Inflation

Notes based on Teaching Company lectures, and associated undergraduate text – with some additional material added.

Independent Founders of Inflation Theory

Alexei Starobinsky
b. 1948

Alan Guth
b. 1947

An early vacuum phase

Guth suggested a period before radiation dominance in which a dense "vacuum" (scalar field) dominated the energy density.

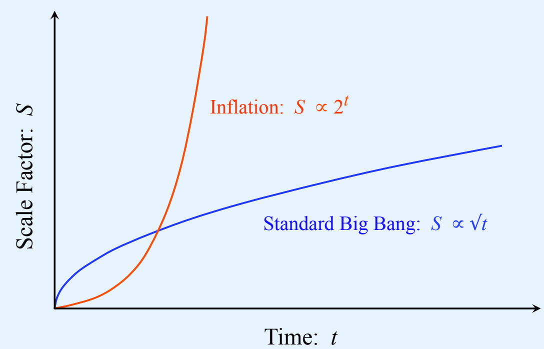
The Freidmann equation tells us that the scale factor grows exponentially with time.

More and more vacuum is made, while other components quickly dilute away, leaving pure exponential expansion.

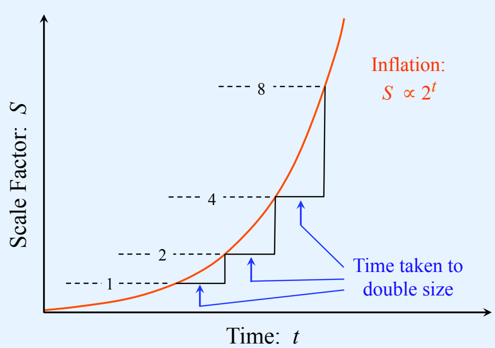
Vacuum: $E(a) = \Omega_v^{1/2} \approx 1$ so $da/dt = aH$ → $a \sim \exp(Ht)$

Note: H, here, is a <u>constant</u> = $(3/8\pi G\rho_v)^{½}$, So the e-folding time is the Hubble time, or equivalently, roughly the cosmic age at that time (assuming the standard model).

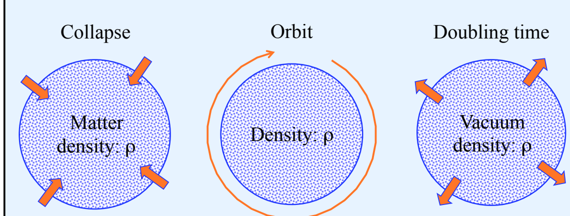
Exponential expansion is radically different!



The region size (diameter) doubles every 0.69 t_H [volume and total mass go up by factor 8]



Doubling time for inflation ≈ "Falling" Time



"Falling" time: $t_{\text{fall}} \approx \dfrac{1}{\sqrt{(G \times \rho)}}$ ← Density of the system

Collapse time   Orbit time   Doubling time

Short doubling time

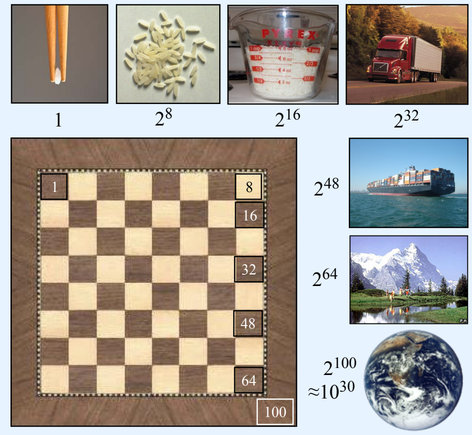
Inflation is thought to occur at high energy density (early time).

Example: at 1 pico-sec, $\rho \sim 10^{24}$ tons/cm^3, and $t_H \sim 1$ ps
For duration of 100 ps, a grows by $\sim 2^{100} \approx 10^{30}$ volume by 10^{90}
(note, in radiation universe, a grows by $100^{1/2} = 10$, Vol by 10^3)

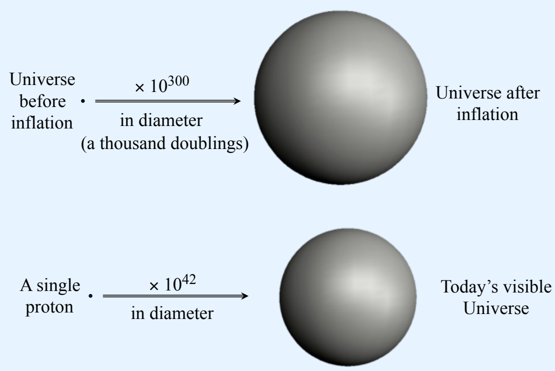
We currently don't know how long (how many e-folds) inflation lasted – maybe 100, maybe 10^6, maybe 10^{10}?

As you know, exponentials grow *enormously* fast.

It doesn't even matter what units you use to describe the final size of the region: $10^{1,000,031}$ nm = $10^{1,000,000}$ Mpc



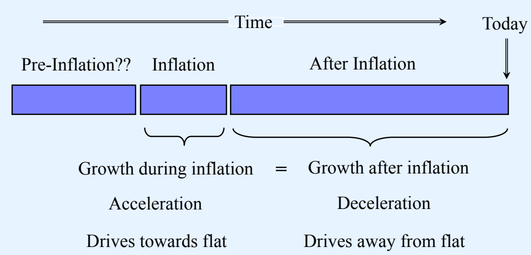
Inflation can be incredibly efficient



Minimum expansion needed: solving the Flatness Problem

Recall: $\Omega_t - 1 = (\Omega_{t,0} - 1)/v^2$

During inflation: $v^2 \sim a^2$ ← drives towards flat
radiation era: $v^2 \sim a^{-2}$ ← drives away from flat
Hence:



Example: GUT inflation

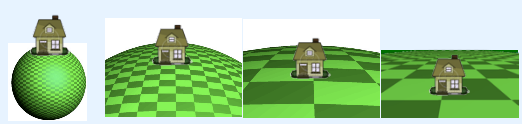
Example: if inflation occurs at the GUT era: $T \approx 10^{16}$ GeV $\approx 10^{28}$ K, so end of inflation has $a \approx 10^{-28}$ ($T \sim 1/a$ with $T \approx 1$K at $a \approx 1$).

So after inflation to today: a grows by 10^{28}.
So during inflation: a must grow by 10^{28} leaving $1-\Omega_t \approx 10^{-56}$.
(note: we're ignoring the $\sim 10^{3.5}$ growth during matter era).

Two periods of equal growth:
first lasts ~90 x 10^{-36} sec, (~90 e-fold is ~10^{28})
second lasts 13.7 Gyr.

Standard way to display this using inflating sphere/balloon:

Inflation flattens the geometry



Inflation flattens the geometry →

One way to look at this: recall, curvature radius $R = a R_0$

During inflation: scale factor & curvature radius grow much faster than the horizon → we see a relatively smaller patch

During radiation/matter era: horizon grows faster than scale factor & curvature radius → we see larger fraction of "sphere"

Solving the Horizon Problem

Keep example of GUT inflation:
current horizon size ≈ 45 Gly ≈ 10^{18} light-seconds.
So, at the GUT era this region is ≈10^{-10} ls across (10^{28} x smaller)

In the standard model, the horizon size at 10^{-36} s is ≈10^{-36} ls, so our current universe was ~10^{26} times larger than the horizon size at that time, which is enormously causally disconnected.

However, with inflation, our region is a further x10^{28} smaller, or 10^{-38} ls across, before inflation, which is 100 x smaller than the horizon at that time.

→ We've just solved the horizon problem: a coherent causally connected region ultimately created our visible Universe.

Note: same 90 e-fold minimum solves both flatness & horizon.

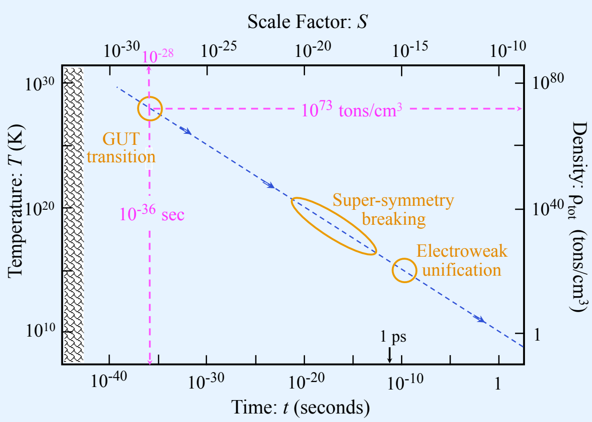
The Physics of Inflation

Relevant branches of physics:
- High energy physics
- Quantum field theory

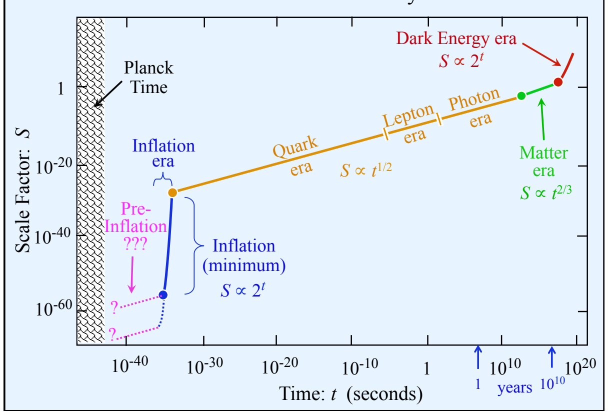
Quantum Fields:
- The Universe is filled with fields -- one for each particle
- Most fields have no energy
- Scalar fields can have energy -- acts like a vacuum energy

Field responsible for inflation is not yet known
It is called: "The Inflaton Field"

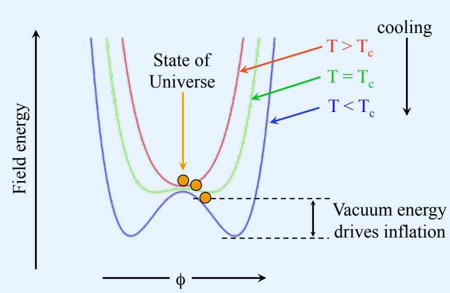
The necessary conditions may be associated with a phase change (see below), but exactly when is not known. Three main possibilities are suggested: GUT; Supersymmetry; EW transition.



Full cosmic history

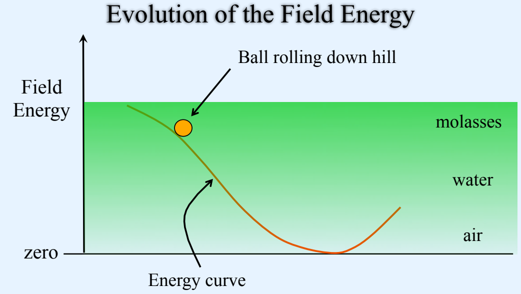


Changing inflaton field leads to a vacuum energy



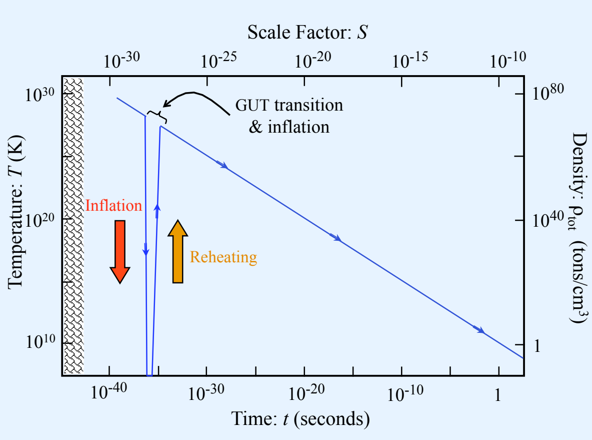
Quantum fields are usually expressed V(ϕ): uniform in space, but evolving in time. It transpires, that the evolution of the state is similar to the dynamics of a ball rolling on the potential "hill".

Evolution of the Field Energy



One oddity: the ball experiences a height (energy) dependent drag. Hence it hangs for a while (during inflation), then rapidly settles.

When it settles, it rolls back and forth, and the energy of the field is transferred to all other quantum fields → radiation era.



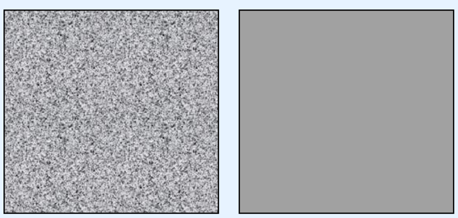
Origin of structure

About a year after Guth introduced inflation, it was realized that inflation could solve the “structure problem” – i.e. it could naturally generate the right kind of density fluctuations that over time could turn into stars and galaxies.

Many people regard this as one of the greatest successes of inflation.

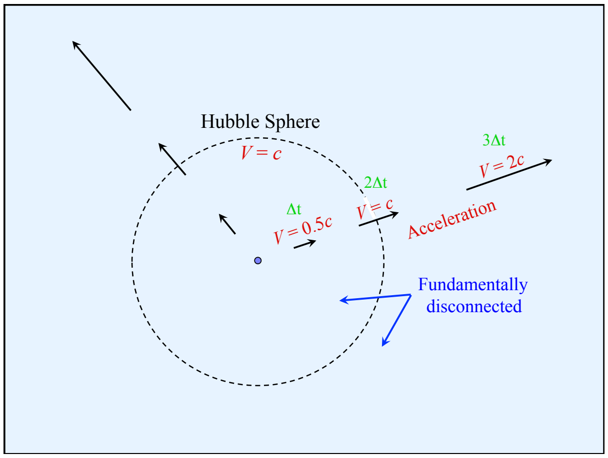
The key idea is that quantum fluctuations of the inflaton field get turned into real fluctuations as they leave the Hubble sphere. They then get stretched by expansion. The ongoing nature of inflation creates a heirarchy of fluctuations on all scales.

Fluctuations in a quantum field

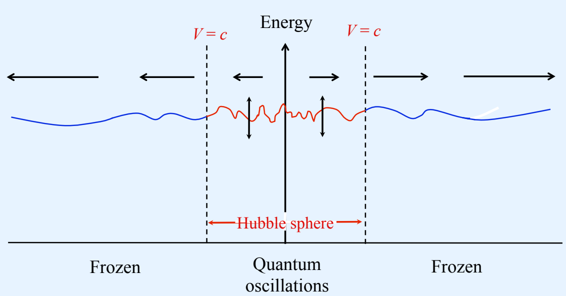


At any instant: rough

Long term average: smooth

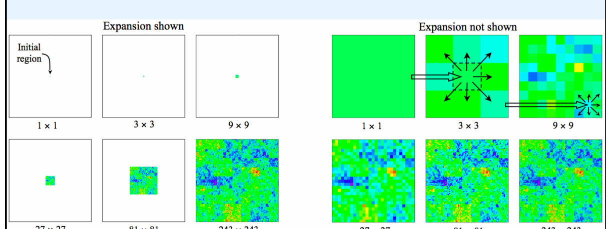


Freezing out the quantum fluctuations

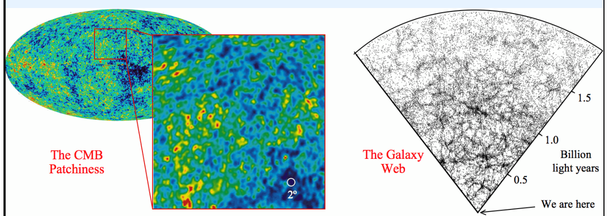


Inflation Creates Fluctuations

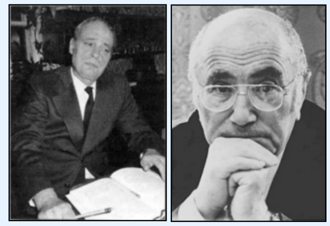
Expansion not shown



Quantum patterns in the CMB and galaxy web



The initial fluctuation spectrum has the Harrison-Zel'dovich "scale invariant" form



Ed Harrison (1919 - 2007)

Yakov Zel'dovich (1914 - 1987)

Rather simple arguments suggest that the primordial fluctuations should be "scale invariant" in the potential.

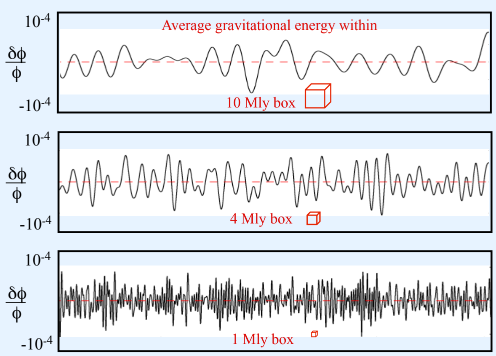
$$\langle\delta(\phi)/\phi\rangle_k = \text{const}$$

In practice, this means:

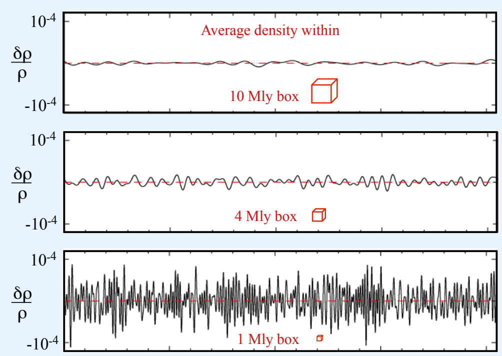
a) Density variations are greater on smaller scales. $P(k) = \langle\delta(\rho)/\rho\rangle_k^2 \propto k$

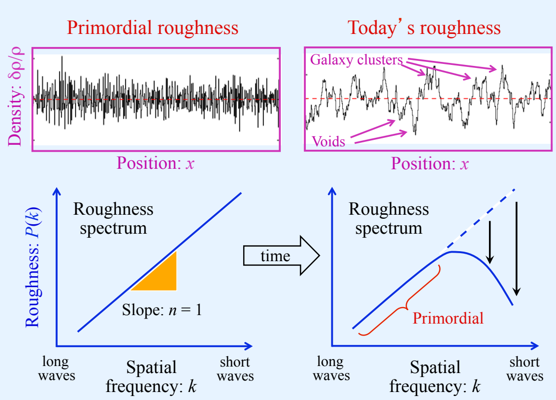
b) The density variation on the scale of the horizon is always the same: $[\delta(\rho)/\rho]_{Hor} \approx 10^{-5}$

Gravitational roughness is similar on all scales

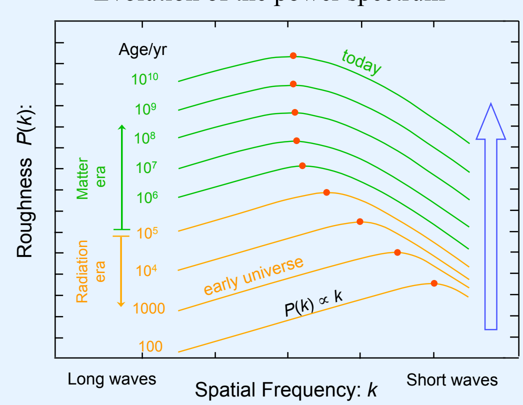


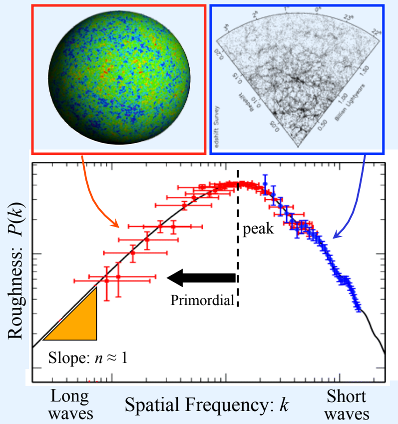
Density roughness is larger on smaller scales

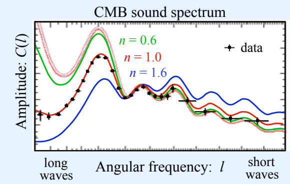




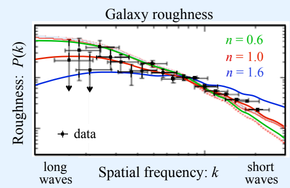
Evolution of the power spectrum







$n = 1$ fits best to both



Further tests of inflation

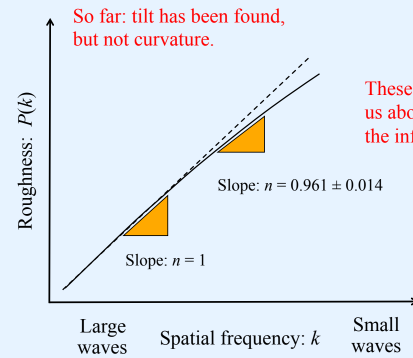
The fact that the initial power spectrum is of roughly the correct form, P(k) ∝ k, is encouraging, but there are further signatures of inflation.

1) As inflation ends, the inflaton field energy drops, and the expansion slows from pure exponential. This makes the power spectrum curve over, so we expect an index *n* slightly less than 1, decreasing to higher k. This is called *tilt* and *curvature*.

2) The quantum origin means the *phases* of the waves are *random*. This in turn leads to a *Gaussian random field*.

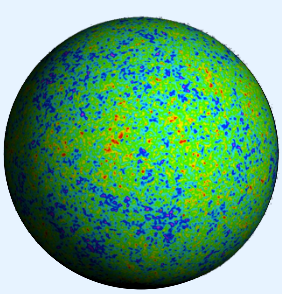
3) Inflation generates *gravity waves* (tensor fluctuations), with amplitude that depends on when inflation ocurred. These induce "B-mode" patterns in the CMB *polarization*.

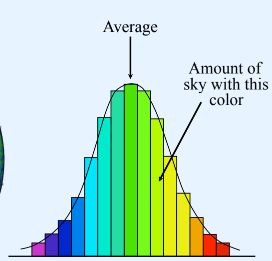
End of Inflation: Tilt & Curvature



These deviations tell us about the shape of the inflaton potential.

A Gaussian Random Field



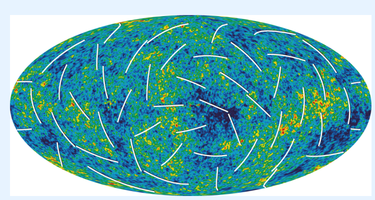


CMB brightness

Gaussian (Bell) Curve

So far: no deviations from Gaussian have been found.

Polarization on the CMB



Sketch (exaggerated) of polarization patterns on the CMB

B-mode polarization pattern made by gravity waves

The hunt is on for B-mode polarization; maybe Planck?

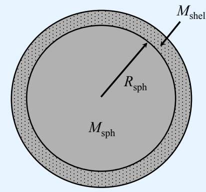
Inflation is stunningly creative

1) Inflation makes the universe out of (almost) nothing.
2) Inflation makes a creative universe with low-entropy.
3) Inflation can make many universes, not just ours.

1) Recall, a Newtonian sum of all total energy within a Hubble volume suggests zero: negative gravitational binding energy equals the rest-mass energy within the sphere. In GR the zero curvature measurement is equivalent to this. So, it suggests the birth process may have started from ~nothing.

It transpires, a tiny seed is needed to start inflation going. Using Newtonian language: the gravitational energy released by expansion must be enough to make the new shell of vacuum.

A seed is needed to start inflation



gravitational energy released — greater than — new energy in shell

$$\frac{G \times M_{sph} \times M_{shell}}{R_{sph}} > M_{shell} \times c^2$$

$$\frac{G \times M_{sph}}{R_{sph}} > c^2$$

$$\frac{G \times M_{sph}}{c^2} > R_{sph}$$

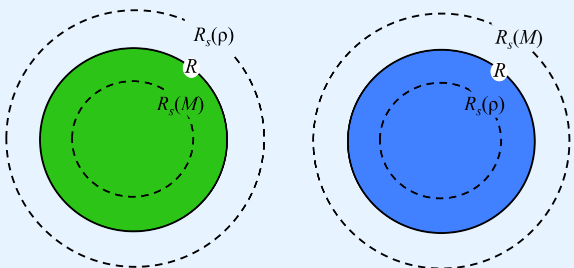
This is in fact the "black hole" condition. If the sphere contained normal matter or radiation (with w > 0) it would collapse.

However, since vacuum has w = −1, inflation occurs instead.

E.g. for GUT inflation, $\rho \sim 10^{73}$ tons/cm^3, $R \sim 10^{-11}$ fm, M~1 kg

Normal Condition: $R_s(M) < R < R_s(\rho)$

Inflation & Black Hole Condition: $R_s(M) > R > R_s(\rho)$



Sphere: *M, R, ρ*

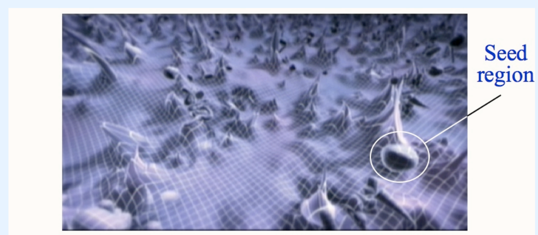
$R_s(M) = 2GM/c^2$

$R_s(\rho) = \sqrt{(3c^2/8\pi G\rho)}$

Sphere: *M, R, ρ*

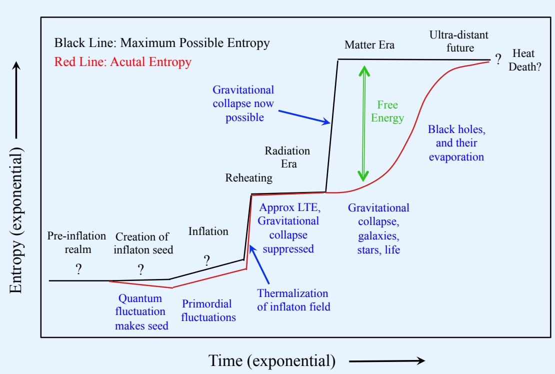
How do such seeds arise?

Not well known: possibly via quantum fluctuations



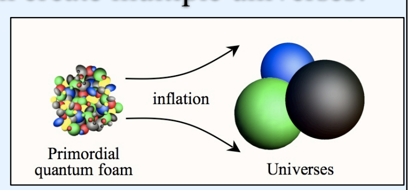
If true, then the Universe may have had no explicit cause: It just happened!

Entropy Evolution in the Universe



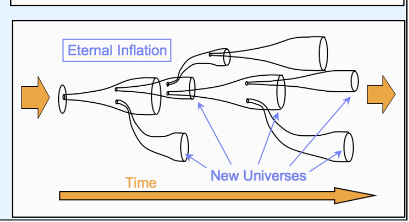
Inflation can create multiple universes!

Chaotic Inflation: Multiple seeds can make a multiverse

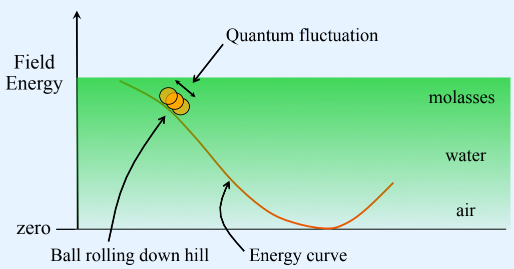


Much more speculative

Eternal Inflation: High quantum fluctuations keep inflating

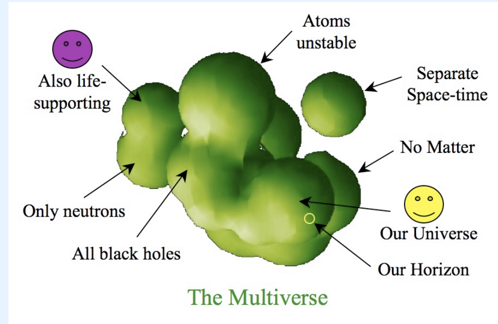


Evolution of the Field Energy



The high fluctuatios expand faster and ultimately dominate the total volume. I.e. there is always somewhere inflating.

Each universe may have different laws of physics



Only a few universes have "fertile" laws of physics.
We (obviously) live in one of those….

A Special Planet Among Many

Equivalent to the minority of fertile universes.

End of inflation