Self-Organized Criticality

Large interactive systems naturally evolve toward a critical state in which a minor event can lead to a catastrophe. Self-organized criticality may explain the dynamics of earthquakes, economic markets and ecosystems

by Per Bak and Kan Chen

When catastrophe strikes, analysts typically blame some rare set of circumstances or some combination of powerful mechanisms. When a tremendous earthquake shook San Francisco, geologists traced the cataclysm to an immense instability along the San Andreas fault. When the stock market crashed on Black Monday in 1987, economists pointed to the destabilizing effect of computer trading. When fossil records revealed the demise of the dinosaurs, paleontologists attributed the extinction to the impact of a meteorite or the eruption of a volcano. These theories may well be correct. But systems as large and as complicated as the earth's crust, the stock market and the ecosystem can break down not only under the force of a mighty blow but also at the drop of a pin. Large interactive systems perpetually organize themselves to a critical state in which a minor event starts a chain reaction that can lead to a catastrophe.

Traditionally, investigators have analyzed large interactive systems in the same way as they have small, orderly systems, mainly because the methods developed for simple systems have proved so successful. They believed they could predict the behavior of a large interactive system by studying its elements separately and by analyzing its microscopic mechanisms individually. For lack of a better theory, they assumed that the response of a large interactive system was proportional to the disturbance. It was believed that the dynamics of large interactive systems could be described in terms of an equilibrium state that is disturbed now and then by an external force.

During the past few decades, it has become increasingly apparent, however, that many chaotic and complicated systems do not yield to traditional analysis. In 1987 one of us (Bak)—in collaboration with Kurt A. Wiesenfeld, now at the Georgia Institute of Technology, and Chao Tang, now at the Institute for Theoretical Physics in Santa Barbara—developed a concept to explain the behavior of composite systems, those containing millions and millions of elements that interact over a short range. We proposed the theory of self-organized criticality: many composite systems naturally evolve to a critical state in which a minor event starts a chain reaction that can affect any number of elements in the system. Although composite systems produce more minor events than catastrophes, chain reactions of all sizes are an integral part of the dynamics. According to the theory, the mechanism that leads to minor events is the same one that leads to major events. Furthermore, composite systems never reach equilibrium but instead evolve from one metastable state to the next.

Self-organized criticality is a holistic theory: the global features, such as the relative number of large and small events, do not depend on the microscopic mechanisms. Consequently, global features of the system cannot be understood by analyzing the parts separately. To our knowledge, self-organized criticality is the only model or mathematical description that has led to a holistic theory for dynamic systems.

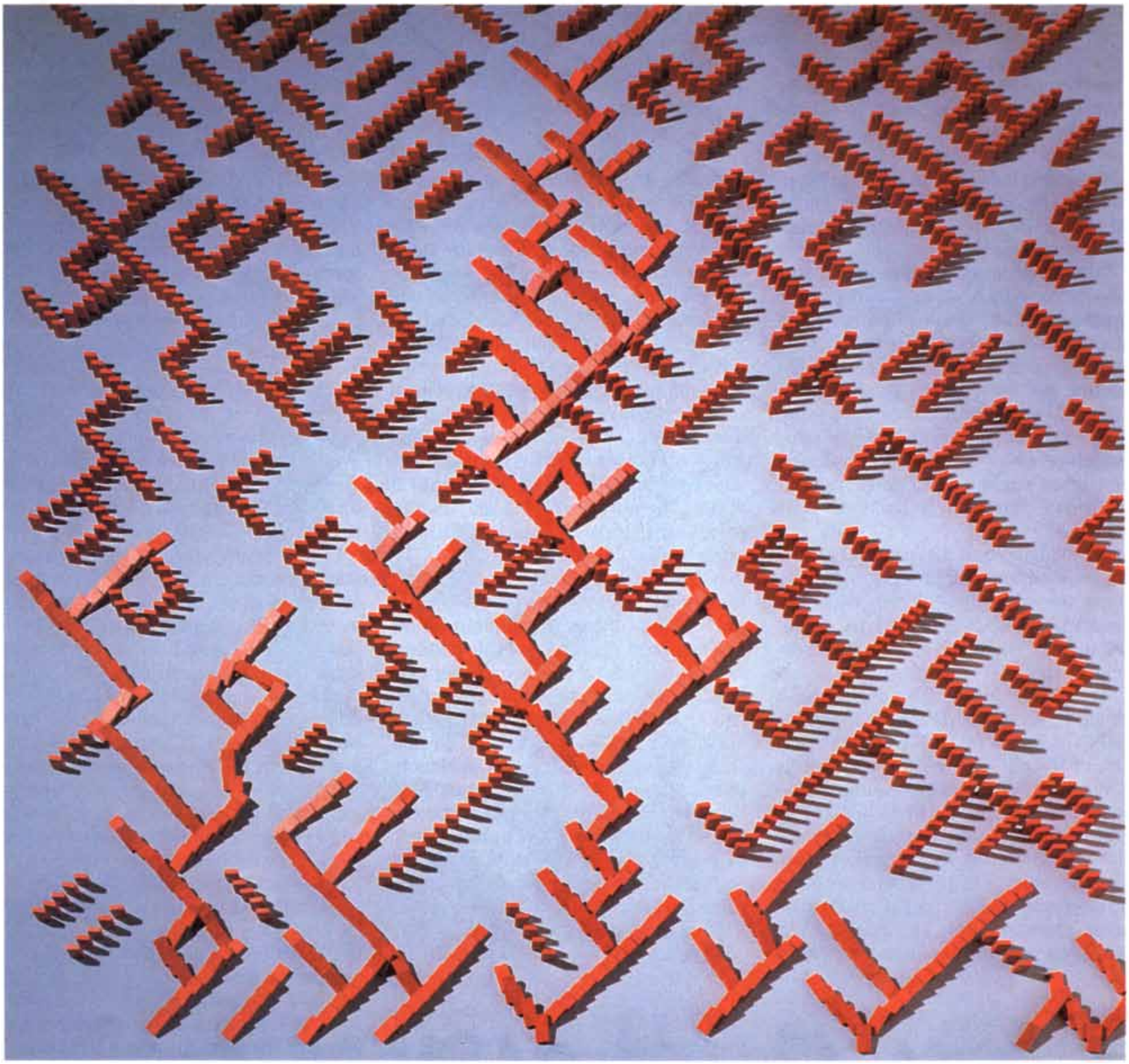
During the past four years, experiments and models have demonstrated that many composite systems at the heart of geology, economy, biology and meteorology show signs of self-organized criticality. These insights have improved our understanding of the behavior of the earth's crust, stock markets, ecosystems and many other composite systems.

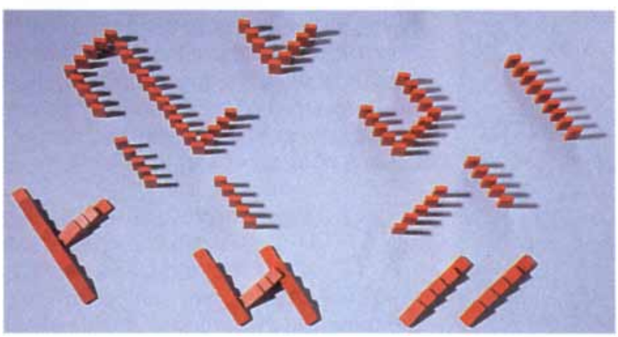
Because composite systems contain many components and are governed by many interactions, analysts cannot possibly construct mathematical models that are both totally realistic and theoretically manageable. They therefore have to resort to simple, idealistic models that capture the essential features of real systems. If the simple models are robust with respect to various modifications, they might be able to extrapolate the findings to real situations. (This approach has been very successful in equilibrium statistical mechanics, where universal phenomena in systems with many degrees of freedom can be understood from studies of simple models.)

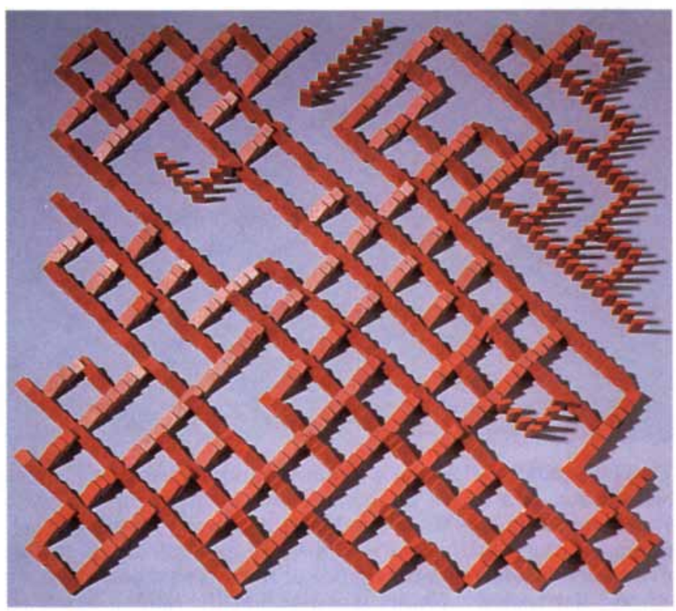
A deceptively simple system serves as a paradigm for self-organized criticality: a pile of sand. Some investigators have simulated the dynamics of sandpiles with computer programs; others such as Glenn A. Held and his colleagues at the IBM Thomas J. Watson Research Center have performed experiments. Both models and experiments reveal the same features.

Held and his co-workers devised an apparatus that pours sand slowly and uniformly, one grain at a time, onto a flat, circular surface. At first the grains stay close to the position where they land. Soon they rest on top of one another, creating a pile that has a gentle slope. Now and then, when the slope becomes too steep somewhere on the

PER BAK and KAN CHEN have studied self-organized criticality together since 1988. Bak is a senior scientist at Brookhaven National Laboratory. In 1974 he received his Ph.D. in physics at the Technical University of Denmark. Working for NORDITA Denmark, the IBM Thomas J. Watson Research Center and the University of Copenhagen, he pursued his interests in statistical physics, dynamic systems (chaos), low-dimensional conductors and quasicrystals. Bak was recently elected to the Danish Academy of Sciences. Chen is a postdoctoral fellow at Simon Fraser University, British Columbia. In 1983 he graduated from the University of Science and Technology in China, and in 1988 he obtained his Ph.D. in physics from Ohio State University. As a research associate at Brookhaven, he investigated self-organized criticality and dynamic optimization processes.







DOMINOES demonstrate criticality, subcriticality and supercriticality. In the critical system (*top*), dominoes were randomly placed on about half of the segments in a diamond grid. When the dominoes in the bottom row were tipped over, the critical system produced many sizes of chain reactions. The subcritical system (*bottom left*)—in which the density of dominoes was much less than the critical value—produced small chain reactions. The supercritical system (*right*)—in which the density was much greater than the critical value—exploded with activity.

pile, the grains slide down, causing a small avalanche. As more sand is added and the slope of the pile steepens, the average size of the avalanches increases. Some grains begin to fall off the edge of the circular surface. The pile stops growing when the amount of sand added is balanced, on average, by the amount of sand falling off the edge. At that point, the system has reached the critical state.

When a grain of sand is added to a pile in the critical state, it can start an avalanche of any size, including a "catastrophic" event. But most of the time, the grain will fall so that no avalanche occurs. We have found that even the largest avalanches involve only a small proportion of grains in the pile, and therefore even catastrophic avalanches cannot cause the slope of the pile to deviate significantly from the critical slope.

An avalanche is a type of chain reaction, or branching process. By simplifying the dynamics of the avalanche somewhat, one can identify the major features of the chain reaction and develop a model.

At the beginning of an avalanche, a single grain slides down the slope because of some instability on the surface of the pile. The grain will stop only if it falls into a stable position; otherwise, it will continue to fall. If it hits grains that are almost unstable, it will cause them to fall. As the process continues, each moving grain may stop or continue to fall, and it may cause other grains to fall. The process will cease when all the active particles have stopped or have moved off the sandpile. To measure the size of the avalanche, one can simply count the total number of fallen particles.

The pile maintains a constant height and slope because the probability that the activity will die is on average balanced by the probability that the activity will branch. Thus, the chain reaction maintains a critical state.

If the pile is shaped so that the slope is less than the critical value—the subcritical state—then the avalanches will be smaller than those produced by the critical state. A subcritical pile will grow until it reaches the critical state. If the slope is greater than the critical value—the supercritical state—then the avalanches will be much larger than those generated by the critical state. A supercritical pile will collapse until it attains the critical state. Both subcritical and supercritical piles are naturally attracted to the critical state.

What will happen if one uses wet sand instead of dry or if one tries to prevent avalanches by building "snow screens"? At first the wet pile produces smaller avalanches at a slower rate than those of a comparable dry pile. After a while the wet pile builds up to a state steeper than the dry pile. In that state the wet pile supports avalanches of all sizes; it has evolved to a critical state. Similar dynamics can be observed for a pile that has snow screens. In general the critical state is robust with respect to any small change in the rules for the system.

The sandpile has two seemingly incongruous features: the system is unstable in many different locations; nevertheless, the critical state is absolutely robust. On the one hand, the specific features, such as the local configurations of sand, change all the time because of the avalanches. On the other, the statistical properties, such as the size distribution of avalanches, remain essentially the same.

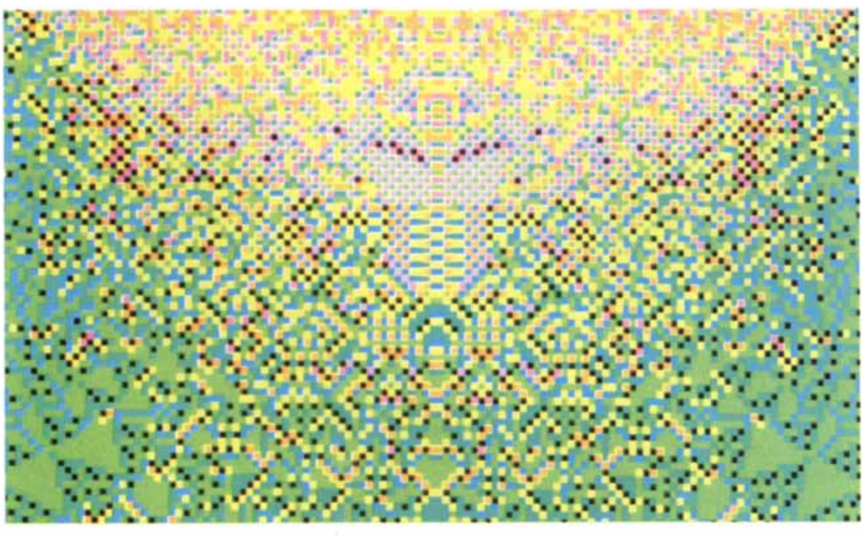
An observer who studies a specific area of a pile can easily identify the mechanisms that cause sand to fall, and he or she can even predict whether avalanches will occur in the near future. To a local observer, large avalanches would remain unpredictable, however, because they are a consequence of the total history of the entire pile. No matter what the local dynamics are, the avalanches would mercilessly persist at a relative frequency that cannot be altered. The criticality is a global property of the sandpile.

Even though sand is added to the pile at a uniform rate, the amount of sand flowing off the pile varies greatly over time. If one graphed the flow versus time, one would see a very erratic signal that has features of all durations. Such signals are known as flicker noise, or $1/f$ noise (pronounced "one over 'ef' noise"). Scientists have long known that flicker noise suggests that the dynamics of a system are strongly influenced by past events. In contrast, white noise, a random signal, implies no correlation between the current dynamics and past events.

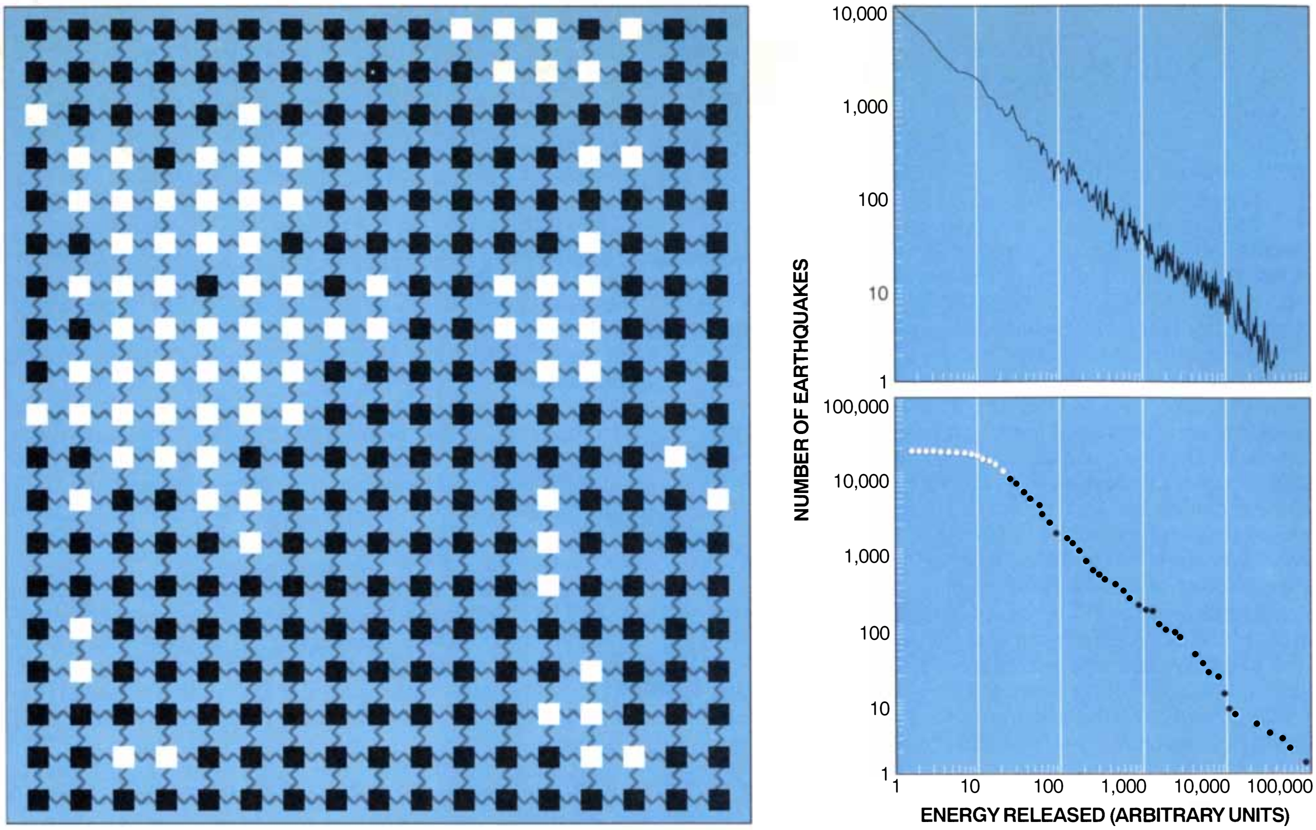
Flicker noise is extremely common in nature. It has been observed in the activity of the sun, the light from galaxies, the current through a resistor and the flow of water through a river. Indeed, the ubiquitousness of flicker noise is one of the great mysteries in physics. The theory of self-organized criticality suggests a rather general interpretation: flicker noise is a superposition of signals of all sizes and durations—signals produced when a dynamic system in the critical state produces chain reactions of all sizes and durations.

We and our colleagues have built many computer models that exhibit self-organized criticality. These models have helped us understand the dynamics of earthquakes, ecosystems and turbulence in fluids.

Models of earthquakes have perhaps been the most successful. In 1956 geologists Beno Gutenberg and Charles F. Richter, who is famous for devising the Richter scale, discovered that the number of large earthquakes is related to the number of small ones (known as the Gutenberg-Richter law). The number of earthquakes each year that re-



COMPUTER SIMULATION of a sandpile naturally evolves to a critical state in which the addition of a single grain can cause avalanches throughout the system. As grains were added to the pile (along the top row of the image), the computer determined where each grain would move and calculated the slope of the pile at several points. Pink squares represent the steepest parts of the pile; black squares indicate flat regions. Grains that reached the sides or bottom fell off the pile.



EARTHQUAKE MODEL simulates the forces on blocks of the earth's crust. Whenever the force on a block exceeds a critical value, the block slides, and the force is transferred to neighboring blocks. Each white square represents sliding blocks; each cluster represents an earthquake. The model produces earthquakes of all sizes, ranging from a single sliding event to "catastrophic" clusters extending throughout the system. The total number of sliding events in a cluster is a measure of the energy released during that earthquake. The graph at the top left shows the results accumulated after 10,000 model earthquakes. For comparison, the graph at the bottom right displays real earthquake measurements collected by Arch C. Johnston and Susan Nava at the New Madrid seismic zone in the U.S. The results from both the earthquake model and the measurements can be described by a power law: the number of earthquakes of energy, *E*, is proportional to *E* to the power of some constant. Power laws can be taken as evidence of self-organized criticality.

lease a certain amount of energy, *E*, is proportional to one divided by *E* to the power of *b* where the exponent *b* is about 1.5. The exponent *b* is universal in the sense that it does not depend on the particular geographic area. Hence, large earthquakes are much more rare than small ones. For example, if an area is hit each year, say, by one earthquake of energy 100 (in some units), it will experience approximately 1,000 earthquakes of energy 1 each year.

Because the number of small earthquakes is systematically related to the number of large earthquakes, one might suspect that small and large events arise from the same mechanical process. We and our co-workers believed the power-law distribution was evidence of self-organized criticality. Before we could test the theory, however, we needed to understand how we could simulate the process that produces earthquakes.

It is generally assumed that earthquakes are caused by a stick-slip mechanism: regions of the crust stick and then slide against other regions, creating faults. When one region slides against another, stress is released and propagates to adjacent regions.

To replicate this mechanism in the laboratory, Vladimir Bobrov and Mihail Lebyodkin of the Institute of Solid State Physics in Chernogolovka, Moscow, performed an experiment in which they applied pressure to an aluminum rod, representing a region of the earth's crust. The pressure caused a transition from elastic flow (at which point the rod would return to its original shape once the pressure was released) to plastic flow (at which point the deformation was irreversible). In the plastic phase the rod developed a "fault" region where two parts of the rod would slide against each other. Bobrov and Lebyodkin observed "earthquakes" whose size and frequency were related by a power law. When they conducted the experiments with niobium bars instead of aluminum, they obtained the same results, even though the microscopic mechanisms for the two materials are different.

We have constructed a simple computer model of the earth's crust that reproduces important features of earthquakes. Our model is composed of one elastic plate and one rigid plate, for simplicity's sake. The elastic plate is represented by a two-dimensional array of blocks, each connected to four neighbors by springs. As the array of blocks is squeezed, the springs exert a force on the blocks proportional to the compression. (We later included other types of forces in the model, resulting in little change in the dynamics.) The blocks of the elastic plate interact with the rigid plate through friction.

Whenever the spring force exerted on a particular block exceeds a critical value, the block slides. It continues to move until the spring force has been reduced below the critical value. The force lost by the block is transferred equally to its four neighbors. (During this process, the potential energy stored in the springs is first converted to kinetic energy and then dissipated when the blocks are decelerated by frictional forces.) The model describes the force distribution before

Self-Organized Criticality and Sandpiles

The theory of self-organized criticality makes a simple prediction about sandpiles: when a single grain of sand is dropped on a pile, it usually causes a few grains to fall, but every so often it will initiate a large avalanche. To test that prediction on real sandpiles, Glenn A. Held and his colleagues at the IBM Thomas J. Watson Research Center recently devised an ingenious experiment.

The most difficult challenge for the IBM group was constructing an apparatus that would add sand to a pile slowly, essentially one grain at a time. Held and his co-workers mounted a variable-speed motor on a laboratory stand and clamped a funnel to the motor's drive shaft. The funnel consisted of a 250-milliliter leveling bulb and a capillary tube 23 centimeters long and 2.0 millimeters in inner diameter. They then filled the funnel with sand and angled it about two degrees lower than horizontal so that sand grains slid into the capillary tube but did not fall through.

As the motor rotated the funnel about its axis at approximately 60 revolutions per minute, the sand grains lined up in the capillary tube and traveled single file to the end. By adjusting the angle of the funnel and the speed of the motor, the team could tune the apparatus so that one grain would fall every 15 seconds. They positioned the mouth of the capillary tube about 10 centimeters above the pan of a high-precision balance.

The balance had a precision of .0001 gram and a capacity of 100 grams. Each grain of sand weighed about .0006 gram; a sandpile whose base was four centimeters in diameter weighed approximately 15 grams.

To support the sandpiles, Held and his colleagues built circular plates ranging from one to eight centimeters in diameter. They attached each plate to a post 2.5 centimeters long and .5 centimeter in diameter. The post, in turn, was connected to a circular base four centimeters in diameter. The whole assembly—which looked like a tiny cake stand or a spool—rested on top of the balance pan. They built a metal skirt around the post of the "spool" to keep the sand that fell off the pile away from the balance pan so that the balance would record only the weight of the pile itself.

The balance was enclosed by a shield to prevent drafts from blowing the sand around. The equipment sat on a heavy table to damp vibrational disturbances. It took Held about 10 hours to put the whole apparatus together.

In the first experiments Held and his co-workers used aluminum oxide particles, but they found that sand from Long Island Beach worked just as well. They prepared the sand by drying it in an oven and filtering the grains through a coarse and then a fine sieve. They kept the grains that passed through the coarse mesh (eight cross wires per centimeter) but removed the grains that flowed through the fine mesh (10 cross wires per centimeter).

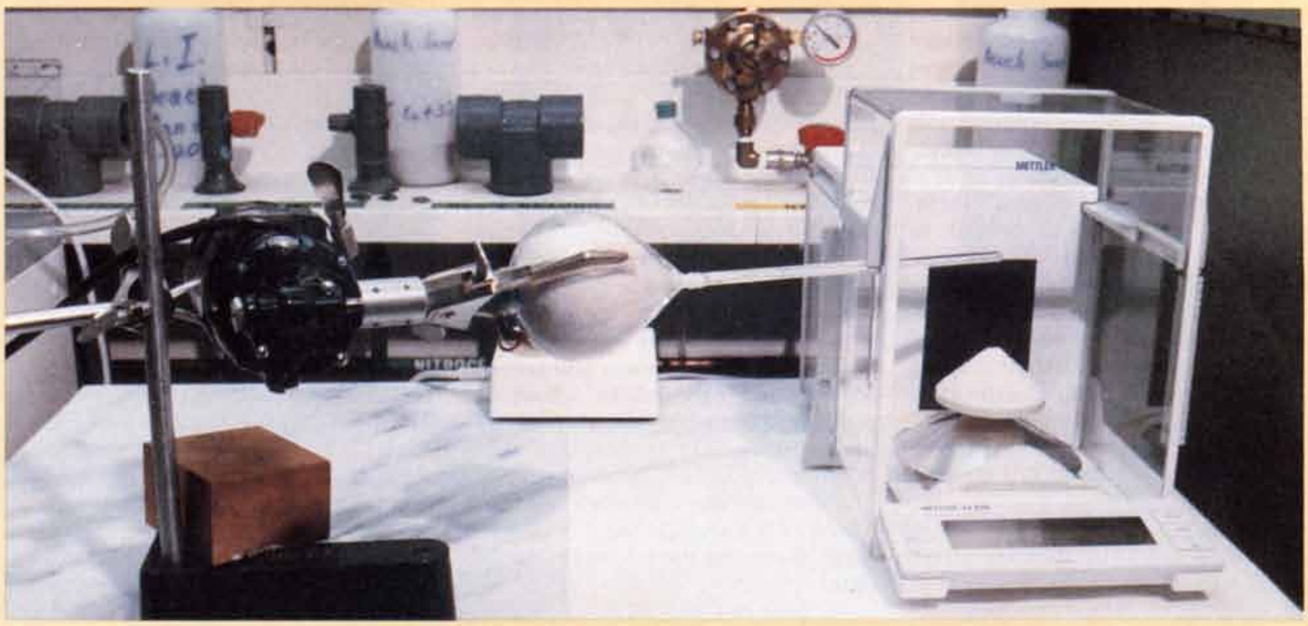
They filled the funnel with sand and crudely molded a pile on a four-centimeter circular plate. To ensure that the sandpile would settle into its natural form, they allowed grains to fall on the pile continuously for a few hours. They then watched avalanches of sand cascade down the pile. As sand fell off the edges of the plate, they measured the fluctuations in the mass of the pile.

The group used a personal computer to control the motor and to monitor the balance. When the computer detected a change in mass comparable to a grain of sand, it stopped the rotation of the funnel and thus the flow of sand. After the balance stabilized, the computer recorded the mass. It then restarted the motor to resume the dropping process.

Held and his colleagues ran the system for two weeks, dropping more than 35,000 grains on the four-centimeter plate. They observed avalanches in a range of sizes. The mass of the pile fluctuated by one to several hundred grains when as few as one and as many as several thousand grains were added to the pile. This result strongly suggested that the sandpile had indeed organized itself to a critical state.

When the team increased the diameter of the sandpile's base by using an eight-centimeter plate, however, they found that the pile produced only large avalanches (about four grams). They concluded that sandpiles of this size do not exhibit self-organized criticality. They do not yet understand why only small piles naturally evolve to a critical state.

More information about the sandpile experiment can be found in "Experimental Study of Critical-Mass Fluctuations in an Evolving Sandpile," by Glenn A. Held et al., in *Physical Review Letters*, Vol. 65, No. 9, pages 1120–1123; August 27, 1990.



and after each event but not the motions of the block or other details of the dynamic process.

When the model is driven by increasing the force at a uniform, low rate on all blocks in the same direction, the model begins to produce a series of earthquakes. At first the model produces only small earthquakes, but eventually it evolves to a critical state in which both small and large earthquakes are generated. The uniform increase in force is balanced by the release of force at the boundary.

We studied the model most intensely after the system had evolved to the critical state. We assume that the crust of the earth has already evolved to the stationary, critical state, and, therefore, real earthquakes can be simulated by the critical state of the model.

In the model, the energy released during an earthquake is related to the number of sliding events that follow a single instability at some "epicenter." Indeed, if one counts the number of earthquakes of each size during an extended period, one obtains a power-law distribution similar to the Gutenberg-Richter law [*see illustration on page 49*]. The catastrophic earthquakes are represented by the high-energy part of the power-law curve, which can be extrapolated smoothly from the low-energy part, which represents small earthquakes. There is no separate mechanism for the large earthquakes.

We have created models in two, three and four dimensions in which four, six or eight springs, respectively, are connected to each block. The dimension determines the exponent *b* of the power law. In the critical chain reaction picture, different *b* values correspond to different couplings between the individual branching processes. Sergei P. Obukhov of the Landau Institute in Moscow has shown that in four or more dimensions the individual branching processes are essentially independent, and the *b* value can be determined to be 1.5 by analytical methods.

Of course, the active regions of real earthquakes are three-dimensional, and computer simulations are so far the only way to predict the real *b* value. One cannot expect a grossly simplified model to yield the correct exponents for the distribution of real earthquakes. Nevertheless, the model suggests that power laws should arise from self-organized critical systems, and, conversely, the Gutenberg-Richter law can be taken as evidence that the earth's crust is indeed locked in a perpetually critical state.

Investigators around the world have applied the theory of self-organized criticality to account for many other features of earthquakes. Keisuke Ito and Mitsuhiro Matsuzaki of Kobe University were able to explain the spatial distribution of epicenters with a slightly modified model. They also explained a simple empirical law for the number of aftershocks of a given magnitude known as Omori's law. Anne and Didier Sornette of the University of Nice studied the time intervals between large earthquakes and found patterns that may have important implications for long-term forecasting of earthquakes. Jean M. Carlson and James S. Langer of the Institute for Theoretical Physics created a one-dimensional model that simulated the motions of the earth's crust along a fault. They found that the model did indeed evolve to a critical state.

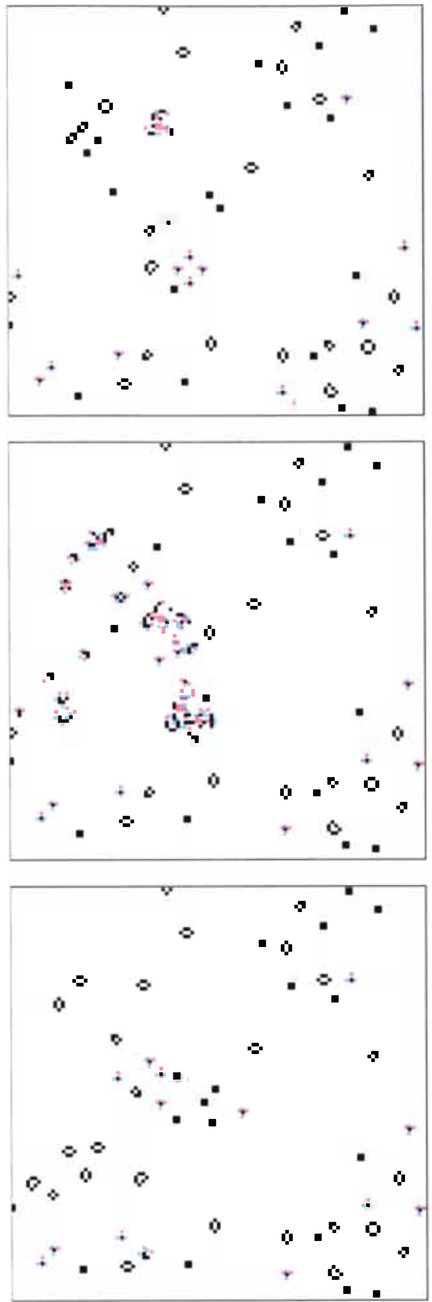
The theory of self-organized criticality has been successful not only in explaining the evolution of earthquakes but also in describing the distribution of the epicenters of earthquakes. For more than a decade, workers have known that power laws can describe the distribution of objects such as mountains, clouds, galaxies and vortices in turbulent fluids. The number of objects within, say, a sphere of radius r is proportional to r to the power of some constant D. Such a distribution of objects is generally called a fractal [see "The Language of Fractals," by Hartmut Jürgens, Heinz-Otto Peitgen and Dietmar Saupe; SCIENTIFIC AMERICAN, August 1990]. We find that fractals describe the distribution of the epicenters of earthquakes.

Although fractals appear throughout nature, investigators have only begun to understand the dynamics that create fractals. We and our colleagues suggest that fractals can be viewed as snapshots of self-organized critical processes. Fractal structures and flicker noise are the spatial and temporal fingerprints, respectively, of self-organized criticality.

Earthquake prediction still remains a difficult task. The stability of the earth's crust appears to be quite sensitive to the initial conditions of the system. Sometimes conditions far away from the epicenter can affect the evolution of an earthquake.

To evaluate the accuracy of predictions for a dynamic system, one must know the initial conditions with some precision as well as the rules of the dynamics. In nonchaotic systems, such as the earth orbiting around the sun, the uncertainty remains constant at all times: one can determine the earth's position in one million years with almost as much precision as one can know the earth's position today.

In chaotic systems, a small initial uncertainty grows exponentially with time. Furthermore, as one attempts to make predictions further and further into the future, the amount of information one needs to gather about the initial conditions increases exponen-



GAME OF LIFE simulates the evolution of a colony of organisms and suggests that the theory of self-organized criticality can explain the dynamics of ecosystems. Black squares indicate living organisms, and red squares show dying ones; blue squares represent the imminent birth of an organism. The first frame shows the colony a short time after a single organism was added to a stable configuration. In the second and third frames, the colony evolves to a new, stable state.

AVALANCHE DYNAMICS may be explained by the theory of self-organized criticality, which states that snow piles and other large complex systems naturally evolve to a critical state in which minor events can cause chain reactions of many sizes. If the theory proves correct, analysts may improve predictions of catastrophes.

tially with time. For the most part, this exponential growth prevents long-term predictions.

To check the accuracy of predictions in our earthquake model, we conducted two simulations of the critical state. The simulations differ by a small random force on each block, representing a small uncertainty about the initial conditions. When we run the two simulations, the uncertainty grows with time but much more slowly than it does for chaotic systems. The uncertainty increases according to a power law rather than an exponential law. The system evolves on the border of chaos. This behavior, called weak chaos, is a result of self-organized criticality.

Weak chaos differs significantly from fully chaotic behavior. Fully chaotic systems are characterized by a time scale beyond which it is impossible to make predictions. Weakly chaotic systems lack such a time scale and so allow long-term predictions.

Because we find that all self-organized critical systems are weakly chaotic, we expect weak chaos to be very common in nature. It would be interesting indeed to know whether the inaccuracy of earthquake predictions, economic forecasts and weather forecasts generally increases with time according to a power law rather than an exponential law.

For example, if weather is chaotic and if 100 observatories gather enough information to predict the weather two days in advance, then 1,000 observatories might allow predictions four days in advance. If weather is weakly chaotic, then 1,000 observatories might allow predictions 20 days in advance.

By a change of language (and a little bit of imagination), one can transform the sandpile or the earthquake model into many situations. It has been shown, for instance, that traffic on a highway is flicker noise. The stop-and-go pattern can be thought of as critical avalanches propagating in the traffic.

Models of traffic, sandpiles and earthquakes are similar in the sense that the number of elements is always conserved. For instance, the number of grains of sand in the pile always equals the number that was dropped on the pile minus the number that fell off. The conservation of elements is an important feature of many systems that naturally evolve to a critical state. The theory of self-organized criticality is not limited to systems that have local conservation laws, however, as can be shown by the "game of life."

In 1970 mathematician John H. Conway invented the game, which was subsequently popularized by Martin Gardner in a series of articles in these pages [see "Mathematical Games," SCIENTIFIC AMERICAN, October 1972]. The game of life simulates the evolution of a colony of living organisms and mimics the generation of complexity in nature.

To start the game, the pieces, or organisms, are placed at random on a board composed of square sites. Each site is occupied by at most one organism and is surrounded by eight neighboring sites. To determine the status of a site at each turn, one must count the number of organisms that occupy the eight neighboring sites. If one counts two live sites around an empty or an occupied site, then the status of the site does not change. If one counts three live sites around a site, the live sites give birth to a new organism or sustain the life of an old organism. In all other cases, an organism will die from overcrowding or loneliness.

The game continues according to the rules until it comes to "rest" in a simple periodic state, containing stable colonies. When the game is perturbed by adding an extra "live" cell, the system often sustains long transients of activity.

We and Michael Creutz of Brookhaven National Laboratory recently studied the game of life to determine whether the number of live sites would fluctuate over time, like the size of the avalanches in the sandpile model. Once the system settled into its rest state, we added a single organism at a random position, waited until the system settled and then repeated the procedure. Next we measured the total number of births and deaths in the "avalanche" after each additional perturbation. Indeed, the distribution was found to be a power law, indicating that the system had organized itself to a critical state.

We also found that the distribution of live sites is a fractal that can be described by a power law. The average number of active sites within a distance r from a given active site was proportional to r to the power of D, where D turned out to be about 1.7.

But is the criticality accidental in the sense that it occurs only for the very particular rules invented by Conway? To find the answer, we have constructed models that are variations of the game of life. Some variations were three-dimensional; in others, organisms were added to the system as it was evolving, or they were introduced at specific rather than random sites. All variations evolved to a critical state and could be described by power laws that seem to depend only on the spatial dimension.

We speculate that our models may have important ramifications in real biology. One may view the game of life as a toy model of a coevolutionary system. Each site can represent a gene of a very simple species. This gene can assume the values of 1 or 0. The stability of each value depends on the environment as expressed by the values of the genes of nearby species. The coevolutionary process then takes the system from an initial random state to the highly organized, critical state with complex static and dynamic configurations. The complexity of the global dynamics is intimately related to the criticality of the dynamics. In fact, the theory of complexity and the theory of criticality may generically be one and the same thing.

Biologist Stuart Kauffmann of the University of Pennsylvania has suggested a model for evolution in which species are represented by strings of numbers (genes). The genes interact both within and between species. The fitness of individual species is thus coupled to the fitness of other species. Kauffmann speculated that the complexity of life might be intimately related to the existence of a critical state. Our studies indicate that evolution may indeed automatically lead a simple, more or less random interactive dynamic system to precisely such a critical state. If this scenario is correct, then evolution operates at the border of chaos. The extinction of the dinosaurs, for instance, may be thought of as an avalanche in the dynamics of evolution, and an external force, such as a meteorite or volcano, would not be necessary.

Philip W. Anderson of Princeton University, Brian W. Arthur of Stanford University, Kauffmann and one of us (Bak) became aware that fluctuations in economics might indeed be avalanches in a self-organized critical state of that system. Benoit B. Mandelbrot of IBM has analyzed indicators such as the Dow Jones index and found fluctuations similar to flicker noise. The various metastable stationary states of economics might correspond to the various metastable states of a sandpile or the earth's crust.

Conventional models assume the existence of a strongly stable equilibrium position for the economy, whereby large aggregate fluctuations can result only from external shocks that simultaneously affect many different sectors in the same way. Yet it is often difficult to identify the reasons for such large-scale fluctuations as the depression of the 1930s. If, on the other hand, the economy is a self-organized critical system, more or less periodic large-scale fluctuations are to be expected even in the absence of any common jolts across sectors.

To check the viability of those ideas, we and José A. Scheinkman and Michael Woodford of the University of Chicago have created a simple model in which companies that make different products occupy positions in a two-dimensional lattice, or economic web. Each company buys supplies from two companies located at adjacent positions. Each then produces new products, which it tries to sell on the open market. If the demand for each company's product varies at random by a small amount, many companies may experience an "avalanche" in sales and production. Simulations indicate that such a model tends toward a self-organized critical state in the way the sandpile model does. Large fluctuations are intrinsic and unavoidable properties of the dynamics of this model economy.

The theory of self-organized criticality may also find applications in fluid dynamics. It has long been assumed that energy in turbulent fluids is stored in swirling vortices of all sizes. Mandelbrot has suggested that the energy dissipation would be confined to a tiny part of the space, located on a complex fractal structure. Although this seems to be in accordance with experiments, no theory or calculation that supports this picture has been performed.

In collaboration with Tang, we have constructed a simple "toy" model of turbulence that operates at the self-organized critical state. The model simulates forest fires where "trees" grow uniformly and burn (energy dissipated) on a fractal. The energy dissipation may be viewed as caused by a sequence of fires propagating as avalanches. In the critical state, there is a distribution of fires and forests of all sizes, corresponding to the fact that during turbulence energy is stored in vortices of all sizes. Although the model is quite unrealistic for turbulence in liquids, we nevertheless speculate that turbulence may indeed be a self-organized critical phenomenon. One consequence of this hypothesis (which can be readily studied experimentally) is that fully developed turbulence is not a "strong" chaotic phenomenon, as often assumed. Turbulence would then be only weakly chaotic, as is the case in the earthquake model.

One might think of more exotic examples of self-organized criticality. Throughout history, wars and peaceful interactions might have left the world in a critical state in which conflicts and social unrest spread like avalanches. Self-organized criticality might even explain how information propagates through the neural networks in the brain. It is no surprise that brainstorms can be triggered by small events (for instance, we hope, reading this article).

FURTHER READING

SELF-ORGANIZED CRITICALITY. Per Bak, Chao Tang and Kurt Wiesenfeld in *Physical Review A*, Vol. 38, No. 1, pages 364–374; July 1, 1988.

EARTHQUAKES AS A SELF-ORGANIZED CRITICAL PHENOMENON. Per Bak and Chao Tang in *Journal of Geophysical Research*, Vol. 94, No. B11, pages 15635–15637; November 10, 1989.

THE PHYSICS OF FRACTALS. Per Bak and Kan Chen in *Physica D*, Vol. 38, pages 5–12; 1989.

SELF-ORGANIZED CRITICALITY AND EARTHQUAKES. Anne Sornette and Didier Sornette in *Europhysics Letters*, Vol. 9, No. 3, pages 197–202; 1989.

SELF-ORGANIZED CRITICALITY IN THE 'GAME OF LIFE.' Per Bak, Kan Chen and Michael Creutz in *Nature*, Vol. 342, No. 6251, pages 780–782; December 14, 1989.

A FOREST-FIRE MODEL AND SOME THOUGHTS ON TURBULENCE. Per Bak, Kan Chen and Chao Tang in *Physics Letters*, Vol. 147, Nos. 5–6, pages 297–300; 1990.