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Electronic Transport in Weakly Disordered Metals

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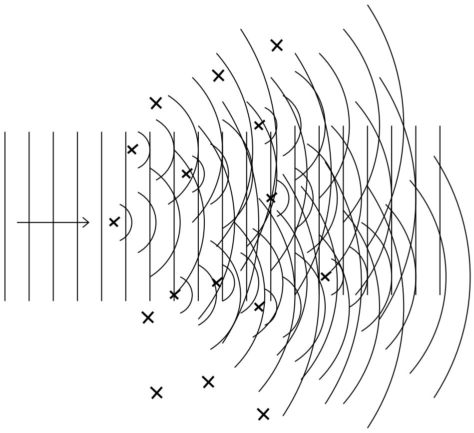
What is this course about ?

Wave propagation in a weakly disordered medium.



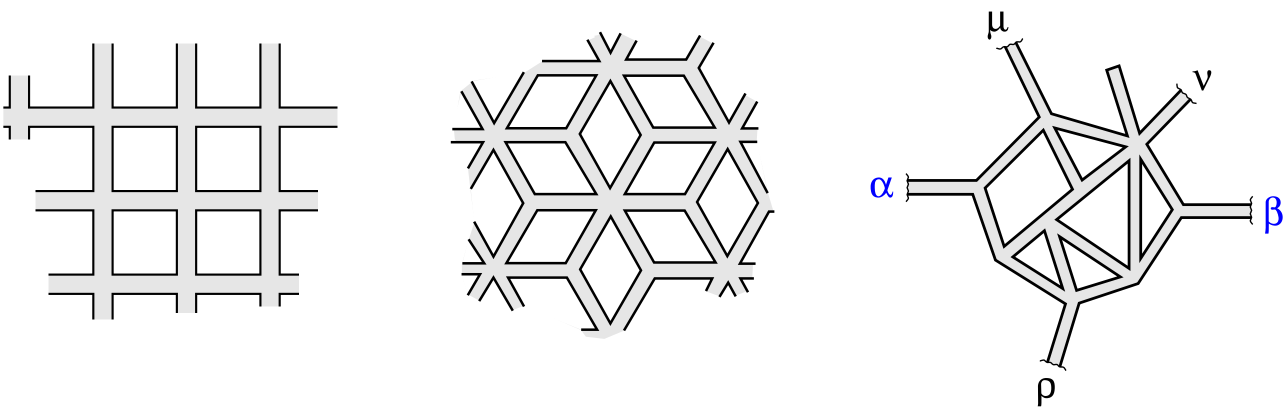
⋆ Electronic waves

⋆ Optical waves ⟶ Éric Akkermans

⋆ · · ·

• Systems ? weakly disordered metals

• low dimension : wire, networks of wires,...



• Quantities ? Transport properties : $\langle G\rangle$, $\langle\delta G^2\rangle$, ...

Outline :

1/ Introduction

- ⋆ Length scales & basic phenomena
- ⋆ Linear response & perturbation theory

2/ Perturbation theory for transport

- ⋆ Classical transport & current conservation
- ⋆ Weak localization
- ⋆ Simple examples : plane, wire, ring,...

3/ Networks

- ⋆ Nonlocality of quantum transport
- ⋆ Large regular networks

4/ Electron-electron interaction

- ⋆ Altshuler-Aronov correction

5/ Phase coherence

- ⋆ Dephasing ($\mathcal{B}$ field, spin-orbit)
- ⋆ Decoherence due to e-e interaction

6/ Fluctuations

A short bibliography :

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▷ Washburn & Webb, Adv. Phys. **35**, 375 (1986).

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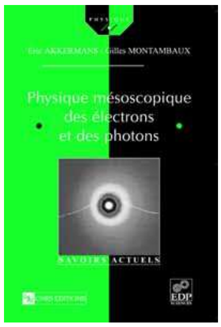

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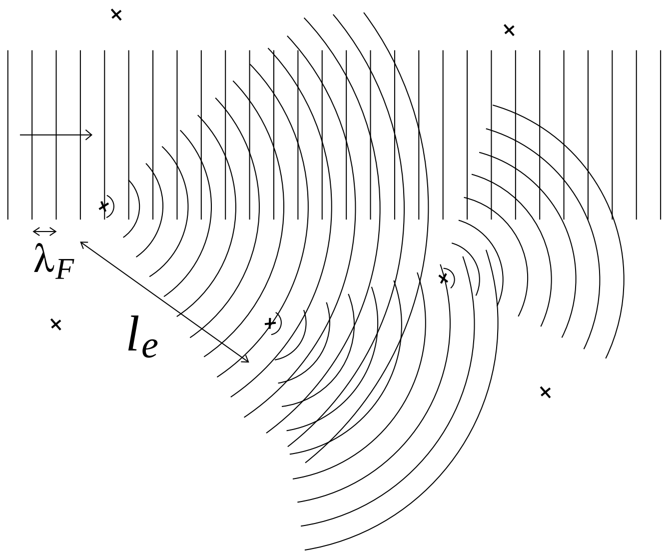
1. Introduction

Length scales

- Fermi wavelength $\lambda_F = 2\pi/k_F$

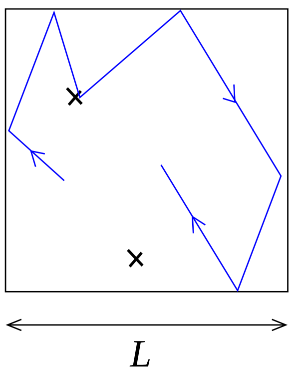
- Disorder $\longrightarrow$ elastic mean free path : $\ell_e = v_F\tau_e$



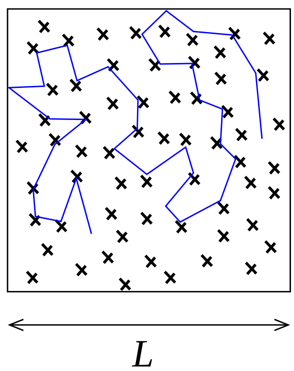
Weak disorder : $k_F\ell_e \gg 1$

▶ Ballistic / Diffusive

$L \ll \ell_e$



$\ell_e \ll L$



- $\boxed{\text{Phase coherence length}}$: $\boxed{L_\varphi}$

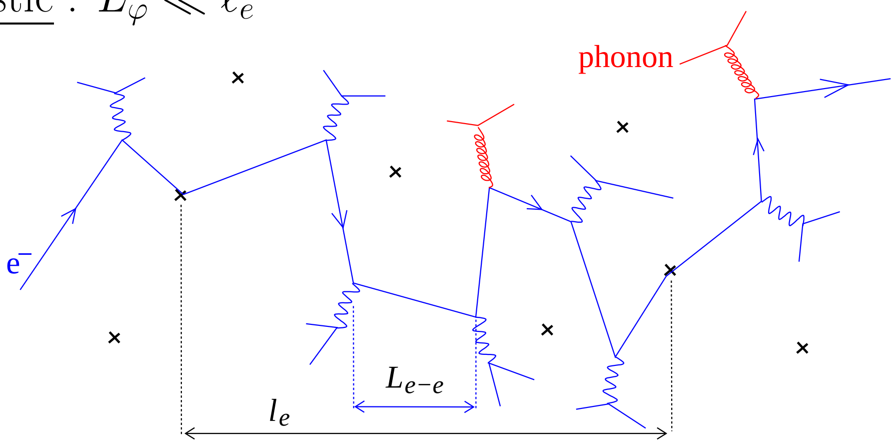
  Interaction with other degrees of freedom :

  $\longrightarrow$ Electron-electron interaction : $L_{\mathrm{e-e}}$

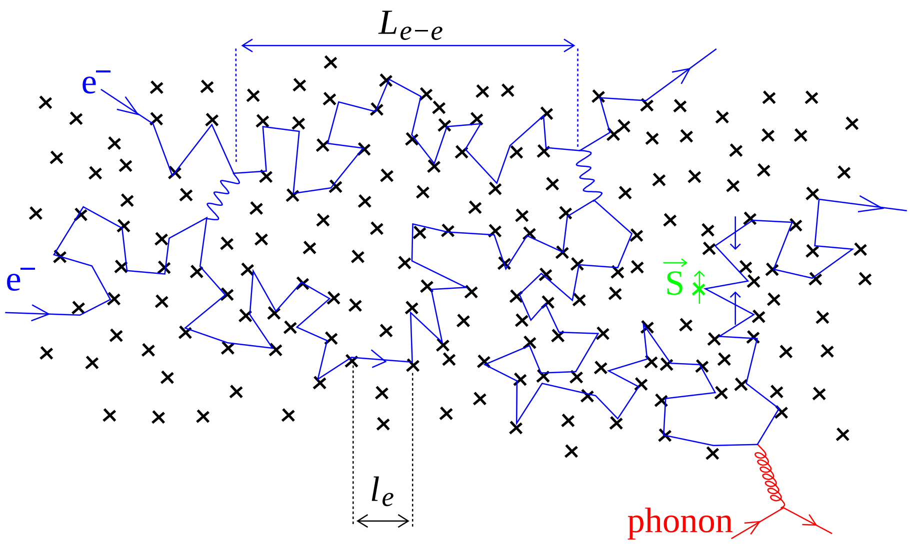
  $\longrightarrow$ Electron-phonon interaction : $L_{\mathrm{e-ph}}$

  $\longrightarrow$ Interaction with magnetic impurities : L_{m}

► <u>Ballistic</u> : $L_\varphi \ll \ell_e$



► <u>Diffusive</u> : $\boxed{\ell_e \ll L_\varphi}$



Diffusive regime : $k_F^{-1} \ll \ell_e \ll L,\ L_\varphi$

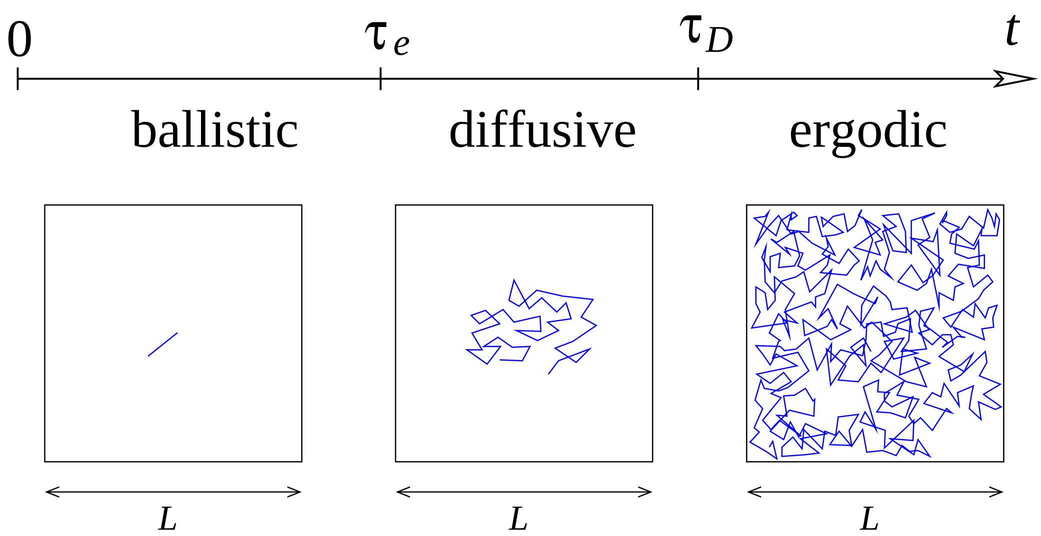
e^- experiences many elastic collisions before losing its phase memory

Interference phenomena occur over lengths $\gg \ell_e$

Dimensionality

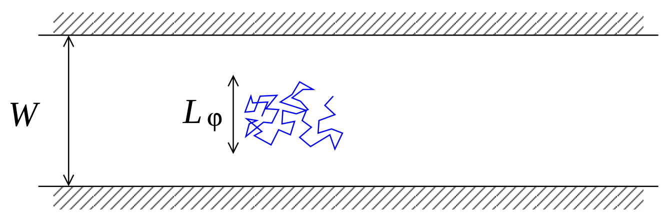
Coherent diffusion ($r \sim \sqrt{Dt}$) is limited by L and/or L_φ

Thouless time : $\boxed{\tau_D = L^2/D}$ with $\boxed{D = \frac{1}{d} v_F \ell_e = \frac{\ell_e^2}{d\tau_e}}$

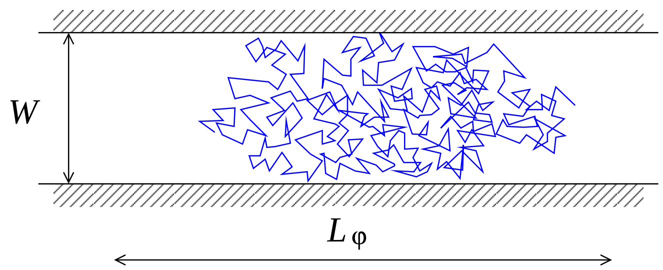


▶ Effective dimensionality :

3D :



1D :



Examples of order of magnitude in quasi 1d wires :

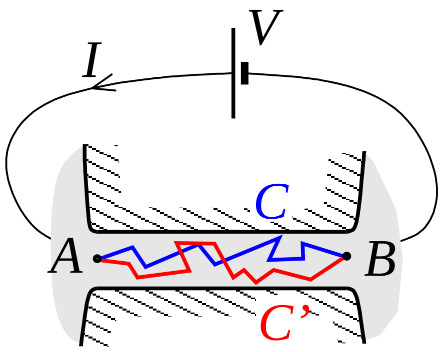
	Metal[a] (Au)	Semiconductor[b] (GaAs/GaAlAs)
	3d	2d
Fermi wave length : k_F^{-1}	0.085 nm	6 nm
Elastic mean free path : ℓ_e	30 nm	220 nm
Diffusion constant : D	0.013 m^2/s	0.030 m^2/s
$k_F \ell_e$	360	37
spin-orbit : L_{so}	50 nm	∞
e-e scattering : L_{ee} (at 50 mK)	$\sim 8\ \mu$m	$\sim 2\ \mu$m

→ electron-phonon scattering is negligible below 1 K

(a) metallic networks : Schopfer, Bäuerle, Saminadayar, Mailly.

(b) semiconducting networks : Ferrier, Guéron, Bouchiat, Mailly.

Conductance of weakly disordered metals



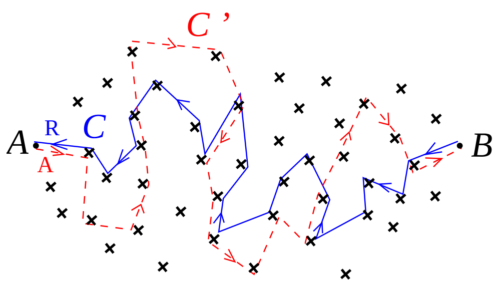
Landauer :

$$G = \frac{I}{V} \sim \overbrace{\left| \sum_{\mathcal{C}} \mathcal{A}_{\mathcal{C}}(A \to B) \right|^2}^{\text{Proba}(A \to B)}$$
$$= \sum_{\mathcal{C}} |\mathcal{A}_{\mathcal{C}}|^2 + \sum_{\mathcal{C} \neq \mathcal{C}'} \mathcal{A}_{\mathcal{C}} \mathcal{A}^*_{\mathcal{C}'}$$

amplitude $\mathcal{A}_{\mathcal{C}} \sim \mathrm{e}^{\mathrm{i}k_F l(\mathcal{C})}$ has a large random phase

Average conductance :

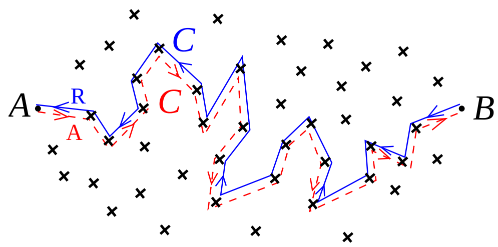
$$\langle G \rangle = \underbrace{G_{\text{Drude}}}_{\text{classical}} + \underbrace{\langle \Delta G \rangle}_{\text{quantum correction}}$$



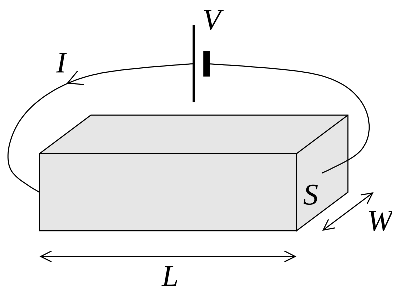
$$\propto \quad \mathrm{e}^{\mathrm{i}k_F[l(\mathcal{C}) - l(\mathcal{C}')]}$$

Does not survive disorder averaging

- Classical transport : $\mathcal{C} = \mathcal{C}'$



$$G_{\rm Drude} = \sigma_0 \frac{\rm section}{\rm length} \sim \langle \sum_{\mathcal{C}} |\mathcal{A}_{\mathcal{C}}|^2 \rangle$$



Dimensionless conductance

$$g_{\rm Drude} = \frac{G_{\rm Drude}}{e^2/h} = \frac{h}{e^2} \frac{n_e e^2 \tau_e}{m} \frac{S}{L}$$

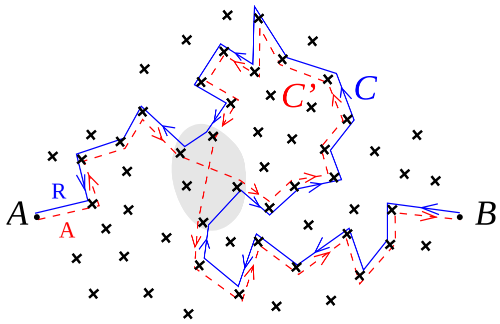
$$\swarrow \quad n_e \sim k_F^d$$

$$g_{\rm Drude} \sim \frac{k_F \tau_e}{m} \frac{k_F^{d-1} S}{L} \quad \Rightarrow \quad \boxed{g_{\rm Drude} \sim N_c \frac{\ell_e}{L} \gg 1}$$

Exercice : a coherent gold wire

$$S = 50\,{\rm nm} \times 50\,{\rm nm}$$
$$L = L_\varphi(25{\rm mK}) \sim 10\mu{\rm m}$$
$$\Rightarrow N_c = 27500 \qquad\qquad g_{\rm Drude} \simeq 110$$

• Quantum interferences : $\mathcal{C} \neq \mathcal{C}'$



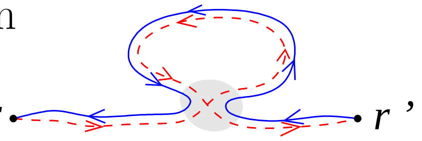
Crossing

Phase $e^{ik_F[l(\mathcal{C})-l(\mathcal{C}')]}$ is small $\Rightarrow$ survives disorder averaging

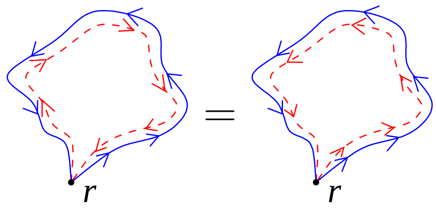
Quantum correction to the probability : some properties

$$P(r,r') = \left\langle \left| \sum_{\mathcal{C}} \mathcal{A}_{\mathcal{C}}(r' \to r) \right|^2 \right\rangle = P_{\text{class}}(r,r') + P_{\text{quant}}(r,r')$$

- Crossing $\Rightarrow$ A small correction

$P_{\text{quant}}(r,r') =$ 

- Increase of backscattering



$\Rightarrow$ $P(r,r) = 2\,P_{\text{class}}(r,r)$ (fully coherent)

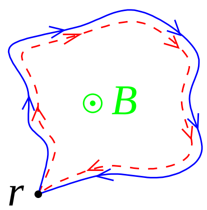
$\longrightarrow$ WL for the wire : $\langle \Delta G \rangle < 0$

- COHERENT contribution

$\longrightarrow$ Only loops smaller than L_φ contributes

Experimental probe of phase coherence

- Magnetic field sensitivity

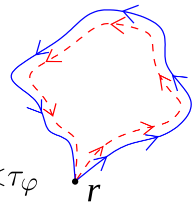
Loop 

carries a phase $e^{2ie\phi/\hbar}$ where ϕ is the flux.

Weak localization : heuristic point of view

We will show that

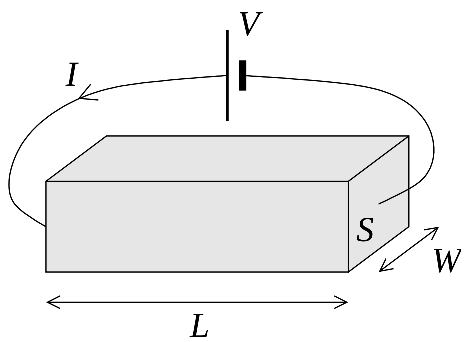
$$\langle \Delta\sigma \rangle \sim -\sum_{\text{loops for } t<\tau_\varphi}$$



$$\sim -\frac{e^2 D}{\hbar}\int_{\tau_e}^{\tau_\varphi} \mathrm{d}t\, \mathcal{P}(r,r;t)$$

$\mathcal{P}(r,r;t)$: return probability

- Conductance : $G = \frac{e^2}{h} g = \sigma \frac{S}{L}$



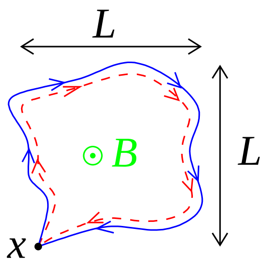
$$\boxed{\langle \Delta g \rangle \sim -\frac{S}{L} D \int_{\tau_e}^{\tau_\varphi} \mathrm{d}t\, \mathcal{P}(r,r;t)}$$

Exercice : For an effective dimension d : $\mathcal{P}(r,r;t) \propto 1/(4\pi D t)^{d/2}$

$$\begin{aligned} \langle \Delta g \rangle &\sim -\frac{1}{L}(L_\varphi - \ell_e) && \text{for } d=1 \\ &\sim -\frac{W}{L}\ln(L_\varphi/\ell_e) && \text{for } d=2 \\ &\sim -\frac{S}{L}\left(\frac{1}{\ell_e} - \frac{1}{L_\varphi}\right) && \text{for } d=3 \end{aligned}$$

Probing weak localization with magnetic field

⋆ **Magnetoconductance of a plane**



$\mathcal{B}$ field destroys contributions for $Dt = L^2 \gtrsim \phi_0/\mathcal{B}$.

$$\text{cutoff at } t \sim \tau_B = \phi_0/(D|\mathcal{B}|)$$

$$\downarrow \qquad 1/\tau_\varphi \longrightarrow 1/\tau_\varphi + 1/\tau_B$$

$$\frac{1}{L_\varphi(\mathcal{B})^2} \simeq \frac{1}{L_\varphi^2} + C_2 \frac{e|\mathcal{B}|}{\hbar}$$

WL correction is

$$\langle \Delta g \rangle \sim -\ln(L_\varphi(\mathcal{B})/\ell_e) \sim \ln \mathcal{B}$$

Positive magnetoconductance

Consistent temperature and field dependence in weak localization

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(Received 25 February 1983)

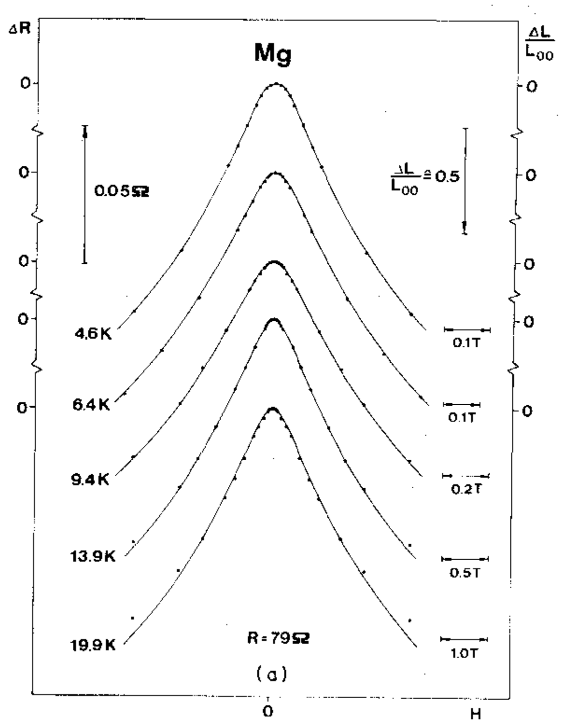
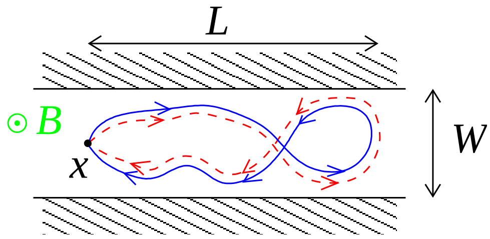


FIG. 2. (a) Magnetoresistance (i.e., $[L(H)-L(0)]/L_{00}$ using the right scale) of a Mg film ($d=8.4$ nm) as a function of the field The units of the field are shown beside each magnetoresistance curve. The points represent the experimental results. The solid cur are calculated using the characteristic fields $H_i(T)$ plotted in Fig. 5 and $H_{so}=0.0046$ T.

$\star$ Magnetoconductance of a wire



$\mathcal{B}$ field destroys contributions for $W\sqrt{Dt} = WL \gtrsim \phi_0/\mathcal{B}$.

$$\text{cutoff at } t \sim \tau_B = \frac{1}{D}\left(\frac{\phi_0}{W\mathcal{B}}\right)^2$$

$$\downarrow$$

$$\frac{1}{L_\varphi(\mathcal{B})^2} = \frac{1}{L_\varphi^2} + C_1\left(\frac{e\mathcal{B}W}{\hbar}\right)^2$$

WL correction is

$$\langle \Delta g \rangle \sim -\frac{L_\varphi(\mathcal{B})}{L} \sim -\frac{1}{\mathcal{B}}$$

Weakly Localized Behavior in Quasi-One-Dimensional Li Films

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AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 3 January 1985)

The low-temperature magnetoresistance of quench-condensed Li films of varying widths is studied in order to observe the one-dimensional localization effects first predicted by Thouless. The localization effects are dominant and clearly differentiated from other contributions to the resistivity. The roles of the various scattering mechanisms controlling the localization contribution are determined.

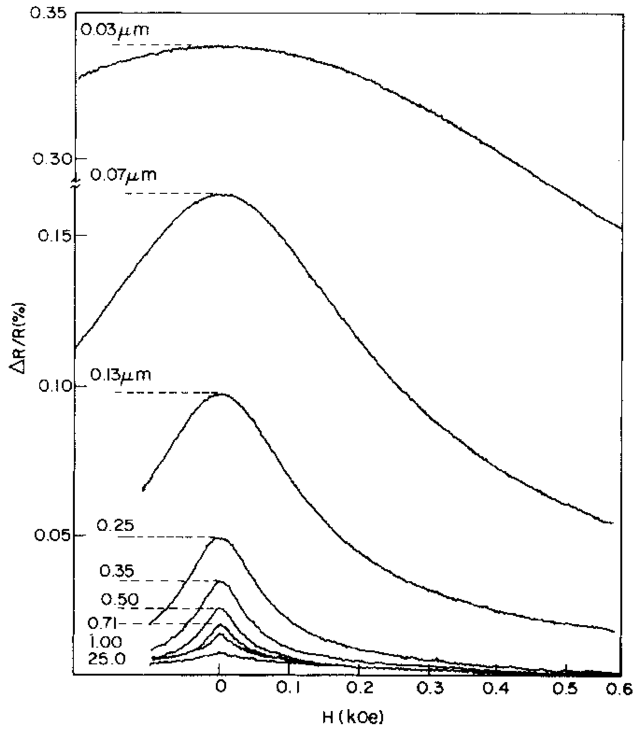
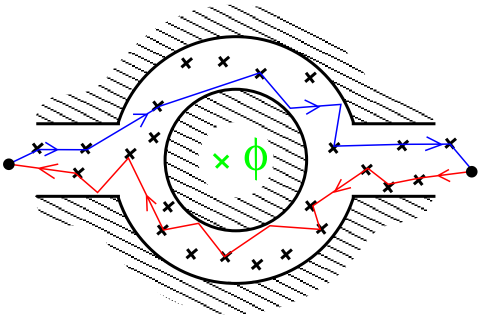


FIG. 1. Magnetoresistance data for L_i films varying in width, W, down to 0.03 ± 0.01 μm.

In networks : Magnetoconductance oscillations

- Aharonov-Bohm (AB) oscillations

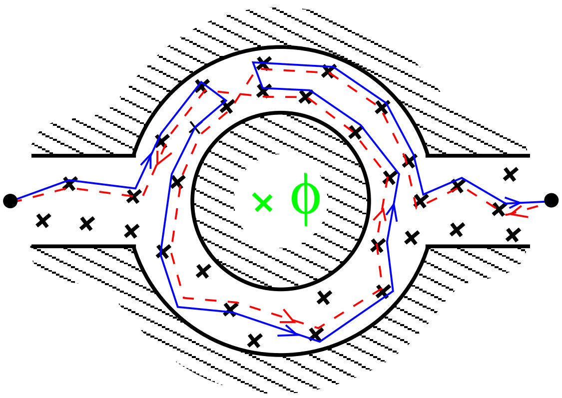


$$\mathcal{A}_{\mathcal{C}}\mathcal{A}^*_{\mathcal{C}'} \propto \mathrm{e}^{\mathrm{i}e\phi/\hbar}$$

$$\Rightarrow G(\phi) : h/e \text{ AB oscillations}$$

$\Downarrow$ disorder averaging

- Al'tshuler-Aronov-Spivak (AAS) oscillations

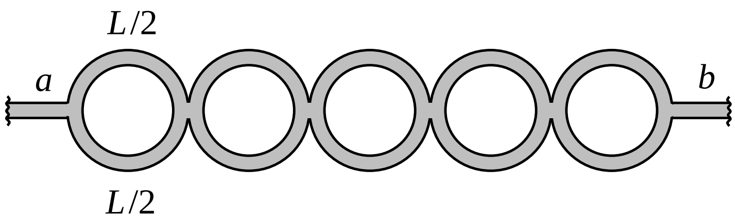


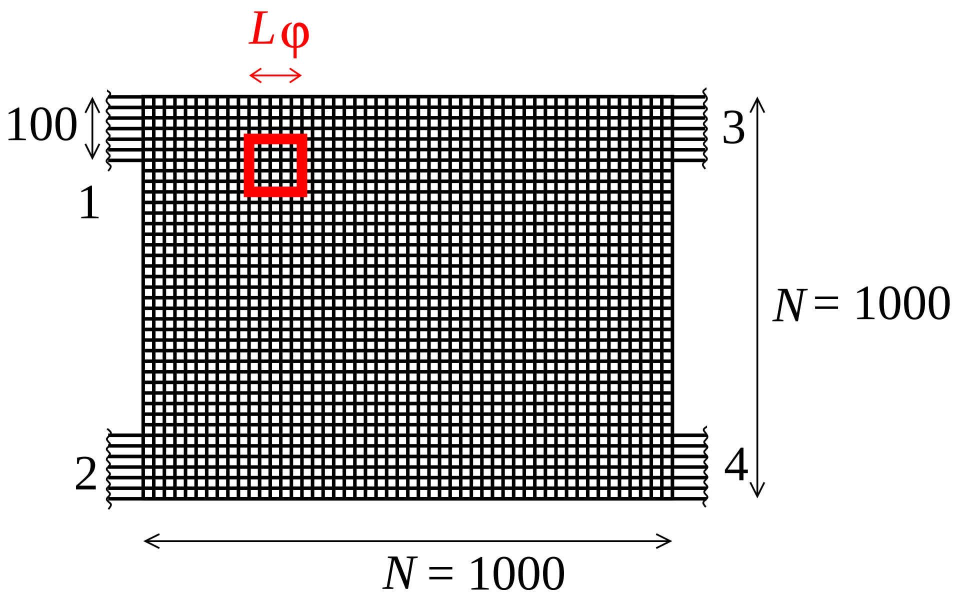
$$\mathcal{A}_{\mathcal{C}}\mathcal{A}^*_{\mathcal{C}'} \propto \mathrm{e}^{2\mathrm{i}e\phi/\hbar}$$

$\langle \Delta G(\phi) \rangle$: $h/2e$ AAS oscillations

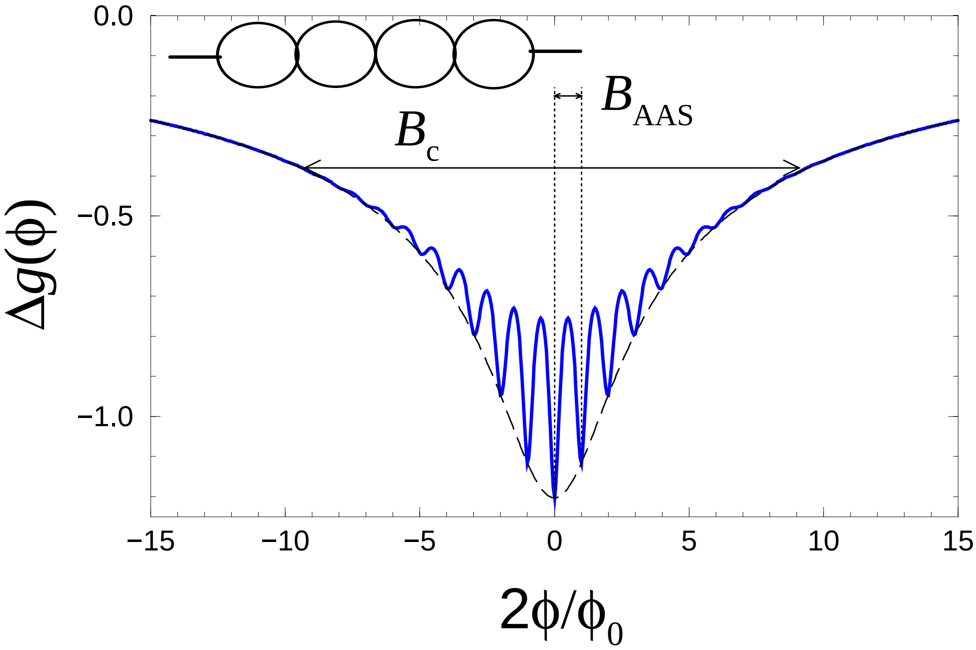
- How to realize disorder averaging ?

$\rightarrow$ Large networks : size $\gg L_\varphi$





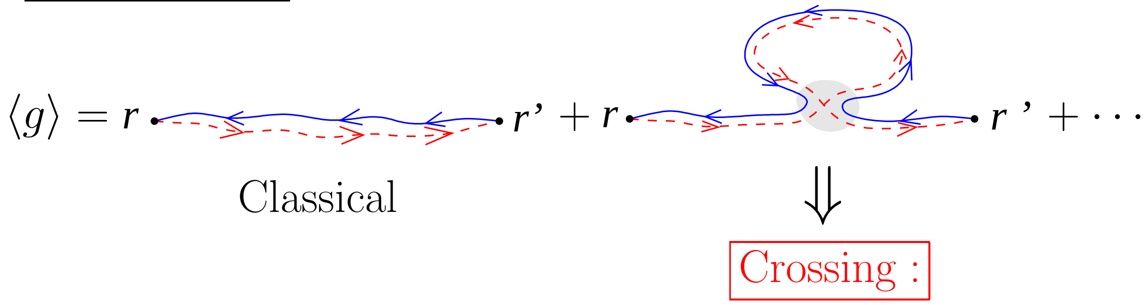
AAS oscillations & penetration of $\mathcal{B}$ in the wires



AAS oscillations $\qquad \mathcal{B}_{\text{AAS}} = \dfrac{\phi_0}{2 \times \text{area}}$

penetration of $\mathcal{B}$ field $\qquad \mathcal{B}_c = \dfrac{\phi_0}{W L_\varphi}$

Summary

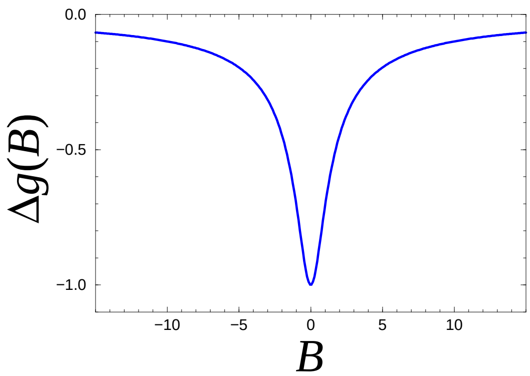


Quantum correction (WL) : $\langle \Delta g \rangle$

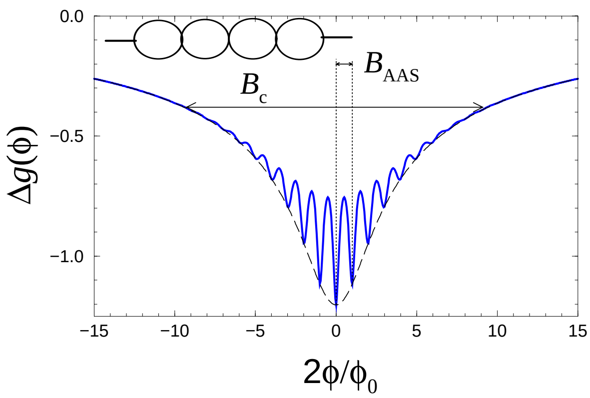
- Coherent wire ($L \sim L_\varphi$) : $g \sim N_c \frac{\ell_e}{L} \gg 1$

  $\langle \Delta g \rangle \sim 1$ and $\langle \Delta g \rangle < 0$

- Positive magnetoconductance



- AAS oscillations



Linear response & Kubo

- Linear response theory

$$\hat{H} = \hat{H}_0 - \hat{A}\, f(t)$$

How observable $\hat{B}$ responds to the force coupled to $\hat{A}$?

$$\langle \hat{B}(t)\rangle^{(1)} = \int \mathrm{d}t'\, \chi_{BA}(t-t')\, f(t')$$

with

$$\chi_{BA}(t) = \mathrm{i}\,\theta(t)\,\langle[\hat{B}(t), \hat{A}]\rangle_0$$

- Conductivity (Kubo) : Response to an external electric field

$$j_\alpha(r) = \int \mathrm{d}r'\, \sigma_{\alpha\beta}(r,r')\,\mathcal{E}_\beta(r')$$

$\sigma_{\alpha\beta}(r,r')$ is a current-current correlation function $\sim \langle[j(t), j]\rangle$

At $\omega = 0$ and $T = 0$:

$$\sigma_{\alpha\beta}(\vec{r},\vec{r}\,') = -\frac{e^2}{4\pi m_e^2}\,\Delta G(\vec{r},\vec{r}\,';E_F)\ \overset{\leftrightarrow}{\nabla}_\alpha \overset{\leftrightarrow}{\nabla}{}'_\beta\,\Delta G(\vec{r}\,',\vec{r};E_F)$$

Current operator : $\overset{\leftrightarrow}{\nabla} = \frac{1}{2}(\overset{\rightarrow}{\nabla} - \overset{\leftarrow}{\nabla})$

$$\Delta G(E_F) = G^{\mathrm{R}}(E_F) - G^{\mathrm{A}}(E_F) = -2\mathrm{i}\pi\,\delta(E_F - H)$$

$\Rightarrow$ Fermi surface property.

For our purpose :

$$\sigma_{\alpha\beta}(\vec{r},\vec{r}\,') = \frac{e^2}{2\pi m_e^2} G^{\mathrm{R}}(\vec{r},\vec{r}\,')\ \overset{\leftrightarrow}{\nabla}_\alpha \overset{\leftrightarrow}{\nabla}{}'_\beta\, G^{\mathrm{A}}(\vec{r}\,',\vec{r})$$

$$\boxed{\text{Transport} \quad \Leftrightarrow \quad G^{\mathrm{R}}(r,r')\, G^{\mathrm{A}}(r',r)}$$

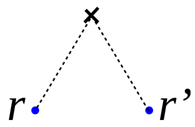
Perturbation Theory for Random Potentials

Weak disorder $\Rightarrow$ Only $\langle V(\vec{r})V(\vec{r}\,')\rangle$ is needed

- Gaussian disorder + local correlations

$$\langle V(\vec{r})\rangle = 0$$

$$\langle V(\vec{r})V(\vec{r}\,')\rangle = w\,\delta(\vec{r}-\vec{r}\,') \equiv$$

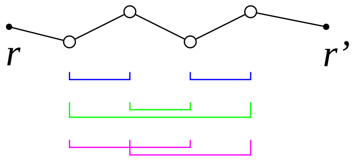


- Expansion of the Green function $\quad G = \frac{1}{E-H_0-V}$:

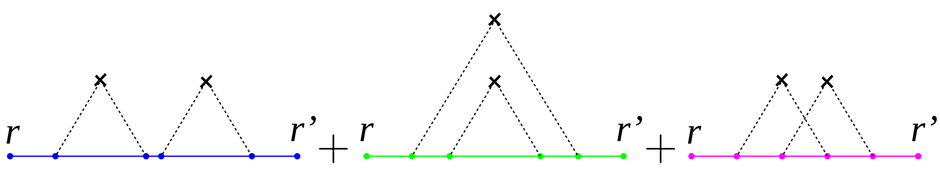
$$G = G_0 + G_0VG_0 + G_0VG_0VG_0 + \cdots$$

example : the 4st-order correction $\delta^4 G = G_0VG_0VG_0VG_0VG_0$

$$\delta^4 G(r,r') =$$



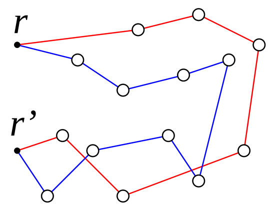
$$\langle \delta^4 G(r,r')\rangle =$$



- Expansion of the conductivity

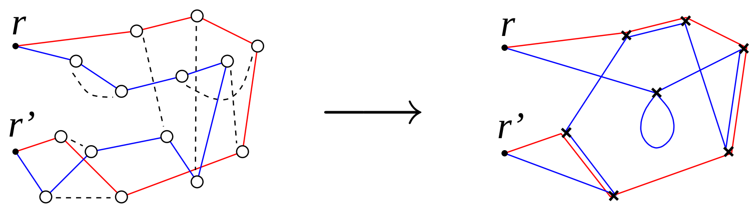
Four point Green function : $\sigma \sim G^R\, G^A$

$$\delta^8 G^R(r,r')\ \delta^6 G^A(r',r) =$$



disorder average $\Downarrow$

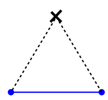
A possible contribution to $\langle G^R(r,r')\, G^A(r',r)\rangle$:



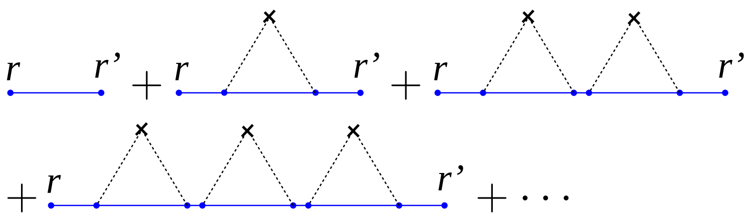
- Self energy : A series of diagrams for the price of 1 diagram

$$\langle G\rangle = \frac{1}{G_0^{-1}-\Sigma} = G_0 + G_0\Sigma G_0 + G_0\Sigma G_0\Sigma G_0 + \cdots = G_0 + G_0\Sigma\,\langle G\rangle$$

<u>example :</u> If $\Sigma^{(2)} =$



$\langle G\rangle \simeq$

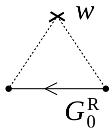


Average Green function

- Free Green function

$$G_0^{\mathrm{R}}(\vec{r},\vec{r}\,') = \langle \vec{r}\,| \frac{1}{E_F - H_0 + \mathrm{i}0^+} |\, \vec{r}\,' \rangle \sim \frac{\mathrm{e}^{\mathrm{i}k_F||\vec{r}-\vec{r}\,'||}}{||\vec{r}-\vec{r}\,'||^{\frac{d-1}{2}}}$$

- Average Green function

$$\frac{1}{2\tau_e} = -\,\mathrm{Im}\,\Sigma^{\mathrm{R}}(E_F) = -\,\mathrm{Im}$$ 

$$= -\,\mathrm{Im}\left[w\, G_0^{\mathrm{R}}(\vec{0},\vec{0})\right]$$

$$\boxed{\frac{1}{\tau_e} = 2\pi\rho_0\, w}$$

elastic mean free path :

$$\ell_e = v_F \tau_e$$

$$\boxed{\overline{G}^{\mathrm{R}}(\vec{r},\vec{r}\,') \simeq G_0^{\mathrm{R}}(\vec{r},\vec{r}\,')\,\mathrm{e}^{-||\vec{r}-\vec{r}\,'||/2\ell_e}}$$

Short range object

2. Perturbation theory for transport

Our purpose :

Compute the average nonlocal conductivity $\langle \sigma_{ij}(r, r') \rangle$

$$\Longrightarrow \quad \langle G^{\mathrm{R}}(r, r')\, G^{\mathrm{A}}(r', r) \rangle$$

Nonlocal / local :

- Nonlocal conductivity $\langle \sigma_{ij}(r, r') \rangle$

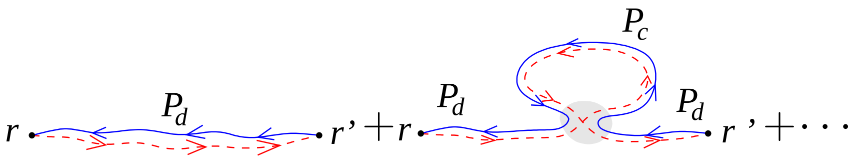
  $\longrightarrow$ required to study networks

  $\longrightarrow$ discussion of current conservation

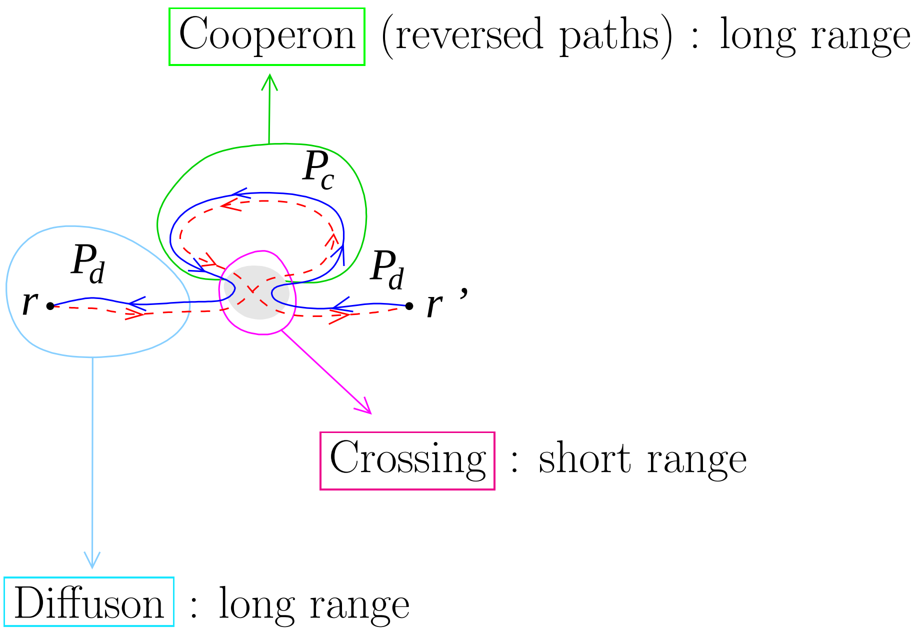
- If distribution of current is uniform (wire, plane,...)

  $\longrightarrow$ local conductivity $\sigma = \int \frac{\mathrm{d}r \mathrm{d}r'}{\mathrm{Vol}}\, \sigma(r, r')$ is sufficient

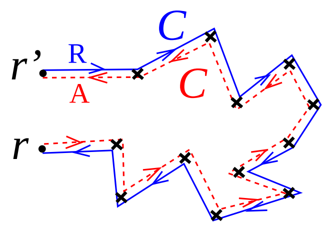
What diagrams ?

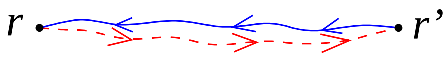
$\langle G^{\mathrm{R}}(r,r')\, G^{\mathrm{A}}(r',r)\rangle =$ 

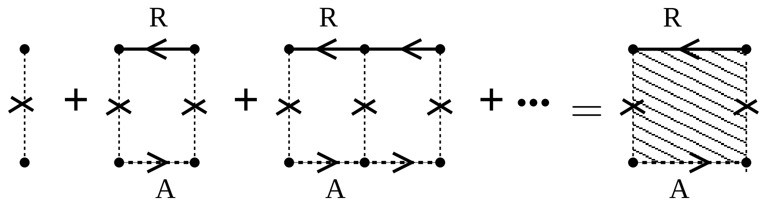
Strategy : use scale separation



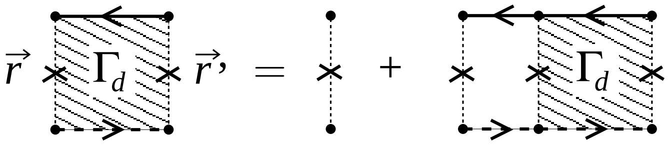
Diffuson : ladder



▷ Diffuson $P_d(r,r') =$ 

$\Gamma_d(\vec r, \vec r\,') =$ 

Bethe-Salpether equation :



$$= w\,\delta(\vec r - \vec r\,') + w \int \mathrm{d}\vec r\,'' \; \underbrace{\overline{G}^{\mathrm{R}}(\vec r, \vec r\,'')\,\overline{G}^{\mathrm{A}}(\vec r\,'', \vec r)}_{\delta(\vec r - \vec r\,'')\frac{1}{w}\left[1 + \tau_e D\,\Delta + \cdots\right]} \; \Gamma_d(\vec r\,'', \vec r\,')$$

$$\Gamma_d(\vec r, \vec r\,') = w\,\delta(\vec r - \vec r\,') + \Gamma_d(\vec r, \vec r\,') + \tau_e D\,\Delta\,\Gamma_d(\vec r, \vec r\,') + \cdots$$

$$\Gamma_d(\vec r, \vec r\,') = \frac{w}{D\tau_e} P_d(\vec r, \vec r\,')$$

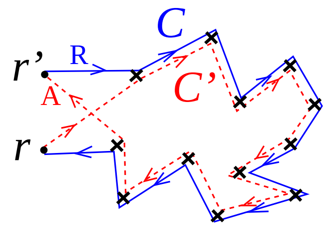
This is the diffusion approximation

$$\boxed{-\Delta P_d(\vec r, \vec r\,') = \delta(\vec r - \vec r\,')}$$

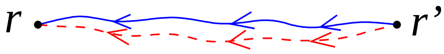
The diffusion approximation :

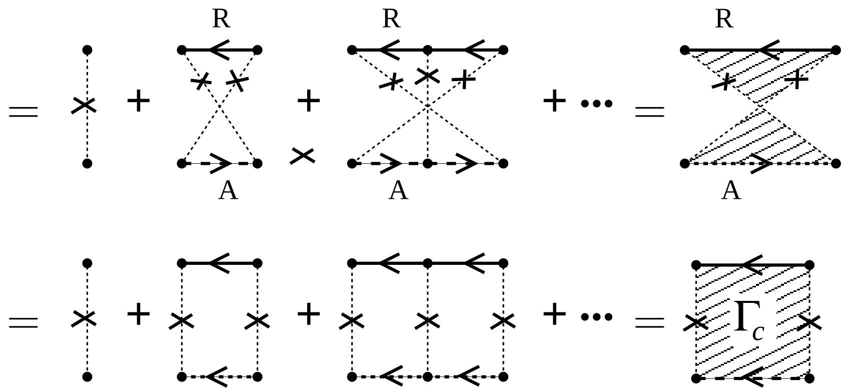
$$\boxed{\text{physics on length scales} \gg \ell_e \gg k_F^{-1}}$$

Cooperon : Maximally crossed



(reversed paths)

▷ Cooperon $P_c(r, r') =$ 

$\Gamma_c(\vec{r}, \vec{r}\,') =$ 

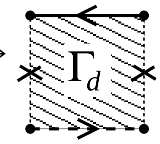
$\longrightarrow$ Magnetic field sensitive

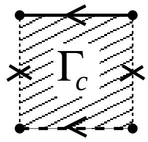
$\longrightarrow$ Loops longer than L_φ do not contribute

$$\left[\frac{1}{L_\varphi^2} - \left(\vec{\nabla} - 2ie\vec{A}\right)^2\right] P_c(\vec{r}, \vec{r}\,') = \delta(\vec{r} - \vec{r}\,')$$

Summary : Building blocks

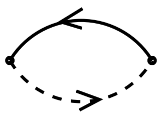
Long range objects :

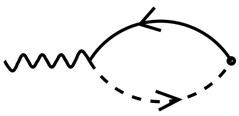
Diffuson : $\frac{w}{D\tau_e} P_d(r, r') = \vec{r}$  $\vec{r}\,' \sim 1/|r - r'|^{d-2}$

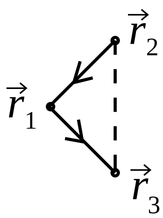
Cooperon : $\frac{w}{D\tau_e} P_c(r, r') =$  $\sim \mathrm{e}^{-|r-r'|/L_\varphi}$

Short range objects (Hikami boxes) :

(see appendix)

$\vec{r}$  $\vec{r}\,' = \delta(\vec{r} - \vec{r}\,') \frac{1}{w} \left[1 + \tau_e D \, \Delta + \cdots \right]$

$\vec{r}, i$  $\vec{r}\,' = -\mathrm{i} \frac{\sqrt{2\pi}}{d} e \rho_0 \ell_e^2 \, \delta(\vec{r} - \vec{r}\,') \, \nabla_i$



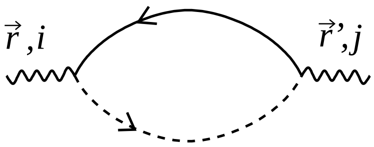
$= -2\mathrm{i}\pi \rho_0 \tau_e^2 \, \delta(\vec{r}_1 - \vec{r}_2) \, \delta(\vec{r}_1 - \vec{r}_3)$

⋮ ⋮ ⋮

Nonlocal Conductivity

Drude contribution : $\overline{G}^{\mathrm{R}}\overline{G}^{\mathrm{A}}$

$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\mathrm{Drude}} = \frac{e^2}{2\pi m_e^2}\,\overline{G}^{\mathrm{R}}(\vec{r},\vec{r}\,')\;\overset{\leftrightarrow}{\nabla}_i\overset{\leftrightarrow}{\nabla}{}'_j\,\overline{G}^{\mathrm{A}}(\vec{r}\,',\vec{r})$$

$=$ 

Exercice : use $\nabla_i\overline{G}^{\mathrm{R}}\,\nabla'_j\,\overline{G}^{\mathrm{A}} \simeq \delta_{ij}\,\frac{k_F^2}{d}\,\overline{G}^{\mathrm{R}}\overline{G}^{\mathrm{A}}$

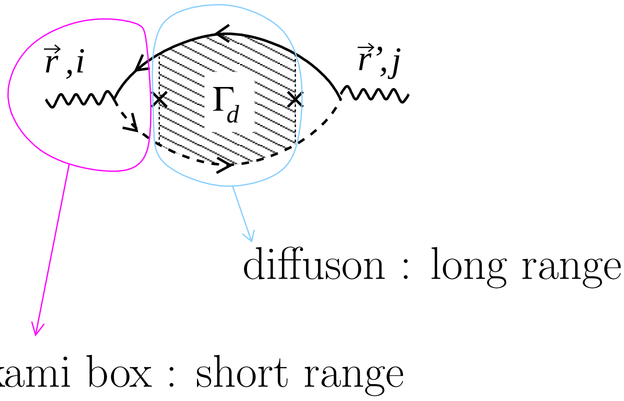
$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\mathrm{Drude}} \simeq \underbrace{\frac{e^2k_F^2}{2\pi m_e^2 d}\,\frac{1}{w}}_{e^2\tau_e\frac{\rho_0 v_F^2}{d}}\,\delta_{ij}\,\delta(\vec{r}-\vec{r}\,')$$

$$\boxed{\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\mathrm{Drude}} = \sigma_0 \times \delta_{ij}\,\delta(\vec{r}-\vec{r}\,')}$$

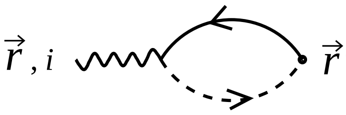
is short range

Diffuson contribution (Incoherent)

$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{diffuson}} =$$



We combine the box



$$= -\mathrm{i}\frac{\sqrt{2\pi}}{d} e\rho_0\ell_e^2\,\delta(\vec{r}-\vec{r}\,')\,\nabla_i$$

and the diffuson

$$\Gamma_d(r,r') = \frac{w}{D\tau_e}\,P_d(r,r')$$

$$\frac{2\pi}{d^2}e^2\rho_0^2\ell_e^4\frac{w}{D\tau_e} = \frac{2\pi}{d}e^2\rho_0^2\ell_e^2\frac{1}{2\pi\rho_0\tau_e} = e^2\tau_e\frac{v_F^2\rho_0}{d} = \frac{n_e e^2\tau_e}{m}$$

$$\boxed{\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{diffuson}} = -\sigma_0\,\nabla_i\nabla'_j P_d(\vec{r},\vec{r}\,')}$$

is long range

Classical conductivity & Current conservation

$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{classical}} = \langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{Drude}} + \langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{diffuson}}$$

► Current conservation : $\boxed{\nabla_i \sigma_{ij}(\vec{r},\vec{r}\,') = 0}$

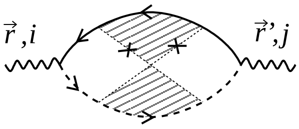
$$\begin{array}{ccccc} \text{Classical} = \text{Drude} & & + & & \text{Diffuson} \\ \downarrow & & & & \downarrow \end{array}$$

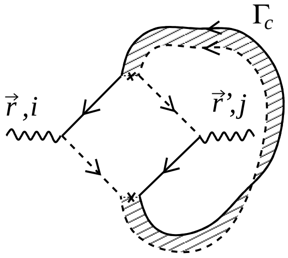
$$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{classical}} = \sigma_0 \left[\delta_{ij}\, \delta(\vec{r}-\vec{r}\,') - \nabla_i \nabla_j' P_d(\vec{r},\vec{r}\,')\right]$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \text{short range} & & \text{long range} \\ & \searrow \quad \swarrow & \end{array}$$

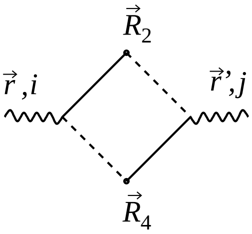
$$\nabla_i \sigma_{ij}(\vec{r},\vec{r}\,') = 0$$

Cooperon contribution (Coherent)

$$\langle \sigma_{ij}(\vec{r}, \vec{r}\,')\rangle_{\text{cooperon}} =$$


$$=$$


We combine the box



$$= -\sigma_0\, 2\tau_e^2 \delta_{ij}\, \delta(\vec{r} - \vec{r}\,')\, \delta(\vec{r} - \vec{R}_2)\, \delta(\vec{r} - \vec{R}_4)$$

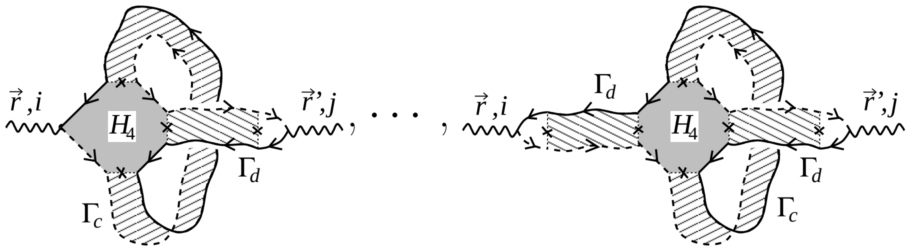
and the cooperon $\Gamma_c(r, r') = \frac{w}{D\tau_e} P_c(r, r')$

$$\boxed{\langle \sigma_{ij}(\vec{r}, \vec{r}\,')\rangle_{\text{cooperon}} = -\frac{e^2}{\pi}\delta_{ij}\, \delta(\vec{r} - \vec{r}\,')\, P_c(\vec{r}, \vec{r})}$$

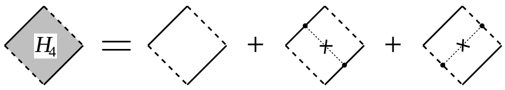
is short range

▷ Divergencies in long range diagrams :

Long range diagrams :



Hikami box :



$\sim \vec{\nabla}_1 \cdot \vec{\nabla}_3 + \vec{\nabla}_2 \cdot \vec{\nabla}_4 - \frac{1}{2} \sum_{i=1}^{4} \vec{\nabla}_i^2$

$$\downarrow$$

Divergency : $\vec{\nabla}_2 \cdot \vec{\nabla}_4 P_c(\vec{r}_2, \vec{r}_4)|_{\vec{r}_2=\vec{r}_4} \sim \delta(\vec{0})$

$$\downarrow$$

$$\Delta g_{\text{wire}} = -\frac{1}{3} + \frac{1}{3} L\delta(0)$$

$$\delta(\vec{0}) \longrightarrow 1/\ell_e^d$$

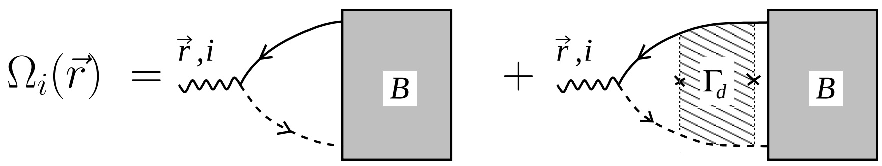
$\longrightarrow$ Volumic divergency Vol/ℓ_e^d

Problem with current conservation

The procedure of KSL

Kane, Serota & Lee, PRB (1988).

$\Omega_i(\vec{r}) = \langle j_i(r) \cdots \rangle$ a correlation function



$$= A_i(\vec{r}) \qquad + \int d\vec{r}\,'\, \nabla_i P_d(\vec{r}, \vec{r}\,')\, B(\vec{r}\,')$$

$\downarrow$ short range $\qquad$ $\downarrow$ long range

current conservation $(\nabla_i j_i(\vec{r}) = 0) \Rightarrow \nabla_i \Omega_i(\vec{r}) = 0$

$$\Downarrow$$

$$0 = \nabla_i A_i(\vec{r}) - B(\vec{r})$$

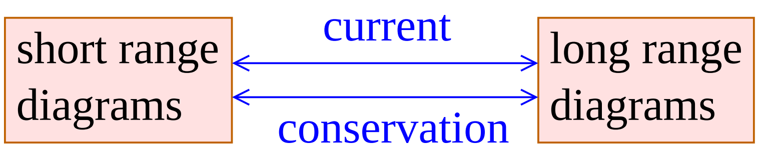
$$\boxed{\Omega_i(\vec{r}) = \int d\vec{\rho}\, \phi_{ij}(\vec{r}, \vec{\rho})\, A_j(\vec{\rho})}$$

where

$$\phi_{ij}(\vec{r}, \vec{r}\,') = \delta_{ij}\delta(\vec{r} - \vec{r}\,') - \nabla_i \nabla'_j P_d(\vec{r}, \vec{r}\,') = \frac{1}{\sigma_0}\langle \sigma_{ij}(\vec{r}, \vec{r}\,')\rangle_{\text{classical}}$$

- Current conserving weak localization :

$\longrightarrow$ use current conservation (Kane, Serota & Lee, (1988) for $\langle \delta\sigma^2 \rangle$).



$$\langle \Delta\sigma_{ij}(\vec{r},\vec{r}\,')\rangle \;=\; \sum_{i',j'} \int \mathrm{d}\vec{\rho}\,\mathrm{d}\vec{\rho}\,'\,\phi_{ii'}(\vec{r},\vec{\rho})\,\phi_{jj'}(\vec{r}\,',\vec{\rho}\,')\,\langle \sigma_{i'j'}(\vec{\rho},\vec{\rho}\,')\rangle_{\text{cooperon}}$$

$$\boxed{\langle \Delta\sigma_{ij}(\vec{r},\vec{r}\,')\rangle = -\frac{e^2}{\pi}\sum_k \int \mathrm{d}\vec{R}\,\phi_{ik}(\vec{r},\vec{R})\,P_c(\vec{R},\vec{R})\,\phi_{kj}(\vec{R},\vec{r}\,')}$$

where $\phi_{ij}(\vec{r},\vec{r}\,') = \delta_{ij}\,\delta(\vec{r}-\vec{r}\,') - \nabla_i \nabla_j' P_d(\vec{r},\vec{r}\,')$

$$\boxed{\text{equivalent to replace } H \text{ by } \widetilde{H} \sim 2\vec{\nabla}_1\cdot\vec{\nabla}_3}$$

$\longrightarrow$ current conservation is satisfied : $\nabla_i \langle \Delta\sigma_{ij}(\vec{r},\vec{r}\,')\rangle = 0$

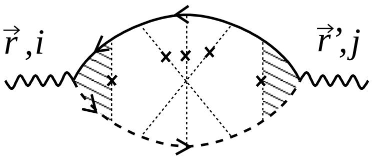
Set of diagrams satisfying current conservation

→ Other approach :

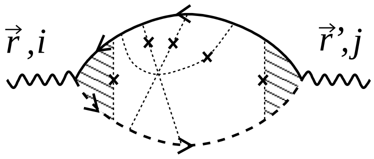
generate the correct set of diagrams satisfying current conservation

(For UCF : Hershfield, Ann. Phys. (1989))

For WL : Hastings, Stone & Baranger, PRB (1994).

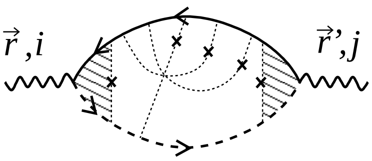
Add to 

the diagrams :

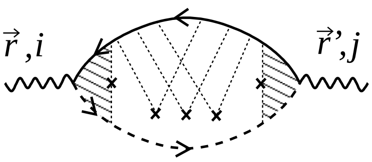


$+$ c.c.

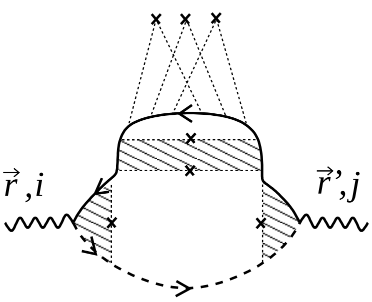
(which is in the Hikami box) and higher order contributions in $1/(k_F \ell_e)$:



$+$ c.c.

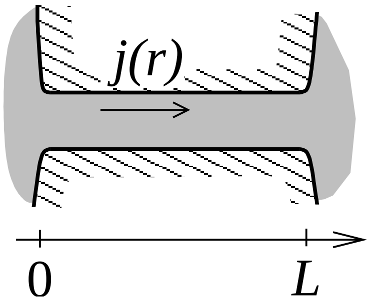


$+$ c.c.



$+$ c.c.

Local conductivity



translation invariance :

$$j(r) = j = \frac{I}{\text{section}}$$

and $\mathcal{E}(r) = \mathcal{E} = \frac{V}{L}$.

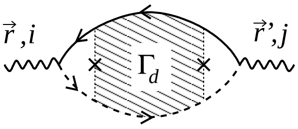
Nonlocal conductivity : $j(r) = \int \mathrm{d}r'\, \sigma(r,r')\, \mathcal{E}(r')$

$$\Downarrow$$

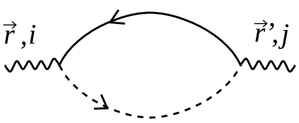
local conductivity : $j = \sigma\mathcal{E}$ with $\sigma = \int \frac{\mathrm{d}r\mathrm{d}r'}{\text{Vol}}\, \sigma(r,r')$

Conductance : $G = \sigma\frac{\text{section}}{L}$

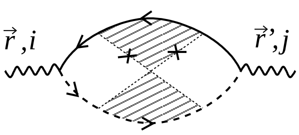
Long range diagrams do not contribute to the local conductivity :

$\int \mathrm{d}\vec{r}$  $= 0$

- Classical transport :

$\langle\sigma\rangle_{\text{classical}} = \int \frac{\mathrm{d}r\mathrm{d}r'}{\text{Vol}}$  $= \sigma_0$

- Weak localization correction :

$\langle\Delta\sigma\rangle = \int \frac{\mathrm{d}r\mathrm{d}r'}{\text{Vol}}$  $= -\frac{e^2}{\pi}\int \frac{\mathrm{d}\vec{r}}{\text{Vol}}\, P_c(\vec{r},\vec{r})$

Why it is simpler to consider $\langle\sigma\rangle$ instead of $\langle\sigma(r,r')\rangle$?

Spectrum of the diffusion equation : E_n, $\psi_n(r)$

$$-D\left(\nabla - 2\mathrm{i}eA\right)^2\psi_n(r) = E_n\psi_n(r)$$

$$\frac{1}{D}\int \mathrm{d}r\, P_c(r,r) = \int \mathrm{d}r\,\langle r|\frac{1}{1/\tau_\varphi - D\left(\nabla - 2\mathrm{i}eA\right)^2}|r\rangle$$

$$= \sum_n \frac{1}{1/\tau_\varphi + E_n} = \int_0^\infty \mathrm{d}t\,\mathrm{e}^{-t/\tau_\varphi}\sum_n \mathrm{e}^{-E_n t}$$

$$\boxed{\langle\Delta\sigma\rangle = -\frac{e^2}{\pi \mathrm{Vol}}\,\mathrm{Tr}\left\{\frac{1}{1/L_\varphi^2 - \Delta}\right\} = -\frac{e^2 D}{\pi}\int_0^\infty \mathrm{d}t\,\mathcal{P}(t)\,\mathrm{e}^{-t/\tau_\varphi}}$$

where

$$\mathcal{P}(t) = \frac{1}{\mathrm{Vol}}\sum_n \mathrm{e}^{-E_n t}$$

We only need the eigenvalues E_n

What is $\mathcal{P}(t)$?

In a time representation

$$\left[\frac{\partial}{\partial t} - D\left(\nabla - 2\mathrm{i}eA\right)^2\right]\mathcal{P}_c(r,r';t) = \delta(r-r')\,\delta(t)$$

$\mathcal{P}(t) = \int \frac{\mathrm{d}r}{\mathrm{Vol}}\,\mathcal{P}_c(r,r;t)$: return "probability" averaged over space

The infinite wire

$$\mathcal{P}(t) = \frac{1}{\sqrt{4\pi D t}}$$

WL correction is

$$\langle \Delta\sigma \rangle = -\frac{e^2 D}{\pi} \int_0^\infty \mathrm{d}t \, \frac{\mathrm{e}^{-t/\tau_\varphi}}{\sqrt{4\pi D t}} = -\frac{e^2}{h} L_\varphi$$

$$\boxed{\langle \Delta g \rangle = -\frac{L_\varphi}{L}}$$

THE FINITE WIRE

$$\mathcal{P}(t) = \frac{1}{L}\sum_n \mathrm{e}^{-E_n t}$$

Wire connected to reservoirs $\Rightarrow$ Dirichlet boundaries

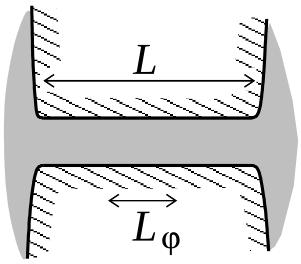
$$E_n = D\left(\frac{n\pi}{L}\right)^2 \quad ; \quad n \in \mathbb{N}^*$$

$$\langle \Delta g \rangle = -\frac{2D}{L^2}\int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t/\tau_\varphi} \sum_{n=1}^{\infty} \mathrm{e}^{-D(\frac{n\pi}{L})^2 t} = -\frac{2}{L^2}\sum_{n=1}^{\infty} \frac{1}{1/L_\varphi^2 + (n\pi/L)^2}$$

$$\boxed{\langle \Delta g \rangle = -\frac{L_\varphi}{L}\left(\coth\frac{L}{L_\varphi} - \frac{L_\varphi}{L}\right)}$$

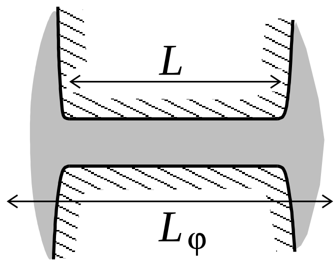
Al'tshuler, Aronov & Zyuzin, 1984.

⋆ Long wire ($L \gg L_\varphi$)



$\Rightarrow$ $\boxed{\langle \Delta g \rangle \simeq -\frac{L_\varphi}{L}}$

⋆ Short wire ($L \ll L_\varphi$)



$\Rightarrow$ $\boxed{\langle \Delta g \rangle \simeq -\frac{1}{3}}$

Universal result : Mello & Stone, 1991.

Effect of a magnetic field

Al'tshuler & Aronov, 1981.

Solve

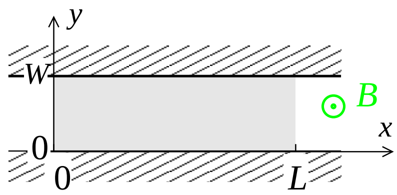
$$\left[\frac{1}{L_\varphi^2} - \left(\vec{\nabla} - 2\mathrm{i}e\vec{A}\right)^2\right] P_c(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$$

perturbatively in $\vec{A} = \vec{u}_x A_x(y)$

⋆ quasi 1d and diffusive limit : $L \gg W \gg \ell_e$.

⋆ weak magnetic field : $W \ll \sqrt{\hbar/e\mathcal{B}}$

choose the gauge $\boxed{A_x(W-y) = -A_x(y)}$:



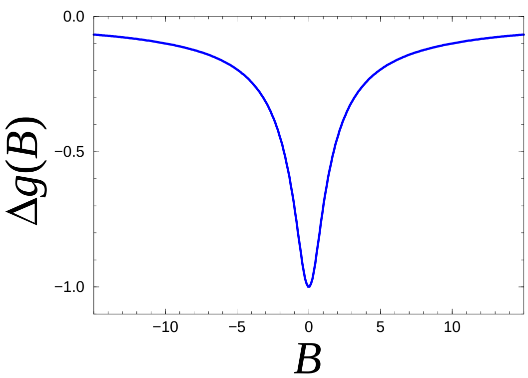
$$\begin{aligned}
&\langle\,\vec{r}\,|\frac{1}{\gamma-(\vec{\nabla}-2\mathrm{i}e\vec{A})^2}|\,\vec{r}\,'\rangle \\
=\;& \langle\,\vec{r}\,|\frac{1}{\gamma-\Delta}|\,\vec{r}\,'\rangle - 4e^2\langle\,\vec{r}\,|\frac{1}{\gamma-\Delta}\vec{A}^2\frac{1}{\gamma-\Delta}|\,\vec{r}\,'\rangle + \cdots \\
\simeq\;& \frac{1}{W}\langle\,x\,|\frac{1}{\gamma-\mathrm{d}_x^2}|\,x'\,\rangle - \frac{(e\mathcal{B}W)^2}{3}\frac{1}{W}\langle\,x\,|\frac{1}{(\gamma-\mathrm{d}_x^2)^2}|\,x'\,\rangle + \cdots \\
=\;& \frac{1}{W}\langle\,x\,|\frac{1}{\gamma+(e\mathcal{B}W)^2/3-\mathrm{d}_x^2}|\,x'\,\rangle
\end{aligned}$$

$$\boxed{\frac{1}{L_\varphi^2} \longrightarrow \frac{1}{L_\varphi^2} + \frac{1}{3}\left(\frac{e\mathcal{B}W}{\hbar}\right)^2}$$

Effective phase coherence length :

$$\frac{1}{L_\varphi(\mathcal{B})^2} = \frac{1}{L_\varphi^2} + \frac{1}{3}\left(\frac{e\mathcal{B}W}{\hbar}\right)^2$$

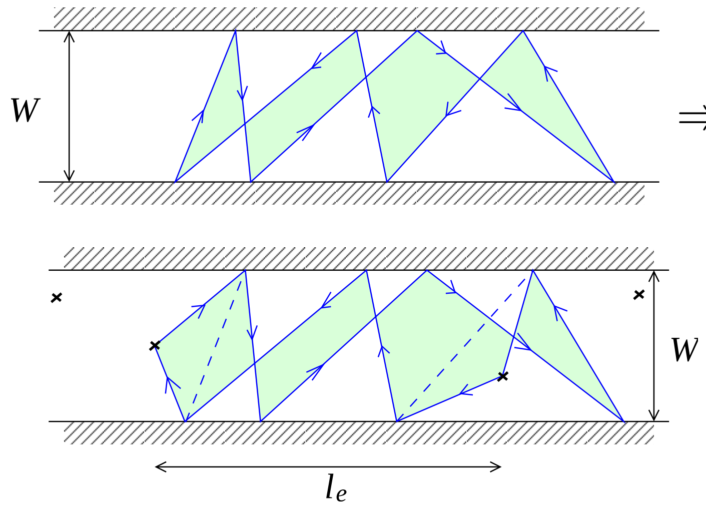
$$\Delta g_{\text{wire}} = -\frac{L_\varphi(\mathcal{B})}{L}$$



- If $\boxed{W \lesssim \ell_e}$

(wires etched at GaAl/GaAlAs interface)

$\longrightarrow$ Flux cancellation :



$\Rightarrow \oint \mathrm{d}\vec{r}\cdot\vec{A} = 0$

$$\frac{1}{L_\varphi^2} \longrightarrow \frac{1}{L_\varphi^2} + C\left(\frac{e\mathcal{B}W}{\hbar}\right)^2\left(\frac{W}{\ell_e}\right)$$

Dugaev & Khmel'nitskiĭ, Sov. Phys. JETP (1984)

Beenakker & van Houten, PRB (1988)

The plane

Solution of the diffusion equation is :

$$\mathcal{P}(t) = \frac{1}{4\pi D t}$$

$\longrightarrow$ introduce a cutoff at short time

WL correction is

$$\langle \Delta\sigma \rangle = -\frac{e^2 D}{\pi} \int_{\tau_e}^{\infty} \mathrm{d}t \, \frac{\mathrm{e}^{-t/\tau_\varphi}}{4\pi D t} \simeq -\frac{e^2}{4\pi^2} \ln(\tau_\varphi/\tau_e)$$

$$\boxed{\langle \Delta\sigma \rangle \simeq -\frac{e^2}{\pi h} \ln(L_\varphi/\ell_e)}$$

Magnetoconductance of the plane

Spectrum of

$$-D\,(\nabla - 2\mathrm{i}eA)^2\psi_n(r) = E_n\psi_n(r)$$

is the Landau spectrum

$$\begin{cases} \frac{\hbar^2}{2m} & \to D \\ e\mathcal{B} & \to 2e\mathcal{B} \end{cases}$$

$$\mathcal{P}(t) = \frac{e\mathcal{B}}{2\pi}\frac{1}{\sinh(2e\mathcal{B}Dt)} \xrightarrow{\text{Laplace}} \quad \text{Digamma function } \psi$$

$$\langle\Delta\sigma(\mathcal{B})\rangle - \langle\Delta\sigma(0)\rangle = \frac{e^2}{4\pi^2\hbar}\left[\psi\left(\frac{1}{2} + \frac{\phi_0}{8\pi\mathcal{B}L_\varphi^2}\right) - \ln\left(\frac{\phi_0}{8\pi\mathcal{B}L_\varphi^2}\right)\right]$$

Low field :

$$\langle\Delta\sigma(\mathcal{B})\rangle - \langle\Delta\sigma(0)\rangle \simeq \frac{2}{3}\frac{e^2}{\hbar}\left(\frac{\mathcal{B}L_\varphi^2}{\phi_0}\right)^2$$

High field :

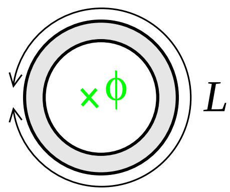
$$\langle\Delta\sigma(\mathcal{B})\rangle - \langle\Delta\sigma(0)\rangle \simeq \frac{e^2}{4\pi^2\hbar}\ln\left(\frac{8\pi\mathcal{B}L_\varphi^2}{\phi_0}\right)$$

$\longrightarrow$ Experiments : Bergmann, Phys. Rep. (1984).

Isolated ring : AAS oscillations

Al'tshuler, Aronov & Spivak, Sov. Phys. JETP (1981)

$\theta = 4\pi\phi/\phi_0$



$$\underbrace{\mathcal{P}_c(x,x';t) = \frac{\mathrm{e}^{-(x-x')^2/4t}}{\sqrt{4\pi t}}}_{\text{infinite wire}} \longrightarrow \sum_{n\in\mathbb{Z}} \mathcal{P}_c(x+nL,x';t)\,\mathrm{e}^{\mathrm{i}n\theta}$$

$$\mathcal{P}(t) = \frac{1}{\sqrt{4\pi Dt}} \sum_{n=-\infty}^{\infty} \mathrm{e}^{-(nL)^2/4Dt}\,\mathrm{e}^{\mathrm{i}n\theta}$$

Then

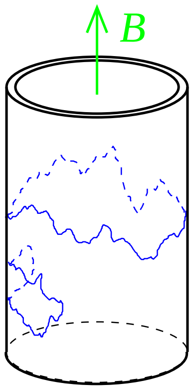
$$\langle\Delta\sigma(\theta)\rangle = -\frac{e^2}{h}\,L_\varphi\,\frac{\sinh(L/L_\varphi)}{\cosh(L/L_\varphi)-\cos\theta}$$

The harmonic $\langle\Delta\sigma_n\rangle = \int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\,\langle\Delta\sigma(\theta)\rangle\,\mathrm{e}^{-\mathrm{i}n\theta}$ involves trajectories that wind n times around the flux

$$\langle\Delta\sigma_n\rangle = -\frac{e^2D}{\pi}\int_0^\infty \mathrm{d}t\,\frac{\mathrm{e}^{-(nL)^2/4Dt}}{\sqrt{4\pi Dt}}\,\mathrm{e}^{-t/\tau_\varphi} = -\frac{e^2}{h}\,L_\varphi\,\mathrm{e}^{-|n|L/L_\varphi}$$

The cylinder

Al'tshuler, Aronov & Spivak, Sov. Phys. JETP (1981)



$$\langle \Delta\sigma_n \rangle = -\frac{e^2 D}{\pi} \int_0^\infty \mathrm{d}t \, \frac{\mathrm{e}^{-(nL)^2/4Dt}}{4\pi D t} \, \mathrm{e}^{-t/\tau_\varphi} = -\frac{e^2}{2\pi^2} \underbrace{\mathrm{K}_0(nL/L_\varphi)}_{\text{modified Bessel}}$$

Finally

$$\boxed{\langle \Delta\sigma \rangle = -\frac{e^2}{\pi h} \left(\ln(L_\varphi/\ell_e) + 2 \sum_{n=1}^{\infty} \mathrm{K}_0(nL/L_\varphi) \, \cos 4\pi n\phi/\phi_0 \right)}$$

Observation of the Aaronov-Bohm effect in hollow metal cylinders

B. L. Al'tshuler, A. G. Aronov, B. Z. Spivak, D. Yu. Sharvin, and Yu. V. Sharvin

B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR and Institute of Solid State Physics, Academy of Sciences of the USSR and Institute of Physical Problems, Academy of Sciences of the USSR

(Submitted 22 April 1982)

Pis'ma Zh. Eksp. Teor. Fiz. **35**, No. 11, 476–478 (5 June 1982)

The oscillatory dependence of the resistance on the magnitude of the magnetic flux in the cross section of a specimen with period $hc/2e$ and negative longitudinal magnetoresistance are observed in cylindrical lithium films at helium temperatures. The phase of the oscillations and the sign of the magnetoresistance are opposite to those observed for magnesium,[4] which is attributed to the smallness of the spin-orbital interaction in lithium. The results agree well with the theoretical predictions.

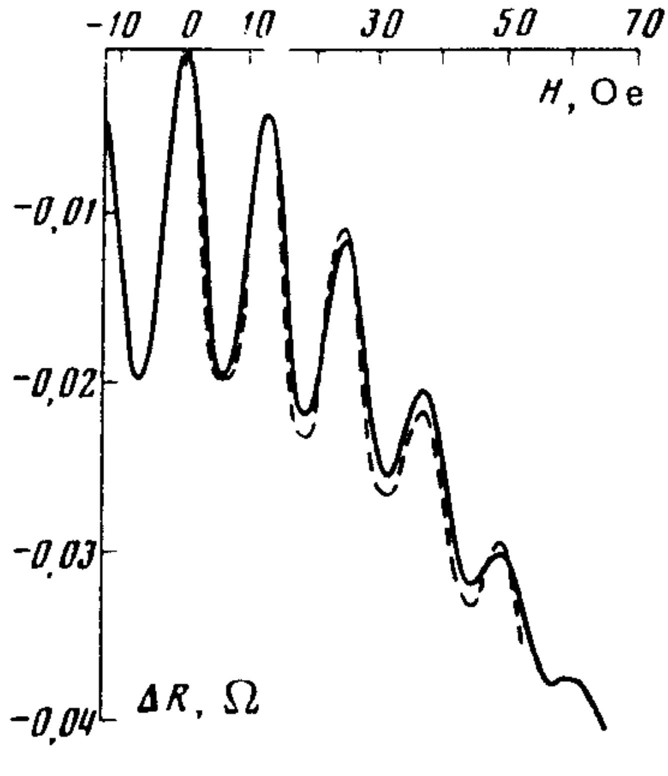
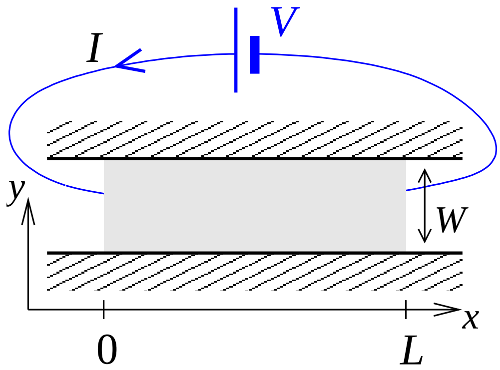


FIG. 1.

Landauer approach

Diffusive wire :



Transverse mode decomposition

$$-\frac{1}{2m}\frac{\mathrm{d}^2}{\mathrm{d}y^2}\chi_n(y) = \epsilon_n \chi_n(y)$$

Write conductance as

$$G = \frac{I}{V} = \frac{e^2}{h}\sum_{n,m} T_{nm}$$

T_{nm} : Transmission probability from channel m to channel n

<u>Relation between T_{nm} and Green functions :</u>

$G^{\mathrm{R}}(r, r'; E_F) \sim$ proba. amplitude to go from r' to r with an energy E_F.

$$\Downarrow$$

$$T_{nm} \sim |G^{\mathrm{R}}(L, 0; E_F)|^2$$

More precisely :

$$G = \int \mathrm{d}y\mathrm{d}y' \, \sigma_{xx}(L, y; 0, y') = \frac{e^2}{h} \sum_{n,m} T_{nm}$$

Transverse mode decomposition

$$G^{\mathrm{R}}_{nm}(L, 0) = \int \mathrm{d}y\mathrm{d}y' \, \chi^*_n(y) \, G^{\mathrm{R}}(L, y; 0, y') \, \chi_m(y')$$

Transmission probability from channel m to channel n :

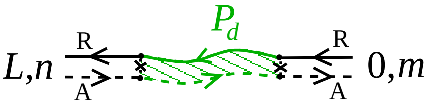
$$\boxed{T_{nm} = v_n v_m \, G^{\mathrm{R}}_{nm}(L, 0; E_F) \, G^{\mathrm{A}}_{mn}(0, L; E_F)}$$

where $v_n = \sqrt{\frac{2}{m}(E_F - \epsilon_n)}$

- Drude conductance

Long range term ⇒ diffuson

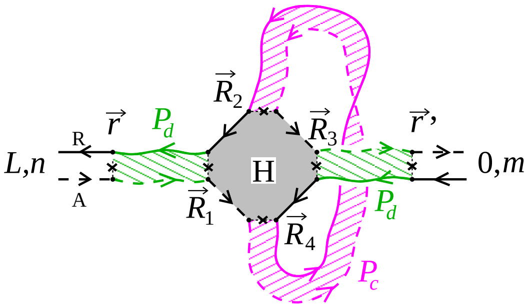
$$T_{nm}^{\rm cl} = \langle T_{nm}\rangle_{\rm diffuson} =$$



$$T_{nm}^{\rm cl} = \frac{1}{\alpha_d N_c \ell_e} P_d\left(L - v_n\tau_e, v_m\tau_e\right)$$

- WL correction

$$\Delta T_{nm} =$$



$$\Delta T_{nm} = \frac{2}{(\alpha_d N_c)^2}\frac{1}{\ell_e^2}\int_0^L {\rm d}x \, \frac{\rm d}{{\rm d}x}P_d(L - v_n\tau_e, x)\; P_c(x,x)\; \frac{\rm d}{{\rm d}x}P_d(x, v_m\tau_e)$$

This is not a uniform integration of P_c

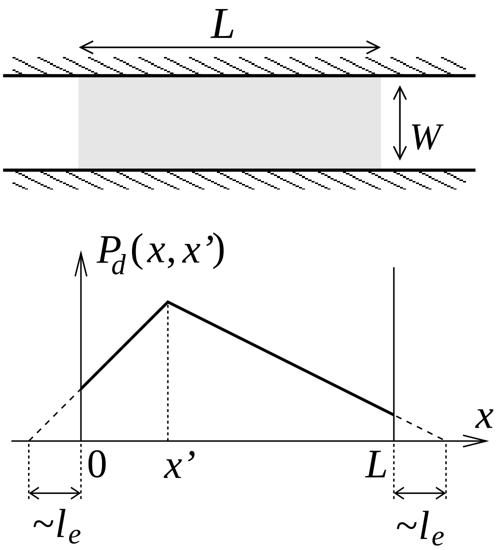
• Boundary conditions

→ Diffuson must be evaluated near the boundary

→ Careful treatment of boundary conditions

For the finite wire :

$$P_d(x, x') = \min\left(x + x_d, x' + x_d\right) - \frac{(x + x_d)(x' + x_d)}{L + 2x_d}$$



with $x_d = \alpha_d \ell_e / 2$
($\alpha_1 = 2$, $\alpha_2 = \pi/2$ and $\alpha_3 = 4/3$)

THE WIRE

$$g_{\rm cl} = \sum_{n,m} T^{\rm cl}_{nm} = \sum_{n,m} \frac{1}{\alpha_d N_c} \frac{\ell_e}{L} \left(\frac{v_n}{v_F} + \frac{\alpha_d}{2} \right) \left(\frac{v_m}{v_F} + \frac{\alpha_d}{2} \right) = \alpha_d N_c \frac{\ell_e}{L}$$

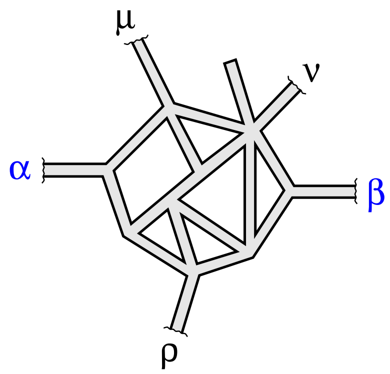
$$\begin{aligned} \langle \Delta g \rangle &= \sum_{n,m} \Delta T_{nm} = 2 \int_0^L {\rm d}x \left(-\frac{1}{L} \right) P_c(x,x) \left(\frac{1}{L} \right) \\ &= -\frac{2}{L^2} \,{\rm Tr} \left\{ \frac{1}{1/L_\varphi^2 - \Delta} \right\} = -\frac{2}{L^2} \sum_{n=1}^{\infty} \frac{1}{1/L_\varphi^2 + (n\pi/L)^2} \end{aligned}$$

$$\langle \Delta g \rangle = -\frac{L_\varphi}{L} \left(\coth \frac{L}{L_\varphi} - \frac{L_\varphi}{L} \right)$$

Al'tshuler, Aronov & Zyuzin, 1984.

→ We have recovered the result obtained from the *local* conductivity

3. Networks



Landauer-Büttiker : Conductance matrix

$$I_\alpha = \sum_\beta G_{\alpha\beta} U_\beta$$

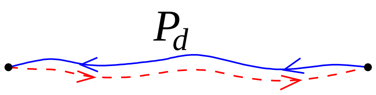
$$G_{\alpha\beta} = -\frac{e^2}{h} T_{\alpha\beta} \quad \text{for } \alpha \neq \beta$$

Fisher & Lee formula : $T_{\alpha\beta} \sim G^{\mathrm{R}}(\alpha, \beta; E_F)\, G^{\mathrm{A}}(\beta, \alpha; E_F)$

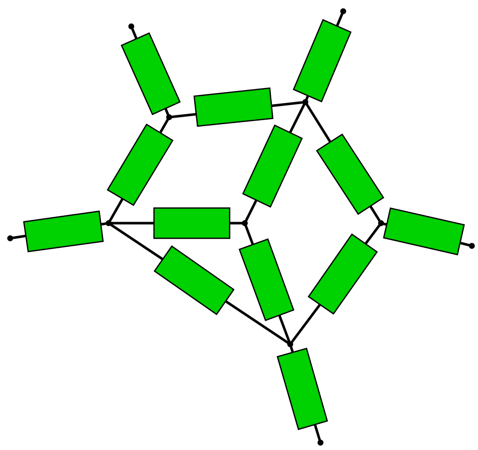
$$\langle T_{\alpha\beta} \rangle = T^{\mathrm{cl}}_{\alpha\beta} + \Delta T_{\alpha\beta}$$

↙ classical ↘ quantum correction

Classical transport

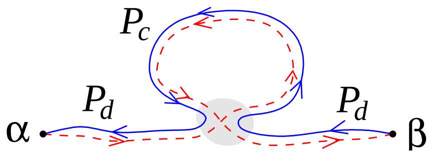
$T^{\mathrm{cl}}_{\alpha\beta} = \alpha$  $\beta \sim P_d(\alpha, \beta)$

→ Network of classical resistances



WEAK LOCALIZATION CORRECTION

$$\Delta T_{\alpha\beta} \;=$$



$$= \frac{2}{\ell_e^2}\int_{\text{Network}} \mathrm{d}x \,\frac{\mathrm{d}}{\mathrm{d}x} P_d(\alpha, x)\; P_c(x,x)\; \frac{\mathrm{d}}{\mathrm{d}x} P_d(x,\beta)$$

$$\boxed{\Delta T_{\alpha\beta} = \frac{2}{\alpha_d N_c \ell_e} \sum_{\text{wire }(\mu\nu)} \frac{\partial T^{\text{cl}}_{\alpha\beta}}{\partial\, l_{\mu\nu}} \times \int_{(\mu\nu)} \mathrm{d}x \, P_c(x,x)}$$

for $d = 1,\ 2,\ 3$:

N_c : number of conducting channels

$\alpha_d = 2, \pi/2, 4/3$

C. T. & G. Montambaux, PRL **92** (2004)

This is not a uniform integration of P_c

Weights depend on topology and connection

EQUIVALENT LENGTH

Wire :

$$\langle \Delta g \rangle = -\frac{2}{L^2} \int_0^L \mathrm{d}x \, P_c(x,x)$$

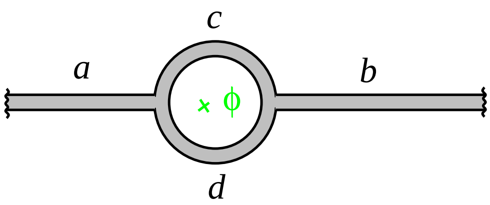
Networks :

Classsical conductance involves the equivalent length $\mathcal{L}$

$$g_{\rm cl} = \frac{\alpha_d N_c \ell_e}{\mathcal{L}}$$

$$\langle \Delta g \rangle = -\frac{2}{\mathcal{L}^2} \sum_i \frac{\partial \mathcal{L}}{\partial l_i} \int_{\text{wire } i} \mathrm{d}x \, P_c(x,x)$$

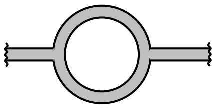
Exercice :



$$\langle \Delta g \rangle = -\frac{2}{(l_a + l_{c\|d} + l_b)^2}$$

$$\times \left[\int_a + \frac{l_d^2}{(l_c+l_d)^2} \int_c + \frac{l_c^2}{(l_c+l_d)^2} \int_d + \int_b \right] \mathrm{d}x \, P_c(x,x)$$

An heuristic argument :

used by Santhanam (1991) for 

$$1/T_{\alpha'\beta'} = \mathcal{R}_{\rm cl}(R_{\mu\nu}, \cdots)$$

Resistance of the wire $(\mu\nu)$: $\quad R_{\mu\nu} = R^{\rm cl}_{\mu\nu} + \Delta R_{\mu\nu}$

$\star$ Drude : $R^{\rm cl}_{\mu\nu} = l_{\mu\nu}/(2\ell_e) \quad (d=1)$

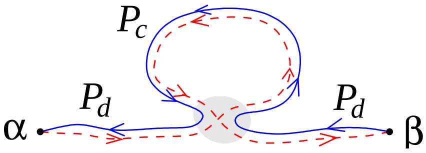
$\star$ Weak loc. : $\frac{\Delta R_{\mu\nu}}{R^{\rm cl}_{\mu\nu}} = \frac{e^2}{\pi\sigma_0}\int_{\text{wire }(\mu\nu)} \frac{{\rm d}x}{l_{\mu\nu}} P_c(x,x).$

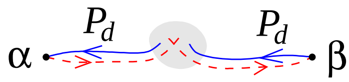
$$\Delta R = \sum_{(\mu\nu)} \frac{\partial \mathcal{R}_{\rm cl}}{\partial R_{\mu\nu}} \Delta R_{\mu\nu} = \underset{\substack{\downarrow \\ 1/\ell_e \text{ (in } d=1)}}{\frac{e^2}{\pi\sigma_0}} \sum_{(\mu\nu)} \frac{\partial \mathcal{R}_{\rm cl}}{\partial l_{\mu\nu}} \int_{(\mu\nu)} {\rm d}x \; P_c(x,x)$$

$\longrightarrow$ This result is non trivial due to nonlocality

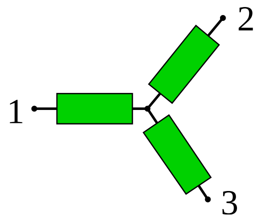
- starts from a classical formula for transport (no quantum interferences).
- A formula for the quantum resistance of the network as a function of quantum resistances of the wires does not exist
- $\Delta T_{\alpha'\beta'}$ is global
  contributions of the wires cannot be computed separatively :
  the Cooperon in a given wire depends on the whole network

2 Nonlocalities

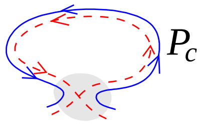
$\Delta T_{\alpha\beta} =$ 

1. "Classical" nonlocality 

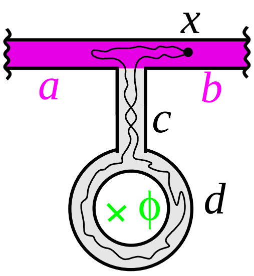
→ similar origin as in



$T^{\mathrm{cl}}_{\alpha\beta} =$ 

2. "Quantum" nonlocality 

→ Nonlocality of $P_c(x,x)$



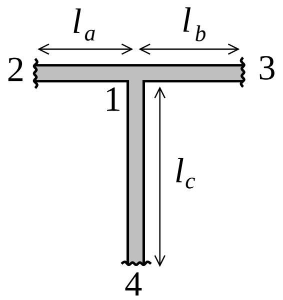
$$T^{\mathrm{cl}} = \frac{\alpha_d N_c \ell_e}{l_a + l_b}$$

⇒ P_c integrated over $a+b$ only

$$\Delta T = -\frac{2}{(l_a+l_b)^2}\int_{\boxed{a+b}} \mathrm{d}x\, P_c(x,x)$$

How weak localization can increase a transmission ?

C. Wire with arms :

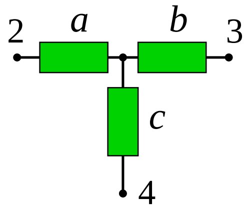


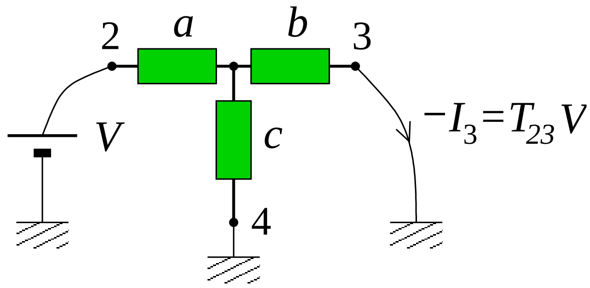
• one long arm

⋆ Classical transport :

3-terminal network : conductance matrix $\to I_\alpha = \sum_\beta G_{\alpha\beta} V_\beta$

$$G = \frac{e^2}{h} \begin{pmatrix} R_{22} & T_{23} & T_{24} \\ T_{23} & R_{33} & T_{34} \\ T_{24} & T_{34} & R_{44} \end{pmatrix}$$





$$T_{23}^{\rm cl} = \alpha_d N_c \ell_e \frac{l_c}{l_a l_b + l_b l_c + l_c l_a} \qquad \Rightarrow \qquad \boxed{\frac{\partial T_{23}^{\rm cl}}{\partial l_c} > 0}$$

⋆ Weak localization correction :

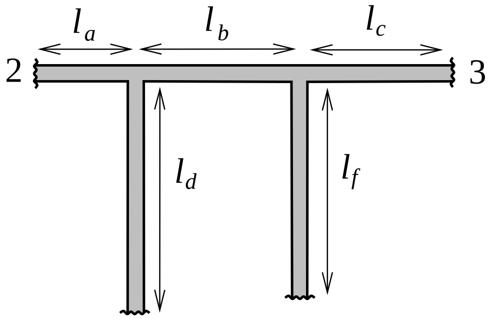
$\longrightarrow$ In the fully coherent limit $L_\varphi \to \infty$

$$\Delta T_{23} = \frac{1}{3}\left(-1 + \frac{l_{a//b//c}}{l_c} + \frac{l_{a//b//c}^2}{l_a l_b}\right) \xrightarrow[l_c \gg l_a, l_b]{} \frac{1}{3}\left(-1 + \frac{l_{a//b}}{l_a + l_b}\right)$$

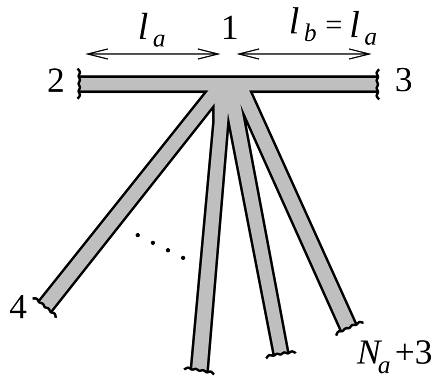
(-1 $\downarrow$ wire; $\frac{l_{a//b}}{l_a + l_b}$ $\downarrow$ arm)

- How to make $\Delta T_{23} > 0$?

  ⋆ With several arms :



$\Rightarrow$



N_a long arms

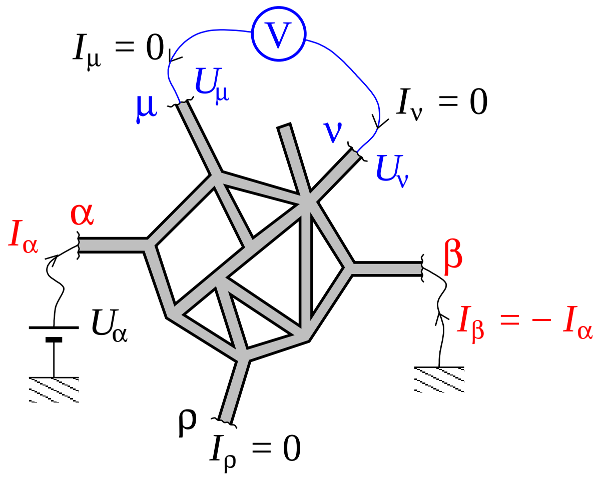
For $l_a \ll l_{\rm arm} \ll L_\varphi$:

$$\Delta T_{23} \simeq \frac{1}{3}\left(-1+\frac{N_a}{4}\right)$$

$\rightarrow$ for $N_a > 4 \Rightarrow$ $\boxed{\Delta T_{23} > 0}$

$\rightarrow$ Purely geometrical effect

4 TERMINAL RESISTANCES



$$\mathcal{R}_{\alpha\beta,\mu\nu} = \frac{U_\mu - U_\nu}{I_\alpha}$$

- Weak localization correction

$$\Delta\mathcal{R}_{\alpha\beta,\mu\nu} = \frac{2}{\alpha_d N_c \ell_e} \sum_{(\rho\sigma)} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha\beta,\mu\nu}}{\partial\, l_{\rho\sigma}} \int_{(\rho\sigma)} \mathrm{d}x\, P_c(x,x)$$

Proof of the relation

- Conductance matrix : $I_\alpha = \sum_\beta G_{\alpha\beta} U_\beta$

current conservation $\Rightarrow \sum_\alpha G_{\alpha\beta} = 0$

- Resistance matrix : $U_\alpha = \sum_\beta R_{\alpha\beta} I_\beta$

gauge invariance $\Rightarrow R_{\alpha\beta}$ not unique

$$\sum_\gamma R_{\alpha\gamma} G_{\gamma\lambda} = \sum_\gamma G_{\alpha\gamma} R_{\gamma\lambda} = \delta_{\alpha\lambda} - \frac{1}{N_{\text{term}}}$$

- Four-terminal resistance :

$$\begin{aligned} \mathcal{R}_{\alpha\beta,\mu\nu} &= \frac{U_\mu - U_\nu}{I_\alpha} \\ &= R_{\mu\alpha} - R_{\mu\beta} - R_{\nu\alpha} + R_{\nu\beta} \end{aligned}$$

$$\Downarrow$$

$$\mathcal{R}_{\alpha\beta,\mu\nu} = f(\{G_{\rho\sigma}\})$$

- Weak localization correction

$$\begin{aligned} \langle f(G) \rangle &= f(\langle G \rangle) + O(1/N_c^2) \\ &= f(G^{\text{cl}} + \Delta G) + O(1/N_c^2) \end{aligned} \quad \Rightarrow \Delta f = \frac{\partial f(G^{\text{cl}})}{\partial G} \Delta G$$

$$\Delta\mathcal{R}_{\alpha\beta,\mu\nu} = -\sum_{\gamma,\lambda} (R^{\text{cl}}_{\mu\gamma} - R^{\text{cl}}_{\nu\gamma})\, \Delta G_{\gamma\lambda} (R^{\text{cl}}_{\lambda\alpha} - R^{\text{cl}}_{\lambda\beta})$$

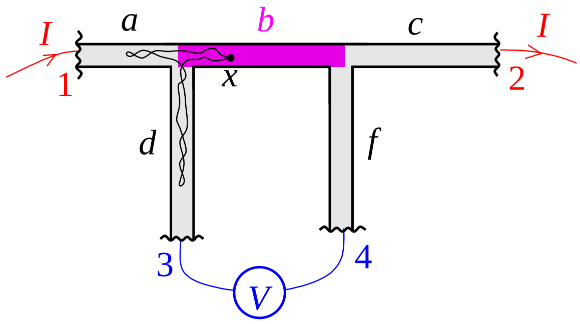
$$\Delta\mathcal{R}_{\alpha\beta,\mu\nu} = \frac{2}{\alpha_d N_c \ell_e} \sum_{(\rho\sigma)} \frac{\partial \mathcal{R}^{\text{cl}}_{\alpha\beta,\mu\nu}}{\partial\, l_{\rho\sigma}} \int_{(\rho\sigma)} \mathrm{d}x\, P_c(x,x)$$

How weak localization can be large ?

WL correction to the four terminal resistance :

$$\mathcal{R}^{\rm cl}_{12,34} = \frac{l_b}{\alpha_d N_c \ell_e}$$

$$\frac{\Delta\mathcal{R}_{12,34}}{(\mathcal{R}^{\rm cl}_{12,34})^2} = \frac{2}{l_b^2}\int_{\text{wire } b} \mathrm{d}x\, P_c(x,x)$$



$\longrightarrow$ Long connecting wires $l_a, l_c, l_d, l_f \gg L_\varphi$:

$$\frac{\Delta\mathcal{R}_{12,34}}{(\mathcal{R}^{\rm cl}_{12,34})^2} = \frac{L_\varphi}{l_b}\,\frac{5\coth(l_b/L_\varphi)+4-3L_\varphi/l_b}{4\coth(l_b/L_\varphi)+5}$$

Santhanam, 1987.

$\star$ Long wire $l_b \gg L_\varphi$

$$\frac{\Delta\mathcal{R}_{12,34}}{(\mathcal{R}^{\rm cl}_{12,34})^2} \simeq \frac{L_\varphi}{l_b}$$

$\star$ Short wire $l_b \ll L_\varphi$

$$\frac{\Delta\mathcal{R}_{12,34}}{(\mathcal{R}^{\rm cl}_{12,34})^2} \simeq \frac{1}{2}\frac{L_\varphi}{l_b} \gg 1$$

Reminiscent of the large fluctuations of resistances measured by

Benoit *et al* PRL **58** (1987).

Skocpol *et al* PRL **58** (1987).

Length-Independent Voltage Fluctuations in Small Devices

A. Benoit,[a] C. P. Umbach, R. B. Laibowitz, and R. A. Webb
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 25 August 1986)

Conductance fluctuations in one-dimensional lines of length L shorter than the phase-coherence length L_ϕ are *not* universal but diverge as L^{-2}. Using the Onsager relations and voltage additivity, we show that the voltage fluctuations are independent of the distance between voltage probes. The antisymmetric (Hall-type) contribution to the voltage fluctuations is constant for all values of L. Measurements of the voltage fluctuations and correlation function between different regions in Au and Sb lines confirm these results.

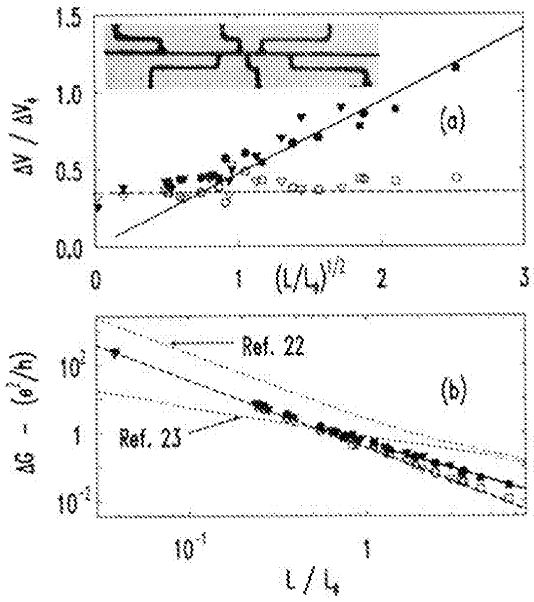


FIG. 2. (a) Measured rms voltage fluctuations normalized by ΔV_ϕ, as a function of $(L/L_\phi)^{1/2}$. The symmetric contributions are represented by solid symbols. The solid line represents the expected behavior for $L > L_\phi$. The antisymmetric part of the voltage fluctuations is represented by the open symbols and the dashed line is the predicted constant behavior. The symbols refer to different samples and temperatures: circles, Sb at $T = 40$ mK and $L_\phi = 1.05$ μm; inverted triangles, Sb at $T = 300$ mK and $L_\phi = 0.60$ μm; squares, Au at $T = 40$ mK and $L_\phi = 2.0$ μm. Inset: A photograph of the Sb sample. (b) Conductance fluctuations in units of e^2/h on a logarithmic scale for the data displayed in (a). Dotted lines are weak-localization predictions for two different boundary conditions.

Nonlocal Potential Measurements of Quantum Conductors

W. J. Skocpol, P. M. Mankiewich, R. E. Howard, L. D. Jackel, and D. M. Tennant
AT&T Bell Laboratories, Holmdel, New Jersey 07733

and

A. Douglas Stone
Department of Applied Physics, Yale University, New Haven, Connecticut 06520
(Received 14 October 1986)

Multiterminal measurements of magnetoresistance fluctuations in silicon inversion-layer nanostructures are extended to probe spacings $L \ll L_\phi$, the phase-preserving diffusion length. Unlike for $L > L_\phi$, the sizes of the voltage fluctuations are independent of L, and have novel correlations consistent with independent potential fluctuations of each probe. The corresponding "conductance" fluctuations $\delta G(L)$ are $\gg e^2/h$; however, this can be understood if each pair of probes effectively measures voltage fluctuations at scale L_ϕ, determined by the condition $\delta G(L_\phi) = e^2/h$.

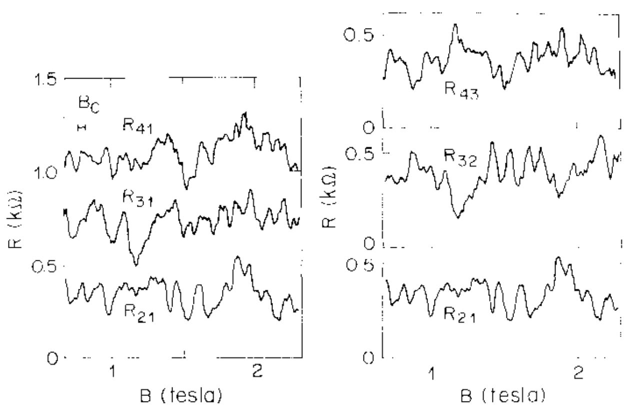


FIG. 1. Resistance measured between various pairs of probes for the short device with 0.15 μm probe spacing.

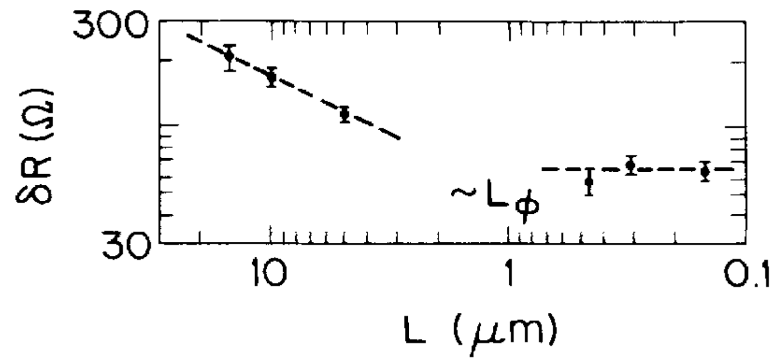
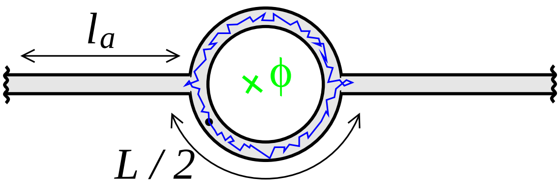


FIG. 2. Amplitude of resistance fluctuations as a function of probe spacing for the long and short devices, showing distinctly different dependence.

Effect of the arms on the AAS oscillations

A consequence of the nonlocality of P_c

- Small coherence length : $\boxed{L_\varphi \ll l_a, L}$



$$\boxed{\langle \Delta g_n \rangle \simeq -\frac{L\,L_\varphi}{4(2l_a + L/4)^2}\left(\frac{2}{3}\right)^{2|n|} \mathrm{e}^{-|n|\,L/L_\varphi}}$$

$$\langle \Delta g_n \rangle \sim \int_0^\infty \mathrm{d}t \quad \underbrace{\frac{1}{\sqrt{t}}\,\mathrm{e}^{-\frac{(nL)^2}{4t}}}_{\text{Proba to wind } n \text{ times}} \quad \mathrm{e}^{-t/\tau_\varphi} \sim \mathrm{e}^{-|n|\,L/L_\varphi}$$

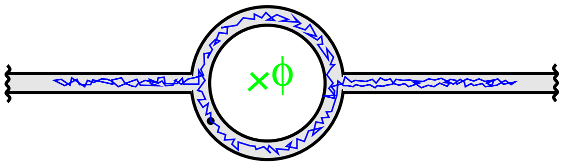
Winding around the loop : $n_t \sim t^{1/2}/L$

- Large coherence length : $\boxed{L \ll L_\varphi \ll l_a}$ $\Rightarrow$ New behaviour.

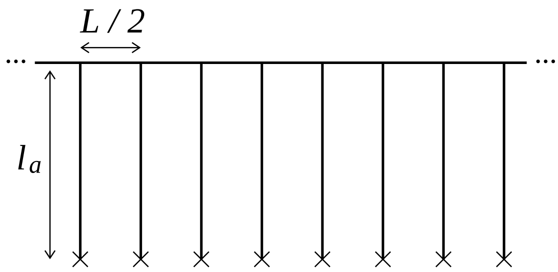
$$\boxed{\langle \Delta g_n \rangle \simeq -\left(\frac{L_\varphi}{2l_a}\right)^2 \sqrt{\frac{L}{2L_\varphi}}\ \mathrm{e}^{-|n|\sqrt{2L/L_\varphi}}}$$

C. T. & G. Montambaux, J. Phys. A **38** (2005).

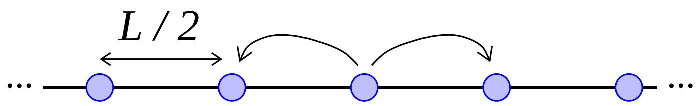
Origin of the behaviour for $L \ll L_\varphi$: slow winding



mapping to the diffusion in a comb :



trapping by the arms



Trapping time distribution :

first return probability in 1d : $Q(t) \sim 1/t^{3/2}$

for a trapping time distribution $Q(t) \sim 1/t^{\mu+1} \Rightarrow n_t \sim t^{\mu/2}$

$$0 < \mu < 1$$

$$\boxed{n_t \sim t^{1/4}/\sqrt{L}}$$

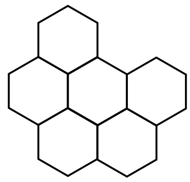
Tail of winding distribution is :

$$\mathcal{P}_n(t) \propto \exp -3\left(\frac{n\sqrt{L}}{4\,t^{1/4}}\right)^{4/3}$$

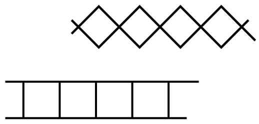
$$\langle \Delta g_n \rangle \sim \int_0^\infty \mathrm{d}t \; \underbrace{\frac{1}{t^{1/6}}\,\mathrm{e}^{-3\left(\frac{n\sqrt{L}}{4t^{1/4}}\right)^{4/3}}}_{\text{Proba to wind } n \text{ times}} \; \mathrm{e}^{-t/\tau_\varphi} \sim \mathrm{e}^{-|n|\sqrt{2L/L_\varphi}}$$

WL in large regular networks

- large networks :



AAS oscillations in honeycomb metallic lattices :
Pannetier *et al*, 1984.

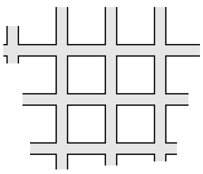
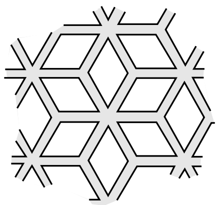


arrays of rings, ladders, square lattice...
Bishop, Dolan & Licini 1985, 1986.

large square lattices

2DEG : Ferrier, Bouchiat *et al* (2003).

metal : Bäuerle, Mallet, Saminadayar, Schopfer (2004).

 and 

Interest of networks to probe phase coherence

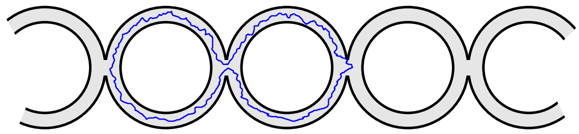
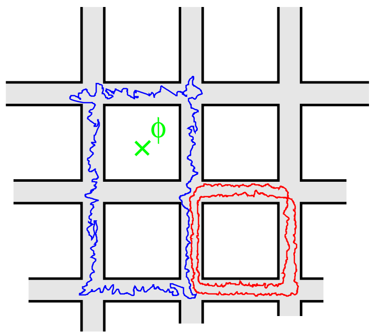
→ good disorder averaging

→ rich AAS harmonic content

Comparison of AAS harmonics in rings and square network

<u>For $L_\varphi \ll L$</u> $\longrightarrow$ periodic orbits analysis

Akkermans, Comtet, Desbois, Montambaux & C. T., Ann. Phys. (2000)

Harmonic 1	Harmonic 2	Harmonic 3
		
$\mathrm{e}^{-L/L_\varphi}$	$\frac{3}{4}\,\mathrm{e}^{-2L/L_\varphi}$	$\frac{5}{8}\,\mathrm{e}^{-3L/L_\varphi}$
		
$\mathrm{e}^{-L/L_\varphi}$	$\frac{3}{4}\,\mathrm{e}^{-3L/2L_\varphi}$	$\frac{3}{4}\,\mathrm{e}^{-2L/L_\varphi}$

$\rightarrow$ harmonics decay faster in chain of rings

Quantum Interference Effects in Lithium Ring Arrays

G. J. Dolan, J. C. Licini, and D. J. Bishop
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 22 July 1985)

We report detailed measurements on the weak-localization corrections in well-defined and -characterized arrays of quasi one-dimensional normal-metal rings. The Bohm-Aharonov oscillations are very well resolved. Measurements on sets of samples of varying ring size and geometry determine the distinct effects of these parameters and allow an unambiguous, quantitative comparison to the weak localization theory for ring geometries.

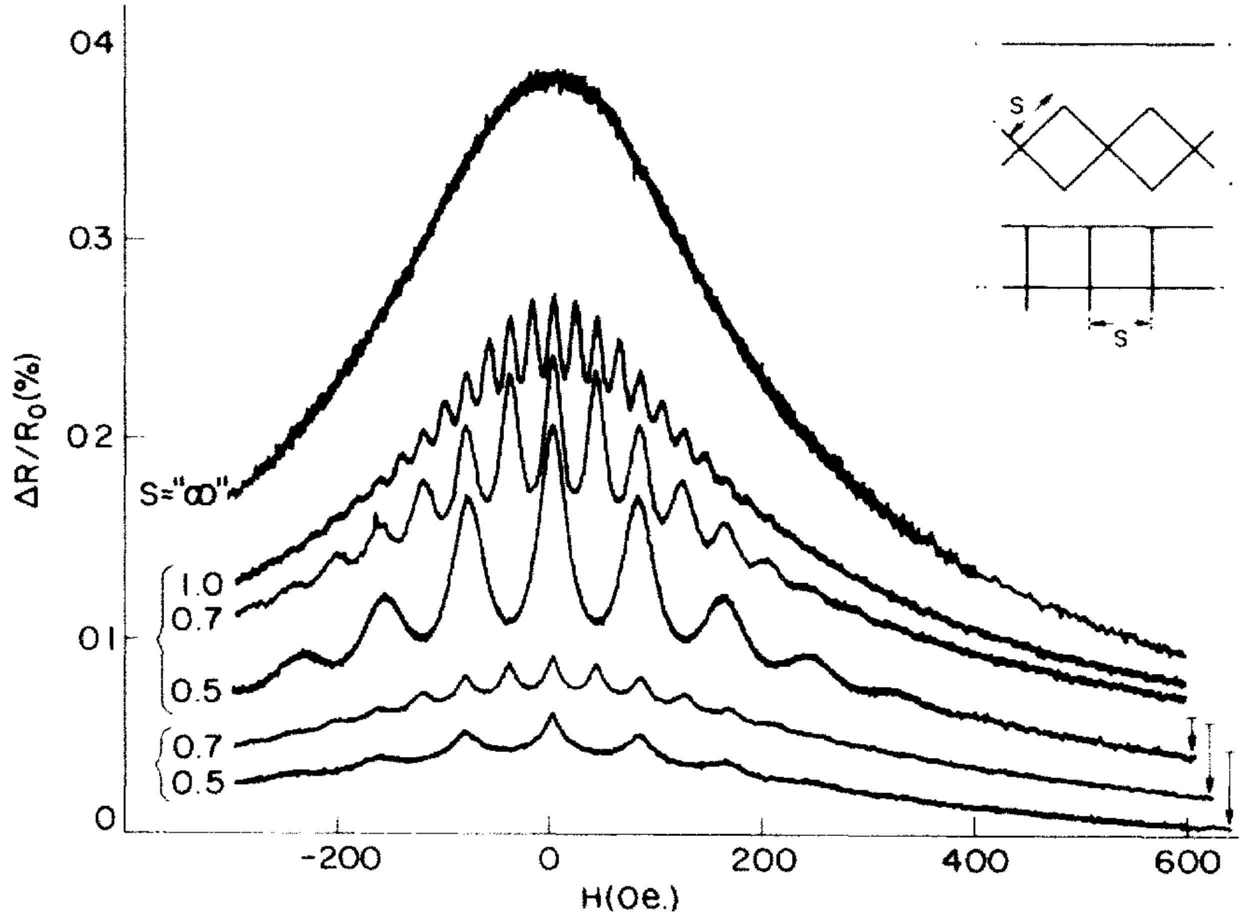


FIG. 1. The magnetoresistance $R(T,H)$ for $T=0.13$ K for the wire control sample (top), three necklace arrays (next three curves), and two meshes (bottom two curves). The upper-right-hand sketches define the control, necklace, and mesh geometry. The size, S, of the unit-cell side is indicated next to each curve. Some of the curves have been displaced vertically for clarity.

Magnetic Flux Quantization in the Weak-Localization Regime of a Nonsuperconducting Metal

B. Pannetier, J. Chaussy, R. Rammal, and P. Gandit
Centre de Recherches sur les Trés Basses Températures, Centre National de la Recherche Scientifique, F-38042 Grenoble-Cédex, France
(Received 7 May 1984)

The magnetic flux quantization effect, with period $\phi_0 = hc/2e$, is observed with high accuracy in the resistance of a Mg honeycomb network at temperatures 50 mK $\leqslant T \leqslant$ 6 K. As expected, the phase of the oscillations and the sign of the magnetoresistance are dominated by the spin-orbit interaction. Our results confirm, for a new geometry, the reality of the Bohm-Aharonov effect for a nonsuperconducting metal in the weak-localization regime.

Quantum Oscillations in Normal-Metal Networks

B. Douçot and R. Rammal
Centre de Recherches sur les Très Basses Températures, Centre National de la Recherche Scientifique, F-38042 Grenoble Cédex, France
(Received 8 April 1985)

A general formalism is outlined for the calculation of the transport coefficients of a normal-metal network in the weak-localization regime. Simple circuits such as loops and ladders are used to illustrate our approach. A closed expression for the magnetoresistance of an infinite regular network is derived. We find that, in contrast with superconducting networks, no fine structure due to interference effects between adjacent loops is expected. Our results agree very well with the recently observed oscillations in normal-metal networks.

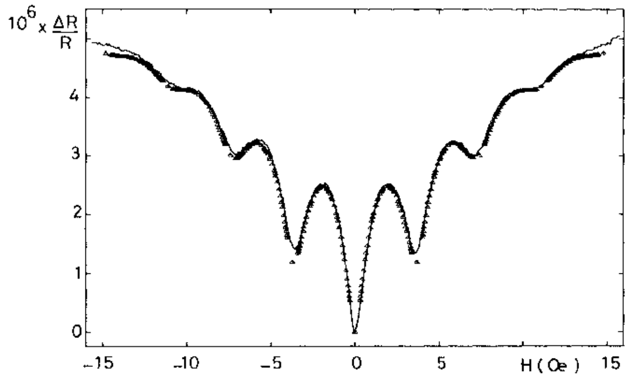
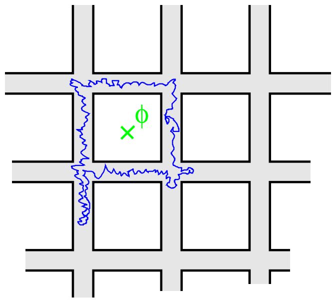
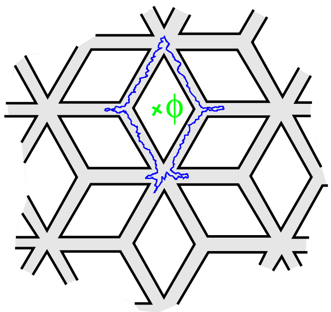
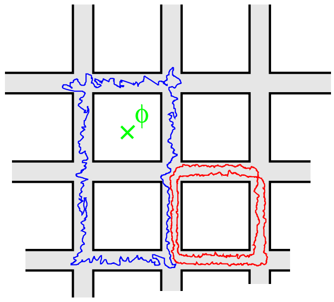
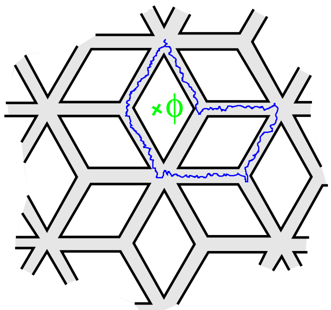
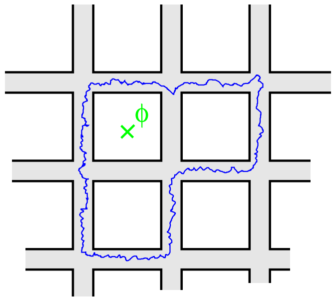
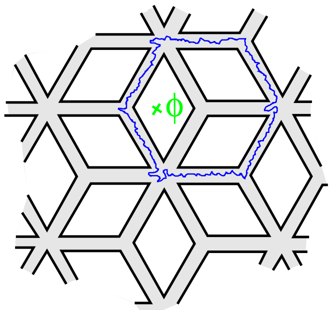


FIG. 2. Quantitative comparison between the theoretical results (triangles) and experimental data (solid line), for Cu at $T = 133$ mK, taken from Ref. 2. The hexagonal elementary cells (side $a = 1.5$ μm) are made of wires of width 0.42 μm. In this fit, we have $L_\varphi = 5.36$ and $L_{s.o.} = 3.12$ μm, respectively ($L_{s.o.}$ is the spin-orbit length).

Comparison between Square & $\mathcal{T}_3$ lattices

Diffusive trajectories		
	Square network	$\mathcal{T}_3$ network
harmonic 1		
harmonic 2		
harmonic 3		

Therefore : harmonic 3 is larger in $\mathcal{T}_3$

$\longrightarrow$ Experiment : C. Bäuerle, F. Mallet, L. Saminadayar & F. Schopfer

Weak localization & spectral determinant

Pascaud & Montambaux, (1999).

$\longrightarrow$ Improvement of the work of Douçot & Rammal, (1985).

$$\langle\Delta\sigma\rangle = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\int_{\mathrm{Network}} \mathrm{d}x\, P_c(x,x)$$

with $(\gamma - \mathrm{D}_x^2)P_c(x,x') = \delta(x-x')$

$\mathrm{D}_x = \frac{\mathrm{d}}{\mathrm{d}x} - 2\mathrm{i}eA(x)$

$\gamma = 1/L_\varphi^2$

$$\int \mathrm{d}x\, P_c(x,x) = \mathrm{Tr}\{\frac{1}{\gamma - \mathrm{D}_x^2}\} = \frac{\partial}{\partial\gamma}\mathrm{Tr}\{\ln(\gamma - \mathrm{D}_x^2)\} = \frac{\partial}{\partial\gamma}\ln\det(\gamma - \mathrm{D}_x^2)$$

Spectral determinant :

$$S(\gamma) \stackrel{\mathrm{def}}{=} \det(\gamma - \mathrm{D}_x^2)$$

$$\boxed{\langle\Delta\sigma\rangle = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)}$$

The spectral determinant for graphs

M. Pascaud & G. Montambaux, PRL **82** (1999)

Akkermans, Comtet, Desbois, Montambaux & C.T., Ann. Phys. **284** (2000)

$$S(\gamma) = \prod_{(\alpha\beta)} \frac{\sinh\sqrt{\gamma}l_{\alpha\beta}}{\sqrt{\gamma}} \det\mathcal{M}$$

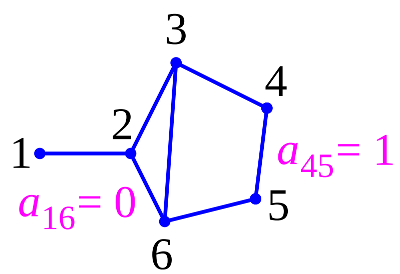
where

$$\mathcal{M}_{\alpha\beta} = \delta_{\alpha\beta}\left(\lambda_\alpha + \sqrt{\gamma}\sum_\mu a_{\alpha\mu}\coth(\sqrt{\gamma}l_{\alpha\mu})\right) - a_{\alpha\beta}\frac{\sqrt{\gamma}\,\mathrm{e}^{-\mathrm{i}\theta_{\alpha\beta}}}{\sinh(\sqrt{\gamma}l_{\alpha\beta})}$$

$a_{\alpha\beta}$: connectivity matrix

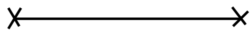
$a_{\alpha\beta} = 1$ if $(\alpha\beta)$ is a wire

$a_{\alpha\beta} = 0$ otherwise



$\lambda_\alpha = 0$ for an internal vertex

$\lambda_\alpha = \infty$ for a vertex connected to a reservoir.

Exercice : The connected wire 

$$\mathcal{M} = \begin{pmatrix} \lambda_1 + \sqrt{\gamma}\coth\sqrt{\gamma}L & -\frac{\sqrt{\gamma}}{\sinh\sqrt{\gamma}L} \\ -\frac{\sqrt{\gamma}}{\sinh\sqrt{\gamma}L} & \lambda_2 + \sqrt{\gamma}\coth\sqrt{\gamma}L \end{pmatrix}$$

Dirichlet $\Rightarrow \lambda_1 = \lambda_2 = \infty \Rightarrow \det\mathcal{M} \simeq \lambda_1\lambda_2 = \text{cste}$

$$S_{\text{wire}}(\gamma) = \frac{\sinh\sqrt{\gamma}L}{\sqrt{\gamma}}$$

$$\langle\Delta\sigma\rangle = -\frac{e^2}{\pi}\frac{1}{L}\frac{\partial}{\partial\gamma}\ln S(\gamma) = -\frac{e^2}{h}L_\varphi\left(\coth\frac{L}{L_\varphi} - \frac{L_\varphi}{L}\right)$$

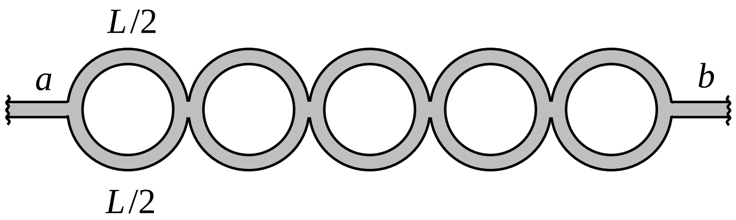
Exercice 2 : The isolated ring

$$S(\gamma) = 2(\cosh\sqrt{\gamma}L - \cos\theta)$$

$$\theta = 4\pi\phi/\phi_0$$

$$\langle\Delta\sigma\rangle = -\frac{e^2}{h}L_\varphi\frac{\sinh L/L_\varphi}{\cosh L/L_\varphi - \cos\theta}$$

Chain of N_r **symmetric** rings



$$g^{\rm cl} \simeq \frac{4\alpha_d N_c \ell_e}{N_r L}$$

- Weak localization : AAS oscillations

$$\langle \Delta g \rangle = -\frac{2}{(l_a + \frac{N_r}{4} L + l_b)^2} \left(\int_a P_c + \frac{1}{4} \int_{\rm rings} P_c + \int_b P_c \right)$$

$$\simeq -\frac{8}{(N_r L)^2} \frac{\partial}{\partial \gamma} \ln S_{\rm rings}(\gamma)$$

$$= -\frac{2L_\varphi}{N_r L} \left[\coth(L/2L_\varphi) - \frac{2L_\varphi}{L} + \frac{\sinh(L/2L_\varphi)}{\sqrt{\cosh^2(L/2L_\varphi) - \cos^2(\theta/2)}} \right]$$

Amplitude of AAS oscillations :

$$\boxed{\frac{\langle \Delta g_{\rm AAS} \rangle}{g^{\rm cl}} \simeq \frac{L_\varphi}{2\alpha_d N_c \ell_e} \mathrm{e}^{-L/L_\varphi}}$$

Measurement of L_φ in large networks

Direct measurement of L_φ in large GaAs/GaAlAs square networks

↘ One-parameter fit.

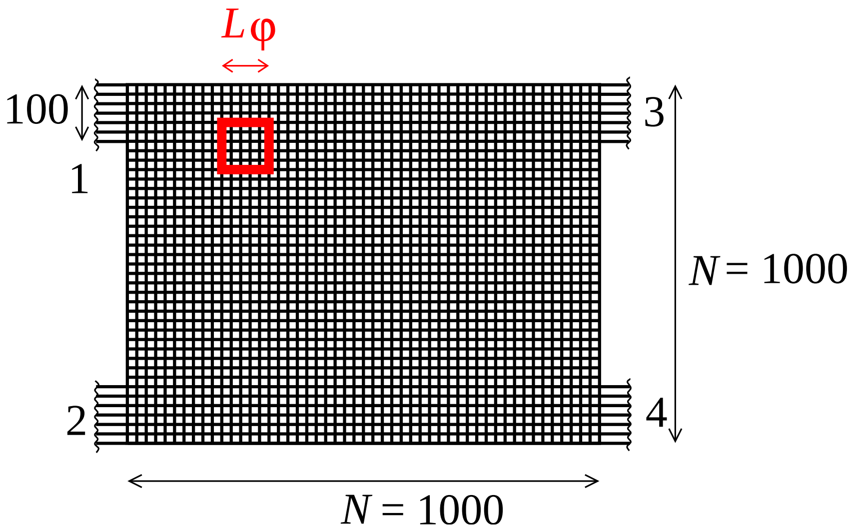
M. Ferrier, L. Angers, A. Rowe, S. Guéron & H. Bouchiat (2004).

Lattice spacing :

$a = 1\ \mu$m

Number of channels :

$N_c \sim 10$



→ Probe : Magnetic field

Analysis of large networks' Magnetoconductance

- Theory :

$$\Delta\sigma(\mathcal{B}, L_\varphi) = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)$$

$\downarrow$ AAS only

▷ $\mathcal{B} = 0$: Envelope of MC curve

$$\Delta\sigma(0, L_\varphi) = -\frac{e^2}{h}\frac{L_\varphi}{2}\left[\coth(\frac{a}{L_\varphi}) - \frac{L_\varphi}{a} + \frac{2}{\pi}\tanh(\frac{a}{L_\varphi})\underbrace{\mathrm{K}\left(\frac{1}{\cosh(a/L_\varphi)}\right)}_{\text{Elliptic integral}}\right]$$

dimensional crossover

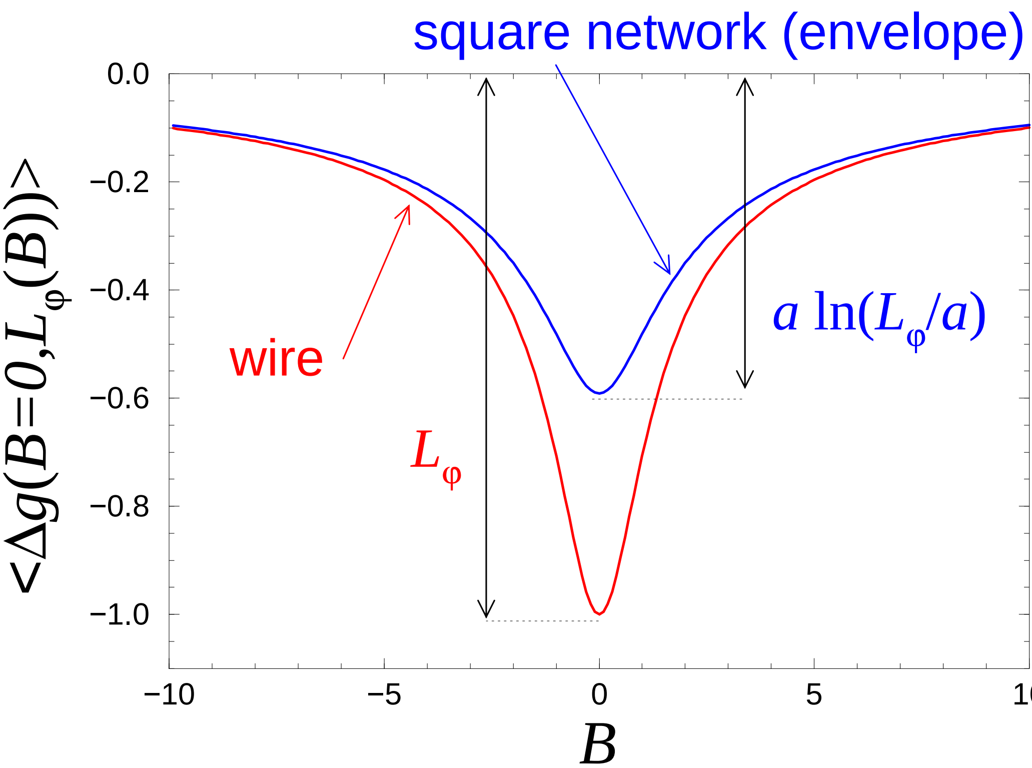
- limit $L_\varphi \ll a$

$$\Delta\sigma \simeq -\frac{e^2}{h}L_\varphi \qquad \longrightarrow \qquad \text{1d result}$$

- limit $a \ll L_\varphi$

$$\Delta\sigma \simeq -\frac{e^2}{\pi h}a\left[\ln(4L_\varphi/a) + \frac{\pi}{6}\right] \qquad \longrightarrow \qquad \text{2d result}$$

Envelope for $L_\varphi \gtrsim a$:



▷ $\boxed{\mathcal{B} \neq 0 : \text{ AAS oscillations}}$

- small L_φ (periodic orbit) expansion :

$$\langle \Delta\sigma \rangle = -\frac{e^2}{h}\frac{L_\varphi}{2}\Bigg[\ 2 - \frac{L_\varphi}{a} + \mathrm{e}^{-2a/L_\varphi} + \frac{7}{4}\,\mathrm{e}^{-4a/L_\varphi} + \cdots$$

$$+\frac{1}{2}\cos\theta\,\mathrm{e}^{-4a/L_\varphi}\left(1 - \frac{3}{2}\,\mathrm{e}^{-2a/L_\varphi} + \cdots\right)$$

$$+\frac{3}{8}\cos 2\theta\,\mathrm{e}^{-6a/L_\varphi}\left(1 - \frac{19}{12}\,\mathrm{e}^{-2a/L_\varphi} + \cdots\right)$$

$$+\frac{3}{8}\cos 3\theta\,\mathrm{e}^{-8a/L_\varphi}\left(1 - \frac{15}{8}\,\mathrm{e}^{-2a/L_\varphi} + \cdots\right)$$

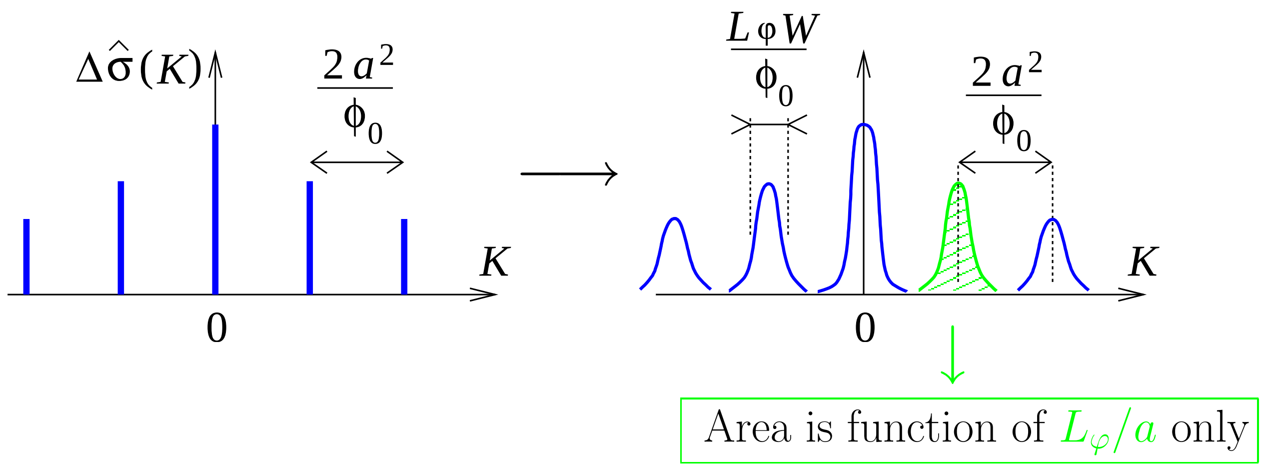
$$+\cdots\Bigg]$$

- arbitrary L_φ : numerics

▷ Penetration of $\mathcal{B}$ into the wires : $L_\varphi \longrightarrow L_\varphi^{\text{eff}}(\mathcal{B})$

$$\Delta\sigma(\mathcal{B}, L_\varphi) \quad \text{(AAS only)} \quad \longrightarrow \quad \Delta\sigma(\mathcal{B}, L_\varphi^{\text{eff}}(\mathcal{B}))$$

Fourier transform $\Downarrow$ $\quad \Delta\hat\sigma(K) = \int d\mathcal{B}\, e^{-i\mathcal{B}K}\, \Delta\sigma(\mathcal{B})$



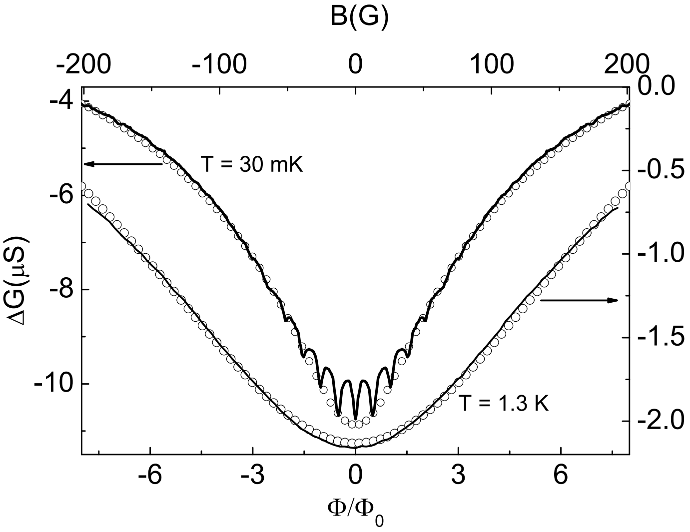
- Experimental fit :

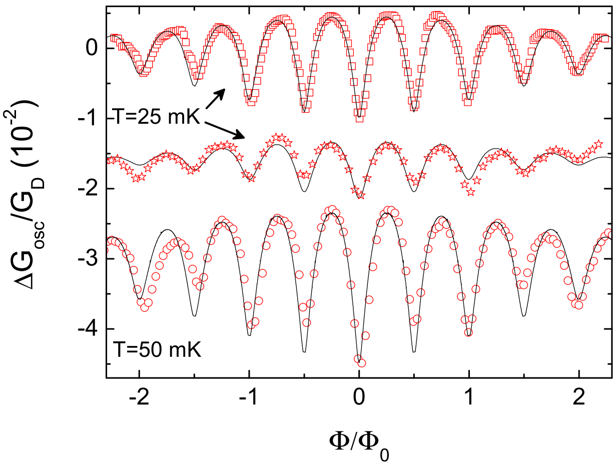
  Step 1 : Extract L_φ from ratio of AAS harmonics of $\Delta\sigma(\mathcal{B}, L_\varphi)$

  Step 2 : Extract W from fit of the envelope $\Delta\sigma(0, L_\varphi^{\text{eff}}(\mathcal{B}))$

- <u>Comparison Experiment-Theory :</u>

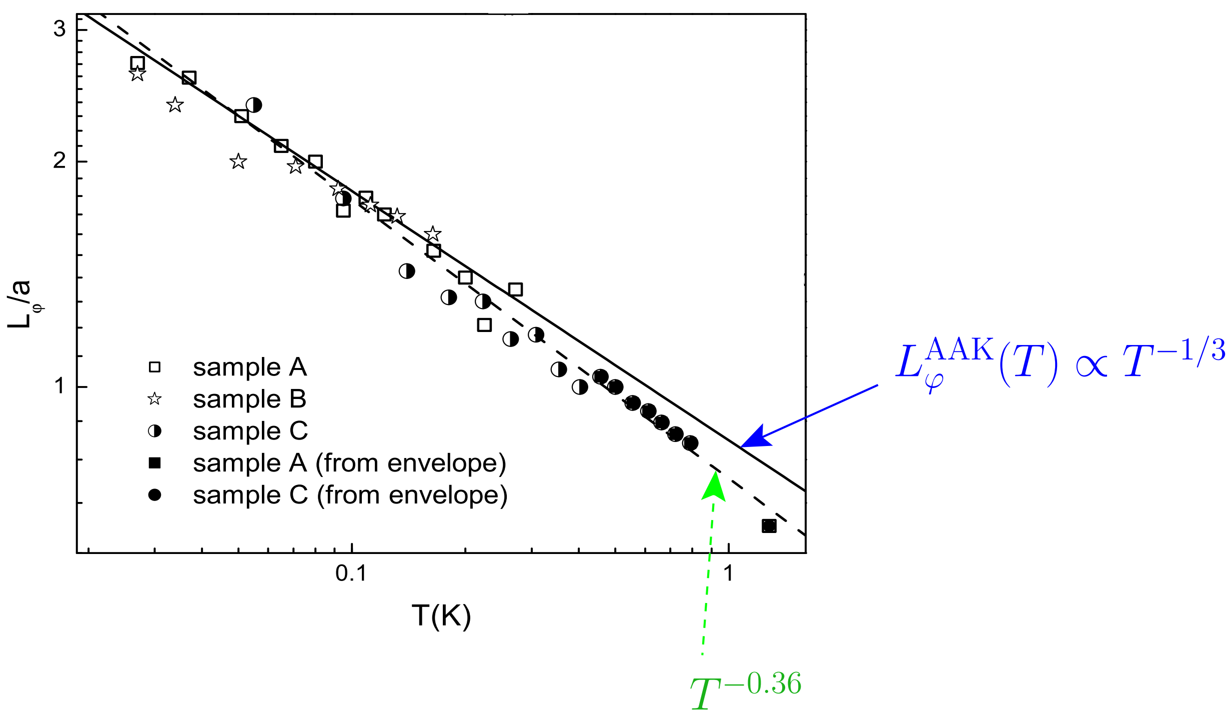
  → Meydi Ferrier's thesis (2004); Ferrier *et al* PRL **93** (2004)





- <u>Phase coherence length :</u>

  → Dominated by electron-electron interaction for $T \to 0$



→ No saturation down to 25 mK

4. Electron-electron interaction

Diffusive motion increases effect of interaction.

→ Lifetime of quasiparticle ? Still Fermi liquid ?

→ electron feels ⋆ disordered (impurity) potential

⋆ the potential created by other electrons

Hartee-Fock :

$$\delta\epsilon_n^{\mathrm{H}} = \int \mathrm{d}r\mathrm{d}r' \, |\phi_n(r)|^2 \, U(r-r') \, \overbrace{\sum_m f(\epsilon_m)|\phi_m(r')|^2}^{n(r')}$$

$$\delta\epsilon_n^{\mathrm{F}} = -\int \mathrm{d}r\mathrm{d}r' \, \phi_n^*(r)\,\phi_n(r')\,U(r-r')\,\sum_m f(\epsilon_m)\phi_m^*(r')\,\phi_m(r)$$

Average displacement of levels :

$$\Delta(\epsilon) = \frac{1}{\rho_0}\left\langle \sum_n \delta(\epsilon-\epsilon_n)\,\delta\epsilon_n \right\rangle$$

Dos correction :

$$\frac{\delta\rho(\epsilon)}{\rho_0} = -\frac{\partial\Delta(\epsilon)}{\partial\epsilon}$$

Al'tshuler-Aronov correction to the DoS

Hartree contribution :

$$\Delta^{\mathrm{H}}(\epsilon) \;=\; \frac{1}{\rho_0}\int \mathrm{d}r\mathrm{d}r'\, U(r-r')\left\langle \sum_{n,m}\delta(\epsilon-\epsilon_n)f(\epsilon_m)\,|\phi_n(r)|^2\,|\phi_m(r')|^2\right\rangle$$

$$= \frac{1}{\rho_0}\int \mathrm{d}\epsilon'\, f(\epsilon')\int \mathrm{d}r\mathrm{d}r'\, U(r-r')$$

$$\times\frac{1}{-4\pi^2}\left\langle \Delta G(r,r;\epsilon)\,\Delta G(r',r';\epsilon')\right\rangle$$

Fock contribution :

$$\Delta^{\mathrm{F}}(\epsilon) \;=\; -\int\cdots\cdots\left\langle \Delta G(r,r';\epsilon)\,\Delta G(r',r;\epsilon')\right\rangle$$

$$\langle G^{\mathrm{R}}G^{\mathrm{A}}\rangle \longrightarrow \text{Diffuson}$$

$\longrightarrow$ Screened interaction $U(r-r') = \frac{1}{2\rho_0}\delta(r-r')$

$$\boxed{\delta\rho(\epsilon) = -\frac{\lambda_\rho}{2\pi}\int_0^\infty \mathrm{d}t\,\frac{\pi T t}{\sinh \pi T t}\,\mathcal{P}(t)\,\cos\epsilon t}$$

$\mathcal{P}(t) = \int \frac{\mathrm{d}r}{\mathrm{Vol}}\mathcal{P}_d(r,r;t)$: diffuson

For $T \ll \epsilon$

$$\delta\rho(\epsilon) \simeq -\frac{\lambda_\rho}{2\pi}\int_0^\infty \mathrm{d}t\,\mathcal{P}(t)\,\cos\epsilon t = -\frac{\lambda_\rho}{2\pi}\,\mathrm{Re}\left[\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)\Big|_{\gamma=\mathrm{i}D\epsilon}\right]$$

$\boxed{d = 1:\ \text{The wire}}$

$$\delta\rho(\epsilon) \simeq -\frac{\lambda_\rho}{4\pi}\frac{1}{\sqrt{2D\epsilon}}$$

$$\delta\rho(\epsilon) \sim -1/\sqrt{T}\ \text{for}\ \epsilon \ll T$$

$\boxed{d = 2:\ \text{The plane}}$

$$\delta\rho(\epsilon) \simeq \frac{\lambda_\rho}{8\pi^2 D}\ln\epsilon\tau_e$$

$\boxed{\text{Coulomb dip}}$

Electrodynamic Dip in the Local Density of States of a Metallic Wire

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(Received 15 January 1999; revised manuscript received 3 February 2000)

We have measured the differential conductance of a tunnel junction between a thin metallic wire and a thick ground plane, as a function of the applied voltage. We find that near zero voltage, the differential conductance exhibits a dip, which scales as $1/\sqrt{V}$ down to voltages $V \sim 10k_BT/e$. The precise voltage and temperature dependence of the differential conductance is accounted for by the effect on the tunneling density of states of the macroscopic electrodynamics contribution to electron-electron interaction, and not by the short-ranged screened-Coulomb repulsion at microscopic scales.

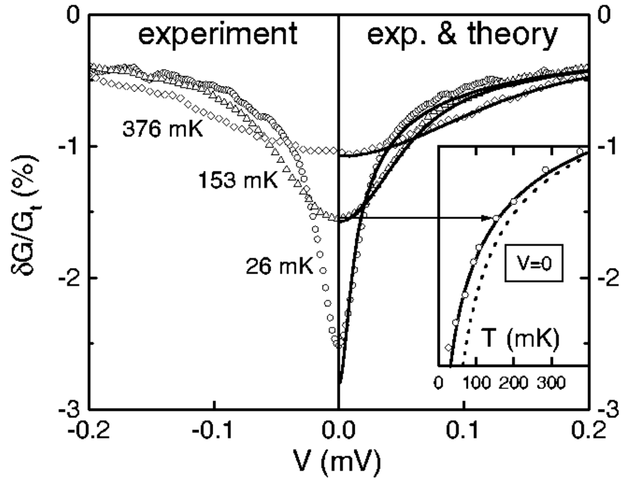

FIG. 3. Symbols in main panel: same experiment as in Fig. 2, but with data near $V = 0$ plotted on linear scale. Solid lines: Predictions for our finite length wire. Inset: $V = 0$ differential conductance. Solid line: Prediction for our finite length wire. Dotted line: $T^{-1/2}$ dependence expected for an infinite wire.

Al'tshuler-Aronov correction to σ

From Einstein relation $\sigma = e^2\rho_0 D$:

$$\frac{\delta\sigma(T)}{\sigma_0} = \int \mathrm{d}\epsilon \left(-\frac{\partial f(\epsilon)}{\partial\epsilon}\right) \frac{\delta\rho(\epsilon)}{\rho_0}$$

$$\boxed{\Delta\sigma_{\mathrm{ee}}(T) = -\lambda_\sigma \frac{e^2 D}{\pi} \int_0^\infty \mathrm{d}t \left(\frac{\pi T t}{\sinh \pi T t}\right)^2 \mathcal{P}(t)}$$

Incoherent contribution

Insensitive to a $\mathcal{B}$ field

In a wire :

$$\boxed{\Delta g_{\mathrm{ee}} \simeq -0.782\,\lambda_\sigma \frac{L_T}{L} \propto -\frac{1}{\sqrt{T}}}$$

Interest : Local probe of the temperature.

Evidence for Interaction Effects in the Low-Temperature Resistance Rise in Ultrathin Metallic Wires

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and

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(Received 12 April 1982)

New measurements are reported of the low-temperature resistance rise in ultrathin wires of Cu, Ni, and AuPd, which confirm the proportionality to $T^{-1/2}$ predicted by the interaction model. Moreover, these results and those in the literature show an absolute magnitude consistent within a factor of ~2 with the predictions of this model, using independently determined parameters of similar accuracy. It is inferred that interaction effects are at least as important as localization effects in these systems.

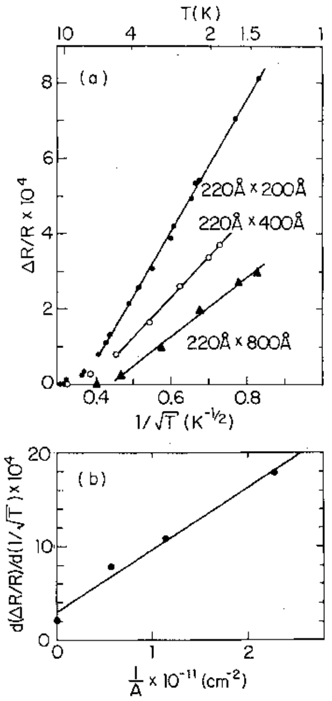

FIG. 2. (a) Data of Fig. 1, plotted vs $1/\sqrt{T}$ to isolate the interaction effect. Only the low-temperature points, where the T^3 term is negligible, are fitted. (b) Plot of slopes found in (a) vs $1/A$ to separate the one-dimensional effect from any bulk effect also giving a resistance rise at low temperatures. The point at $1/A = 0$ is an *upper bound* set by the experimental accuracy.

Temperature dependence of the resistance of one-dimensional metal films with dominant Nyquist phase breaking

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(Received 15 October 1993; revised manuscript received 28 March 1994)

We report the results of a comprehensive study of the temperature dependences of the resistance of one-dimensional narrow gold films with respect to the effects of weak localization (WL) and electron-electron interaction (EEI) in a wide temperature range. The electron wave-function coherence for such samples is limited by electron-electron collisions with small energy transfer (the Nyquist phase-breaking mechanism). It is shown that the temperature dependence of the WL contribution to the resistance, obtained for such a case, is in excellent agreement with the theory by Altshuler, Aronov, and Khmelnitskii. The experimental dependences $\Delta R(T)$ for one-dimensional samples are described quantitatively by the sum of the WL contribution, the contribution of the singlet part of the diffusion channel of EEI, and the contribution of the quantum interference between electron-phonon and electron-impurity scattering.

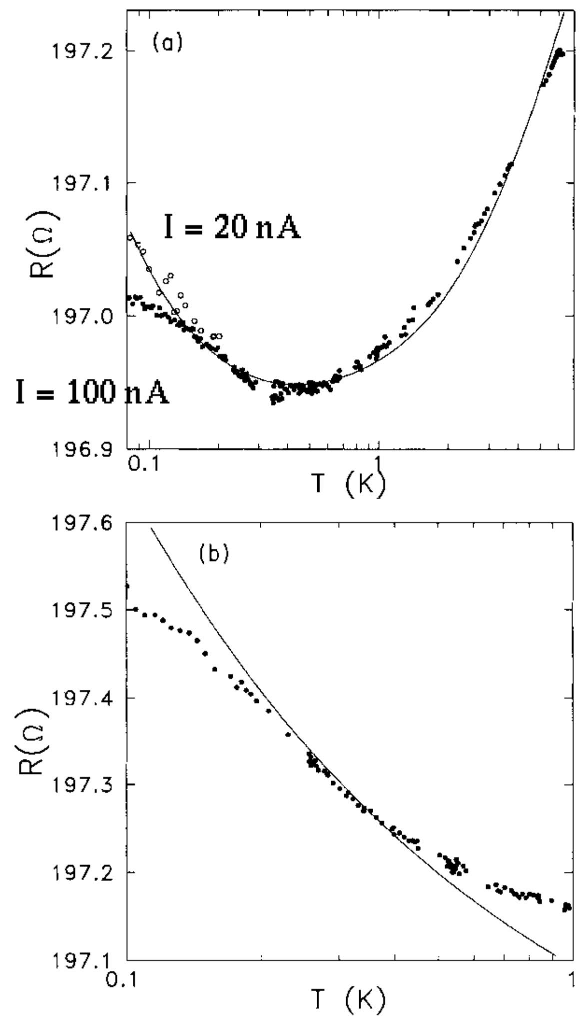
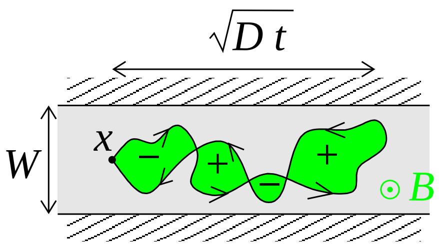


FIG. 1. Temperature dependences of the resistance of sample No. 1 at $H=0$ (a) and 100 Oe (b). The solid line in (a) is the theoretical dependence [Eq. (5)] calculated from Eqs. (2), (3), and (4). In (a) the full circles are measurements with measuring current 100 nA and open circles 20 nA. The solid line in (b) is the theoretical prediction for the EEI contribution [Eq. (3)].

5. Coherence

Dephasing (1) : magnetic field

Penetration of the $\mathcal{B}$ field in the wires



$$L_\varphi \longrightarrow L_\varphi^{\text{eff}}(\mathcal{B})$$

$$\frac{1}{L_\varphi^{\text{eff}}(\mathcal{B})^2} = \frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2} = \frac{1}{L_\varphi^2} + \frac{1}{3}\left(\frac{e\mathcal{B}W}{\hbar}\right)^2$$

Al'tshuler & Aronov, 1981.

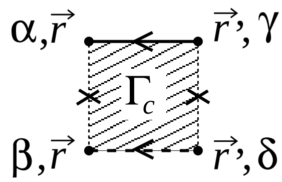
$$\langle \Delta g \rangle = -\frac{1}{L}\left(\frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2}\right)^{-1/2}$$

Dephasing (2) : spin-orbit scattering

WL : $\longrightarrow$ Hikami, Larkin & Nagaoka, Prog. Theor. Phys. (1980)

UCF : $\longrightarrow$ Chandrasekhar, Santhanam & Prober, PRB (1990)

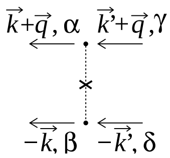
$\longrightarrow$ Interaction vertex on disorder depends on spin indices :



$w_{\rm so} \longrightarrow L_{\rm so}$: spin orbit

$w_{\rm m} \longrightarrow L_{\rm m}$: spin flip on localized (static) magnetic impurities

In the Bethe-Salpether equation : vertex for cooperon (for WL) is now



$$\longrightarrow b^{(c)}_{\alpha\beta,\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta} + \frac{-w_{\rm so}+w_{\rm m}}{3w}\,\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}$$

In the basis $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$:

$$b^{(c)} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} + \frac{-w_{\rm so}+w_{\rm m}}{3w}\begin{pmatrix} 1 & & & \\ & -1 & 2 & \\ & 2 & -1 & \\ & & & 1 \end{pmatrix}$$

Bethe-Salpether equation is now (in Fourier space)

$$\Gamma_c(\vec{q}) = w\, b^{(c)} + \frac{w}{w_{\text{tot}}} \left(1 - D\tau_{\text{tot}}\, \vec{q}^{\,2}\right) b^{(c)}\, \Gamma_c(\vec{q})$$

with $w_{\text{tot}} = w + w_{\text{so}} + w_{\text{m}}$

$$\Gamma_c(\vec{q}) = \frac{w}{[b^{(c)}]^{-1} - \frac{w}{w_{\text{tot}}}\,(1 - D\tau_{\text{tot}}\, \vec{q}^{\,2})}$$

is a matrix

$\longrightarrow$ Diagonalization of $b^{(c)}$: Singlet and Triplet channels decouple.

$$P_c(\vec{q}) = \underbrace{\Pi_0}_{\text{projector on singlet}} \frac{1}{1/L_S^2 + \vec{q}^{\,2}} + (1 - \Pi_0)\,\frac{1}{1/L_T^2 + \vec{q}^{\,2}}$$

for $w \gg w_{\text{so}},\, w_{\text{m}}$:

$$\frac{1}{L_S^2} = \frac{2}{L_{\text{m}}^2} \quad \text{and} \quad \frac{1}{L_T^2} = \frac{4}{3L_{\text{so}}^2} + \frac{2}{3L_{\text{m}}^2}$$

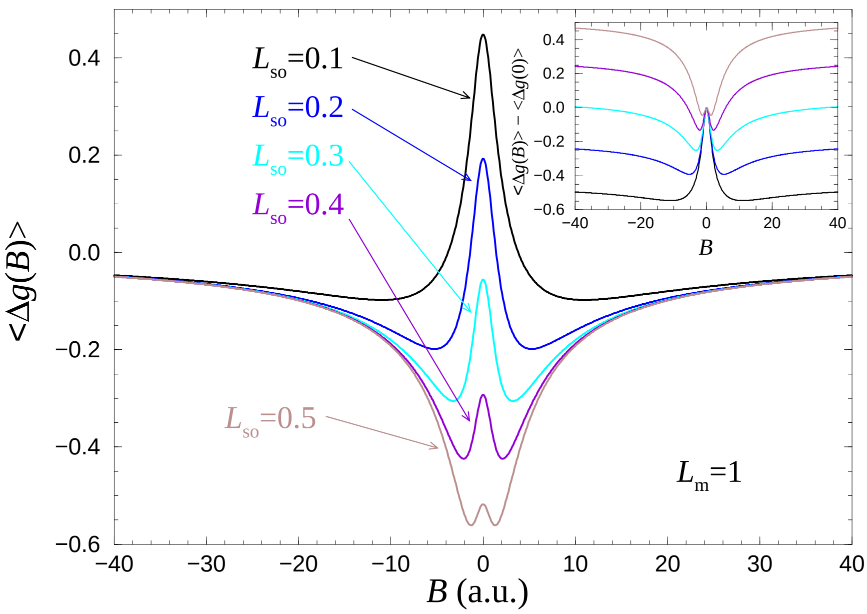
Reversing of trajectories $\Rightarrow$ reversing the singlet $\Rightarrow$ $\boxed{-}$

$$\langle \Delta\sigma \rangle = (\text{singlet } > 0) + (\text{triplet } < 0)$$

Weak antilocalization

Infinite wire

$$\langle \Delta g(\mathcal{B}) \rangle = \frac{1}{L}\Bigg[+ \overbrace{\left(\frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2} + \frac{2}{L_\mathrm{m}^2} \right)^{-1/2}}^{\text{singlet}} - \underbrace{3\left(\frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2} + \frac{4}{3L_\mathrm{so}^2} + \frac{2}{3L_\mathrm{m}^2} \right)^{-1/2}}_{\text{triplet}} \Bigg]$$



Magnesium ($Z = 12$) : weak spin-orbit

Gold ($Z = 79$: heavy metal) : strong spin-orbit

PHYSICAL REVIEW B VOLUME 28, NUMBER 2 15 JULY 1983

Consistent temperature and field dependence in weak localization

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(Received 25 February 1983)

The temperature and magnetic field dependence of the resistivity of thin Mg and Au films are measured. The parameters of weak localization such as the inelastic lifetime $\tau_i(T)$ and the spin-orbit (so) coupling time τ_{so} are evaluated. The Mg film which has a small spin-orbit coupling can be changed into a strong spin-orbit coupler by covering it with 0.25 monolayers of Au. Since the inelastic lifetime is not affected by the small amount of Au only one parameter is changed. This does not only alter the magnetoresistance but also the temperature dependence of the film resistance. The whole set of magnetoresistance curves is well described by the theory. The change of the spin-orbit coupling essentially allows one to separate the temperature-dependent resistance caused by weak localization from other temperature-dependent contributions. It is consistent with the theory.

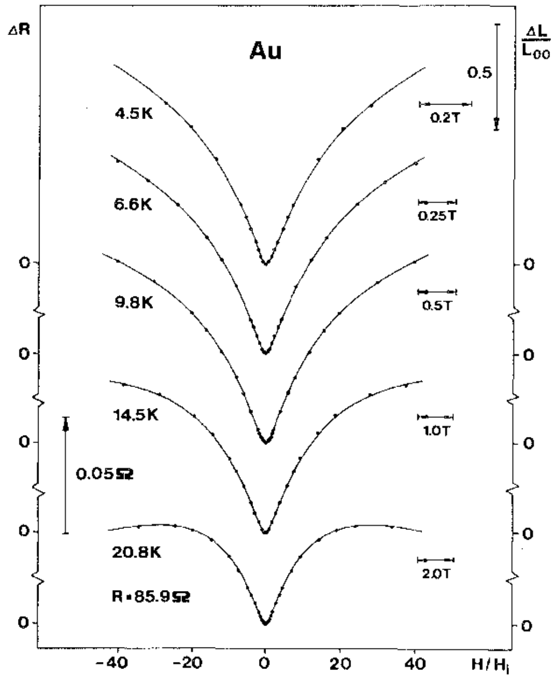


FIG. 4. Magnetoresistance (or $[L(H)-L(0)]/L_{00}$) of a Au film as a function of the field H measured in units of the inelastic field $H_i(T)$. The points represent the experimental results. The solid curves are calculated using the characteristic fields plotted in Fig. 5.

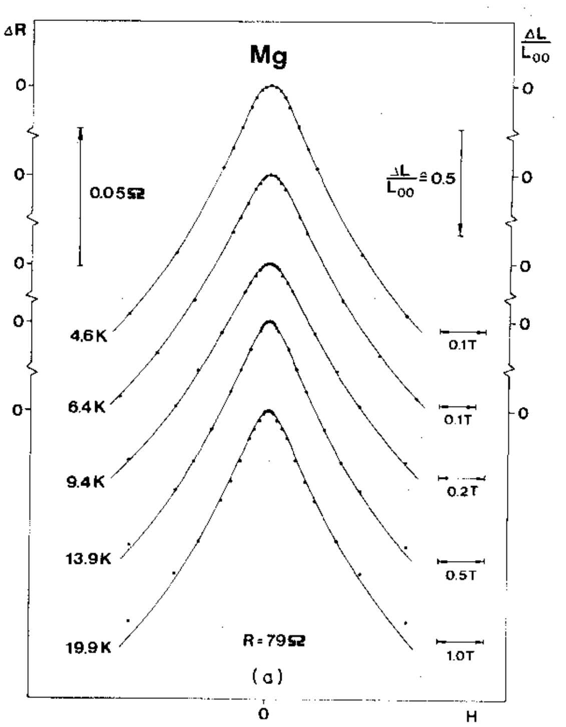


FIG. 2. (a) Magnetoresistance (i.e., $[L(H)-L(0)]/L_{00}$ using the right scale) of a Mg film ($d=8.4$ nm) as a function of the field H. The units of the field are shown beside each magnetoresistance curve. The points represent the experimental results. The solid curves are calculated using the characteristic fields $H_i(T)$ plotted in Fig. 5 and $H_{so}=0.0046$ T.

Decoherence due to electron-phonon interaction

→ Chakravarty & Schmid, 1986.

→ Lin & Bird, J. Phys. C (2002).

$$\boxed{\tau_{\mathrm{e-ph}} \propto T^{-3}}$$

→ negligible below 1 K

Decoherence due to electron-electron interaction

Let us come back on the simple modelization used up to now

A. "Exponential relaxation" : Effective parameter $L_\varphi = \sqrt{D\tau_\varphi}$

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t\, \mathcal{P}_c(r,r;t)\, \mathrm{e}^{-t/\tau_\varphi}$$

$$\text{where } \left(\tfrac{\partial}{\partial t} - \Delta_r\right) \mathcal{P}_c(r,r';t) = \delta(t)\,\delta(r-r')$$

- Conductance of a long wire : Al'tshuler & Aronov, 1981.

$$\langle \Delta g \rangle = -\frac{1}{L}\int_0^\infty \frac{\mathrm{d}t}{\sqrt{\pi t}}\, \mathrm{e}^{-t/\tau_\varphi}\, \mathrm{e}^{-t/\tau_\mathcal{B}} = -\frac{1}{L}\left(\frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2}\right)^{-1/2}$$

$$\text{where } \tfrac{1}{L_\mathcal{B}^2} = \tfrac{1}{3}\left(\tfrac{e\mathcal{B}W}{\hbar}\right)^2$$

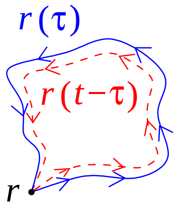
- Exp. relax. describes → Penetration of $\mathcal{B}$ field
  → Spin-orbit scattering
  → Spin-flip (magnetic impurities)

- Does not describe correctly e-e interaction

B. Electron in a fluctuating potential $V(r,t)$:

Al'tshuler, Aronov & Khmel'nitzkiĭ (AAK), J. Phys. C **15** (1982).

→ Electron feels the random electromagnetic field created by other electrons

The loop 

receives a random phase $\mathrm{e}^{\mathrm{i}\Phi}$

$$\Phi[\vec{r}(\tau)] = \int_0^t \mathrm{d}\tau \left[V(\vec{r}(\tau),\tau) - V(\vec{r}(\tau), t-\tau) \right]$$

where $V(r,t)$ is the electric potential due to other electrons

$$\langle \Delta\sigma \rangle \sim \text{Cooperon} \sim \sum_{\text{loops } \mathcal{C}} \langle \mathrm{e}^{\mathrm{i}\Phi[\mathcal{C}]} \rangle_V$$

- Fluctuation-dissipation theorem :

$$\langle V(\vec{r},t)\,V(\vec{r}\,',t')\rangle_V = \frac{2e^2}{\sigma_0} T\,\delta(t-t')\,P_d(\vec{r},\vec{r}\,')$$

Nyquist length : $\boxed{L_N = \left(\frac{\sigma_0 S D}{e^2 T}\right)^{1/3}} = \left(\frac{\alpha_d}{\pi} N_c \ell_e L_T^2\right)^{1/3}$

$$L_T = \sqrt{D/T}.$$

L_N characterizes the efficiency of e-e interaction

Exercice : a gold wire

$S = 50\,\mathrm{nm}\times 50\,\mathrm{nm}$

$\ell_e = 30\,\mathrm{nm}$

$N_c = 27500$

$$L_N(T) \simeq 3.2\,\mu\mathrm{m} \times T^{-1/3}$$

- Phase averaged over potential fluctuations :

$$\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_V = \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_V}$$

where

$$\frac{1}{2}\langle\Phi^2\rangle_V = \frac{2}{L_N^3}\int_0^t \mathrm{d}\tau\, W(x(\tau), x(\bar{\tau}))$$

with

$$W(x,x') = \frac{1}{2}[P_d(x,x) + P_d(x',x')] - P_d(x,x')$$

$$\langle \Delta\sigma \rangle \sim \text{Cooperon} \sim \sum_{\text{loops } \mathcal{C} < L_\varphi} \langle \mathrm{e}^{\mathrm{i}\Phi[\mathcal{C}]} \rangle_V$$

- Path integral formulation :

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi} \int_0^\infty \mathrm{d}t \; \overbrace{\mathrm{e}^{-t/\tau_\varphi}}^{\text{exp. relax.}}$$

$$\times \underbrace{\int_{x(0)=x}^{x(t)=x} \mathcal{D}x(\tau)\, \mathrm{e}^{-\frac{1}{4}\int_0^t \mathrm{d}\tau\, \dot{x}^2}}_{\text{diffusion}} \; \underbrace{\mathrm{e}^{-\frac{2}{L_N^3}\int_0^t \mathrm{d}\tau\, W(x(\tau),x(\bar{\tau}))}}_{\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_V\text{: e-e interaction}}$$

$\longrightarrow$ Electron-electron interaction enters through a complicate functional.

This functional is network dependent

In general $\Delta\sigma(L_\varphi, \infty)$ and $\Delta\sigma(\infty, L_N)$ do not coincide

INFINITE WIRE

$$P_d(x,x') = -\tfrac{1}{2}|x-x'| \Rightarrow W(x,x') = \tfrac{1}{2}|x-x'|$$

$$W(x,x') = f(x-x') \Rightarrow \text{get rid of time nonlocality}$$

Cooperon is solution of :

$$\left[\frac{1}{L_\varphi^2} - \frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{L_N^3}|x|\right] P_c(x,x') = \delta(x-x')$$

$$\langle \Delta\sigma \rangle = -\frac{2e^2}{\pi}\, P_c(0,0)$$

One finds (AAK, 1982)

$$\langle \Delta g \rangle = \frac{L_N}{L}\,\frac{\mathrm{Ai}(L_N^2/L_\varphi^2)}{\mathrm{Ai}'(L_N^2/L_\varphi^2)} \simeq -\frac{1}{L}\left(\frac{1}{2L_N^2} + \frac{1}{L_\varphi^2}\right)^{-1/2}$$

A (exp. relax.) $\longrightarrow$ B (AAK)

$$L_\varphi \longrightarrow \sqrt{2}L_N = L_\varphi^{\mathrm{AAK}}$$

One-Dimensional Conduction in the 2D Electron Gas of a GaAs-AlGaAs Heterojunction

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(Received 17 September 1985)

We present results on the transport properties of the 2D electron gas in a narrow channel formed by the split gate of a GaAs-AlGaAs heterojunction field-effect transistor. There are both quantum-interference and interaction corrections to the conductivity. We find that the temperature dependence of the phase relaxation length is in agreement with a recent theory based on scattering by electromagnetic fluctuations. Beyond the regime of quantum interference the conductivity varies with temperature as T^2.

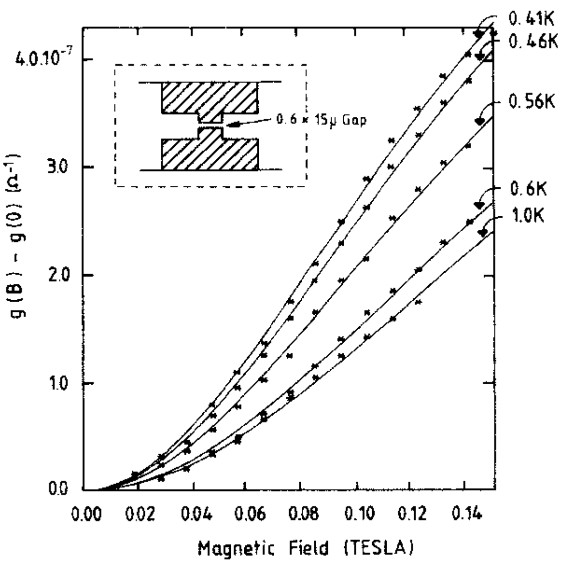

FIG. 1. The values of conductance as a function of magnetic field, indicated by crosses. The lines indicate the best fit of Eq. (2) at each temperature. Inset: The gate defining the narrow channel in the underlying heterojunction.

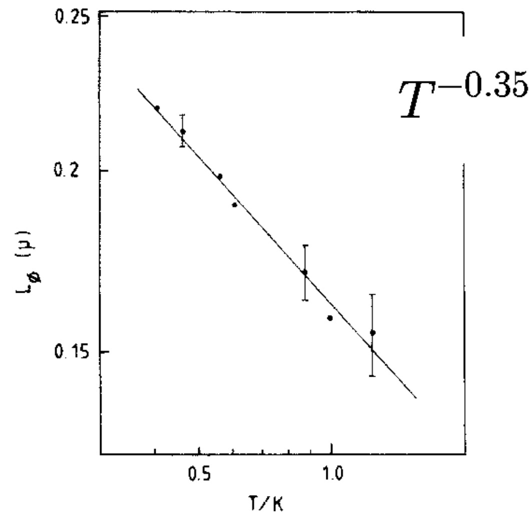

FIG. 2. The phase relaxation length plotted against temperature on a log-log scale.

One-Dimensional Electron-Electron Scattering with Small Energy Transfers

S. Wind,[a] M. J. Rooks, V. Chandrasekhar, and D. E. Prober
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(Received 28 March 1986)

We report magnetoresistance studies of Al and Ag wires of width 35 to 110 nm which probe the electron phase-breaking rate. We find that this rate at low temperatures is determined by one-dimensional electron-electron scattering with small energy transfers. This confirms the importance of this mechanism for electron energy loss in one-dimensional systems, as suggested by Al'tshuler *et al.*, and defines clearly the relevant dimensional length scales.

$$\tau_{ee}^{-1}=\left[\frac{R_{\square}}{\sqrt{2}(\hbar/e^2)}\left(\frac{k_B}{\hbar}\right)\frac{\sqrt{D}}{W}\right]^{2/3}T^{2/3}. \qquad (1)$$

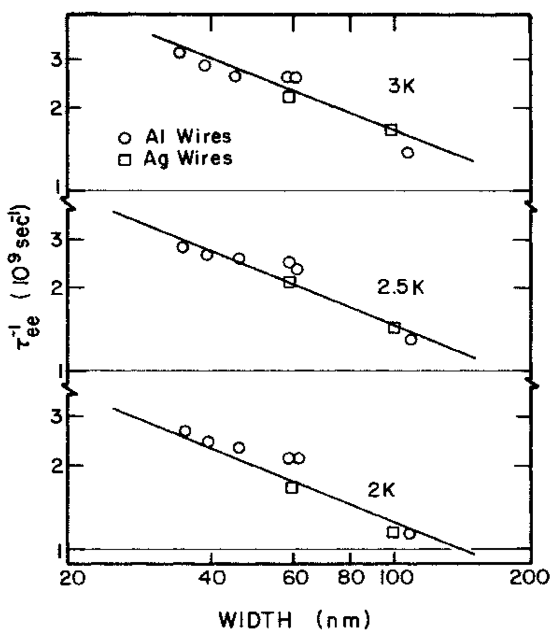

FIG. 3. Electron-electron contribution to $\tau_{\phi}^{-1}=$ [(total phase-breaking rate) – (electron-phonon rate $=A_{ep}T^3$)] as a function of wire width. The solid lines give the theoretical prediction of Eq. (1). The data are normalized to the $R_{\square}$ and D of samples Al2, according to Eq. (1).

PHYSICAL REVIEW B **68**, 085413 (2003)

Dephasing of electrons in mesoscopic metal wires

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[2]*Service de Physique de l'Etat Condensé, Direction des Sciences de la Matière, CEA-Saclay, 91191 Gif-sur-Yvette, France*
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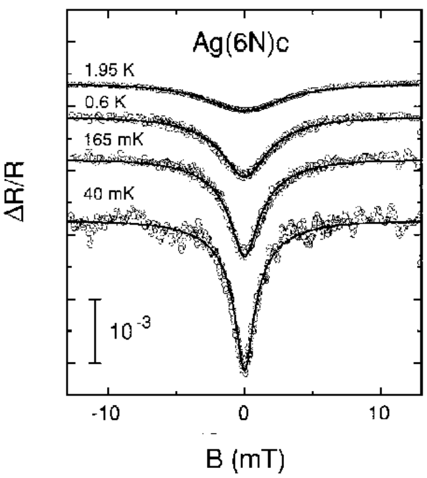
(Received 11 February 2003; published 26 August 2003)

We have extracted the phase coherence time τ_ϕ of electronic quasiparticles from the low field magnetoresistance of weakly disordered wires made of silver, copper, and gold. In samples fabricated using our purest silver and gold sources, τ_ϕ increases as $T^{-2/3}$ when the temperature T is reduced, as predicted by the theory of electron–electron interactions in diffusive wires. In contrast, samples made of a silver source material of lesser purity or of copper exhibit an apparent saturation of τ_ϕ starting between 0.1 and 1 K down to our base temperature of 40 mK. By implanting manganese impurities in silver wires, we show that even a minute concentration of magnetic impurities having a small Kondo temperature can lead to a quasisaturation of τ_ϕ over a broad temperature range, while the resistance increase expected from the Kondo effect remains hidden by a large background. We also measured the conductance of Aharonov–Bohm rings fabricated using a very pure copper source and found that the amplitude of the h/e conductance oscillations increases strongly with magnetic field. This set of experiments suggests that the frequently observed "saturation" of τ_ϕ in weakly disordered metallic thin films can be attributed to spin–flip scattering from extremely dilute magnetic impurities, at a level undetectable by other means.



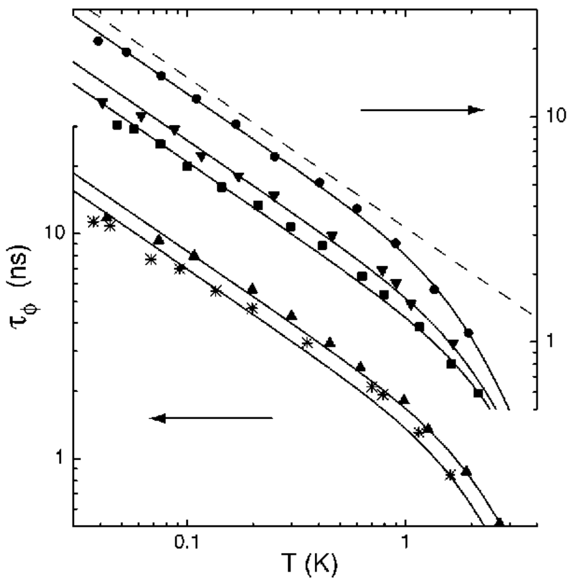


FIG. 4. Phase coherence time vs temperature in samples Ag(6N)a (■), Ag(6N)b (▼), Ag(6N)c (●), Ag(6N)d (▲), and Au(6N) (*), all made of 6N sources. Continuous lines are fits of the data to Eq. (4). For clarity, the graph has been split in two part, shifted vertically one with respect to the other. The quantitative prediction of Eq. (3) for electron–electron interactions in sample Ag(6N)c is shown as a dashed line.

Time analysis : relaxation of phase coherence

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t\, \mathcal{P}_c(r,r;t)\, \underbrace{\mathrm{e}^{-t/\tau_\varphi}}$$

↓

What replaces the exponential ?

- AAK : non exponential phase coherence relaxation

G. Montambaux & É. Akkermans, PRL **95** (2005)

→ Inverse Laplace transform of AAK's result

$$\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}} \simeq 1 - \frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2} \qquad \text{for } t \ll \tau_N$$

$$\simeq \sqrt{\frac{\pi t}{\tau_N}}\,\frac{1}{|u_1|}\,\exp - |u_1|\frac{t}{\tau_N} \qquad \text{for } \tau_N \ll t$$

($|u_1| \simeq 1.019$ is the first zero of $\mathrm{Ai}'(z)$).

Nonexponential Quasiparticle Decay and Phase Relaxation in Low-Dimensional Conductors

G. Montambaux[1] and E. Akkermans[2]
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[2]Department of Physics, Technion Israel Institute of Technology, 32000 Haifa, Israel
(Received 15 April 2004; revised manuscript received 29 July 2004; published 29 June 2005)

We show that in low-dimensional disordered conductors, the quasiparticle decay and the relaxation of the phase are not exponential processes. In the quasi-one-dimensional case, both behave at small time as $e^{-(t/\tau_{in})^{3/2}}$ where the inelastic time, τ_{in}, identical for both processes, is a power $T^{-2/3}$ of the temperature. The nonexponential quasiparticle decay results from a modified derivation of the Fermi golden rule. This result implies the existence of an unusual distribution of relaxation times.

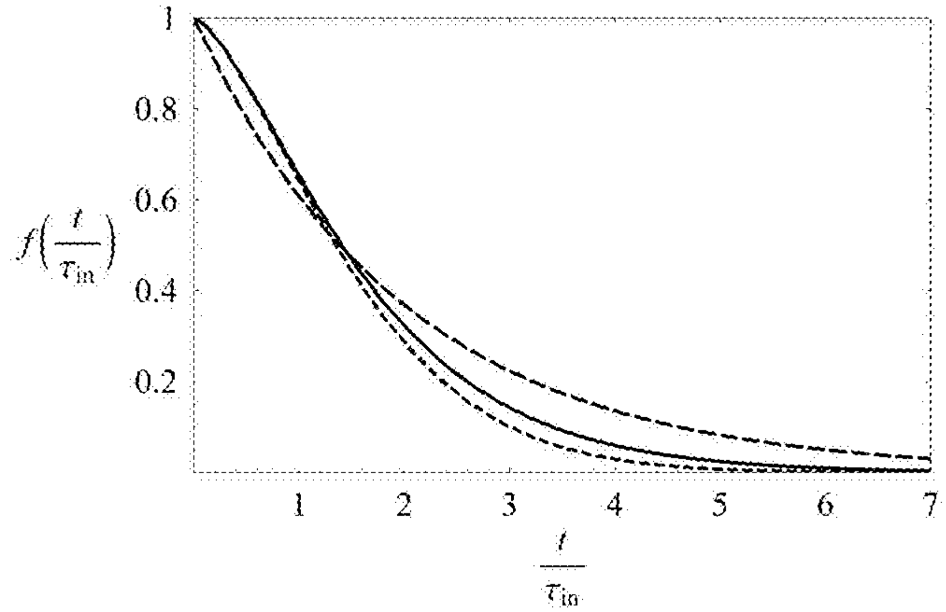
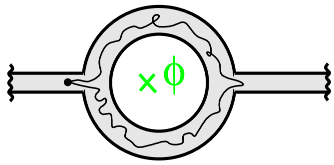


FIG. 2. Behavior of $\langle e^{i\Phi(t)}\rangle_{T,C}$. The continuous line is the exact result (20). The dotted line is obtained from the small time expansion (17). The dashed line shows the exponential fit $e^{-t/2\tau_{in}}$.

MC harmonics of a ring



$\longrightarrow$ Magnetoconductance's harmonics

$$\Delta g_n = \int_0^{\phi_0/2} \frac{\mathrm{d}\phi}{\phi_0/2}\, \mathrm{e}^{-4\mathrm{i}\pi n\phi/\phi_0}\, \Delta g(\phi)$$

A (exp. relaxation) gives $\Delta g_n \propto \exp -|n|\frac{L}{L_\varphi}$

Question is : Does B give $\Delta g_n \propto \exp -|n|\frac{L}{L_N}$?

Answer is No

Ludwig & Mirlin (LM), PRB (2004) :

$$\Delta g_n \propto \exp -|n| \left(\frac{L}{L_N}\right)^{3/2} \qquad \text{for } L \gg L_N$$

How to characterize phase coherence relaxation in the ring ?

- Harmonic n of WL can be written

$$\Delta\sigma_n = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t \quad \underbrace{\mathcal{P}_n(x,x;t)}_{\text{Proba to wind } n \text{ times}} \quad \langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n}$$

$\mathcal{C}_n$: diffusive trajectories with winding n

$\Rightarrow$ We analyze

$$\boxed{\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n} = \langle \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_V}\rangle_{\mathcal{C}_n}}$$

Why phase coherence relaxation is different for $n = 0$ and $n \neq 0$?

- Diffusion of the phase : Johnson-Nyquist

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi = V$$

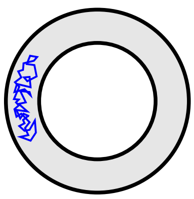
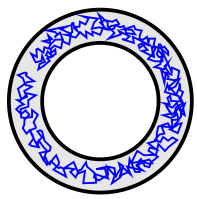
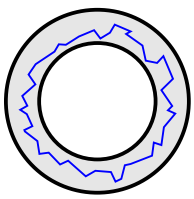
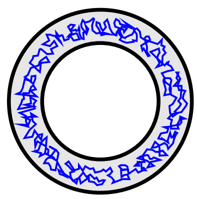
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\Phi^2\rangle_V = \int \mathrm{d}\tau \langle V(\tau)V(0)\rangle_V = 2e^2 T R_t \sim e^2 T \frac{r(t)}{\sigma_0 S} = \frac{r(t)}{\sqrt{D}\,\tau_N^{3/2}}$$

Small time $t \ll \tau_D$ $\Rightarrow r(t) \sim \sqrt{Dt}$ $\qquad \langle\Phi^2\rangle_V \approx (\frac{t}{\tau_N})^{3/2}$

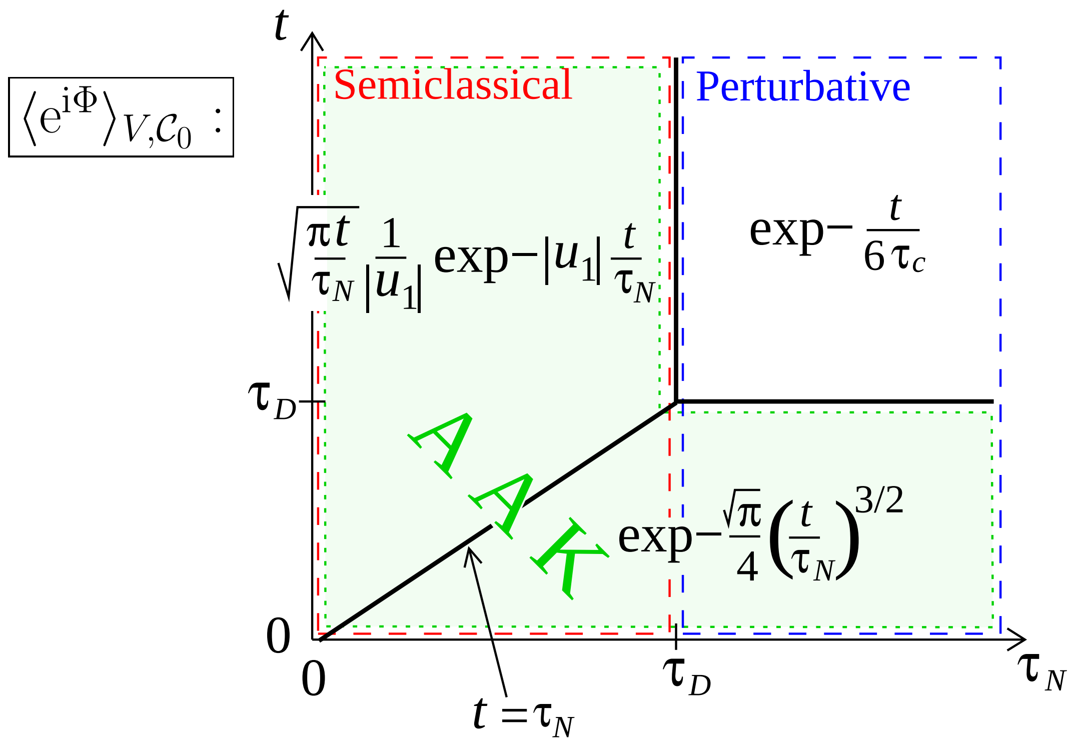
Long time $t \gg \tau_D$ $\Rightarrow r(t) \sim L$ $\qquad \langle\Phi^2\rangle_V \approx \frac{\sqrt{\tau_D}}{\tau_N^{3/2}}\, t = \frac{t}{\tau_c}$

New length scale $L_c = L_N^{3/2}/L^{1/2} \propto T^{-1/2}$

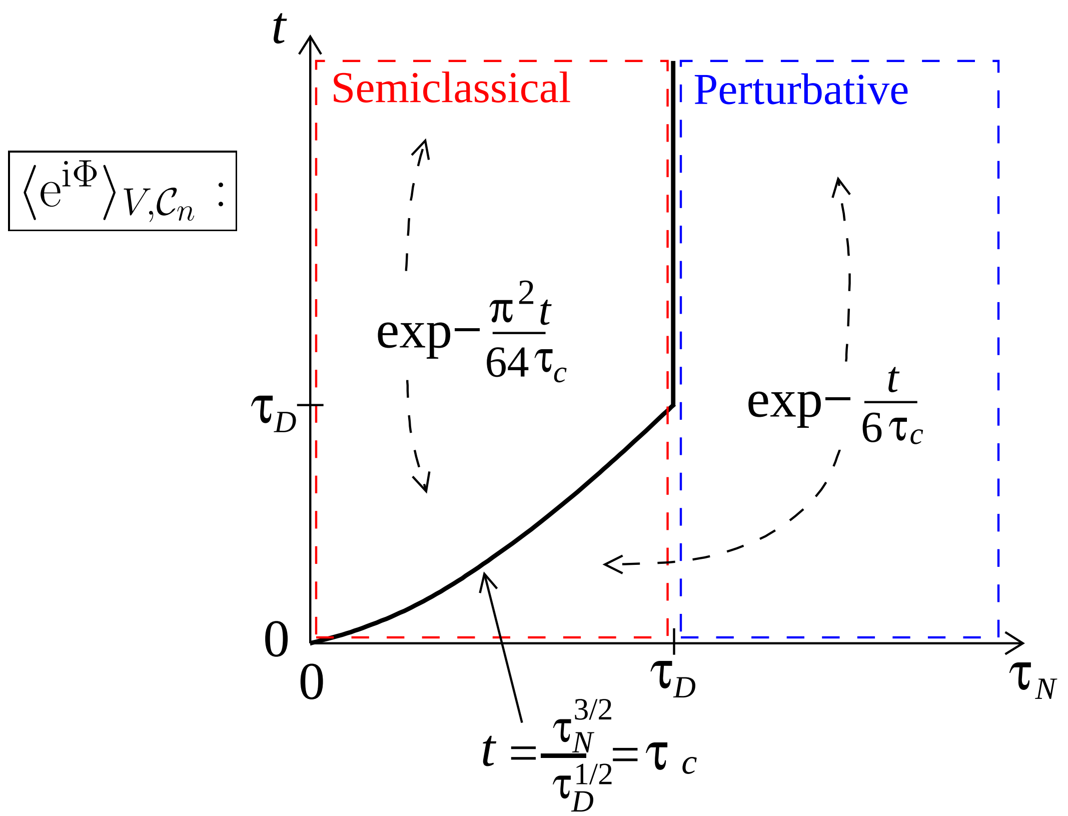
- For $\tau_N \gg \tau_D$: $\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n} = \langle \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_V}\rangle_{\mathcal{C}_n} \simeq \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_{V,\mathcal{C}_n}}$

	$t \ll \tau_D$		$t \gg \tau_D$	
Harmonic $n = 0$		$\mathrm{e}^{-\frac{\sqrt{\pi}}{4}(\frac{t}{\tau_N})^{3/2}}$ (AAK)		$\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_c}}$
Harmonics $n \neq 0$		$\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_c}}$		$\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_c}}$

For harmonics $n = 0$, phase relaxation is non exponential



For harmonics $n \neq 0$, phase relaxation is always exponential



Origin of $\Delta\sigma_n \propto \mathrm{e}^{-n(\frac{L}{L_N})^{3/2}}$: Exponential phase relaxation

For $n \neq 0$: $\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} \simeq \exp -\beta t/\tau_c$

with $\tau_c = \tau_N^{3/2}/\tau_D^{1/2}$

$\beta = 1/6$ or $\pi^2/64$

- Weak localization :

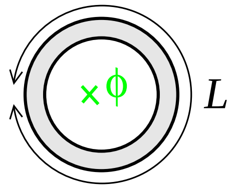
$$\Delta\sigma_n \sim \int_0^\infty \mathrm{d}t \, \overbrace{\mathrm{e}^{-\beta t/\tau_c}}^{\text{phase relax.}} \overbrace{\frac{1}{\sqrt{t}} \mathrm{e}^{-(nL)^2/(4t)}}^{\text{diffusion}}$$

$$\sim \exp -\sqrt{\beta}|n|\frac{L}{L_c} = \exp -\sqrt{\beta}|n|(\frac{L}{L_N})^{3/2}$$

$$\boxed{\Delta\sigma_n \sim \exp -nL^{3/2}T^{1/2}} \quad \text{for } L \gg L_N$$

Exact result for isolated ring of perimeter L

C. T. & G. Montambaux, PRB **72** (2005)



$\longrightarrow$ One can compute exactly the path integral

$W(x,x') = f(x-x') \Rightarrow$ get rid of time nonlocality

Cooperon is solution of :

$$\left[\frac{1}{L_\varphi^2} - \left(\frac{\mathrm{d}}{\mathrm{d}x} - 2\mathrm{i}eA\right)^2 + \frac{1}{L_N^3}|x|\left(1-\frac{|x|}{L}\right)\right] P_c(x,x') = \delta(x-x')$$

$$\langle \Delta\sigma \rangle = -\frac{2e^2}{\pi} P_c(0,0)$$

$$\boxed{\langle \Delta\sigma_n \rangle \propto \exp -|n|\ell_{\text{eff}}}$$

$\longrightarrow$ Analytical expressions for the prefactor and ℓ_{eff}

- Effective perimeter ℓ_{eff} for $L_\varphi = \infty$:

$$\ell_{\text{eff}} \simeq \frac{1}{\sqrt{6}}\left(\frac{L}{L_N}\right)^{3/2} \qquad \text{for } L \ll L_N$$

$$\simeq \frac{\pi}{8}\left(\frac{L}{L_N}\right)^{3/2} \qquad \text{for } L \gg L_N \qquad \text{(LM)}$$

- Combination of L_N (e-e interaction) and L_φ (exp. relaxation) :

$$\boxed{\frac{1}{\tau_\varphi} \longrightarrow \cancel{\frac{1}{\tau_\varphi} + \frac{1}{\tau_N}}}$$

$$\boxed{\ell_{\text{eff}} = \left(\frac{L}{L_N}\right)^{3/2} \times \eta(L_c^2/L_\varphi^2)} \qquad \text{with} \quad L_c = \frac{L_N^{3/2}}{L^{1/2}}$$

$\star$ <u>For $L \ll L_N$:</u> $\ell_{\text{eff}} = \sqrt{\frac{1}{6}\left(\frac{L}{L_N}\right)^3 + \left(\frac{L}{L_\varphi}\right)^2}$

$\star$ <u>For $L \gg L_N$:</u> $\ell_{\text{eff}} \simeq \sqrt{\frac{\pi^2}{64}\left(\frac{L}{L_N}\right)^3 + \left(\frac{L}{L_\varphi}\right)^2}$

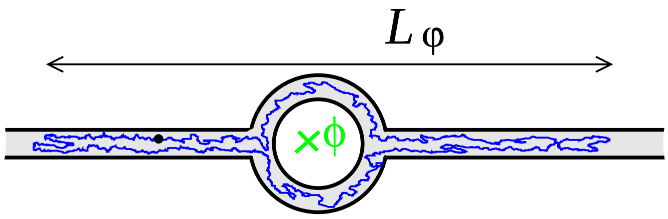
$$\Delta\sigma_n \simeq \underbrace{\frac{e^2}{\pi} L_N \frac{\text{Ai}(L_N^2/L_\varphi^2)}{\text{Ai}'(L_N^2/L_\varphi^2)}}_{\text{AAK}} \, \mathrm{e}^{-|n|\ell_{\text{eff}}}$$

Connected ring

$L_\varphi,\, L_N \ll L$: Arms have almost no effect

- Exp. phase coherence relaxation : $\Delta g_n \propto L_\varphi \exp -|n| L/L_\varphi$
- Electron-electron interaction : $\Delta g_n \propto L_N \exp -|n| (L/L_N)^{3/2}$

$L_\varphi,\, L_N \gtrsim L$: The arms strongly manifest



Winding is anomalously slow : $n_t \propto t^{1/4}$

(in the isolated ring : $n_t \propto t^{1/2}$)

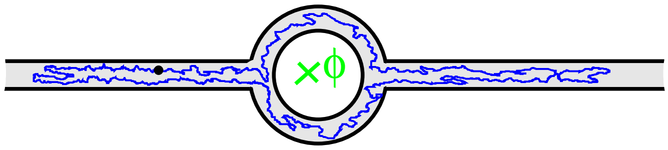
- For exponential phase coherence relaxation :

$$\langle \Delta g_n \rangle \sim \int_0^\infty \mathrm{d}t\, \frac{1}{t^{1/6}}\, \mathrm{e}^{-3\left(\frac{n\sqrt{L}}{4t^{1/4}}\right)^{4/3}}\, \mathrm{e}^{-t/\tau_\varphi}$$

$$\Delta g_n \propto \exp -|n| (2L/L_\varphi)^{1/2}$$

- $\boxed{L_N \gtrsim L}$ For electron-electron interaction :

Phase coherence relaxation occurs mostly in the wires



$$\Rightarrow \langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} \simeq \langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}}\big|_{\infty \text{ wire}}$$

$$\langle \Delta g_n \rangle \sim \int_0^\infty \mathrm{d}t\, \frac{1}{t^{1/6}}\, \mathrm{e}^{-3\left(\frac{n\sqrt{L}}{4t^{1/4}}\right)^{4/3}} \langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}}\big|_{\infty \text{ wire}}$$

$$\boxed{\Delta g_n \propto \exp -\kappa |n| (L/L_N)^{1/2}} \sim \exp -L^{1/2} T^{1/6}$$

$$\kappa \simeq 1.421$$

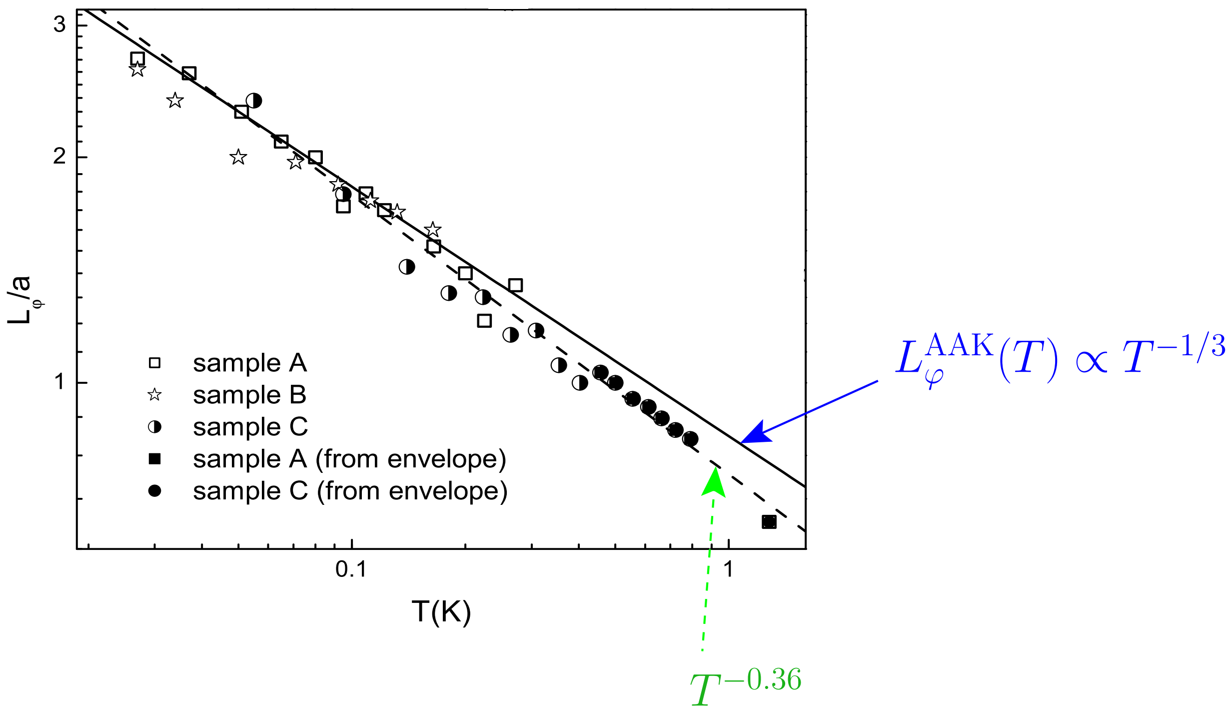
Let us come back to experiments

→ Meydi Ferrier's thesis (2004) ; Ferrier *et al* PRL **93** (2004)

→ Large square network

• Phase coherence length : Analysis of ratio of AAS harmonics

→ Analysis with model A. (Exp. phase relaxation)



Why we do not see LM's behaviour ?

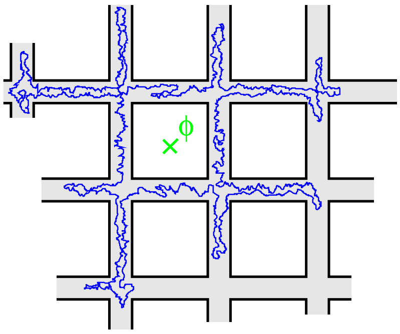
$\longrightarrow$ Exp. phase relaxation analysis should give $L_\varphi \rightarrow L_c \propto T^{-1/2}$

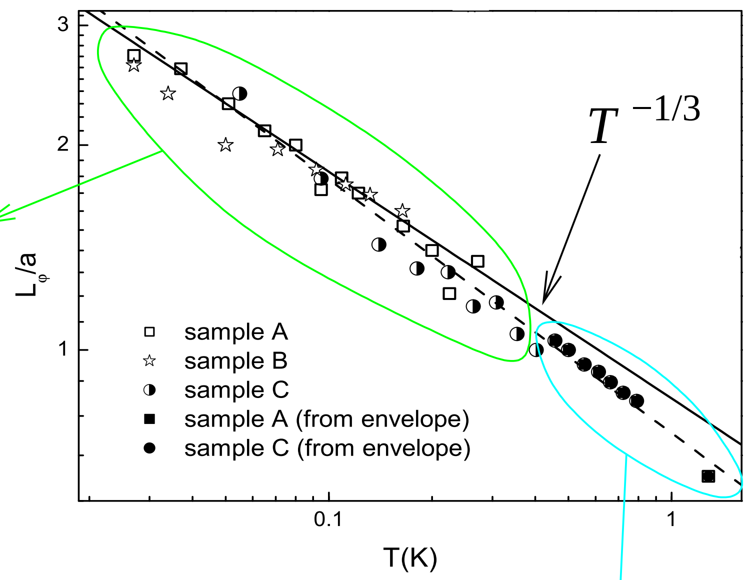
(for $L_\varphi \ll L$)

From harmonics

$\Downarrow$

For $L_N \gtrsim a$:





From envelope ($\approx$ AAK)

$\longrightarrow$ Experiment : behaviour of Δg_n at higher T ?

$\longrightarrow$ Theory : behaviour of Δg_n at low T (when $L \ll L_N$) ?

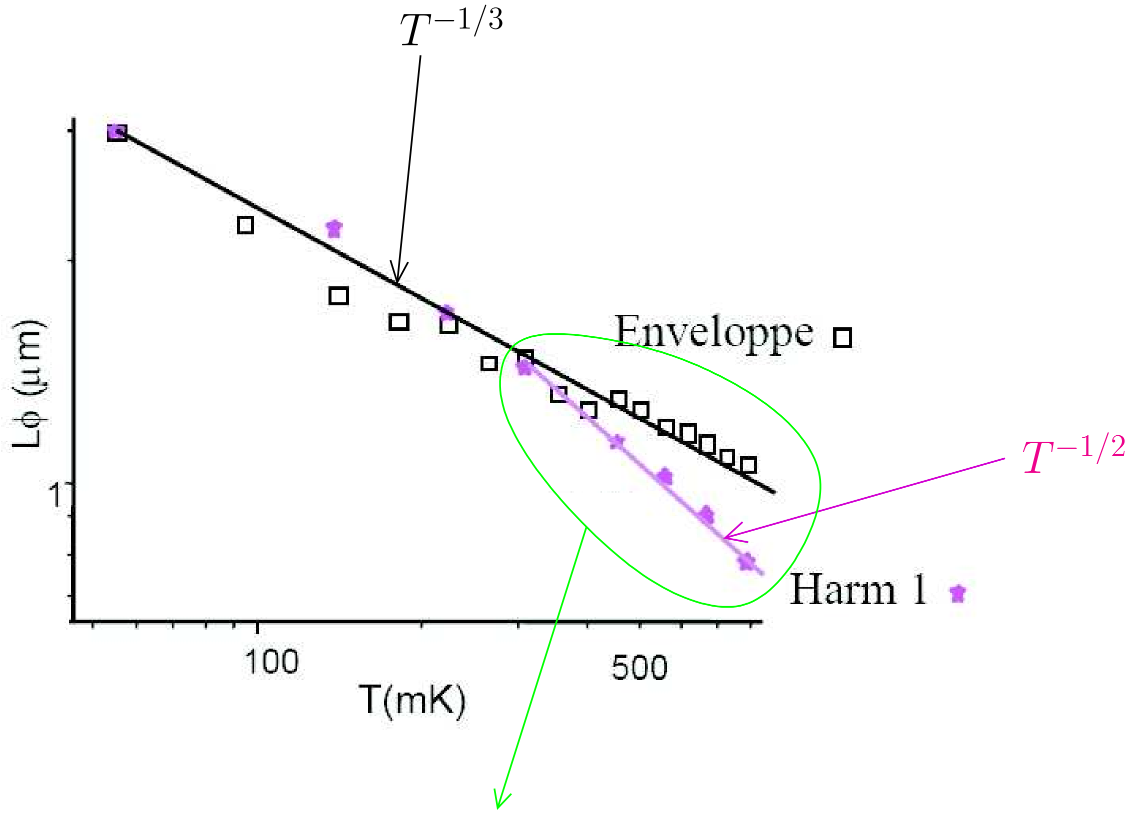
Observation of LM's result ?

- (for $L \gg L_N$) $\Delta g_n \propto \exp -\frac{\pi}{8}|n|(L/L_N)^{3/2} \sim \mathrm{e}^{-nL^{3/2}T^{1/2}}$

- New analysis of experimental data (M. Ferrier & H. Bouchiat)

  $\longrightarrow$ Extract parameters W, ℓ_e from envelope

  $\longrightarrow$ Then follow the 1st harmonic (instead of ratios)

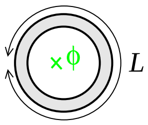
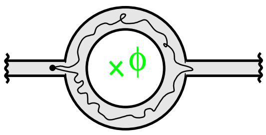
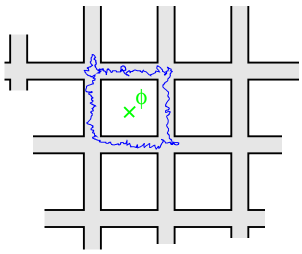


For $L \gtrsim L_\varphi$:

$$\Delta g_n \sim \exp -|n|L/L_\varphi \qquad \text{(Exp. relax.)}$$

$$\frac{L}{L_\varphi} \longrightarrow \frac{L}{L_c} = (\frac{L}{L_N})^{3/2} \propto T^{1/2}$$

Current theoretical understanding about e-e interaction in networks

Weak Localization Δg_n		
	Modelization A	Modelization B
	L_φ	AAK : $-L_N \frac{\mathrm{Ai}(L_N^2/L_\varphi^2)}{\mathrm{Ai}'(L_N^2/L_\varphi^2)}$
	$L_\varphi \, \mathrm{e}^{-n\, L/L_\varphi}$	$L_N \, \mathrm{e}^{-n\sqrt{\beta}\,(L/L_N)^{3/2}}$
	$\mathrm{e}^{-n\, L/L_\varphi}$ for $L_\varphi \ll L$ $\mathrm{e}^{-n(2L/L_\varphi)^{1/2}}$ for $L_\varphi \gg L$	$L_N \, \mathrm{e}^{-\frac{\pi}{8} n (L/L_N)^{3/2}}$ $\mathrm{e}^{-\kappa\, n (L/L_N)^{1/2}}$
	envelope : analytic harmonics : numerics	$L_N \ll L$: AAK $L_N \gg L$: ??? $L_N \ll L$: $\mathrm{e}^{-C\, n (L/L_N)^{3/2}}$ $L_N \gg L$: ???

Experiment is required on :

What experiments show ?

- Meydi Ferrier's thesis (2004); Ferrier *et al* PRL **93** (2004)

  → semiconducting networks

  → regime $L_N \lesssim L$

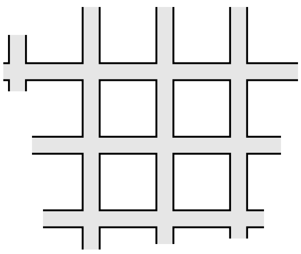
- C. Bäuerle, F. Mallet, L. Saminadayar & F. Schopfer (2004)

  → Metallic networks (Ag & Au)

  → regime $L_N \gtrsim L$

Oscillating part of MC curve is well described
by modelization A (with spectral determinant).

→ analysis of Grenoble's experiments within modelization A



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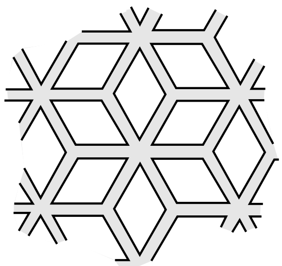

Ratios of harmonics (1/2 & 2/3) give the same $L_\varphi(T)$

⇒ Meaningful procedure

Limit of the AAK theory

Nyquist length

$$L_N = \left(\frac{\alpha_d}{\pi} N_c \ell_e L_T^2\right)^{1/3} \quad \text{with} \quad L_T = \sqrt{D/T}$$

- Strong localization in a fully coherent weakly disordered wire

  localization length is (DMPK) :

$$L_{\text{loc}} \sim N_c \ell_e$$

AAK is only valid for $L_N \ll L_{\text{loc}}$ *i.e.* $\boxed{L_T \ll L_N}$

- Strong localization threshold : $T < T^*$

$$L_N \sim L_T \sim L_{\text{loc}}$$

$$\boxed{T^* \sim 1/(N_c^2 \tau_e d)}$$

Exercice : a **metallic** gold wire

$S = 50\,\text{nm}\times 50\,\text{nm}$

$\ell_e = 30\,\text{nm}$

$D = 0.013\,\text{m}^2/\text{s}$ $\qquad$ $L_T \simeq 0.3\ \mu\text{m} \times T^{-1/2}$

$N_c = 27500$ $\qquad$ $L_N(T) \simeq 3.2\mu\text{m}\times T^{-1/3}$

$$T^* \simeq 0.7\,\mu\text{K}$$

Exercice 2 : a **semiconducting** narrow wire

$$T^* \simeq \text{few } 10\,\text{mK}$$

Strong localization of electrons in quasi-one-dimensional conductors

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(Received 7 May 1998)

We report on an experimental study of electron transport in submicrometer-wide ''wires'' fabricated from Si δ-doped GaAs. These quasi-one-dimensional (Q1D) conductors demonstrate the crossover from weak to strong localization with decreasing temperature. On the insulating side of the crossover, the resistance has been measured as a function of temperature, magnetic field, and applied voltage for different values of the electron concentration, which was varied by applying the gate voltage. The activation temperature dependence of the resistance has been observed with the activation energy close to the mean energy spacing of electron states within the localization domain. The study of nonlinearity of the current-voltage characteristics provides information on the distance between the critical hops that govern the resistance of Q1D conductors in the strong localization (SL) regime. We observe the exponentially strong negative magnetoresistance; this orbital magnetoresistance is due to the universal magnetic-field dependence of the localization length in Q1D conductors. The method of measuring the single-particle density of states (DOS) in the SL regime has been suggested. Our data indicate that there is a minimum of DOS at the Fermi level due to the long-range Coulomb interaction.
[S0163-1829(98)03936-8]

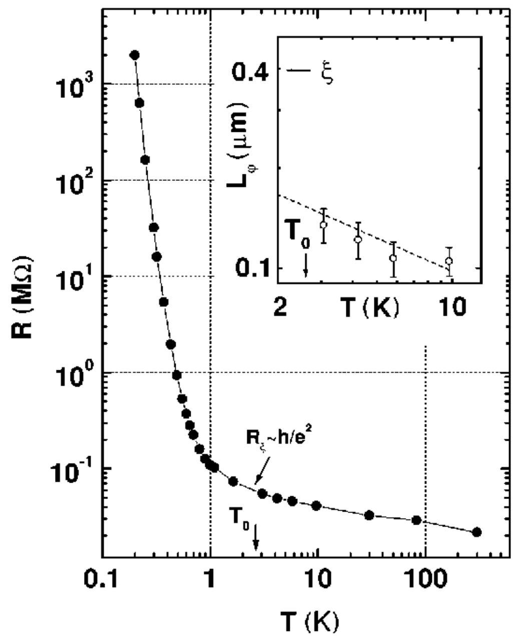

FIG. 1. The temperature dependence of the resistance of the 0.05-μm-wide wires (sample 1) in zero magnetic field without the gate, the solid curve is a guide to the eye. The arrow indicates the temperature T_0 that corresponds to the activation energy of hopping transport on the insulating side of the crossover. Inset: the temperature dependence of the phase-breaking length L_φ. The dashed line is the Nyquist phase-breaking length [Eq. (2)].

6. Fluctuations

PHYSICAL REVIEW B VOLUME 35, NUMBER 3 15 JANUARY 1987-II

Universal conductance fluctuations in metals: Effects of finite temperature, interactions, and magnetic field

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The conductance of any metallic sample has been shown to fluctuate as a function of chemical potential, magnetic field, or impurity configuration by an amount of order e^2/h independent of sample size and degree of disorder at zero temperature. We discuss the relationship of these results to other results in the theory of weak and strong localization, and discuss its physical implications. We discuss the physical assumptions underlying the ergodic hypothesis used to relate theory to experiment. We review the zero-temperature theory and provide a detailed discussion of the conductance correlation functions in magnetic field and Fermi energy. We show that the zero-temperature amplitude of the fluctuations is unaffected by electron-electron interactions to lowest order in $(k_f l)^{-1}$, and at finite temperature interactions only enter insofar as they contribute to the inelastic scattering rate. We calculate the effects of finite temperature on both the amplitude of the fluctuations and their scale. We discuss the conditions for dimensional crossover at finite temperature, and the behavior of different experimental measures of the fluctuation amplitude, in order to facilitate quantitative comparisons of experiment and theory.

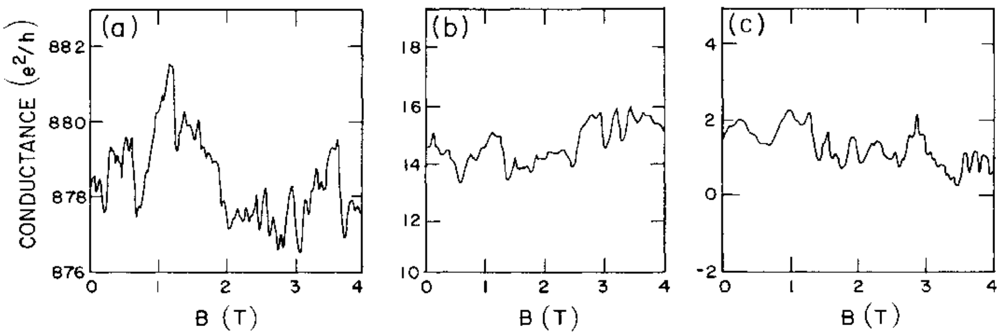


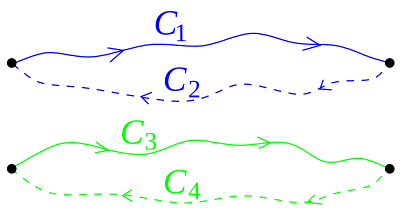
FIG. 1. Comparison of aperiodic magnetoconductance fluctuations in three different systems. (a) $g(B)$ in 0.8-μm-diam gold ring, analysis of data from Refs. 3 and 4, reprinted with the permission of Webb *et al.* (the rapid Aharonov-Bohm oscillations have been filtered out). (b) $g(B)$ for a quasi-1D silicon MOSFET, data from Ref. 9, reprinted with the permission of Skocpol *et al.* (c) Numerical calculation of $g(B)$ for an Anderson model using the technique of Ref. 11. Conductance is measured in units of e^2/h, magnetic field in tesla. Note the 3 order-of-magnitude variation in the background conductance while the fluctuations remain order unity.

$$\delta G \sim \frac{e^2}{h}$$

Reproducible structures : Magnetofingerprints

Conductance fluctuations

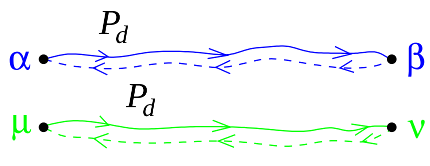
$$g^2 = \sum_{\mathcal{C}_1}\sum_{\mathcal{C}_2}\sum_{\mathcal{C}_3}\sum_{\mathcal{C}_4} \mathcal{A}_{\mathcal{C}_1}\,\mathcal{A}^*_{\mathcal{C}_2}\,\mathcal{A}_{\mathcal{C}_3}\,\mathcal{A}^*_{\mathcal{C}_4}$$

$=$ 

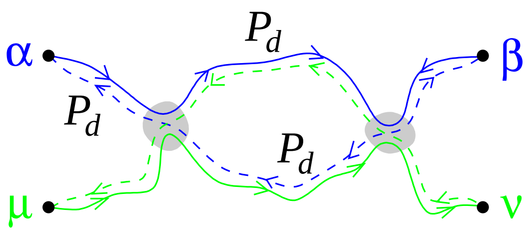
Survives disorder averaging iff
lines follow same sequences of scattering events
or reversed sequences.

How to pair the lines ?

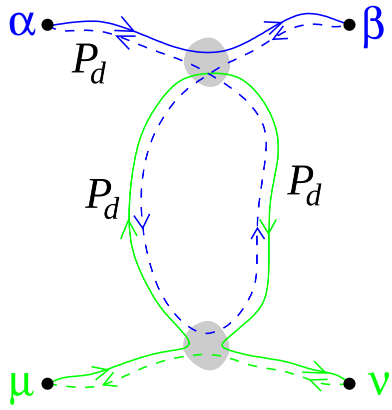
uncorrelated pairing :

$\langle g\rangle^2 \simeq$  $= g^2_{\mathrm{Drude}}$

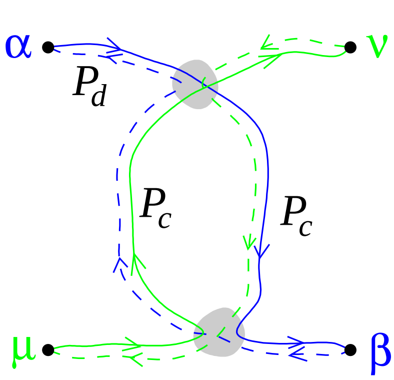
Quantum crossings :



and



and two similar diagrams with cooperon P_c :



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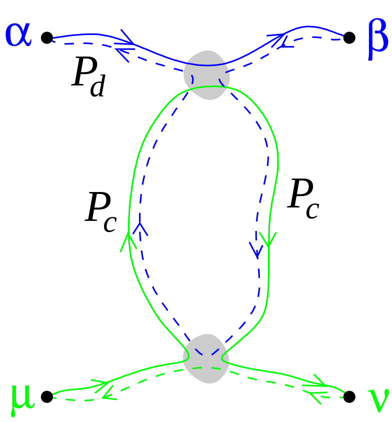
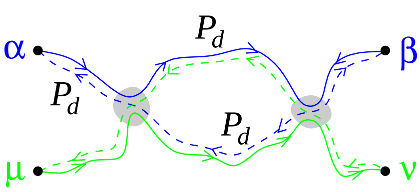


Order of magnitude :

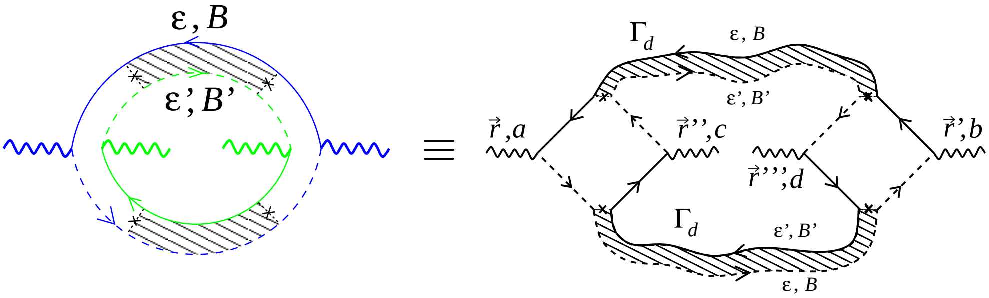
$\langle \delta g^2 \rangle =$  $\sim g^2 \times (1/g)^2 \sim 1$

$$\boxed{\langle \delta g^2 \rangle \sim 1}$$

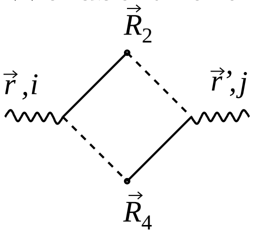
Conductivity correlations (Short range terms)

$$\langle \delta\sigma_{ab}(r,r')\,\delta\sigma_{cd}(r'',r''')\rangle$$

Contribution 1



We use the box :



$$= -\sigma_0\, 2\tau_e^2 \delta_{ij}\, \delta(\vec{r}-\vec{r}\,')\, \delta(\vec{r}-\vec{R}_2)\, \delta(\vec{r}-\vec{R}_4)$$

and

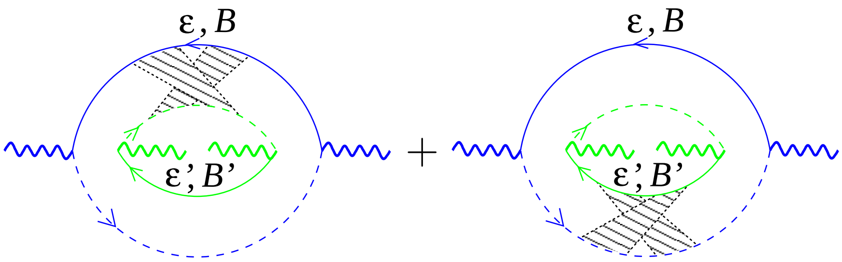
$$\sigma_0 \tau_e^2 \frac{w}{D\tau_e} = \frac{e^2}{h}$$

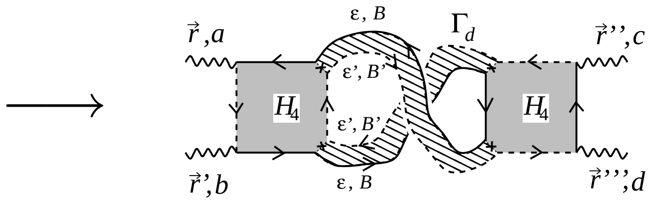
$$\text{Contrib. } 1 = 4\left(\frac{e^2}{h}\right)^2 \delta_{ac}\delta_{bd}\,\delta(\vec{r}-\vec{r}\,'')\,\delta(\vec{r}\,'-\vec{r}\,''')|P_d(\vec{r},\vec{r}\,')|^2$$

Contribution 2 : $P_d \longrightarrow P_c$

$$\text{Contrib. } 2 = 4\left(\frac{e^2}{h}\right)^2 \delta_{ad}\delta_{bc}\,\delta(\vec{r}-\vec{r}\,''')\,\delta(\vec{r}\,'-\vec{r}\,'')|P_c(\vec{r},\vec{r}\,')|^2$$

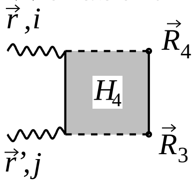
Contribution 3





$+ (\mathrm{R} \leftrightarrow \mathrm{A})$

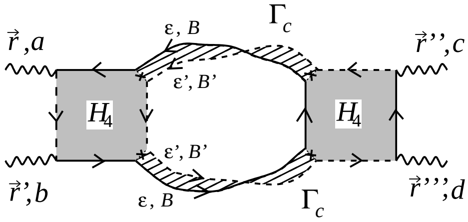
We use the box :



$$= \sigma_0\, \tau_e^2 \delta_{ij}\, \delta(\vec{r} - \vec{r}\,')\, \delta(\vec{r} - \vec{R}_2)\, \delta(\vec{r} - \vec{R}_4)$$

$$\text{Contrib. } 3 = 2\left(\frac{e^2}{h}\right)^2 \delta_{ab}\delta_{cd}\, \delta(\vec{r} - \vec{r}\,')\, \delta(\vec{r}\,'' - \vec{r}\,''')\, \mathrm{Re}\left[P_d(\vec{r}, \vec{r}\,'')\, P_d(\vec{r}\,'', \vec{r})\right]$$

Contribution 4 : $P_d \longrightarrow P_c$



$+ (\mathrm{R} \leftrightarrow \mathrm{A})$

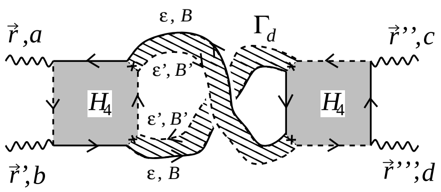
$$\text{Contrib. } 4 = 2\left(\frac{e^2}{h}\right)^2 \delta_{ab}\delta_{cd}\, \delta(\vec{r} - \vec{r}\,')\, \delta(\vec{r}\,'' - \vec{r}\,''')\, \mathrm{Re}\left[P_c(\vec{r}, \vec{r}\,'')\, P_c(\vec{r}\,'', \vec{r})\right]$$

Einstein relation : $\sigma_{ij} = e^2 \rho_0 D_{ij}$

Correlations :

$$\langle \delta\sigma_{ab}\, \delta\sigma_{cd} \rangle = e^4 \langle \delta\rho^2 \rangle D^2\, \delta_{ab}\delta_{cd} + e^4 \rho_0^2 \langle \delta D_{ab}\, \delta D_{cd} \rangle$$

Contribution 3+4 : DoS correlations $e^4 \langle \delta\rho^2 \rangle D^2$



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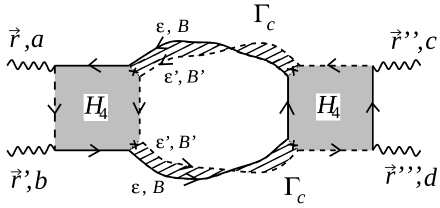
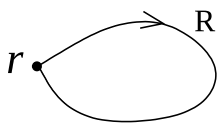
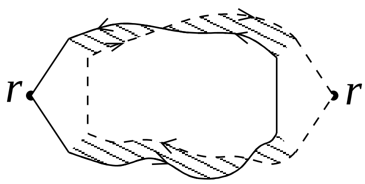


Dos is $\rho =$ 

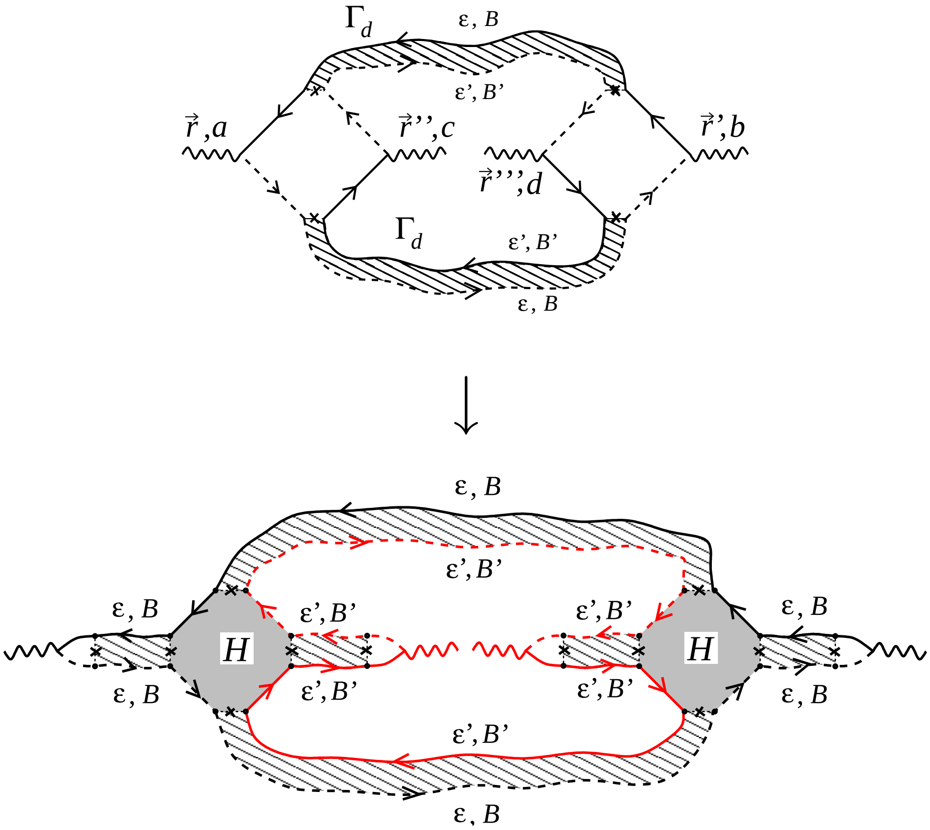
$\langle \delta\rho(r;\epsilon)\, \delta\rho(r';\epsilon') \rangle =$ 

$+ \cdots$

Contribution 1+2 : diffusion constant correlations $e^4 \rho_0^2 \langle \delta D_{ab}\, \delta D_{cd} \rangle$

Last step :

→ Following KSL, construct long range terms (divergenceless) :



Fluctuations of local conductivity

$$\langle \delta\sigma^2 \rangle = 6 \left(\frac{e^2}{h}\right)^2 \int \frac{\mathrm{d}r\mathrm{d}r'}{\mathrm{Vol}^2} \left[P_d(r,r')^2 + P_c(r,r')^2\right]$$

Spectral determinant

$$\begin{aligned} \int \mathrm{d}r\mathrm{d}r'\, P(r,r')^2 &= \int \mathrm{d}r\mathrm{d}r' \,\langle r \,|\frac{1}{\gamma-\Delta}|\, r' \rangle \,\langle r' \,|\frac{1}{\gamma-\Delta}|\, r \rangle \\ &= \sum_n \frac{1}{(\gamma+E_n)^2} = -\frac{\partial^2}{\partial\gamma^2} \ln S(\gamma) \end{aligned}$$

The diffuson in UCF is also affected by decoherence

Diffuson : $\gamma_d = 1/L_\varphi^2$

Cooperon : $\gamma_c = 1/L_\varphi^2 + 1/L_{\mathcal{B}}^2$

$$\boxed{\langle \delta\sigma^2 \rangle = -6 \left(\frac{e^2}{h}\right)^2 \frac{1}{\mathrm{Vol}^2} \left[\frac{\partial^2}{\partial\gamma_d^2} \ln S(\gamma_d) + \frac{\partial^2}{\partial\gamma_c^2} \ln S(\gamma_c)\right]}$$

The connected wire

$$S(\gamma) = \frac{\sinh \sqrt{\gamma} L}{\sqrt{\gamma}}$$

- Incoherent wire $L_\varphi \ll L$

$$\langle \delta g^2 \rangle \simeq 6 \left(\frac{L_\varphi}{L} \right)^3$$

- Coherent wire $L \ll L_\varphi$: UCF

$$\langle \delta g^2 \rangle \simeq \frac{2}{15}$$

- Reduction in strong magnetic field

If $L_{\mathcal{B}} \ll L_\varphi$: Contributions of the cooperon vanish

$$\boxed{\langle \delta g(\mathcal{B})^2 \rangle \simeq \frac{1}{2} \langle \delta g(0)^2 \rangle}$$

Crossover for the long wire :

$$\langle \delta g(\mathcal{B})^2 \rangle \simeq 3 \left[\left(\frac{L_\varphi}{L} \right)^3 + \frac{1}{L^3} \left(\frac{1}{L_\varphi^2} + \frac{1}{L_{\mathcal{B}}^2} \right)^{-3/2} \right]$$

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Classification
Physics Abstracts
72.15G — 72.15R — 72.70

Short Communication

Sensitivity of quantum conductance fluctuations and of $1/f$ noise to time reversal symmetry

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(Received 17 January 1992, accepted 12 February 1992)

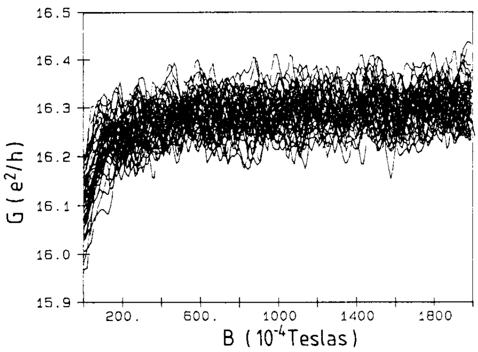


Fig. 2. — 46 reproducible MC curves at $T = 45$ mK in the same wire.

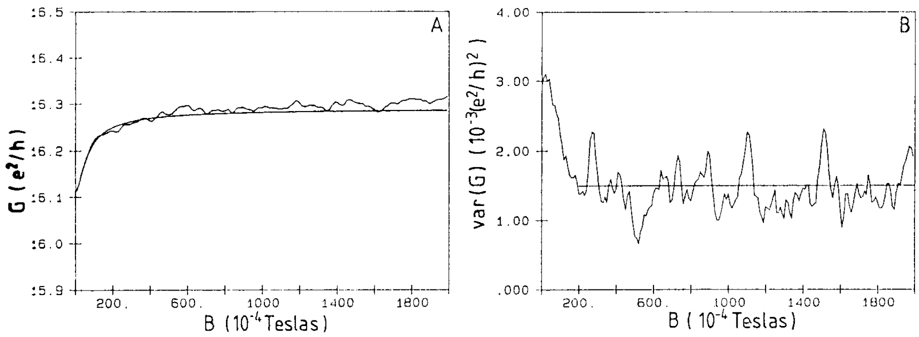


Fig. 3. — a) The mean conductance deduced from figure 2 and the weak localisation fit [10]. b) The variance over the 50 disorder configurations (at fixed magnetic field) as a function of the magnetic field. Note the reduction by a factor 2, for the same field range than for the mean MC effect in figure 3a.

The mesoscopic regime

- Coherent wire $L \ll L_\varphi$:

$$\boxed{\langle \Delta g \rangle \sim 1 \text{ and } \delta g \sim 1}$$

Mesoscopic regime

- Incoherent wire $L_\varphi \ll L$:

$$\langle \Delta g \rangle \simeq -\frac{L_\varphi}{L}$$

$$\delta g \sim \left(\frac{L_\varphi}{L}\right)^{3/2} \ll |\langle \Delta g \rangle|$$

$$\boxed{\Delta g = g - g_{\text{cl}} \text{ is self averaging}}$$

EFFECT OF TEMPERATURE

At finite temperature :

$$\sigma(E_F) \longrightarrow \int \mathrm{d}\epsilon \left(-\frac{\partial f}{\partial \epsilon}\right) \sigma(\epsilon)$$

We have to consider

$$\langle \delta\sigma_{ab}(r, r'; \epsilon)\, \delta\sigma_{cd}(r'', r'''; \epsilon') \rangle$$

New length scale : thermal length $L_T = \sqrt{D/T}$

REGIME $L_T \ll L_\varphi$

Contributions 3 & 4 are negligible

$$\langle \delta\sigma^2 \rangle \simeq \frac{2\pi}{3} \left(\frac{e^2}{h}\right)^2 \frac{L_T^2}{\mathrm{Vol}^2} \int \mathrm{d}r\, [P_d(r,r) + P_c(r,r)]$$

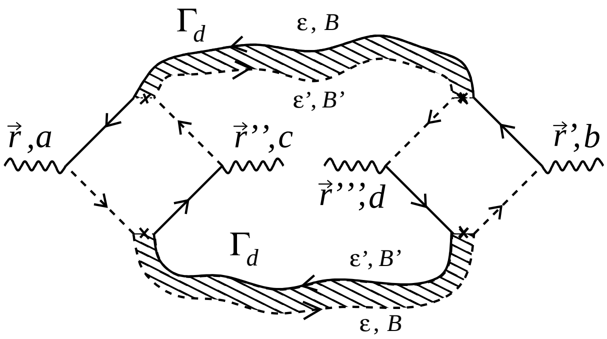
$$\langle \delta\sigma^2 \rangle \simeq \frac{2\pi}{3} \left(\frac{e^2}{h}\right)^2 \frac{L_T^2}{\mathrm{Vol}^2} \left[\frac{\partial}{\partial \gamma_d} \ln S(\gamma_d) + \frac{\partial}{\partial \gamma_c} \ln S(\gamma_c)\right]$$

Exercice : Long wire

$$\langle \delta g^2(\mathcal{B}) \rangle \simeq \frac{\pi}{3} \frac{L_T^2}{L^3} \left[L_\varphi + \left(\frac{1}{L_\varphi^2} + \frac{1}{L_\mathcal{B}^2} \right)^{-1/2} \right]$$

PARAMETRIC CORRELATIONS

Study $\langle \delta\sigma(\mathcal{B})\, \delta\sigma(\mathcal{B}')\rangle$



Diffuson :

$$\left[\frac{1}{L_\varphi^2} - (\nabla - \mathrm{i}e(A - A'))\right] P_d(r, r') = \delta(r - r')$$

Cooperon :

$$\left[\frac{1}{L_\varphi^2} - (\nabla - \mathrm{i}e(A + A'))\right] P_c(r, r') = \delta(r - r')$$

If $L_T \ll L_\varphi$: Relation between WL and fluctuations

$$\langle \delta\tilde{\sigma}(\mathcal{B})\, \delta\tilde{\sigma}(\mathcal{B}')\rangle \simeq -\frac{e^2}{h}\frac{\pi}{3}\frac{L_T^2}{\mathrm{Vol}}\left[\langle \Delta\sigma\left(\frac{\mathcal{B}-\mathcal{B}'}{2}\right)\rangle + \langle \Delta\sigma\left(\frac{\mathcal{B}+\mathcal{B}'}{2}\right)\rangle\right]$$

AB AMPLITUDE

Conductance of a network :

$$g(\phi) = g_0 + \delta g_{\mathrm{AB}} \cos(2\pi\phi/\phi_0 + \delta_1) + \cdots$$

Correlations of conductance :

$$\langle g(\phi) g(\phi') \rangle \simeq \langle g_0^2 \rangle + \frac{1}{2} \langle \delta g_{\mathrm{AB}}^2 \rangle \cos[2\pi(\phi - \phi')/\phi_0] + \cdots$$

δg_{AB} appears in parametric correlations

δg_{AB} involves both L_T and L_φ

(Δg_{AAS} is only function of L_φ)

For $L_T \ll L_\varphi$: Relation between AAS & AB amplitudes

$$\langle \delta\sigma_{\mathrm{AB}}^2 \rangle \simeq \frac{e^2}{h} \frac{2\pi}{3} \frac{L_T^2}{\mathrm{Vol}} \langle \Delta\sigma_{\mathrm{AAS}} \rangle$$

Exercice : The isolated ring

$$\langle \delta g_{\mathrm{AB}}^2 \rangle \sim L_T^2 L_\varphi \, \mathrm{e}^{-L/L_\varphi}$$

FROM MESO- TO MACRO-

Disorder averaging in a chain of N_r rings :

→ Consider **symmetric** rings → spectral determinant

Amplitude of AAS oscillations :

$$\frac{\langle \Delta g_{\mathrm{AAS}} \rangle}{g^{\mathrm{cl}}} \simeq \frac{L_\varphi}{2\alpha_d N_c \ell_e} \, \mathrm{e}^{-L/L_\varphi}$$

For $L_T \ll L_\varphi$:

$$\langle \delta g(\theta) \delta g(\theta') \rangle \simeq \frac{32\pi L_T^2}{3(N_r L)^4} \left[\frac{\partial}{\partial \gamma_d} \ln S(\gamma_d) + \frac{\partial}{\partial \gamma_c} \ln S(\gamma_c) \right]$$

Amplitude of AB oscillations :

$$\left(\frac{\delta g_{\mathrm{AB}}}{g^{\mathrm{cl}}} \right)^2 \simeq \frac{1}{N_r} \frac{\pi}{3} \frac{L_T^2 L_\varphi}{(\alpha_d N_c \ell_e)^2 L} \, \mathrm{e}^{-L/L_\varphi}$$

⇒ AB oscillations vanish with N_r :

$$\boxed{\frac{\delta g_{\mathrm{AB}}}{g^{\mathrm{cl}}} \propto \frac{1}{\sqrt{N_r}}}$$

Direct Observation of Ensemble Averaging of the Aharonov-Bohm Effect in Normal-Metal Loops

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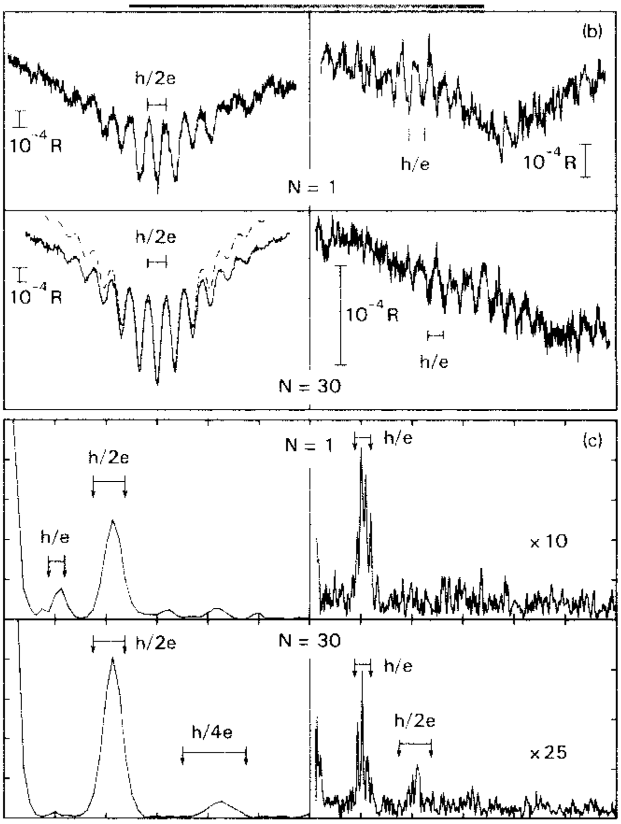
(Received 6 November 1985)

Aharonov-Bohm magnetoconductance oscillations have been measured in series arrays of one, three, ten, and thirty submicron-diameter Ag loops. At constant temperature, the amplitude of the h/e oscillations is observed to decrease as the square root of the number of loops, while the amplitude of the $h/2e$ conductance oscillations, measured in the same samples, is independent of the number of series loops. This is direct confirmation of the ensemble-averaging properties of h/e oscillations in multiloop systems. The amplitude of the h/e oscillations is in good agreement with recent calculations.



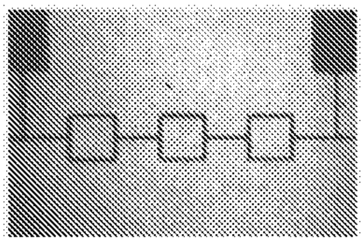


FIG. 1. (a) Transmission electron micrograph of the three-loop sample. (b) Magnetoresistance data at $T = 0.32$ K. Clockwise from the lower right-hand corner: the thirty-loop sample for $0.2 < H < 0.3$ T; the thirty-loop sample for $-0.02 < H < 0.02$ T (the dash-dotted line is the fit by the AAS theory); the single-loop sample for $-0.02 < H < 0.02$ T; the single-loop sample for $0.15 < H < 0.25$ T. (c) The Fourier transforms of the data in (b). The arrows in the figure indicate the bounds for the flux periods h/e and $h/2e$ based on the measured inside and outside areas of the loop.

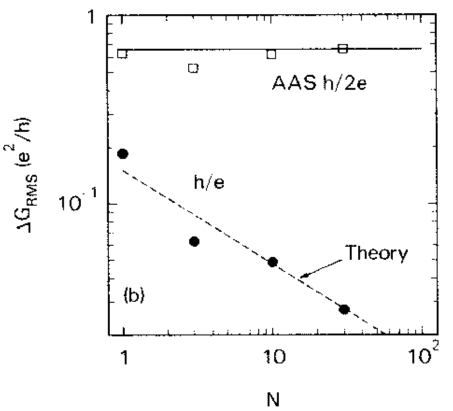
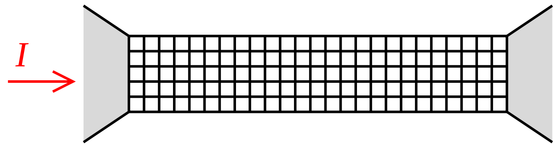


FIG. 2. (a) The h/e oscillations near zero field for the four samples extracted from the raw data by a digital filter set to include all periods allowed by the measured inside and outside areas. (b) The rms conductance amplitude of the h/e (circles) and the $h/2e$ (squares) oscillations as a function of the number of loops in the sample. The dashed line represents the calculated amplitude of the h/e oscillations, and the solid line is the theoretical value for the $h/2e$ oscillations predicted by AAS for $L_\phi = 2.2$ μm, $L_{SO} = 0.47$ μm, and $L_S = 3.1$ μm.

Disorder averaging in large networks :

Large network of dimension $L_x \times L_y$ with N cells



$$\frac{e^2}{h} g = \sigma \, \frac{L_y S}{L_x a} \qquad \Rightarrow \qquad \langle \Delta g \rangle = -2 \frac{L_y S}{L_x a} \int \frac{\mathrm{d}r}{\mathrm{Vol}} P_c(r,r)$$

$$\delta g_{\mathrm{AB}}^2 = \frac{2\pi L_T^2}{3 L_x^2} \, \Delta g_{\mathrm{AAS}}$$

$\longrightarrow$ Vary N with $L_x/L_y = 10$ fixed

- 2d regime : $L_\varphi \ll L_x,\ L_y$

$$\Delta g_{\mathrm{AAS}} \propto N^0$$

$$\delta g_{\mathrm{AB}} \propto N^{-1/2}$$

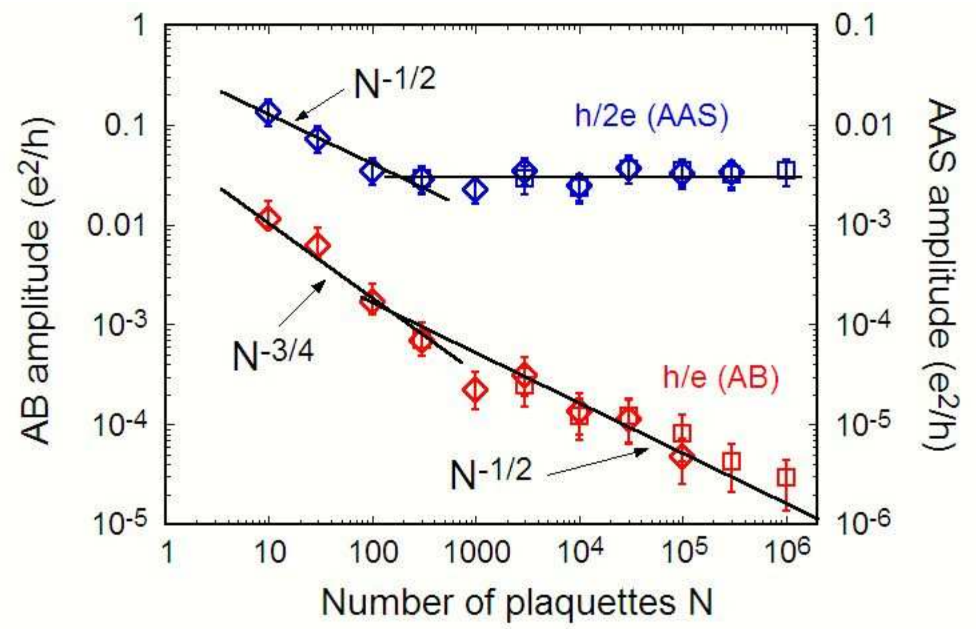
- quasi 1d regime : $L_y \ll L_\varphi \ll L_x \Rightarrow P_c \sim 1/L_y$

$$\Delta g_{\mathrm{AAS}} \propto N^{-1/2}$$

$$\delta g_{\mathrm{AB}} \propto N^{-3/4}$$

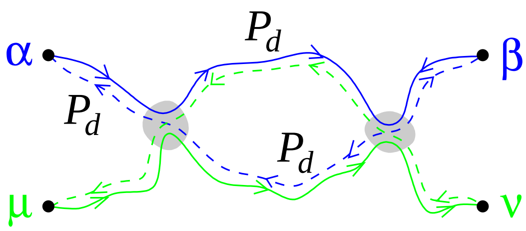
→ C. Bäuerle, F. Mallet, L. Saminadayar & F. Schopfer (2005)

→ Ag networks



CONDUCTANCE CORRELATIONS $\langle \delta G_{\alpha\beta} \delta G_{\mu\nu} \rangle$

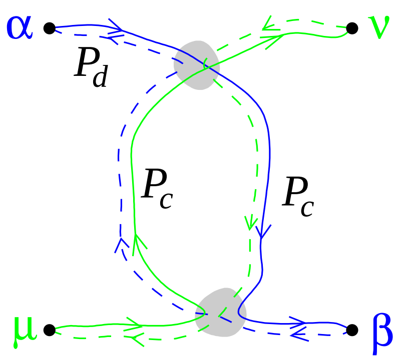
Network $\longrightarrow$ nonlocal effects $\longrightarrow$ Landauer approach



$$\propto \sum_{i,j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, \frac{\mathrm{d}}{\mathrm{d}x} P_d(\alpha, x) \, \frac{\mathrm{d}}{\mathrm{d}x} P_d(\mu, x) \, [P_d(x, x')]^2$$

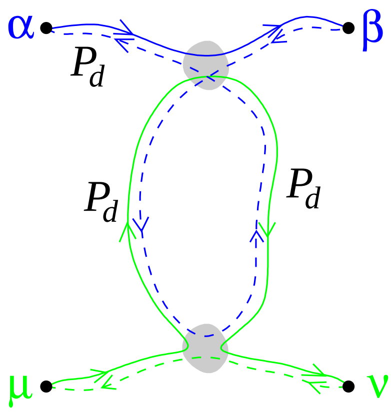
$$\frac{\mathrm{d}}{\mathrm{d}x'} P_d(x', \beta) \, \frac{\mathrm{d}}{\mathrm{d}x'} P_d(x', \nu)$$

$$\langle \delta G_{\alpha\beta} \delta G_{\mu\nu} \rangle^{(1)} \propto \sum_{i,j} \frac{\partial G^{\mathrm{cl}}_{\alpha\mu}}{\partial l_i} \frac{\partial G^{\mathrm{cl}}_{\beta\nu}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_d(x, x')]^2$$

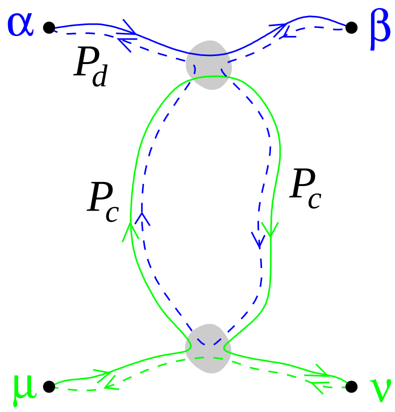


$$\langle \delta G_{\alpha\beta} \delta G_{\mu\nu} \rangle^{(2)} \propto \sum_{i,j} \frac{\partial G^{\mathrm{cl}}_{\alpha\nu}}{\partial l_i} \frac{\partial G^{\mathrm{cl}}_{\beta\mu}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_c(x, x')]^2$$

Dos (3 & 4) : no correlation of indices



$$\langle \delta G_{\alpha\beta} \delta G_{\mu\nu} \rangle^{(3)} \propto \sum_{i,j} \frac{\partial G^{\rm cl}_{\alpha\beta}}{\partial l_i} \frac{\partial G^{\rm cl}_{\mu\nu}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_d(x,x')]^2$$



$$\langle \delta G_{\alpha\beta} \delta G_{\mu\nu} \rangle^{(4)} \propto \sum_{i,j} \frac{\partial G^{\rm cl}_{\alpha\beta}}{\partial l_i} \frac{\partial G^{\rm cl}_{\mu\nu}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_c(x,x')]^2$$

Four-terminal resistances

$$\langle \delta\mathcal{R}_{\alpha\beta,\mu\nu}\, \delta\mathcal{R}_{\alpha'\beta',\mu'\nu'} \rangle$$

Contribution 1

$$\sum_{i,j} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha\beta,\alpha'\beta'}}{\partial l_i} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\mu\nu,\mu'\nu'}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_d(x,x')]^2$$

Contribution 2

$$\sum_{i,j} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha\beta,\mu'\nu'}}{\partial l_i} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\mu\nu,\alpha'\beta'}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_c(x,x')]^2$$

Contribution 3

$$\sum_{i,j} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha\beta,\mu\nu}}{\partial l_i} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha'\beta',\mu'\nu'}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_d(x,x')]^2$$

Contribution 4

$$\sum_{i,j} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha\beta,\mu\nu}}{\partial l_i} \frac{\partial \mathcal{R}^{\mathrm{cl}}_{\alpha'\beta',\mu'\nu'}}{\partial l_j} \int_i \mathrm{d}x \int_j \mathrm{d}x' \, [P_c(x,x')]^2$$

RESISTANCES $\mathcal{R}_S$ AND $\mathcal{R}_A$

Casimir-Onsager : $\mathcal{R}_{12,34}(-\mathcal{B}) = \mathcal{R}_{34,12}(\mathcal{B})$

$$\mathcal{R}_{S,A} = \frac{1}{2}\left(\mathcal{R}_{12,34} \pm \mathcal{R}_{34,12}\right)$$

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Length-Independent Voltage Fluctuations in Small Devices

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(Received 25 August 1986)

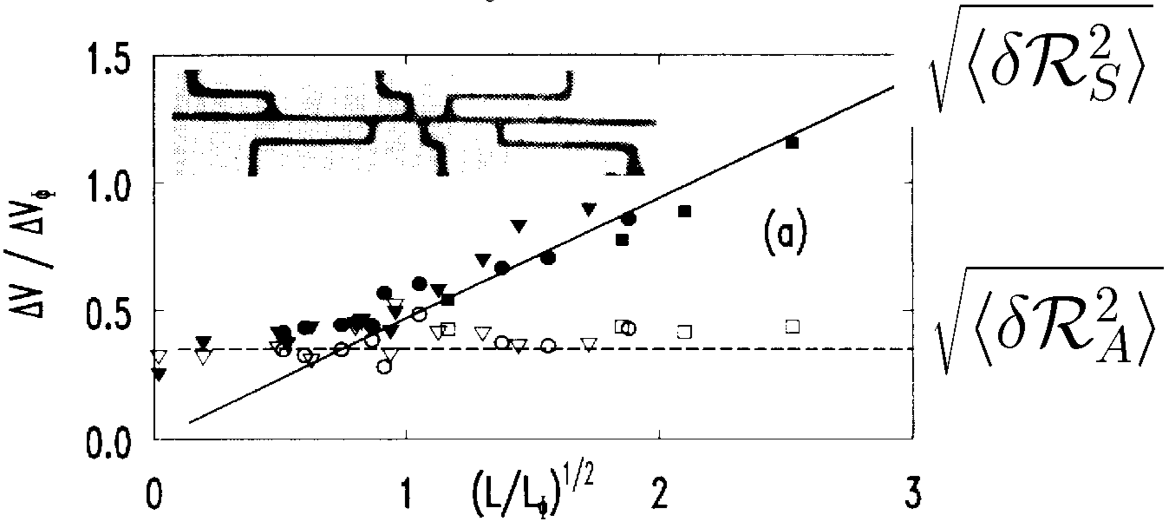


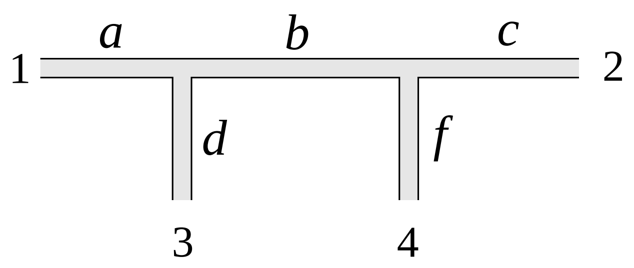
FIG. 2. (a) Measured rms voltage fluctuations normalized by ΔV_ϕ, as a function of $(L/L_\phi)^{1/2}$. The symmetric contributions are represented by solid symbols. The solid line represents the expected behavior for $L > L_\phi$. The antisymmetric part of the voltage fluctuations is represented by the open symbols and the dashed line is the predicted constant behavior. The symbols refer to different samples and temperatures: circles, Sb at $T=40$ mK and $L_\phi = 1.05$ μm; inverted triangles, Sb at $T=300$ mK and $L_\phi=0.60$ μm; squares, Au at $T=40$ mK and $L_\phi=2.0$ μm. Inset: A photograph of the Sb sample.

→ Kane, Lee & DiVincenzo, PRB (1988)

→ Hershfield & Ambegaokar, PRB (1988)

$\rightarrow$ $\boxed{\text{At strong } \mathcal{B} \text{ field} \Rightarrow \text{Contributions 2 \& 4} = 0}$

$$\langle \delta\mathcal{R}^2_{S,A} \rangle \;=\; \frac{1}{2}\left(\langle \delta\mathcal{R}^2_{12,34} \rangle \pm \langle \delta\mathcal{R}_{12,34}\delta\mathcal{R}_{34,12} \rangle \right)$$



$$\mathcal{R}^{\text{cl}}_{12,34} \;\propto\; l_b$$

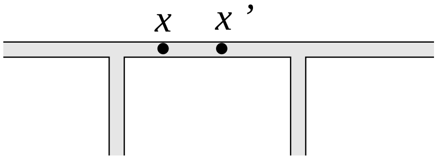
$$\mathcal{R}^{\text{cl}}_{12,12} \;\propto\; l_a + l_b + l_c$$

$$\mathcal{R}^{\text{cl}}_{34,34} \;\propto\; l_d + l_b + l_f$$

- Fluctuations $\langle \delta\mathcal{R}^2_{12,34}\rangle$

Weight of $\int_i \int_j [P_d(x,x')]^2$ for contribution (3) :

$$\frac{\partial \mathcal{R}^{\text{cl}}_{12,34}}{\partial l_i}\frac{\partial \mathcal{R}^{\text{cl}}_{12,34}}{\partial l_j} \quad \Rightarrow \quad \boxed{x,\, x' \in b}$$

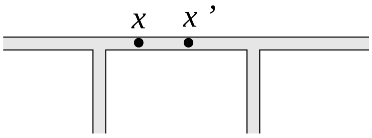


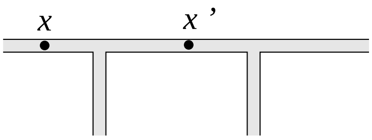
$$\mathcal{C}_3 = \int_b \mathrm{d}x \int_b \mathrm{d}x' \, [P_d(x,x')]^2 \sim L_\varphi^3 l_b \quad \text{for } \boxed{L_\varphi \ll l_b}$$

Weights for contribution (1) :

$$\frac{\partial \mathcal{R}^{\text{cl}}_{12,12}}{\partial l_i}\frac{\partial \mathcal{R}^{\text{cl}}_{34,34}}{\partial l_j} \quad \Rightarrow \quad \boxed{x \in a,\, b,\, c \text{ and } x' \in d,\, b,\, f}$$

9 contributions

$\mathcal{C}_{1,1} \equiv \mathcal{C}_3 =$  $\sim L_\varphi^3 l_b$

$\mathcal{C}_{1,2} =$  $= \int_a \mathrm{d}x \int_b \mathrm{d}x' \, [P_d(x,x')]^2 \sim L_\varphi^4$

etc. ⋮ ⋮

$$\boxed{\langle \delta\mathcal{R}^2_{12,34}\rangle = 2 \underbrace{\mathcal{C}_3}_{\sim L_\varphi^3 l_b} + \underbrace{\mathcal{C}_{1,2} + \cdots \mathcal{C}_{1,9}}_{\sim L_\varphi^4}}$$

- $\boxed{\text{Correlations } \langle \delta\mathcal{R}_{12,34}\delta\mathcal{R}_{34,12}\rangle}$

Weights for contribution (1) :

$$\frac{\partial \mathcal{R}^{\rm cl}_{12,34}}{\partial l_i}\frac{\partial \mathcal{R}^{\rm cl}_{34,12}}{\partial l_j} \quad \Rightarrow \quad \boxed{x,\, x' \in b}$$

Weights for contribution (3) :

$$\frac{\partial \mathcal{R}^{\rm cl}_{12,34}}{\partial l_i}\frac{\partial \mathcal{R}^{\rm cl}_{34,12}}{\partial l_j} \quad \Rightarrow \quad \boxed{x,\, x' \in b}$$

$$\boxed{\langle \delta\mathcal{R}_{12,34}\delta\mathcal{R}_{34,12}\rangle = 2 \underbrace{\mathcal{C}_3}_{\sim L_\varphi^3 l_b}}$$

- $\boxed{\text{Symmetric \& antisymmetric resistances}}$

In the limit $L_\varphi \ll l_b$

Symmetric resistance fluctuations increase with l_b :

$$\boxed{\langle \delta\mathcal{R}_S^2\rangle \sim L_\varphi^3 l_b}$$

Antisymmetric resistance fluctuations saturate :

$$\boxed{\langle \delta\mathcal{R}_A^2\rangle \sim L_\varphi^4}$$

Electron-electron interaction

Relation between WL and CF (since $L_T \ll L_N$ necessary) :

→ Aleiner & Blanter, PRB (2002).

AB harmonics for $L_N \ll L$:

$$\boxed{\delta g_{\mathrm{AB}} \sim L_T \sqrt{L_N}\, \mathrm{e}^{-(L/L_N)^{3/2}}}$$

Ludwig & Mirlin, PRB (2004)

C. T. & G. Montambaux PRB (2005).

What I have not mentioned

- Thermodynamics → magnetization, persistent current.

- Shot noise

- Role of the dynamic of magnetic impurities on decoherence

  Kondo impurities (Saclay & Grenoble's experiments)

- Measurements of local distribution functions in diffusive wires

  (Saclay's experiments)

- Superconducting fluctuations : contributions to transport.

  (DoS, Maki-Thomson, Alsamazov-Larkin)

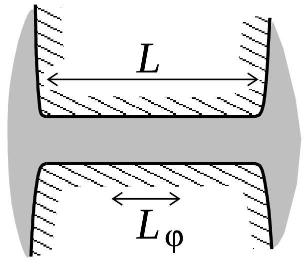
- Technical aspects :

  → Semiclassical approach (Boltzmann-Langevin)

  → Field theory and nonlinear-σ-model

- · · ·

7. Conclusion



In a wire

- Classical transport : $g_{\text{Drude}} \propto N_c \ell_e / L$

- Weak localization :

$$\langle \Delta g(\mathcal{B}, L_\varphi) \rangle = -\frac{1}{L}\left(\frac{1}{L_\varphi^2}+\right)^{-1/2} \quad \text{where } \frac{1}{L_{\mathcal{B}}^2} = \frac{e^2\mathcal{B}^2W^2}{3\hbar^2}$$

- Interaction : Al'tshuler-Aronov correction

$$\langle \Delta g(L_T) \rangle_{\text{ee}} \simeq -0.782\lambda_\sigma \frac{L_T}{L} \propto -\frac{1}{\sqrt{T}}$$

- Fluctuations :

  ⋆ For $L_\varphi \ll L_T$:

$$\langle \delta g^2(\mathcal{B}, L_\varphi) \rangle \simeq 3\left[\left(\frac{L_\varphi}{L}\right)^3 + \frac{1}{L^3}\left(\frac{1}{L_\varphi^2}+\frac{1}{L_{\mathcal{B}}^2}\right)^{-3/2}\right]$$

  ⋆ For $L_T \ll L_\varphi$:

$$\langle \delta g^2(\mathcal{B}, L_\varphi, L_T) \rangle \simeq \frac{\pi}{3}\frac{L_T^2}{L^3}\left[L_\varphi + \left(\frac{1}{L_\varphi^2}+\frac{1}{L_{\mathcal{B}}^2}\right)^{-1/2}\right]$$

AAS/AB HARMONICS

For exponential phase coherence relaxation (L_φ) :

$\longrightarrow$ spin-orbit

$\longrightarrow$ spin-flip

amplitude of AAS/AB oscillations for a ring are :

$$\Delta g_{\mathrm{AAS}} \sim L_\varphi \, \mathrm{e}^{-L/L_\varphi}$$

$$\delta g_{\mathrm{AB}} \sim L_T \sqrt{L_\varphi} \, \mathrm{e}^{-L/2L_\varphi}$$

This is true only for $L_\varphi \ll L$

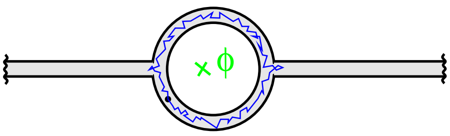
Does not apply to $L_\varphi \to L_{\mathrm{e-e}}$

$$\Delta g_{\mathrm{AAS}} \sim L_\varphi \, \mathrm{e}^{-L/L_\varphi}$$

$$\delta g_{\mathrm{AB}} \sim L_T \sqrt{L_\varphi} \, \mathrm{e}^{-L/2L_\varphi}$$

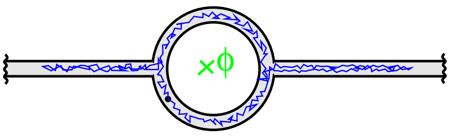
1. Network effect

- $L_\varphi \ll L$



$$\Delta g_{\mathrm{AAS}} \sim L_\varphi \, \mathrm{e}^{-L/L_\varphi}$$

- $L_\varphi \gg L$



$$\Delta g_{\mathrm{AAS}} \sim L_\varphi \, \mathrm{e}^{-\sqrt{2L/L_\varphi}}$$

2. electron-electron interaction effect $L_N \propto T^{-1/3}$

- $L_N \ll L$

$$\Delta g_{\mathrm{AAS}} \sim L_N \, \mathrm{e}^{-(L/L_N)^{3/2}}$$

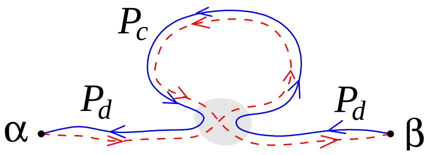
$$\delta g_{\mathrm{AB}} \sim L_T \sqrt{L_N} \, \mathrm{e}^{-(L/L_N)^{3/2}}$$

- $L_N \gg L$ (network effect)

$$\Delta g_{\mathrm{AAS}} \propto \mathrm{e}^{-(L/L_N)^{1/2}}$$

Networks

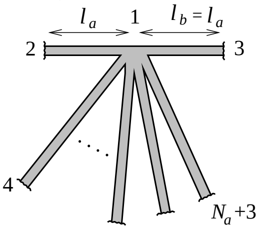
$$\Delta T_{\alpha\beta} =$$



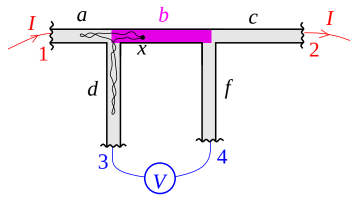
$$= \frac{2}{\alpha_d N_c \ell_e} \sum_{\text{wire }(\mu\nu)} \frac{\partial T^{\text{cl}}_{\alpha\beta}}{\partial\, l_{\mu\nu}} \int_{(\mu\nu)} \mathrm{d}x\, P_c(x,x)$$

- Classical non locality

  ⋆ $\Delta T_{13} > 0$ in



  ⋆ large WL and large fluctuations for $\mathcal{R}_{12,34}$



- Quantum nonlocality of P_c

- For large regular networks : spectral determinant

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi} \frac{1}{\text{Vol}} \int_{\text{Network}} \mathrm{d}x\, P_c(x,x) = -\frac{e^2}{\pi} \frac{1}{\text{Vol}} \frac{\partial}{\partial\gamma} \ln S(\gamma)$$

- Dephasing due to e-e interaction is not well understood

Appendices

A. Free Green function

$$G_0^{\rm R}(\vec{r},\vec{r}\,';E_F) \stackrel{\rm def}{=} \langle\,\vec{r}\,|\frac{1}{E_F+\frac{1}{2m}\Delta+\mathrm{i}0^+}|\,\vec{r}\,'\,\rangle$$

In Fourier space

$$G_0^{\rm R}(\vec{k}) = \frac{1}{E_F-\frac{1}{2m}\vec{k}^2+\mathrm{i}0^+}$$

In real space

$$G_0^{\rm R}(x,x') = \frac{1}{\mathrm{i}v_F}\mathrm{e}^{\mathrm{i}k_F|x-x'|} \qquad \text{in } d=1$$

$$G_0^{\rm R}(\vec{r},\vec{r}\,') = \frac{m_e}{2\mathrm{i}}H_0^{(1)}\left(k_F||\vec{r}-\vec{r}\,'||\right) \qquad \text{in } d=2$$

$$G_0^{\rm R}(\vec{r},\vec{r}\,') = -\frac{m_e}{2\pi||\vec{r}-\vec{r}\,'||}\mathrm{e}^{\mathrm{i}k_F||\vec{r}-\vec{r}\,'||} \qquad \text{in } d=3$$

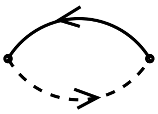
DoS & IDoS

$$n_e = \frac{V_d}{(2\pi)^d}k_F^d = \frac{k_F v_F}{d}\,\rho_0$$

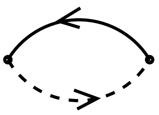
B. Hikami boxes

Hikami boxes are short range objects

Example :

$\vec{r}$  $\vec{r}$' $= \overline{G}^{\mathrm{R}}(\vec{r},\vec{r}\,')\,\overline{G}^{\mathrm{A}}(\vec{r}\,',\vec{r}) \sim \mathrm{e}^{-||\vec{r}-\vec{r}\,'||/\ell_e}$

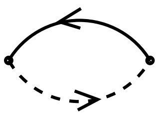
The poor's man Hikami box : $d = 1$

$\vec{r}$  $\vec{r}$' $= \left|\overline{G}^{\mathrm{R}}(x,x')\right|^2 = \dfrac{1}{v_F^2}\,\mathrm{e}^{-|x-x'|/\ell_e} = \underbrace{\dfrac{2\tau_e}{v_F}}_{1/w}\,\dfrac{1}{2\ell_e}\,\mathrm{e}^{-|x-x'|/\ell_e}$

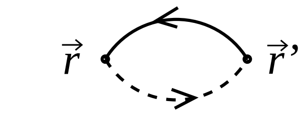
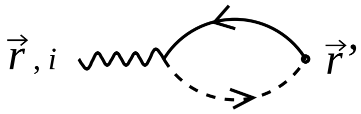
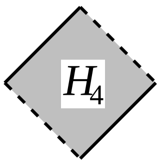
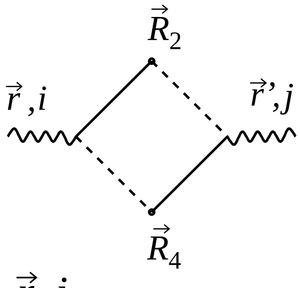
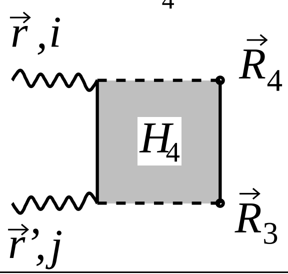
The sharp function can be expanded as a series of distributions :

$$\frac{1}{2\ell_e}\,\mathrm{e}^{-|x-x'|/\ell_e} = \delta(x-x') + \ell_e^2\,\delta''(x-x') + \cdots$$

One can show that in any dimension :

$\vec{r}$  $\vec{r}$' $= \delta(\vec{r}-\vec{r}\,')\dfrac{1}{w}\left[1 + \tau_e D\,\Delta + \cdots\right]$

Hikami boxes

	$\delta(\vec r-\vec r')\frac{1}{w}\left[1+\tau_e D\,\Delta+\cdots\right]$
	$-\mathrm{i}\frac{\sqrt{2\pi}}{d}e\rho_0\ell_e^2\,\delta(\vec r-\vec r')\,\nabla_i$
	$2\pi\rho_0 D\tau_e^4\int \mathrm{d}\vec R\,\prod_{i=1}^4\delta(\vec R-\vec R_i)$ $\times(\vec\nabla_1\cdot\vec\nabla_3+\vec\nabla_2\cdot\vec\nabla_4-\frac{1}{2}\sum_i\vec\nabla_i^2)$
	$-\sigma_0\,2\tau_e^2\delta_{ij}\,\delta(\vec r-\vec r')\,\delta(\vec r-\vec R_2)\,\delta(\vec r-\vec R_4)$
	$\simeq\sigma_0\,\tau_e^2\delta_{ij}\,\delta(\vec r-\vec r')\,\delta(\vec r-\vec R_3)\,\delta(\vec r-\vec R_4)$