Lorentz Invariant Breaking of CPT

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Sources:

1) M.Chaichian, A.Dolgov, V.Novikov, A.Tureanu, Phys.Lett. B699(2011) 177

2) A.Dolgov, V.Novikov, Pis'ma ZhETF, v.95,is.11, (2012) 672

3) Numerous papers by A.Dolgov and V.Novikov, unpublished in (2013)

Summary

- We present a class of nonlocal quantum field theories, in which all discrete symmetries, i.e. C, P, CP, $T-$ and CPT, are violated while the Lorentz invariance is not !!
- We rule out a standard claim in the literature that the CPT violation implies the violation of Lorentz invariance!!
- We demonstrate that at one-loop level the masses for particle and antiparticle remain equal due to Lorentz symmetry only.
- We demonstrate that an inequality of masses implies non-conservation of the usually conserved charges.

Introduction

- The weak interactions break both C and P symmetries.
- Individual CP and T symmetries have been observed to be violated in hadrons.
- Combined product, CPT, remarkably remains as an exact symmetry (still).
- The interplay of Lorentz symmetry and CPT symmetry was considered in the literature for decades

- Prehistory of CPT
  J.Schwinger
- First Proof of CPT
  Lüders and Pauli (Bell?) within the Hamiltonian formulation of quantum field theory with local and Lorentz invariant interaction.
- General Proof of CPT
  Jost within the axiomatic formulation of quantum field theory. The "local commutativity" condition was relaxed to "weak local commutativity".
- Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian QFT and in the axiomatic QFT.

- Violation of Lorentz symmetry and CPT.
  A long list of references includes Coleman,Glashow, Okun,Colladay,Kostelecky, Cohen,Lehner ...
- Relation between the *CPT* and Lorentz invariance.
  Does the violation of any of symmetry automatically imply the violation of the other one?
- This issue has recently become a topical one due to the growing phenomenological importance of *CPT* violating scenarios in neutrino physics and in cosmology.

- Different masses for neutrino and antineutrino. First phenomenological consideration by Murayama and Yanagida(2001).(See *MINOS* data (2010)).
- *CPT*-violating quantum field theory with a mass difference between neutrino and antineutrino
  First by Barenboim et al (2001) and later by Greenberg (2002) .
- Greenberg conclusion: *CPT* violation implies violation of Lorentz invariance.
- This result was given as a "theorem"!! The dispute on the validity of the theorem is the subject of this talk.

CPT-violating Quantum Mechanics

- There is widely spread habit to parametrize *CPT* violation by attributing different masses to particle and antiparticle.
- This tradition is traced to a good old theory of $K - \bar{K}$-mesons oscillation.
- For a given momenta **q** the theory of oscillation is equivalent to a non-hermitian Quantum Mechanics (QM) with two degrees of freedom.
- Diagonal elements of 2×2 Hamiltonian matrix represent masses for particle and antiparticle. Their unequality breaks CPT-symmetry.
- Such strategy has no explicit loop-holes and is still used for parametrization of CPT-symmetry violation in D and B meson oscillations.

CPT-violating Quantum Field Theory

(QFT) deals with an infinite sum over all momenta.
The set of plane waves with all possible momenta for particle and antiparticle is a complete set of orthogonal modes and an arbitrary field operator can be decomposed over this set.
Naive generalization of CPT-conserving QFT to CPT-violating QFT was to attribute different masses for particle and antiparticle.

- Bose commutation relations for particle $a(p), a^{+}(p^{'})$ with mass m;
- Bose commutation relations for antiparticle $b(p), b^{+}(p^{'})$ with masse $\tilde{m}$
- Hamiltonian as a sum over free oscillators

The Simplest Example

For a complex scalar field one gets the infinite sum

$$\phi(x) = \sum_{\mathbf{q}} \left\{ a(\mathbf{q}) \frac{1}{\sqrt{2E}} e^{-i(Et-\mathbf{q}\mathbf{x})} + b^{+}(\mathbf{q}) \frac{1}{\sqrt{2\tilde{E}}} e^{i(\tilde{E}t-\mathbf{q}\mathbf{x})} \right\} \quad , \tag{1}$$

where $(a(\mathbf{q}), a^{+}(\mathbf{q}))$, $(b(\mathbf{q}),\ b^{+}(\mathbf{q}))$ are annihilation and creation operators, and (m, E) and $(\tilde{m}, \tilde{E})$ are masses and energies of particle and antiparticle respectively.

Greenberg arguments

- Propagator is not Lorentz covariant, unless the masses of particle and antiparticle coincide.
- Theory is nonlocal and acausal: the $\Delta(x, y)$-function, i.e. the commutator of two fields, does not vanish for space-like separation, unless the two masses are the same, thus violating the Lorentz invariance.
- These arguments support a general "theorem" that interacting fields that violate CPT symmetry necessarily violate Lorentz invariance.

Not a QFT

Such theory can not be considered as a quantum field theory!!!

- There are no differential equations of motion.
- Conjugate momenta do not exist and, as a result, there are no canonical equal-time commutation relations
- "Free fields" separated by a space-like distance do not commute. They do not anticommute as well.
- One has no rule whether to apply commutation or anticommutation relations in quantizing the fields!
- There are no conservation laws,i.e. no conservation of electric charge.

CPT-violating, Lorentz invariant non-local model

We propose a model

- which preserves Lorentz invariance
- and breaks the *CPT* symmetry through a (nonlocal) interaction.
- Free field theory is a local one.
- Nonlocal field theories appear, in general, as effective field theories of a larger theory.

$$\mathcal{H}_{int}(x) = g \int d^4y \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0)\theta((x-y)^2)\phi(y) + h.c., \quad (2)$$

where $\phi(x)$ is a Lorentz-scalar field and θ is the Heaviside step function, with values 0 or 1.

- The combination $\theta(x_0 - y_0)\theta((x-y)^2)$ ensures the Lorentz invariance under the proper orthochronous Lorentz transformations
- The order of the times x_0 and y_0 remains unchanged for time-like intervals, while for space-like distances the interaction vanishes
- The same combination makes the nonlocal interaction causal at the tree level.
- There is no interaction when the fields are separated by space-like distances and thus there is a maximum speed of $c = 1$ for the propagation of information
- C and P invariance are trivially satisfied, while T invariance is broken due to the presence of $\theta(x_0 - y_0)$ in the integrand.

One can always insert into the Hamiltonian a form-factor $F((x-y)^2)$, for instance of a Gaussian type:

$$F = \exp\left(-\frac{(x-y)^2}{l^2}\right), \tag{3}$$

with l being a nonlocality length in the considered theory. Such a weight function would smear out the interaction and would guarantee the desired behaviour of the integrand in this equation; in the limit of fundamental length $l \to 0$, the Hamiltonian would correspond to a local, CPT- and Lorentz-invariant theory.

Quantum theory of nonlocal interactions

The S-matrix in the interaction picture is obtained as solution of the Lorentz-covariant Tomonaga- Schwinger equation :

$$i\frac{\delta}{\delta\sigma(x)}\Psi[\sigma] = \mathcal{H}_{int}(x)\Psi[\sigma]\,, \tag{4}$$

with σ a space-like hypersurface, and the boundary condition:

$$\Psi[\sigma_0] = \Psi\,. \tag{5}$$

where $\mathcal{H}_{int}$ is the Hamiltonian in the interaction picture. Then Eq. (4) with the boundary condition (5) represent a well-posed Cauchy problem.

The existence of a unique solution for the Tomonaga-Schwinger equation is ensured if the integrability condition

$$\frac{\delta^2\Psi[\sigma]}{\delta\sigma(x)\delta\sigma(x')} - \frac{\delta^2\Psi[\sigma]}{\delta\sigma(x')\delta\sigma(x)} = 0, \tag{6}$$

with x and x' on the surface σ, is satisfied. The integrability condition (6), inserted into (4), requires that the commutator of the interaction Hamiltonian densities vanishes at space-like separation:

$$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] = 0\,, \qquad \text{for } (x-y)^2 < 0\,. \tag{7}$$

Inserting our decomposition into (7), we get:

$$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] =$$

$$= \int d^4 a d^4 b \theta((x-a)^2)\theta(x^0 - a^0)\theta((y-b)^2)\theta(y^0 - b^0)\times$$

$$\times \left[\phi(x)\phi^2(a) + h.c., \phi(y)\phi^2(b) + h.c.\right]. \tag{8}$$

The commutator on the r.h.s. is a sum of products of field at the points x, y, a, b, multiplied by commutators of free fields like $[\phi(x), \phi(y)]$, $[\phi(x), \phi(b)]$, $[\phi(a), \phi(y)]$, $[\phi(a), \phi(b)]$.

A straightforward calculation shows that the terms containing these fields:

$$\int d^4a\, d^4b\ \theta((x-a)^2)\theta(x^0-a^0)\theta((y-b)^2)\theta(y^0-b^0)\times$$

$$2\Delta(a-b)\{\phi(a),\phi(b)\}\phi(x)\phi(y)+h.c. \qquad (9)$$

does not vanish at space-like distances between x and y and thus the causality condition (7) is not satisfied.

This, in turn, implies that the field operators in the Heisenberg picture, $\Phi_H(x)$ and $\Phi_H(y)$, do not satisfy the locality condition

$$[\Phi_H(x), \Phi_H(y)] = 0, \quad \text{for } (x-y)^2 < 0, \tag{10}$$

when the quantum corrections are taken into account. This is in accord with the requirement of locality condition (10) for the validity of *CPT* theorem both in the Hamiltonian proof (Luders,Pauli)and as well in the axiomatic one (Jost, Bogoliubov), taking into account that there is no example of a QFT, which satisfies the weak local commutativity condition (WLC) but not the local commutativity (LC).

Non-local QED

We propose a class of (slightly!) non-local Lorentz invariant field theories with explicit breakdown of CPT symmetry and with the same masses for particle and antiparticle.
A good example is a non-local QED with the Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{n.l.}$, where $\mathcal{L}_0$ is the usual QED Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{4}F^2_{\mu\nu}(x) + \bar{\psi}(x)[i\hat{\partial} - e\hat{A}(x) - m]\psi(x) \ , \qquad (11)$$

and $\mathcal{L}_{n.l}$ is a small non-local addition:

$$\mathcal{L}_{n.l.}(x) = g\int dy\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(y)K(x-y) \ , \qquad (12)$$

Here $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the electromagnetic field strength tensor, $A_\mu(x)$ is the four-potential, and $\psi(x)$ is the Dirac field for electrons.

Non-local form-factor $K(x-y)$ is chosen in such a way that it explicitly breaks T-invariance, e.g.

$$K(x-y) = \theta(x_0 - y_0)\theta[(x-y)^2]e^{-(x-y)^2/l^2} \quad , \qquad (13)$$

where l is a scale of the non-locality and the Heaviside functions $\theta(x_0 - y_0)\theta[(x-y)^2]$ are equal to the unity for the future light-cone and are identically zero for the past light-cone.

Non-local interaction breaks T-invariance, preserves C- and P-invariance and, as a result, breaks CPT-invariance. This construction demonstrates that CPT-symmetry can be broken in Lorentz-invariant non-local field theory! The masses of an electron, m, and of a positron, $\tilde{m}$, remain identical to each other in this theory despite breaking of CPT-symmetry. The evident reason is that the interaction $\mathcal{L}_{n.l.}(x)$ is C-invariant and its exact C-symmetry preserves the identity of masses and anti-masses.

A New Step

To study further the relation between mass difference for a particle and an antiparticle and CPT-symmetry we introduce a non-local interaction that breaks the whole set of discrete symmetries, i.e. C, P, CP, T, and CPT.

There is no discrete symmetry which preserves equality of m to $\tilde{m}$ in this case.

We start from the standard local free field theory of electrons with the usual dispersion relation between energy and momentum:

$$p_\mu^2 = p_0^2 - \mathbf{p}^2 = m^2 = \tilde{m}^2 \qquad (14)$$

In principle the non-local interaction can shift m from $\tilde{m}$. But an explicit one-loop calculation demonstrates that this is not true. So we conclude that it is Lorentz-symmetry that keeps the identity

$$m = \tilde{m} \quad . \qquad (15)$$

This conclusion invalidates the experimental evidence for CPT-symmetry based on the equality of masses of particles and antiparticles. CPT may be strongly broken in a Lorentz invariant way and in such a case the masses must be equal. Another way around, if we assume that the masses are different, then Lorenz invariance must be broken. Lorentz and CPT violating theories would lead not only to mass difference of particles and antiparticles but to much more striking phenomena such as violation of gauge invariance, current non-conservation, and even to a breaking of the usual equilibrium statistics (for the latter see our unpublished papers).

C, CP and CPT violating QFT

To formulate a model we start with the standard QED Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}(x)F_{\mu\nu}(x) + \bar{\psi}(x)[i\hat{\partial} - e\hat{A}(x) - m]\psi(x) \quad , \tag{16}$$

and add the interaction of a photon, A_μ, with an axial current

$$\mathcal{L}_1 = g_1\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)A_\mu(x) \tag{17}$$

and with the electric dipole moment of an electron

$$\mathcal{L}_2 = g_2\bar{\psi}(x)\sigma_{\mu\nu}\gamma_5\psi(x)F_{\mu\nu}(x) \quad . \tag{18}$$

The first interaction, $\mathcal{L}_1$, breaks C and P-symmetry and conserves CP-symmetry. The second interaction breaks P- and CP-symmetry. Still the sum of Lagrangians

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 \tag{19}$$

preserves CPT-symmetry. To break the CPT we modify the interaction $\mathcal{L}_1$ to a non-local one $\tilde{\mathcal{L}}_1$:

$$\mathcal{L}_1 \to \tilde{\mathcal{L}}_1(x) = \int dy g_1 \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)K(x-y)A_\mu(y) \ . \tag{20}$$

With this modification the model

$$\mathcal{L} = \mathcal{L}_0 + \bar{\mathcal{L}}_1 + \mathcal{L}_2 \tag{21}$$

breaks all discrete symmetries.

One-loop calculation

In general to calculate high order perturbative contributions of a non-local interaction into S-matrix one has to modify the Dyson formulae for S-matrix with T-ordered exponential

$$S = T\left\{\exp\left(i\int d^4x\mathcal{L}_{int}\right)\right\} \tag{22}$$

and the whole Feynman diagram techniques.
But in the first order in the non-local interaction one can work with the usual Feynman rules in the coordinate space. The only difference is that one of the vertices becomes non-local.

Mass and wave function renormalization for particle and antiparticle

We start with the standard free field theory for an electron, i.e.

$$\mathcal{L} = \bar{\psi}[i\hat{\partial} - m]\psi \tag{23}$$

that fixes the usual dispersion law

$$p^2 = p_0^2 - \mathbf{p}^2 = m^2 \quad . \tag{24}$$

The self-energy operator, $\Sigma(p)$, contributes both to the mass renormalization and to the wave function renormalization. In general one-loop effective Lagrangin can be written in the form:

$$\mathcal{L}_{eff}^{(1)} = \bar{\psi}[i(A\gamma_\mu + B\gamma_\mu\gamma_5)\partial_\mu - (m_1 + im_2\gamma_5)]\psi \quad . \tag{25}$$

It is useful to rewrite the same one-loop effective Lagrangian in terms of the field for antiparticle ψ_c:

$$\psi_c = (-i)[\bar{\psi}\gamma^0\gamma^2]^T \quad , \tag{26}$$

$$\mathcal{L}^{(1)}_{eff} = \bar{\psi}_c[i(A\gamma_5 - B\gamma_\mu\gamma_5)\partial_\mu - (m_1 + im_2\gamma_5)]\psi_c \quad . \tag{27}$$

We see that the mass term is the same for ψ and for ψ_c, but the wave function renormalization is different: the coefficient in front of the pseudovector changes its sign. This change is unobservable since one can remove $B\gamma_\mu\gamma_5$ and $im_2\gamma_5$ terms by redefining of variables.

Indeed

$$\bar{\psi}(A + B\gamma_5)\gamma_\mu\psi \equiv \bar{\psi}'\sqrt{A^2 + B^2}\gamma_\mu\psi' \quad , \tag{28}$$

where

$$\psi = (\cosh\alpha + i\gamma_5 \sinh\alpha)\psi' \quad , \tag{29}$$

$$\tanh 2\alpha = B/A \quad , \tag{30}$$

and

$$\bar{\psi}(m_1 + i\gamma_5 m_2)\psi \equiv \sqrt{m_1^2 + m_2^2}\bar{\psi}'\psi' \ , \tag{31}$$

where

$$\psi = \exp(i\gamma_5\beta)\psi' \ , \tag{32}$$

$$\tan 2\beta = m_2/m_1 \ . \tag{33}$$

This simple observation is sufficient to conclude that technically there is no possibility to write one-loop corrections that produce different contributions for particle and antiparticle. Still it is instructive to check directly that the difference is zero.

Explicit one-loop calculation

We are looking for a one-loop contribution into self-energy operator $\Sigma(p)$ that breaks C, CP, and CPT symmetries and that changes the chirality of the fermion line. It is clear that this contribution potentially can be different (opposite in sign) for particle ψ and antiparticle ψ_c.
To construct such contribution we need both anomalous interactions $\tilde{\mathcal{L}}_1$ and $\mathcal{L}_2$. Indeed interaction $\mathcal{L}_2$ changes chirality and breaks CP symmetry, while non-local interaction $\tilde{\mathcal{L}}_1$ breaks C and CPT and leaves the chirality unchanged. In combination they break all discrete symmetries and change chirality. There are two diagrams that are proportional to $g_1 g_2$ (see Fig. 1).

Figure: The diagram contributing to the mass difference of electron and positron. The blob represents a non-local form-factor.

We will calculate these diagrams in two steps. The first step is a pure algebraic one. Self-energy $\Sigma(p)$ is 4×4 matrix that was constructed from a product of three other 4×4 matrices, i.e. two vertices and one fermion propagator. Notice that any 4×4 matrix can be decomposed as a sum over complete set of 16 Dirac matrices. In this decomposition of $\Sigma(p)$ we need terms that are odd in C and changes chirality. Fortunately there is only one Dirac matrix with these properties. That is $\sigma_{\mu\nu}$. So

$$\Sigma(p) = \sigma_{\mu\nu} I_{\mu\nu} \ (p), \tag{34}$$

where $I_{\mu\nu}$ represents Feynman (divergent) integral. We could obtain eq. (24) after some long explicit algebraic transformation, but the net result is determined by the symmetry only.

The second step is the calculation of Feynman integrals. Again fortunately we do not need actual calculations. Indeed due to the Lorentz symmetry of the theory this $I_{\mu\nu}$ should be a tensor that depends only on the momentum of fermion line p. The general form for $I_{\mu\nu}$ is

$$I_{\mu\nu} = A g_{\mu\nu} + B p_\mu p_\nu \quad . \tag{35}$$

As a result we get

$$\Sigma(p) = \sigma_{\mu\nu} I_{\mu\nu} \equiv 0 \tag{36}$$

and we conclude that the one-loop contribution into possible mass difference is identically zero.[1]

[1] Recently our former collaborators published a paper where they demonstrated that for a particle with a non-standard dispersion law the quantity which they define as mass can be different for particle and antiparticle [?].

CPT and charge non-conservation

Naive generalization of CPT-conserving QFT to CPT-violating QFT is to attribute different masses for particle and antiparticle. For a complex scalar field one has to use the infinite sum

$$\phi(x) = \sum_{\mathbf{q}} \left\{ a(\mathbf{q}) \frac{1}{\sqrt{2E}} e^{-i(Et - \mathbf{q}\mathbf{x})} + b^{+}(\mathbf{q}) \frac{1}{\sqrt{2\tilde{E}}} e^{i(\tilde{E}t - \mathbf{q}\mathbf{x})} \right\} \quad , \tag{37}$$

where $(a(\mathbf{q}), a^{+}(\mathbf{q}))$, $(b(\mathbf{q}),\ b^{+}(\mathbf{q}))$ are annihilation and creation operators, and (m, E) and $(\tilde{m}, \tilde{E})$ are masses and energies of particle and antiparticle respectively.

Greenberg found that this construction runs into trouble. The dynamic of fields determined according to eq. (27) cannot be a Lorentz-invariant one.
We'd like to notice that for charged particles (say for electrons and positrons) similar generalization of the field theory breaks not only the Lorentz symmetry but the electric charge conservation as well. The reason is very simple. For the standard QED the operator of electric charge $\hat{Q}(t)$ can be written in the form

$$\hat{Q}(t) = \sum_{\mathbf{q}} \left\{ a^{+}(\mathbf{q}) a(\mathbf{q}) - b^{+}(\mathbf{q}) b(\mathbf{q}) \right\} \quad . \tag{38}$$

Operator $\hat{Q}(t)$ is a diagonal one, i.e. there are no mixed terms with different momenta. The modes with different momenta are orthogonal to each other and disappear after integration over space. This is a technical explanation why one can construct a time-independent operator.

If one shifts the mass of electron from the mass of positron the situation drastically changes. For electron the modes with different momenta are still orthogonal to each other. The same is true for the modes of positron, they are also orthogonal among themselves. But there is no reason for wave function of electron with mass m be orthogonal to wave functions for positron with mass $\tilde{m}$. As a result one obtaines

$$Q(t) = Q_{loc} + \ C \sum_{\mathbf{q}} \frac{(E - \tilde{E})}{\sqrt{4E\tilde{E}}} \left[b(\mathbf{q}) a(-\mathbf{q}) e^{-i(E+\tilde{E})t} + \mathrm{h.c.} \right] ,$$

where constant C depends on the sorts of particles and on the definition of the charge.

We can conclude from this equation that non-conservation of charge exhibits itself only in annihilation processes but not in the scattering processes. So there is no immediate problem with the Coulomb law. Nevertheless non-conservation of this type is also absolutely excluded by the experiment. In a case of charge-nonconservation annihilation of particle and antiparticle with a creation of the infinite number of soft massless photons creates a terrible infrared problem. Infrared catastrophe can not be avoided by usual summation over infrared photons. On the other hand, as is argued by Okun, Voloshin, Zeldovich, the electron decay might be exponentially suppressed due to vanishing of the corresponding formfactor created by virtual longitudinal photons. Similar arguments lead to the conclusion that conservation of energy cannot survive as well in a theory with different masses of particles and antiparticles.

Conclusion

We have shown that in the framework of a Lorentz invariant field theory it is impossible to have different masses of particles and antiparticles, even if CPT (together with C and P) invariance is broken. On the other hand, unequal masses of particles and antiparticles imply breaking of the Lorentz invariance. Moreover, in such theories charge and energy conservation seem to be broken as well.