Dissipative Optical Flow in a Nonlinear Fabry-Pérot Cavity

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We describe a classical nonlinear optical system that displays superfluidity and its breakdown. The system consists of a self-defocusing refractive medium inside a Fabry-Pérot cavity with a cylindrical obstacle. We have numerically solved for the transmitted beam when an incident plane wave strikes the cavity at an oblique angle. The presence of the incident beam pins the steady-state phase of the output, preventing the formation of vortices or time-dependent flow. When the incident beam is switched off, a transient wake of moving optical vortices is produced. This is analogous to the breakdown of superfluidity above a critical velocity.

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The nonlinear high-finesse Fabry-Pérot cavity (NLFP) displays a wealth of interesting dynamics [1], such as pattern formation [2], symmetry breaking [3], and bifurcations to chaos [4]. This is despite the apparent simplicity of the equation of motion for the transmitted field Ψ [5],

$$i\,\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\nabla_\perp^2\Psi - \Delta\Psi + V(x,y)\Psi + |\Psi|^2\Psi + i\Gamma(\Psi_i - \Psi)\,. \tag{1}$$

Here $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, Δ is a cavity detuning, Γ is proportional to the mirror transmittance, and $V(x,y)$ is an obstacle potential to be described below.

Afficionados of Bose-Einstein condensates (BEC) can hardly help but notice the similarity between this equation and the Gross-Pitaevskii equation for the order parameter of a superfluid far below the critical temperature. Apart from the last two terms, and the restriction in optics to two dimensions, the equations are identical. The term $i\Gamma\Psi_i$ is a coherent source while the term $-i\Gamma\Psi$ is a linear loss. These terms resemble models of pumping and output coupling of an atom laser. Several recent experiments on trapped atomic BECs have demonstrated their superfluid behavior [6] and dissipation above a critical flow velocity [7]. This has renewed interest in the theory of superfluidity, since accurate solutions of the Gross-Pitaevskii equation can be carried out, starting from the s-wave scattering length and maximum atomic density [8,9]. Because of the similarity of Eq. (1) to the Gross-Pitaevskii equation, it is natural to consider how superfluidity might manifest itself in the NLFP. Finding an optical analog of a superfluid may add to the understanding of BEC, since the parameters in an optical experiment can easily be varied to study different regimes. We emphasize that we consider nonlinear optics at the purely classical level, which is similar to the Gross-Pitaevskii treatment of BEC. Thermal or quantum aspects of superfluidity in BEC may be expected to differ from an optical system.

Hydrodynamic analogies in optics have been made for lasers with many transverse modes [10]. An example more relevant to superfluids is the diffraction around a wire in a Kerr nonlinear medium [11]. In that case, a beam traveling at a slight angle to the wire in a nonlinear medium can produce stationary optical vortices, since the time t in the above equation is replaced by the longitudinal coordinate z (and $\Gamma = 0$). However, this does not lead to dynamics of the light or actual dissipation (forces on the wire). In this Letter we propose an optical experiment where the vortex motion occurs in real time and therefore leads to a dynamical transition between stationary and dissipative flow. Thus, a drag force can be exerted on an obstacle, indicating the breakdown of superfluidity of light. This is a nonlinear-optical analog of dragging a laser beam through a BEC [7].

We consider a high-finesse Fabry-Pérot cavity (Fig. 1) consisting of two parallel plane mirrors of transmittance T separated by a distance L. The z axis is taken normal to the mirrors; we refer to the transverse coordinates by $\mathbf{x}_\perp = (x,y)$. A linearly polarized plane wave of frequency ω is incident at an oblique angle θ on the left mirror; we write its field as $E_i(\mathbf{x},t) = \mathrm{Re}[\psi_i(\mathbf{x}_\perp,t)e^{i(k_z z-\omega t)}]$,

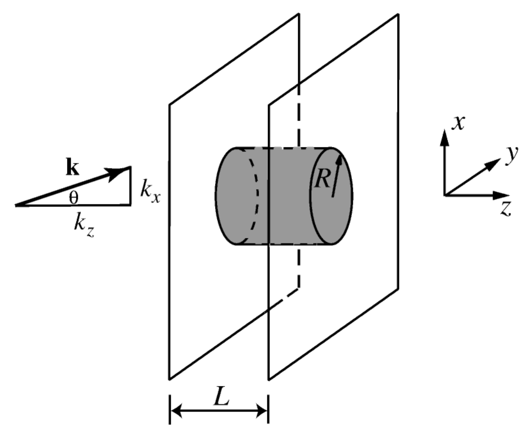

FIG. 1. Proposed experimental configuration for observing dissipative optical flow. A cylindrical obstacle (shaded) with a known index of refraction is placed between parallel mirrors of the Fabry-Pérot cavity. The remaining space between the mirrors is filled with an atomic vapor. Light detuned close to atomic and cavity resonances impinges at an angle θ on the left mirror, and is observed after transmission through the right mirror.

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where ψ_i is the slowly varying envelope field and $k_z = \frac{\omega}{c}\cos\theta$. Similarly, for the intracavity light, we write $E_c(\mathbf{x}, t) = \mathrm{Re}[\psi_c(\mathbf{x}_\perp, t)\sin n_0 k_z z\, e^{-i\omega t}]$ (where n_0 is the linear part of the refractive index inside the cavity) and, for the transmitted light that is to be observed, $E_t(\mathbf{x}, t) = \mathrm{Re}[\psi(\mathbf{x}_\perp, t)e^{i(k_z z - \omega t)}]$. At normal incidence, exact resonance (on the longitudinal mode m) occurs at the frequency $\omega_c = \pi mc/(n_0 L)$ and we allow a cavity detuning $\tilde{\Delta} = \omega - \omega_c$. The transverse component $k_x = (\omega/c)\sin\theta$ of the wave vector is included in the slowly varying field. In the cavity between the mirrors is placed a cylindrical obstacle of radius R with index of refraction:

$$n_l(\mathbf{x}_\perp) = n_0 + \delta n(\mathbf{x}_\perp) = \begin{cases} n_0 + n_m & \text{if } |\mathbf{x}_\perp| \le R\,, \\ n_0 & \text{otherwise.} \end{cases} \tag{2}$$

The entire cavity has an optical Kerr effect described by [12]

$$n_{nl}(\mathbf{x}, t) = n_2|\psi(\mathbf{x}, t)|^2. \tag{3}$$

We consider only the self-defocusing case ($n_2 < 0$) here. (This corresponds to repulsive elastic collisions between atoms in a BEC.) One way to achieve a large negative n_2 is to fill the cavity with an atomic vapor, and tune the light just below an atomic transition [13].

We make the following assumptions: all fields are paraxial along the z axis, and their spectra lie close to a single longitudinal mode of the cavity. The field inside the cavity has the profile $\sin n_0 kz$ regardless of transverse shape (mean-field approximation). The nonlinear response needs to be fast compared with the round-trip time of the cavity, i.e., we can adiabatically eliminate the atomic degrees of freedom. Under these assumptions, we obtain [5,14]

$$i\,\frac{\partial\psi}{\partial t} = -\frac{c^2}{2\omega n_0^2}\nabla_\perp^2\psi - \tilde{\Delta}\psi - \omega\,\frac{\delta n(\mathbf{x}_\perp)}{n_0}\,\psi - \frac{3n_2\omega}{2Tn_0}|\psi|^2\psi + i\,\frac{cT}{2n_0 L}(\psi_i - \psi)\,. \tag{4}$$

The intracavity field is just $1/\sqrt{T}$ times the transmitted field. By choosing an incident electric field magnitude $|\mathcal{E}|$, the transverse width scale is the vortex core size $a = c/\omega[2T/(3n_0|n_2|\mathcal{E}^2)]^{1/2}$ (the analog of the "healing length" of BEC) and the time scale $\tau = (n_0/c)^2\omega a^2$ is the time it takes a beam of size a to diffract in the absence of nonlinearity. Thus we make the physical scaling

$$x = a\tilde{x},\ t = \tau\tilde{t},\ \psi = \mathcal{E}\Psi,\ \psi_i = \mathcal{E}\Psi_i\,, \tag{5}$$

and setting $\Delta = \tilde{\Delta}\tau$, $\Gamma = cT\tau(n_0 L)$, and $V(\mathbf{x}_\perp) = -\omega\tau\delta n(\mathbf{x}_\perp)/n_0$ results in the scaled equation (1). This equation is the starting point for the analysis and simulations of the dynamics of the NLFP. The term proportional to Γ is due to the incident field driving the cavity and the transmitted field leaking out at the ring-down rate. We note that we could just as well consider the obstacle to move in a normally incident beam, by changing to coordinates moving with a velocity proportional to the transverse wave vector.

The transverse momentum of the field is [15]

$$\mathbf{J}_\perp = \frac{i}{2}\int(\Psi\nabla_\perp\Psi^* - \Psi^*\nabla_\perp\Psi)\,dx\,dy\,, \tag{6}$$

and it follows from Eq. (1) that for time-independent fields the momentum is constant. Therefore, a force is exerted on the obstacle only in the case of a time-varying field. Although this force is probably too difficult to measure experimentally, one can infer it from the transverse dynamics of the transmitted field.

We show below that the transition to dissipative flow occurs by shedding optical vortices. A vortex is characterized by a zero in intensity and a 2π phase slip around a curve enclosing the zero.

We solve Eq. (1) numerically by a modification of the split-step method for inhomogeneous equations. We assume periodic boundary conditions at the edge of the spatial grid in x and y; when the region of interest is small compared with the total grid these have no effect.

To illustrate the transition between superfluid and dissipative flow, we applied a uniform oblique incident field constant for a time t_o that switched on and off smoothly in a time t_r:

$$\Psi_i(\mathbf{x}_\perp, t) = \begin{cases} Ae^{-i\kappa_x x}\sin^2\frac{\pi t}{2t_r} & \text{for } 0 < t < t_r\,, \\ Ae^{-i\kappa_x x} & \text{for } t_r < t < t_o\,, \\ Ae^{-i\kappa_x x}\cos^2\frac{\pi(t - t_o)}{2t_r} & \text{for } t_o < t < t_o + t_r\,, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The numerical solution is shown in Figs. 2–4. The chosen parameters were $\Gamma = 0.05$, $\Delta = 0.5$, $V(|\mathbf{x}_\perp| < R) = 50$, $R = 5$ [units a], $A = 10.05$, $\kappa_x = -0.7$ [units a^{-1}], $t_o = 200$, and $t_r = 10$. (The value $A = 10.05$ results, in the absence of the obstacle, in a steady-state output $|\Psi| = 1$. Also note that $V > 0$ corresponds to a defocusing obstacle.)

Initially the transmitted field goes through an underdamped transient (Fig. 2, $t < \approx 100$). Waves (in intensity and phase) radiate from the front of the obstacle; when they are of large magnitude they produce vortex pairs but these undergo pair annihilation quickly as the waves propagate.

After the oscillations have been damped, the field approaches a steady state ($100 \approx < t < 200$), flowing around the obstacle with little change in local phase from the $\kappa_x x$ background (Fig. 3). The intensity is nearly constant, except near the obstacle boundary. We term this "phase-pinned" superfluid flow because the transmitted phase is simply related to the incident field. No force

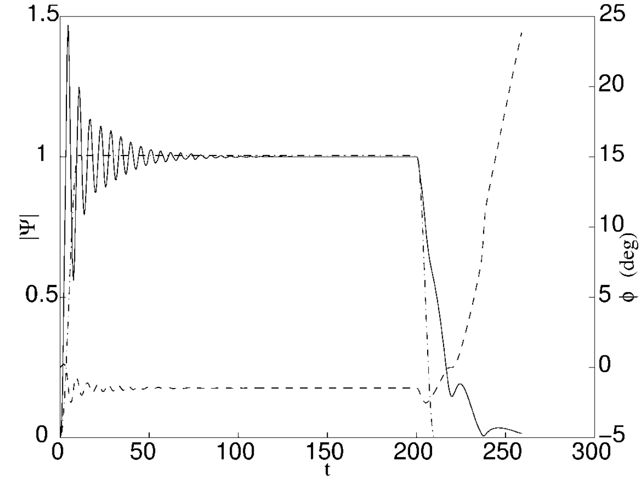

FIG. 2. Numerical solution of transmitted field vs time at the point $(-10, -5)$ behind the obstacle. *Dash-dotted curve, left scale* $\times 10$: incident pulse magnitude. *Solid curve, left scale:* transmitted magnitude. *Dashed curve, right scale:* transmitted phase.

can be exerted on the obstacle, because of the time independence of the steady state.

Finally, when the incident field is switched off at $t = 200$, vortex pairs are emitted from the back of the obstacle (Fig. 4). The dynamics in this stage resembles that produced by dragging an obstacle through a condensate at greater than the critical velocity [8]. The vortex cross sections are about 1 in the scaled units. The first pair appears at $t = 213.5$ and the second pair at $t = 223.5$. They are carried away from the obstacle along with the transverse flow of the field. More vortex pairs are emitted later, but by then the intensity is very small ($|\Psi| \sim 0.1$) and the dynamics is approaching a linear, geometrical optics regime. We can compare the dynamics after the end of the pulse with the field leaking from a NLFP with $V(x, y) = 0$. If the transmitted field at t_0 is $|\Psi_0| e^{i(\phi_0 + \kappa_x x)}$ and there is no light incident, without the obstacle one would have

$$|\Psi(\mathbf{x}_\perp, t)| = |\Psi_0| e^{-\Gamma(t - t_o)}, \tag{8}$$

$$\phi(\mathbf{x}_\perp, t) = \phi_0 + \kappa_x x + \left(\Delta - \frac{1}{2}\kappa_x^2\right)(t - t_o) - \frac{|\Psi_0|^2}{2\Gamma}\left[1 - e^{-2\Gamma(t - t_o)}\right]. \tag{9}$$

The deviation in Fig. 2 from these simple curves is due mainly to the vortices flowing past the observation point.

The main practical limits on an experiment are that the vortex core and the oblique incidence angle must not be too small, for a given wavelength. As an example, we consider a 1-cm-long cavity with $T = 0.001$ filled with atomic ^{87}Rb vapor, detuned close to the $D2$ transition. In this case $k = 8.0554 \times 10^4$ cm^{-1}, and a typical value of the nonlinear index change inside the cavity is 2.5×10^{-7} [13]. For these values, the core size is $a = 0.025$ cm and the angle θ corresponding to $\kappa_x = 0.7$ is 3.4×10^{-4} rad. The time scale $\tau = 1.7$ ns and the scaled $\Gamma = 0.05$. Observation at a given instant of the intensity and phase (by interference with a reference beam) should be possible by imaging the output mirror face. The time dependence of the light after a single pulse may be difficult to follow on this time scale, but by repeated experiments a complete picture of the dynamics may be built up.

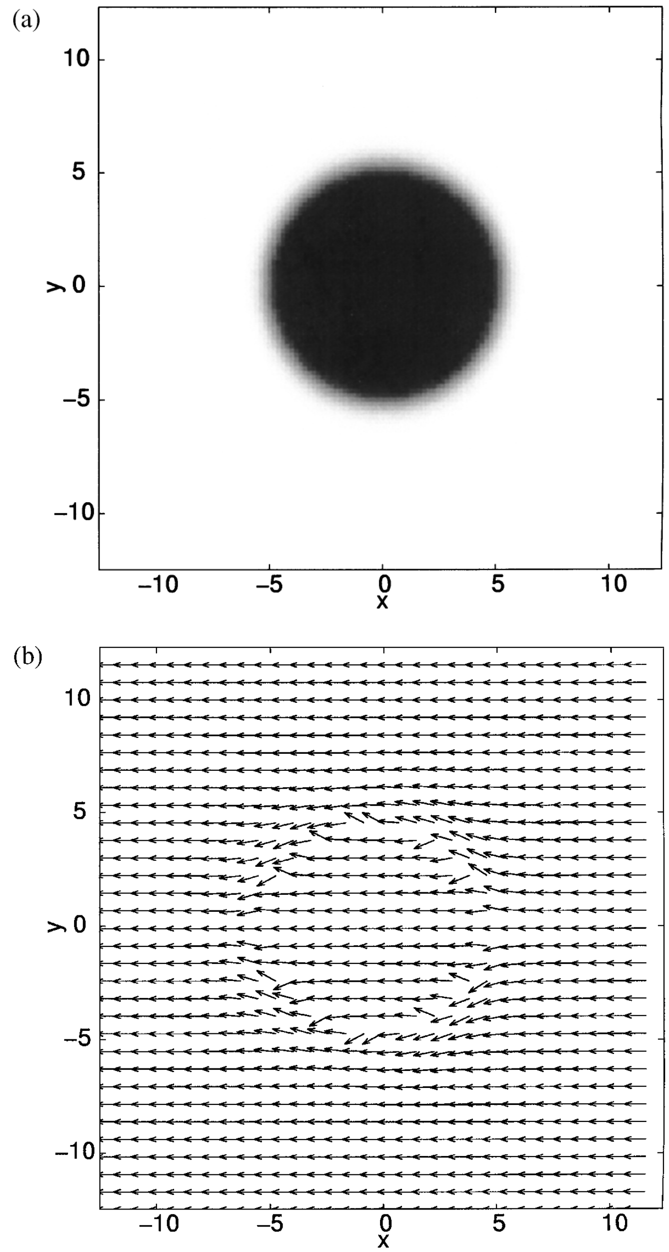

FIG. 3. Transmitted field before the end of incident pulse ($t = 200$). Scaled parameters are $\Gamma = 0.05$, $\Delta = 0.5$, $V_0 = 50$, $R = 5$, $A = 10.05$, $\kappa_x = -0.7$, $t_o = 200$, $t_r = 10$. (a) Magnitude; lighter regions correspond to higher intensity. (b) Direction of transverse gradient of phase.

We have proposed a nonlinear optical experiment which would demonstrate superfluid and dissipative flow of light. Our numerical solution shows that although initially pinning of the transmitted field to the incident field occurs,

(a)

(b)

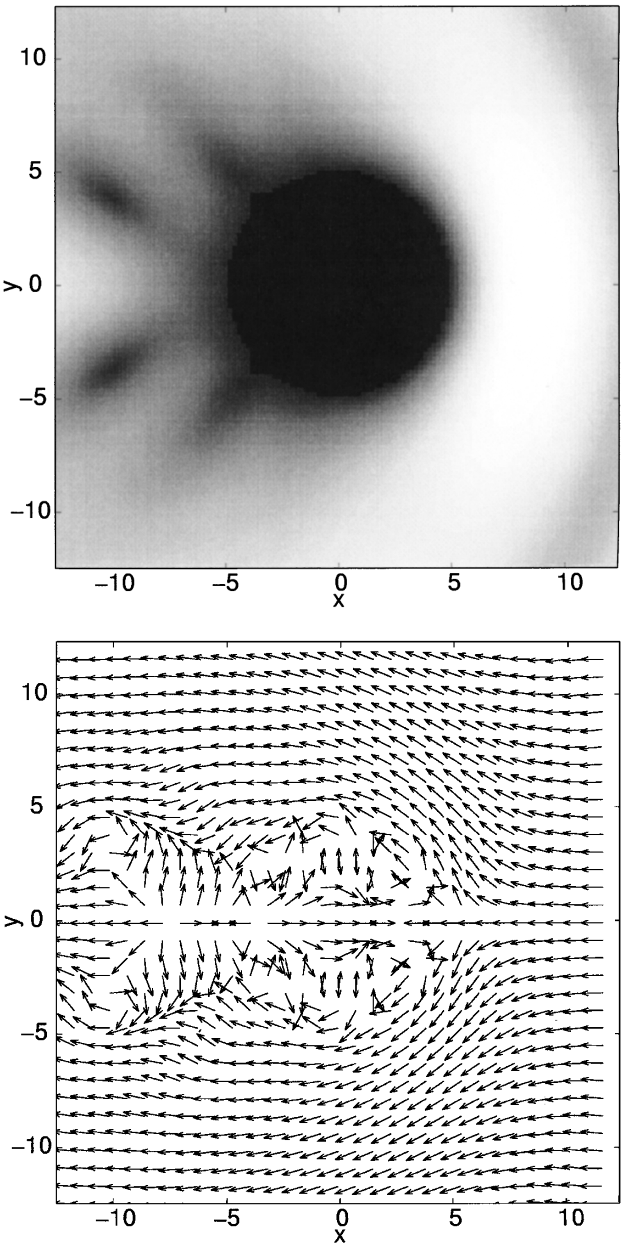

FIG. 4. Transmitted field after end of incident pulse ($t = 221$). Parameters are the same as in Fig. 3. (a) Magnitude. (b) Direction of transverse gradient of phase. Optical vortices are evident as dark spots with rotational flow, centered near $(-10, \pm 3.5)$.

there is a transition to vortex shedding and drag on an obstacle after the incident beam is switched off.

We have stressed the importance of actual time dynamics in the NLFP (as opposed to z dependence in a traveling wave) in producing dissipation. This is even more essential at the quantum level, which for this system has only been studied in the two limits of large and small photon number. A quantum theory of superfluidity based on many interacting photons in the self-defocusing cavity was outlined in [16]. At the quantum level, this optical system may allow observation of quantum phase transitions predicted for the weakly interacting Bose gas as its orbital angular momentum is increased [17].

When an object is moved rapidly enough through a BEC, heating of the condensate is observed [7]. This corresponds to large beam angles in our nonlinear optical analog, where a turbulent or incoherent transmitted beam might result. Thus we may study the coherence of the light as a function of incident angle. Finally, to make an analogy with a two-dimensional system at finite temperature, one could already introduce incoherent light at the input mirror of the NLFP.

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