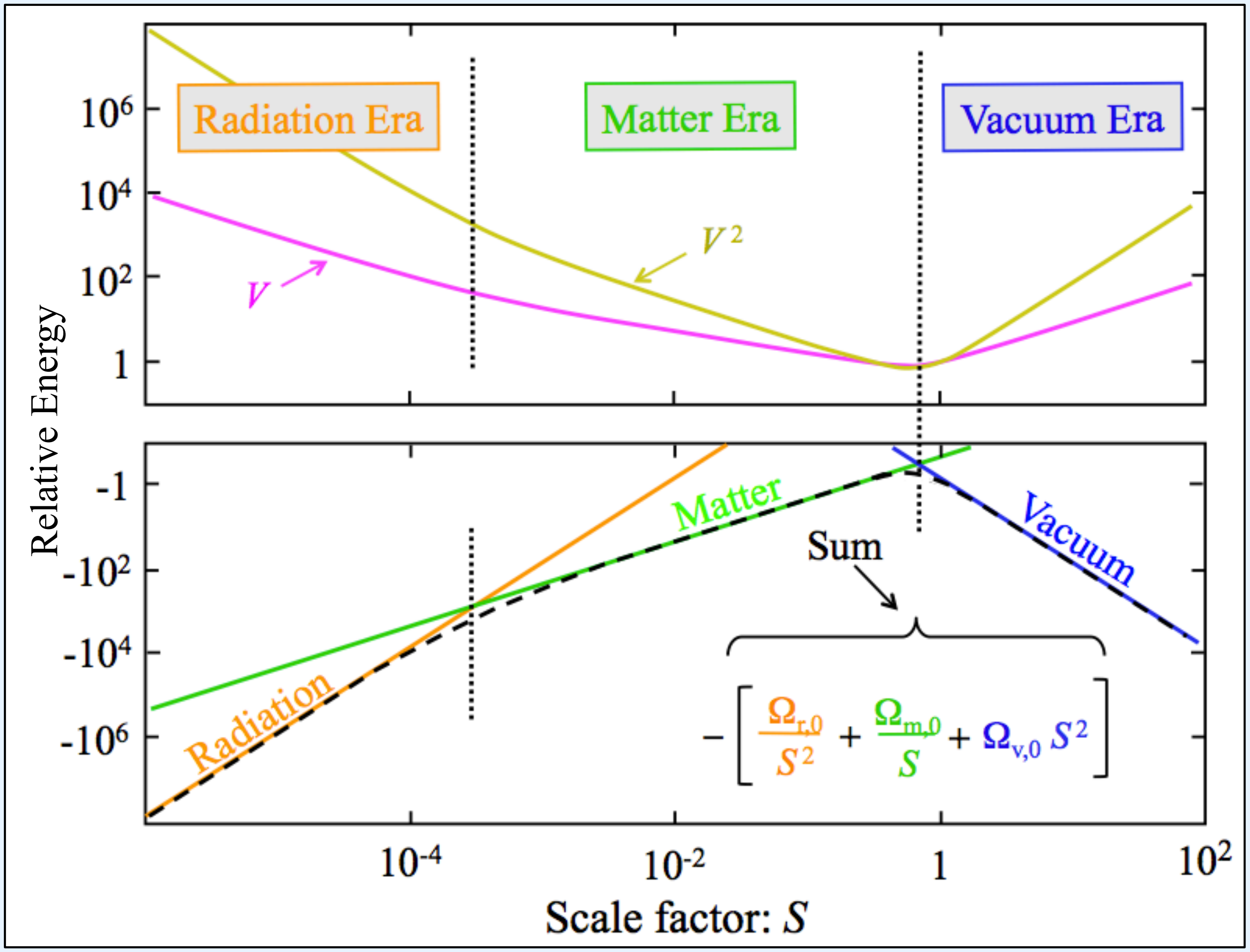
$$\frac{\Omega_{m,0}}{S_{da}^2} = 2\Omega_{v,0} S_{da} \quad \rightarrow \quad S_{da} = \left(\frac{\Omega_{m,0}}{2\Omega_{v,0}}\right)^{1/3}$$

For $\Omega_{m,0} = 0.3$ and $\Omega_{v,0} = 0.7$, $S_{da} \approx 0.75$

Cosmic Age: $t_0 = 0.96\, t_{H0}$

The Real Universe: All Three

Energy diagram with log axes



The Friedman equation gives the form for $V(S)$ or $V(z)$:

$$V(S) = \left[\frac{\Omega_{m,0}}{S} + \frac{\Omega_{r,0}}{S^2} + \Omega_{v,0}\, S^2 + 1 - \Omega_{t,0}\right]^{1/2}$$

$$V = \left[\Omega_{m,0}(1+z) + \Omega_{r,0}(1+z)^2 + \frac{\Omega_{v,0}}{(1+z)^2} + 1 - \Omega_{t,0}\right]^{1/2}$$

Note: often H is used as main variable:

$$\frac{H}{H_0} = \frac{V}{S} \equiv E(S) \text{ or } E(z)$$

Intuiting vacuum's accelerating expansion

	$S = ½$	$S = 1$	$S = 2$	
Radius	½	1	2	S
Volume	⅛	1	8	S^3
Density, with $\rho_0 = 1$				
Matter	8	1	⅛	$1/S^3$
Radiation	16	1	$^1/_{16}$	$1/S^4$
Vacuum	1	1	1	Const.
Mass, with $M_0 = 1$				
Matter	1	1	1	Const.
Radiation	2	1	½	$1/S$
Vacuum	⅛	1	8	S^3
Gravitational Energy, $-GM/r$, with $GE_0 = -1$				
Matter	–2	–1	–½	$-1/S$
Radiation	–4	–1	–¼	$-1/S^2$
Vacuum	–¼	–1	–4	$-S^2$
Acceleration, $-d(GE)/dr$, with $A_0 = -1$ (deceleration)				
Matter	–4	–1	–¼	$-1/S^2$
Radiation	–8	–1	–⅛	$-1/S^3$
Vacuum	+½	+1	+2	+S

Given that vacuum's density is constant, then the mass of an expanding sphere increases.

In this case the GE for unit mass at the surface decreases (more negative) as sphere expands.

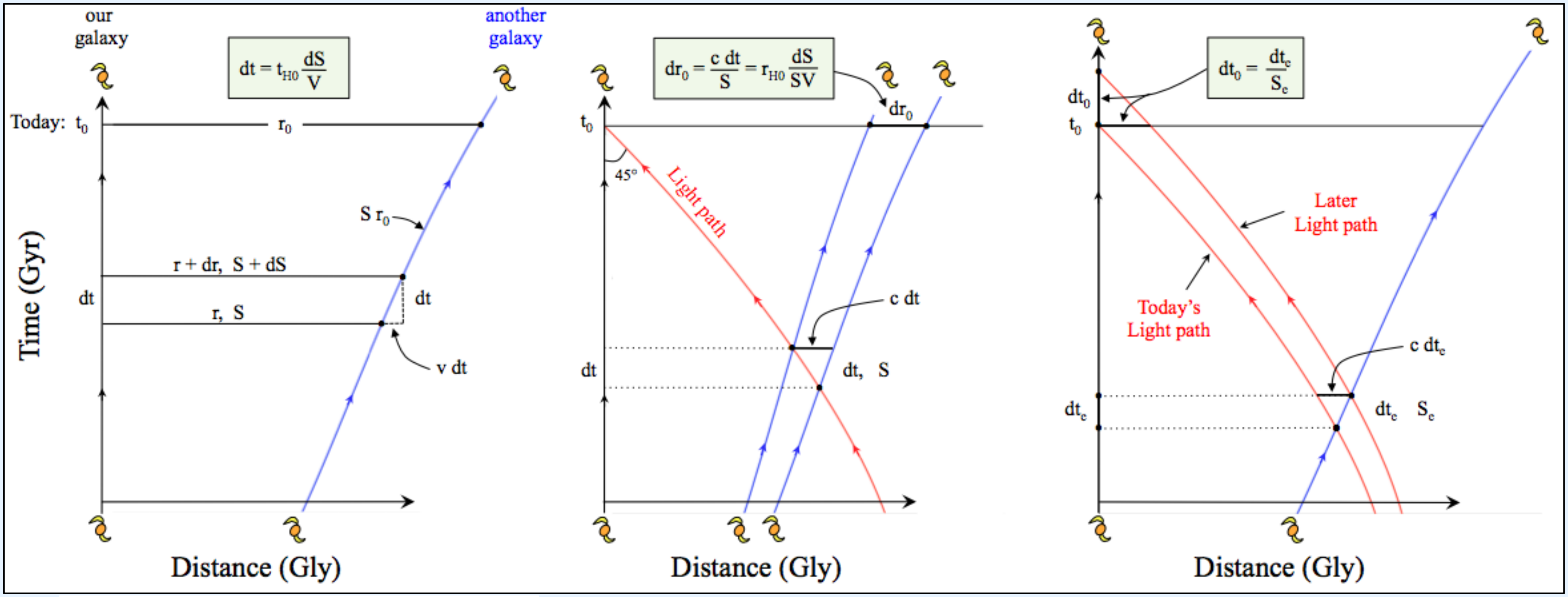
Thus: outwards is "downhill".
Vacuum sphere's fall outwards, naturally!

Where does the new mass come from??
Several possible answers:
Newtonian: new mass created from negative global gravitational energy ($-GM^2/R$).
E.g. a sphere with $R \sim r_{H0}$ has $GM^2/R \sim Mc^2$.

Einstein: $G_{i,j} = 8\pi G/c^4 \times T_{i,j}$ so gain on RHS has matching gain on LHS – i.e. the new energy comes from the changing metric.

Three important differentials

Simple space-time diagrams yield three useful differential relations.
Blue lines are paths of receding galaxies (world lines).
Red line is "past light cone". All today's visible galaxies lie on this line.



$$S + dS = \frac{r + v_0 V dt}{r_0} = S + \frac{V dt}{t_{H0}}$$

$$dr_0 = \frac{c\,dt}{S} = r_{H0}\frac{dS}{SV}$$

$$dt_0 = \frac{dt_e}{S_e}$$

$$dt = t_{H0}\frac{dS}{V}$$

Simple kinematics

Distance between two close objects on our light path.

Cosmological time dilation: distant events appear slowed down.

Three important differentials (cont.)

If you prefer to work with redshift, z, the differential relations become:

$$S_e = \frac{1}{1+z} \quad \Rightarrow \quad dS = \frac{-dz}{(1+z)^2} = -S^2 dz$$

$$dt = t_{H0}\frac{-dz}{(1+z)^2 V(z)} \qquad dr_0 = (1+z)\,c\,dt = r_{H0}\frac{-dz}{(1+z)V(z)} \qquad dt_0 = dt_e(1+z)$$

Direct application of the differential relations:

a) Time interval corresponding to given redshift interval, dz:
$dt = t_{H0}\, S^2/V\, dz$. Clearly, this gets very short at high-z (small S):
e.g. at $z \approx 20$, $S \approx 1/21$, $V \approx 2.4$, $S^2/V \sim 10^{-5}$ so $dz = 1$ gives $dt \approx 1.3$ Myr

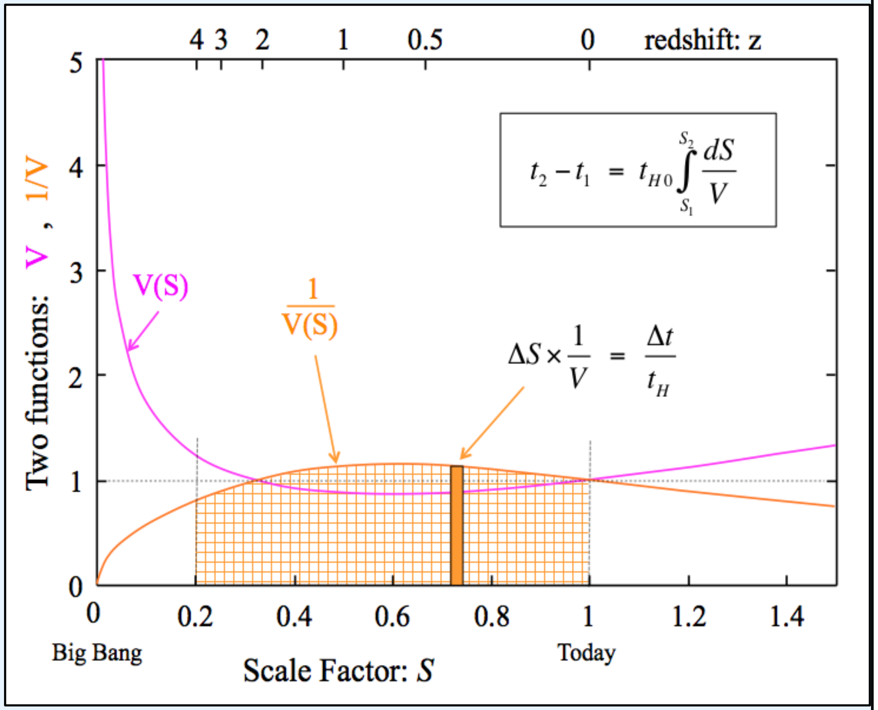
b) Comoving separation corresponding to given redshift interval dz:
$dr_0 = r_{H0} dS/SV = -r_{H0}\, S/V\ dz$.
e.g. At $z \approx 20$, $S \approx 1/21$, $V \approx 2.4$, $S/V \approx 0.02$, so $dr_0 \approx 0.02\, r_{H0} \approx 0.26$ Gly.

c) Redshift is simply Cosmological time dilation: $dt_0 = dt_e/S_e$
For light wave periods: $1/f_0 = 1/f_e \,/\, S_e$ or $S_e = f_0/f_e = \lambda_e/\lambda_0$

Calculating times

Use first differential to calculate time intervals:

$$t_2 - t_1 = \int_{t_1}^{t_2} dt = t_{H0} \int_{S_1}^{S_2} \frac{dS}{V}$$



Basically, duration of journey with speed v(x) from x_1 to x_2: use v = dx/dt, so dt = dx/v so integrate dx/v from x_1 to x_2 to find duration.

a) Lookback time: $t_{lt} = t_{H0} \int_{S_e}^{1} \frac{dS}{V}$

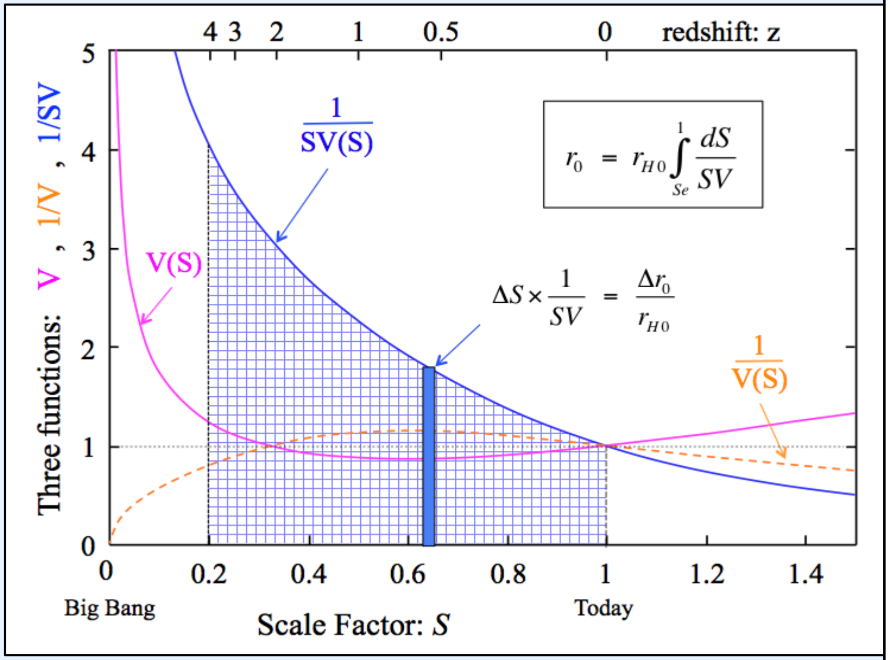
b) Age at S_e:
Invert to get $S(t)$ $t(S_e) = t_{H0} \int_{0}^{S_e} \frac{dS}{V}$

c) Today's age: $t_0 = t_{H0} \int_{0}^{1} \frac{dS}{V}$

Example: $z = 4$ galaxy ($S_e = 0.2$). $V = [0.3/S + 0.7\, S^2]^{1/2}$. See figure.
<u>Lookback time</u>: integrate from $S_e = 0.2$ to 1.0 gives $t_{lt} \approx 0.85\, t_{H0} \approx 11.5$ Gyr.
<u>Age</u> at $z = 4$: Integrate from 0 to 0.2, gives $t(0.2) \approx 0.11\, t_{H0} \approx 1.5$ Gyr.

Calculating Distances: r_0 and r_e

Integrate dr_0 differential to calculate distances:



a) Comoving distance, r_0:
$$r_0 = r_{H0} \int_{S_e}^{1} \frac{dS}{SV}$$

Previous example ($z = 4$; $S_e = 0.2$):
Integration gives $r_0 = 1.67\ r_{H0} \approx 22.7$ Gly

b) Emission distance, r_e:
$$r_e = S_e\, r_0$$
$r_e = 0.33\ r_{H0} \approx 4.5$ Gly

c) Light travel distance: $r_{lt} = ct_{lt}$
Same as lookback time: 11.5 Gly. Notice $r_e < r_{lt} < r_0$.

d) Particle horizon, $r_{ph,0}$, = furthest we can see:
Integration gives: $r_{ph,0} \approx 3.3\ r_{H0} \approx 46$ Gly.
$$r_{ph,0} = r_{H0} \int_{0}^{1} \frac{dS}{SV}$$

e) Event horizon, $r_{eh,0}$, = furthest we can signal:
Also, distance beyond which we will never see an event that happens today.
Integration gives: $r_{eh,0} \approx 1.14\ r_{H0} \approx 15.4$ Gly.
$$r_{eh,0} = r_{H0} \int_{1}^{\infty} \frac{dS}{SV}$$

Calculating Sizes from Angles

What's the size, Δ, of an object at distance r that subtends a small angle θ radians?

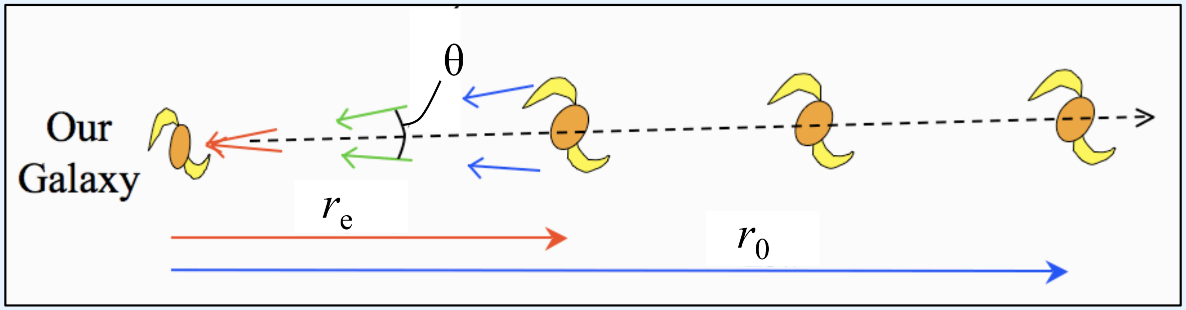
For a static, Euclidean space: $$\Delta = C \times \frac{\theta}{2\pi} = r \times \theta$$

where C is the circumference of a circle of radius r.

In cosmology, we have two complications:

a) Which r do we use? Answer: we use $r_e = S_e\, r_0$ (see figure)

b) Space may not be Euclidean



For non-Euclidean space, a circle's circumference-radius relation is modified:

$C = 2\pi r \times F_c$ where F_c is a correction factor.

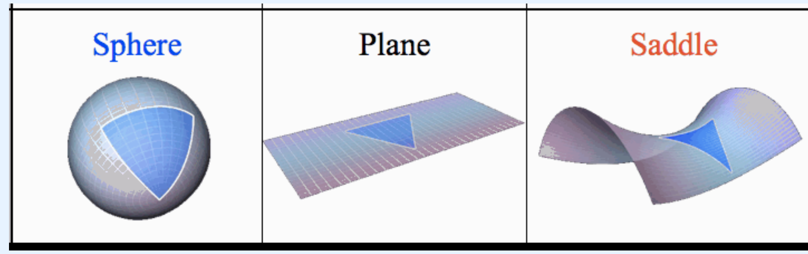
Hence our true relation is: $$\Delta = S_e r_0 F_c \theta \equiv D_A \theta$$

Where $D_A = S_e\, r_0\, F_c$ is an angular diameter distance: a pseudo-distance that gets Δ correct using the static Euclidean relation.

What is F_c ? …..

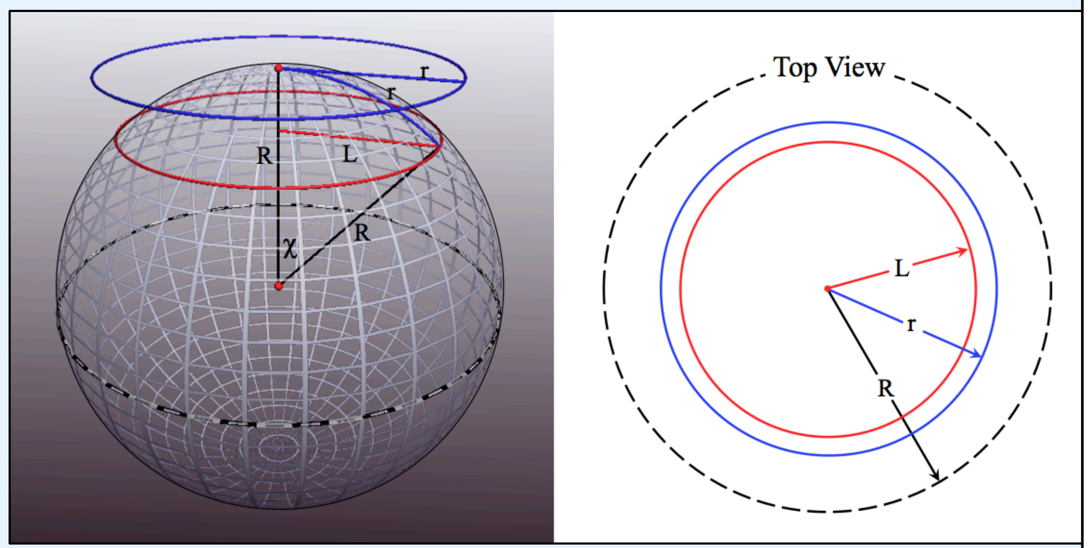
Curved Geometry

Einstein's GR: space-time geometry linked to mass/energy and momentum:
Fortunately in cosmology, isotropy & homogeneity → geometry is simple.
Time separates out; space has single curvature parameter, $\mathcal{R}$ (positive or negative).



Rules of geometry are *different* from Euclidean:
Equivalent to geometry on a *curved* surface.

a) Circle circumference $C = 2\pi r\, F_c$
b) Sphere area: $A = C^2/\pi = 4\pi r^2\, F_c^{\,2}$
c) Sphere volume: $\mathrm{Vol} = {}^4/_3\, \pi r^3\, F_{cv}$
d) Triangle angle sum: $\pi \pm A/\mathcal{R}^2$



Deviations large when $r \sim \mathcal{R}$
Use parameter $\chi = r/\mathcal{R} = r_0/\mathcal{R}_0$

Circle circumference:
$C = 2\pi L = 2\pi \mathcal{R} \sin\chi = 2\pi r\, (\sin\chi)/\chi$
We have:

$$F_c = \frac{\sin\chi}{\chi} \qquad F_c = 1 \qquad F_c = \frac{\sinh\chi}{\chi}$$

See next →

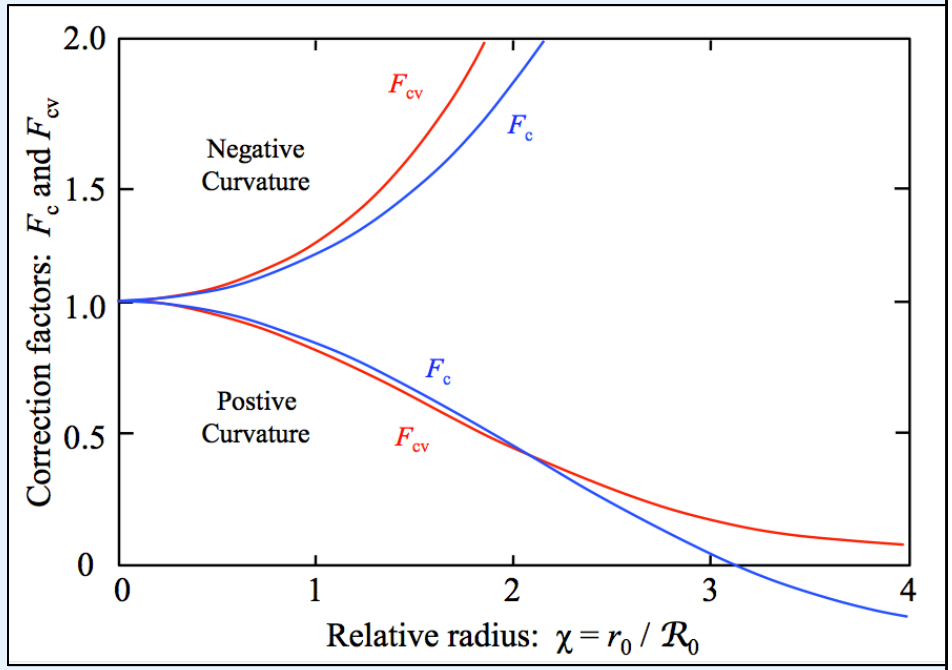
Curvature: Positive (sphere) Euclidean (flat) Negative (saddle)

Curvature Radius

The correction factors F_c & F_{cv} are shown →
For small $\chi = r_0 / \mathcal{R}_0$ we have fractional changes:

Circumference: $\pm\frac{1}{6}\chi^2$
Sphere area: $\pm\frac{1}{3}\chi^2$
Sphere volume: $\pm\frac{1}{5}\chi^2$

Note: at $\chi = \pi$ (antipode), $F_c = 0$ and the circumference goes to zero!



Two things set the value of $\mathcal{R}_0$:

a) Cosmic density (ρ)
b) Cosmic expansion (H)

Start from GR Friedman Eqn: $(dS/dt)^2 - {}^8/_3\,\pi G\,\rho\, S^2 = -kc^2/\mathcal{R}_0^2$

$c^2/\mathcal{R}_0^2$ linked to Newtonian TE.

Several ways to express $\mathcal{R}$

$c^2/\mathcal{R}^2 = 8\pi G\rho/3 - H^2$

$r_H^2/\mathcal{R}^2 = \Omega_{tot} - 1$

$c^2/\mathcal{R}^2 = 8\pi G/3 \times (\rho - \rho_{crit})$

$r_H^2/\mathcal{R}^2 = (v_{esc}/v)^2 - 1$

Here, sign of $\mathcal{R}^2$ sets sign of curvature

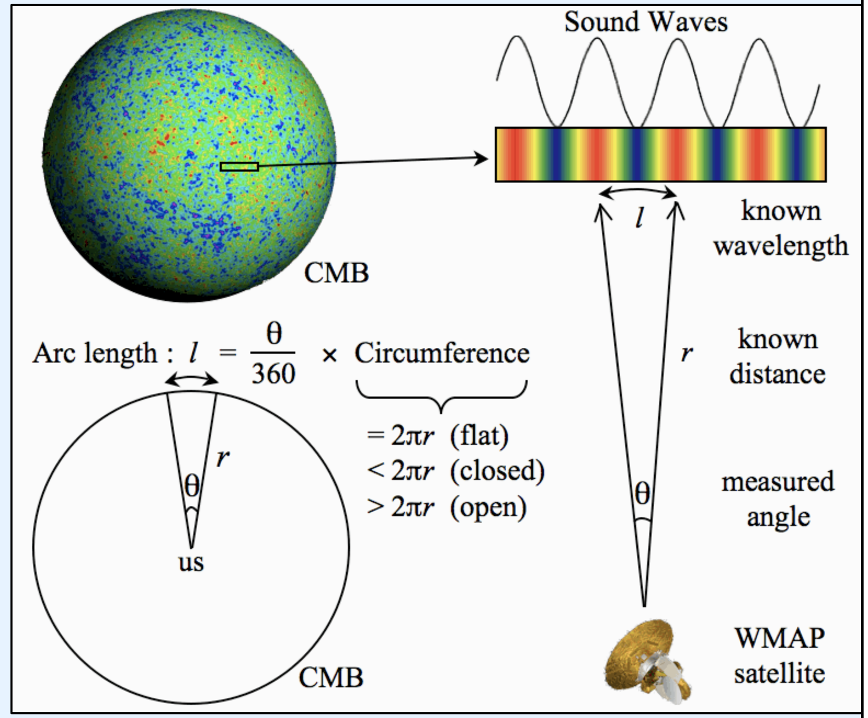
Static region ($H = 0$) has $\mathcal{R} \sim ct_{grav} \sim c/\sqrt{G\rho}$. E.g. Earth has $\mathcal{R} \sim 1$ lt hr. so $\chi \sim 10^{-4}$ so $\delta C/C = \frac{1}{6}\chi^2 \sim 2{\times}10^{-9}$ so C_E is 8 cm smaller than $2\pi R_E$.

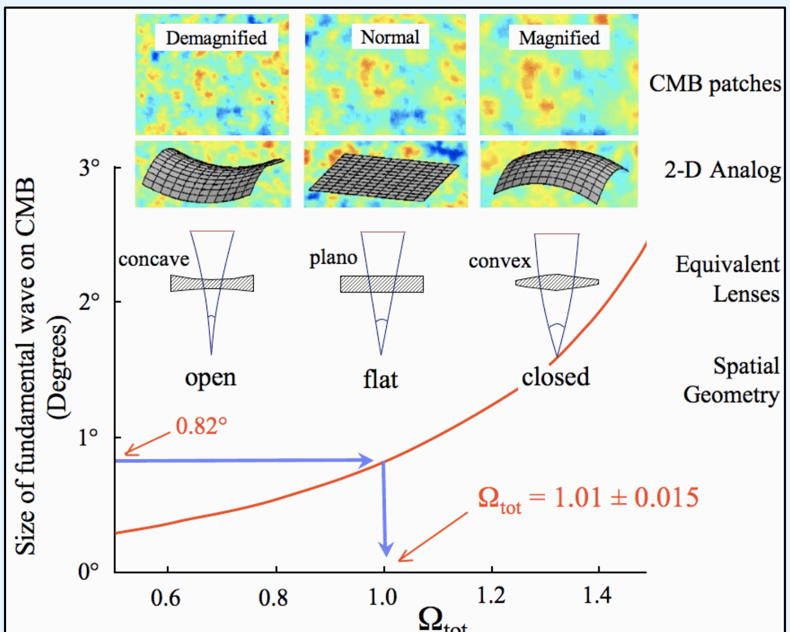
Measuring $\mathcal{R}$ for the Universe

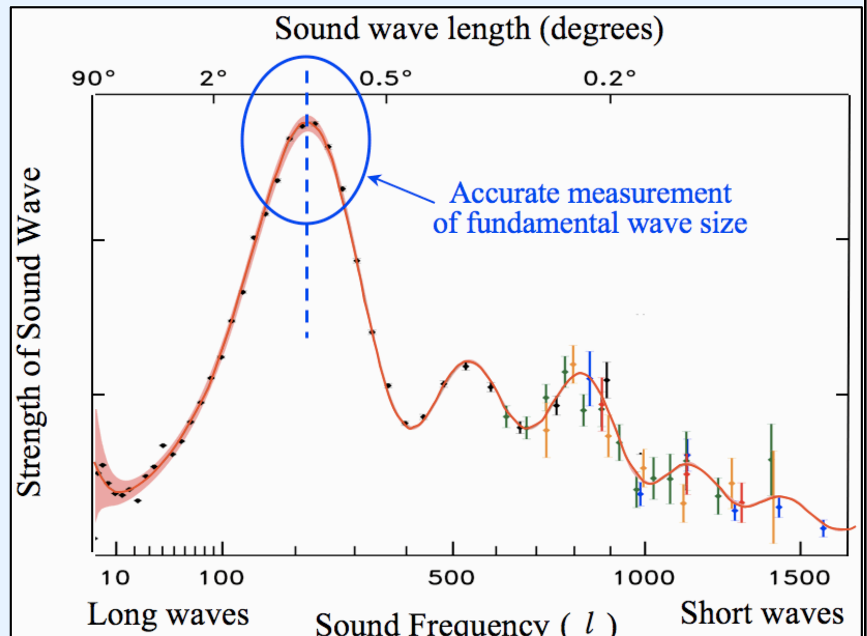
Pick biggest circle possible (out to CMB)
Measure angle subtended by known length – provided by sound wavelength, ℓ.

$$\frac{l}{C} = \frac{\theta^\circ}{360} = \frac{l}{2\pi r_e F_c}$$

Gives F_c and hence $\mathcal{R}_0$ (or $\Omega_{tot,0}$)
→ find $\Omega_{tot,0} = 1.001 \pm 0.004$ ($\mathcal{R}_0 \approx \infty$)
So, Euclidean geometry ($F_c \approx 1$).





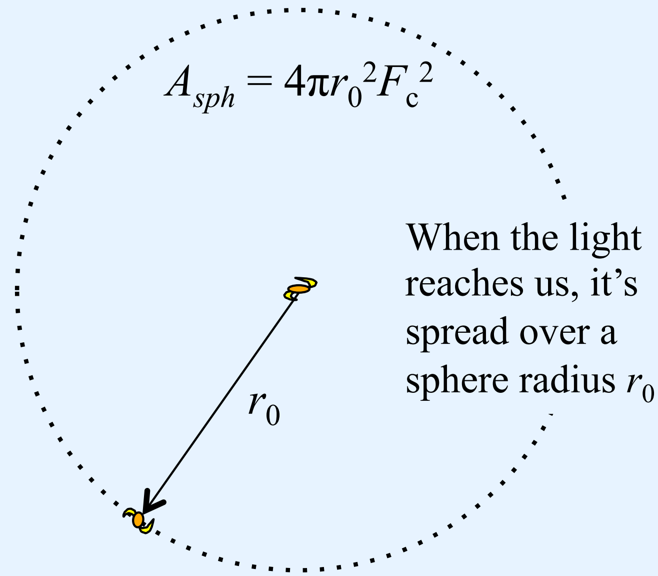


Calculating Luminosity from Flux

What's the Luminosity, L, of an object at distance r with flux f?

For a static, Euclidean space: $L = 4\pi\, r^2 \times f$

Where $4\pi\, r^2 = A_{\rm sph}$ is the area of a sphere of radius r.



In cosmology, we have <u>three</u> complications:

a) Which r do we use? Answer: we use r_0

b) Space may not be Euclidean: $A_{\rm sph} = 4\pi r_0^2\, F_c^2$

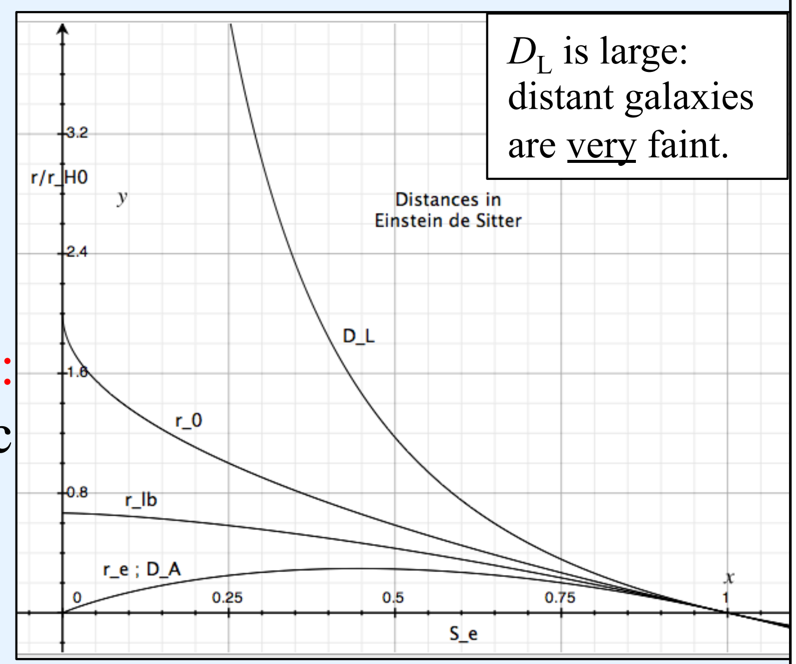
c) Redshift reduces photon <u>rate</u> and <u>energy</u>: $\times S_e^{-2}$

Combining these, we have:

$$L = 4\pi\, r_0^2\, F_c^2\, S_e^{-2} \times f$$

We may write this as: $L = 4\pi\, D_L^2 \times f$

where $D_L = r_0\, F_c \,/\, S_e$ is the Luminosity Distance: a pseudo-distance that gets L correct using the static Euclidean relation.



Note: need extra power of S_e ($1/S_e$) for L_λ (L_ν)

Calculating Surface Brightness

Consider an object of luminosity L and physical area Δ.
Famously, its surface brightness, μ, is independent of distance:

$$\mu = \frac{f}{d\Omega} = \frac{L}{4\pi r^2} \div \frac{\Delta}{r^2} = \frac{L}{4\pi\Delta}$$

However, in cosmology, expansion adds a new quality:

$$\mu = \frac{f}{d\Omega} = \frac{L}{4\pi D_L^2} \div \frac{\Delta}{D_A^2} = \frac{L}{4\pi\Delta}\frac{D_A^2}{D_L^2} = \mu_0 S_e^4$$

Recall: $D_A = r_0 F_c S_e$
and $D_L = r_0 F_c / S_e$

Thus the surface brightness drops rapidly with redshift (note: $S_e^4 = 1/(1+z)^4$).
Bad news: makes studying high-z galaxies very hard, however,
Good news: saves us from lethal CMB (10^6 W/m^2 → 10^{-6} W/m^2)

Note: the factor S_e^4 comes explicitly from cosmic expansion (S_e^2 from object being closer when light set out; S_e for redshift; S_e from reduced rate). Showing galaxies follow $\mu \propto S_e^4$ nicely *proves* the Universe is expanding (Tolman test).

Calculating Volumes

Knowing the volume of a survey is very important – e.g. for calculating the space density of a particular class of objects.

Almost always, we use the comoving volume – the volume out to r_0 in today's universe. For Euclidean space, this is easy: $\mathrm{Vol}_0 = {}^4/_3\, \pi\, r_0^3$.
For non-Euclidean, each spherical shell has area $4\pi r_0^2 F_c^2$ so the total volume is:

$$Vol_0(r_0) = \int_0^{r_0} 4\pi r^2 F_c^2(r)\, dr \;\; = \tfrac{4}{3}\pi r_0^3 \times F_{cv}$$

$$F_{cv} = \frac{3}{2}\frac{[\chi - \frac{1}{2}\sin(2\chi)]}{\chi^3} \;\;;\;\; 1 \;\;;\;\; \frac{3}{2}\frac{[\frac{1}{2}\sinh(2\chi) - \chi]}{\chi^3}$$

Recall: $\chi = r_0/\mathcal{R}_0$

See earlier figure

Survey area $d\Omega$ from z_1 to z_2: $\Delta Vol_0 = \frac{d\Omega}{4\pi}\,[Vol_0(r_0(z_2)) - Vol_0(r_0(z_1))]$

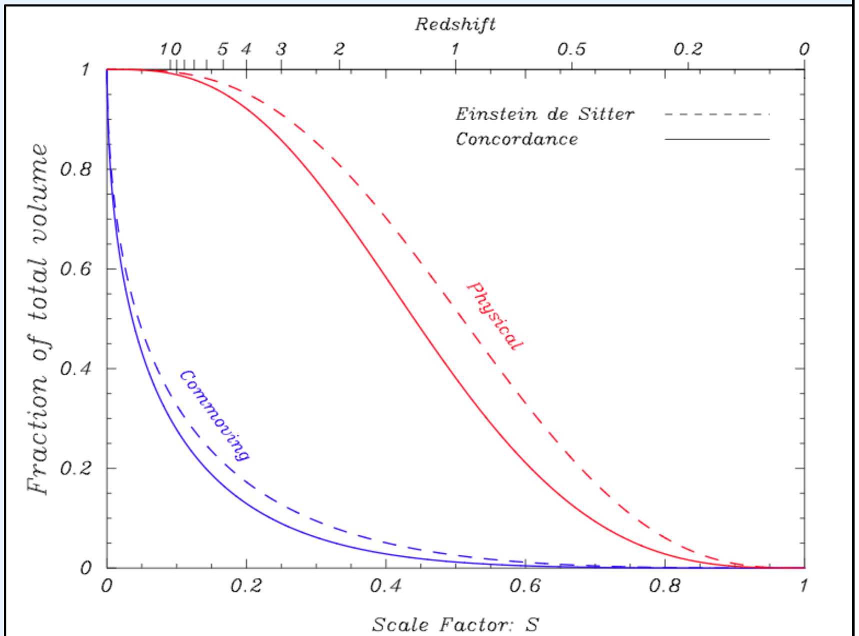
Visible Universe: $Vol_{tot,0} = \frac{4}{3}\pi r_{ph,0}^3 \times F_{cv}$ $= 160\; r_{H0}^3$

Physical (non-comoving) volume out to S_e:

$$Vol(S_e) = \int_{S_e}^{1} 4\pi\,[S r_0(S)]^2\,[F_c(r_0(S))]^2\,\frac{r_{H0}\,dS}{V(S)}\,.$$

Total volume: $\mathrm{Vol}_{tot} = 1.2\; r_{H0}^3$ (= $\mathrm{Vol}_{tot,0}/134$)

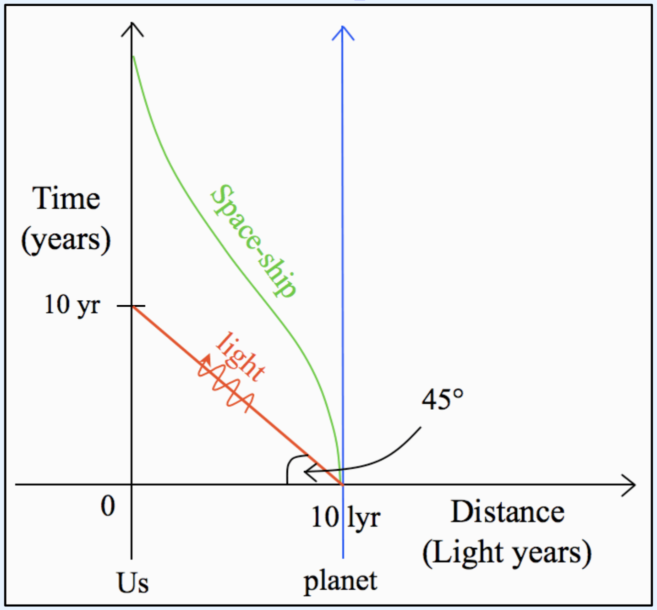
Progress in mapping the entire visible universe:
z(survey): SDSS ~ 0.2; BOSS ~ 1; QSOs ~ 2.5
Mapping the entire visible realm is within reach.



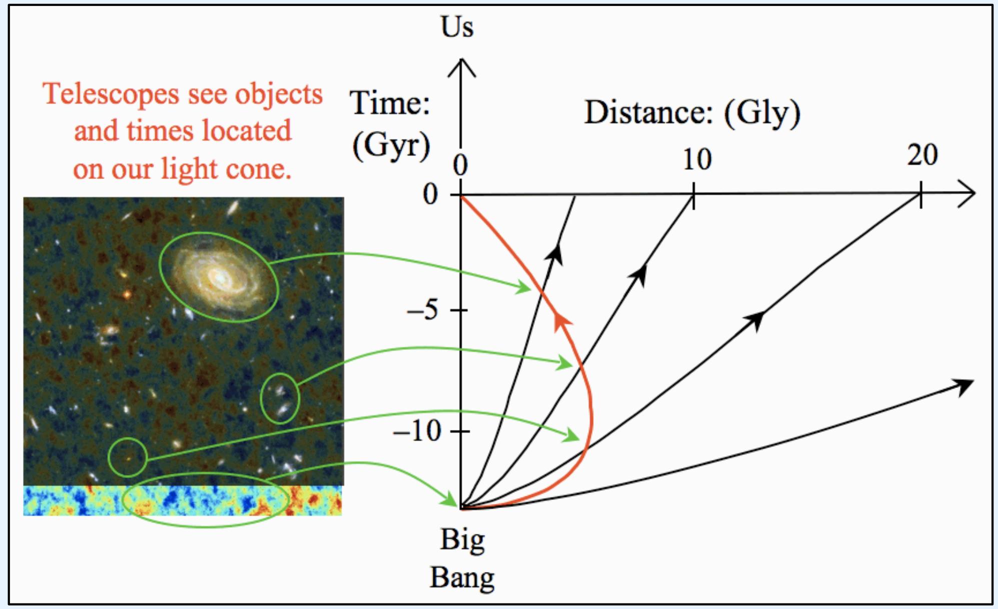
Space-Time Diagrams

These help us think about a number of aspects of our expanding Universe.

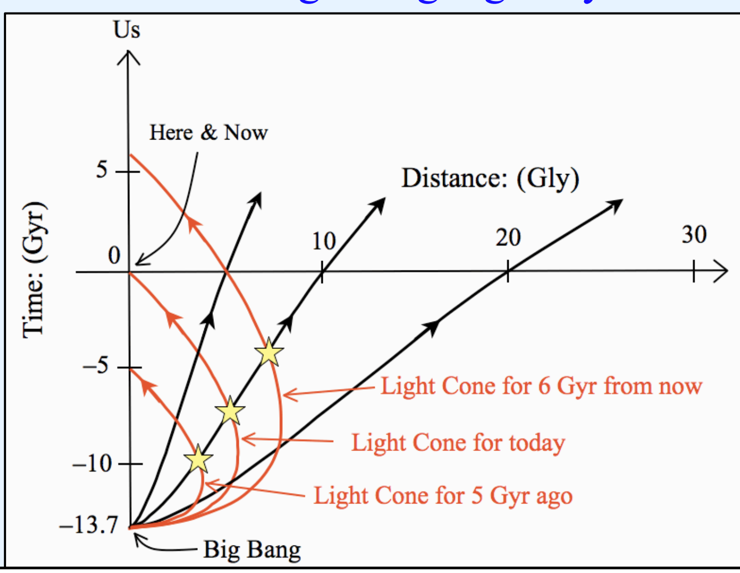
Static Space



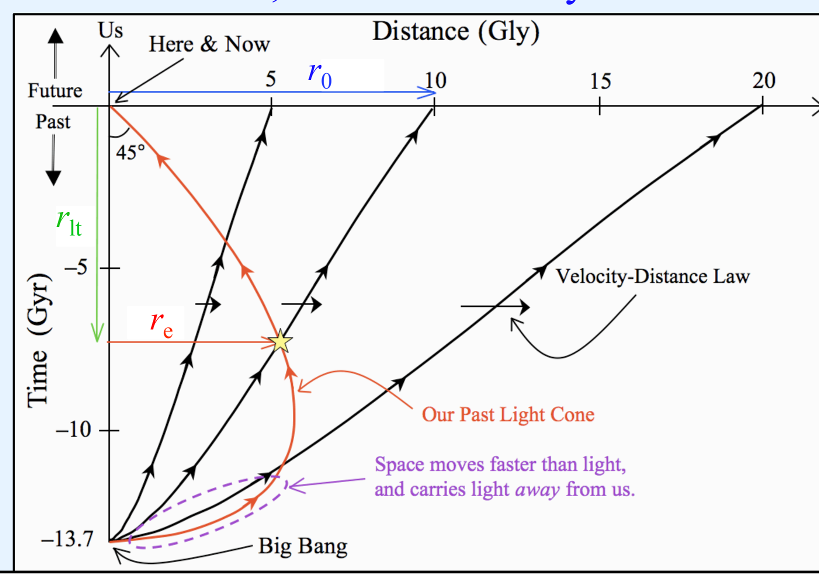
Expanding Space and today's past light path (cone)



Observing a single galaxy

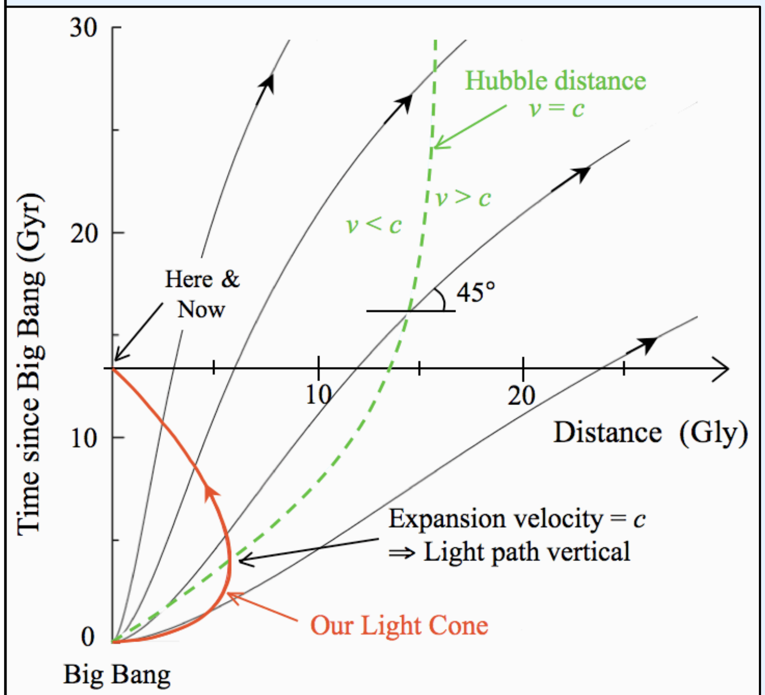


Three distances, and the Velocity-Distance Law

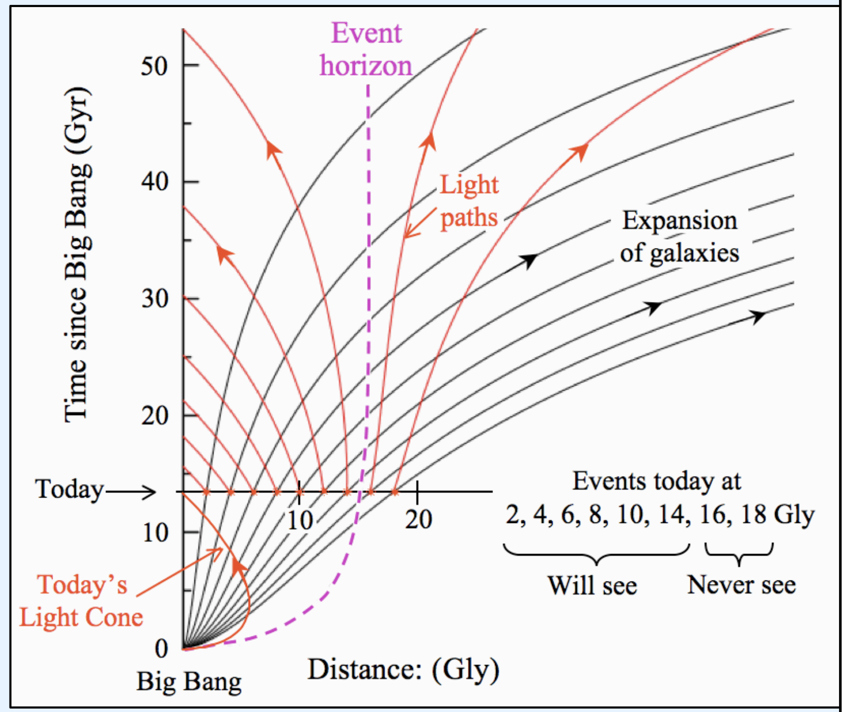


Space-Time Diagrams (cont.)

Various horizons are visible on the space-time diagrams.

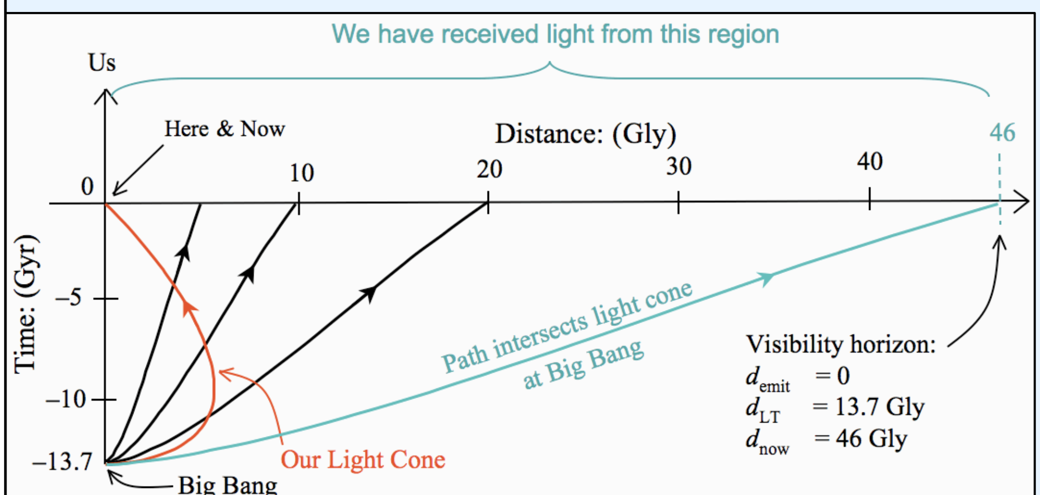


Hubble distance:
$r_H = c/H$
$= r_{H0}\, S/V$



Event horizon:

$$r_{eh}(S) = S\, r_{H0} \int_S^\infty \frac{dS'}{S'V}$$



Particle horizon
(visible limit)

$$r_{ph}(S) = S\, r_{H0} \int_0^S \frac{dS'}{S'V}$$

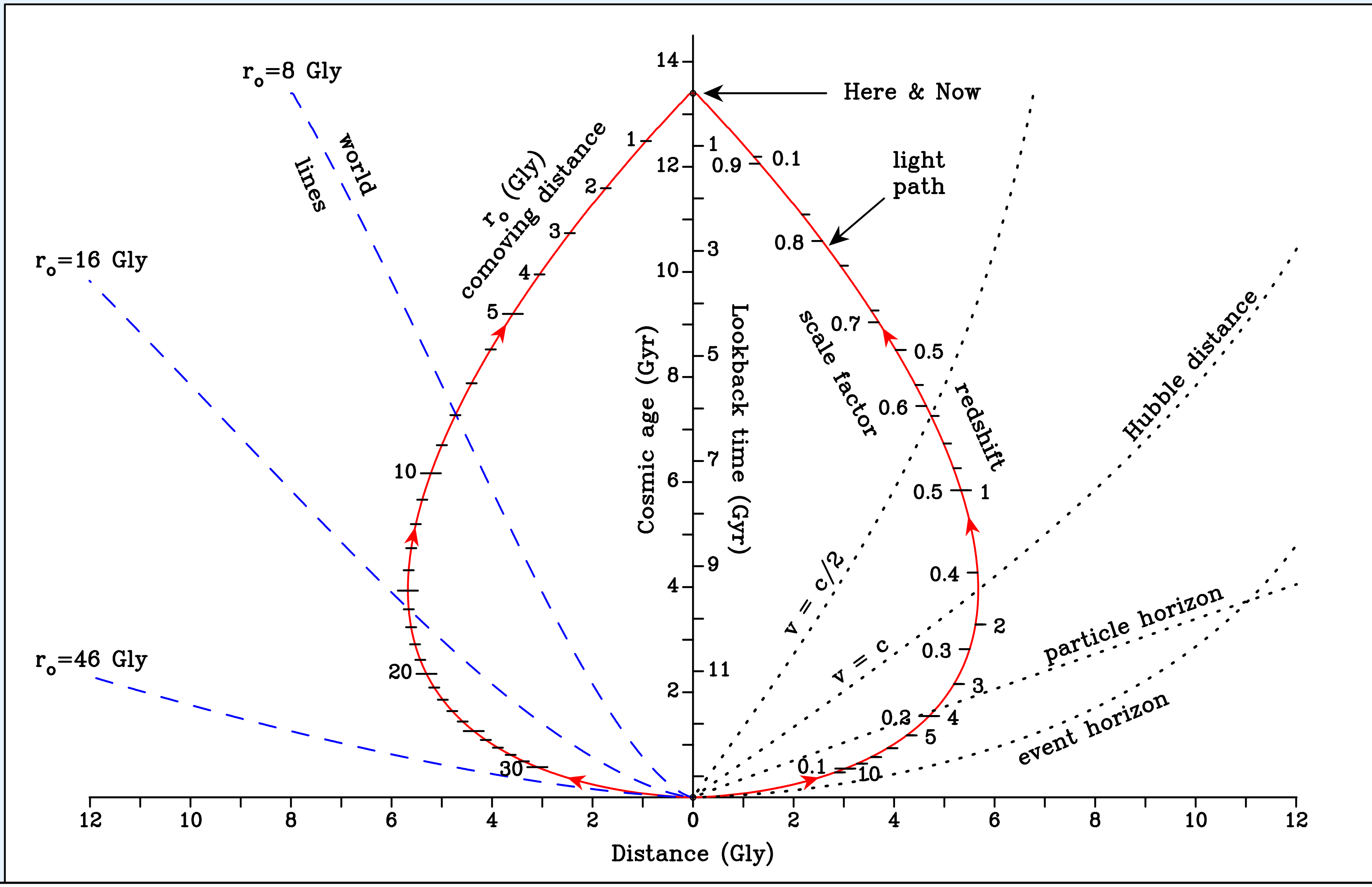
Space-Time Diagrams (cont.)

Full version, showing the light cone, three world lines and three horizons:

Curves plotted as parametric, in S.

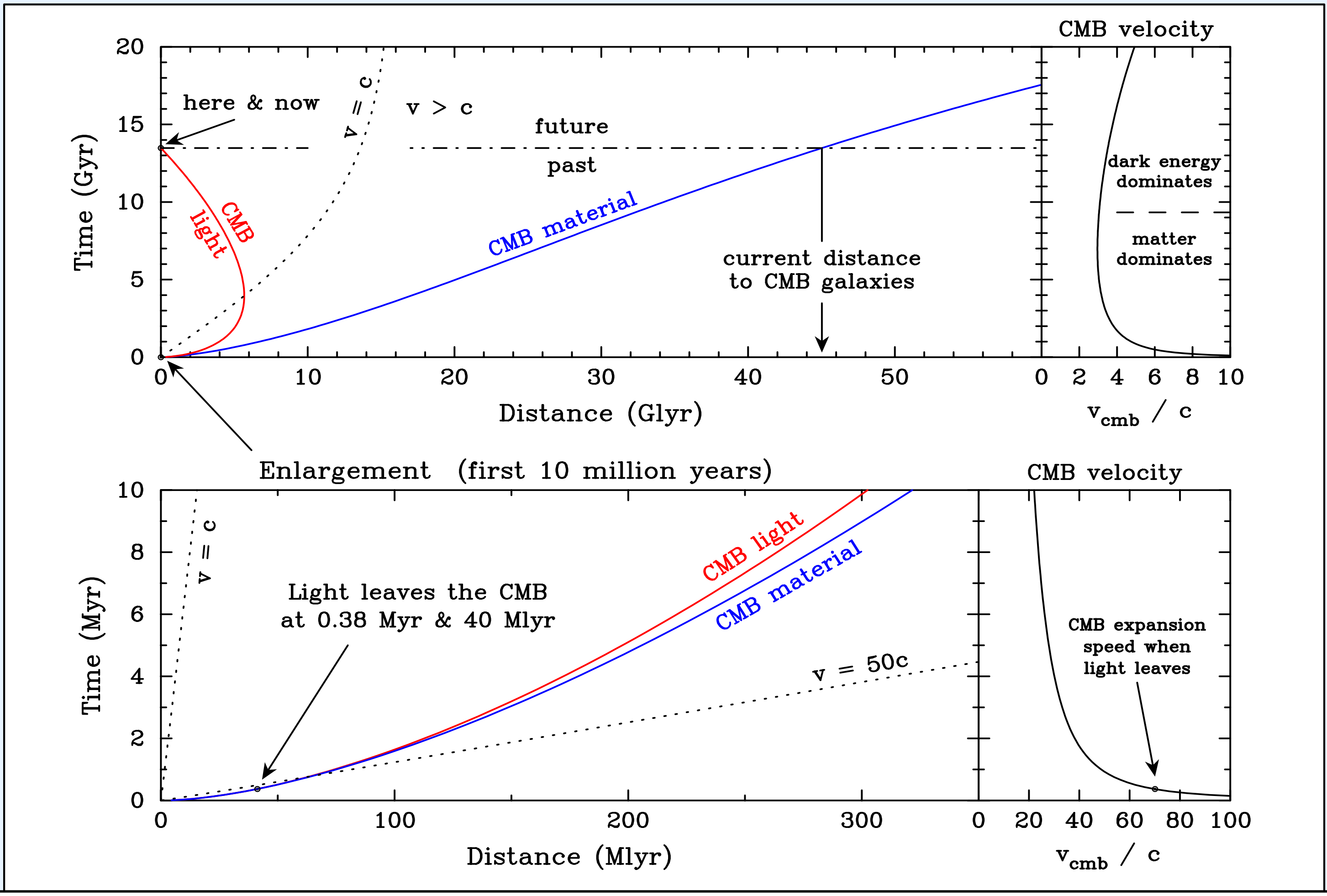
E.g. light cone is: $x = r_e(S) = S\, r_0(S)\,; \quad y = t(S)\,;\ S = 0 \rightarrow 1$ light cone

Marked on the light cone are r_0, z, S_e



Space-Time Diagram for CMB

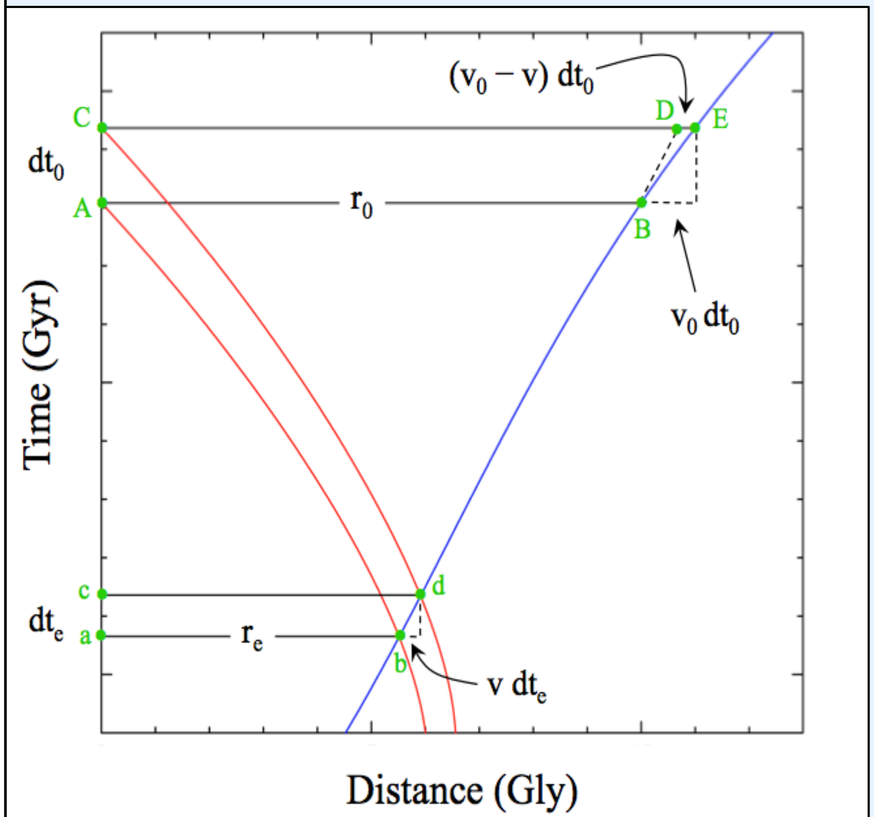
The space time diagram for the CMB and its light is an interesting example.
Light leaves at 380,000 yrs moving in our direction.
At that time, the CMB is at r = 40Mly moving at 65c so light *recedes* from us at 64c.
Recedes for 3.5 Gyr (crosses r_H) then approaches and arrives today.
Notice: velocity of CMB material has always been >c and is also outside r_{eh}.

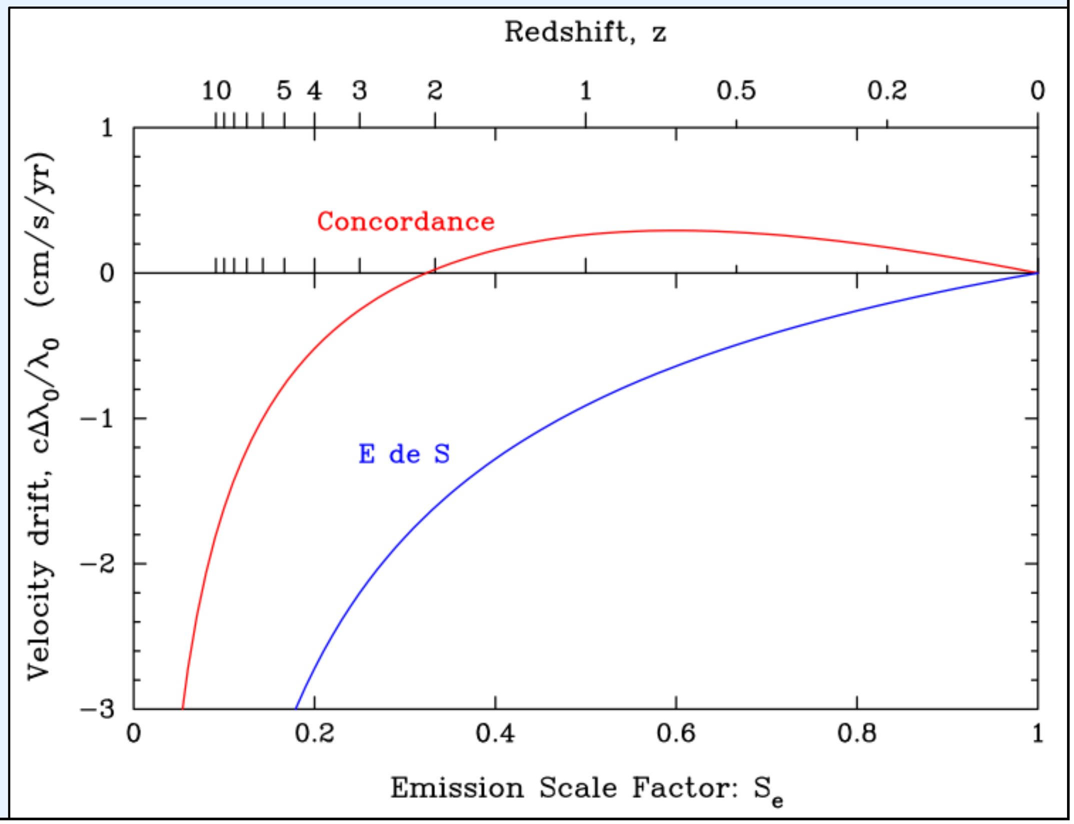


Redshift Drift

An example of real-time cosmology is the steady drift in redshift over cosmic time.

Notice: *abcd* and *ABCD* are congruent (sides and depth stretched by $1/S_e = 1+z$)
But wavelengths stretch the same as distances: so $AB/ab = CD/cd = \lambda_0 / \lambda_e$
But true stretch has increased (by DE), so $d\lambda/\lambda_0 = (v_0 - v)dt_0 / r_0 = (1 - V)dt_0/t_{H0}$
Express $d\lambda/\lambda_0$ as dv/c so $dv/dt_0 = (1 - V)c/t_{H0} = (1 - V) \times 2.2$ cm/s/yr (see graph).
Use QSO absorption lines, laser comb spectrographs, and ELT: needs ~10yrs





End of Cosmology Lite