SIMONS CENTER FOR GEOMETRY AND PHYSICS

Stony Brook University

Part I-Lyapunov exponent and out-of-time-ordered correlator in chaotic systems

Part II-(Time permitting) Dynamical many body localization in integrable kicked rotor

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Efim B. Rozenbaum, SG, and Victor Galitski
Phys. Rev. Lett. **118**, 086801

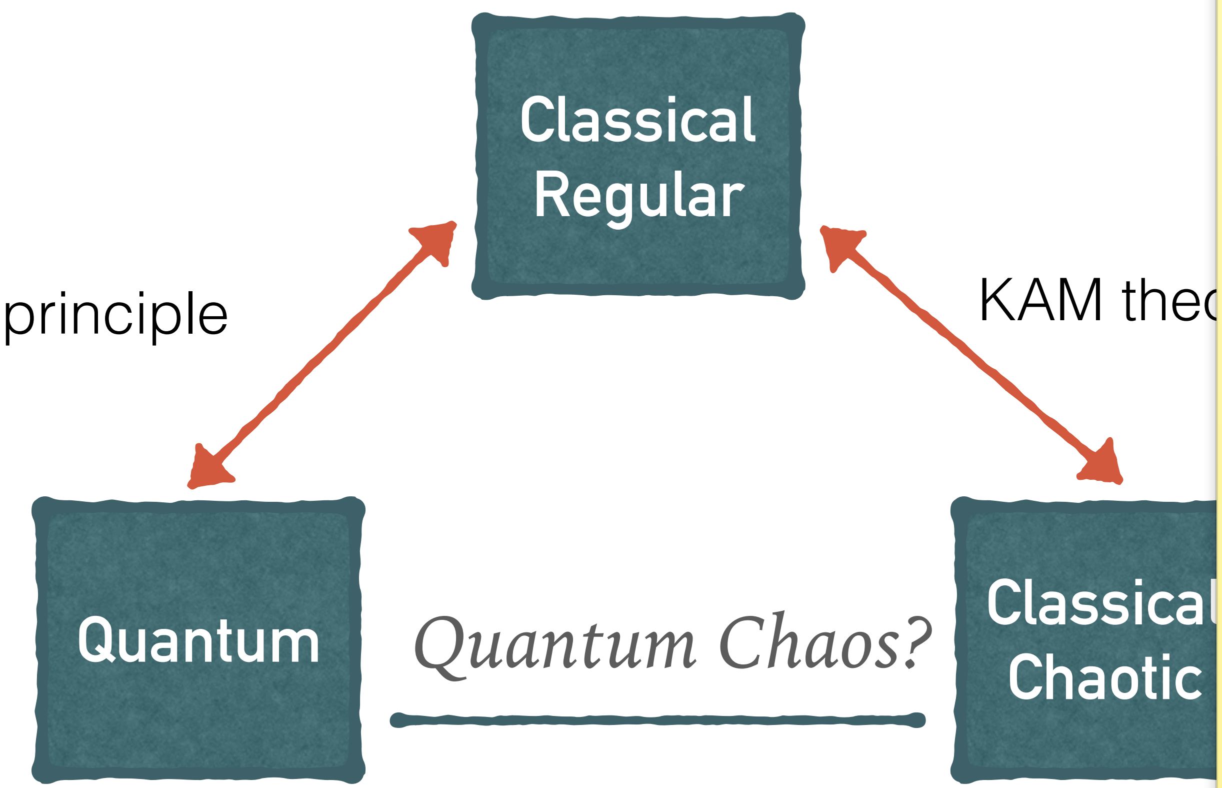
Aydin Cem Keser, SG, Gil Refael, Victor Galitski
Phys. Rev. B 94, 085120 (2016)

Quantum chaos

Gutzwiller, Prange, Fishman, Grempel, Srednicki, Deutch, Be

Correspondence principle

KAM theo



Correspondence principle?

M. Gutzwiller, Scientific American

Key question: How does chaotic classical dynamics emerge from quantum physics

The subject of my talk falls under the field of quantum chaos. One of the motivation of this subject is to define what it means. One motivation is to define fingerprints of chaos in quantum regime, like level statistics etc.

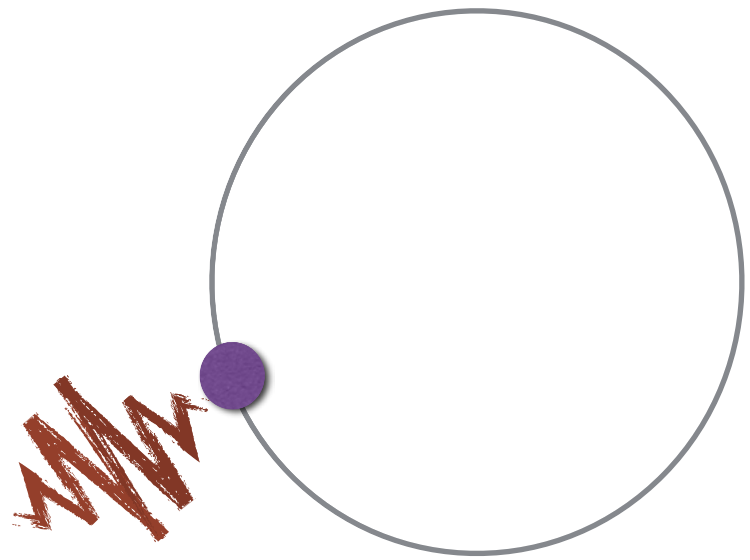
The second motivation which is closer to this talk is how classical chaotic dynamics emerges from an underlying quantum dynamics.

The three pillars of our understanding of microscopic world is understanding is classical regular, classical chaotic and quantum physics. Usually a more general theory does not cannibalize the existing theory, but contains the past understanding as a limiting case. A program of research follows where you recover all existing physics starting from a most fundamental theory, which in this case is quantum theory.

Model for classical chaotic system: Kicked rotor

Chirikov et al ‘78

$$f(\theta)\sum_n \delta(t-nT)$$



Kicked rotor

The simplest model that facilitates the interplay of all these theory is that of a kicked rotor.

$$H = \frac{p^2}{2m} + K\cos(\theta)\sum_n \delta(t-n)$$

standard map

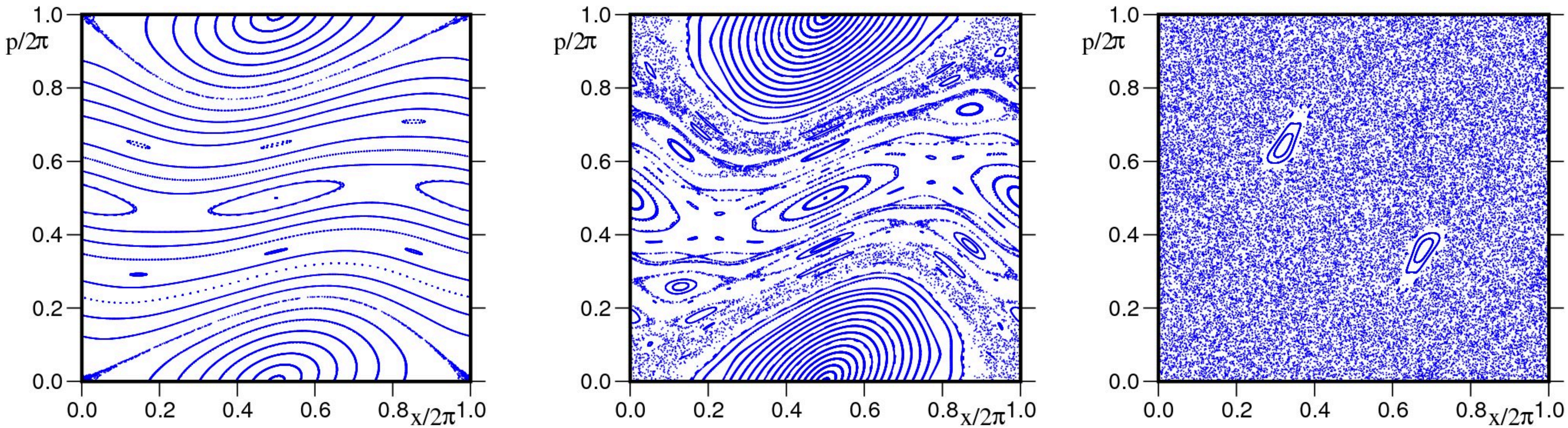
$$p_{n+1} = p_n + K\sin x_n \qquad x_{n+1} = x_n + p_{n+1}$$

K is the only parameter

Standard map (contd.)

$$p_{n+1} = p_n + K \sin x_n \qquad x_{n+1} = x_n + p_{n+1}$$

Shepalyansky et al Scholarpedia



non-chaotic — $K=0.5$

Critical — $K \sim 0.97$

Chaotic — $K=5$

$$\frac{d(t)}{d(0)} \sim e^{\lambda t}$$

$$d(t) = \sqrt{(x'(t) - x(t))^2 + (p'(t) - p(t))^2}$$

Quantum kicked rotor: quantized standard map

Prange, Grempel, Fishman '82

$$H = \frac{\hat{p}^2}{2m} + K\hat{V}(\hat{\theta})\sum_n \delta(t-n) \longrightarrow \sum_r W_{m+r}u_r + \tan[\omega - 2\pi\alpha m^2]u_m = Eu_m$$

Manifests classical chaos for sufficiently strong driving

Disordered Anderson Insulator in momentum space

Suppression of chaos in quantum regime via Anderson mechanism

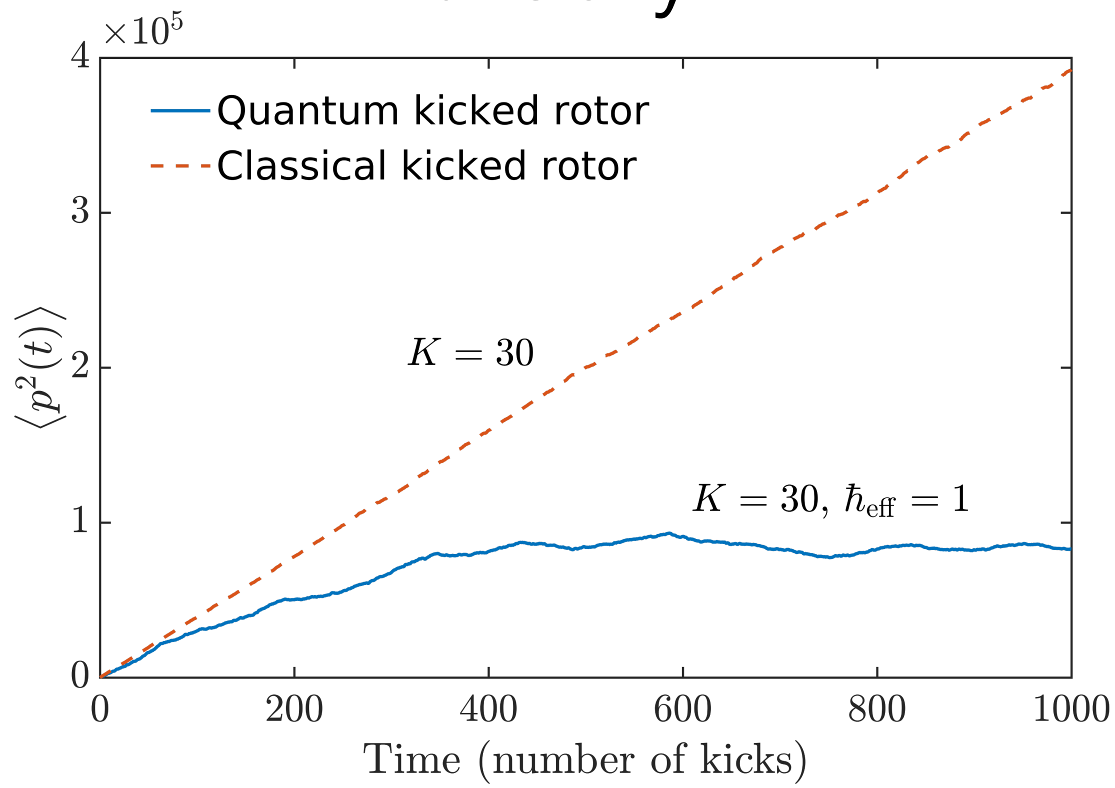
Prange, Grempel, Fishman '82

Parameters in the quantum regime $\hbar_{\text{eff}}$ K

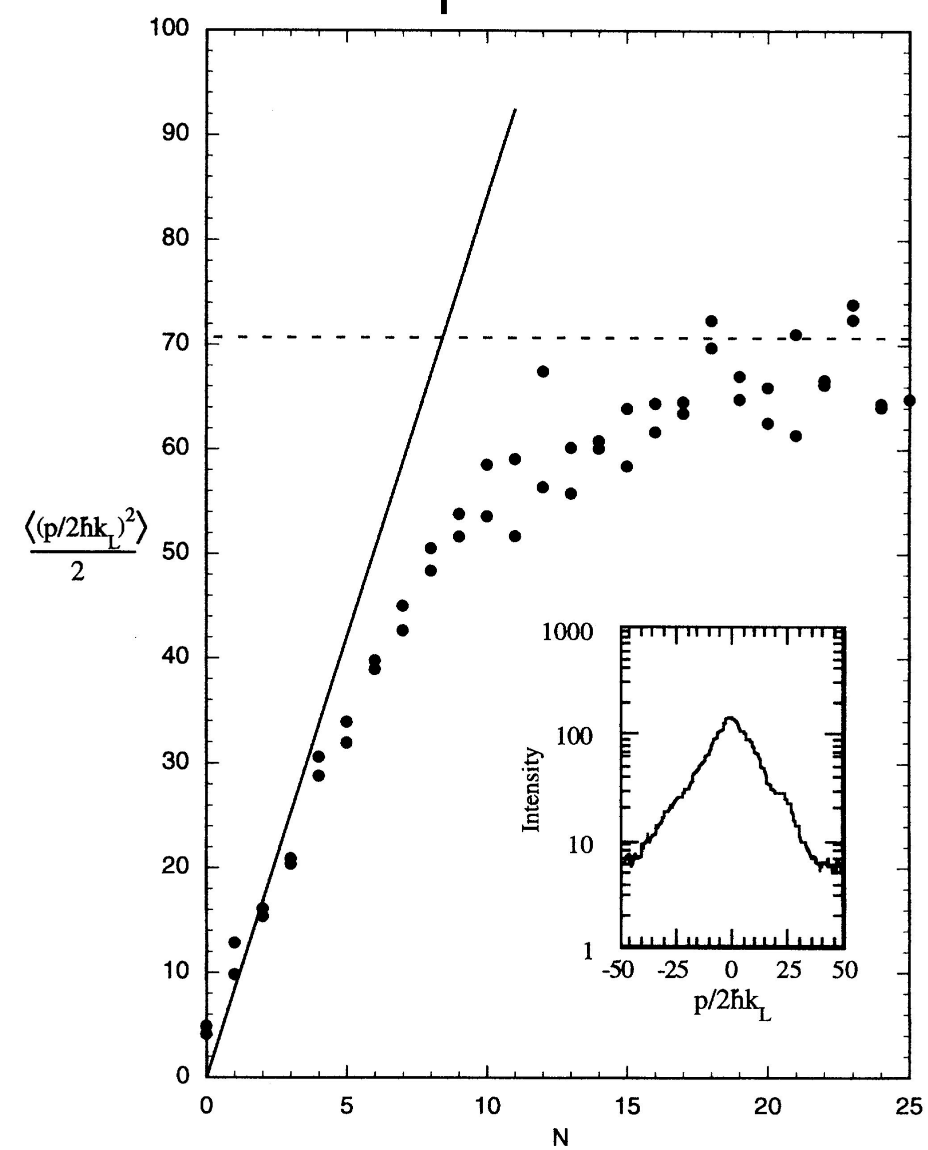
Emergence of classical limit in QKR: Diffusion

Prange, Grempel, Fishman '82

theory



experiment



Aleiner, Larkin, 1996 Tian, Kamenev, Larkin 2005

F. L. Moore, J. C. Robinson, C. F. Bharucha, Bala Sundaram and M. G. Raizen, Atom Optics Realization of Realization of the Quantum δ -Kicked Rotor, Phys. Rev. Lett. **75**, 4598 (1995)

Jean-Claude Garreau group

Many more......

Lyapunov exponent (Classical) of KR

$$\lambda_{cl} = \left\langle \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \ln \frac{d(t)}{d(0)} \right\rangle_{\text{phase space}}$$

$$d(t) = \sqrt{\Delta x^2 + \Delta p^2}$$

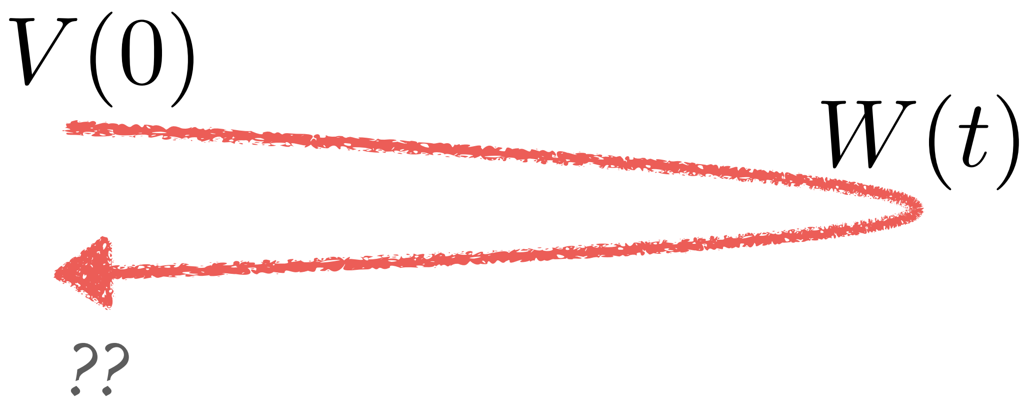
Chirikov estimate (linearized standard map) $\lambda_{cl} \sim \ln K/2 \, (K > 4)$

Can we extract the "Lyapunov exponent" in the semiclassical limit of QKR?

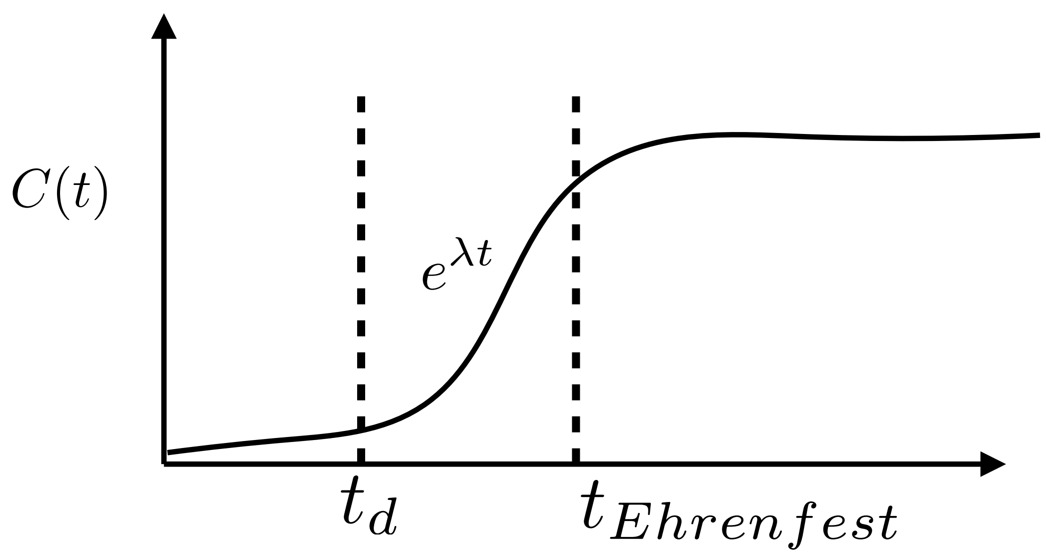
Out of time ordered four point correlator

Larkin and Ovchchinnikov-1967, Aleiner Larkin, 1996 Shenker, Stanford and Maldacena-2013, Kitaev-2013

$$C(t) = -\langle [W(t), V(0)]^2 \rangle$$



Remark: To and fro evolution is with the same Hamiltonian unlike Loschmidt echo



$$\lambda \leq \frac{2\pi k_B T}{\hbar}$$

Time scales

$$t_d \sim \frac{1}{\lambda_{cl}}$$

$$t_{Ehrenfest} \sim \frac{|\ln \hbar_{eff}|}{\lambda_{cl}}$$

Aleiner, Larkin 1996

Remark: $t_{Ehrenfest} \sim \frac{\ln N}{\lambda_{cl}}$ for many body systems like SYK models

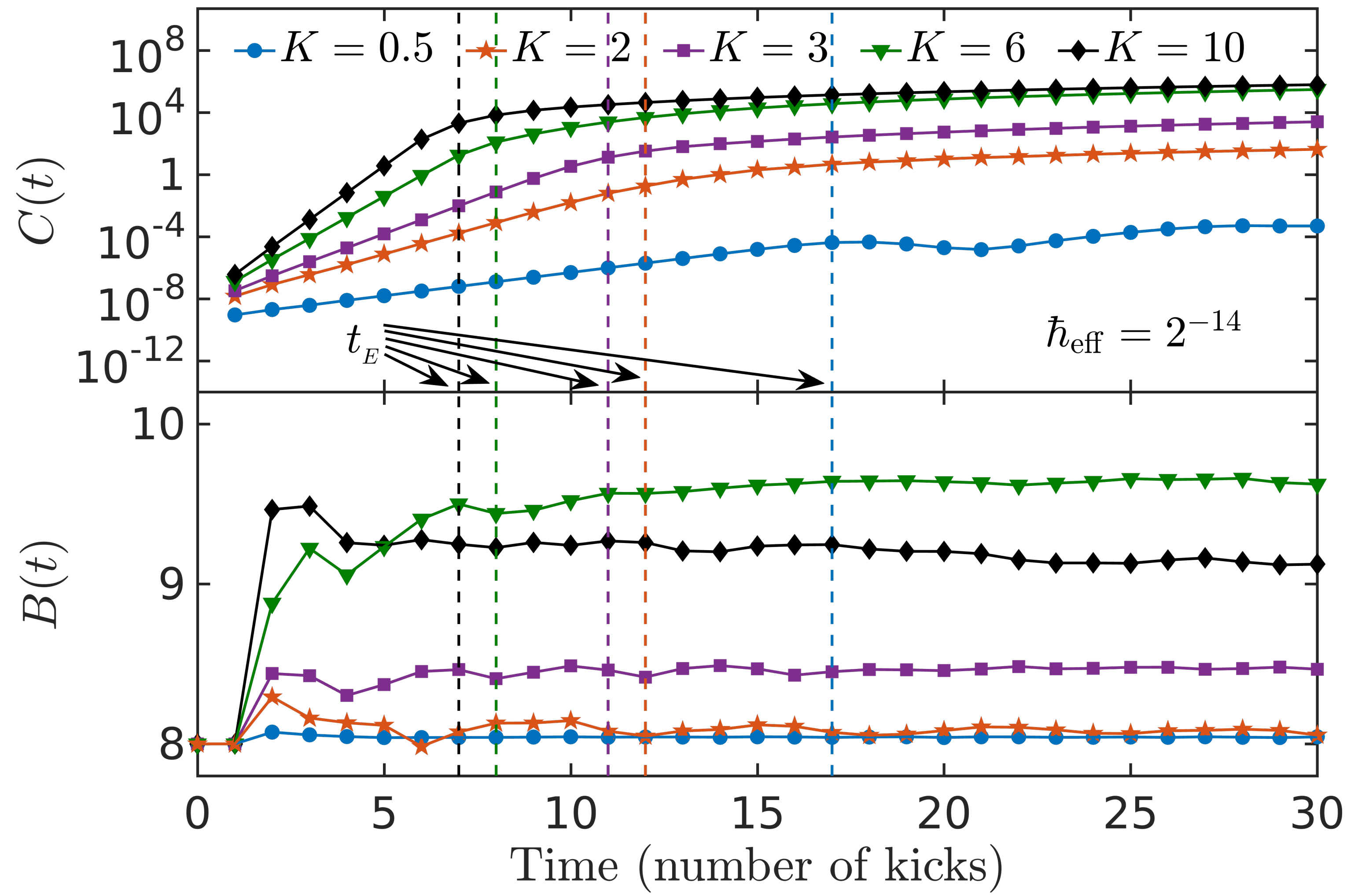
OTOC for quantum kicked rotor

$$|\psi(t+1)\rangle = e^{i\frac{\hat{p}^2}{2m}} e^{iK\cos\hat{\theta}} |\psi(t)\rangle$$

$$C(t) = -\langle [p(t), p(0)]^2 \rangle$$

$$B(t) = -Re\langle p(t)p(0)\rangle$$

$$t_d \sim \ln K/2 \qquad t_{Ehrenfest} \sim \frac{|\ln \hbar_{eff}|}{\ln K/2}$$



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Exponential growth of OTOC is indeed present between the two time scales

Classical to quantum crossover from growth rate

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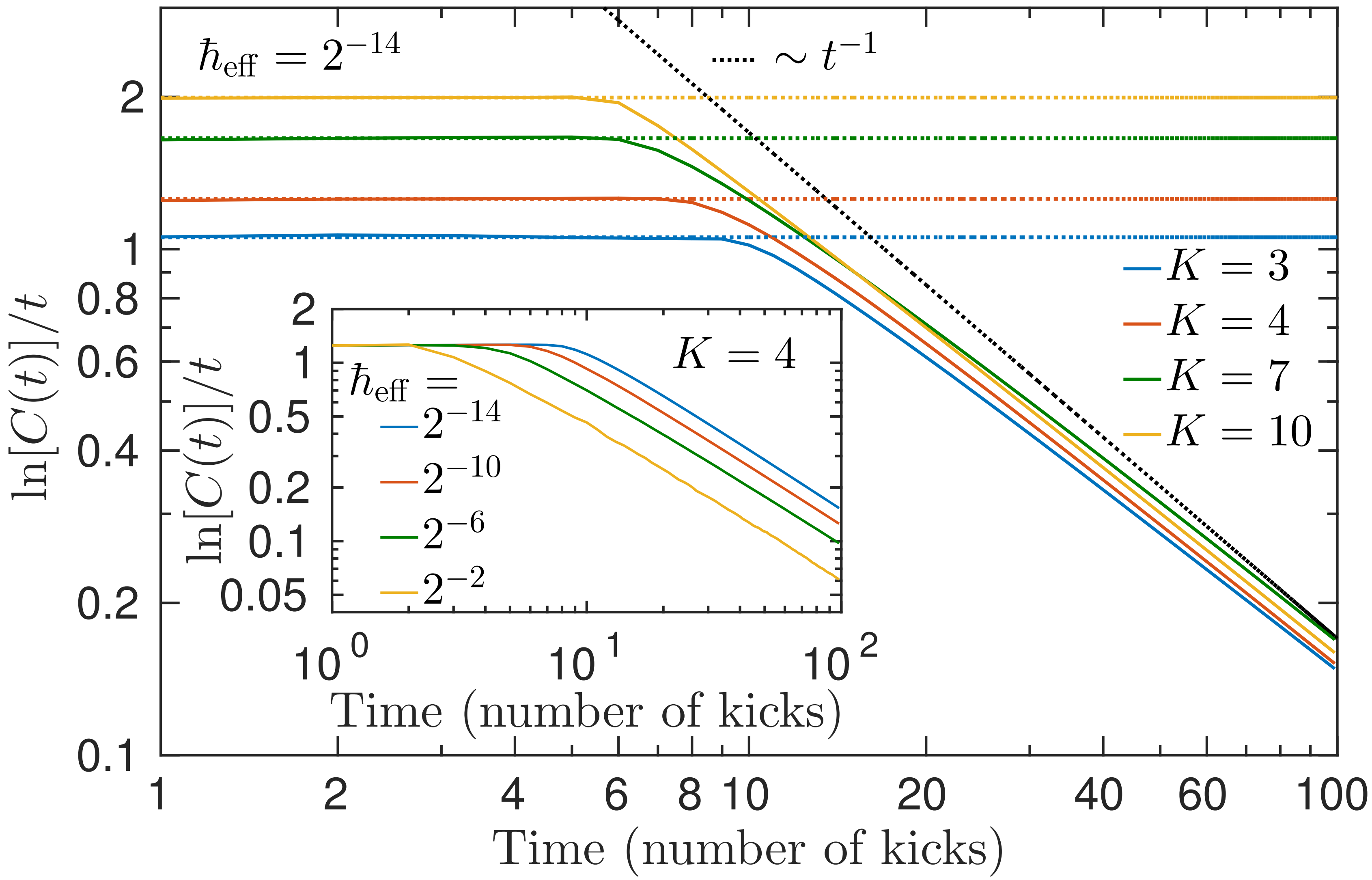

$t_d \sim \ln K/2$

$t_{Ehrenfest} \sim \frac{|\ln \hbar_{eff}|}{\ln K/2}$

OTOC growth rate post Ehrenfest time slows down due to quantum interference effects (weak localization physics)!!!

Aleiner, Larkin, 1996 Tian, Kamenev, Larkin 2005

Lyapunov exponent and OTOC growth rate

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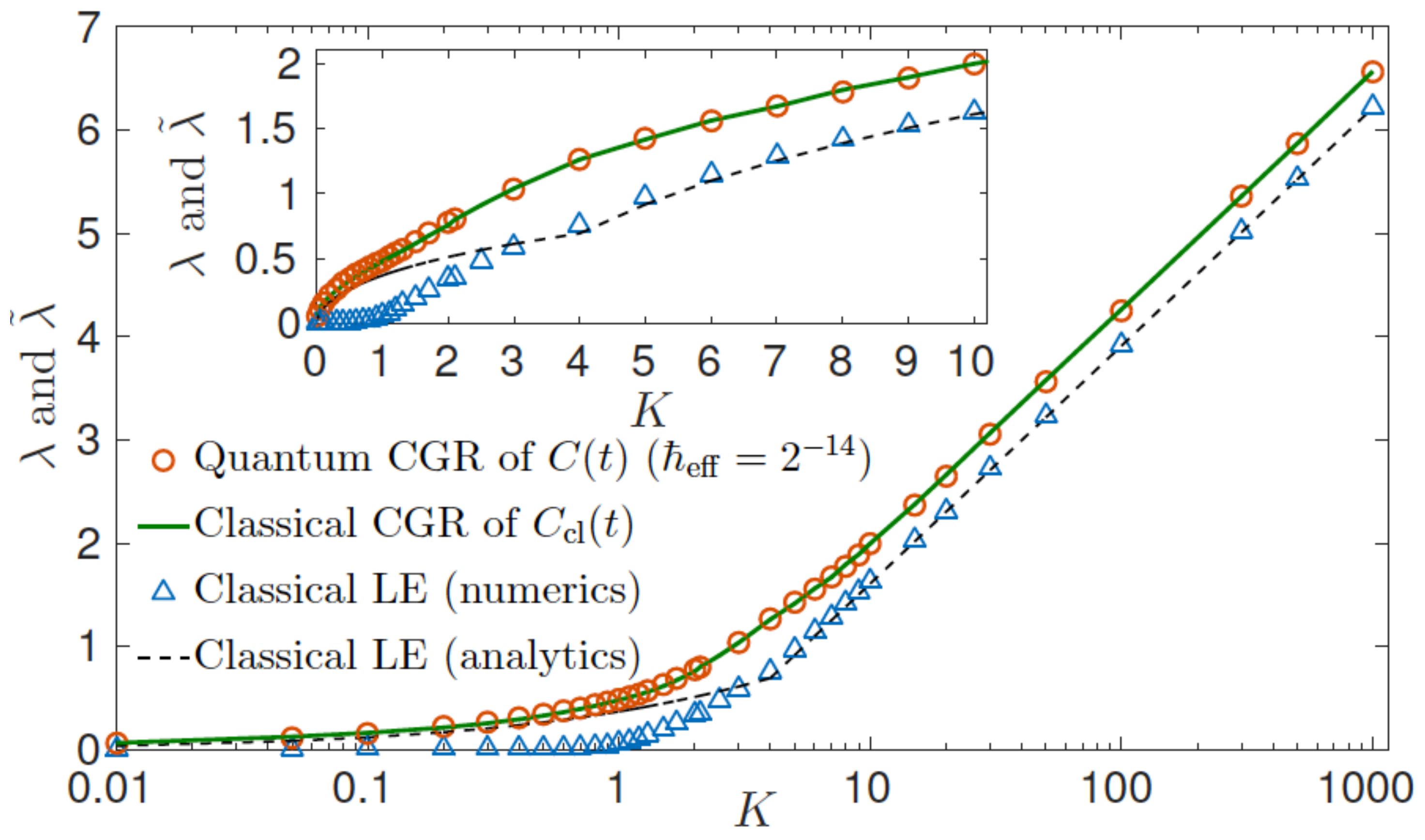
$$\lambda_{cl} = \left\langle \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \ln \frac{d(t)}{d(0)} \right\rangle_{\text{phase space}}$$

$$\lambda_{cl} \sim \ln K/2 \, (K > 4)$$

$$C(t) = -\langle [p(t), p(0)]^2 \rangle$$

$$C_{cl}(t) = -\hbar_{eff}^2 \langle \{p(t), p(0)\}^2 \rangle_{\text{phase space}}$$



Remark: OTOC diagnoses local phase space chaos!

Signature of a classical transition in two point function

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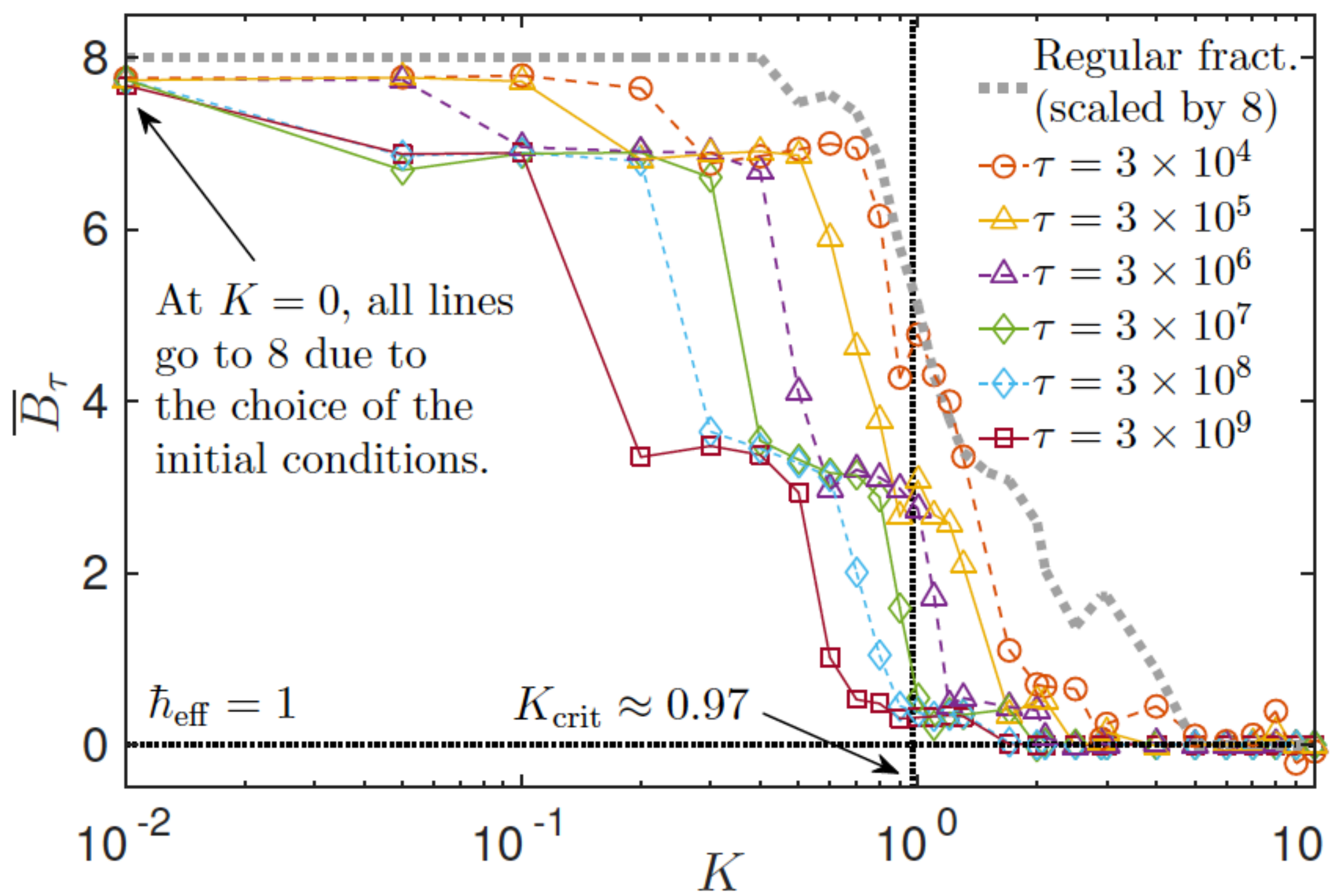
$$\bar{B}_\tau = \sum_{t=0}^{\tau} Re\langle p(t)p(0)\rangle$$

$$t_d >> t_{Ehrenfest}$$



Summary and Future direction

Using OTOC we extracted “Lyapunov exponent” like exponent , a key fingerprint of chaos in a quantum calculation of QKR

We quantitatively understand the difference between the OTOC growth rate and classical Lyapunov exponent

We would like to extend this analysis to static chaotic systems where a thermal expectation has a bound on the OTOC growth rate

Extract this exponential growth of OTOC from the effective field theory of QKR ala Altland and Zirnbauer 98