Lévy flight of Photons in Hot Atomic Vapors

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CLXXIII International School of Physics "Enrico Fermi"
"Nano optics and atomics: transport of light and matter waves«
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Overview of this lecture :

1. Multiple Scattering of Light in Atomic Vapors
   - 1.1 Scattering Properties of Atoms
   - 1.2 Radiation Trapping of Light in Cold Atoms

2. Lévy Flight of Photons in Hot Atomic Vapors
   - 2.1 Random Walk
   - 2.2 Radiation Trapping of Light in Hot Atoms
   - 2.3 Step Size distribution of Photons

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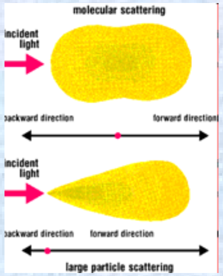
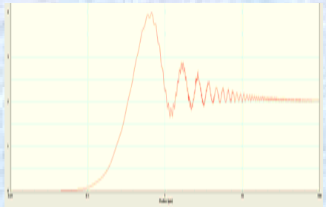
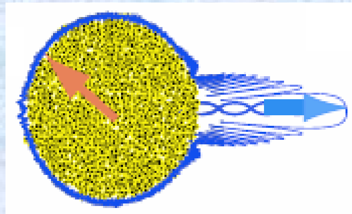
1.1 Scattering Properties of Atoms

• Rayleigh scattering (elastic process) $\propto \omega^4$

• Mie scattering (elastic process)



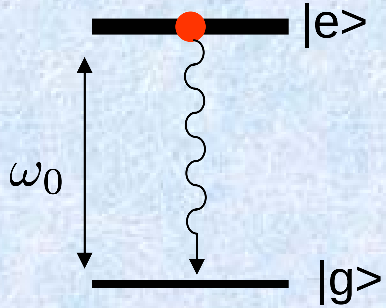


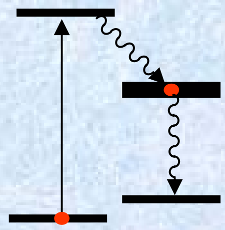


• **Resonant scattering** (elastic and inelastic)
atoms at rest (☺)
moving atoms (Doppler)
multilevel atoms (Raman)

Photon Emission

Spontaneous Emission : $\Gamma = 1/\tau_{nat}$



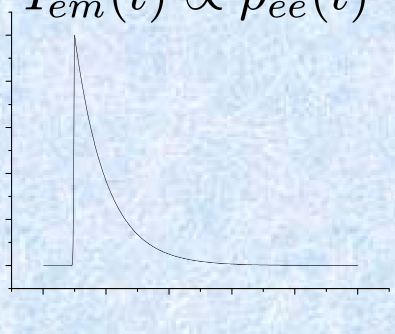


<u>Optical Bloch Equation</u>
(RWA)

$$\dot{\tilde{\rho}}_{ee} = -\Gamma \tilde{\rho}_{ee}$$
$$\dot{\tilde{\rho}}_{eg} = -\frac{\Gamma}{2} \tilde{\rho}_{eg}$$
$$\dot{\tilde{\rho}}_{gg} = \Gamma \tilde{\rho}_{ee}$$

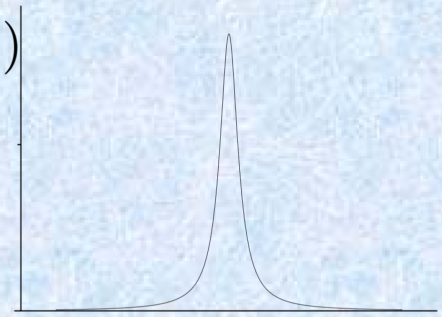
initial condition $\tilde{\rho}_{ee}(t=0) = 1$

$I_{em}(t) \propto \tilde{\rho}_{ee}(t)$



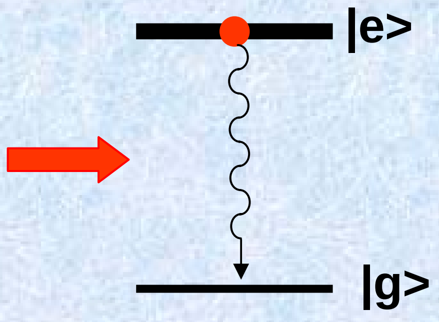
$$I_{em}(t) \propto e^{-t/\tau_{nat}}$$
$$I_{em}(\omega) \propto \frac{1}{1+4\delta^2/\Gamma^2}$$

Fourier Transform



Photon Emission

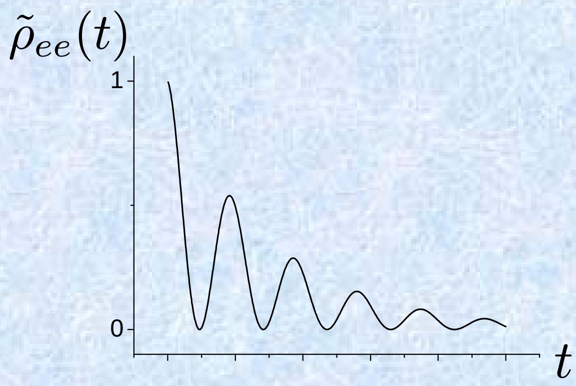
Stimulated Emission



<u>Optical Bloch Equation</u>
(RWA)

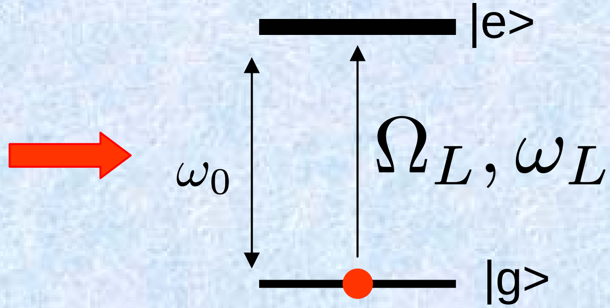
$$\dot{\tilde{\rho}}_{ee} = -\Gamma\tilde{\rho}_{ee} + i\frac{\Omega_L}{2}(\tilde{\rho}_{eg} - \tilde{\rho}_{ge})$$
$$\dot{\tilde{\rho}}_{ge} = -(i\delta_L + \frac{\Gamma}{2})\tilde{\rho}_{ge} - i\frac{\Omega_L}{2}(\tilde{\rho}_{ee} - \tilde{\rho}_{gg})$$
$$\dot{\tilde{\rho}}_{gg} = \Gamma\tilde{\rho}_{ee} - i\frac{\Omega_L}{2}(\tilde{\rho}_{eg} - \tilde{\rho}_{ge})$$

initial condition $\tilde{\rho}_{ee}(t=0) = 1$



$$\hbar\Omega_L = -\mathbf{dE}$$
$$s_0 = \frac{2\Omega_L^2}{\Gamma^2} = \frac{I}{I_{sat}}$$

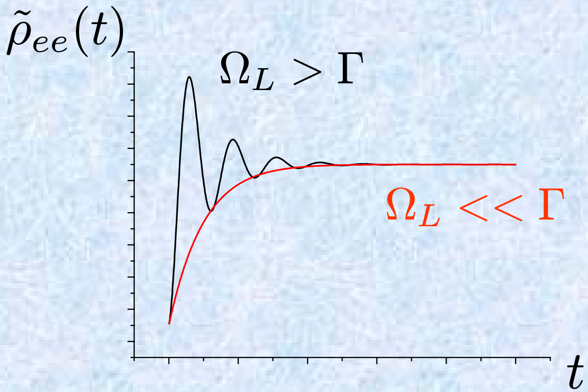
Photon Absorption



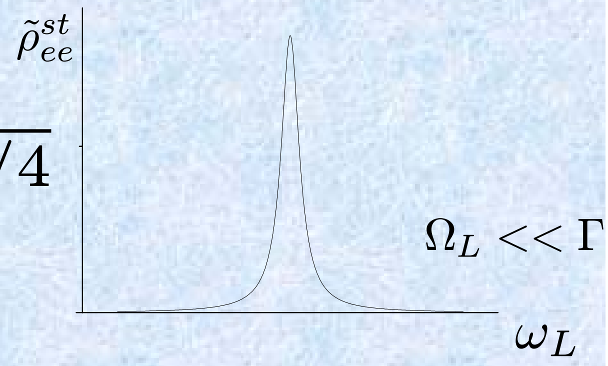
Optical Bloch Equation

$$
\begin{aligned}
\dot{\tilde{\rho}}_{ee} &= -\Gamma\tilde{\rho}_{ee} + i\frac{\Omega_L}{2}(\tilde{\rho}_{eg} - \tilde{\rho}_{ge}) \\
\dot{\tilde{\rho}}_{ge} &= -(i\delta_L + \frac{\Gamma}{2})\tilde{\rho}_{ge} - i\frac{\Omega_L}{2}(\tilde{\rho}_{ee} - \tilde{\rho}_{gg}) \\
\dot{\tilde{\rho}}_{gg} &= \Gamma\tilde{\rho}_{ee} - i\frac{\Omega_L}{2}(\tilde{\rho}_{eg} - \tilde{\rho}_{ge})
\end{aligned}
$$

initial condition $\tilde{\rho}_{gg}(t=0) = 1$



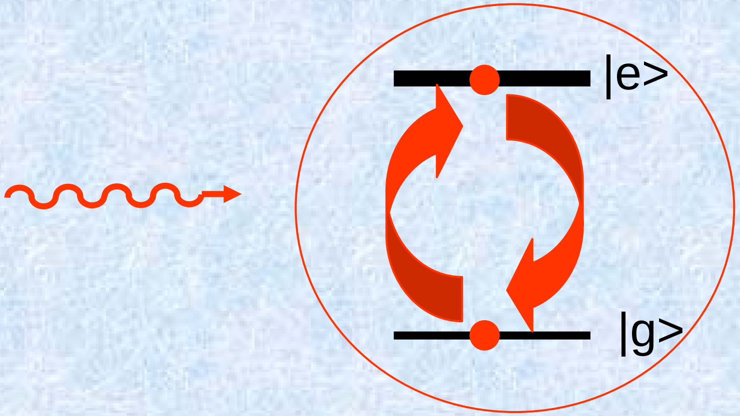
$$\tilde{\rho}^{st}_{ee} = \frac{\Omega_L^2/2}{\delta^2 + \Omega_L^2/2 + \Gamma^2/4}$$

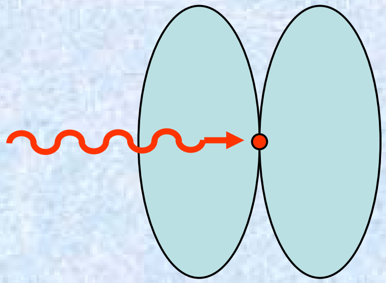


absorption spectrum = emission spectrum

Photon Scattering

scattering of photon ≠ absorption + emission !!!





Induced dipole $< \mathbf{d} >= \mathbf{Tr}(\rho \mathbf{d}) = ||\mathbf{d}||(\rho_{\mathbf{ge}} - \rho_{\mathbf{eg}})$

$$\mathbf{d} = ||\mathbf{d}||(|\mathbf{e} >< \mathbf{g}| + |\mathbf{g} >< \mathbf{e}|)$$

$$\mathbf{d} = \alpha \mathbf{E_L}$$

$$\alpha = Re(\alpha) + iIm(\alpha)$$

index of refraction : $n = 1 - \rho \frac{6\pi}{k^3} \frac{\delta/\Gamma}{1+4(\delta/\Gamma)^2} + i\rho \frac{6\pi}{k^3} \frac{1}{1+4(\delta/\Gamma)^2}$

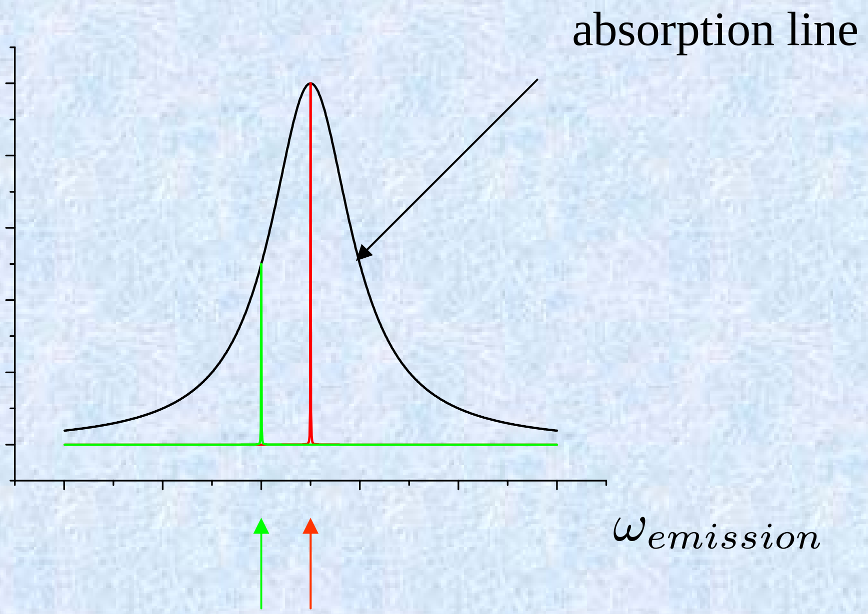
$n - 1 < 10^{-4}$ (MOT)
$n - 1 < 0.1$ (BEC)

Scattered field $E_{sc} \propto < \mathbf{d} >$

induced dipole with precise phase relation to incident field
scattered field with precise phase relation to induced dipole
} ⇒ elastic scattering

Photon Scattering

Spectrum of Scattered Field



elastic scattering for

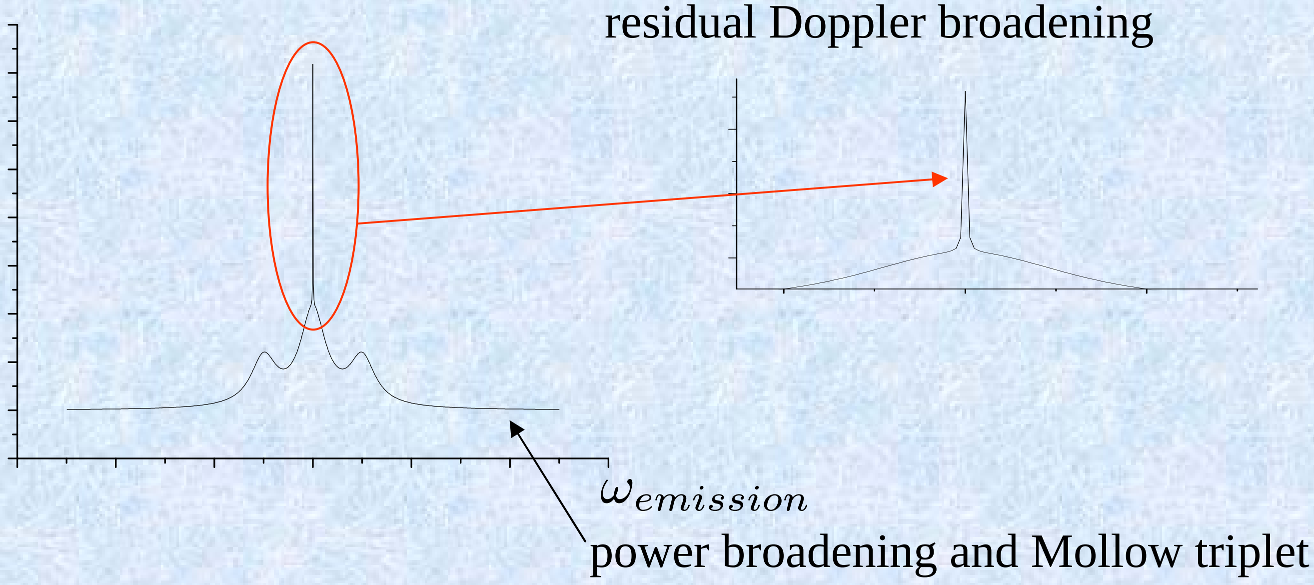
cold atoms ($kv \ll \Gamma$)

low incident intensity ($I \ll I_{sat}$)

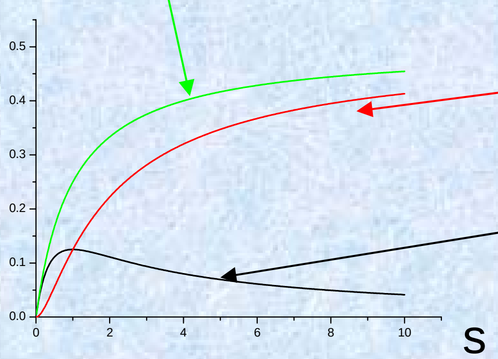
no internal structure (Raman scattering)

Photon Scattering

Inelastic Scattering



$$I_{sc} = I_{coh} + I_{incoh} \propto \frac{1}{2}\frac{s}{1+s}$$



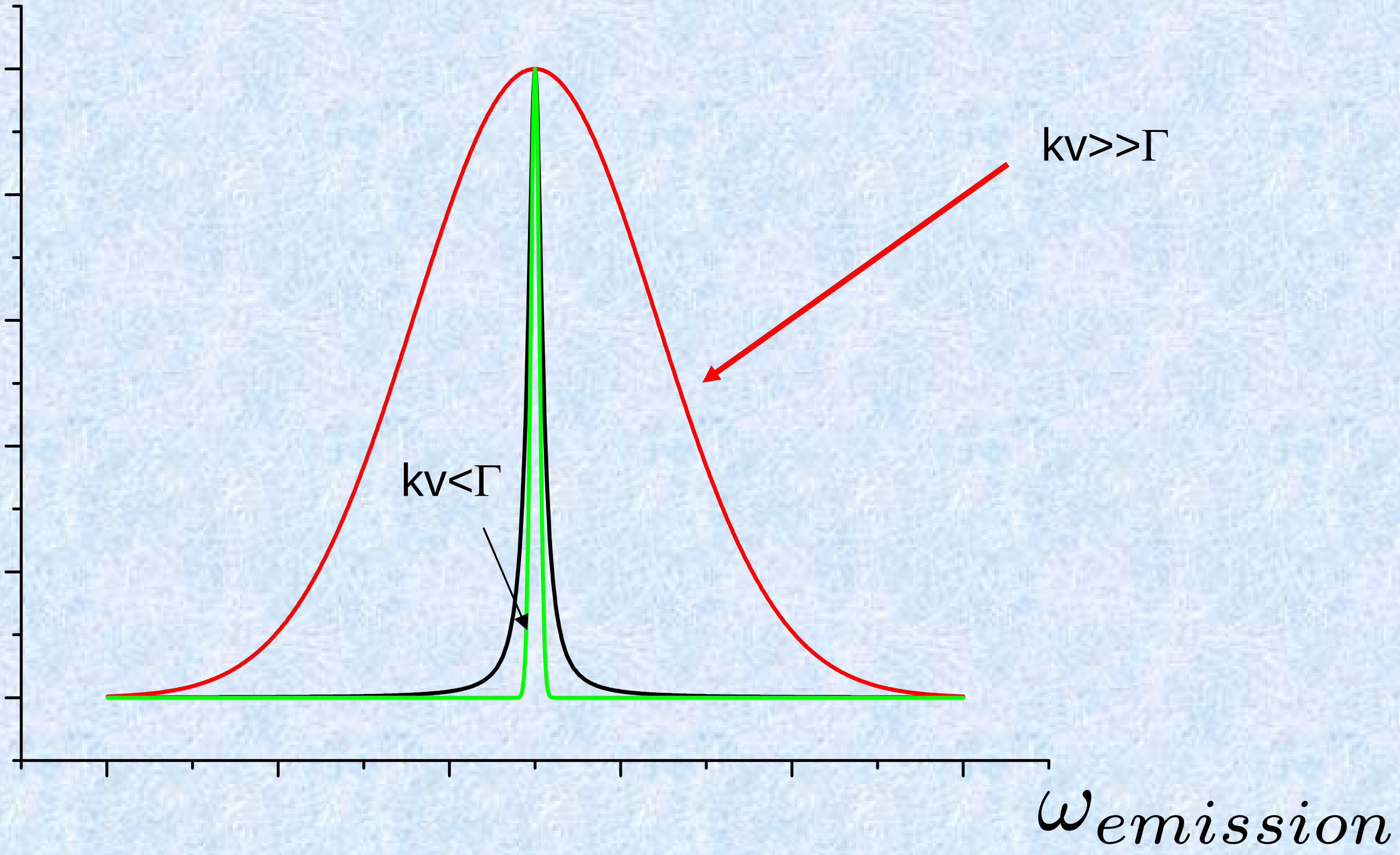
$$I_{incoh} \propto \frac{1}{2}\frac{s^2}{(1+s)^2}$$

$$I_{coh} \propto \frac{1}{2}\frac{s}{(1+s)^2}$$

Photon Scattering

Inelastic Scattering

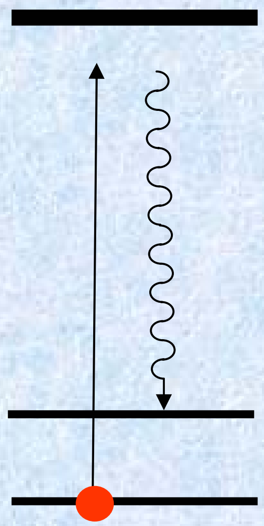
Hot Atoms : Doppler broadening



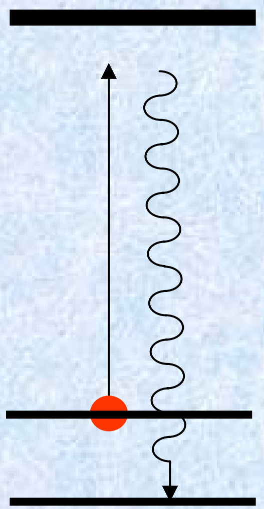
Photon Scattering

Inelastic Scattering

Internal Structure : Raman transitions



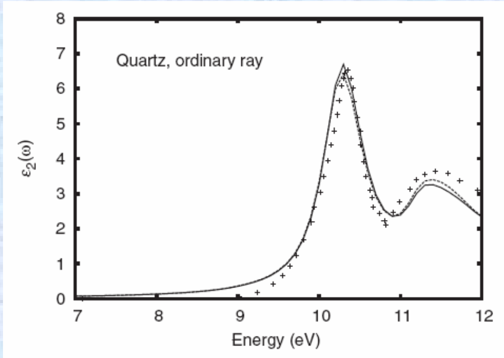
Raman Stokes

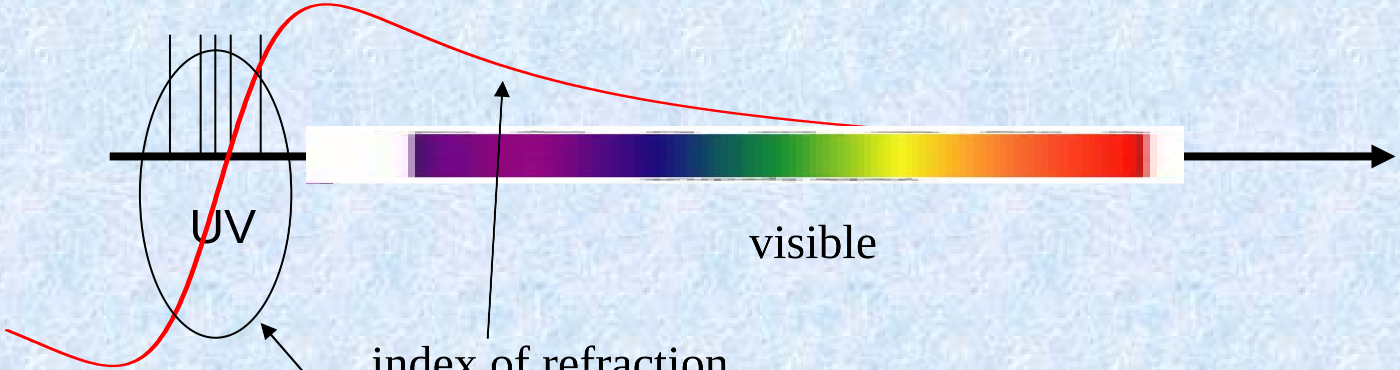


Raman anti-Stokes

From Atoms to condensed matter

Optical Resonances of glass





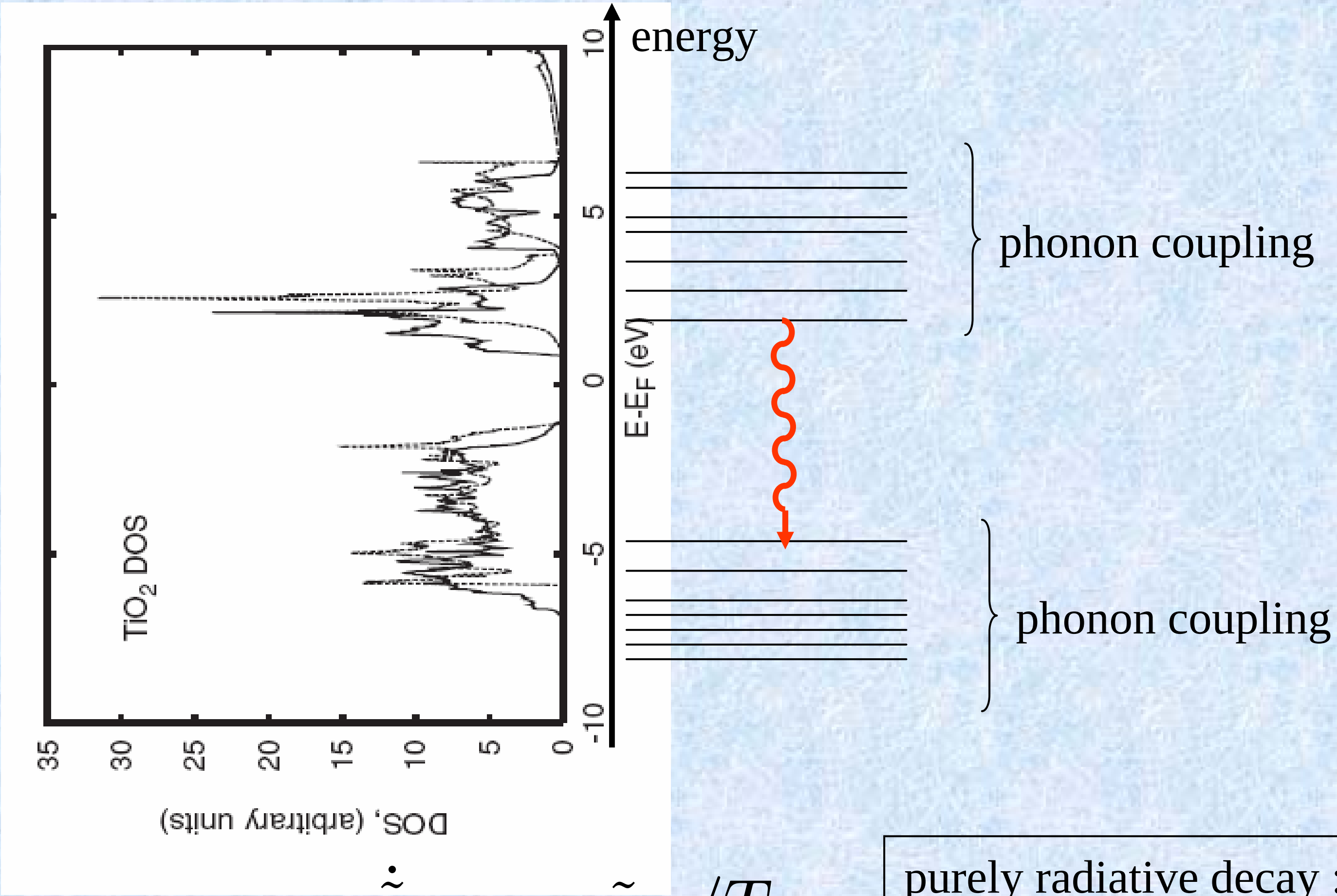
visible

index of refraction

‘anomolous dispersion’

From Atoms to condensed matter

Energy levels



$$\dot{\tilde{\rho}}_{ee} = -\tilde{\rho}_{ee}/T_1$$
$$\dot{\tilde{\rho}}_{eg} = -\tilde{\rho}_{eg}/T_2$$
$$\dot{\tilde{\rho}}_{gg} = \tilde{\rho}_{ee}/T_1$$

purely radiative decay :	$T_1 = T_2/2 = 1/\Gamma$
collisions / phonons :	$T_1 >> T_2$

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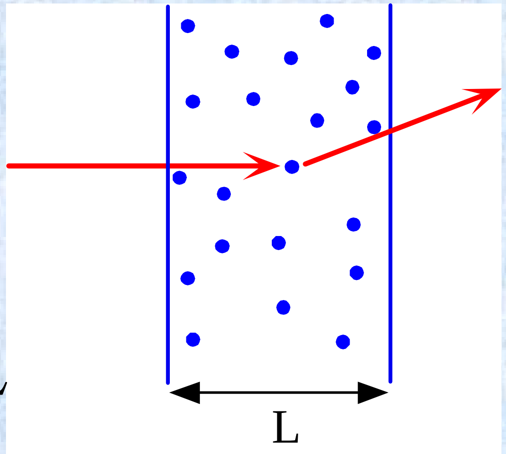
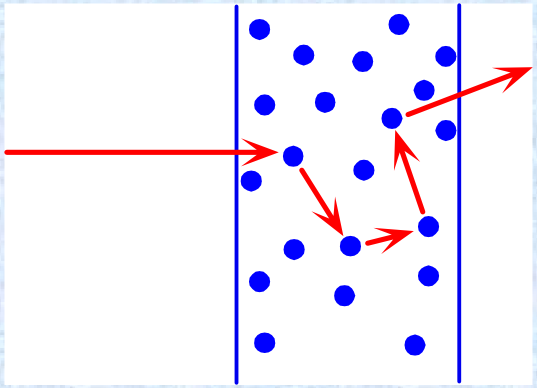
Single scattering (b<<1)



$$\ell_{\text{scat}} = \frac{1}{n\sigma}$$

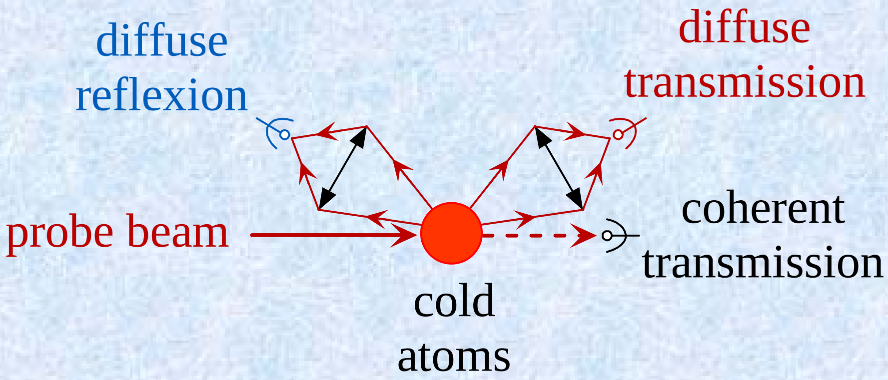
Beer-Lambert law : $T_{coh} = e^{-b} = e^{-n\sigma L}$

Multiple scattering (b>>1)



Ohm's law : $T_{\text{diff}} \propto \ell_{\text{scat}} / L$

'Static' Experiments



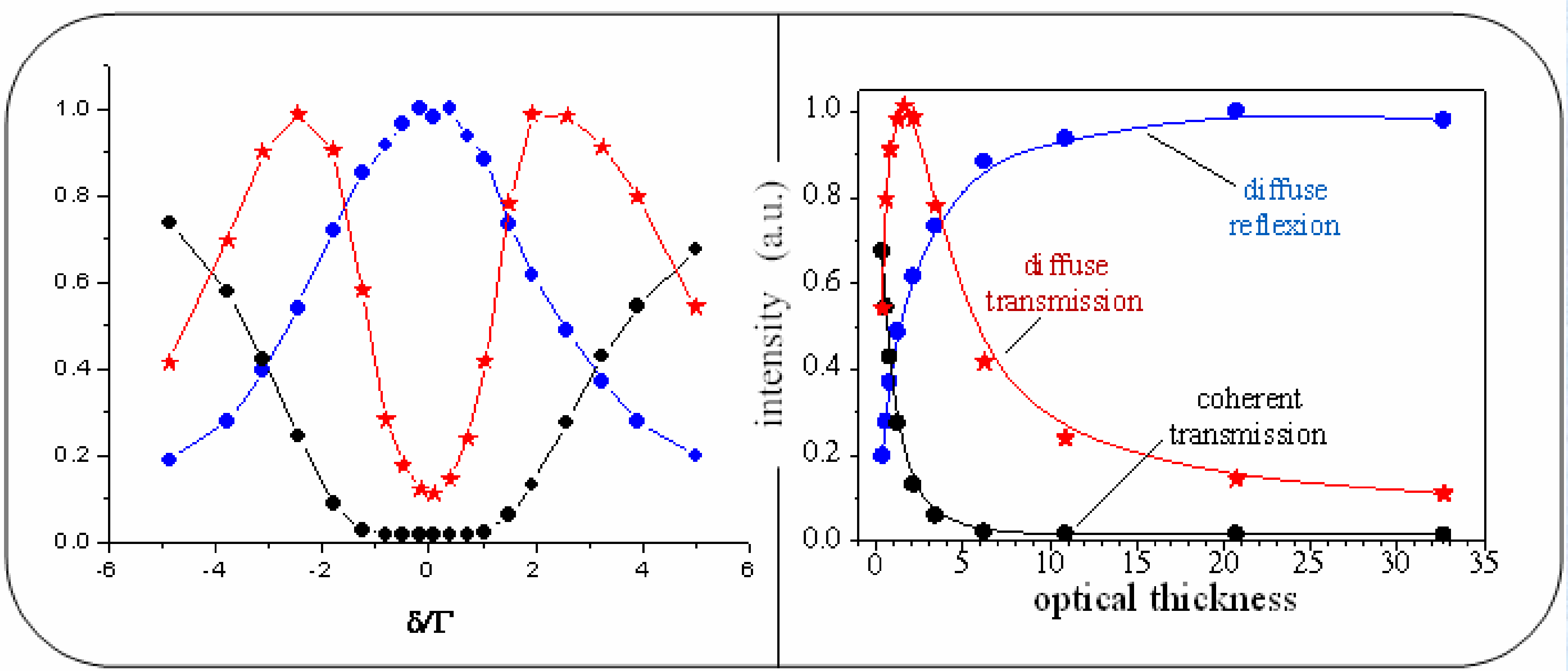
MOT parameters :

MOT diameter ≈ few mm

N ≈ few 10^9

b ≈ 20

velocities ≈ 0.1m/s



Time Resolved Experiments

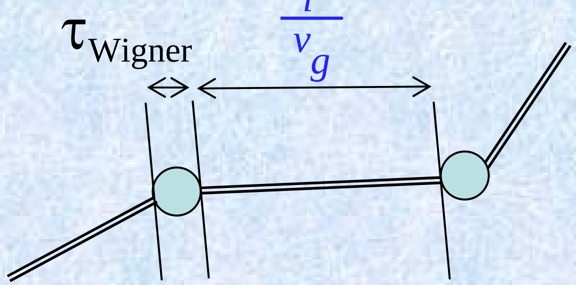
◦ Phase velocity : $c = \frac{c_0}{n}$ propagation of phase for a monochromatic wave

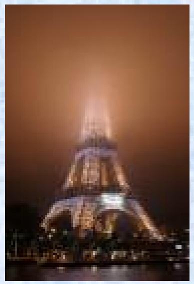
$c > 0 \qquad c \lessgtr c_0$

◦ Group velocity : $v_g = \frac{\partial \omega}{\partial k}$ propagation of transmitted gaussian pulse with slowly varying envelope

cold atoms on resonance : $v_g < 0 \qquad |v_g| << 0$

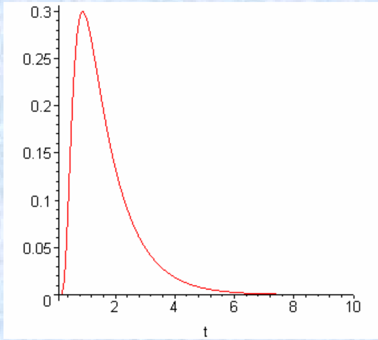
◦ Transport velocity : propagation of scattered wave energy $0 < v_{tr} < c_0$

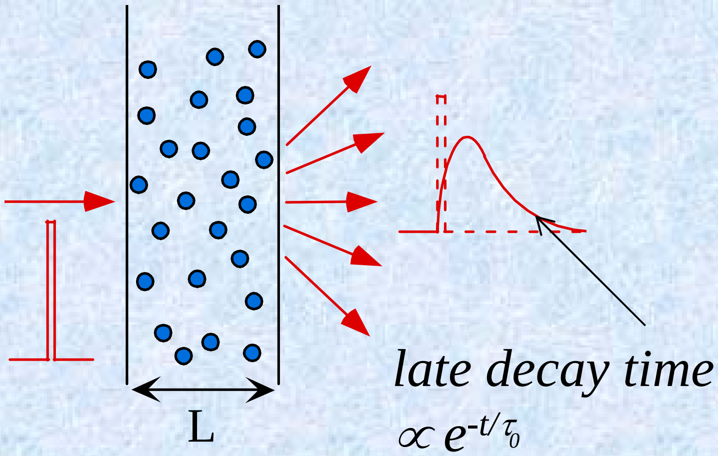




Diffusion Theory :

slab : $T(t) \propto \Sigma_n (-1)^{n+1} n^2 e^{-n^2\pi^2 Dt/L^2}$





$$D = \frac{1}{3}\frac{l_{tr}^2}{\tau_{tr}}$$

transport mean-free path (l_{tr})

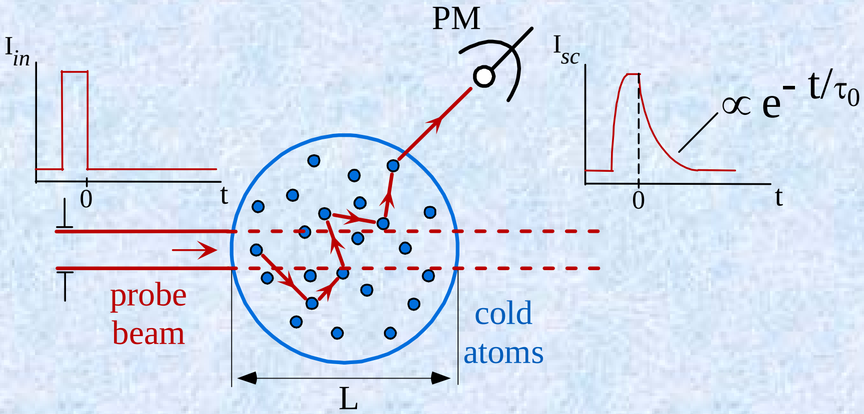
transport time (τ_{tr})

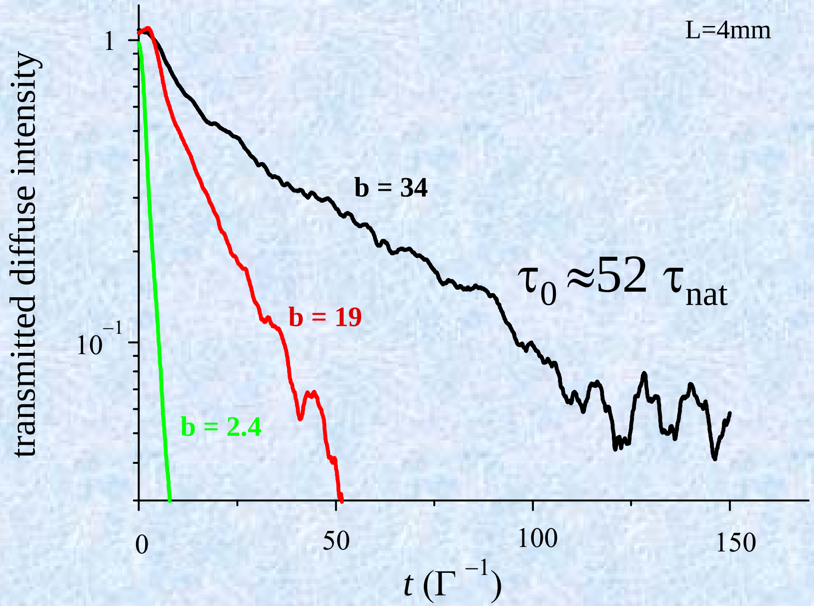
$$= \frac{1}{3} l_{tr} v_{tr}$$

transport velocity (v_{tr})

$$\tau_0 \approx \frac{L^2}{\pi^2 D} = \frac{3}{\pi^2} b^2 \tau_{tr}$$

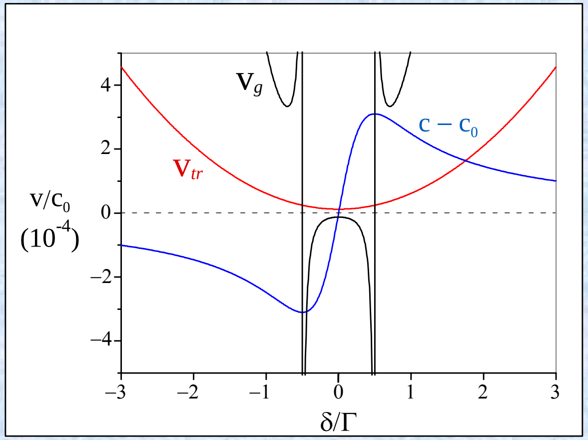
$$b = \frac{L}{l}$$ optical thickness





$$\tau_0 \approx \frac{L^2}{\pi^2 D} \Rightarrow \quad D \approx 0.66 m^2/s$$

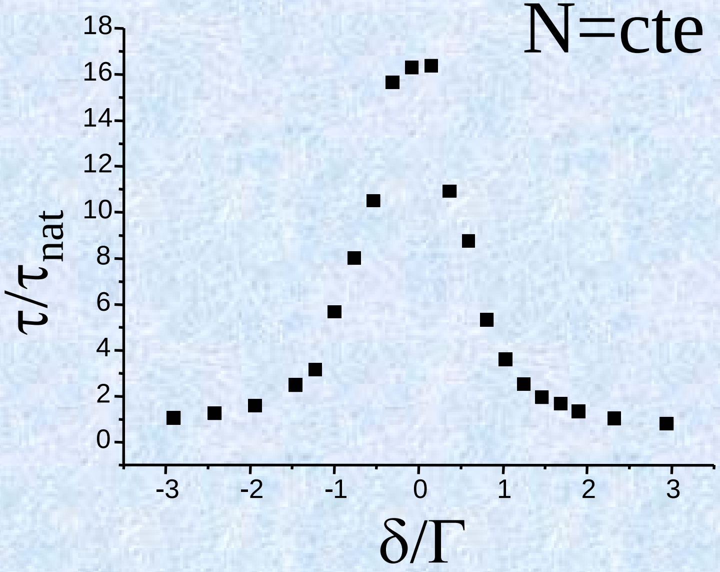
$$\frac{V_{tr}}{c_0} = \frac{\ell_{tr}}{c_0 \tau_{tr}} \approx 3 \cdot 10^{-5}$$

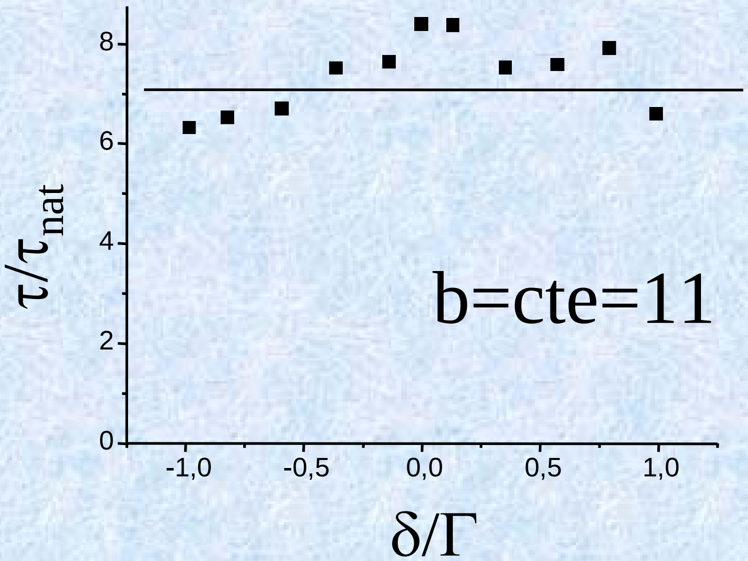


D : smaller than in TiO_2 with kl ≈1 (D≈4m^2/s)

NOT due to interferences in multiple scattering (≠ Localization)

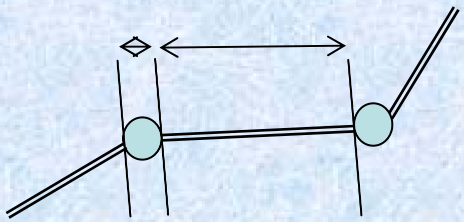
$$\tau_0 \approx \frac{L^2}{\pi^2 D} = \frac{3}{\pi^2}\, b^2\, \tau_{tr}$$



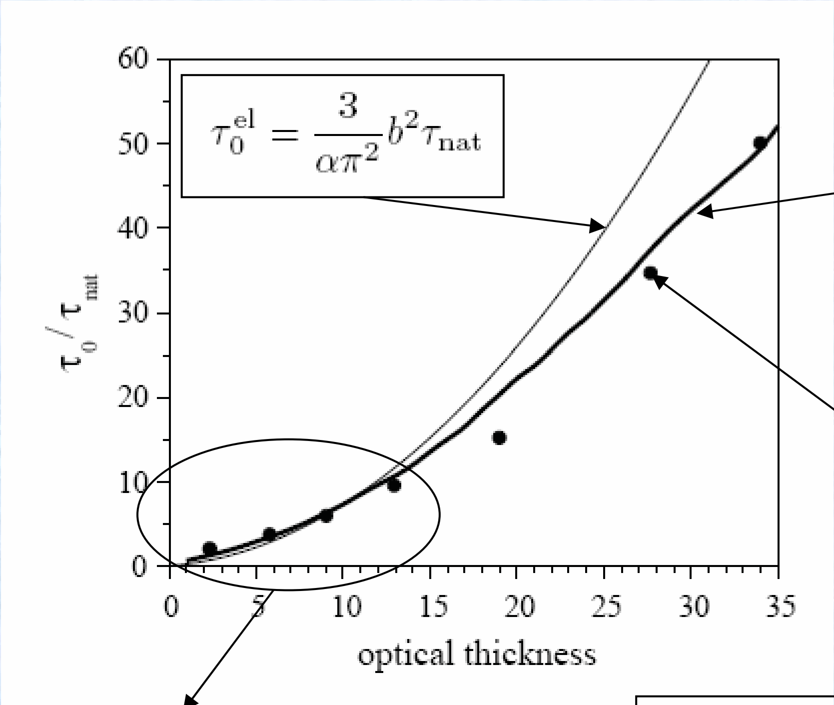


Transport Time :
~Independent of δ

$$\tau_{tr} \approx \tau_{Wigner}(\delta) + \frac{\ell(\delta)}{v_{gr}(\delta)}$$



From Coherent to Incoherent Radiation Trapping



Monte Carlo Simulation

Experiments

$kv <<<< \Gamma$

atoms at rest

atomic motion $\rightarrow$ 3 regimes :

0 — 1 / b — 1 — kv / Γ

elastic

$\tau_0 \propto b^2 \tau_{tr}$

frequency diffusion

$\tau_0 = f\,(b\ kv/\Gamma)$

Doppler

$\tau_0 \propto b\ [\log(b)]^{1/2}$

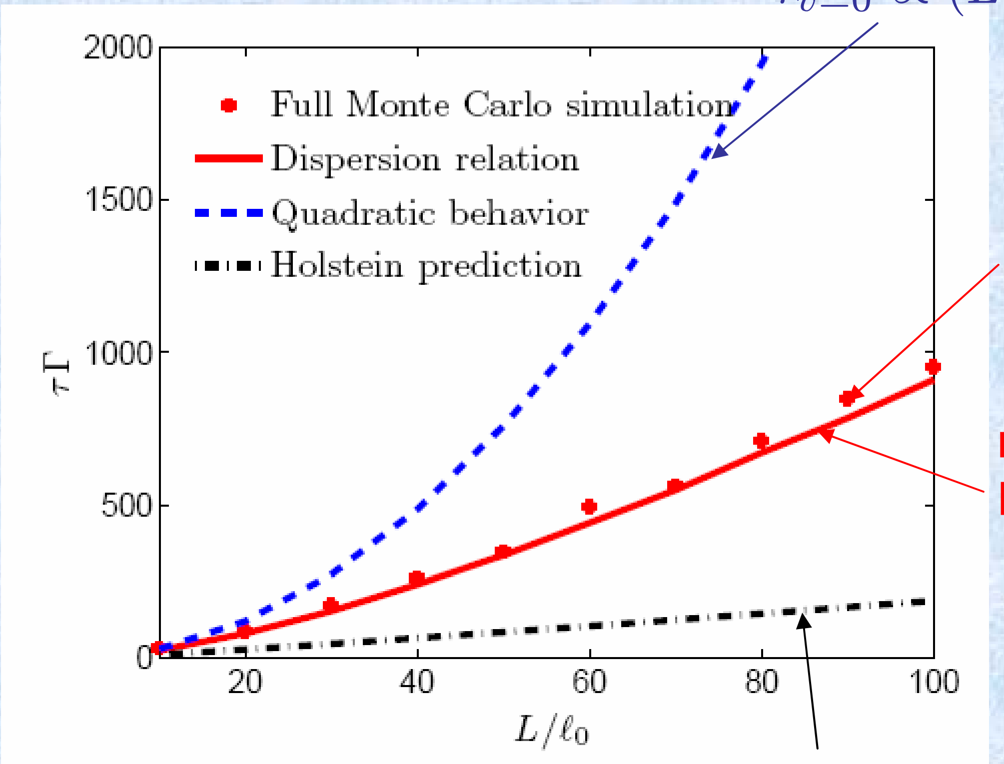
From Coherent to Incoherent Radiation Trapping

± cold atoms : Doppler shift of scattered photons

$$\Delta\nu \sim \sqrt{D_\nu t} \sim \sqrt{kv^2 b^2 \tau_{nat}} << \Gamma$$

$$\tau_{v=0} \propto (L/l_0)^2 \Gamma^{-1}$$

elastic scattering



Monte Carlo Simulation

modal expansion of Radiative Transfer Equation

from R. Pierrat et al., arXiv:0904.0936v1

$$\tau_{Holstein} \propto (L/l_0)\sqrt{log[L/(2l_0)]}\Gamma^{-1}$$

CFR : complete frequency redistribiution

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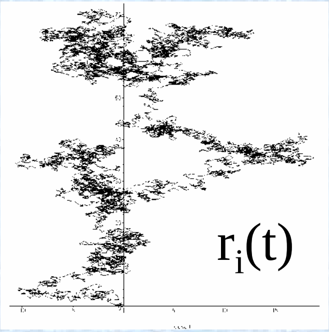
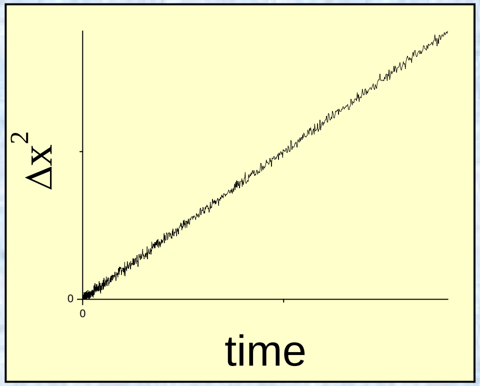
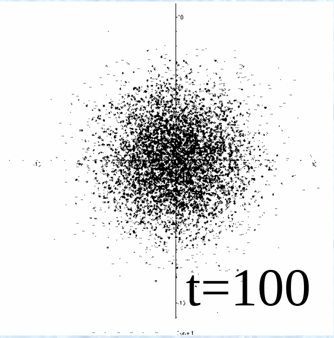
Mean Free Path -- Diffusion

Normal Diffusion : $\Delta x^2 = \frac{1}{2} Dt$ $\quad$ $D \propto \frac{l^2}{\tau}$







For a step size distribution P(x) :

$$l = < x > = \frac{\int dx x P(x)}{\int dx P(x)} \qquad D = < x^2 > = \frac{\int dx x^2 P(x)}{\int dx P(x)}$$

Example :

$$\mathrm{P(x)} \propto exp(-x/l_0) \Rightarrow l = < x > = l_0$$

$$D \propto < x^2 > \propto l_0^2$$

if $< x >$ and $< x^2 >$ are finite
the distribution of a large number of steps Σx_i
converges to a Gaussian distribution
(Central Limit Theorem)

Random Walk of Step Size Distribtion

$P(x) \propto 1/x^{\alpha}$

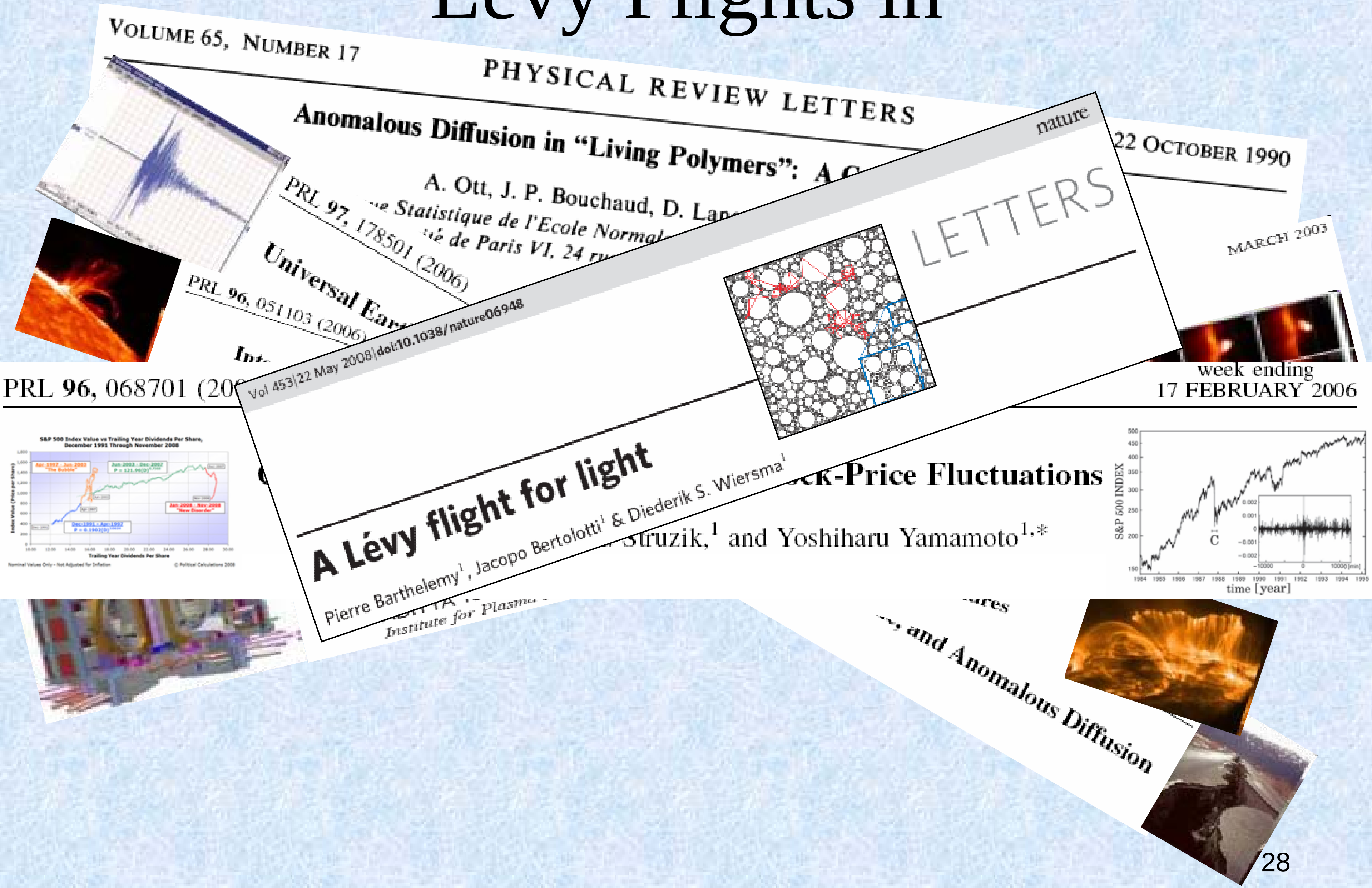
non stationnary	'ballistic'	'super-diffusive'	normal
$\int P(x)dx=\infty$	$\langle \mathbf{x} \rangle = \infty$	$\langle \mathbf{x}^2 \rangle = \infty$	$\langle \mathbf{x}^2 \rangle < \infty$

1 2 3 α

← Lévy Flights (for $\alpha < 3$)

how to measure $\langle x \rangle \pm \sqrt{\langle x^2 \rangle}$: weak ergodicity breaking

Lévy Flights in



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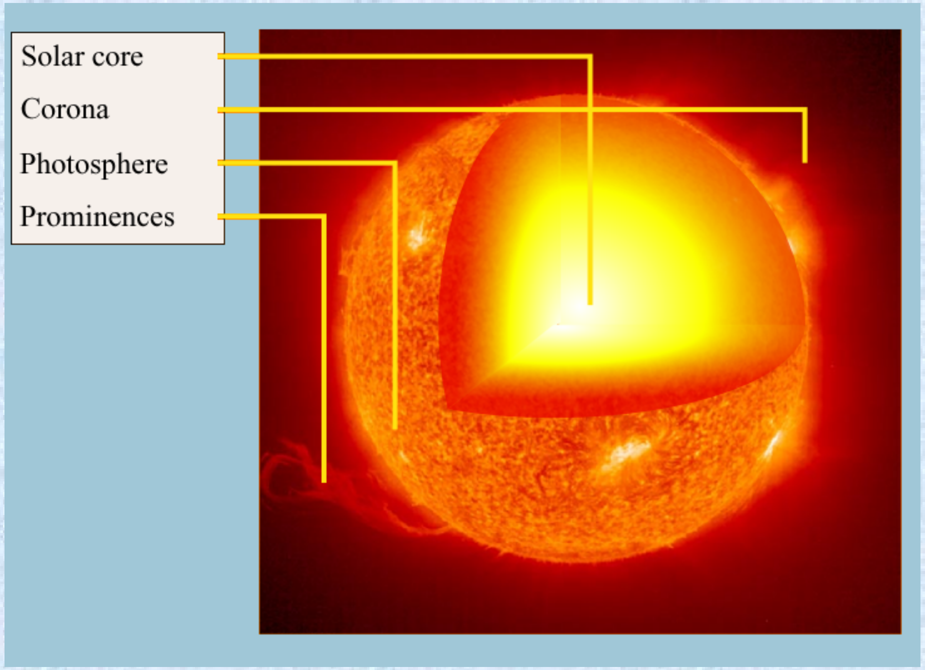
Multiple Scattering of Light in **Hot** Atomic vapors :



Random walk of photons / Radiation trapping in

- dense atomic vapours
- discharge
- hot plasmas
- gas lasers
- stars
- intergalactic scattering

Milne Equation

$$\nabla^2[n(\mathbf{r},\mathbf{t}) + \tau \frac{\partial \mathbf{n}(\mathbf{r},\mathbf{t})}{\partial \mathbf{t}}] = \mathbf{4\bar{k}^2}\tau \frac{\partial \mathbf{n}(\mathbf{r},\mathbf{t})}{\partial \mathbf{t}}$$

diffusion equation for excited state population (and photons)

nice idea :

estimatation of **photon escape time from the sun**

L_{sun} = 10^6 km = 10^9 m and l = 1mm = 0.001m

D=lc/3 L_{sun}^2 = D t_{escape} t_{escape} = 3 10^{18}/ (10^{-3} 3 10^8) = 10^{13}s = **300 000 years**

nice but : wrong !!! (assumes box-shaped atomic lineshape : no wings)
(±used in many estimation of photon life time in the sun I have seen)

A random walk of photons in hot atomic vapours
is NOT correctly described by a diffusion equation

DECEMBER 15, 1932 *PHYSICAL REVIEW* *VOLUME 42*

On Radiation Diffusion and the Rapidity of Escape of Resonance Radiation from a Gas

By Carl Kenty
General Electric Vapor Lamp Company, Hoboken, N. J.

(Received August 25, 1932)

The radiation diffusion process is considered from the standpoint of the free paths of the diffusing resonance quanta as influenced by the Doppler and other line broadening effects. Abnormally long free paths are found to be of such importance as to enable resonance radiation to escape from a body of gas faster than has usually been supposed. It is assumed that a large concentration of diffusing resonance quanta will, on the basis of Doppler broadening only, give rise to a characteristic excitation of atoms, as dependent on their speeds, which can be represented by a distribution function which will lie between two limiting distribution functions, namely (1) Maxwell's distribution function and (2) a distribution function expressing a lower relative excitation of the high speed atoms than that of Maxwell, based on the excitation of all atoms as if by absorption of the core of the line. On the basis of (1) and (2), limiting expressions are derived for: (a) the fraction of emitted quanta traversing at least a given distance before absorption, (b) the diffusion coefficient, (c) the average square free path, (d) the average free path. A fundamental difference between radiation diffusion and molecular diffusion appears in that whereas (a) decreases exponentially with the distance in the latter case it is found to decrease only linearly (roughly) with the distance in the former case. For this reason very long free paths are found to be of relatively great importance in radiation diffusion. It is found that, for a gas container of infinite size, (b), (c), and (d) are all infinite. For a gas container of finite size, esti-

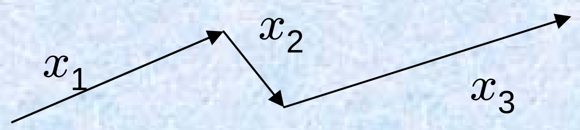
Holstein Equation

$$\frac{\partial n(\mathbf{r},\mathbf{t})}{\partial t} = -\frac{1}{\tau} n(\mathbf{r},\mathbf{t}) + \frac{\mathbf{1}}{\tau}\int_{\mathbf{V}} \mathbf{n}(\mathbf{r}',\mathbf{t})\mathbf{G}(\mathbf{r},\mathbf{r}')\mathbf{dr}'$$

modal expansion of n(**r**,t)   estimation of escape factors of modes

extensivley studied for time dependant photon escape in a large variety of situations

Important ingredient G(r,r') :
i.e. how far flies a photon between two successive scattering events



P(x) = ?

Overview of this lecture :

1. Multiple Scattering of Light in Atomic Vapors
   - 1.1 Scattering Properties of Atoms
   - 1.2 Radiation Trapping of Light in Cold Atoms

2. Lévy Flight of Photons in Hot Atomic Vapors
   - 2.1 Random Walk
   - 2.2 Radiation Trapping of Light in Hot Atoms
   - 2.3 Step Size distribution of Photons

VOLUME 93, NUMBER 12 PHYSICAL REVIEW LETTERS week ending 17 SEPTEMBER 2004

Photon Trajectories in Incoherent Atomic Radiation Trapping as Lévy Flights

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(Received 19 November 2003; published 13 September 2004)

Photon trajectories in incoherent radiation trapping for Doppler, Lorentz, and Voigt line shapes under complete frequency redistribution are shown to be Lévy flights. The jump length (r) distributions display characteristic long tails. For the Lorentz line shape, the asymptotic form is a strict power law $r^{-3/2}$, while for Doppler the asymptotic is $r^{-2}(\ln r)^{-1/2}$. For the Voigt profile, the asymptotic form always has a Lorentz character, but the trajectory is a self-affine fractal with two characteristic Hausdorff scaling exponents.

$$T(x) = \frac{I(L)}{I(0)}$$

$$T(x) = < e^{-x/l(\omega)} > = \int d\omega f_{inc}(\omega) e^{-f_{abs}(\omega)x}$$

$$P(x) = \frac{\partial T(x)}{\partial x} = \int d\omega f_{inc}(\omega) f_{abs}(\omega) e^{-f_{abs}(\omega)x}$$

everything depends on the precise forms of
$l(\omega)$ / $f_{abs}(\omega)$ and $f_{inc}(\omega)$ / $f_{em}(\omega)$

Example : pure Doppler emission and absorption lines
assume $f_{em} = f_{abs}$ (Complete Frequency Distribution)

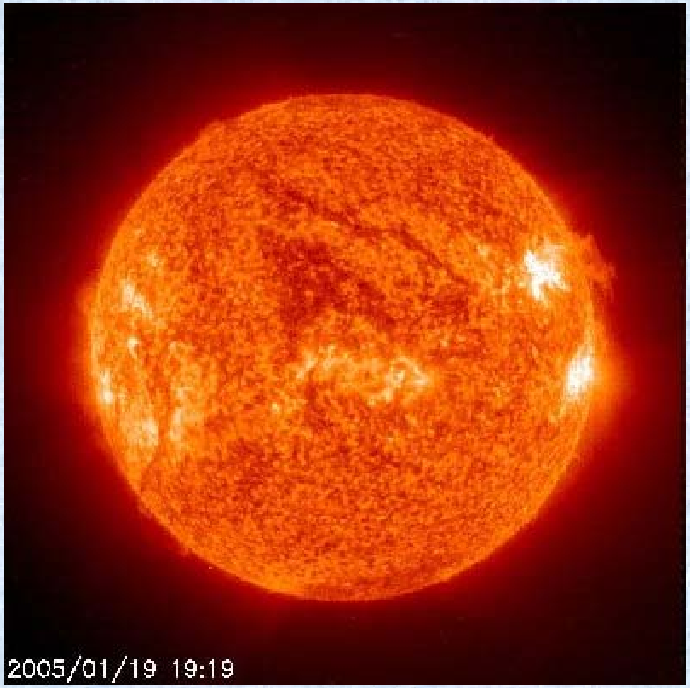
$$\mathrm{P(x)} \propto \frac{1}{x^2\sqrt{Ln(x)}}$$

$$< x > \text{ is finite}$$

$$< x^2 > \propto \int dx x^2 P(x) = \infty$$

⟹ **superdiffusion / Lévy flight**

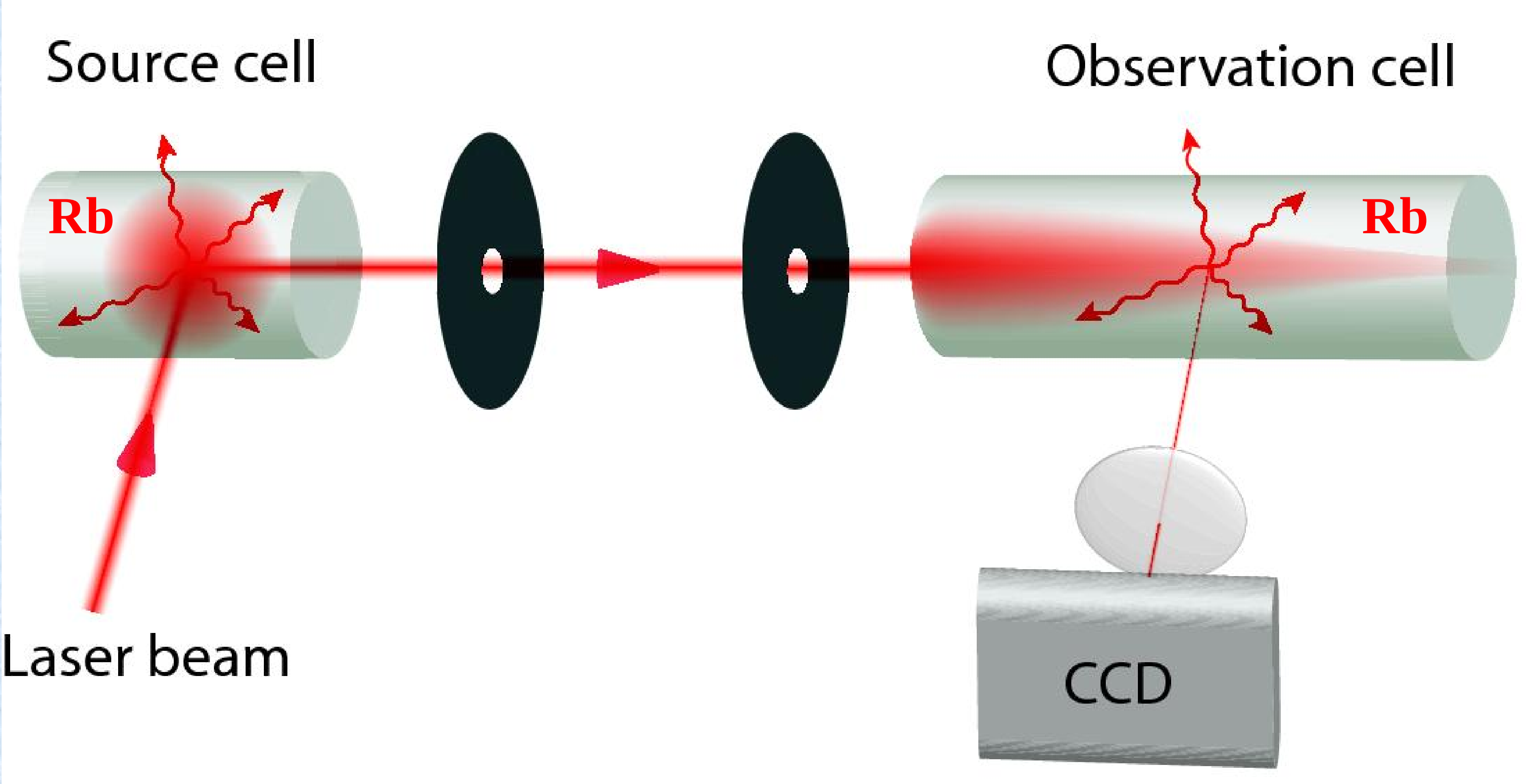
How to measure P(x) ?

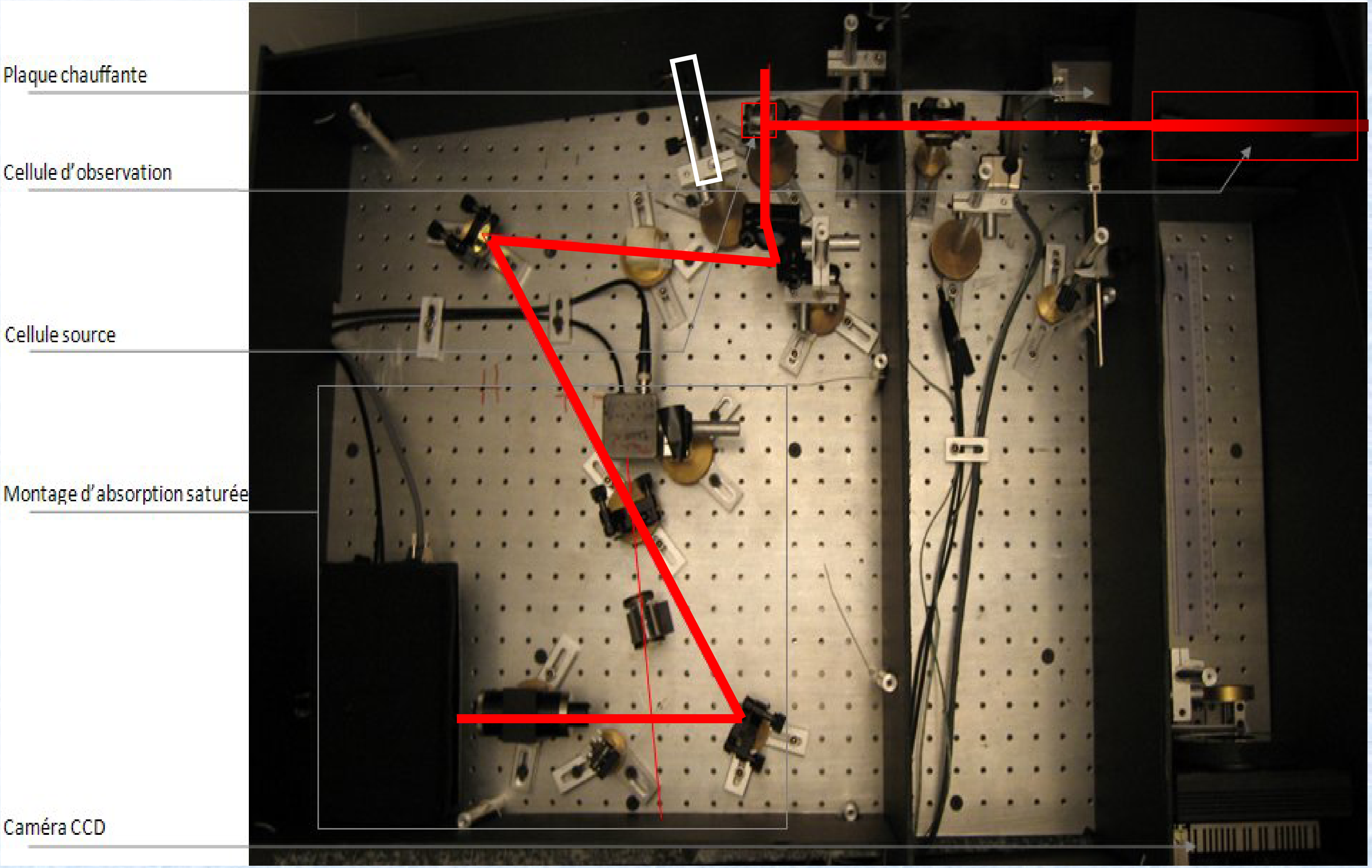


How to track a Photon ????

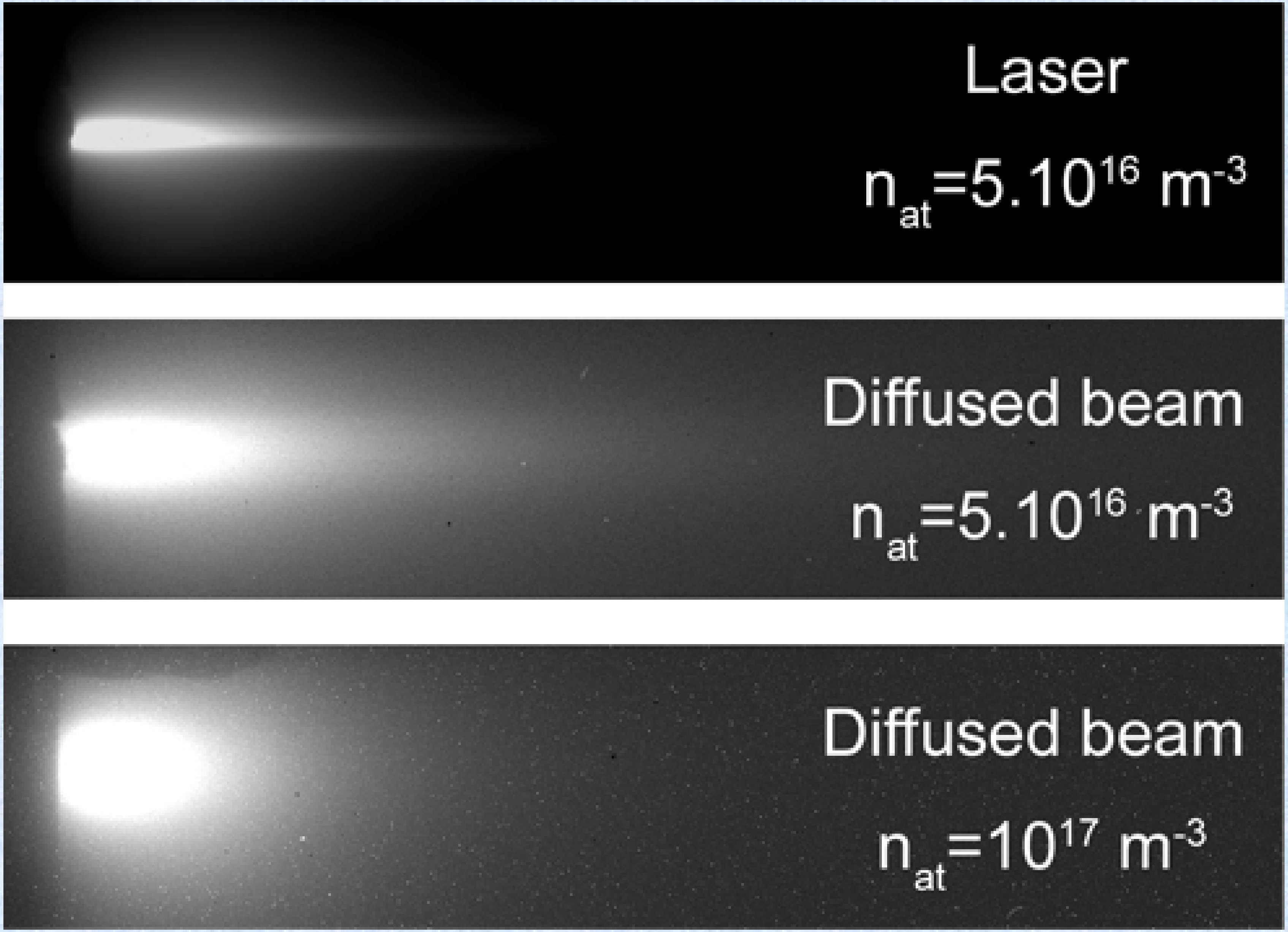
In Stars … ☹

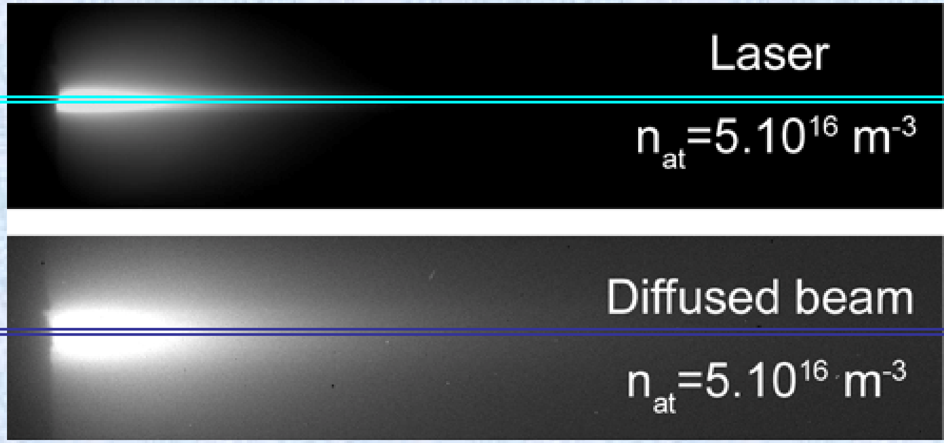
in the lab …. 


Source cell
Observation cell
Rb
Rb
Laser beam
CCD

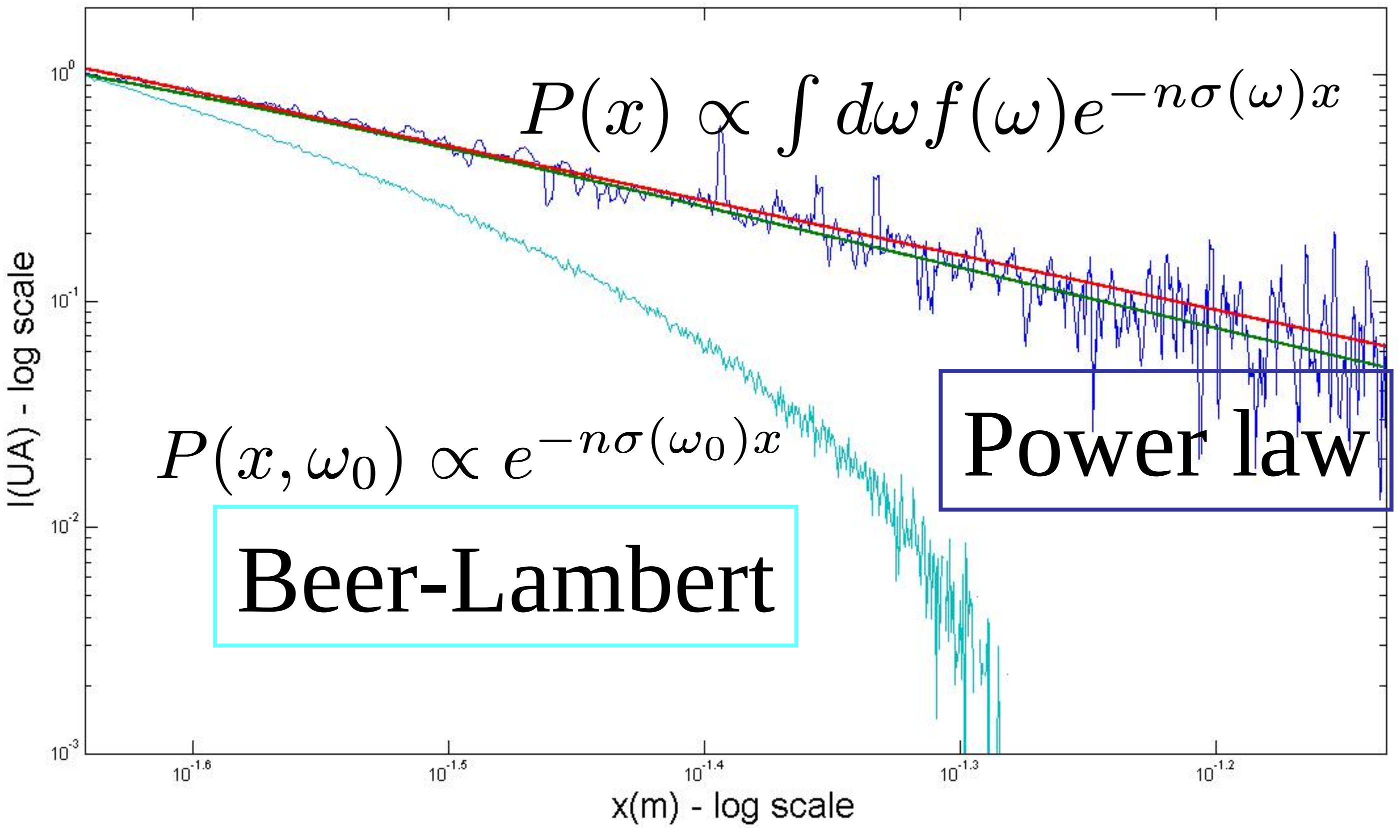


2.3 Step Size Distribution of Photons


Laser
n$_{at}$=5.10^{16} m^{-3}
Diffused beam
n$_{at}$=5.10^{16} m^{-3}
Diffused beam
n$_{at}$=10^{17} m^{-3}

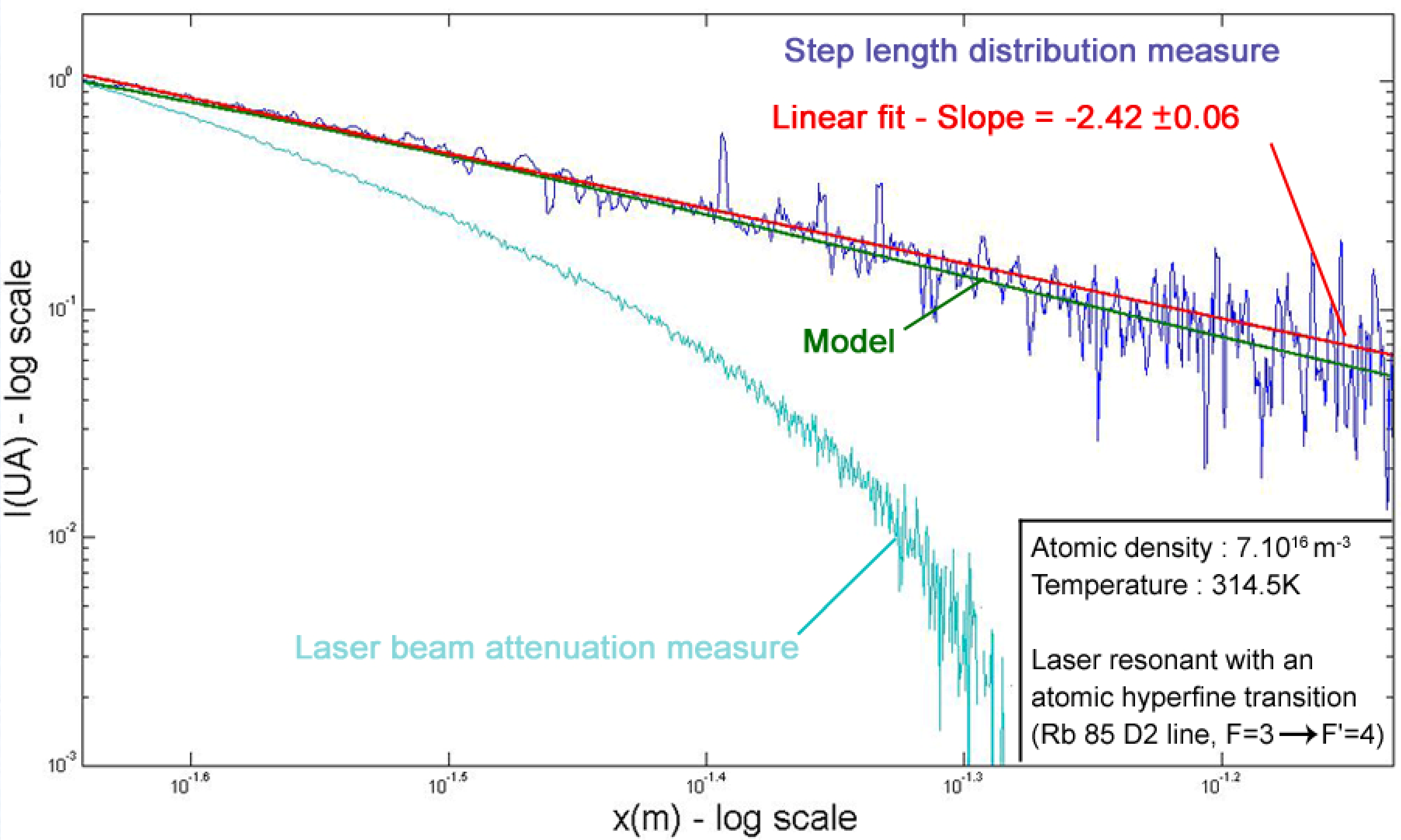


$T_{exposure}$=30min



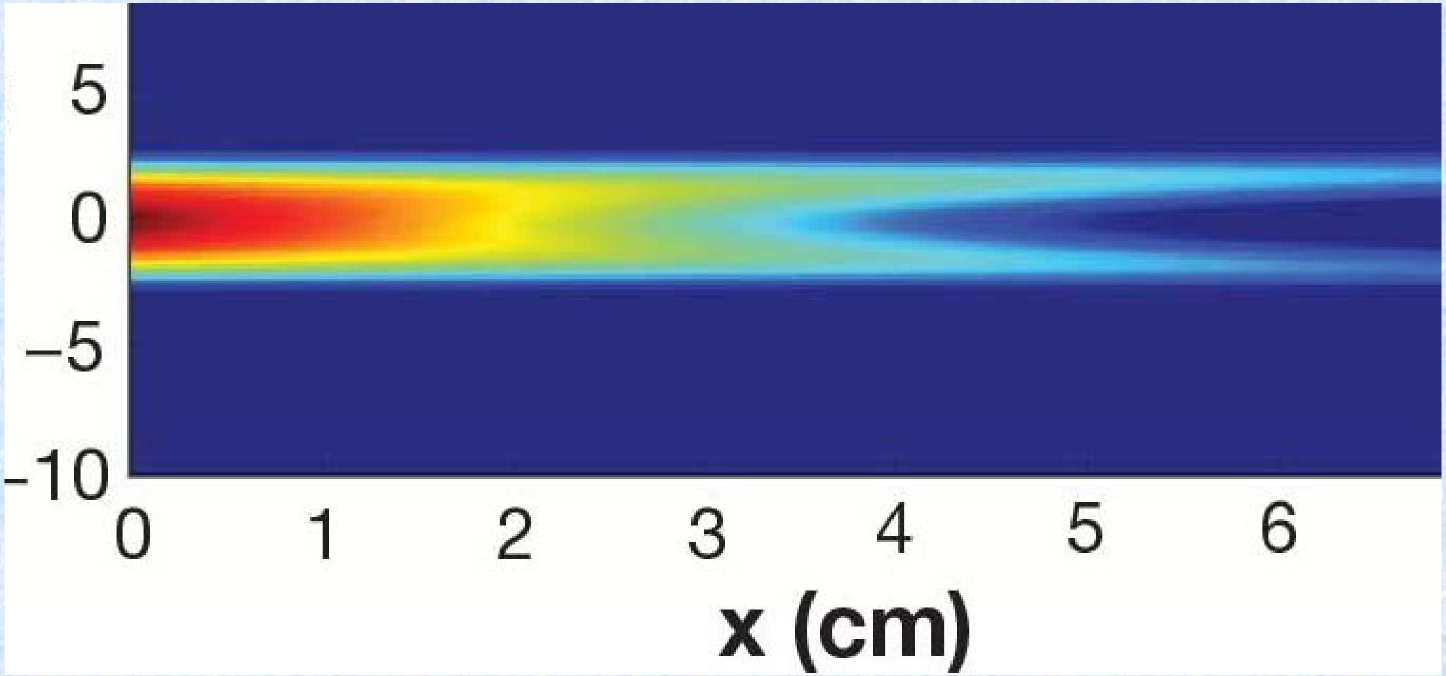
2.3 Step Size Distribution of Photons

Step length distribution

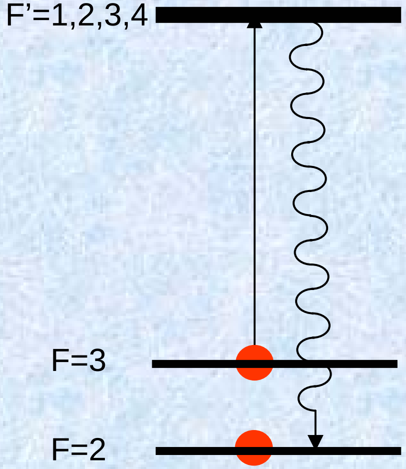


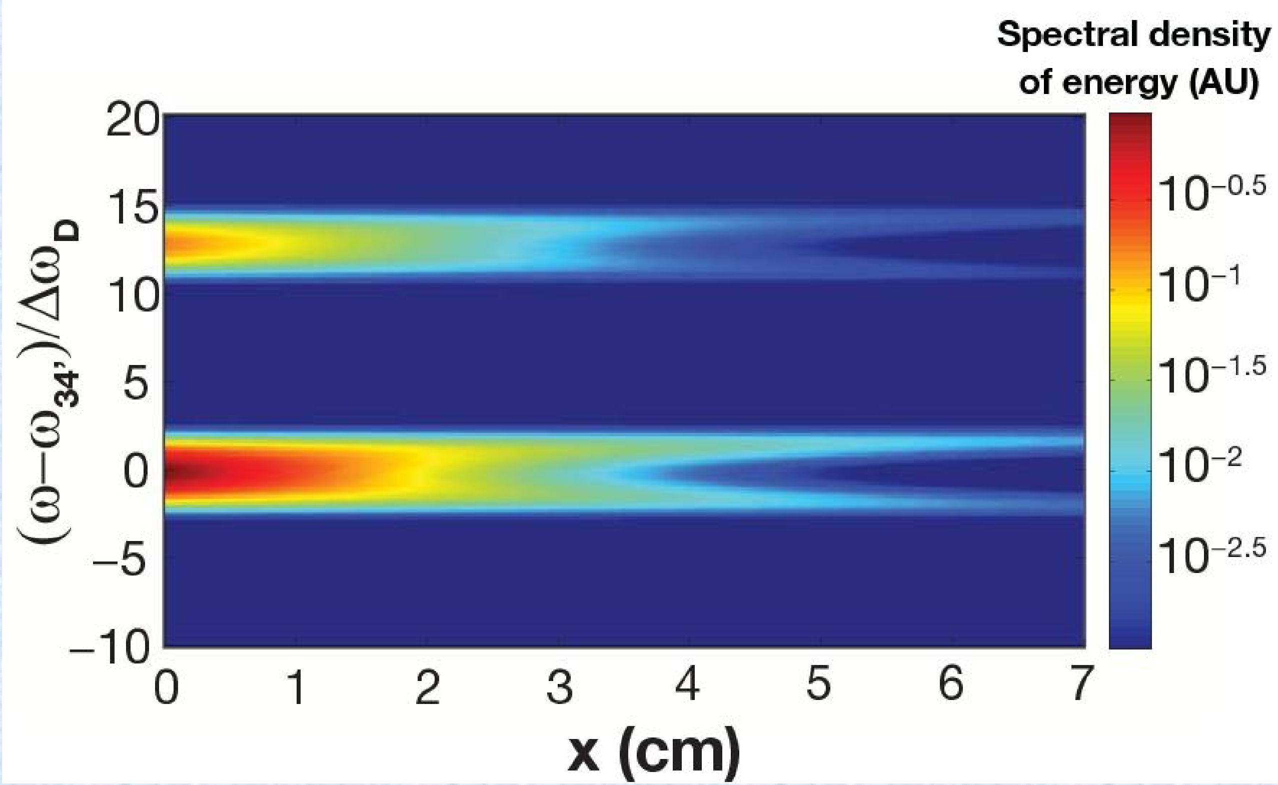
neglecting the natural width of the atoms : α=2

Spatial evolution of the spectrum

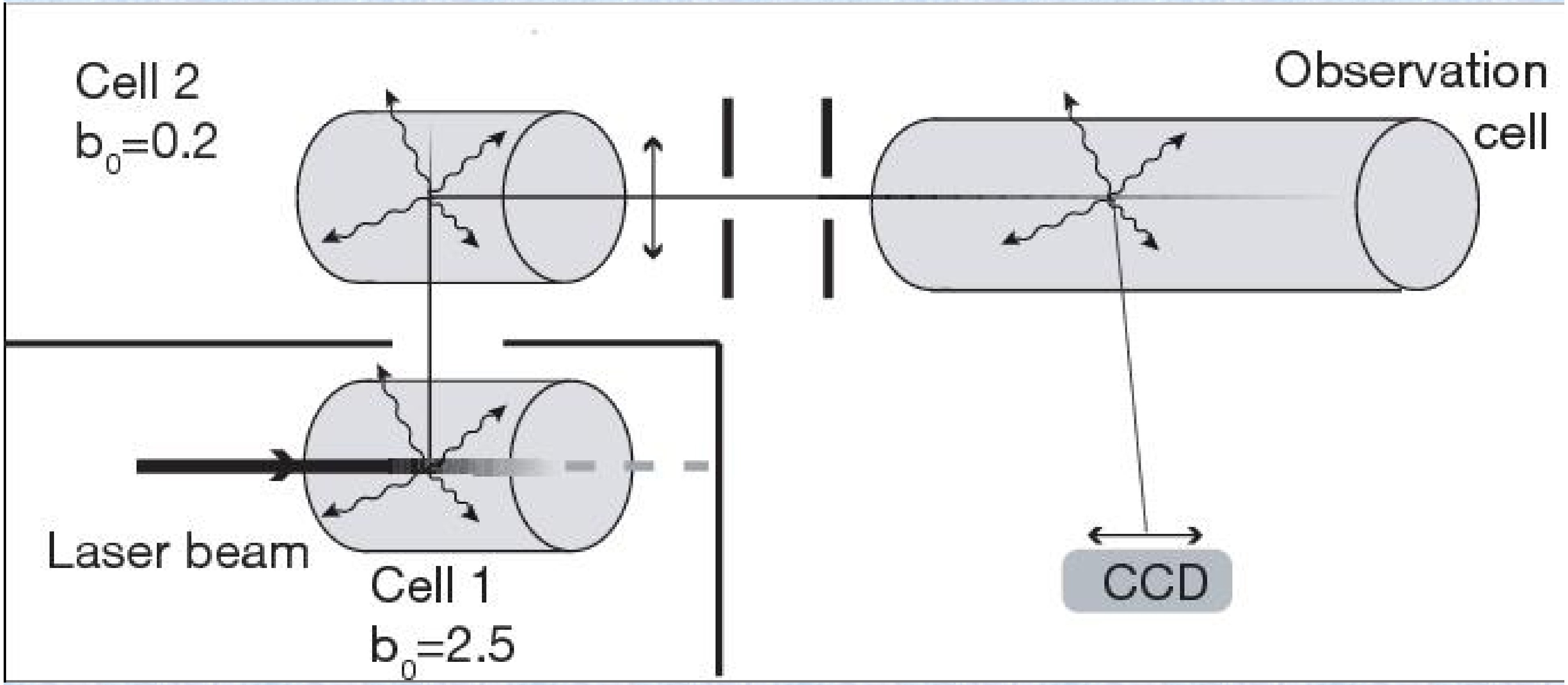


Multilevel calculation of spectral evolution

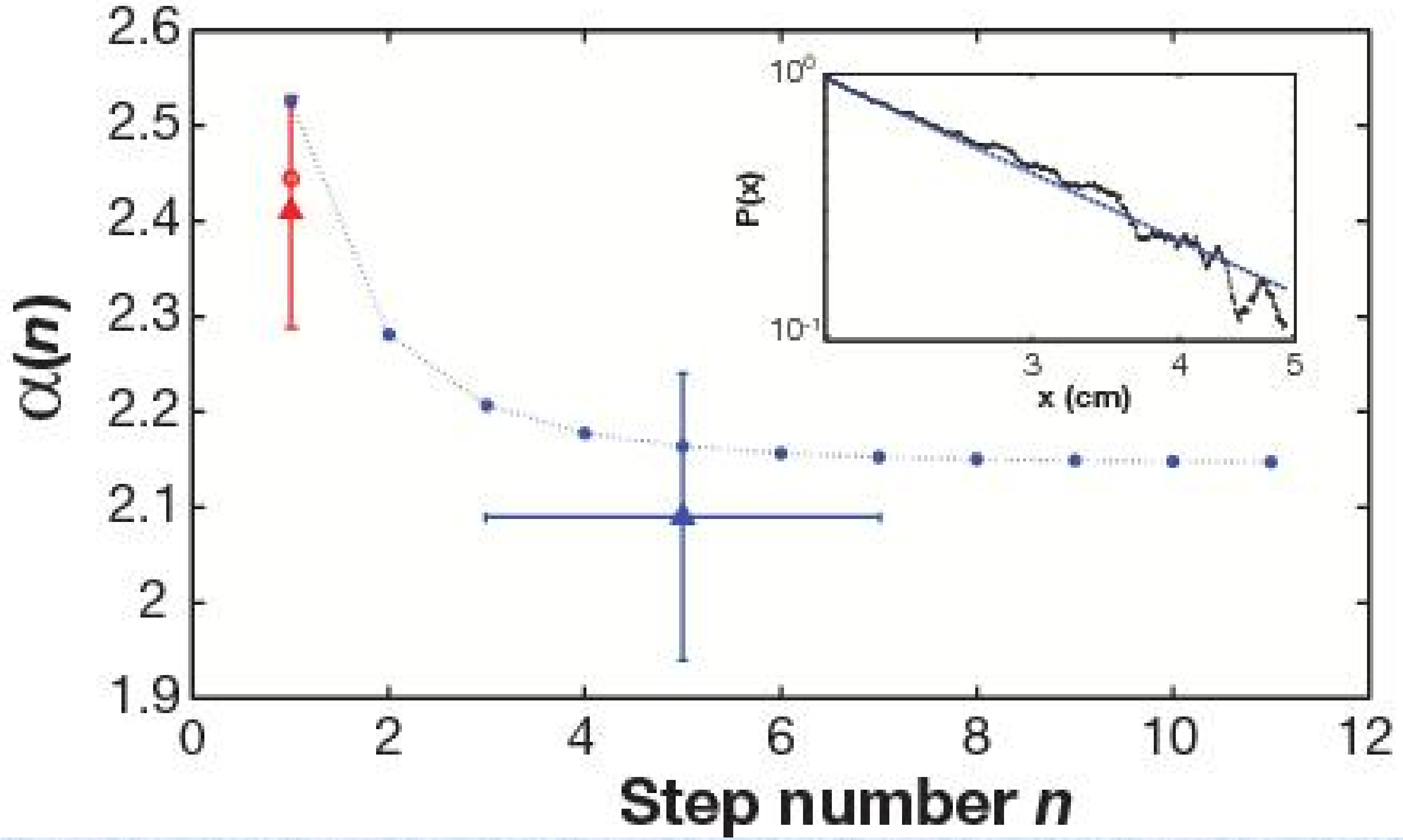




Steady state random walk :
P(x) after many scattering events
(to forget initial memory / partial frequency distribution)



signal/noise limit : photon shot noise, cosmic rays !
(combine 6 images of 5hours each)



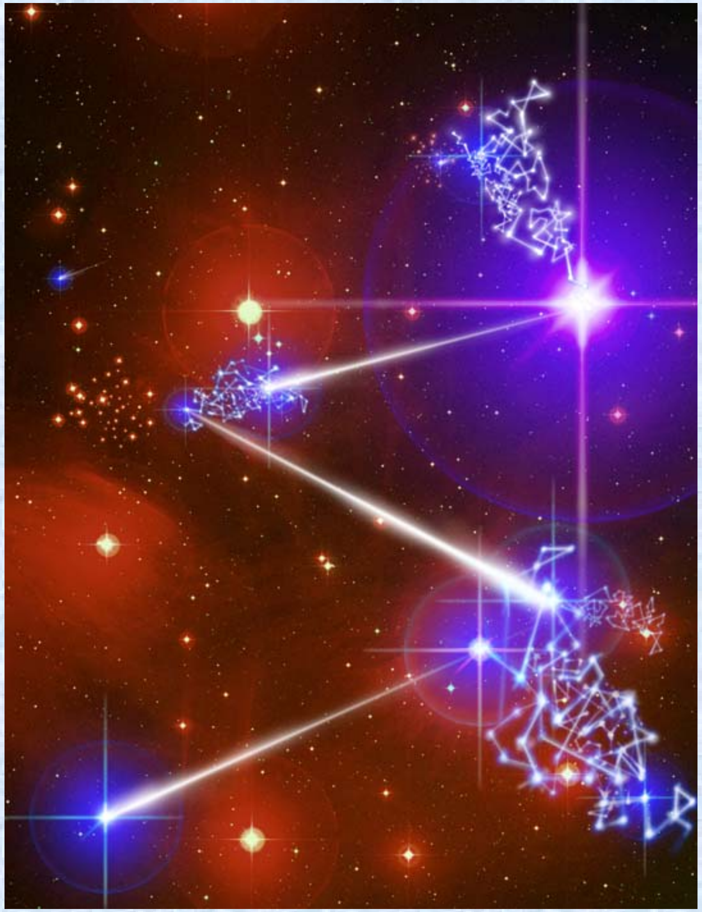
α < 3

Experimental evidence of heavy tail P(x)

Lévy flight of photons

Lévy Flights with atomic vapours : Further developments :

- **Tune the power law exponent ? (detuning, magnetic field, saturation, collisional broadening)**
- **Truncated Lévy Flights (spatial, frequency)**
- **Ergodicity : $x^2 \propto t^\gamma$ (time resovled exp.)**



Conclusions

- Light scattering by atoms is more than
  - spontaneous emission
  - classical dipole emission

- Radiation trapping in atomic vapours
  - cold atoms : slow diffusion
  - hot atoms : Lévy flights

Be careful when using average values !