SUPERCONDUCTIVITY AND OTHER MACROSCOPIC QUANTUM PHENOMENA

A diverse class of physical systems—including superconductors, superfluid helium, lasers and quasi-one-dimensional conductors—derive their unusual properties from the macroscopic occupation of a single quantum state.

John Bardeen

As first suggested by Fritz London, superconductivity and superfluid flow in liquid helium are macroscopic quantum phenomena. They depend on the fact that the energy states of even macroscopic objects, although closely spaced, are discrete, and on the statistical mechanics of systems made up of identical particles. The electrons in a superconducting metal, with a spin of one-half, obey Fermi–Dirac statistics and the exclusion principle. Helium atoms of isotopic mass 4 obey Einstein–Bose statistics, in which there can be many particles in the same quantum state, as is the case with photons, the quanta of radiation, if they are regarded as particles.

A common feature of these systems, as well as other macroscopic quantum systems, such as lasers and quasi-one-dimensional metals that undergo a Peierls transition, is macroscopic occupation of a state of the system. In superfluid helium at rest, there is a finite probability of finding the helium atoms with exactly zero velocity and momentum in spite of the large zero-point motion and thermal agitation (see figure 1). The probability is finite at temperatures below the λ transition to the superfluid state.

In superconductors as well as in superfluid helium of isotopic mass 3, there is a pairing such that the momentum of each pair is exactly the same as that for all other pairs. In lasers one of the modes of electromagnetic radiation possible in a cavity is macroscopically occupied. In quasi-one-dimensional metals at temperatures below the Peierls transition, it is a phonon mode that is macroscopically occupied. I shall discuss some common properties and ways of thinking about these systems.

Superconductivity was first observed at Leiden by Heike Kamerlingh Onnes in 1911, when he noted that the resistance of a rod of frozen mercury suddenly drops to zero when cooled to the boiling point of helium, 4.2 K. To show that the resistance is really zero for all practical purposes, he placed a lead ring in a magnetic field, cooled it below the transition temperature to the superconducting state, and then removed the external field. Currents induced to keep the flux through the ring from changing persisted indefinitely as long as the ring was kept in the superconducting state.

It was not until 1961 that William Fairbank and Bascom Deaver found experimentally that the flux threading a superconducting ring is quantized (see figure 2). The total flux Φ cannot have just any value, but is quantized so that it is an integral multiple of a small flux unit, $hc/2e = 2\times10^{-7}$ gauss cm^2. In his book *Superfluids*, published in 1950, London had predicted such a relation, with e rather than $2e$ in the denominator.

The superfluid properties of liquid helium when cooled below the λ transition at 2.2 K were discovered in 1938 by Jack Allen and collaborators and by Peter Kapitsa. A superfluid cannot be kept in an open beaker. A thin film will form on the inner and outer surfaces of the beaker. The helium in the film will flow up the inner surface and down the outer surface, and drip off the bottom until the beaker is empty. Superfluid helium flows through narrow channels without friction.

Semiconductor laser

The semiconductor laser illustrates the essential features of the systems I am discussing—discrete rather than continuous states with macroscopic occupation of one of them—in their simplest form. A semiconductor cavity with reflecting walls defines a set of normal modes for electromagnetic radiation. By current flow from a p–n junction, an excess population of electrons in the conduction band and holes in the valence band is created. These

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tend to recombine, emitting photons whose energy $h\nu$ is approximately equal to the energy gap. For excess population below a threshold value, the emitted radiation goes to a continuum of closely spaced modes. Above the threshold, a large part, a macroscopic fraction, of the radiation goes to a single mode (see figure 3).

The explanation is stimulated emission of radiation by the chosen mode. The probability of stimulated emission is proportional to the number of quanta of radiation N_ν present in the mode. As the excess population of electrons and holes increases above the threshold, N_ν increases exponentially until a steady state is reached in which a sizable fraction of the total emitted energy goes to the single mode. While the propagation of the electromagnetic radiation can be understood classically in terms of Maxwell's equations, the concentration of energy in a single mode of precise frequency cannot.

Quantization of circulation

In recent years, experiments have been done by John Reppy, William Zimmerman Jr and R. De Bruyn Ouboter that are analogous to those that show flux quantization. A capillary filled with liquid helium is formed into a circular ring. The ring is set in rotation above the superfluid transition temperature, cooled while rotating until the helium has reached the superfluid state, and then the ring is stopped. The superfluid helium keeps its original circulation around the ring. The angular momentum of the circulating helium can be detected from torque developed if the axis of circulation is tilted. These are the same forces that keep a spinning top from falling. From measurements of the very small forces from the circulating helium, it has been found that the circulation (the line integral of the velocity around the ring) is quantized:

$$C = \oint \mathbf{v}_s \cdot d\mathbf{l} = \frac{nh}{m_4}$$

Here n is an integer and m_4 the mass of a helium atom.

If written in terms of the momentum $\mathbf{p}_s$ the quantum condition is

$$\oint \mathbf{p}_s \cdot d\mathbf{l} = nh$$

the same condition suggested by Niels Bohr in 1913 to describe the allowed orbits in atomic hydrogen. Bohr's condition follows from the wave nature of the electron and the de Broglie relation between momentum and wavelength, $p = h/\lambda$. It reflects the requirement that there be an integral number of wavelengths around the path of integration.

In an ingenious experiment, W. F. "Joe" Vinen measured the circulation around a wire lying along the axis of a cylindrical tube containing superfluid helium. Under the conditions of his experiment, he observed either zero or one unit of circulation, never anything in between.

Quantized vortex lines with one unit of circulation, h/m_4, are observed in superfluid helium. If a bucket of helium is rotated and then cooled below the λ transition, the helium tends to rotate as a rigid body with the vorticity provided by an array of quantized vortex lines.

Very recently Richard Packard and his students have done an experiment similar to that of Vinen to measure the circulation in superfluid He^3-B. The He^3 atoms obey Fermi–Dirac statistics and undergo a pairing transition to a superfluid phase below $T_c \sim 0.3$ mK. The phase change was first observed by David Lee, Robert Richardson and Douglas Osheroff in 1972. The theory, due mainly to Tony Leggett, is analogous to the theory of superconductivity, but with triplet rather than singlet spin pairing. Packard and his collaborators found that below $T/T_c \sim 0.2$, the circulation around the wire is stable for zero or ± 1 units of circulation, $h/2m_3$, where $2m_3$ is the mass of a pair (see figure 4).

Fritz London (1900–1954), for many years a professor at Duke University, was the first to suggest that both superconductivity and superfluid flow in liquid helium are manifestations of quantum effects operating on the scale of macroscopic objects.

Superconductivity

The electrons in a superconducting metal form a charged superfluid. A current produces a magnetic field that affects the electrons' motion. In a magnetic field described by a vector potential $\mathbf{A}$, the drift velocity of an electron, $\mathbf{v}_s$, is related to the canonical momentum $\mathbf{p}_s$ by

$$m^*\mathbf{v}_s = \mathbf{p}_s - \frac{e^*}{c}\mathbf{A}$$

where $m^* = 2m_e$ and $e^* = 2e$, the mass and charge of a pair of electrons. The current density is $\mathbf{J}_s = \rho_s \mathbf{v}_s$, where ρ_s is the charge density of the superconducting electrons.

The supercurrent in a superconducting ring flows only in a thin penetration region near the inner surface. In the interior of a ring of reasonable thickness, $\mathbf{J}_s = \mathbf{v}_s = 0$, so

$$\oint \mathbf{p}_s \cdot d\mathbf{l} = \frac{e^*}{c}\oint \mathbf{A}\cdot d\mathbf{l} = \frac{e^*}{c}\Phi = nh$$

where Φ is the total magnetic flux threading the ring. This leads to the flux quantization mentioned earlier.

A superconductor is not simply a body with infinite conductivity. As shown by Walther Meissner in 1933, the state with the flux excluded is the unique stable state of a simply connected superconductor in a magnetic field. If the material is cooled into the superconducting state in the presence of a field, the flux is expelled from the interior, making the superconductor a perfect diamagnet ($\mathbf{B} = 0$).

In a class of superconductors called type II, above a lower threshold field H_{c1} flux enters the interior in the form of an array of quantized flux lines. A flux line may be regarded as a normal core surrounded by the superconducting bulk. Currents circulating in the superconductor around the axis of a flux line give a magnetic field along the axis. The total flux along the line is one flux unit, hc/e^*. If the flux lines are "pinned" so they cannot move, a supercurrent will flow around the normal cores with no dissipation of energy.

In a type-I superconductor it is not energetically favorable to form flux lines. To get a persistent current in a superconducting ring to decay and flux to leak out of the ring would require the equivalent of the passage of a vortex line through the body of the ring. It requires much more than thermal energy to form a flux line, so a supercurrent is extraordinarily stable. Decays have been observed only in rings of small thickness at temperatures very close to the transition temperature.

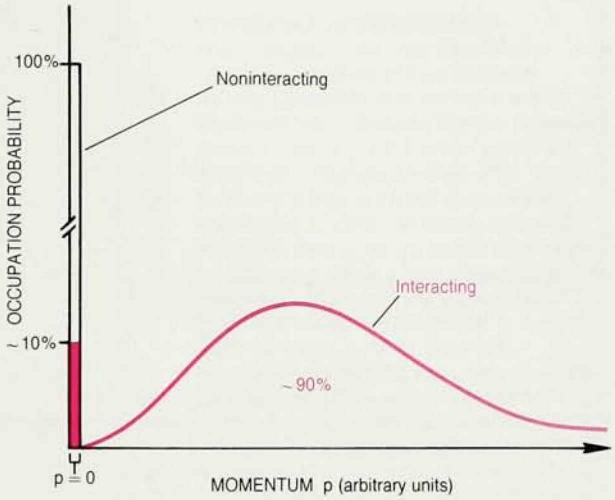
In type-II superconductors there is no dissipation as long as the flux lines remain pinned. In a large superconducting magnet one meter in diameter with a flux density of 4 tesla, the total number of flux quanta can be as large as 10^{15}. It is the difficulty of changing this large number by one flux quantum that prevents decay of the currents flowing around the superconducting coil.

Two-fluid model and superfluid flow in He⁴

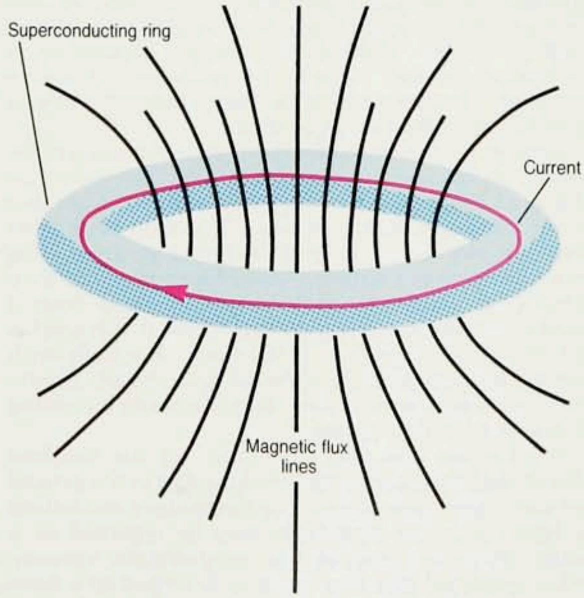
In 1934 Hendrik Casimir and Cornelis Gorter proposed a phenomenological two-fluid model to account for the thermal properties of superconductors. They suggested a model of interpenetrating normal and superfluids such that the total charge density of the electrons is $\rho = \rho_n + \rho_s$. The current density is $\mathbf{J} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$, with the normal component $\rho_n \mathbf{v}_n$ subject to the usual dissipation in an electric field. At high frequencies, such a dissipation is observed in the penetration depth of the magnetic field.

Casimir and Gorter were able to get an approximate fit to the thermal properties by assuming that ρ_n increases with the fourth power of the absolute temperature. There would be a second-order transition to the normal state when $\rho_n = \rho$ and $\rho_s = 0$, with a jump in specific heat but no latent heat. The results of experiments on surface impedance at microwave frequencies could be accounted for if it were assumed that the normal component is subject to the usual scattering and loss.

In 1938 Laszlo Tisza suggested a similar model for superfluid helium. The normal component flows down a temperature gradient, and there is a counterflow of the superfluid component. In steady state, a pressure difference develops between a heated component and the bulk of the fluid. A striking example of this behavior is the fountain effect of Jack Allen and H. Jones, in which the in-

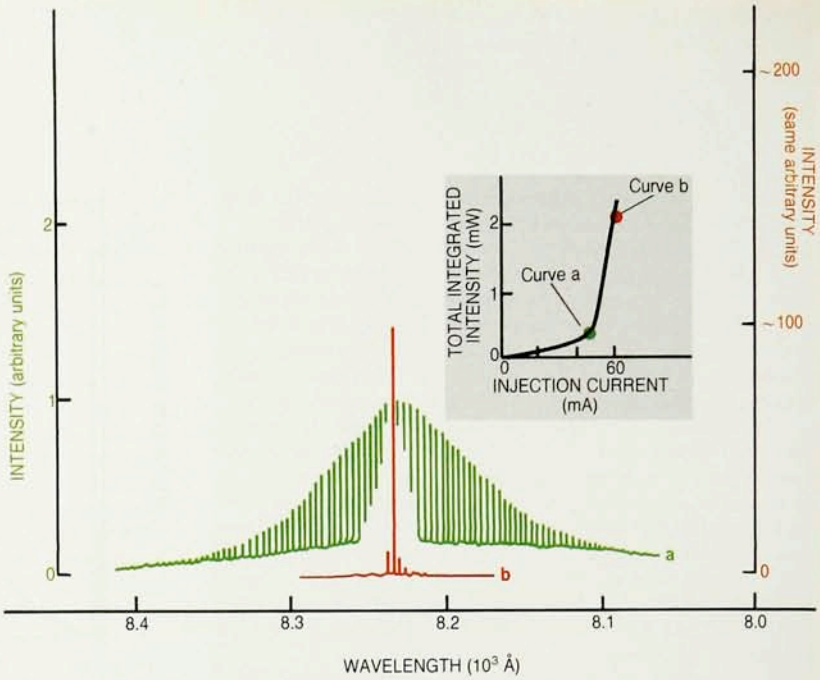


Momentum distributions of atoms in the ground state of superfluid helium-4 at $T = 0$ K. In a noninteracting Bose gas (shown in black), 100% of the particles are in the zero momentum state. In an interacting system (shown in magenta), because of zero-point motion the occupation probability is reduced to about 10%, but it is still finite. The probability remains finite up to the λ-transition temperature. **Figure 1**



Quantized flux. The total flux threading a superconducting ring is an integral multiple of a small flux unit, $\Phi_0 = hc/2e = 2 \times 10^{-7}$ gauss cm². **Figure 2**

Light intensity as a function of wavelength in a $Ga_{1-x}Al_xAs$–GaAs quantum well heterostructure laser. Curve a (green) was obtained using an injection current just below the threshold for lasing, curve b (red) using a current about 30% above threshold. As shown in the inset, the total light intensity of curve b is almost an order of magnitude larger than that of curve a, with nearly all the power going into a single mode that is macroscopically occupied. (Adapted from L. J. Guido *et al.*, *Appl. Phys. Lett.* **50**, 609, 1987, a paper by Nick Holonyak and associates at the University of Illinois, Urbana–Champaign, and Robert Burnham and associates at the Xerox Palo Alto Research Center.) **Figure 3**



side of a vessel is heated to make a fountain as much as 30 centimeters in height.

The two-fluid model may be illustrated by a model of flow between parallel walls with which the normal fluid is in thermal equilibrium. If the superfluid is at rest and the walls are moving with velocity $\mathbf{v}_n$, the flow is $\rho_n \mathbf{v}_n$. The superfluid at rest is a state of macroscopic occupation of the momentum $\mathbf{p}_s = 0$. If the walls are at rest and the superfluid is moving with velocity $\mathbf{v}_s$, the flow is $\rho_s \mathbf{v}_s$. If both are moving, the flow is $\rho \mathbf{v} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$.

Both $\mathbf{v}_n$ and $\mathbf{v}_s$ are required to specify the state of the fluid for thermal equilibrium. In a classical system, from Galilean invariance, it is necessary to give only $\mathbf{v}_n$, the velocity of the reference frame in which the system is in equilibrium. The state of macroscopic occupation, a nonclassical concept, leads to a breakdown of Galilean invariance. In the case of a superconductor, this is referred to as broken gauge symmetry.

Kapitsa did beautiful experiments to demonstrate the two-fluid properties in dramatic ways. The superfluid component behaves as a perfect fluid with potential flow (no vorticity) and zero viscosity. The fluid thus flows around an obstacle in its path, exerting no force. The normal component exerts the usual forces expected for a viscous fluid. If the normal component flows from a heated container into the bulk superfluid, it will exert a force on a vane immersed in the fluid. A spindle with vanes can be made to rotate from the force of a jet. The reaction from jets of normal fluid can put a spider consisting of a ring of jets into rotation.

Lev Landau was able to account for the two-fluid model of superfluid helium by invoking flow in the ground state and a spectrum of low-lying elementary excitations (see figure 5). The excitations may be regarded as a weakly interacting gas in the ground-state vacuum. Landau suggested that they could be described by a wave vector $\mathbf{k}$ of magnitude $2\pi/\lambda$. Those with wavelengths long compared with the interatomic spacing are phonons, the quanta of sound waves, with density oscillations. Those in the vicinity of an energy minimum at a larger wave vector Landau called rotons. It was not clear why only these two excitations would exist at low temperatures, nor was the role of statistics. The phonon modes are what might be expected for a solid body, not a liquid.

Richard Feynman attempted to understand how the phenomenological equation of the two-fluid model and Landau's excitation spectrum for superfluid helium follow from first principles. He suggested that the ground-state wavefunction Ψ_g could be taken as a real, everywhere-positive, symmetric function of the particle coordinates $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$, where symmetry means that Ψ_g is unchanged by every permutation of the particle coordinates. The square of Ψ_g gives the probability density for the particle distribution and is presumed to be small for distributions with any two particles close together.

Superfluid flow may be described by multiplying by a phase factor

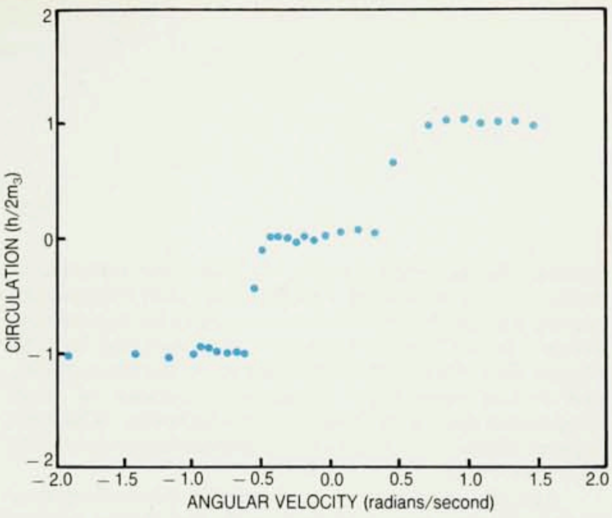
$$\Psi = \Psi_g \exp\Big(i \sum_j \phi(\mathbf{x}_j)\Big)$$

where $\mathbf{p}_s = m_4 \mathbf{v}_s = \hbar \,\mathrm{grad}\, \phi(\mathbf{x})$. This function thus describes potential flow with a velocity potential $(\hbar/m_4)\phi$. Superfluid flow is that of a perfect fluid with zero viscosity and no vorticity. It carries no heat or entropy.

Roger Penrose and Lars Onsager argued that the pair distribution function derived by integrating $\Psi_g{}^2$ over the coordinates of all particles except two should have a coherent part of the form $\psi(\mathbf{x}_1)\psi(\mathbf{x}_2)$, which they estimated to be about 10% of the total. This coherent part comes from macroscopic occupation of the state with momentum $p = 0$, with a probability of about 10%.

The conjecture of Penrose and Onsager was verified in a computer calculation done by a former student of mine, William McMillan, for his PhD thesis. It was the first application of Monte Carlo methods to a quantum problem. He found that the occupation of the zero-momentum state is about 11%, close to the value suggested by Penrose and Onsager. Since then, more elaborate calculations have been done, without significant changes in the results. Thus London's thought that the macroscopic occupation predicted for a Bose gas would not be qualitatively altered by the large interactions present

Average value of circulation in superfluid He^3-B (in units of $h/2m_3$) measured for 10 minutes after the angular velocity of the cryostat has been ramped to Ω_{max} and back to 0. Data for positive angular velocities were first taken. Then the cell was warmed up above T_c to remove any vorticity. Data for negative angular velocities were then taken after cooling down again. (Adapted from a paper by W. Davis, R. Zieve, J. Close and R. E. Packard, given at the International Conference on Low-Temperature Physics in Sussex, UK, in August.) **Figure 4**



in superfluid helium has proved to be correct.

The phonon part of Landau's spectra of elementary excitations comes from longitudinal density oscillations. The theory of the roton part, important for $T > 0.5$ K, proved to be more elusive. Progress was made by Feynman and Michael Cohen in 1957 and much more by David Pines and his students in the 1980s. The complete microscopic explanation is still subject to debate.

Microscopic theory of superconductivity

The breakthroughs that led to an understanding of superconductivity occurred in the 1950s. Most important was the discovery of the isotope effect (the dependence of the transition temperature on isotopic mass) and Herbert Fröhlich's independent suggestion that interactions between electrons and phonons are involved.

Fröhlich and I both attempted to develop theories based on the self-energy of the electrons in the field of the phonons. It soon turned out that these energies are included in the normal state. The energies of states near the Fermi surface are changed by the interaction, but not in such a way as to introduce an energy gap or otherwise give a departure from normal behavior.

The states for electrons in a metal are characterized by a wave vector **k** and spin σ in one-to-one correspondence with the electron states in a noninteracting Fermi sea. At $T = 0$ K, those states below the Fermi surface are occupied, those above, empty. With increasing temperature, electrons are thermally excited to quasiparticle states above the Fermi surface, leaving unoccupied states below. If a number of particles are excited, they are not exact eigenstates of the system because of residual interactions between them. These include an attractive electron–phonon interaction and a screened Coulomb interaction. The criterion for superconductivity is that the net interaction be attractive for quasiparticles with energies close to the Fermi surface.

In 1955 Leon Cooper came to the University of Illinois as a postdoctoral research associate to join one of my graduate students, Robert Schrieffer, in attacking the problem from this point of view. Less than two years later, we found the long-sought explanation of the mystery of superconductivity. The ground-state wavefunction of a superconductor can be considered to be a sum of low-lying normal configurations in which the quasiparticle states are paired such that if the state $\mathbf{k}\uparrow$ is occupied, then $-\mathbf{k}\downarrow$ is also occupied. The states of opposite spin and momentum are either both occupied or both empty.

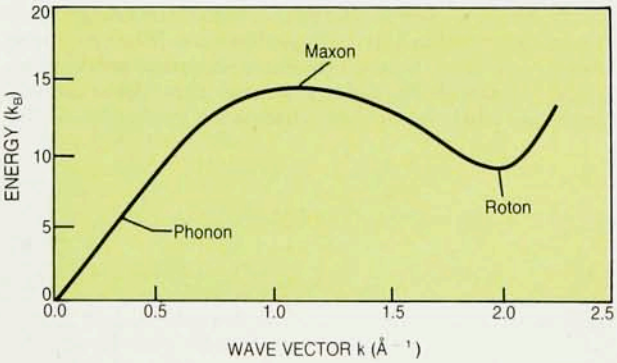
When there is current flow, the paired states $(\mathbf{k}_1\uparrow, \mathbf{k}_2\downarrow)$ all have exactly the same net momentum, $m^*\mathbf{v}_s = 2m\mathbf{v}_s = \hbar(\mathbf{k}_1 + \mathbf{k}_2)$. It is the common momentum of the paired states that gives the long-range order in momentum required by London. Scattering of individual particles does not change the common momentum of the paired states, so the current persists in time.

Cooper, Schrieffer and I gave an excitation spectrum for quasiparticles in a superconductor in one-to-one correspondence with those of the metal in the normal state (see figure 6). A minimum amount of energy, the energy gap, is required to create a pair of quasiparticle excitations from the ground state.

There is strong evidence that pairing is responsible for superconductivity in the high-T_c ceramic oxides, although the lifetimes of the states that are paired differ from those of ordinary metallic superconductors.

Transport in quasi-one-dimensional metals

As a final example of discrete states and macroscopic occupation, I shall discuss transport of electrons by



Landau spectrum of elementary excitations in superfluid He^4. Excitations of small wave vector (long wavelength) are phonons, quanta of longitudinal density fluctuations. Those of short wavelength are similar to particles moving in a fluid. Excitations at the energy maximum are called maxons, at the minimum, rotons. Above 0 K, the normal component is a weakly interacting gas of elementary excitations. **Figure 5**

moving charge-density waves in quasi-one-dimensional metals. As early as 1954, Fröhlich suggested transport by moving charge-density waves as a model for a superconductor. In 1977 such transport was observed by Pai-Phuam Ong, Pierre Menceau and Alan Portis in $NbSe_3$, and it has since been found in a number of other compounds that form linear-chain structures. While not superconductors, they have many interesting properties in their own right.

At high temperatures, conduction in quasi-one-dimensional compounds is metallic, with the Fermi surface consisting of fairly flat sheets normal to the chain direction at $\pm k_F$. The electrons couple strongly with a phonon of wave number $2k_F$. As a result of the electron-phonon interaction, the $2k_F$ phonon mode becomes softened. The frequency of the mode decreases with decreasing temperature and finally goes to zero at the Peierls transition temperature. The mode becomes macroscopically occupied at lower temperatures, with formation of a charge-density wave of electron and ion displacements.

An energy gap (the Peierls gap) opens up at $\pm k_F$ and the electrical conductivity becomes thermally activated. Since the wavelength is in general incommensurate with the lattice period in an ideal crystal, there is no preferred position for the charge-density wave in space. The phase can have any value. What Fröhlich suggested is that the charge-density wave could move and transport electricity.

For a drift velocity v_d, the charge density is

$$\rho = \rho_1 \sin[2k_F(x - v_d t)]$$

One may regard the Fermi distribution, along with the Peierls gaps, as being displaced by a wave vector $\mathbf{q}$ such that $\hbar\mathbf{q} = m\mathbf{v}_d$.

Experiments showed that the wave does not move freely, but is pinned by the phase-dependent energy from impurities or other lattice imperfections. What has been observed is that there is no charge-density-wave motion below a threshold field E_T and that the current associated with motion increases only gradually as the field is increased above threshold. The conductance, $G_{CDW} = I_{CDW}/E$, varies approximately as $G_b \exp(-E_0/E)$ for $E > \sim 2E_T$ (see figure 7).

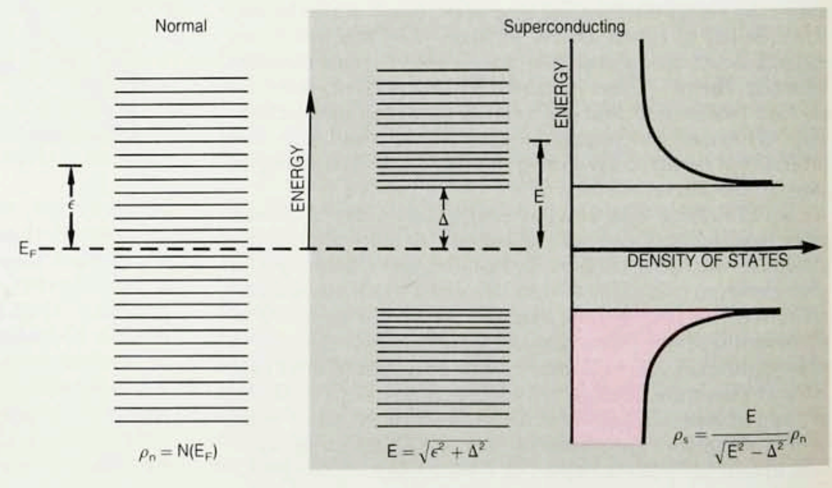
The exponential form suggests that depinning above threshold occurs by a tunneling process. The original investigators, Ong and his collaborators, proposed a model in which depinning occurs by Zener tunneling through a small pinning gap. They abandoned the theory when they found that when applied to a single chain of atoms the pinning energy is far smaller than the thermal energy. In 1979 I pointed out that a phase-coherent volume in which the $2k_F$ phonons are defined contains many parallel chains. The pinning energy in the volume can be far larger than $k_B T$, so that depinning does not occur thermally.

A quantum tunneling step is required to increase the momentum of the charge-density wave in an electric field because the momentum is quantized in units of $2\hbar k_F$. As Fröhlich pointed out, the momentum is given by the difference between the number of $2k_F$ phonons moving to the right and the number moving to the left:

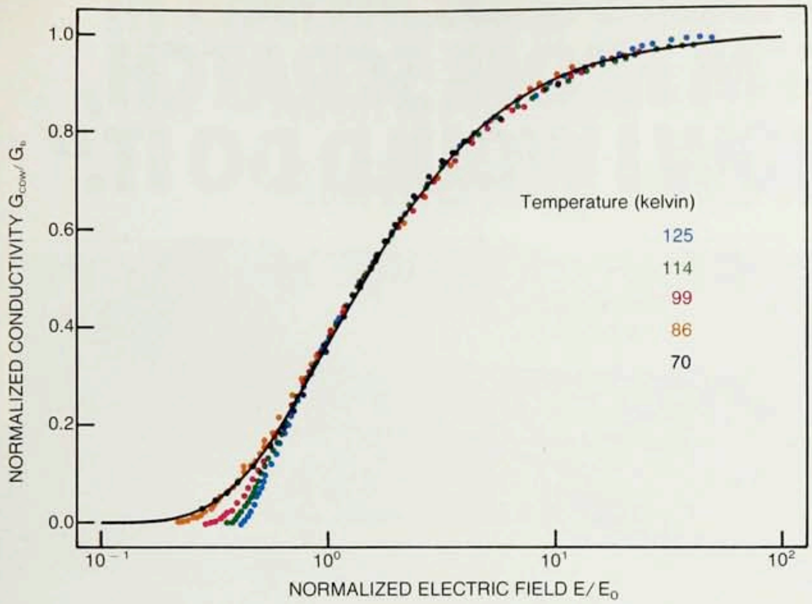
$$(N_R - N_L)2\hbar k_F = N_e M_F v_d$$

where N_R is the number moving right and N_L the number moving left. On the right-hand side of the equation, N_e is the number of electrons in the volume concerned, M_F is the Fröhlich mass ($M_F \sim 1000 m_e$), which includes the momentum associated with ion motion, and v_d is the drift velocity of the wave.

The number $N_R - N_L$ is large. Even for a modest current density of 1 amp/cm^2 in a phase-coherent volume of 10^{-12} cm^3 containing 10^9 electrons, $N_R - N_L$ is about 10^3. But with impurity pinning, even to increase the momentum by one unit of $2\hbar k_F$ requires a tunnel step. The step requires taking an electron from the left of the Fermi sea to a ground-state configuration on the right, with the added momentum being shared with the ion motion. The $2\hbar k_F$ momentum is with the electrons only a fraction m_e/M_F of the time. Although the tunneling is from one ground state to another moving slightly faster,



Energies of quasiparticle states near the Fermi energy E_F for normal and superconducting metals. The energy levels in the superconducting state are in one-to-one correspondence with those in the normal state, but are changed by introduction of an energy gap of magnitude 2Δ. The distribution of states in energy is shown on the right. **Figure 6**



Transport in $NbSe_3$. The dc conductivity, defined by $G_{CDW} = I_{CDW}/E$, is approximately $G_b \exp(-E_0/E)$. The black curve is the Zener factor, $\exp(-E_0/E)$. (Adapted from the PhD thesis of R. E. Thorne, "Charge-Density-Wave Transport in Quasi-One-Dimensional Conductors," University of Illinois, Urbana–Champaign, 1987.) **Figure 7**

the tunneling probability is the same as that for interband Zener tunneling across a small pinning gap:

$$P(E) = \exp(-E_0/E)$$

Many quasi-classical calculations have been made that ignore this factor. Such calculations start with the equation for free acceleration, which is derived from semiclassical equations that do not include the tunnel step.

The field E should be regarded as a voltage gradient that includes the effects of concentration gradients corresponding to elastic deformations of the charge-density wave in addition to forces resulting from pinning.

Free acceleration occurs only over a relaxation time τ, which is larger by a factor of about M_F/m_e than the relaxation time τ_e for electrons above the Peierls transition. The current density is then

$$n_e e v_d = \frac{n_e e^2 \tau E\, P(E)}{M_F}$$

where n_e is the density of electrons.

The tunnel step allows the possibility of photon-assisted tunneling. From the theory of photon-assisted tunneling as developed by John Tucker for superconducting tunnel junctions, one can express the ac conductivity, and effects of combined ac and dc fields, in terms of the dc current–voltage characteristic. This theory has been remarkably successful in accounting in a quantitative way for a wide range of experiments on transport in quasi-one-dimensional metals. One consequence is that the ac conductivity scales with the dc, with the conductivity due to a charge-density wave of angular frequency ω proportional to $\exp(-\omega_s/\omega)$.

In spite of the remarkable success of the tunneling model over more than a decade, many theorists in the field still try to account for the data with classical theories that ignore the tunnel step.

Concluding remarks

All of the remarkable macroscopic quantum phenomena I have discussed depend on the fact that the quantum states of macroscopic bodies are discrete, although closely spaced, and that there can be macroscopic occupation of one or a selected group of them. To define the system, it is necessary to specify the state of macroscopic occupation (or vacuum state) as well as the velocity of the walls with which the system comes to thermal equilibrium. As it becomes possible to design and build structures on smaller and smaller scales, it is well to keep these essential features of macroscopic quantum systems in mind. If many electrons are involved in a phase-coherent step, the energy involved can be much larger than thermal energy even though the energy per electron is smaller.

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This article is based on the Julian Mack lecture given at the University of Wisconsin in April 1990, and on my lecture sponsored by Sony Corporation, given in Tokyo in May 1990.

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