DRESSING UP A REISSNER NAKED SINGULARITY

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Spontaneous pair creation in the field of a large Reissner singularity (a point-like charge e whose mass M is such that $MG^{1/2} < e$) is here considered. Using as a guide the definition of the positive and negative energy states of a classical particle in this field, a particular basis of quantum states is chosen which contains resonance states – these are interpreted by invoking particle creation. Extremely energetic particles are shown to burst out to infinity whereas the antiparticles dress up and neutralize the singularity. This result is contrasted with the process of pair production by black holes and compared with the isotropization of the early universe by creation of matter.

Introduction. The vacuum polarization by external gravitational and electromagnetic fields has been extensively studied both in cosmology and in black hole physics. A variety of processes have been predicted, in particular:

(i) the isotropization of the early universe by creation of bursts of particles via a parametric resonance of the vacuum modes [1],

(ii) the loss of angular momentum and charge from black holes via pair creation at constant rates [2].

Pair production by a Reissner naked singularity (this problem was first considered in ref. [3]), although taking place in a stationary background, is more similar to the cosmological than to the black hole pair production process. Indeed we shall show in this paper that if we are given a time-like singularity of the Reissner type at time $t = 0$ (as in cosmology we are given a space-like singularity) pairs are not created at constant rates; rather, extremely energetic particles burst out to infinity whereas the corresponding antiparticles dress up and neutralize the singularity; the underlying reason is that the vacuum modes which give rise to particle creation are resonance modes whose time dependence is crucial, instead of the usual running waves.

We shall use the effective potential approach [4, 5] (a generalization of the Klein paradox [6] as interpreted by Heisenberg and Euler [7]) which gives a direct understanding of the physical mechanism responsible for the production of particles[‡1]. We decompose this method into three steps:

(1) *Zeroth quantization* where the Dirac "positive" and "negative" energy states of a classical particle are defined. The positive energy states ω^+ describe particles of charge ϵ, energy ω^+; the negative energy states ω^- have no direct meaning but are readily reinterpreted "à la Dirac" to describe antiparticles of charge $(-\epsilon)$, energy $(-\omega^-)$. There is a "level crossing" between the positive and negative energy states if the same ω can be considered as a ω^+ state in one region of space and as an ω^- state in another region.

(2) *First quantization* where the classical $\omega^\pm$ states serve as a guide to construct at each time t an orthonormal basis of quantum states whose elements can be classified into "positive" and "negative" frequency states. When there is level crossing, a wave initially of negative frequency (which, classically, would be confined in the ω^- region) tunnels at later times through the classically forbidden region towards the ω^+ region, leading to a Klein paradox. Spontaneous pair production is then easily predicted using for instance the heuristic concept of the Dirac sea.

(3) *Second quantization* which formalizes the previous results. The Heisenberg quantum fields $\psi(t)$ are expanded at each time t on the basis we have con-

‡1 For the application of this method to the loss of angular momentum and charge from black holes, see ref. [8].

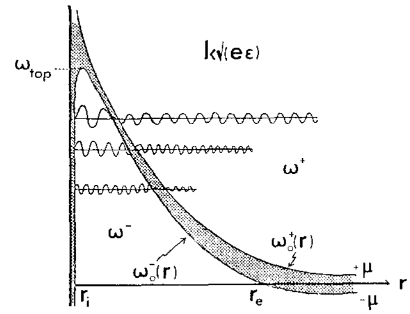

Fig. 1. Effective potentials $\omega_0^{\pm}(r)$ in the field of a large Reissner naked singularity ($e\epsilon \gg 1$). $\omega_0^+(r) = \omega$ gives the turning points of a particle of energy ω, charge ϵ; $\omega_0^-(r) = \omega$ gives the turning points of an antiparticle of energy $(-\omega)$, charge $(-\epsilon)$. When $l < e\epsilon$, $\omega_{\text{top}} > \mu$: there is a wide domain of level crossing between the negative ($\omega^- < \omega_0^-(r)$) and the positive ($\omega^+ > \omega_0^+(r)$) states (when $l \ll e\epsilon$, $r_\mathrm{i} \sim l/\epsilon$, $r_\mathrm{e} \sim e\epsilon/\mu$ and $\omega_{\text{top}} \sim e\epsilon^2/4l$). Classically the ω^- states such that $\mu < \omega^- < \omega_{\text{top}}$ form a continuous spectrum and are confined in the ω^- region. In quantum mechanics the spectrum becomes a discrete spectrum of resonances which tunnel towards the ω^+ region thereby creating pairs. Their lifetimes characterize the process and show that only pairs such that $l < (e\epsilon)^{1/2}$ and $\mu(e\epsilon)^{1/2} < \omega < \omega_{\text{top}}$ are created in significant number.

structed and the creation and annihilation operators and the vacuum are defined at time t. Hence, taking as the initial state for the system the vacuum state at $t = 0$, one obtains directly at each time the mean value of the particle number operator.

Zeroth quantization. We first define more precisely the positive and negative energy states $\omega^{\pm}$ of a classical particle of mass μ, charge ϵ, in the field of a Reissner naked singularity of mass M, charge e such that $M < e$ [‡2] and $e\epsilon \gg 1$ (we have chosen the units $G = c = h = 1$ [‡3]).

The metric is:

$$\mathrm{d}s^2 = -\delta\,\mathrm{d}t^2 + \mathrm{d}r^2/\delta + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\psi^2)$$
$$\delta = 1 - 2M/r + e^2/r^2, \tag{1}$$

and the electromagnetic potential is:

$$A = -e\,\mathrm{d}t/r. \tag{2}$$

The action S of the particle can be separated as:

$$S = -\omega t + m\varphi + \int p_\theta\,\mathrm{d}\theta + \int p_r\,\mathrm{d}r, \tag{3}$$

which leads to the following Hamilton–Jacobi equation:

$$p_r^2 = [\omega - \omega_0^+(r)]\,[\omega - \omega_0^-(r)]/\delta^2, \tag{4}$$

where

$$\omega_0^{\pm}(r) = e\epsilon/r \pm [\delta(\mu^2 + l^2/r^2)]^{1/2}. \tag{5}$$

(ω and l are the energy and the total angular momentum of the particle.)

The "classical effective potential" $\omega_0^{\pm}(r)$ are represented in fig. 1; they are the loci of the turning points of the particles moving in the field (1), (2). The positive energy states are such that $\omega^+ > \omega_0^+(r)$ and describe particles (ω^+, ϵ); the negative energy states are such that $\omega^- < \omega_0^-(r)$ and describe antiparticles ($-\omega^-$, $-\epsilon$).

A few comments must be made about these classical effective potentials:

(a) If we are given a fully formed singularity at $r = 0$ the whole diagram of $\omega_0^{\pm}(r)$ has a meaning for $t \geqslant 0$ from $r = 0$ to $r = \infty$. On the other hand, if the singularity forms by a spherically symmetric collapse, the effective potentials are valid only outside $r = R(t)$, where $R(t)$ is the radius of the collapsing object. In the following we shall only consider "God given" singularities.

(b) Because of the potential wall near the origin, neither the particles nor the antiparticles can ever reach the singularity; they do not collapse to the centre (unless $l = 0$) as special relativistic antiparticles of low angular momentum ($l < e\epsilon$) would, because they "see" a repulsive effective mass $M_{\text{eff}} = M - e^2/2r$, where $e^2/2r$ is the electromagnetic mass between r and infinity.

(c) When $l < e\epsilon$, negative bound states can be found such that:

$$+\mu < \omega^- < \omega_{\text{top}}, \quad (\omega_{\text{top}} \approx e\epsilon^2/4l \text{ for } l \ll e\epsilon). \tag{5}$$

For these states there is level crossing between the ω^- region and the ω^+ region at spatial infinity. Classically they form a continuous spectrum and are confined in the ω^- region, but in quantum mechanics they will be discrete, will tunnel towards spatial infinity and pair creation will occur.

[‡2] In fact we shall impose $M \ll e$ and neglect M in all the practical calculations.
[‡3] In these units $1/e^2 \approx 137$ and $\epsilon/\mu \approx 2 \times 10^{21}$ for electrons.

First quantization. Using the above discussion and quantizing the states by introducing both orbital and internal (spin) quantum numbers we can predict the qualitative features of the pair production process.

We suppose that the singularity is given to us at $t = 0$, that is, we assume that the negative bound states of the Dirac sea are confined in the ω^- region at $t = 0$. When $t > 0$, the discrete states satisfying eq. (6) start to leak towards the ω^+ region as resonance states of energy ω and lifetime $1/\Gamma$. This leakage, being associated with level crossing, is interpreted by invoking particle creation.

In the case of fermions (e.g. electrons) each negative state (n, l, m, σ) where $n = 1, 2, 3, \ldots$ labels the bound states, l and m the angular momentum and σ the spin, is occupied by one inobservable electron of the Dirac sea (exclusion principle). After a time $t \gtrsim 1/\Gamma_{n,l}$ this electron has flown out to the ω^+ region as a real electron we can detect, leaving a hole in the ω^- region, i.e. a positron orbiting around the singularity. Even if the back reaction were not taken into account a finite number of pairs would be created, equal to the total number of resonance states.

In the case of bosons there is no exclusion principle and the number of pairs created in each state (n, l, m, σ) increases exponentially with time [9] but should however remain finite if one takes into account the back reaction and/or the Coulomb repulsion between the created antiparticles orbiting around the singularity [10].

These considerations serve as a guide to choose a basis of quantum states.

For the sake of simplicity we use the Lagrangian density:

$$\mathcal{L} = -\tfrac{1}{2} g^{1/2} [|D_\alpha \phi|^2 + (\mu^2 + \lambda R)|\phi|^2], \tag{7}$$

where ϕ is a complex scalar field, $D_\alpha = \nabla_\alpha - \mathrm{i}\epsilon A_\alpha$, $|B_\alpha|^2 = g^{\alpha\beta} B^*_\alpha B_\beta$ and where R is the scalar Riemannian curvature (λ being a numerical constant). The extra term $\lambda R|\phi|^2$ is introduced to model the interaction between the curvature of space–time and the spatial extension of the particle (non-minimal coupling). ϕ satisfies the Klein–Gordon equation:

$$D_\alpha D^\alpha \phi = (\mu^2 + \lambda R)\phi, \tag{8}$$

and the stress energy tensor is

$$\begin{aligned} T_{\alpha\beta} = \tfrac{1}{2} [&(D_\alpha\phi)^* D_\beta \phi + (D_\beta \phi)^* D_\alpha \phi \\ &- g_{\alpha\beta}(|D_\gamma \phi|^2 + \mu^2 |\phi|^2)] \\ &+ \lambda [R_{\alpha\beta} - \tfrac{1}{2} R g_{\alpha\beta} + g_{\alpha\beta} \nabla^\sigma \nabla_\sigma - \nabla_\alpha \nabla_\beta] |\phi|^2. \end{aligned} \tag{9}$$

In a Reissner background $R = 0$ but $R_{\alpha\beta} \neq 0$; therefore ϕ fulfils the usual Klein–Gordon equation but its stress energy tensor exhibits extra terms.

Thanks to the staticity and spherical symmetry of the Reissner geometry, ϕ can be chosen as:

$$\phi_{\omega l m} = \mathrm{e}^{-\mathrm{i}\omega t} Y_l^m(\theta, \varphi) u_{\omega l}(r)/(r \delta^{1/2}), \tag{10}$$

so that eq. (8) reduces to:

$$\mathrm{d}^2 u_{\omega l}/\mathrm{d}r^2 = W u_{\omega l}, \quad \text{where } W = W^{\text{class}} + \widetilde{W}, \tag{11a}$$

with

$$\begin{aligned} W^{\text{class}} &= -[\omega - \omega_0^+(r)]\, [\omega - \omega_0^-(r)]/\delta^2 \\ &= \{[\mu^2 r^2 + l(l+1)]\, r^2 \delta - (\omega r^2 - e\epsilon r)^2\}/(r^4 \delta^2), \end{aligned} \tag{11b}$$

$$\widetilde{W} = (e^2 - M^2)/(r^4 \delta^2). \tag{11c}$$

$\omega_0^\pm(r)$ are given by eq. (5) where l^2 is replaced by $l(l+1)$. The "tidal term" $\widetilde{W}$ (a quantum correction to W^{class} consisting in replacing $l(l+1)$ by $l(l+1) + (e^2 - M^2)/r^2\delta$ can be eliminated by a proper choice of the radial coordinate [11] (for instance $Z = \int \mathrm{d}r/r^2\delta$). On the contrary the relative correction $\widetilde{W}/W^{\text{class}}$ can be spuriously blown up by using a coordinate of the type $r^* = \int \mathrm{d}r/\delta$ [3]. In any case in the r coordinate $\widetilde{W}/W^{\text{class}}$ is negligible if $l \gg 1$ which we shall henceforth assume.

The origin $r = 0$ being an ordinary point of eq. (11) the choice of a boundary condition at the origin is not obvious. However, since we are considering test fields in a given background, the total energy of the wave ϕ must be finite. Now because of the extra terms, the energy density $-T_0{}^0$ deduced from eq. (9) is integrable only if $|\phi|^2 \to 0$ as $r \to 0$; the total energy is then easily shown to be equal to $\omega(\phi, \phi)$ where (,) denotes the usual Klein–Gordon scalar product. Therefore we shall impose[‡4]:

‡4 This boundary condition should be contrasted with the corresponding condition in special relativity where for $Z > 137/2$ ($Z > 137$ for electrons in the Dirac equation) and for small enough angular momenta there is a collapse to the centre so that physical assumptions about what happens near the singularity must be made in order to choose a boundary condition. For details see e.g. ref. [12].

$$\phi(r) = u(r) = 0 \qquad \text{when } r = 0, \tag{12}$$

which is in agreement with the fact that, classically, the singularity is repulsive.

Before proceeding we emphasize the following point: the Klein–Gordon eq. (8) is meaningful only if the external field approach is correct, and this is certainly not the case in the Planck region of radius $r_{\mathrm{P}} \sim e^{1/2}$ (such that $\int_0^{r_{\mathrm{P}}} (g_{rr})^{1/2}\, \mathrm{d}r \sim 1$). When $r \gg e^{1/2}$ general relativity is valid and the linear and non linear radiative corrections to the Coulomb law are negligible[‡5]. When solving eq. (8) we shall therefore have to keep in mind that the solutions are meaningless for $r \leqslant e^{1/2}$. Note that $e^{1/2} < e\epsilon/\omega$ (the radius at which the pairs are created) as long as $\omega < \epsilon(e\epsilon)^{1/2}$ and that $e^{1/2} < l/\epsilon$ (the radius of the potential wall) as long as $l > \epsilon(e\epsilon)^{1/2}$.

We can now construct an orthonormal basis of positive and negative frequency states.

When $l > e\epsilon$, or $l < e\epsilon$ with either $\omega > \omega_{\mathrm{top}}$ or $\omega < \mu$, the $\phi_{\omega lm}$ are readily classified, with the help of fig. 1, into positive frequency states $\phi^{(+)}_{\omega lm}$ (if $\omega > \mu$ when $l > e\epsilon$, and $\omega > \omega_{\mathrm{top}}$ if $l < e\epsilon$) and negative frequency states $\phi^{(-)}_{\omega lm}$ (if $\omega < \mu$). The latter states will in fact be bound and therefore discrete when $-\mu < \omega < +\mu$.

When $l < e\epsilon$ and $\mu < \omega < \omega_{\mathrm{top}}$ we separate the frequencies by means of the following formal trick which is the mathematical expression of the physical assumption that the negative bound states are confined inside the ω^- region at $t = 0$: we suppose that for $t < 0$ there is an "impermeable" membrane between the ω^- and the ω^+ region; more precisely we impose the further condition on ϕ:

$$\begin{aligned} &\text{if } l < e\epsilon,\ \mu < \omega < \omega_{\mathrm{top}}, \\ &\phi_{\omega lm}(r) = 0 \quad \text{for } r = e\epsilon/\omega \text{ when } t < 0, \end{aligned} \tag{13}$$

which consists in changing eq. (11) in an infinitesimal neighbourhood of $e\epsilon/\omega$ by increasing W to an infinitely high value when $t < 0$. (The locus $r = e\epsilon/\omega$ is chosen because it is always located in the classically forbidden region between the ω^- and the ω^+.) Thanks to condition (13) we obtain a continuous spectrum of positive frequency states: $\phi^{(+)}_{\omega lm} = \phi_{\omega lm}\, Y(r - e\epsilon/\omega)$ (the $\phi_{\omega lm}$ are the solutions of eq. (11) on the interval $[e\epsilon/\omega, +\infty]$ satisfying condition (13) and Y is the Heaviside function) and a discrete spectrum of negative frequency states: $\phi^{(-)}_{nlm} = \phi_{\omega_n lm}\, Y(e\epsilon/\omega_n - r)$ (here the $\phi_{\omega_n lm}$ are the solutions of eq. (11) on the interval $[0, e\epsilon/\omega_n]$ satisfying conditions (12), (13).

We have thus constructed a complete orthonormal set: $\{\phi^{(+)}_{\omega lm}, \phi^{(-)}_{\omega lm}, \phi^{(-)}_{nlm}\}$.

The energy levels of the negative bound states ϕ_{nlm} are given in the WKB approximation by:

$$I(\omega_n, l) = 2\pi(n - \gamma), \tag{14}$$

where $n = 1, 2, 3, \ldots$ and where $I(\omega, l) = 2 \int (-W)^{1/2}\, \mathrm{d}r$, the integral being extended to the interval where $W \leqslant 0$. When $l \gg (e\epsilon)^{1/2}$, the WKB approximation is valid in the confining barriers (that is near $r = 0$ and $r = e\epsilon/\omega$) so that $\gamma = 1/2$ because one has to add $\pi/4$ to the phase of the wave function at each end of the interval. But, as we shall see, we shall be interested in the case $l \ll (e\epsilon)^{1/2}$ for which the WKB approximation is not valid in the confining barriers; using in this case a long wavelength approximation and approximating the confining barriers by low parabolic barriers [5], it is easily shown that one has to add $\pi/8$ to the phase at each end of the interval so that $\gamma = 1/4$.

When $l \ll (e\epsilon)^{1/2}$, the explicit expression for $I(\omega, l)$ is:

$$I(\omega, l) = e\epsilon[\ln(1 + \epsilon^2/\omega^2) - 2 + 2(\omega/\epsilon)\tan^{-1}(\epsilon/\omega)]\,. \tag{15}$$

For the highest bound states ($1 \lesssim n \ll e\epsilon$):

$$\omega_n \approx \epsilon[e\epsilon/6\pi(n - 1/4)]^{1/2}, \tag{16}$$

that is $\omega_n \sim [e\epsilon/(n - 1/4)]^{1/2}\, \omega_{\mathrm{P}}/50$, where $\omega_{\mathrm{P}} \approx 1.22 \times 10^{28}$ eV.

At $t = 0$ the impermeable membrane (condition (13)) is removed. The negative frequency bound states such that $l < e\epsilon$, $\mu < \omega_n < \omega_{\mathrm{top}}$ then become resonances which leak towards infinity thereby creating pairs; their "lifetime" $1/\Gamma_{nl}$[‡6] is given by:

$$|1/\Gamma_{nl}| \sim |\mathrm{d}I/\mathrm{d}\omega| \exp \zeta_{nl}, \tag{17}$$

with

[‡5] The linear radiative correction is the Uehling effect which is less than 10% as long as the proper radius is greater than $10^{-28}\,\mu^{-1} \sim 10^{-6}$ Planck lengths. The non linear radiative correction is the Heisenberg–Euler correction which is less than 10% as long as the proper radius is greater than $10^{-56}\,\epsilon\mu^{-2} \sim 10^{-13}$ Planck lengths. See e.g. ref. [13].

[‡6] See next page.

$$\zeta_{nl} = 2 \int_{\text{barrier}} (W)^{1/2}\,\mathrm{d}r \approx \pi[l^2/e\epsilon + \mu^2 e\epsilon/\omega^2]\,.$$

Eq. (17) shows that only pairs with $l < (e\epsilon)^{1/2}$ and $\mu(e\epsilon)^{1/2} < \omega < \omega_{\text{top}}$ will be created in significant number. For the most energetic resonances:

$$|1/\Gamma_n| \sim (2/3)[6\pi(n-1/4)]^{3/2}\,\epsilon^{-1}(e\epsilon)^{-1/2}, \qquad (18)$$

that is $|1/\Gamma_n| \sim 3.4\,(n-1/4)^{3/2}(e\epsilon)^{-1/2} \times 10^{-41}$ sec.

Second quantization. Let us denote by $\psi(t)$ the quantum field which satisfies the wave equation written as first-order equation in time. (In the scalar case we considered above, ψ has two components: ϕ and $\partial\phi/\partial t$.)

Using the preceeding analysis and putting $t = 0$ in expression (10) and its derivative with respect to time, we get a time independent set of functions $\{\psi^{(+)}_{\omega lm\sigma}(r,\theta,\varphi), \psi^{(-)}_{\omega lm\sigma}(r,\theta,\varphi), \psi^{(-)}_{nlm\sigma}(r,\theta,\varphi)\}$ classified according to the sign of the frequency. This set of functions forms at each time a complete orthonormal basis on which we can expand the second quantized field $\psi(t,r,\theta,\varphi)$. Suppressing the indices l, m, σ, we can write:

$$\psi(t,r,\theta,\varphi) = \int_{\mu}^{\infty} \mathrm{d}\omega\, a_\omega(t)\,\psi^{(+)}_\omega(r,\theta,\varphi) + \int_{-\infty}^{-\mu} \mathrm{d}\omega\, b^+_\omega(t)\,\psi^{(-)}_\omega(r,\theta,\varphi) + \sum_n b^+_n(t)\psi^{(-)}_n(r,\theta,\varphi), \qquad (19)$$

the discrete sum being extended over all the bound states whose energies are contained between $-\mu$ and ω_{top}. Among these states only those for which $\mu < \omega_n < \omega_{\text{top}}$ will give rise to pair creation. We then interpret the time dependent coefficients $a(t)$, $b^+(t)$, appearing in eq. (19) as the annihilation and creation operators at time t for particles and antiparticles. They will satisfy the usual (anti)commutation relations.

The vacuum at time t is defined as the vector $|t\rangle$, supposed unique, such that:

$$a_\omega(t)|t\rangle = b_\omega(t)|t\rangle = b_n(t)|t\rangle = 0. \qquad (20)$$

The particle number operator at time t is then defined as:

$$N(t) = \int_{\mu}^{\infty} \mathrm{d}\omega\, a^+_\omega(t)\,a_\omega(t). \qquad (21)$$

The mean value of $N(t)$ in the vacuum state at $t = 0$, $\langle 0|N(t)|0\rangle$, is the total number of particles created at time t given that at $t = 0$ there were no particles present in the field. An expression for this quantity is readily obtained by introducing the functions $\psi^{(\pm)}_\omega(t,r,\theta,\varphi)$ which evolve in time according to the wave equation in the external field considered and which take the $\psi^{(\pm)}_\omega(r,\theta,\varphi)$ as initial values when $t = 0$. These functions are just the first quantized solutions we considered above. One easily gets:

$$\langle 0|N(t)|0\rangle = \sum_n \int \mathrm{d}\omega\, |(\psi^{(+)}_\omega, \psi^{(-)}_n(t))|^2. \qquad (22)$$

Each term in this sum is in fact the part of the norm of $\psi^{(-)}_n(t)$ which has leaked out of $e\epsilon/\omega$. Now, by the definition of Γ_n, the part of the norm of $\psi^{(-)}_n(t)$ which did not leak out $\approx \pm\exp(-\Gamma_n t)$, where the upper sign holds for fermions and the lower one for bosons. The total norm ± 1 being conserved we get, reinserting all the indices:

$$\langle 0|N(t)|0\rangle \approx \sum_{nlm\sigma} \pm[1-\exp(-\Gamma_{nlm\sigma}t)]\,, \qquad (23)$$

as expected and previously described. In this formula Γ is positive for fermions and negative for bosons (see footnote 6).

Let us now take into account the back reaction. It is clear that the process will stop when the singularity is neutralized i.e. when $e/\epsilon = 137(e\epsilon)$ pairs are created. If we consider the production of electrons only, one pair is created per state (n, l, m, σ) where $\sigma = +1/2$, $-l \leqslant m \leqslant l$ and $0 \leqslant l \leqslant (e\epsilon)^{1/2}$ so that roughly $2\Sigma_0^{(e\epsilon)^{1/2}}(2l+1) \sim 2(e\epsilon)$ pairs are created per state ω_n. Therefore pair creation in the first 69 bound states is enough to completely discharge the singularity. It is clear that the more different species of particles we consider, the more this number is reduced.

‡6 In fact $1/\Gamma_{nl}$ is positive and can be interpreted as a decay time for fermions only. For bosons, as shown explicitly in ref. [9], $1/\Gamma_{nl}$ is negative and is an "amplification time" (superradiance effect). In both cases it would be impossible to consider as basic modes the exact Gamow resonance (solutions of eq. (11) for complex energies $\omega \sim \omega_n - \mathrm{i}\Gamma_n/2$) because they are not normalizable; their norm is infinite for fermions and null for bosons. That is why we introduced the formal trick (13) which allows us to normalize these resonance states.

The net effect is that $137(e\epsilon)$ particles burst out to infinity in a time of the order of a few $|1/\Gamma_1|$. Their energy is of the order of ω_1 and their angular momenta are equally distributed up to $\sim(e\epsilon)^{1/2}$. The $137(e\epsilon)$ created antiparticles stay in orbit around the singularity neutralizing it. As seen from infinity the singularity appears as a neutral object, stable against further pair creation, and endowed with a negative energy: $E \sim M - e\omega_1/\epsilon \sim -e(e\epsilon)^{1/2}$. This means that the singularity has suddenly released in a burst of particles all the electromagnetic field energy contained in the space outside $r_1 \sim e\epsilon/\omega_1 \sim (e/\epsilon)^{1/2}$.

We must, however, be aware of the crudeness of the results since:

(a) The most energetic pairs are created not far from the Planck region.

(b) We have used the external field approximation and therefore neglected the very important radiative effects of the created particles.

(c) The physical assumption about the confinement of the negative states at $t = 0$ has been crucial to our analysis. This condition cannot be fulfilled if the singularity forms by an adiabatically slow collapse because in this case pair creation occurs progressively as the negative states reach the energy $+\mu$ [14]. Neither could it be fulfilled if the singularity formed by a very rapid collapse because in this case the pair creation process would be mainly due to the time variation of the fields.

Conclusion. The main point of this paper was to show that the process of pair creation by a large Reissner singularity, given at time $t = 0$, is very different from what happens

(a) in the Coulomb problem in special relativity where pair creation is predicted when imposing the condition of "collapse to the center" and where pairs are then created at infinite rates; or

(b) in black hole physics where the horizon behaves like an inner infinity which implies that pairs are created at constant rates.

Rather, the neutralization of the singularity by pair creation is similar to the isotropization of space in early cosmology.

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