No time at the end of the tunnel

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Modern atto-second experiments seek to provide an insight into a long standing question: "how much time does a tunnelling particle spend in the barrier?" Traditionally, quantum theory relates this duration to the delay with which the particle emerges from the barrier. The link between these two times is self-evident in classical mechanics, but may or may not exist in the quantum case. Here we show that it does not, and give a detailed explanation why. The tunnelling process does not lend itself to classical analogies, and its duration cannot, in general, be guessed by observing the behaviour of the transmitted particle.

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The question "how long does it take for a particle to tunnel?" is, probably, one of the oldest problems in elementary quantum mechanics[1], which still awaits its full resolution. The problem, extensively discussed since the 1990's (for a review see refs. [2–5]), has regained some of its prominence, owing to the current progress in experimental atto-second physics[6]. Recently, two conflicting claims were made in refs. [7,8]. While in ref. [7] it is argued a "clear evidence for a nonzero tunnelling time delay", the authors of ref. [8] maintain that "tunnelling is instantaneous within [their] experimental and numerical uncertainty". Both papers mention the so-called Wigner-Smith (WS) time delay[9,10], often considered as an estimate for the duration of a tunnelling process. The WS delay and its variants also proved to be useful tools for understanding ballistic transport in meso-scopic systems (for a recent review see ref. [11]).

There have been other attempts to define tunnelling times, to name a few[12–14]. The first two are of most interest for our discussion: the authors of ref. [12] relate the relevant time to the real part of the complex saddle point of the action integral. Even more relevant, the authors of ref. [13] relate the times, obtained with the help of a Larmor clock, to the complex "weak value" of the net time spent in the barrier. This relation, previously studied in ref. [15], and later scrutinised in ref. [16], suggests that the "weak values" (for a recent review see ref. [17]) might provide the much needed insight into the tunnelling time controversy. This view is not without problems. Firstly, the "weak values" themselves are the subject of an ongoing discussion, and in the following we will rely on the analysis of their meaning, given in ref. [18]. Secondly, the already mentioned WS delay, also discussed in ref. [14], does not appear to be one's usual "weak value". With no external meter involved in its measurement, it is not immediately clear whether it is possible to make the measurement "weak". Or is the WS delay a kind "weak value", nevertheless? And if so, what could it mean for one's hope to find the elusive duration governing a tunnelling process? In the following we focus on the simpler case of a wave packet impinged on a potential barrier, with the expectation that any fundamental difficulties encountered in our analysis, would be likely to persist also in the more complex cases, such as strong field ionisation reviewed in ref. [6].

In a "tunnelling time" discussion, it is not unusual to expect that the sought time might be revealed by a suitable experimental procedure, without stating clearly which theoretical concept should be used to define the said time[6]. This position is sustainable in classical physics, where a particle moves along a predictable classical trajectory. It is far less obvious in the quantum case, where the particle's motion results from constructive or destructive interference between all possible scenarios, and is subject to strict limitations imposed by the uncertainty principle[19]. In what follows we approach the problem from the opposite perspective, and provide a detailed theory of the WS delay, in order to analyse both its meaning and usefulness.

Results

In quantum mechanics, quantities such as the coordinate or momentum always have amplitude distributions (the wave function) and, if measured, also the probability distributions (the absolute square of the wave function). The case of the WS delay appears to be different.

It is well known that the transmitted wave packets leave the scatterer delayed or advanced relative to free propagation, and that for a broad nearly monochromatic wave packet this (Wigner-Smith) delay is given by the energy derivative of the phase of the transmission amplitude S_i[9,10], $\delta\tau^{\mathrm{WS}} = -i\hbar\mathrm{Im}[\partial_E \ln S_i] = \hbar\partial_E\mathrm{Arg}\{S_i\}$. The remaining, and probably most interesting, questions are of a more general nature. Is $\delta\tau^{\mathrm{WS}}$ an averaged quantity, and in
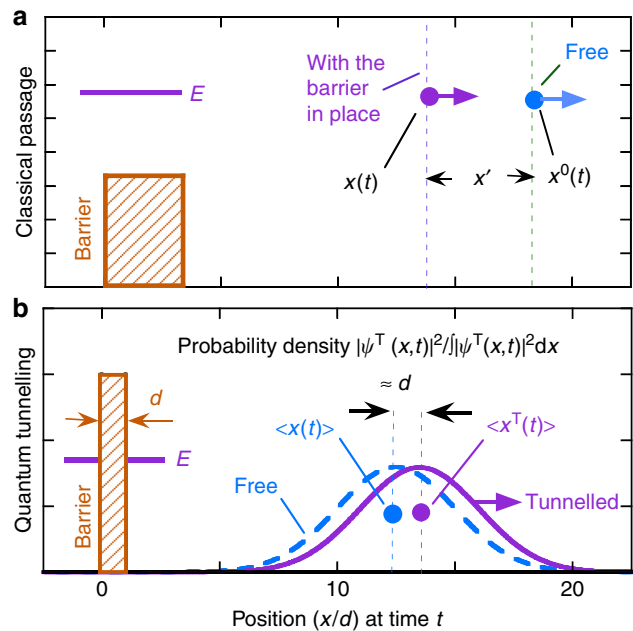

Fig. 1 The spatial delay. a On the snapshot taken at a time t, a classical particle with an energy $E = p^2/2\mu$, having passed over a potential barrier of a height V, lags behind its freely moving counterpart by $x' = \tau - d/v$, where τ is the duration it spends in the potential. b In the case of tunnelling ($pd = 20$, $V/E = 2$, $\Delta x/d = 5$, $pt/\mu\Delta x = 2.5$), the centre of mass of the (greatly reduced) transmitted wave packet, is seen to be advanced by roughly the barrier width d. Does this mean that the particle crossed the barrier region infinitely fast? No, the probability density $|\psi^{\mathrm{T}}(x, t)|^2$, should rather be seen as the distribution of the readings of a highly inaccurate quantum pointer, designed to measure the spatial delay x'

what sense? Does quantum mechanics provide any insight into what happened in the case of tunnelling shown in Fig. 1, rather than just state the fact? Classically, a barrier delays a passing particle (Fig. 1a). Can the advancement of the tunnelled state, evident in Fig. 1b, tell us something about the time the particle has spent in the barrier and, if not, then why? Recent developments in quantum measurement theory, and in particular, the introduction of the so-called "weak measurements" (for a review see ref. [17]) will allow us to answer these questions with sufficient clarity. In order to simplify the narrative, we have consigned relevant mathematics to the Methods section available to the reader.

Above the barrier. We start with a classical experiment we want to mimic in the quantum case. For a free particle with a velocity v it takes $\tau^0 = d/v$ seconds to traverse (for simplicity, in one dimension) a region $0 \leq x \leq d$, and we want to know the duration τ a particle would spend there in the presence of a potential $V(x)$. One can take a snapshot of the particle (passing through $x = 0$ at $t = 0$) at a later time t, large enough for it to leave the region, and compare its positions with and without potential, $x(t)$ and $x^0(t) = vt$, in order to measure the spatial delay, $x' = x(t) - vt$ (see Fig. 1a). Alternatively, one can measure the temporal delay, i.e. the difference between the times the particle and its free counterpart arrive at a chosen location x, $t' = t(x) - x/v$. There is a relation between x', t' and τ, which can be written as

$$x' = -vt' = -v[\tau - \tau^0],\tag{1}$$

or, explicitly, as

$$x(t) - vt = -v[t(x) - x/v] = -v(\tau - d/v). \quad (2)$$

Thus, by measuring either x' or t', we also indirectly measure τ.

Below the barrier. It would be natural to extend both approaches to the case of classically forbidden tunnelling by replacing the classical particle with a broad wave packet $\psi(x, t)$, sharply peaked in the momentum space in order to represent a particle with a momentum p, and a velocity v. Without a barrier, such a wave packet has the form ($\hbar = 1$)

$$\psi^0(x, t) = \int A(k) \exp[ikx - iE(k)t] dk = \exp[ipx - iE(p)t] G(x, t), \quad (3)$$

where we have separated the smooth envelope G, which will play an important part in what follows. With the barrier in place, we will be interested in the transmitted (tunnelled) part of the wave packet, $\psi^T(x, t)$, obtained by multiplying each plane wave in Eq. (3) by the barrier transmission amplitude $T(k)$,

$$\psi^T(x, t) = \int T(k) A(k) \exp[ikx - iE(k)t] dk. \quad (4)$$

Replacing in Eq. (2) $x(t)$, $x^0(t)$, $t(x)$ and $t^0(x)$ by the corresponding mean values (cf ref. [20].),

$$\langle x^{0,T}(t) \rangle = \int x |\psi^{0,T}(x, t)|^2 dx / \int |\psi^{0,T}(x, t)|^2 dx, \quad (5)$$

$$\langle t^{0,T}(x) \rangle = \int t |\psi^{0,T}(x, t)|^2 dt / \int |\psi^{0,T}(x, t)|^2 dt, \quad (6)$$

we find the τ in its r.h.s. given by the so-called "phase time"[2–5], closely related to the Wigner-Smith time delay τ^{WS},

$$\tau(p) \approx \tau^{phase}(p) \equiv v(p)^{-1} \left[d + \partial_p \varphi(p) \right] = d/v + \delta\tau^{WS}, \quad (7)$$

where $T(p) = |T(p)| \exp[i\varphi(p)]$. Intriguingly, for tunnelling across a broad potential barrier, $\tau^{phase}(p)$ can be very short, prompting speculation that a classically forbidden process may "take almost no time"[1]. The conflict with relativity is usually resolved by noting that, due to reshaping, there should be no causal connections between the positions of the incident and the tunnelled wave packets[2]. Possible mechanisms, leading to this effect, have been discussed, for example, in refs. [20–22] and [23]. Yet, to lay this particular "tunnelling time problem" to rest, one needs to explain what happens in the experiment shown in Fig. 1b, and to find a place for the phase time (7) within the formalism of elementary quantum mechanics.

In a nutshell, our argument is as follows. Classically, one who measures the position, or the passage time, of a particle immediately knows also by how much it is delayed (or advanced) relative to a particle that moves freely. Quantally, it is only true to the extent that by determining the average position or passage time, one also performs a highly inaccurate quantum measurement of the corresponding delay. Our next step is to reveal the measurements implied by Eqs. (5) and (6). As with every quantum measurement, there are three things we will have to state clearly.

Firstly, what is measured?
Secondly, how is it measured?
Thirdly, to what accuracy is it measured?

Comparing the final positions. We begin with the "snapshot" version of the experiment, which turns out to be the simpler case. Transmission of a particle with a momentum p is a transition, in which the particle is prepared (pre-selected) in the distant past and found (post-selected) in distant future, in the same momentum's eigenstate $|p\rangle$. Applying the Fourier transform, we can write the transition amplitude $T(p)$ as a sum of sub-amplitudes,

$$T(p) = \int \eta(p, x') dx', \quad \eta(p, x') = (2\pi)^{-1} \exp(-ipx')$$
$$\int T(k) \exp(ikx') dk. \quad (8)$$

Noting that $\eta(p, x') \exp(ipx) \sim \exp[ip(x - x')]$, suggests that x' should represent the spatial delay, experienced by the particle as a result of scattering. The delay is not uniquely defined, and transmission of a plane wave can be seen as a result of interference between all virtual spatial delays. If our understanding is correct, the amplitude distribution of the delays, $\eta(p, x')$, must determine also the spatial delay of a transmitted wave packet with a mean momentum p. This is, indeed, the case, since we can write Eq. (4) as (see Eq. (33) of the Methods)

$$\psi^T(x, t) = \exp[ipx - iE(p)t] \int_{-\infty}^{\infty} G(x - x', t) \eta(p, x') dx'. \quad (9)$$

From now on, we will assume that the barrier potential $V(x)$ does not support bound states, so that $\eta(x)$ must vanish for $x' > 0$,

$$\eta(p, x') \equiv 0, \quad x' > 0, \quad (10)$$

and the envelope of the transmitted wave packet is always built from the envelopes $G(x - x', t)$, none of which are advanced relative to free propagation. [Note that classically, in order to move faster than in free propagation, a particle needs to pass over a potential well. In a similar way, quantally, for a well supporting one or more bound states, one has $\eta(p, x') \neq 0$ for $x' > 0$ and, therefore, also advanced envelopes in the sum (9).]

Recovering the classical limit. The classical Eqs. (1) and (2) are readily recovered from (9) for a (semi)classical particle passing over the barrier. In this limit $\eta(p, x')$ remains highly oscillatory, except in the vicinity of the classical value $x'_{cl} = x(t) - x^0(t) < 0$, where its phase is stationary[23]. As a result, there is a single envelope $G(x - x'_{cl}, t)$, selected in the r.h.s. of Eq. (9), which lags behind the freely propagating $G(x, t)$, as one would expect from a particle passing over a potential hill in Fig. 1a.

We are, however, interested in tunnelling, where $\eta(p, x')$ rapidly oscillates everywhere, and shows no preference for any particular delay. In addition, Heisenberg's uncertainty principle[24] demands that $G(x, t)$ be very broad, if the particle is to have a reasonably well defined momentum. For example, we could choose a Gaussian wave packet whose spreading, if any, we will neglect (see Eq. (40) of the Methods),

$$G(x, t) = G(x - vt) = (2/\pi \Delta x^2)^{1/4} \exp[-(x - vt)^2 / \Delta x^2], \quad (11)$$

and make its width very large, $\Delta x \to \infty$.

Revisiting quantum measurements. Next we turn for assistance to quantum measurement theory.

To convince the reader that, according to Eq. (9), we are in fact performing a highly inaccurate (weak) quantum measurement of the spatial delay x' defined by Eq. (8), we briefly revisit a von

Neumann measurement of an operator $\hat{B} = \sum_\nu |b_\nu\rangle B_\nu \langle b_\nu|$ for a system, initially prepared and finally found in the states $|\psi\rangle$ and $|\phi\rangle$, respectively. Assuming the initial state of a pointer with position x to be some smooth $G(x)$ [e.g., a Gaussian similar to that in Eq. (11)], and neglecting the system's own dynamics, we found the pure state of the pointer after a successful post-selection to be

$$\psi(x) = \sum_\nu \eta_\nu G(x - B_\nu), \quad \eta_\nu \equiv \langle\phi|b_\nu\rangle\langle b_\nu|\psi\rangle, \quad (12)$$

where η_ν is the amplitude for reaching $|\phi\rangle$ by passing first through $|\nu\rangle$. Note that here we measure a quantity which can take values B_ν, by means of coupling it to an external pointer, to an accuracy which is determined by the width Δx of the pointer's initial state $G(x)$. To have an accurate (strong) measurement we need to know the initial pointer position well. Thus, sending $\Delta x \to 0$ we see interference between the η_ν's completely destroyed, so that the the system can be said to "pass through" $|\nu\rangle$ with a probability $|\eta_\nu|^2$. The pointer always points at one of the B_ν's, and its mean position after many trials is given by

$$\langle x \rangle = \int x|\psi(x)|^2 \mathrm{d}x / \int |\psi(x)|^2 \mathrm{d}x = \sum_\nu B_\nu |\eta_\nu|^2 / \sum_{\nu'} |\eta_{\nu'}|^2. \quad (13)$$

In the opposite limit, $\Delta x \to \infty$, inaccurate (weak) measurement leaves the interference almost intact, and all final pointer positions are nearly equally probable. This is a consequence of the uncertainty principle[19], which forbids answering the "which way?" question while the alternatives interfere. A weak pointer just skims over the available virtual scenarios, and the expression for its mean position mimics Eq. (13), but with the probabilities replaced with the probability amplitudes (see Eq. (53) of the Methods),

$$\langle x \rangle = \mathrm{Re}\left[\sum_\nu B_\nu \eta_\nu / \sum_{\nu'} \eta_{\nu'} \right] + O(1/\Delta x). \quad (14)$$

The l.h.s. of Eq. (14) is still the standard quantum mechanical average of the pointer's position. The complex valued sum in its r.h.s., often written in an equivalent form $\bar{B} \equiv \langle\phi|\hat{B}|\psi\rangle/\langle\phi|\psi\rangle$, is known as "the weak value of an operator $\hat{B}$"[17].

As a brief summary, what we learned from the quantum measurement theory is this. A weakly perturbing measurement of the type described above can only yield information about a particular combination of the probability amplitudes. With no a priori restrictions on the signs of the η_ν, $\langle x \rangle$ can take an arbitrary value, depending on the choice of the states $|\psi\rangle$ and $|\phi\rangle$, e.g. be positive even if all B_ν's are not. For example, we can choose one with only one non-zero eigenvalue, $B_\nu = 1$ for $\nu = n$ and 0 otherwise. In an accurate measurement, $\langle x \rangle = |\eta_n|^2 / \sum_{\nu'} |\eta_{\nu'}|^2$ will coincide with the probability, with which the system is seen to pass through $|b_n\rangle$ on its way to the final state $|\phi\rangle$. On the other hand, for an improbable transition $|\sum_\nu \eta_\nu|^2 \ll 1$, a highly inaccurate measurement might yield $\langle x \rangle \approx \mathrm{Re}[\eta_n / \sum_{\nu'} \eta_{\nu'}] = -100$. We would be ill advised to conclude that now the probability to travel the route $|\phi\rangle \leftarrow |b_n\rangle \leftarrow |\psi\rangle$ is a large negative number. We can, however, say "for the chosen transition, the real part of the n-th probability amplitude (divided by the transition amplitude itself) is -100" without raising many eyebrows.

The weak spatial delay. Having set out to evaluate the distance separating the free and the transmitted particle, we can now describe the task in the proper quantum mechanical language. Comparing Eqs. (9) and (12) we conclude that by taking snapshots of the broad wave packets, we measure the spatial delay (8) of the transmitted particle with a momentum p. This is hardly surprising, since this is precisely what we do in the classical version of the experiment, shown in Fig. 1a. We note also that, as in the classical case, the particle's own position plays the role of a pointer. Most importantly, the measurement is, of necessity, inaccurate or weak, since the wave packet envelope (11) must be very broad. If so, the mean pointer's (particle's) position is given by the real part of the corresponding "weak value" [cf. Eq. (14)], and the first difference in the classical Eq. (2) is replaced with

$$\langle x^\mathrm{T}(t) \rangle - vt = \mathrm{Re}\left[\overline{x'}(p) \right], \quad (15)$$

where

$$\overline{x'}(p) \equiv \int_{-\infty}^0 x' \eta(p, x') \mathrm{d}x' / \int_{-\infty}^0 \eta(p, x') \mathrm{d}x' = i\partial_p \ln T(p)$$
$$= i\partial_p \ln|T(p)| - \partial_p \varphi(p) \quad (16)$$

is the "weak value" of the spatial delay, i.e., a particular combination of the delay amplitudes $\eta(p, x')$ in Eq. (8). (In general, the tunnelled wave packet would increase its mean velocity, since its higher momenta tend to tunnel more easily. However, in the limit of a broad nearly monochromatic wave packet the effect can be neglected, and Eq. (15) remains valid at any finite time t.) We note again that a reader, used to associating weak values with systems coupled to other degrees of freedom[17], should not be put off by the absence of an external pointer in our analysis. All that is required for construction of a "weak value" is a set of values associated with each of the virtual scenarios, and the corresponding probability amplitudes[18].

Realising that there is no reason, except in the classical limit, for relating $\overline{x'}(p)$ to a duration $\tau(p)$ spent in the barrier, helps avoid paradoxes, otherwise inevitable. For example, for a non-relativistic particle of a mass μ crossing a zero-range potential $V(x) = \Omega\delta(x)$, we find (putting $\mu = 1$ for simplicity)

$$\eta(p, x' \leq 0) = \delta(x) - \Omega\exp[(\Omega - ip)x'], \quad \text{and} \quad \overline{x'}(p)$$
$$= -\frac{\Omega}{p^2 + \Omega^2} + i\frac{\Omega^2}{p(p^2 + \Omega^2)}. \quad (17)$$

For $\Omega > 0$, there is a spatial delay of approximately $\Omega/(p^2 + \Omega^2)$, which cannot, however, be attributed to the excess time spent in the barrier region, since there is no finite barrier region to spend time in. Neither need we worry about Einstein's relativity, having noticed that in Fig. 1b the (greatly reduced) wave packet, which tunnelled across a broad potential barrier, lies ahead of the freely propagating one. For a broad rectangular barrier, we have (see Eq. (41) of the Methods) $T(p) \sim \exp\left\{ -[2\mu V - p^2]^{1/2}d - ipd] \right\}$, and $\overline{x'}(p) \approx d$, which would have given a nearly zero duration spent in the barrier, had we tried to deduce $\tau(p)$ from Eq. (15). Instead, we find this speed-up effect to be caused by what also causes the weak value of an operator to lie, at times, far from the region containing its eigenvalues. The barrier potential duly delays all the envelopes in Eq. (9), yet the resulting wave packet is advanced relative to free motion, owing to the oscillatory nature of the amplitude distribution $\eta(p, x)$. Finally, using Eqs. (16) and (7), and recalling that $\partial_p \phi = \partial_p E \partial_E \phi = v(p)\delta\tau^\mathrm{WS}$, we can write down the quantum

version of Eq. (1)

$$\mathrm{Re}\big[\overline{x'}(p)\big] = -v\delta\tau^{\mathrm{WS}} = -v\big[\tau^{\mathrm{phase}} - d/v\big],\qquad (18)$$

which now relates the "weak value" of the spatial delay to the Wigner-Smith delay and the phase time.

The weak temporal delay. We could stop here, but the presence of the "weak spatial delay" in Eq. (18) suggests that also the Wigner-Smith delay is a weak value of some kind. To find out more, we revert to the second of our experiments, and return the discussion to the time domain. Considering the transmission amplitude T as a function of the particle's energy $E(p)$, rather than its momentum p, we can study the time dependence of the transmitted wave function at a fixed location x. We have (see Eqs. (30)–(32) of the Methods)

$$T(E) = \int \xi(E,t')\mathrm{d}t', \quad \xi(E,t') \equiv (2\pi)^{-1}\exp[iE(p)t']$$
$$\int_{-\infty}^{\infty} T(E')\exp(-iE't')\mathrm{d}E', \qquad (19)$$

$$\psi^{\mathrm{T}}(x,t) = \exp[ipx - iE(p)t]\int_0^{\infty} \xi(E,t')G(x,t-t')\mathrm{d}t', \quad (20)$$

where we have used the fact that, in the absence of bound states, $\xi(t') \equiv 0$ for $t' < 0$. Now the wave function at x, $\psi^{\mathrm{T}}(x, t)$, is a superposition of the amplitudes, produced by freely propagating wave packets, whose launch was delayed by $0 \leq t' < \infty$, and $\xi(E, t)$ stands for the probability amplitude that a plane wave with an energy E will experience delay by t' seconds when exiting the barrier, $\exp[i(px - Et)] \rightarrow \exp\{i[px - E(t - t)]\}$. Combining Eqs. (8) and (19) we can relate the amplitude distributions of temporal and spatial delays (see Eq. (56) of the Methods),

$$\xi(E,t') = \int_{\infty}^0 Q(x',t',p)\eta(p,x')\mathrm{d}x',$$
$$Q(x,t,p) = -\frac{x}{t}\exp[iE(p)t + ipx]K_0(-x,t), \qquad (21)$$

where $K_0(x,t) \equiv (2\pi)^{-1}\int_{-\infty}^{\infty} \exp[-iE(k)t + ikx]\mathrm{d}k$ is the free-particle propagator. In the absence of dispersion, $v(p) = c = \mathrm{const}$, $E = cp$, $K_0(x, t) = \delta(x - ct)$, the analysis is the same as in the "snapshot" case. We obtain $\xi(E, t') = c\eta(p, -ct')$ which only demonstrates that the envelope whose maximum G in Eq. (19) arrives at x t' seconds after the free wave packet, is displaced backwards by $x' = ct'$ in Eq. (9). For a massive particle, the wave packet spreads, and the relation between $\xi(E, t')$ and $\eta(p, x')$ is no longer simple. Thus, in the case of a zero-range barrier, $[E(p) = p^2/2\mu, K_0(x, t) = (\mu/2it)^{1/2}\exp(i\mu x^2/2t)]$, from Eq. (17) we obtain (using again $\mu = 1$, see Eq. (62) of the Methods)

$$\xi(E,t' \geq 0) = \delta(t') - \Omega\left(\frac{i}{2\pi t'}\right)^{1/2}\exp(iEt') + \frac{i\Omega^2}{2}$$
$$\exp[i(\Omega^2/2 + E)t']\mathrm{erfc}(\sqrt{it'/2\Omega^2}), \qquad (22)$$

where $\mathrm{erfc}(z) = 2\pi^{-1/2}\int_z^{\infty}\exp(-z^2)\mathrm{d}z$ is the complimentary error function[25].

Comparing Eqs. (20) and (12) shows that if the wave packet is broad, $\Delta x \rightarrow \infty$, we have another "weak measurement", this time in the time domain. Thus, quantally, in the classical Eq. (2) $t(x) - x/v$ is replaced with,

$$\langle t^{\mathrm{T}}(x)\rangle - x/v \approx \mathrm{Re}\big[\overline{t'}(E)\big] = \delta\tau^{\mathrm{WS}}, \qquad (23)$$

where

$$\overline{t'}(E) \equiv \int_0^{\infty} t'(E,t')\mathrm{d}t' \bigg/ \int_0^{\infty} \xi(E,t')\mathrm{d}t' = -i\partial_E\ln|T(E)| + \delta\tau^{\mathrm{WS}}$$

$$(24)$$

is the "weak value of the temporal delay", another complex valued quantity, whose real part coincides with the Wigner-Smith delay. Equation (18) can now be seen as a relation between the "weak values" $\overline{x'}(p)$ and $\overline{t'}(p)$, and the phase time, defined by (7),

$$\mathrm{Re}\big[\overline{x'}(p)\big] = -v\mathrm{Re}\big[\overline{t'}(p)\big] = -v\big[\tau^{\mathrm{phase}} - d/v\big]. \qquad (25)$$

Accordingly, we should not try to relate $\tau^{\mathrm{phase}}(p)$ to the duration τ the particle is supposed to spend in the barrier, for the same reason we could not relate to it the $\overline{x'}(p)$ in Eq. (15) or $\overline{t'}(p)$ in Eq. (24). Finally, we note that the quantum Eqs. (18) and (25) rely on the fact that, although the amplitude distributions $\eta(p, x')$ and $\xi(E, t')$ in Eqs. (17) and (22) have very different forms, the complex valued weak delays in Eqs. (15) and (24) always enjoy the same relation (1), as do their classical counterparts, x' and t' (see Eq. (59) of the Methods),

$$\frac{\int_{-\infty}^0 x'\eta(p,x')\mathrm{d}x'}{\int_{-\infty}^0 \eta(p,x')\mathrm{d}x'} = v(p)\frac{\int_0^{\infty} t'\xi(E,t')\mathrm{d}t'}{\int_0^{\infty} \xi(E,t')\mathrm{d}t'}. \qquad (26)$$

This last equation describes nothing more than a property of free motion performed by the transmitted particle after it has left the barrier, which holds in the quantum case as well as in the classical limit.

Discussion

The classical task of inferring the time τ spent in the scatterer from either the position of the transmitted particle, or the time of its arrival to a given location, cannot be completed in the quantum case. Beyond the classical limit, both spatial and temporal delays are distributed quantities, and their destructive interference determines the value of the tunnelling probability. By the uncertainty principle, an experiment designed to measure either delay without destroying the interference is doomed to fail. It fails by providing only a limited information about the probability amplitudes in the form of the complex "weak values" of the delays, while leaving the precise delay indeterminate. Shown in Fig. 1b is the result of such a weakly perturbing measurement. Having found the centre of the transmitted wave packet lying ahead of the freely propagated one by approximately the width of the classically forbidden region, we cannot say that "tunnelling appears to take almost no time". We can, however, say "if we take the probability amplitude for the particle with a momentum p to be shifted by x' upon leaving the barrier, multiply it by x', sum over all shifts, and divide by the sum of all amplitudes, the real part of the result will equal roughly the barrier's width". While disappointing in one respect, this conclusion also offers a relief. Since neither spatial no temporal delays need to be related to the τ, one avoids such spurious "paradoxes" as the "apparent superluminality" in tunnelling, illustrated in Fig. 1b, or the delays accrued in regions whose width tends to zero. A quantum theory of the net duration τ spent by a particle in the barrier region can be constructed by considering its evolution along various Feynman paths[26], which involves the Fourier transform of the transmission amplitude with respect to the barrier potential, rather than to the particle's momentum or energy. It does, however meet with the same difficulty: τ turns out to be a distributed quantity, and only its complex "weak value" is available in an experiment

which perturbs the studied transition only slightly. This "complex time" cannot be interpreted, in the classical sense, as a meaningful duration[26], just like the complex spatial delay $\overline{x'}(p)$ in Eq. (17) is not a meaningful displacement.

The general rule may be stated like this. An attempt to measure a distributed quantity without destroying the interference may produce only combinations of the relevant amplitudes, which cannot simply replace the classical values in an intrinsically quantum situation. Our analysis was not meant to question the importance of the Wigner-Smith time delay for the theory of mesoscopic transport[11]. The delay will enter a calculation, through the derivative of the scattering amplitude, wherever the energy range of interest is narrow enough to justify an expansion in Taylor series. Our analysis did, however, answer a more fundamental question why sometimes it appears to[11] "lead to paradoxal conclusions in the presence of tunnelling barriers?"

Although, in our simple example, quantum interference frustrates our efforts to find "the" tunnelling time, various quantities with units of time will occur both in calculations and experimental situations. We can, however, expect them to lack in generality, and have little application outside the context of a particular experiment. Suppose, for instance, that the proverbial Alice decides to measure "the tunnelling time" as discussed above, while Bob uses a weak Salecker-Wigner-Peres clock[26] to measure the already mentioned net duration τ. While in perfect agreement if the particle is classical, their results would, in general, differ in the full quantum case[27]. Notably, neither Alice nor Bob would be able to claim that their result yields the "true" tunnelling time. Any such claim is predicated on the existence of a single duration which governs the tunnelling process, and is rendered meaningless, if such a duration simply does not exist.

We conclude where we started, with a cautious comment on the discussion in refs. [7,8]. In their experiments, the authors of refs. [7,8] used different atomic species, different laser intensities, as well as different methodologies, neither of which is identical with the simple scheme sketched in Fig. 1. There are, however, certain parallels. As in the case of Alice and Bob, the results of both experiments may be correct. As with Alice and Bob, confronting the "non-zero time" of ref. [7] with the "zero time" of ref. [8] may not lead to a meaningful discussion. A reader wishing only to know whether tunnelling electron crosses the barrier infinitely fast, will not get a yes or no answer, and would need to be referred to the intricacies of interfering amplitudes, and the uncertainty principle. To put it slightly differently, our contribution to the discussion of refs. [7,8] was to indicate that the measurement of the passage time of the ionised electron in the atto-clock experiment is unlikely to provide any conclusion on to the time spent in the barrier.

Methods
Amplitude distributions for spatial and temporal delays. Transmission amplitude $T(k)$ of a short-range barrier $V(x)$, contained in the interval $0 \le x \le d$, is an analytical function of the particle's momentum k, which can only have resonance poles in the lower half of the complex k-plane, and on its positive imaginary axes, provided $V(x)$ supports bound states[28]. Typically, it tends to unity, $T(k) \to 1$, for $|k| \to \infty$. Thus we can evaluate a Fourier transform ($\hbar = 1$), separating the singular term,

$$\eta(x') = (2\pi)^{-1}\int_{-\infty}^{\infty} T(k)\exp(ikx')\mathrm{d}k = \delta(x') + (2\pi)^{-1}\int_{-\infty}^{\infty}[T(k)-1]\exp(ikx')\mathrm{d}k$$
$$\equiv \delta(x') + \tilde{\eta}(x'),$$
(27)

where $\delta(x)$ is the Dirac delta, and $\tilde{\eta}(x)$ is a "smooth" function of x'. In the absence of bound states, i.e., with no poles on the positive imaginary semi-axis, for $x' > 0$ the contour of integration can be closed in the upper half-plane, so that

$$\eta(x') \equiv 0 \quad \text{for} \quad x' > 0.$$
(28)

Inverting the Fourier transform we then have

$$T(p) = \int_{-\infty}^{0}\exp(-ipx')\eta(x')\mathrm{d}x' = \int_{-\infty}^{0}\eta(p,x')\mathrm{d}x' \equiv 1 + \int_{-\infty}^{0}\tilde{\eta}(p,x')\mathrm{d}x',$$
(29)

which is valid also for a complex p, provided $\mathrm{Im}[p] \ge 0$.

Similarly, we can evaluate the Fourier transform of $T(E)$ as a function of the energy E, rather than the momentum p. For a non-relativistic particle of a mass μ, $E(p) = p^2/2\mu$, $T(E)$ is single valued on the physical sheet of the Riemann surface of $\sqrt{E}$, cut along the positive real axis, with the poles, corresponding to the bound states now lying on the negative semi-axis. Without such poles we can integrate $T(E)$ along the whole real E-axis above the cut, to obtain

$$\xi(t') = (2\pi)^{-1}\int_{-\infty}^{\infty}T(E)\exp(-iEt')\mathrm{d}E = \delta(t') + (2\pi)^{-1}\int_{-\infty}^{\infty}[T(E)-1]\exp(iEt')\mathrm{d}E$$
$$\equiv \delta(t') + \tilde{\xi}(t'),$$
(30)

$$\xi(t') \equiv 0 \quad \text{for} \quad t' < 0,$$
(31)

$$T(E) = \int_{0}^{\infty}\exp(iEt')\xi(t')\mathrm{d}t' \equiv \int_{0}^{\infty}\xi(E,t')\mathrm{d}t' \equiv 1 + \int_{0}^{\infty}\tilde{\eta}(E,t')\mathrm{d}t',$$
(32)

where the last equation is valid in the lower half of the complex E-plane, $\mathrm{Im}[E] \le 0$.

The convolution formulae. Inserting Eq. (29) into Eq. (4) of the article, we obtain

$$\psi^{\mathrm{T}}(x,t) = \int\mathrm{d}x'\eta(x')\psi^0(x-x',t) = \int\eta(x')\exp[ip(x-x')-iE(p)t]G(x-x',t)\mathrm{d}x'$$
$$= \exp[ip(x-iE(p)t]\int\eta(p,x')G(x-x',t)\mathrm{d}x'.$$
(33)

Alternatively, Eq. (4) can be re-written as an integral over energies $\mathcal{E}$,

$$\psi^{\mathrm{T}}(x,t) = \int_{\Gamma}T(\mathcal{E})A(\mathcal{E})\exp[ik(E)-i\mathcal{E}t]v^{-1}(\mathcal{E})\mathrm{d}\mathcal{E}, \quad v(\mathcal{E}) \equiv \mathrm{d}\mathcal{E}/\mathrm{d}k.$$
(34)

In the absence of dispersion, $\mathcal{E}(k) = ck$, $c = $ const, the integration proceeds along the real $\mathcal{E}$-axis. For $\mathcal{E}(k) = k^2/2\mu$, the contour Γ runs below and above the cut along the positive $\mathcal{E}$-axis as shown in Fig. 2. Eq. (32) is valid everywhere on Γ and we can insert it into (34) and write $\psi^{\mathrm{T}}(x,t)$,

$$\psi^{\mathrm{T}}(x,t) = \int_{0}^{\infty}\mathrm{d}t'\xi(t')\psi^0(x,t-t') = \int_{0}^{\infty}\xi(t')\exp[ipx-iE(p)(t-t')]G(x,t-t')\mathrm{d}t'$$
$$= \exp[ip(x-iE(p)t]\int_{0}^{\infty}\xi(E,t')G(x,t-t')\mathrm{d}t'.$$
(35)

This is Eq. (20) of the article.

Gaussian wave packets. Negligible spreading. Momentum amplitude distribution for a Gaussian wave packet of a spatial width Δx, and a mean momentum p,
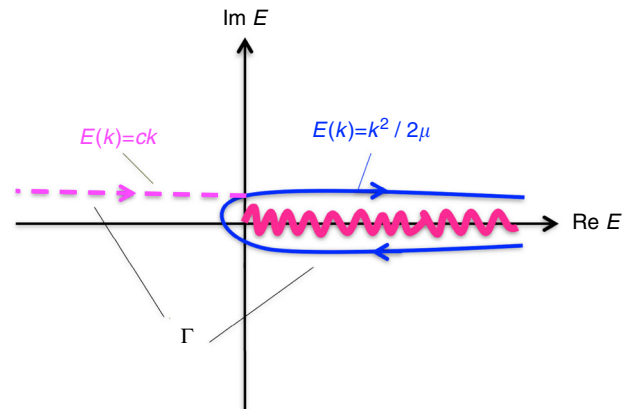

Fig. 2 The choices of the contour Γ in Eq. (34), for $\mathcal{E}(k) = ck$ (dashed) and $\mathcal{E}(k) = k^2/2\mu$ (solid)

whose centre passes through $x = 0$ at $t = 0$ is given by

$$A(k) = \Delta x^{1/2}/(2\pi)^{3/4} \exp[-(k-p)^2 \Delta x^2/4]. \quad (36)$$

Inserting (36) into Eq. (3) of the paper, for the amplitude $G(x, t)$ we find:

(a)   for $E(k) = ck$,

$$G(x, t) = G(x - ct, 0) = (2/\pi \Delta x^2)^{1/4} \exp[-(x-vt)^2/\Delta x^2], \quad (37)$$

(b)   for $E(k) = k^2/2\mu$,

$$G(x, t) = [2\Delta x^2/\pi \Delta x_t^4]^{1/4} \exp[-(x-vt)^2/\Delta x_t^2], \quad (38)$$

where the time dependent "complex width" Δx_t is given by

$$\Delta x_t = \Delta x \sqrt{1 + \frac{2it}{\mu \Delta x^2}}. \quad (39)$$

The spreading of the envelope over a time t can be neglected provided

$$2t/\mu \Delta x^2 \ll 1, \quad (40)$$

which allows a broad wave packet to travel a distance equal to many of its widths Δx, while retaining its shape. Replacing Δx_t with Δx in Eq. (38) yields Eq. (11) of the paper.

The classical limit and the tunnelling regime. Above the barrier, i.e., for $E(p) > V(x)$, the transmission amplitude is an oscillatory function. In the semiclassical limit, $pd \gg 1$, contributions to the integral Eq. (27) come mostly from the stationary points of the phase of the integrand. Thus, for a non-relativistic particle of a mass μ, and $E(k) \gg V(x)$, we have

$$T(k) \sim \exp\left\{ i \int_0^d [k^2 - 2\mu V(x)]^{1/2} dx - ikd \right\} \equiv \exp[i\Phi(k)]. \quad (41)$$

The phase of the integrand in Eq. (27) is stationary provided $\partial_k \Phi(k) + x' = 0$, which defines the value $k_s(x')$ (see Fig. 3a), such that

$$x' = d - v(k_s)\tau(k_s), \quad v(k_s) \equiv k_s/\mu, \quad \tau(k_s) \equiv \mu \int_0^d [k_s^2 - 2\mu V(x)]^{-1/2} dx > d/v(k_s). \quad (42)$$

We recognise in Eq. (42) one of the classical Eq. (3) of the article, with $\tau(k_s)$ being the time it takes a classical particle with a momentum k_s to cross the barrier
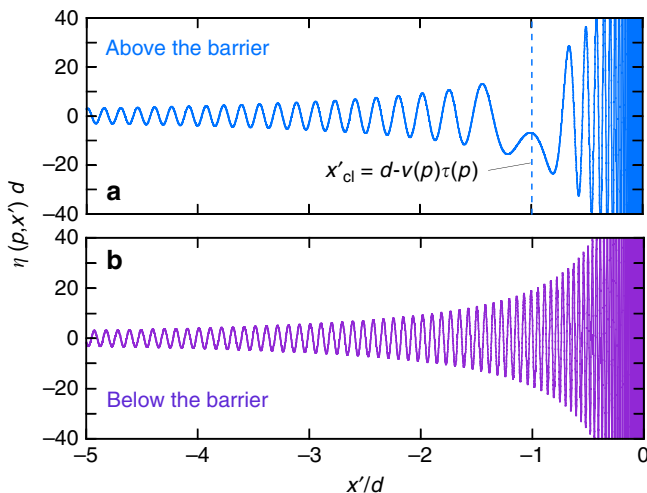

Fig. 3 Amplitude distribution of spatial delays. **a** For a particle passing above a rectangular barrier of a width d, $pd = 300$, and a height V, for $E(p)/V = 4/3$. There is a stationary region centred at the classical delay $x'_{cl}(p)$. **b** Same as **a**, but for the case of tunnelling, $E(p)/V = 2/3$. There is no preferred value for the delay x'

region $[0,d]$. Now the phase of $\eta(p,x')$, $\tilde{\Phi}(x') = \Phi(k_s(x')) + x'k_s(x') - px'$, is stationary for $x' = x'_{cl}(p)$ such that

$$k(x_{cl}'(p)) = p, \quad (43)$$

from which we recover again the classical relation

$$x_{cl}'(p) = d - v\tau(p) < 0, \quad (44)$$

and the integral (33) selects a single envelope, $G(x - x_{cl})$.

The position of the stationary point x_{cl}' depends on the particle's momentum, and in the tunnelling regime, $p < V(x)$, $x_{cl}'(p)$ moves off the real x'. Now all envelopes $G(x - x', t)$ contribute to the r.h.s. of Eq. (33), and destructive interference between them determines the shape of the transmitted wave packet.

As an illustration, we consider a rectangular barrier, $V(x) = V$ for $0 \le x \le d$, and truncate $T(p)$ to the first term in the multiple reflections expansion,

$$T(k) = \frac{4kq \exp(-ikd)}{(k+q)^2 \exp(-iqd) + (k-q)^2 \exp(iqd)} \approx \frac{4kq \exp[-i(k-q)d]}{(k+q)^2}, \quad (45)$$
$$q \equiv \sqrt{k^2 - 2\mu V}.$$

For $p^2 > 2\mu V$ and $x' < 0$, we, therefore, have (see Fig. 3a)

$$k_s = \frac{\sqrt{2\mu V}(d - x')}{\sqrt{x'(x - 2d)}}, \quad \tau(p) \equiv \frac{\mu d}{\sqrt{p^2 - 2\mu V}}, \quad x_{cl}'(p) = d\left[1 - \frac{p}{\sqrt{p^2 - 2\mu V}}\right]. \quad (46)$$

For $p^2 < 2\mu V$, $q = i|q|$ is imaginary, and the $x_{cl}'(p) = d\left[1 + ip(2\mu V - p^2)^{-1/2}\right]$ no longer lies on the real x'-axis, where $\eta(p, x')$ oscillates as shown in Fig. 3b. Note also that, according to Eq. (42), we should have $\tau(p) = i\mu d/|q(p)|$, which is why it is sometimes said (see, for example ref. [29]) that a particle "spends imaginary time" in a classically forbidden region.

Averages for convolutions. Suppose we have a complex valued function

$$\psi(f) = \int_{-\infty}^{\infty} G(f - f')\eta(f')df' = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} G^{(n)}(f)\tilde{f}^n, \quad \tilde{f}^n \equiv \int f^n \eta(f)df, \quad (47)$$

where $G^{(n)}(f) \equiv \partial_f^n G(f)$, and $\int_{-\infty}^{\infty} |G(f)|^2 df = 1$. We are interested in the average

$$\langle f \rangle = \int f |\psi(f)|^2 df / \int |\psi(f)|^2 df, \quad (48)$$

in the limit where $G(f)$ is made very broad, e.g., by rescaling some $G_0(f)$, whose width is of order of unity,

$$G(f) = \Delta^{-1/2} G_0(f/\Delta), \quad G^{(n)}(f) = (\Delta)^{-(n+1/2)} \partial_z^n G_0(z)\big|_{z=f/\Delta}. \quad (49)$$

Then with the help of (47) we obtain

$$\int |\psi(f)|^2 df = \sum_{m,n=0}^{\infty} \frac{(-1)^{m+n}}{m!n!\Delta^{(m+n)}} \tilde{f}^{m*} \tilde{f}^n I_{mn},$$
$$\int f |\psi(f)|^2 df = \sum_{m,n=0}^{\infty} \frac{(-1)^{m+n}}{m!n!\Delta^{(m+n-1)}} \tilde{f}^{m*} \tilde{f}^n J_{mn}, \quad (50)$$

with both I_{mn} and J_{mn} of order of unity,

$$I_{mn} = \int G_0^{(m)*}(z) G_0^{(n)}(z) dz, \quad J_{mn} = \int z G_0^{(m)*}(z) G_0^{(n)}(z) dz. \quad (51)$$

To the leading order in Δ we, therefore, have

$$\langle f \rangle = I_{00}^{-1} \left[J_{00}\Delta - J_{10}\bar{f}^* - J_{01}\bar{f} \right] + O(1/\Delta), \quad (52)$$

where

$$\bar{f} \equiv \tilde{f}^1/\tilde{f}^0 = \int f\eta(f)df / \int \eta(f)df. \quad (53)$$

In the special case of a $G_0(f)$ real, with a zero mean, $G_0^*(f) = G_0(f)$, $J_{00} = 0$, yields $J_{01} = J_{10} = -2I_{00}$ so that we have

$$\langle f \rangle = \text{Re}\bar{f}, \quad (54)$$

which is Eq. (14) of the article.

The relation between $\eta(p, x')$ and $\xi(E, t')$. To express $\xi(E, t')$ in terms of $\eta(p, x')$ for a non-relativistic particle of a mass μ, we substitute the expression for $T(p)$ Eq. (29) into Eq. (30), and change the variable E to $k = \sqrt{2\mu E}$. The contour of integration now runs down the positive imaginary axis, where Eq. (29) holds, and can be deformed to coincide with the whole real k-axis,

$$\xi(E, t') = (2\pi)^{-1} \exp(iTt') \int dx' \eta(x') \int_{-\infty}^{\infty} \partial_k E(k) \exp(-iE(k)t' - ikx')dk. \quad (55)$$

Noting that $\partial_k E(k)\exp(-iE(k)t') = it'^{-1}\partial_k \exp[-iE(k)t']$, and integrating by parts, yields

$$\xi(E, t') = -(t')^{-1} \exp(iEt') \int x' \exp[ip(E)x']K_0(-x', t')\eta(p, x')dx', \quad (56)$$

where $K_0(x, t) \equiv (2\pi)^{-1} \int_{-\infty}^{\infty} \exp[ikx - iE(k)t]dk$ is the free particle propagator.

The relation between the spatial and temporal (WS) delays. At a given time, the centre of mass of a broad transmitted packet with a momentum p lies at

$$\langle x^{\mathrm{T}}(t) \rangle \approx \mathrm{Re} \left[\frac{\int_{-\infty}^{0} x' \exp(-ipx')\eta(x')dx'}{\int_{-\infty}^{0} \eta(p, x')dx'} \right] = \mathrm{Re} \left[i\partial_p T(p)/T(p) \right] = \mathrm{Re} \left[\partial_p \ln T(p) \right]. \quad (57)$$

Similarly, for the "mean passage time", for a given x we have

$$\langle t(x) \rangle \approx \mathrm{Re} \left[\frac{\int_{0}^{\infty} t' \exp(iEt')\xi(t')dt'}{\int_{0}^{\infty} \exp(iEt')\xi(t')dt'} \right] = \mathrm{Re}[-i\partial_E \ln T(E)]. \quad (58)$$

Since $\partial_p = (\partial E/\partial p)\partial_E = v(p)\partial_E$, the complex quantities in the square brackets satisfy the relation [Eq. (3) of the paper] which exists between the classical spatial and temporal delays x' and t',

$$\frac{\int_{0}^{\infty} t' \exp(iEt')\xi(t')dt'}{\int_{0}^{\infty} \exp(iEt')\xi(t')dt'} = v(p)^{-1} \frac{\int_{-\infty}^{0} x' \exp(-ipx')\eta(x')dx'}{\int_{-\infty}^{0} \exp(-ipx')\eta(x')dx'}. \quad (59)$$

This is a simple consequence of the fact that, once transmitted, the particle moves freely, and has nothing to do with the transmission mechanism itself.

Tunnelling across zero-range ($\delta-$) barrier. For a particle of a mass μ, incident on a high narrow barrier, $V(x) = \Omega\delta(x)$, $\Omega > 0$, the transmitted amplitude $T(k)$,

$$T(k) = \frac{k}{k + i\mu\Omega} = 1 - \frac{i\mu\Omega}{k + i\mu\Omega}, \quad (60)$$

has a single pole on the imaginary negative k-axis. Deforming the integration contour in Eq. (27) into the lower half of the k-plane for $x' < 0$, we have

$$\tilde{\eta}(x') = -\mu\Omega \exp(\mu\Omega x'), \quad \text{and} \quad \eta(p, x') = \delta(x') - \mu\Omega \exp[(\mu\Omega - ip)x']. \quad (61)$$

To obtain the temporal delay distribution $\xi(t')$ we need to evaluate the integral Eq. (30) along the contour shown in Fig. 2 by the dashed line, which for $t' > 0$ can be transformed into the one shown there by the solid line. Thus, from Eq. (30) we have

$$\begin{aligned} \tilde{\eta}(t') &= \delta(t') - \frac{i\mu\Omega}{2\pi} \int_{-\infty}^{\infty} \frac{k \exp(-ik^2 t'/2\mu)}{k + i\mu\Omega} dk \\ &= \delta(t') - \Omega\sqrt{\frac{i\mu}{2\pi t'}} + \frac{i\mu\Omega^2}{2} \exp\left(\frac{i\mu\Omega^2 t'}{2}\right) \mathrm{erfc}\left(\sqrt{\frac{i\mu\Omega^2 t'}{2}}\right), \end{aligned} \quad (62)$$

where $\mathrm{erfc}(z) = 2\pi^{-1/2} \int_{z}^{\infty} \exp(-z^2)dz$[25], and we have used the known relation[30] $\int_{-\infty}^{\infty} \exp(-x^2/4\beta - \gamma x)dx = \sqrt{\pi\beta}\exp(\beta\gamma^2)\mathrm{erfc}(\gamma\sqrt{\beta})$. Multiplying Eq. (62) by $\exp(Et')$ yields Eq. (22) of the paper. Note that we can also arrive at Eq. (62) by applying Eq. (21) of the article directly to $\eta(p, x')$ in Eq. (61).

A similar analysis can be applied also to barriers of a more general shape, which have more than one pole in the lower half of the complex k-plane. Then equation for $\tilde{\eta}(x')$ will contain a sum over exponential terms, one for each pole, and a similar sum of complementary error functions will appear in the r.h.s. of Eq. (62).

Data availability. All relevant data are available on reasonable request from the authors.

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D.S. and E.A. both wrote the paper and reviewed it.

Additional information

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