Growth of Structure

Notes based on Teaching Company lectures, and associated undergraduate text – with some additional material added.

For a more detailed discussion, see the article by Peacock taken from a 2002 winter school, which is a briefer version of his textbook:
http://ned.ipac.caltech.edu/level5/Sept03/Peacock/Peacock_contents.html

Preliminaries

Inflation, we think, created very slight variations in density from place to place, which we describe using: $\delta(\mathbf{r}) = \delta\rho/\langle\rho\rangle(\mathbf{r})$.

During expansion, although $\langle\rho\rangle$ decreases, $\delta\rho/\langle\rho\rangle$ can *increase*.

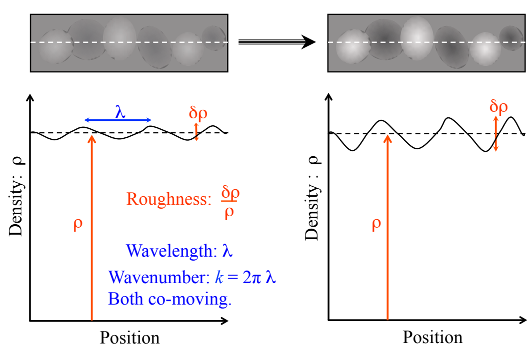
When $\delta\rho/\langle\rho\rangle$ reaches ~1 (i.e. $\rho \sim 2\langle\rho\rangle$), a region breaks away from Hubble expansion, slows, turns around, and collapses – this is now "object formation" (stars, galaxies, clusters).

We describe $\delta(\mathbf{r}) = \delta\rho/\langle\rho\rangle(\mathbf{r})$ using its Power spectrum, P(*k*):

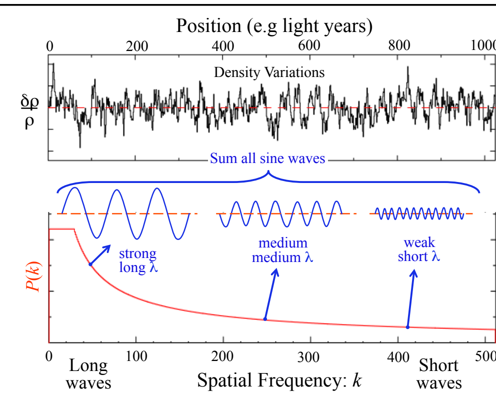
$$\delta(\boldsymbol{r}) = \sum \delta_k \, e^{ik.r} \quad \text{and} \quad P(k) = \langle|\delta_k|\rangle^2 \quad \text{where} \quad k = |\boldsymbol{k}| = 2\pi / \lambda$$

Questions:
What was inflation's P(*k*), and what is P(*k*) for today's Universe?
Can we understand how P(*k*) evolves from one to the other?

Growth in roughness: δρ/ρ

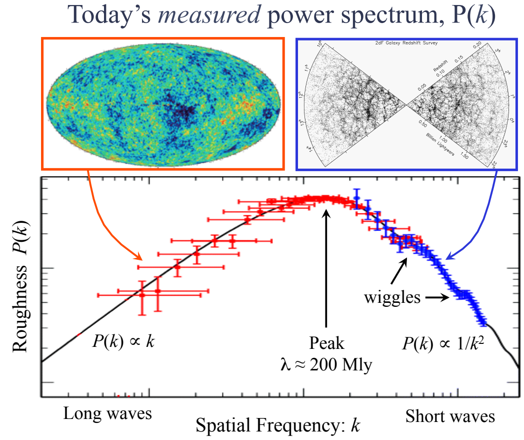


Even though ρ is dropping due to expansion, δρ/ρ can increase.

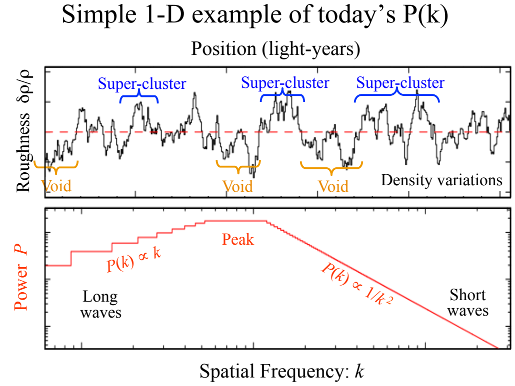


Example 1-D power spectrum, P(*k*), and its density field, δρ/ρ.
In this, and all other cosmological examples, the *phases* are random

Today's *measured* power spectrum, P(*k*)



Simple 1-D example of today's P(k)



Largest features (super-clusters/voids) have scale corresponding to peak of P(*k*). Larger scales are progressively smoother.

Super/Sub-Horizon Growth

$P(k)$ for the density field contains both short and long λ modes.

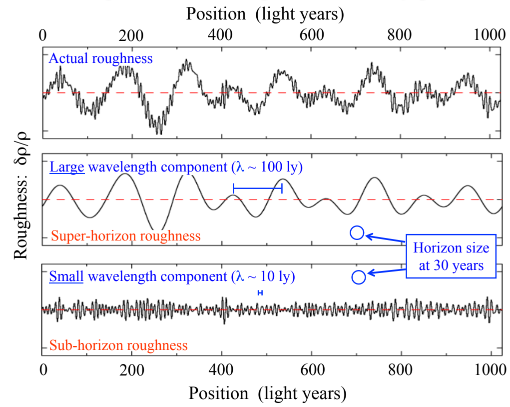
In linear growth (when $\delta\rho/\rho << 1$), the growth of each λ mode is independent of all the other modes.

However, the growth of a mode depends on whether the mode's λ is smaller or larger than the *horizon* at that time.

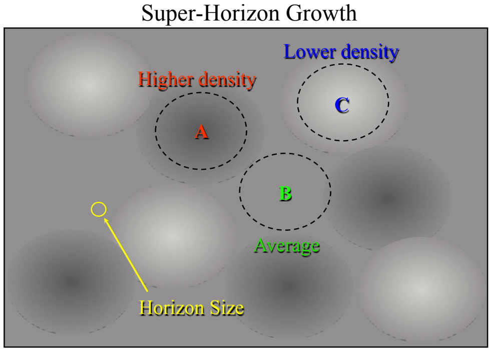
Let's look at super-horizon growth, and sub-horizon growth.

Initially, we are only interested in the dark matter component.
This ultimately dominates the matter components
It is pressureless, so only responds to gravity.

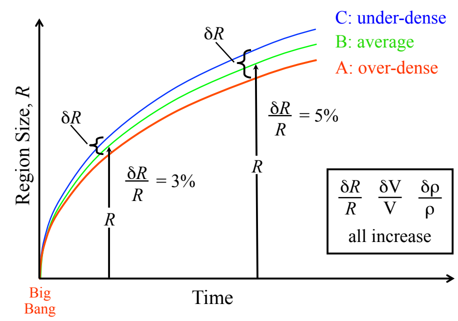
Example of two component density pattern.



Super-Horizon Growth

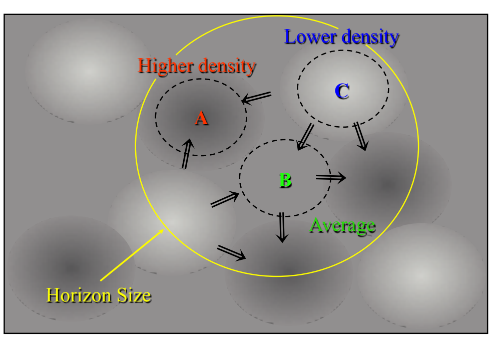


Different expansion rates amplify density variations



During radiation era: $\delta\rho/\rho \propto t$, so from inflation ($10^{-36}$ s) to 1000 yrs $\delta\rho/\rho$ grows by factor $\sim 10^{46}$ (on all scales). Incredible growth!

Sub-Horizon Growth



Local forces at work – gravity pulls material into denser regions away from less dense regions – density contrast *grows*.

Perturbation growth in an expanding medium

This process has a long history – the Newton-Bentley dialogue:
Newton realized infinite universe unstable to local collapse.
For *static* system, collapse proceeds *exponentially*: $\delta\rho/\rho \sim e^t$.
However, *expansion* reduces the density, slowing the growth rate:
Static collapse time: $t_c \sim (G\rho_m)^{-1/2}$,
Cosmic expansion timescale: $1/H \sim (G\rho_c)^{-1/2}$

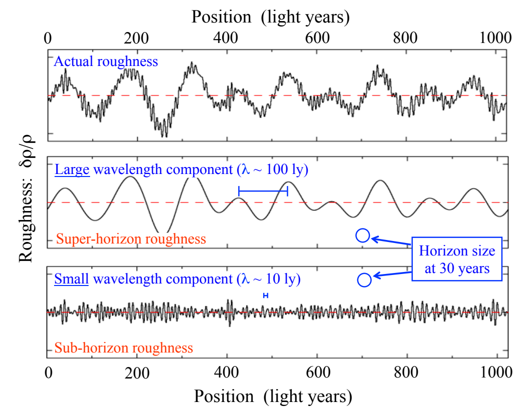
Isaac Newton 1643 - 1727 | Richard Bentley 1662 - 1742

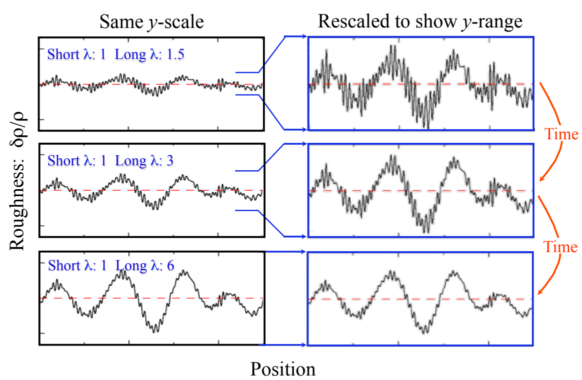
During the matter era:
$\rho_m \approx \rho_c$ and $\delta\rho/\rho \propto a \propto t^{2/3}$

During the radiation era:
$\rho_m << \rho_c$ so $\delta\rho/\rho \sim$ *frozen*
In fact, super-horizon growth slowly redshifted away, so $\delta\rho/\rho \propto \ln a$.
This is the Mésáros effect.

Density Contrast Growth Rates.

Roughness Growth Rates	Radiation Era	Matter Era	Dark Energy Era
Super-horizon	$\delta\rho/\rho \propto a^2 \propto t$	$\delta\rho/\rho \propto a \propto t^{2/3}$	$\delta\rho/\rho$ = const (frozen)
Sub-horizon	$\delta\rho/\rho \propto \ln a$ (frozen)	$\delta\rho/\rho \propto a \propto t^{2/3}$	$\delta\rho/\rho$ = const (frozen)





Because sub-horizon scales have suppressed growth, the overall nature of the density pattern *changes* over time.

Evolution of P(k)

Quantum fluctuations during (exponential) inflation provide an "Initial Power Spectrum" (IPS):

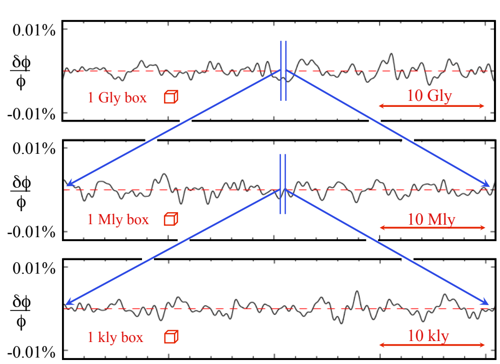
$$P(k) = A\,k^{n} \quad \text{with } n = 1$$

This is also called the Harrison-Zel'dovich spectrum, and/or the scale-invariant spectrum.

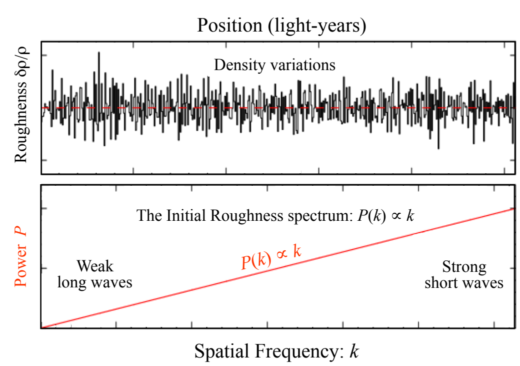
This latter term is used because: $\Delta_k^2(\Phi) = k^3 P(k)$ = constant. This means that the rms variation in gravitational potential, Φ, within regions of size r ~ 1/k, is independent of r. I.e the gravitational "wrinkliness" of the (initial) universe looks the same on all scales.

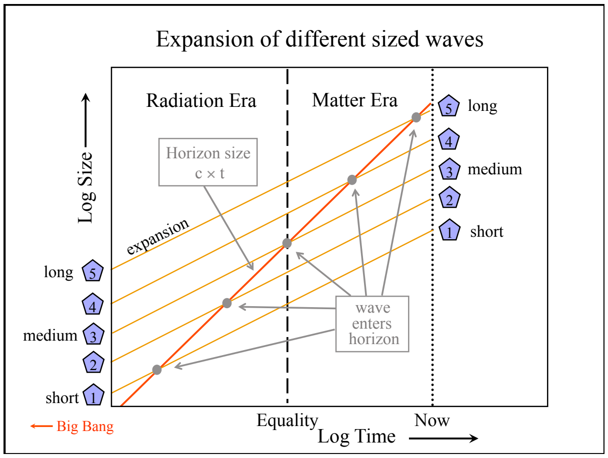
Let's see how an IPS of $P(k) \propto k$ evolves over time.

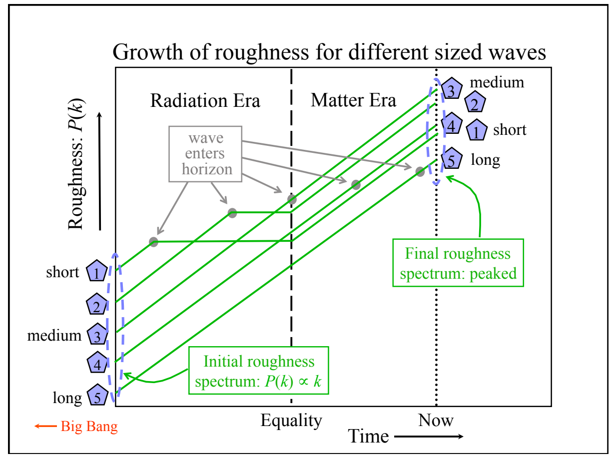
Gravitational roughness is similar on all scales

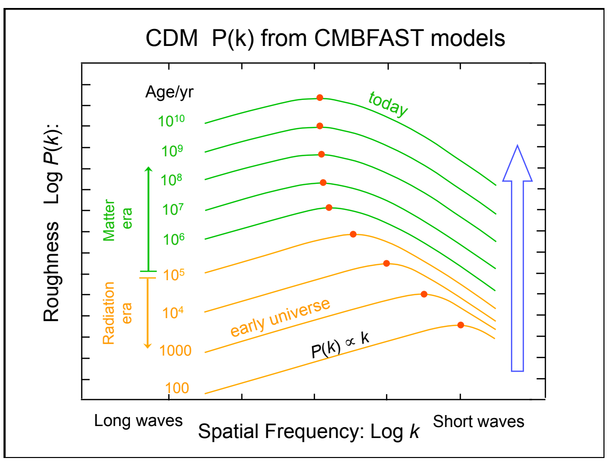


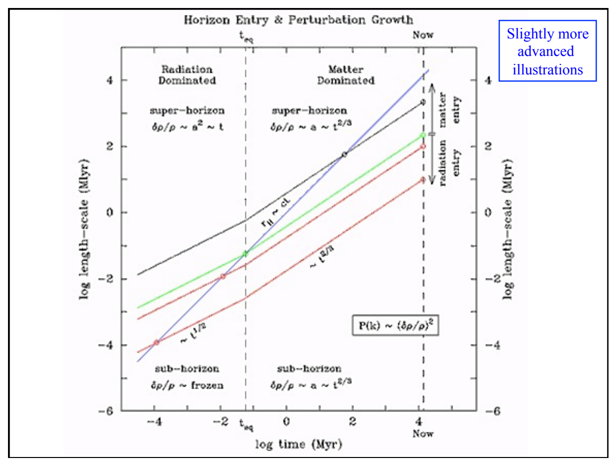
The Initial Power Spectrum and its corresponding density (1-D)

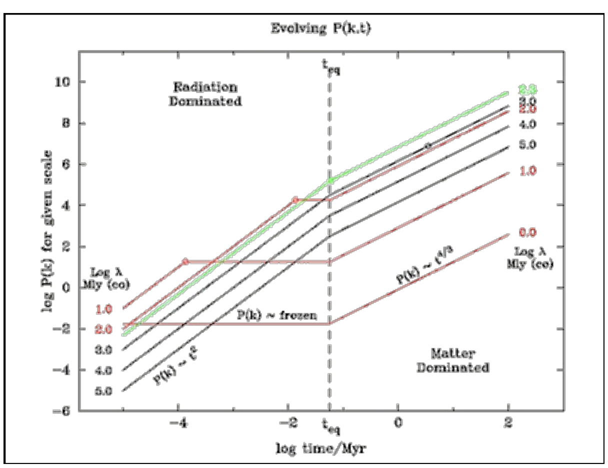


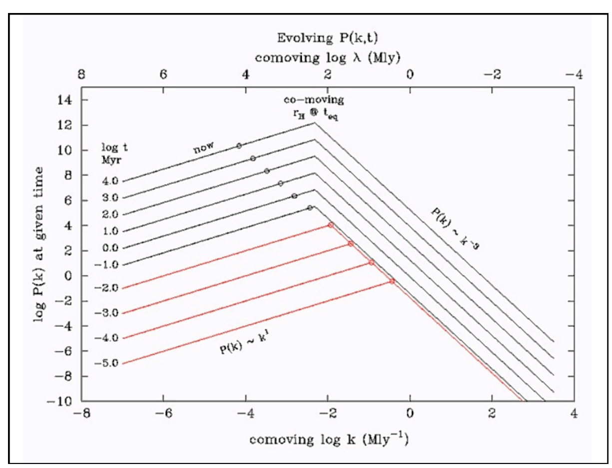

Expansion of different sized waves
Radiation Era
Matter Era
Horizon size
c × t
expansion
wave
enters
horizon
Log Size
long
medium
short
Big Bang
Equality
Now
Log Time

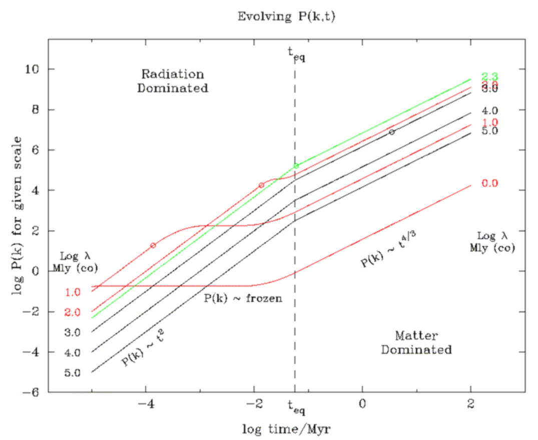

Growth of roughness for different sized waves
Radiation Era
Matter Era
wave
enters
horizon
Roughness: P(k)
medium
short
long
Final roughness
spectrum: peaked
Initial roughness
spectrum: P(k) ∝ k
Big Bang
Equality
Now
Time

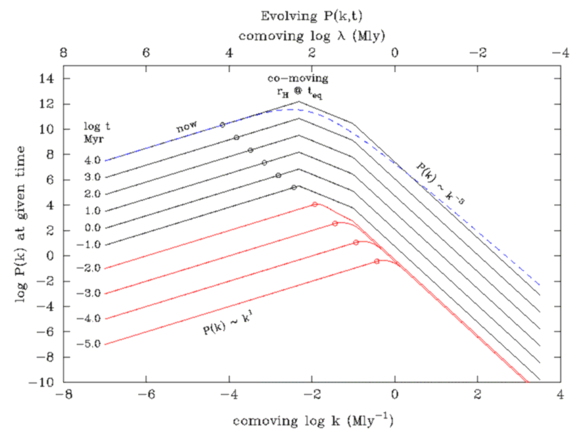

CDM P(k) from CMBFAST models
Age/yr
today
Matter
era
Radiation
era
early universe
P(k) ∝ k
Roughness Log P(k):
Long waves
Spatial Frequency: Log k
Short waves


Horizon Entry & Perturbation Growth
Slightly more
advanced
illustrations
Radiation
Dominated
Matter
Dominated
super-horizon
sub-horizon
P(k) ~ (δρ/ρ)²
log length-scale (Myr)
log time (Myr)
Now


Evolving P(k,t)
Radiation
Dominated
Matter
Dominated
P(k) ~ frozen
log P(k) for given scale
log time/Myr


Evolving P(k,t)
comoving log λ (Mly)
co-moving
now
log P(k) at given time
comoving log k (Mly⁻¹)



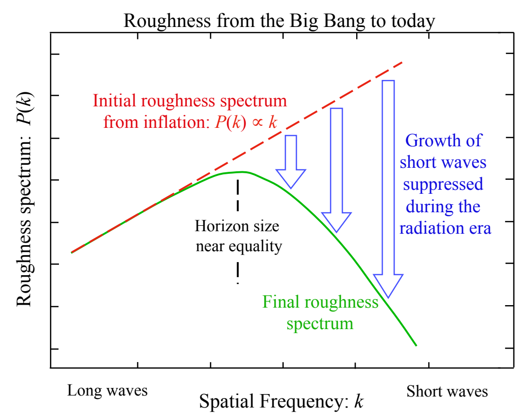


Change in gradient by 4: n = 1 to -3

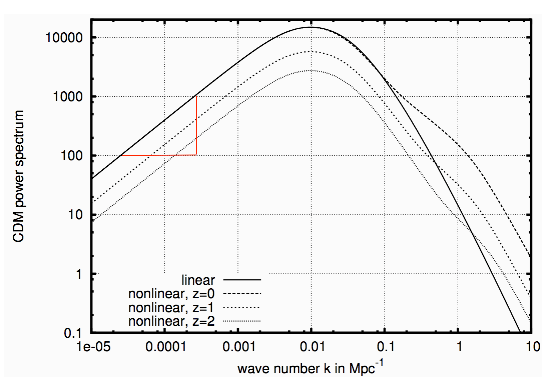
It is relatively easy to see why there is a change by 4 in the gradient of log P(k) vs log k due to suppressed growth in the radiation era:

Consider two waves, 1 dex apart in k. Since wavelengths grow ~ $t^{1/2}$ in the radiation era and the horizon grows ~ t, then there is a *two*-dex delay in time between the horizon entry of the two waves. Since δρ/ρ grows ~ t in the radiation era, then there is a two-dex difference in growth of δρ/ρ, which corresponds to a *four*-dex difference in P(k) (since P(k) ~ $(\delta\rho/\rho)^2$). Hence, the high-k (small-λ) wave now has P(k) four-dex lower due to its extra suppression in the radiation era.

In practice, (i) the peak is so broad that it takes a few dex in k to reach the k^{-3} part, and (ii) non-linear effects make the high-k side less steep anyway.

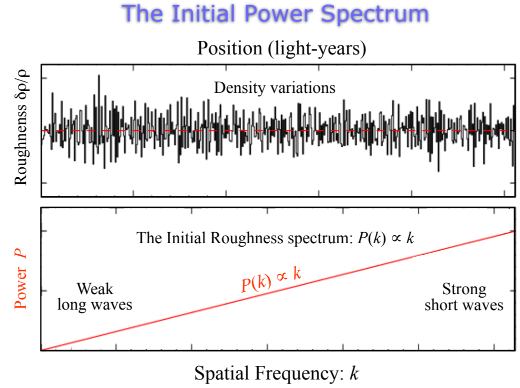


Non-linear effects: high k side less steep

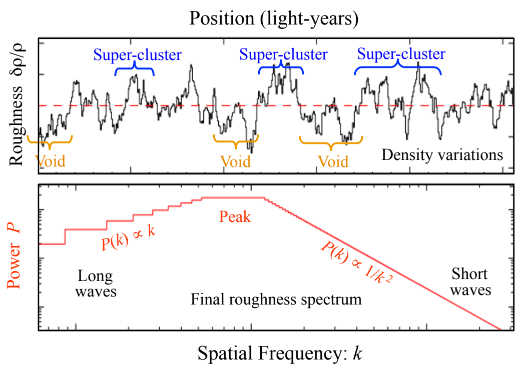


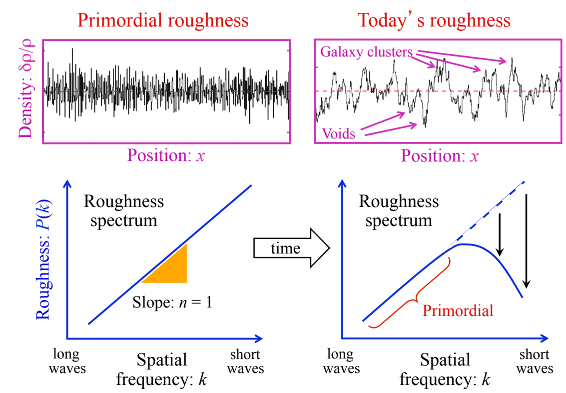
Spatial Frequency: k

Review using 1-D examples:
The Initial Power Spectrum



Today's Power Spectrum





Comparing P(*k*) for CDM & Baryons

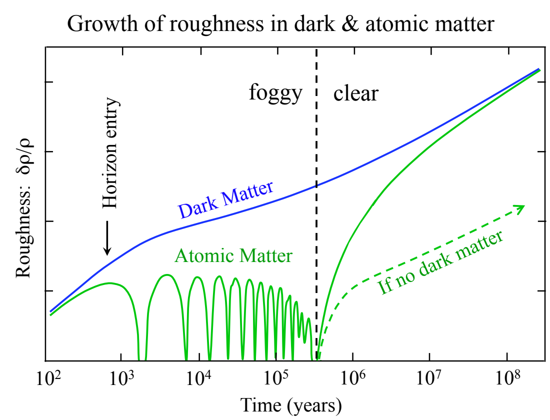
The Baryons are tied to the photons via Thomson opacity, so they experience *pressure*.

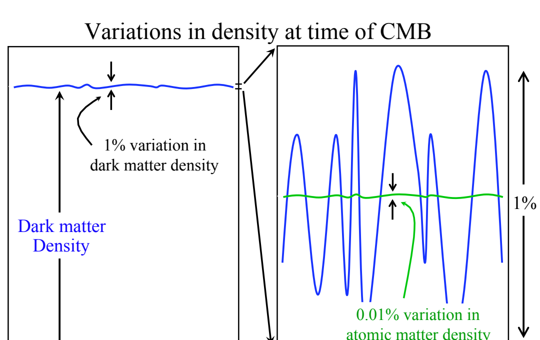
On scales less than the Jean's length, pressure dominates and the photon-baryon gas oscillates as sound waves.

Since $\lambda_J \approx c_s(\pi/G\rho)^{1/2}$ and $c_s \approx c/\sqrt{3}$, then $\lambda_J \approx c/\sqrt{(G\rho)} \approx c/H$.
So essentially all scales within the horizon are dominated by pressure and undergo oscillatory (acoustic) motion, with no steady growth in $\delta\rho/\rho$.

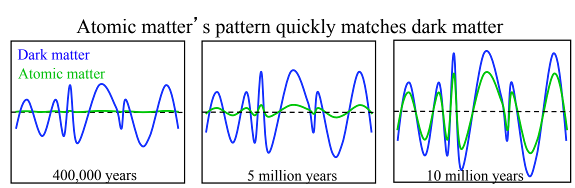
At the time of the CMB, therefore, $\delta\rho/\rho$ is much *less* in the baryons than in the dark matter.

Only when the radiation decouples and the pressure drops do the baryons fall into the DM pockets and inherit its density pattern.



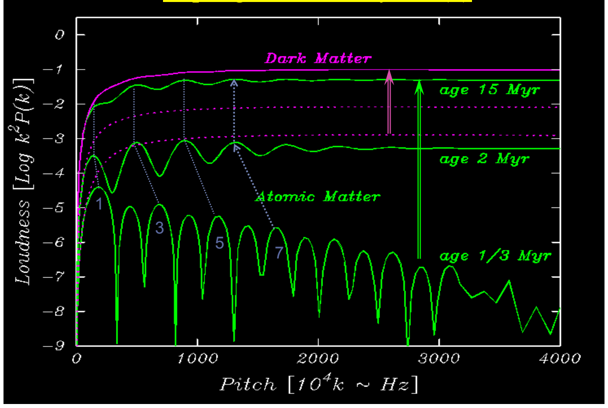


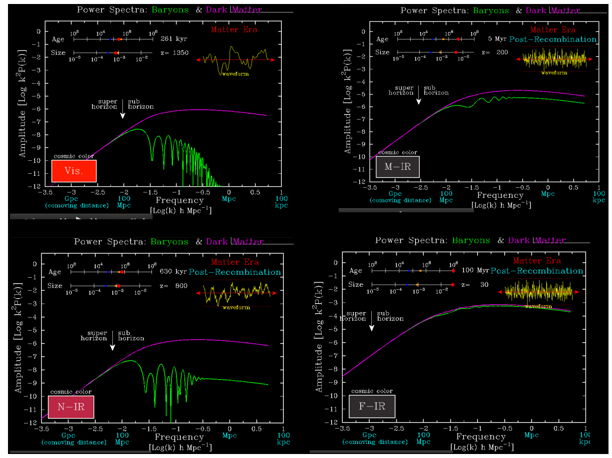
Before CMB: atomic matter *oscillates* in the DM potentials



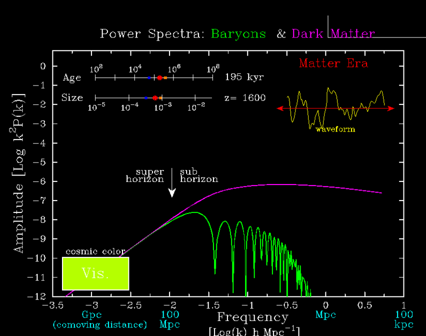
After CMB: atomic matter *falls into* the DM potentials – takes on its 3-D density pattern.

Rapid growth of baryon P(k)





CDM & baryon evolution to z ~ 30



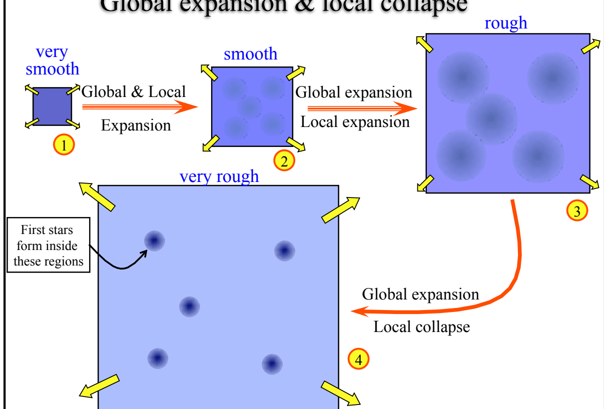
Turnaround and Collapse

Previous discussion was for linear growth regime, so up to $\delta\rho/\rho \sim 1$.

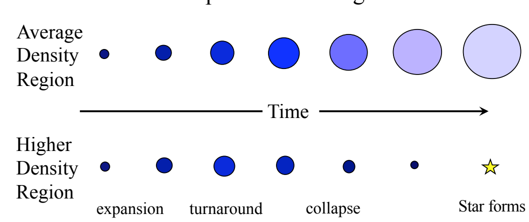
After that, a region deviates significantly from the Hubble flow, and ultimately halts expansion, turns around and collapses.

Here are some figures illustrating the situation.

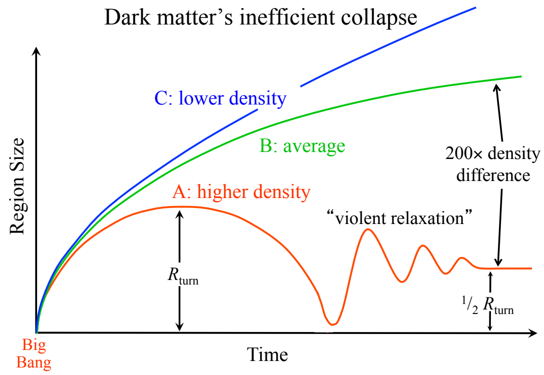
Global expansion & local collapse



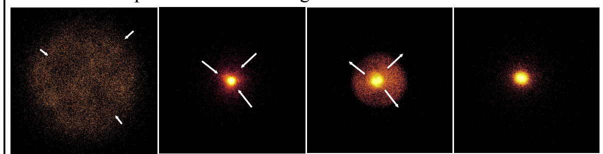
Collapse of denser regions

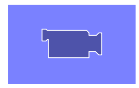


Dark matter's inefficient collapse



Collapse of dark matter region: "violent relaxation"





Once collapse starts in the early Universe, it is a rapid process – early structure formation is *fast*.

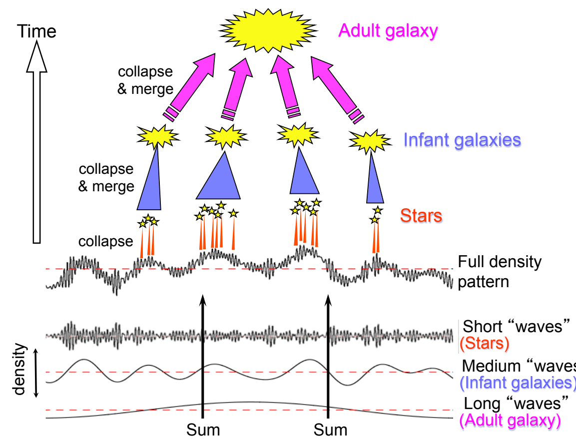
The reason is simple: $t_{coll} \sim 1/\sqrt{G\rho}$, and since ρ is high at early times, collapse times are high (roughly, t_{age}).

Hierarchy of Collapse

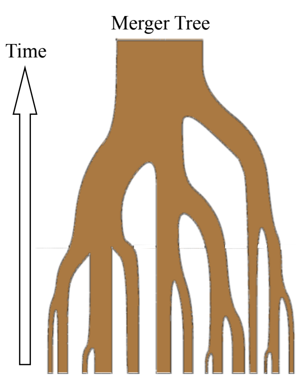
The first objects form where the density is highest.

Where are these peaks in the density field?
They occur where short wave peaks align with medium wave peaks which in turn align with long wave peaks (see figures).

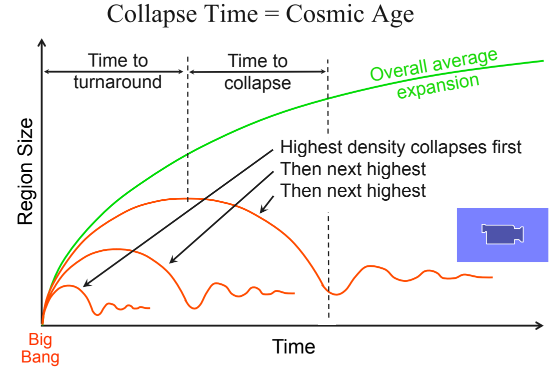
Because of this, the first stars are born in groups, which then fall together in star-clusters, which fall together into infant galaxies, and so on, in a *hierarchy*.



This is often shown as a merger "tree".

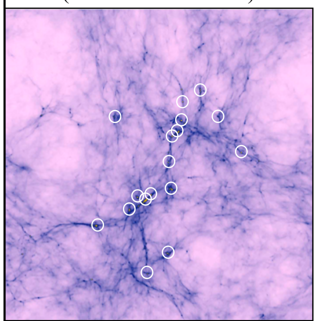


Collapse Time = Cosmic Age



Notice that the earlier objects forming are also the densest.

Location of first stars
(detailed simulation)



150,000 light years

First stars form in densest dark matter regions

Strongly Clustered

Notice, objects forming at high-z are highly clustered because they form within larger over-dense regions.

More quantitative approach

If we want to ask when a region of size ~r will collapse, we simply ask whether $\delta\rho/\rho > 1$ when averaged over the region.

If the answer is "yes" then it will soon collapse and virialize (possibly including many smaller previously virialized objects).

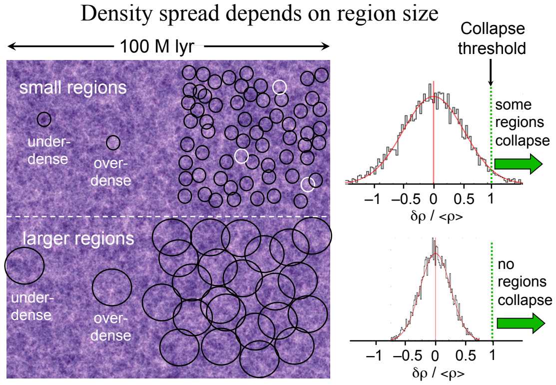
It is important to realize that the relative number of regions of size r that are about to collapse depends strongly on r. In general, at any given time, fewer regions of larger r will be collapsing.

We can get some sense of this by asking what is the *variance* (i.e. σ^2) of $\delta\rho/\rho$ for an ensemble of spheres of radius r.

When $\sigma^2 \sim 1$, then a significant fraction are ready to collapse.

This is illustrated in the next figure.

Density spread depends on region size



The Universe at 200 Myr

More quantitative approach

Placing spheres of radius r at random and evaluating $\delta\rho/\rho$ within the sphere is the same as *convolving* the density field by a spherical "top hat". The variance of this smoothed distribution is what we're after.

Recall: convolving in real space can be done in Fourier space:

$$\sigma^2(r) = \frac{1}{(2\pi)^3}\int P(k)\ W^2(kr)\ 4\pi k^2 dk$$

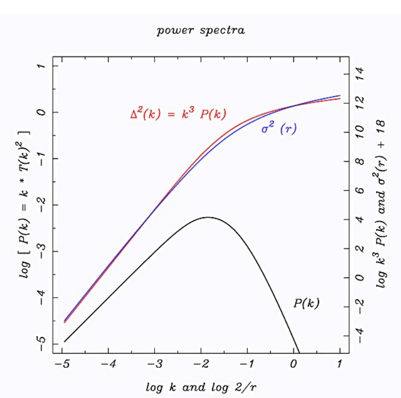
Where $W(kr)$ is the Fourier transform of the spherical top hat:

$$W(kr) = \frac{3}{(kr)^3}\left[\sin(kr) - kr\cos(kr)\right]$$

Since this is a relatively peaked function at $kr \sim 1$, then it turns out:

$$\sigma^2(r) \approx \Delta^2(k) \equiv \frac{1}{2\pi^2}k^3 P(k) \quad \text{where} \quad k \approx \frac{2}{r}$$

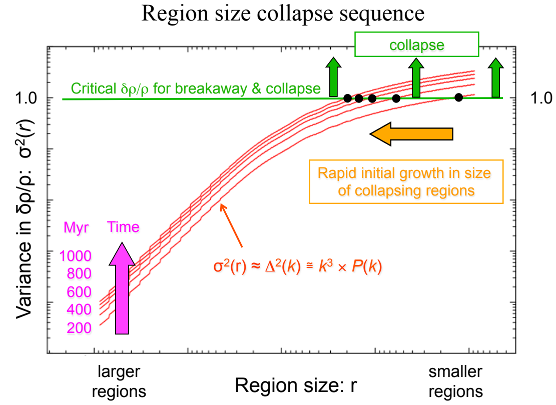
Dimensionless power spectra: $\Delta^2(k)$ and $\sigma^2(r)$

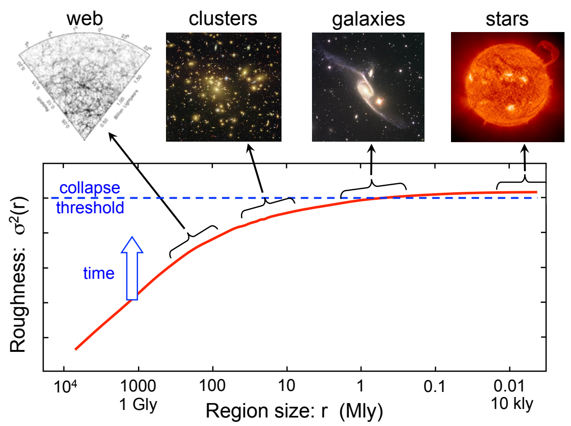


Note: $\Delta^2(k) \sim k^3 P(k)$ is a dimensionless version of the power spectrum, and is often used instead of P(*k*).

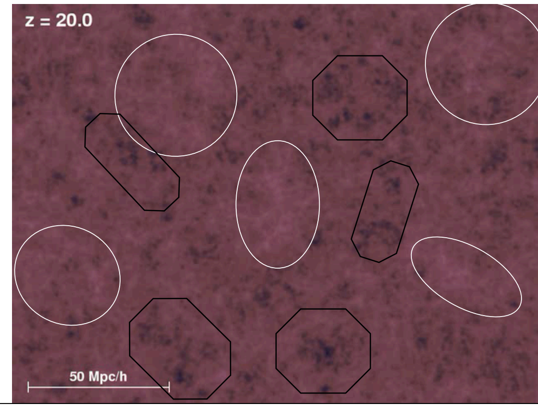
When $\Delta^2(k)$ reaches 1 then regions of size r ~ 1/*k* will soon collapse.

Region size collapse sequence

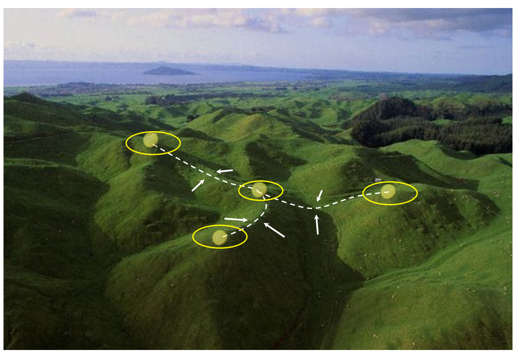




Growth of roughness (expansion not shown!)

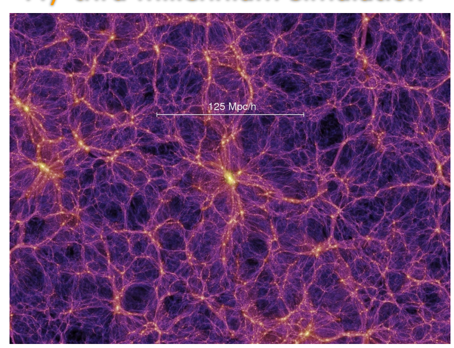


Formation of filaments



Following symmetric peak collapse, next is asymmetric ridge collapse.

Fly-thru millennium simulation



End of Growth of Structure