John Kosinski's answer to https://www.researchgate.net/post/Superposition_does_not_conserve_energy

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The primary error is in assuming that the linear superposition of $\vec{E}_1$ and $\vec{E}_2$ is $\vec{E}_1 + \vec{E}_2$. It is not. The linear superposition of $\vec{E}_1$ and $\vec{E}_2$ is actually $c_1\,\vec{E}_1 + c_2\,\vec{E}_2$ where the constants are determined by the boundary conditions and constraints. The reason is simple : Maxwell's Equations are differential equations, and this is the correct form for the linear superposition of two solutions to a set of linear differential equations. Conservation of energy forms a constraint on the allowable values for the constants.

This highlights an important aspect of the laws of physics : the conservation laws place absolute constraints on the differential equations. The clever designer can mold the solution of the differential equations through selection of the material properties and boundary conditions. No designer can alter the conservation laws, and any result that appears to violate a conservation law is in error.

Here, the Poynting vectors for the two fields would be $\vec{\Pi}_1 = \vec{E}_1 \times \vec{H}_1$ and $\vec{\Pi}_2 = \vec{E}_2 \times \vec{H}_2$ when considered in isolation. When both are present together, the Poynting vector must be

$$\vec{\Pi}_{\mathsf{both}} = \big(c_1\,\vec{E}_1 + c_2\,\vec{E}_2\big) \times \big(c_1\,\vec{H}_1 + c_2\,\vec{H}_2\big)$$

Consider then for the case of $\vec{E}_1 = \vec{E}_2 = \vec{E}$. Conservation of energy requires

$$\vec{\Pi}_1 + \vec{\Pi}_2 = \vec{\Pi}_{\mathsf{both}} \quad \text{then} \quad c_1 = c_2 = \tfrac{1}{\sqrt{2}}$$

Effectively, we see *each electric field being loaded by the other magnetic field*. The same total power flow is carried in terms specific to each field ($\vec{E}_1 \times \vec{H}_1$ and $\vec{E}_2 \times \vec{H}_2$) plus the two interaction terms ($\vec{E}_1 \times \vec{H}_2$ and $\vec{E}_2 \times \vec{H}_1$).

Several earlier answers identified other aspects that should also be noted. The first is that the proportionalities are for energy density, not for energy. Recognition of that leads naturally to a consideration of spatially varying density (*interference*), and some answers proposed that this was the explanation for the apparent contradiction. The spatial variation, while correct, is not the explanation of the apparent contradiction. The spatial variation is present even in an improper calculation that neglects the constants that must be determined. Solving properly for those constants resolves all apparent contradictions including the apparent singularity when two sources are considered to reside at the same point in space.

Note that this same problem occurs in a number of fields and is often ignored for various reasons. Conservation of energy is ignored in many optics texts in the analysis of two-slit diffraction. Indeed, the important characteristics of line spacing can be calculated without the added complication of determining the unknown constants. In engineering, antenna arrays are often analyzed by considering isolated behavior for each of the radiating elements and applying a normalization factor to the calculated array response. These approaches simplify the calculations for teaching, but fail to convey any depth of understanding as to what is actually happening in the EM fields being produced.

To summarize for two identical sources : When the two sources are on simultaneously and interacting, with the sources **constrained** to the same powers P_0, then the total power of the field is $P = P_0 + P_0 = 2\,P_0$ and the field strength is $\vec{E}_0/\sqrt{2} + \vec{E}_0/\sqrt{2}$. In order for the two sources to be on simultaneously and interacting **and** to obtain a field strength of $\vec{E}_0 + \vec{E}_0$, then *the source powers must increase* to $2P_0$ and $2P_0$ for a total $P = 4\,P_0$ (the sources are more loaded). *The boundary conditions can be specified as fixed field strengths or fixed power, but not both.*