Universal Distance Scale

The HUBBLE CONSTANT is the local expansion rate of the universe, local in space and local in time. The equation of uniform expansion is

$$v = H_0 r \tag{1}$$

where v is recession velocity and r is distance from the observer. To measure the proportionality constant, we adopt a definition of the Hubble constant as the asymptotic value of the ratio of recession velocity to distance, in the limit that the effect of random velocities of galaxies is negligible. In galactic structure the velocity of the local standard of rest has wider significance for the dynamics of the Milky Way than the velocity of the Sun. We have to abstract the Hubble constant from local motions in a similar way.

Existence proofs of the Hubble constant so defined, that we would still accept today, include the Hubble diagrams for cluster ellipticals (Sandage and Hardy 1973), for the Arecibo clusters (Aaronson and Mould 1986)[1] and for type Ia supernovae (Hamuy *et al* 1996).

It is also desirable to employ the Hubble constant in models which apply to larger spatial and temporal scales. The present value of the Friedmann model's $\dot{a}/a$, generally acknowledged as the standard model of the universe, is H_0. The expansion age of the universe is H_0^{-1} to a factor of order unity. If the standard model of cosmology were the Einstein–De Sitter model, then that factor would be two-thirds.

How to measure the Hubble constant

Three current schools of thought on this subject are represented in *Critical Dialogues in Cosmology* (Turok 1996). Two of these schools see the CEPHEID PERIOD–LUMINOSITY RELATION for Cepheid variable stars as crucial for establishing the distances of galaxies within 20 Mpc, which then calibrate 'standard candles'. These secondary distance indicators reach out into a region where an asymptotic ratio of recession velocity to distance can be sought.

Supernova Ia standard candles

The SUPERNOVA phenomenon results from instability in the equation of state of the degenerate material in the interior of WHITE DWARFS. According to Branch and Tammann (1992) SNeIa are explosions of a standard maximum power. Cepheid distances can be measured to some of the host galaxies.

The Hubble Space Telescope key project

The key project strategy has been fully described by Kennicutt *et al* (1995), and involves calibration of the infrared TULLY–FISHER RELATION and the equivalent dynamical relation for ELLIPTICAL GALAXIES, the ($D_n\ \sigma$) relation. Also calibratable from key project Cepheid distances are surface brightness fluctuations in galaxies, the globular cluster luminosity function, the planetary nebula luminosity function and the expanding photospheres method for type II supernovae.

One-step methods

Critics of both the above approaches point to the propagation of errors in a three-step ladder from trigonometric parallaxes to Cepheids to secondary distance indicators to H_0. One-step methods to determine H_0 include GRAVITATIONAL LENSING of distant QUASARS by intervening mass concentrations, which results in measurable phase delays between images of the time-varying input signal from the source. Where these phase delays can be accurately determined, and when the model mass distribution can be uniquely inferred, they directly lead to z/H_0, where z is the REDSHIFT of the lens (Turner 1997).

Compton interaction between the microwave background and hot gas in clusters of galaxies (the SUNYAEV–ZELDOVICH EFFECT) allows the physical size and distribution of the absorber/scatterer to be inferred, at least in projection, if the properties of the hot gas are also constrained by direct detection in x-rays. A distance can be determined by comparing the angular and physical projected size (Lasenby and Jones 1996).

The Large Magellanic Cloud (LMC)

The LMC is an anchor-point of the extragalactic distance scale. From the LMC it is possible to calibrate essentially all of the secondary distance indicators using the Cepheid PL relation.

The HIPPARCOS revision of the Cepheid PL relation, designated [H] in table 1, brings trigonometric PARALLAXES (Perryman *et al* 1997) to bear on the calibration in place of cluster main sequence fitting. Although this is a step forward in principle, the signal-to-noise of the parallax measurements, the short periods of the nearby Cepheids and the need for a fuller photometric study of the Hipparcos stars combine to render their impact on the problem uncertain at this stage.

The HST key project has adopted an LMC distance modulus of 18.50 ± 0.13 mag corresponding to 50 ± 3.2 kpc. The adopted reddening is $E(B-V) = 0.10$ mag.

Calibration of the Tully–Fisher relation

The Tully–Fisher relation is a luminosity–linewidth correlation for SPIRAL GALAXIES (Fisher and Tully 1977). Sakai *et al* (1999) present a calibration of $\mathrm{BVRIH}_{-0.5}$ Tully–Fisher relations based on Cepheid distances to 21 galaxies within 25 Mpc, and they go on to determine the Hubble constant based on 23 clusters of galaxies within 10 000 km s^{-1}. Primarily, these are clusters studied by Giovanelli *et al* (1997). They obtain $H_0 = 71 \pm 8$ km s^{-1} Mpc^{-1}.

Like Giovanelli (1997), Sakai *et al* find that the effect of incompleteness biases in their sample is small (cf Sandage *et al* 1995, Teerikoorpi 1997) and correctable.

---

[1] But note that velocity perturbations as large as 5% may exist on these scales (Lauer and Postman 1994).

Table 1. The distance of the Large Magellanic Cloud.

Distance indicator	m–M	Reference
RR Lyrae	18.54 ± 0.07	Walker (1992, 1993a, b)
MACHO RR Lyrae	18.48 ± 0.19	Alcock *et al* (1997)
Cepheids	18.47 ± 0.15	Feast and Walker (1987)
Bump Cepheids	18.51 ± 0.05	Wood *et al* (1997)
Cepheids [H]	18.70 ± 0.10	Feast and Catchpole (1997)
Cepheids [H]	18.57 ± 0.11	Madore and Freedman (1998)
SN1987A	18.50 ± 0.13	Panagia *et al* (1991)
SN1987A [E]	$<18.44 \pm 0.05$	Gould and Uza (1998)
Red clump		Stanek *et al* (1998)
Eclipsing binaries	18.44 ± 0.07	Pritchard *et al* (1998)
Adopted	18.5 ± 0.13	

The (D_n σ) relation for ellipticals

The analogous scaling relation for elliptical galaxies relates velocity dispersion and isophotal diameter (Burstein *et al* 1987). Mould *et al* (1996) selected all the groups with four or more (D_n σ) measurements from Faber *et al* (1989) and renormalized their distances so that the geometric mean of that of the Fornax, Virgo and Leo groups was 15 ± 1 Mpc, which is obtained purely from Cepheid distances. Kelson *et al* (1999) used a similar approach to calibrate the fundamental plane of radius, surface brightness and velocity dispersion. They obtained $H_0 = 78 \pm 10$ km s^{-1} Mpc^{-1} from a sample of clusters with a mean redshift of approximately 6000 km s^{-1}.

Supernovae

There has been significant progress in the discovery, analysis and understanding of supernovae in recent years, not least in the area of theoretical models (Hoflich and Khokhlov 1996). Supernovae of type I are explosions of hydrogen-poor degenerate material, and mark the endpoint of the evolution of a class of white dwarfs, probably accreting in BINARY SYSTEMS (see Branch (1998) for a review).

Type Ia

Independent analyses have teased out some of the systematic differences between these onetime 'standard bombs'. Hamuy *et al* (1996) found improved fits on including decline rate as a parameter in their analysis of the HUBBLE DIAGRAM. Riess *et al* (1996) included intrinsic color in their analysis as well, and found further improvements in the model fit (but see Branch *et al* 1996). The data strongly reject the hypothesis that either or both of these dependences appear by chance. Calibration of this standard candle was initiated by Sandage *et al* (1994) with their distance to the host galaxy of SN 1937C. Gibson *et al* (1999) use the reddening corrected Hubble relations of Phillips *et al* (1999) and obtain $H_0 = 68 \pm 6$ km s^{-1} Mpc^{-1}, significantly higher than the results of Saha *et al* (1996a, b, 1997).

Type II

A fundamental technique for distance determination in both expanding and radially pulsating objects is that pioneered by Baade and Wesselink, in which the angular size of the expanding photosphere is monitored by means of the associated flux and temperature variations, and the linear size is monitored by integrating the radial velocity variations.

Although not blackbodies, as Baade and Wesselink assumed, Type II supernovae have photospheres whose emissivity can be calculated. Distances are derived from a comparison of photospheric angular diameter and the time-integrated expansion velocity (Kirshner and Kwan 1974). In the work of Schmidt *et al* (1994) the expanding photosphere method (EPM) has been employed to cz = 14 600 km s^{-1}.

There is no evidence that any empirical recalibration of EPM is required at present by the Cepheid data. EPM provides independent and consistent constraints on the Hubble constant, currently yielding $H_0 = 73 \pm 11$ km s^{-1} Mpc^{-1}.

Surface brightness fluctuations

Surface brightness fluctuations (SBF) in early-type galaxies result from the counting statistics of the number of stars in a galaxy encompassed by a detector pixel (Tonry and Schneider 1988). If N is the average number of stars in a pixel, and $\bar{f}$ is the average flux from a star, the average brightness in a pixel is

$$\mu = N\bar{f}. \tag{2}$$

Since the number in a pixel of fixed angular size scales with distance as r^2 and the flux scales as r^{-2}, this mean surface brightness is independent of distance. The local rms from pixel to pixel, however, is

$$\sigma = \sqrt{N}\bar{f} \tag{3}$$

and this scales as r^{-1}: the smoothness of otherwise identical galaxies is proportional to distance.

We can derive a distance from a measurement of both μ and σ, provided that we know the average luminosity $\bar{L}$ corresponding to $\bar{f}$:

$$\bar{f} = \frac{\sigma^2}{\mu} = \frac{\bar{L}}{4\pi r^2}. \tag{4}$$

The radial power spectrum of $\sigma/\sqrt{\mu}$ yields $\bar{f}$ for early-type galaxies in a relatively straightforward manner, although the presence of GLOBULAR CLUSTERS in these galaxies adds a source of noise and potential systematic error (Blakeslee and Tonry 1995). The quantity $\bar{f}$ has been measured at a number of wavelengths, and its behavior as a function of galaxy color is reasonably well understood (Worthey 1993, Pahre and Mould 1994, Ajhar 1993) in terms of giant branch systematics in stellar populations. It remains to calibrate $\bar{L}$, and this has been done by Tonry *et al* (1997). Recently it has proved possible to employ the resolution of HST's planetary camera (WFPC2) to measure surface brightness fluctuations to 7000 km s^{-1} redshift (Thomsen *et al* 1997, Lauer *et al* 1998, Pahre *et al* 1998). Ferrarese *et al* (1999) find $H_0 = 69 \pm 7$ km s^{-1} Mpc^{-1}.

Results from the HST key project

We collect the results from calibrating five secondary distance indicators in table 2. There is good consistency between the results from widely different techniques, suggesting that the Hubble constant indeed lies between 60 and 80 km s^{-1} Mpc^{-1}. The key project calibration is entirely based on the Cepheid period–luminosity relation. Combining the constraints from the first four entries in table 2 yields 71 km s^{-1} Mpc^{-1}. Correction for the rather shakily known chemical composition dependence of the period–luminosity relation decreases the key project estimate of H_0 by 4% to 68 ± 6 km s^{-1} Mpc^{-1}.

Table 2. The Hubble constant from secondary distance indicators.

Indicator	H_0 (km s^{-1} Mpc^{-1})	
Tully–Fisher	71 ± 8	Sakai *et al* (1999)
(D_n σ)	78 ± 10	Kelson *et al* (1999)
SBF	69 ± 7	Ferrarese *et al* (1999)
SNIa	68 ± 10	Gibson *et al* (1999)
SNII	73 ± 12	Schmidt *et al* (1994)
Key project	68 ± 6	Mould *et al* (1999)

We caution the reader against the conclusion that the uncertainty in H_0 can be estimated from the consistency of the results in table 2. Uncertainties such as the distance of the LMC affect *all* the entries in Table 2 systematically. Detailed distinctions between random and systematic errors have been made by Madore *et al* (1998), to which we refer the reader.

Cepheid independent methods

Gravitational lensing

At the time Blandford and Narayan (1992) reviewed the cosmological applications of gravitational lensing, the time delay for the prototype lensed QSO 0957+61 was still in dispute at the 30% level. A clear observational determination has now been made (Kundic *et al* 1998), leaving the accurate description of the gravitational potential of this lens as the principal source of uncertainty in the measurement of the Hubble constant from it. Finding a unique model of the mass distribution is in general a difficult task for a system to which a major galaxy, its perturbers and the common cluster halo can all contribute. Turner (2000) asserts that the model for 0957+61 is robust and that the resulting $H_0 = 64 \pm 13$ km s^{-1} Mpc^{-1} for $\Omega = 1$ represents 95% confidence limits. The cosmological geometry is not a major issue for this $z = 0.36$ lens, affecting H_0 by +7% if $\Omega = 0.1$.

Sunyaev–Zeldovich effect

The thermal bremsstrahlung luminosity of the hot gas in x-ray emitting clusters of galaxies is given by

$$L_x = 1.4 \times 10^{42} n_c^2 r_c^3 T_x^{1/2} \text{ erg s}^{-1} \tag{5}$$

where n_c is the number density in the cluster core, r_c is the core radius and T_x is the temperature of the gas.

The Compton effect of electron interactions with the microwave background radiation yields a temperature diminution of the radiation:

$$\delta T/T = -0.0128 n_c r_c \frac{kT_x}{m_e c^2}. \tag{6}$$

One may then solve for r_c, whose angular diameter is also observable. According to Birkinshaw (1998) the current best fit to the clusters observed to that date is for a Hubble constant of about 60 km s^{-1} Mpc^{-1}.

Do we live in a bubble?

Apparently lower values of H_0 from more distant probes of the expansion suggest a model with a locally low density of galaxies. There are, however, constraints on the hypothesis that we live in a bubble, which we should consider. These are the observational determinations that the expansion is linear on 100 to 1000 Mpc scales, such as the work of Sandage and Hardy (1973) on brightest cluster galaxies.

More modern demonstrations include supernovae at intermediate distances and extension of the Tully–Fisher relation to 15 000 km s^{-1} (Dale *et al* 1999). These results limit the difference between the global and local values of the Hubble constant to a few per cent. The rarity of low-density bubbles is also attested by the microwave dipole anisotropy on degree scales. Wang *et al* (1998) find a robust upper limit on the global deviation from the local 10^4 km s^{-1} sphere of 10.5% in H_0 with 95% confidence.

The future

The Hubble constant appears to be a quantity which is well defined on large scales. Homogeneity and isotropy suggest that we are measuring a property of the universe at the current epoch and not some local *ad hoc* parameter. A number of projects therefore suggest themselves:

- Microarcsecond astrometry from space will allow us to measure the trigonometric parallax of the LMC (Shao 1999). A parallax based Cepheid PL relation would be a nearer-term goal, improving on the results of Hipparcos.
- Interferometric measurement of Cepheid angular diameter changes will yield geometrical distances to these standard candles.
- The key project distances to galaxies in that program are now capable of improvement with infrared cameras on HST. This will essentially remove remaining uncertainties arising from extinction.
- One-step distance measurements which avoid the classical extragalactic distance scale will be further developed. The astrophysical models which describe rich clusters and galaxy lenses at large distances need to be probed in more detail.
- Miyoshi *et al* (1995) have shown that H_2O masers orbiting a massive compact object and following a Keplerian rotation curve provide a remarkably good fit to the radial and transverse motions of the components in NGC4258. There is potential in this technique to achieve higher precision and greater distances.

Developments in the subject are logged by Huchra (1998).

Acknowledgments
I wish to acknowledge my fellow team members in the H_0 key project. Team members include Shaun M Hughes, Fabio Bresolin, Shoko Sakai, Barry F Madore, Robert Hill, Laura Ferrarese, Holland C Ford, Garth D Illingworth, Daniel Kelson, John A Graham, Nancy Silbermann, Lucas Macri, Randy Phelps, Brad Gibson, John G Hoessel, Mingsheng Han, John P Huchra, Anne Turner, Abhijit Saha, Kim Sebo and Peter B Stetson. We are grateful to Brian Schmidt and John Tonry for their contributions. Much of the work presented in this paper is based on observations with the NASA/ESA Hubble Space Telescope, obtained by the Space Telescope Science Institute which is operated by AURA Inc under NASA contract no 5-26555.

References

Aaronson M and Mould J 1986 *Astrophys. J.* **303** 1
Ajhar E 1993 *PhD Thesis* MIT
Alcock C *et al* 1997 *Astrophys. J.* **482** 89
Birkenshaw M 1998 *Phys. Rep.* **310** 197
Blandford R and Narayan R 1992 *Ann. Rev. Astron. Astrophys.* **30** 311
Blakeslee J and Tonry J 1995 *Astrophys. J.* **442** 579
Branch D 1998 *Ann. Rev. Astron. Astrophys.* **36** 17
Branch D *et al* 1996 *Astrophys. J.* **470** L7
Branch D and Tammann G 1992 *Ann. Rev. Astron. Astrophys.* **30** 359
Burstein D *et al* 1987 *Astrophys. J.* (suppl) **64** 601
Dale D *et al* 1999 *Astrophys. J.* **510** L11
Faber S M *et al* 1989 *Astrophys. J.* (suppl) **69** 763
Feast M and Catchpole R 1997 *Mon. Not. R. Astron. Soc.* **286** L1
Feast M and Walker A 1987 *Ann. Rev. Astron. Astrophys.* **25** 345
Federspeil M, Sandage A and Tammann G 1994 *Astrophys. J.* **430** 29
Ferrarese L *et al* 1999 *Astrophys. J.* **529** 745
Fisher J R and Tully R B 1977 *Astron. Astrophys.* **54** 661
Gibson B *et al* 1999 *Astrophys. J.* **529** 723
Giovanelli R 1997 *The Extragalactic Distance Scale* ed M Livio, M Donahue and N Panagia (Cambridge: Cambridge University Press) p 113
Giovanelli R, Haynes M, Herter T, Vogt N, da Costa L, Freudling W, Salzer J and Wegner G 1997 *Astron. J.* **113** 53
Gould A and Uza O 1998 *Astrophys. J.* **494** 118
Hamuy M, Phillips M, Suntzeff N, Schommer R, Maza J and Aviles R 1996 *Astron. J.* **112** 2398
Hoflich P and Khokhlov A 1996 *Astrophys. J.* **457** 500
Huchra J P 1998 http://www.cfa-harvard/~huchra
Kelson D *et al* 1999 *Astrophys. J.* **529** 768
Kennicutt R *et al* 1998 *Astrophys. J.* **498** 181
Kennicutt R, Freedman W and Mould J 1995 *Astron. J.* **110** 1476
Kirshner R and Kwan J 1974 *Astrophys. J.* **193** 27
Kundic T, Turner E, Colley W, Gott J R, Rhoads J, Wang Y, Bergeron L, Gloria K, Long D, Malholtra S and Wambsganss J 1998 *Astrophys. J.* **482** 75
Lasenby A and Jones M 1996 *The Extragalactic Distance Scale* ed M Livio, M Donahue and N Panagia (Cambridge: Cambridge University Press) p 76
Lauer T and Postman M 1994 *Astrophys. J.* **425** 418
Lauer T *et al* 1998 *Astrophys. J.* **499** 577
Madore B and Freedman W 1998 *Astrophys. J.* **492** 110
Madore B *et al* 1998 *Astrophys. J.* **515** 29
Miyoshi M *et al* 1995 *Nature* **373** 127
Mould J, Sakai S, Hughes S and Han M 1996 *The Extragalactic Distance Scale* ed M Livio, M Donahue and N Panagia (Cambridge: Cambridge University Press) p 158
Mould J *et al* 1999 *Astrophys. J.* **529** 786
Pahre M and Mould J 1994 *Astrophys. J.* **433** 567
Pahre M *et al* 1998 *Astrophys. J.* **515** 79
Panagia N, Gilmozzi R, Macchetto D, Adorf H M and Kirshner R 1991 *Astrophys. J.* **380** L23
Perryman *et al* 1997 *Astron. Astrophys.* **323** L49
Phillips M *et al* 1999 *Astron. J.* **118** 1766
Pritchard J *et al* 1998 *New Views of the Magellanic Clouds (ASP Conf. Ser.)* ed Y H Chu, J Hesser and N Suntzeff in press
Riess A, Press W and Kirshner R 1996 *Astrophys. J.* **473** 88
Saha A, Sandage A, Labhardt L, Tammann G A, Macchetto F D and Panagia N 1996a *Astrophys. J.* **466** 55
Saha A *et al* 1996b *Astrophys. J.* (suppl) **107** 693
Saha A *et al* 1997 *Astrophys. J.* **486** 1

Sakai S *et al* 1999 *Astrophys. J.* **529** 698
Sandage A and Hardy E 1973 *Astrophys. J.* **183** 743
Sandage A and Tammann G 1997 *Critical Dialogues in Cosmology* ed N Turok (Singapore: World Scientific) p 130
Sandage A, Tammann G and Federspiel M 1995 *Astrophys. J.* **452** 1
Sandage A *et al* 1994 *Astrophys. J.* **423** L14
Schmidt B, Kirshner R and Eastman R 1992 *Astrophys. J.* **395** 366
Schmidt B *et al* 1994 *Astrophys. J.* **432** 42
Shao M 1999 *Space Interferometry Mission* JPL 400-811 (Pasadena: NASA)
Stanek K, Zaritsky D and Harris J 1998 *Astrophys. J.* **500** L141
Teerikoorpi P 1997 *Ann. Rev. Astron. Astrophys.* **35** 101
Thomsen B, Baum W, Hammergren M and Worthey G 1997 *Astrophys. J. Lett.* **483** L37
Tonry J 1996 *The Extragalactic Distance Scale* ed M Livio, M Donahue and N Panagia (Cambridge: Cambridge University Press) p 300
Tonry J and Schneider D 1988 *Astron. J.* **96** 807
Tonry J *et al* 1997 *Astrophys. J.* **475** 399
Turner E 2000 *Gravitational Lensing: Recent Progress and Future Goals* ed T Brainerd and C Kochanek in press
Turok N 1997 *Critical Dialogues in Cosmology* (Singapore: World Scientific)
Walker A 1992 *Astrophys. J.* **390** L81
Walker A 1993b *Astron. J.* **105** 527
Walker A 1993b *Astron. J.* **106** 999
Wang Y, Turner E and Spergel D 1998 *Astrophys. J.* **498** 1
Wood P, Arnold A and Sebo K 1997 *Astrophys. J.* **485** L25
Worthey G 1993 *Astrophys. J.* **409** 530

Jeremy Mould

Brunel Road, Houndmills, Basingstoke, Hampshire, RG21 6XS, UK Registered No. 785998
and Institute of Physics Publishing 2001
Dirac House, Temple Back, Bristol, BS1 6BE, UK