The correct ensemble variance should be estimated instantaneously at a given time t, by considering the N different trajectories at this time t. If one wants to increase the statistics by taking a small time windows δt, there are at least 3 ways to compute the variance from the set of trajectories. These different ways are schematically represented on figure 3.12. We call $x_i(t)$ the position of the particle for the i^{th} quench at the time t:

- The temporal variance σ^2_{time} is obtained by estimating the variance over the time δt for each quench, and then averaging over the N quenches:

$$\sigma^2_{\text{time}}(t) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{\delta t}\int_t^{t+\delta t}\left(x_i(t') - \bar{x}_i(t)\right)^2 \mathrm{d}t'\right] \tag{3.5}$$

  where $\bar{x}_i(t) = \frac{1}{\delta t}\int_t^{t+\delta t} x_i(t')\,\mathrm{d}t'$ is the temporal mean of x for the i^{th} quench, between t and $t+\delta t$.

- The ensemble variance $\sigma^2_{\text{ensemble}}$ is obtained by estimating the variance over the N quenches at a time t and then averaging over the time window δt:

$$\sigma^2_{\text{ensemble}}(t) = \frac{1}{\delta t}\int_t^{t+\delta t}\left[\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i(t') - \langle x(t')\rangle\right)^2\right]\mathrm{d}t' \tag{3.6}$$

  where $\langle x(t')\rangle = \frac{1}{N}\sum_{i=1}^{N} x_i(t')$ is the ensemble mean of the N trajectories $x_i(t')$ at time t'.

- The boxed variance σ^2_{box} is obtained by taking the N segments of trajectory from $x_i(t)$ to $x_i(t+\delta t)$, and then estimating the variance of the whole set of data:

$$\sigma^2_{\text{box}}(t) = \frac{1}{N\delta t}\sum_{i=1}^{N}\int_t^{t+\delta t}\left(x_i(t') - \boxed{x}(t)\right)^2 \mathrm{d}t' \tag{3.7}$$

  where $\boxed{x}(t) = \frac{1}{N\delta t}\sum_{i=1}^{N}\int_t^{t+\delta t} x_i(t')\,\mathrm{d}t'$ is the mean computed on the set of data made of the N segments from $x_i(t)$ to $x_i(t+\delta t)$. It is the variance used in references [64–66].

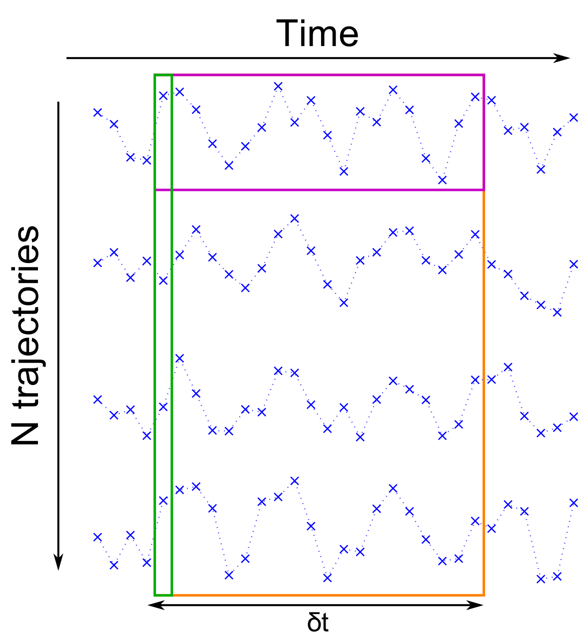

Figure 3.12: Schematic representation of the different ways to estimate the variance for a set of N trajectories with a time window δt. The temporal variance σ^2_{time} is computed by estimating the variance of the points in the fuschia box, and then averaging over the trajectories. The ensemble variance $\sigma^2_{\text{ensemble}}$ is computed by estimating the variance of the points in the green box, and then averaging over the time window δt. The boxed variance σ^2_{box} is computed directly by estimating the variance of all the points in the orange box.

If the system is at equilibrium and δt is big enough to correctly take account of the low-frequency of the motion, all these values should be equal to the equipartition value $k_{\text{B}}T/k$, with k_{B} the Boltzmann constant, T the temperature and k the trap's stiffness.

Unfortunately, when the system is non-stationary (which is the case for an ageing system), these 3 definitions of the variance are not equivalent. Especially, if there's a slow drift existing on each trajectory, the estimations that average over time (*i.e.* temporal and boxed variances) are likely to show a strong artefact.