

The concept of the photon—revisited

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The photon concept is one of the most debated issues in the history of physical science. Some thirty years ago, we published an article in *Physics Today* entitled “The Concept of the Photon,”¹ in which we described the “photon” as a classical electromagnetic field plus the fluctuations associated with the vacuum. However, subsequent developments required us to envision the photon as an intrinsically quantum mechanical entity, whose basic physics is much deeper than can be explained by the simple ‘classical wave plus vacuum fluctuations’ picture. These ideas and the extensions of our conceptual understanding are discussed in detail in our recent quantum optics book.² In this article we revisit the photon concept based on examples from these sources and more.

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The “photon” is a quintessentially twentieth-century concept, intimately tied to the birth of quantum mechanics and quantum electrodynamics. However, the root of the idea may be said to be much older, as old as the historical debate on the nature of light itself – whether it is a wave or a particle – one that has witnessed a seesaw of ideology from antiquity to present. The transition from classical to quantum descriptions of light presents yet another dichotomy, one where the necessity of quantizing the electromagnetic field (over and above a quantization of matter) has been challenged. The resolution lies in uncovering key behavior of quantum light fields that are beyond the domain of the classical, such as vacuum fluctuations and quantum entanglement, which necessitate a quantum theory of radiation.^{2–5} Nevertheless, a precise grasp of the “photon” concept is not an easy task, to quote Albert Einstein:

“These days, every Tom, Dick and Harry thinks he knows what a photon is, but he is wrong.”

We ought to proceed with diligence and caution. In the words of Willis Lamb:⁶

“What do we do next? We can, and should, use the Quantum Theory of Radiation. Fermi showed how to do this for the case of Lippmann fringes. The idea is simple, but the details are somewhat messy. A good notation and lots of practice makes it easier. Begin by deciding how much of the universe needs to be brought into the discussion. Decide what normal modes are needed for an adequate treatment. Decide how to model the light sources and work out how they drive the system.”

We proceed to elucidate the photon concept by specific experiments (real and gedanken) which demonstrate the need for and shed light on the meaning of the “photon.” Specifically, we will start by briefly reviewing the history of the wave-particle debate and then giving seven of our favorite examples, each clarifying some key aspect of the quantum nature of light. The two facets of the photon that we focus

on are vacuum fluctuations (as in our earlier article¹), and aspects of many-particle correlations (as in our recent book²). Examples of the first are spontaneous emission, Lamb shift, and the scattering of atoms off the vacuum field at the entrance to a micromaser. Examples of the second facet include quantum beats, quantum eraser, and photon correlation microscopy. Finally, in the example of two-site downconversion interferometry, the essence of both facets is combined and elucidated.

In the final portions of the article, we return to the basic questions concerning the nature of light in the context of the wave-particle debate: What is a photon and where is it? To the first question, we answer in the words of Roy Glauber:

“A photon is what a photodetector detects.”

To the second question (on the locality of the photon), the answer becomes: “A photon is *where* the photodetector detects it.” In principle, the detector could be a microscopic object such as an atom. Guided by this point of view, we address the much debated issue of the existence of a photon wave function $\psi(\mathbf{r}, t)$.^{2,7,8} Arguments to the contrary notwithstanding, we show that the concept of the photon wave function arises naturally from the quantum theory of photodetection (see Ref. [2], ch. 1). A wealth of insight is gained about the interference and entanglement properties of light by studying such one-photon, and related two-photon, ‘wave functions’.²

Light – wave or particle?

The nature of light is a very old issue in the history of science. For the ancient Greeks and Arabs, the debate centered on the connection between light and vision. The tactile theory, which held that our vision was initiated by our eyes reaching out to “touch” or feel something at a distance, gradually lost ground to the emission theory, which postulated that vision resulted from illuminated objects emitting energy that was sensed by our eyes. This paradigm shift is mainly due to the eleventh-century Arab scientist Abu Ali Hasan Ibn Al-Haitham (or ‘Alhazen’) who laid the groundwork for classical

optics through investigations into the refraction and dispersion properties of light. Later Renaissance thinkers in Europe envisioned light as a stream of particles, perhaps supported by the ether, an invisible medium thought to permeate empty space and all transparent materials.

In the seventeenth century, Pierre de Fermat introduced the *principle of least time* to account for the phenomenon of refraction. Equivalently, his principle states that a ray of light takes the path that minimizes the optical path length between two points in space:

$$\delta \int_{\mathbf{r}_0}^{\mathbf{r}} n ds = 0, \quad (1)$$

where $n = c/v$ is the (spatially varying) refractive index that determines the velocity of the light particle, and δ denotes a variation over all paths connecting \mathbf{r}_0 and \mathbf{r} . Fermat's principle is the foundation for geometrical optics, a theory based on the view that light is a particle that travels along well-defined geometrical rays. The idea of light as particle (or 'corpuscle') was of course adopted by Isaac Newton, who bequeathed the weight of his scientific legacy, including the bearing of his laws of mechanics, on the nature of light.

Christian Huygens on the other hand, a contemporary of Newton, was a strong advocate of the wave theory of light. He formulated a principle (that now bears his name) which describes wave propagation as the interference of secondary wavelets arising from point sources on the existing wavefront. It took the mathematical genius of Augustin Fresnel, 150 years later, to realize the consequences of this discovery, including a rigorous development of the theory of wave diffraction. Light does not form sharp, geometrical shadows that are characteristic of a particle, but bends around obstacles and apertures.

The revival of the wave theory in the early nineteenth century was initiated by Thomas Young. In 1800, appearing before the Royal Society of London, Young spoke for an analogy between light and sound, and declared later that a two-slit interference experiment would conclusively demonstrate the wave nature of light (see Figure 1). It is hard for the modern reader to visualize how counter-intuitive this suggestion was at the time. The idea that a screen uniformly illuminated by a single aperture could develop dark fringes with the introduction of a second aperture – that the addition of *more* light could result in *less* illumination – was hard for Young's audience to digest.

Likewise, Fresnel's diffraction theory was received with skepticism by the judges on the 1819 prize committee in Paris. In particular, the esteemed Pierre Simon de Laplace was very skeptical of the wave theory. His protégé, Siméon-Denis Poisson, highlighted the seemingly absurd fact that the theory implied a bright spot at the center of the shadow of an illuminated opaque disc, now known as Poisson's spot. The resistance to switch from a particle description to a wave description for light by these pre-eminent scientists of the early nineteenth century gives an indication of the great disparity between these two conceptions. It was a precursor of the struggle to come a hundred years later with the advent of

quantum mechanics.

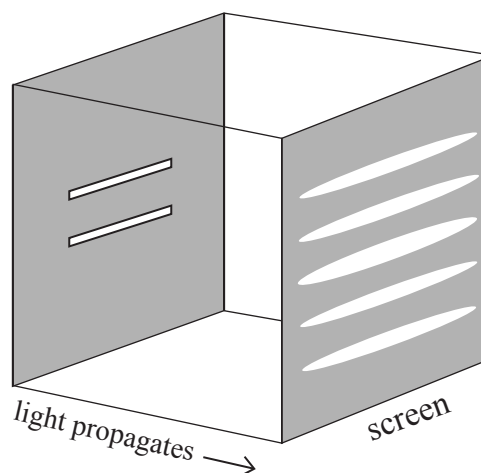


Fig. 1. *Young's two-slit experiment* – Light incident on two slits in a box propagates along two pathways to a given point on the screen, displaying constructive and destructive interference. When a single photon is incident on the slits, it is detected with highest probability at the interference peaks, but never at the interference nodes.

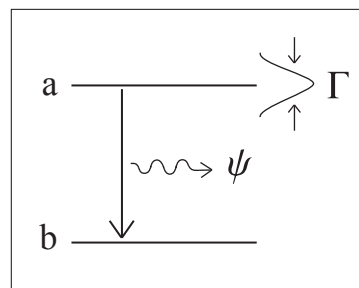


Fig. 2. *Spontaneous emission* – Two-level atom, with upper-level linewidth Γ spontaneously emits a photon. Fluctuations in the vacuum field cause the electron in the excited state to decay to the ground state in a characteristic time Γ^{-1} .

The wave theory really came into its own in the late nineteenth century in the work of James Clerk Maxwell. His four equations, known to all students of undergraduate physics, is the first self-contained theory of radiation. Receiving experimental confirmation by Heinrich Hertz, the Maxwell theory unified the disparate phenomena of electricity and magnetism, and gave physical meaning to the transverse polarizations of light waves. The far-reaching success of the theory explains the hubris of late nineteenth century physicists, many of whom believed that there were really only two "clouds" on the horizon of physics at the dawn of the twentieth century. Interestingly enough, both of these involved light.

The first cloud, namely the null result of the Michelson-Morley experiment, led to special relativity, which is the epitome of classical mechanics, and the logical capstone of

classical physics. The second cloud, the Rayleigh-Jeans ultraviolet (UV) catastrophe and the nature of blackbody radiation, led to the advent of quantum mechanics, which of course was a radical change in physical thought. While both of these problems involved the radiation field, neither (initially) involved the concept of a photon. That is, neither Albert Einstein and Hendrik Lorentz in the first instance, nor Max Planck in the second, called upon the particulate nature of light for the explanation of the observed phenomena. Relativity is strictly classical, and Planck quantized the energies of the oscillators in the walls of his cavity, not the field.⁹

The revival of the particle theory of light, and the beginning of the modern concept of the photon, was due to Einstein. In his 1905 paper on the photoelectric effect,¹⁰ the emission of electrons from a metallic surface irradiated by UV rays, Einstein postulated that light comes in discrete bundles, or quanta of energy, borrowing Planck's five-year old hypothesis: $E = \hbar\nu$, where ν is the circular frequency and \hbar is Planck's constant divided by 2π . This re-introduced the particulate nature of light into physical discourse, not as localization in space in the manner of Newton's corpuscles, but as discreteness in energy. But irony upon irony, it is a historical curiosity that Einstein got the idea for the photon from the physics of the photoelectric effect. In fact, it can be shown that the essence of the photoelectric effect does not require the quantization of the radiation field,¹¹ a misconception perpetuated by the mills of textbooks, to wit, the following quote from a mid-century text:¹²

"Einstein's photoelectric equation played an enormous part in the development of the modern quantum theory. But in spite of its generality and of the many successful applications that have been made of it in physical theories, the equation:

$$\hbar\nu = E + \Phi \quad (2)$$

is, as we shall see presently, based on a concept of radiation – the concept of 'light quanta' – completely at variance with the most fundamental concepts of the classical electromagnetic theory of radiation."

We will revisit the photoelectric effect in the next section and place it properly in the context of radiation theory.

Both the Planck hypothesis and the Einstein interpretation follow from considerations of how energy is *exchanged* between radiation and matter. Instead of an electromagnetic wave continuously driving the amplitude of a classical oscillator, we have the discrete picture of light of the right frequency absorbed or emitted by a quantum oscillator, such as an atom in the walls of the cavity, or on a metallic surface. This seemingly intimate connection between energy quantization and the interaction of radiation with matter motivated the original coining of the word "photon" by Gilbert Lewis in 1926:¹³

"It would seem inappropriate to speak of one of these hypothetical entities as a particle of light, a

corpuscle of light, a light quantum, or light quant, if we are to assume that it spends only a minute fraction of its existence as a carrier of radiant energy, while the rest of the time it remains as an important structural element within the atom... I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an essential part in every process of radiation, the name *photon*."

Energy quantization is the essence of the old quantum theory of the atom proposed by Niels Bohr. The electron is said to occupy discrete orbitals with energies E_i and E_j , with transitions between them caused by a photon of the right frequency: $\nu = (E_i - E_j)/\hbar$. An ingenious interpretation of this quantization in terms of matter waves was given by Louis de Broglie, who argued by analogy with standing waves in a cavity, that the wavelength of the electron in each Bohr orbital is quantized – an integer number of wavelengths would have to fit in a circular orbit of the right radius. This paved the way for Erwin Schrödinger to introduce his famous wave equation for matter waves, the basis for (non-relativistic) quantum mechanics of material systems.

Quantum mechanics provides us with a new perspective on the wave-particle debate, vis á vis Young's two-slit experiment (Figure 1). In the paradigm of quantum interference, we add the probability amplitudes associated with different pathways through an interferometer. Light (or matter) is neither wave nor particle, but an intermediate entity that obeys the superposition principle. When a single photon goes through the slits, it registers as a point-like event on the screen (measured, say, by a CCD array). An accumulation of such events over repeated trials builds up a probabilistic fringe pattern that is characteristic of classical wave interference. However, if we arrange to acquire information about which slit the photon went through, the interference nulls disappear. Thus, from the standpoint of complementarity, both wave and particle perspectives have equal validity. We will return to this issue later in the article.

The semiclassical view

The interaction of radiation and matter is key to understanding the nature of light and the concept of a photon. In the semiclassical view, light is treated classically and only matter is quantized. In other words, both are treated on an *equal footing*: a wave theory of light (the Maxwell equations) is combined self-consistently with a wave theory of matter (the Schrödinger equation). This yields a remarkably accurate description of a large class of phenomena, including the photoelectric effect, stimulated emission and absorption, saturation effects and nonlinear spectroscopy, pulse propagation phenomena, "photon" echoes, etc. Many properties of laser light, such as frequency selectivity, phase coherence, and directionality, can be explained within this framework.¹⁴

The workhorse of semiclassical theory is the two-level atom, specifically the problem of its interaction with a sinusoidal light wave.¹⁵ In reality, real atoms have lots of levels,

but the two-level approximation amounts to isolating a particular transition that is nearly resonant with the field frequency ν . That is, the energy separation of the levels is assumed to be $E_a - E_b = \hbar\omega \approx \hbar\nu$. Such a comparison of the atomic energy difference with the field frequency is in the spirit of the Bohr model, but note that this already implies a discreteness in light energy, $\Delta E = \hbar\nu$. That a semiclassical analysis is able to bring out this discreteness – in the form of *resonance* – is a qualitative dividend of this approach.

Schrödinger's equation describes the dynamics of the atom, but how about the dynamics of the radiation field? In the semiclassical approach, one assumes that the atomic electron cloud $\psi^*\psi$, which is polarized by the incident field, acts like an oscillating *charge* density, producing an ensemble dipole moment that re-radiates a classical Maxwell field. The effects of radiation reaction, i.e., the back action of the emitted field on the atom, are taken into account by requiring the coupled Maxwell-Schrödinger equations to be self-consistent with respect to the *total* field. That is, the field that the atoms see should be consistent with the field radiated. In this way, semiclassical theory becomes a self-contained description of the dynamics of a quantum mechanical atom interacting with a classical field. As we have noted above, its successes far outweigh our expectations.

Let us apply the semiclassical analysis to the photoelectric effect, which provided the original impetus for the quantization of light. There are three observed features of this effect that need accounting. First, when light shines on a photoemissive surface, electrons are ejected with a kinetic energy E equal to \hbar times the frequency ν of the incident light less some work function Φ , as in Eq. (2). Second, it is observed that the rate of electron ejection is proportional to the square of the incident electric field E_0 . Third, and more subtle, there is not necessarily a time delay between the instant the field is turned on, and the time when the photoelectron is ejected, contrary to classical expectations.

All three observations can be nominally accounted for by applying the semiclassical theory to lowest order in perturbation of the atom-field interaction $V(t) = -eE_0r$.¹¹ This furnishes a Fermi Golden Rule for the probability of transition of the electron from the ground state g of the atom to the k th excited state in the continuum:

$$P_k = [2\pi(e|r_{kg}|E_0/2\hbar)^2 t] \delta[\nu - (E_k - E_g)/\hbar], \quad (3)$$

where er_{kg} is the dipole matrix element between the initial and final states. The δ -function (which has units of time) arises from considering the frequency response of the surface, and assuming that t is at least as long as several optical cycles: $\nu t \gg 1$. Now, writing energy $E_k - E_g$ as $E + \Phi$, we see that the δ -function immediately implies Eq. (2). The second fact is also clearly contained in Eq. (3) since P_k is proportional to E_0^2 . The third fact of photoelectric detection, the finite time delay, is explained in the sense that P_k is linearly proportional to t , and there is a finite probability of the atom being excited even at infinitesimally small times.

Thus, the experimental aspects of the photoelectric effect are completely understandable from a semiclassical point of

view. Where we depart from a classical intuition for light is a subtle issue connected with the third fact, namely that there is negligible time delay between the incidence of light and the photoelectron emission. While this is understandable from an *atomic* point of view – the electron has finite probability of being excited even at very short times – the argument breaks down when we consider the implications for the field. That is, if we persist in thinking about the field classically, energy is not conserved. Over a time interval t , a classical field E_0 brings in a flux of energy $\epsilon_0 E_0^2 A t$ to bear on the atom, where A is the atomic cross-section. For short enough times t , this energy is negligible compared to $\hbar\nu$, the energy that the electron supposedly absorbs (instantaneously) when it becomes excited. We just do not have the authority, within the Maxwell formalism, to affect a similar *quantum jump* for the field energy.

For this and other reasons (see next section), it behooves us to supplement the epistemology of the Maxwell theory with a quantized view of the electromagnetic field that fully accounts for the probabilistic nature of light and its inherent fluctuations. This is exactly what Paul Dirac did in the year 1927, when the photon concept was, for the first time, placed on a logical foundation, and the quantum theory of radiation was born.¹⁶ This was followed in the 1940s by the remarkably successful theory of quantum electrodynamics (QED) – the quantum theory of interaction of light and matter – that achieved unparalleled numerical accuracy in predicting experimental observations. Nevertheless, a short twenty years later, we would come back full circle in the saga of semiclassical theory, with Ed Jaynes questioning the need for a quantum theory of radiation at the 1966 conference on Coherence and Quantum Optics at Rochester, New York.

“Physics goes forward on the shoulders of doubters, not believers, and I doubt that QED is necessary,” declared Jaynes. In his view, semiclassical theory – or ‘neoclassical’ theory, with the addition of a radiation reaction field acting back on the atom – was sufficient to explain the Lamb shift, thought by most to be the best vindication yet of Dirac's field quantization and QED theory (see below). Another conference attendee, Peter Franken, challenged Jaynes to a bet. One of us (MOS) present at the conference recalls Franken's words: “You are a reasonably rich man. So am I, and I say put your money where your face is!” He wagered \$100 over whether the Lamb shift could or could not be calculated without QED. Jaynes took the bet that he could, and Willis Lamb agreed to be the judge.

In the 1960s and 70s, Jaynes and his collaborators reported partial success in predicting the Lamb shift using neoclassical theory.¹⁷ They were able to make a qualitative connection between the shift and the physics of radiation reaction – in the absence of field quantization or vacuum fluctuations – but failed to produce an accurate numerical prediction which could be checked against experiment. For this reason, at the 1978 conference in Rochester, Lamb decided to yield the bet to Franken. An account of the arguments for and against this decision was summarized by Jaynes in his paper at the conference.¹⁸ In the end, QED had survived the challenge

of semiclassical theory, and vacuum fluctuations were indeed “very real things” to be reckoned with.

Seven examples

Our first three examples below illustrate the reality of vacuum fluctuations in the electromagnetic field as manifested in the physics of the atom. The “photon” acquires a stochastic meaning in this context. One speaks of a classical electromagnetic field with fluctuations due to the vacuum. To be sure, one cannot “see” these fluctuations with a photodetector, but they make their presence felt, for example, in the way the atomic electrons are “jiggled” by these random vacuum forces.

1. Spontaneous emission

In the phenomenon of spontaneous emission,¹⁹ an atom in the excited state decays to the ground state and spontaneously emits a photon (see Figure 2). This “spontaneous” emission is in a sense stimulated emission, where the stimulating field is a vacuum fluctuation. If an atom is placed in the excited state and the field is classical, the atom will never develop a dipole moment and will never radiate. In this sense, semiclassical theory does not account for spontaneous emission. However, when vacuum fluctuations are included, we can think conceptually of the atom as being stimulated to emit radiation by the fluctuating field, and the back action of the emitted light will drive the atom further to the ground state, yielding decay of the excited state. It is in this way that we understand spontaneous emission as being due to vacuum fluctuations.

2. Lamb shift

Perhaps the greatest triumph of field quantization is the explanation of the Lamb shift²⁰ between, for example, the $2s_{1/2}$ and $2p_{1/2}$ levels in a hydrogenic atom. Relativistic quantum mechanics predicts that these levels should be degenerate, in contradiction to the experimentally observed frequency splitting of about 1 GHz. We can understand the shift intuitively²¹ by picturing the electron forced to fluctuate about its first-quantized position in the atom due to random kicks from the surrounding, fluctuating vacuum field (see Figure 3). Its average displacement $\langle \Delta \mathbf{r} \rangle$ is zero, but the squared displacement $\langle \Delta \mathbf{r} \rangle^2$ is slightly nonzero, with the result that the electron “senses” a slightly different Coulomb pull from the positively charged nucleus than it normally would. The effect is more prominent nearer the nucleus where the Coulomb potential falls off more steeply, thus the s orbital is affected more than the p orbital. This is manifested as the Lamb shift between the levels.

3. Micromaser – scattering off the vacuum

A micromaser consists of a single atom interacting with a single-mode quantized field in a high-Q cavity.²² An interesting new perspective on vacuum fluctuations is given by the recent example of an excited atom scattering off an effective potential barrier created by a vacuum field in the cavity (see

Figure 4).²³ When the atomic center-of-mass motion is quantized, and the atoms are travelling slow enough (their kinetic energy is smaller than the atom-field interaction energy), it is shown that they can undergo reflection from the cavity, even when it is initially empty, i.e. there are no photons. The reflection of the atom takes place due to the discontinuous change in the strength of the coupling with vacuum fluctuations at the input to the cavity. This kind of reflection off an edge discontinuity is common in wave mechanics. What is interesting in this instance is that the reflection is due to an abrupt change in coupling with the vacuum between the inside and the outside of the cavity. It is then fair to view this physics as another manifestation of the effect of vacuum fluctuations, this time affecting the center-of-mass dynamics of the atom.

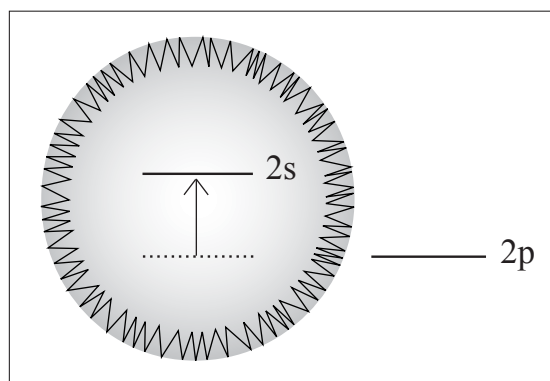


Fig. 3. *Lamb shift* – Schematic illustration of the Lamb shift of the hydrogenic $2s_{1/2}$ state relative to the $2p_{1/2}$ state. Intuitive understanding of the shift as due to random jostling of the electron in the $2s$ orbital by zero-point fluctuations in the vacuum field.

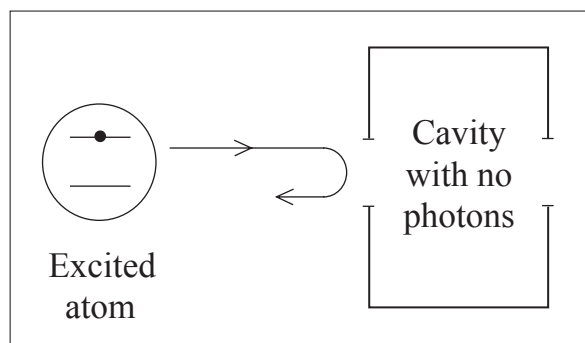


Fig. 4. *Scattering off the vacuum* – An excited atom approaching an empty cavity can be reflected for slow enough velocities. The vacuum cavity field serves as an effective potential barrier for the center-of-mass wave function of the atom.

Our next three examples involve the concept of multi-particle entanglement, which is a distinguishing feature of the quantized electromagnetic field. Historically, inter-particle correlations have played a key role in fundamental tests of quantum mechanics, such as the EPR paradox, Bell inequalities and quantum eraser. These examples illustrate the real-

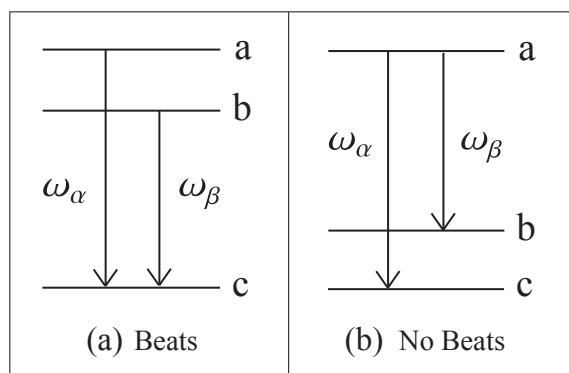


Fig. 5. *Quantum beats* – a) When a single atom decays from either of two upper levels to a common lower level, the two transition frequencies produce a beat note $\omega_\alpha - \omega_\beta$ in the emitted photon. b) No beats are present when the lower levels are distinct, since the final state of the atom provides distinguishing information on the decay route taken by the photon.

ity of quantum correlations in multi-photon physics. In recent years, entangled photons have been key to applications in quantum information and computing, giving rise to new technologies such as photon correlation microscopy (see below).

4. Quantum beats

In general, beats arise whenever two or more frequencies of a wave are simultaneously present. When an atom in the excited state undergoes decay along two transition pathways, the light produced in the process is expected to register a beat note at the difference frequency, $\omega_\alpha - \omega_\beta$, in addition to the individual transition frequencies ω_α and ω_β . However, when a single atom decays, beats are present only when the two final states of the atom are identical (see Figure 5). When the final states are distinct, quantum theory predicts an absence of beats.²⁴ This is so because the two decay channels end in different atomic states [$|b\rangle$ or $|c\rangle$] in Figure 5(b)]. We now have which-path information since we need only consult the atom to see which photon (α or β) was emitted – i.e. the entanglement between the atom and the quantized field destroys the interference. Classical electrodynamics, vis á vis semiclassical theory, cannot explain the “missing” beats.

5. Quantum eraser and complementarity

In the quantum eraser,²⁵ the which-path information about the interfering particle is *erased* by manipulating the second, entangled particle. Complementarity is enforced not by the uncertainty principle (through a measurement process), but by a quantum correlation between particles.²⁶ This notion can be realized in the context of two-photon interferometry.^{27–29} Consider the setup shown in Figure 6, where one of two atoms $i = 1, 2$ emits two photons ϕ_i and γ_i . Interference is observed in ϕ only when the spatial origin of γ cannot be discerned, i.e., when detector D_1 or D_2 clicks. Erasure occurs when the γ photon is reflected (rather than transmitted) at beamsplitter

BS1 or BS2, which in the experiment occurs *after* the ϕ photon has been detected. Thus, quantum entanglement between the photons enables a realization of ‘delayed choice’,³⁰ which cannot be simulated by classical optics.

6. Photon correlation microscopy

Novel interference phenomena arise from second-order correlations of entangled photons, such as arise from the spontaneous cascade decay of a three-level atom (where the emitted photons are correlated in frequency and time of emission).² When two such atoms are spatially separated and one of them undergoes decay, a two-photon correlation measurement enables high-resolution *spectral* microscopy on the atomic level structure.³¹ It can be shown that the resolution of the upper two levels a and b in each atom is limited only by the linewidth Γ_a , and not by Γ_a and Γ_b together (as is usually the case). This phenomenon relies on the path and frequency entanglement between the two photons arising from spatially separated cascade sources.

A further consequence of the two-atom geometry is the enhancement in *spatial* resolution that occurs because the photons are entangled in path – that is, the photon pair arises from one atom or the other, and their joint paths interfere. Coincident detection of the two photons (each of wavelength λ) shows a fringe resolution that is enhanced by a factor of two as compared to the classical Rayleigh limit, $\lambda/2$. This enables applications in high-resolution lithography.^{32,33} The fringe doubling is due to the fact that the two photons propagate along the same path, and their sum frequency, 2ω , characterizes their joint detection probability. Path entanglement cannot be simulated by (co-propagating) classical light pulses.

7. Two-site downconversion interferometry

In what follows, we consider a two-particle interferometry experiment that allows us to elucidate both facets of the photon considered above – vacuum fluctuations and quantum entanglement. The thought experiment we have in mind is based on an actual one that was carried out using parametric downconversion.³⁴ Consider the setup shown in Figure 7, where two atoms $i = 1, 2$ are fixed in position and one of them emits two photons, labeled ϕ_i and γ_i , giving rise to a two-photon state that is a superposition of emissions from each atom:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\phi_1\rangle|\gamma_1\rangle + |\phi_2\rangle|\gamma_2\rangle). \quad (4)$$

This is an entangled state in the sense that an emission of ϕ_i is always accompanied by an emission of γ_i , for $i = 1$ or 2 . Let us suppose that we are interested in interference of the ϕ photon only, as measured by varying the path lengths of ϕ_1 and ϕ_2 to detector D_ϕ . The γ photon serves as a marker that potentially records which atom emitted the ϕ photon. It is found that by inserting (or removing) a beamstop in the path of γ_1 , the interference fringes can be made to vanish (or re-appear) at D_ϕ , even when D_γ is not actually observed.

It is interesting to explain this phenomenon using stochastic electrodynamics³⁵ (as was done with the Lamb shift). Let us replace the two photons ϕ and γ with *classical* light fields

$E_i^\phi(\mathbf{r}, t)$ and $E_i^\gamma(\mathbf{r}, t)$, generated respectively by dipole transitions a - b and b - c in each atom i . If the atoms are initially in a superposition of states a and c , then zero-point fluctuations in the field mode γ will introduce population into level b (from a), with a random phase $\varphi_{\gamma,i}$. The first-order interference in the field mode ϕ will now depend on an ensemble average over the vacuum-induced two-atom phase difference: $\langle E_1^\phi E_2^\phi \rangle \propto \langle \exp[-i(\varphi_{\gamma,1} - \varphi_{\gamma,2})] \rangle$. This quantity goes to zero if the two phases are statistically independent, which is the case when the beamstop is in place between the two atoms. Thus, we have here a connection between vacuum fluctuation physics (which is responsible for spontaneous emission of photons), and two-particle correlation physics (which is the key to quantum erasure).

The quantum field theory view

A quantum theory of radiation²⁻⁵ is indispensable to understanding the novel properties of light mentioned above. Central to the theory is the idea of field quantization, which develops the formal analogy with the quantum mechanics of the harmonic oscillator. The position q and momentum p of an oscillating particle satisfy the commutation relation $[\hat{q}, \hat{p}] = \hat{q}\hat{p} - \hat{p}\hat{q} = i\hbar$. In the case of the radiation field, q and p represent the electric (E) and magnetic (B) fields of the light in a given wave-vector and polarization mode k . Thus, the quantum electromagnetic field consists of an *infinite* product of such generalized harmonic oscillators, one for each mode of the field. A Heisenberg-type uncertainty relation applies to these quantized Maxwell fields:

$$\Delta E \Delta B \geq \hbar/2 \times \text{constant.} \quad (5)$$

Such field fluctuations are an intrinsic feature of the quantized theory. The uncertainty relation can also be formulated in terms of the in-phase (\mathcal{E}_p) and in-quadrature (\mathcal{E}_q) components of the electric field, where $E(t) = \mathcal{E}_p \cos vt + \mathcal{E}_q \sin vt$.

To introduce the notion of a photon, it is convenient to recast the above quantization of the field in terms of a Fourier decomposition, or in terms of the normal modes of a field in a cavity. These correspond to the positive frequency (going like $e^{-i\nu t}$) and negative frequency (going like $e^{i\nu t}$) parts of the electric field respectively (summed over all modes k):

$$\begin{aligned} E(\mathbf{r}, t) &= E^+(\mathbf{r}, t) + E^-(\mathbf{r}, t) \\ &= \sum_k [\alpha_k \mathcal{E}_k(\mathbf{r}) \exp(-i\nu_k t) \\ &\quad + \alpha_k^* \mathcal{E}_k^*(\mathbf{r}) \exp(i\nu_k t)]. \end{aligned} \quad (6)$$

Here α_k is the amplitude of oscillation, and $\mathcal{E}_k(\mathbf{r})$ is a mode function like $\exp(i\mathbf{k} \cdot \mathbf{r})$ for travelling waves in free space and $\sin(\mathbf{k} \cdot \mathbf{r})$ for standing waves in a box. We consider the oscillator amplitudes α_k and α_k^* , corresponding to harmonic motion, to be quantized by replacing $\alpha_k \rightarrow \hat{a}_k$ and $\alpha_k^* \rightarrow \hat{a}_k^\dagger$. By analogy to the quantum mechanics of the harmonic oscillator, the application of \hat{a} produces a field state with one *less* quantum of energy, and the application of \hat{a}^\dagger produces a field state with

one *more* quantum of energy. This naturally leads to discrete energies for the radiation field in each mode: $n_k = 0, 1, 2$, etc.

Both wave and particle perspectives are present in the quantum view – the former in the picture of a stochastic electromagnetic field, and the latter in the language of particle creation and annihilation. Combining these points of view, one can think of the “photon” as a discrete excitation of a set of modes $\{k\}$ of the electromagnetic field in some cavity, where the mode operators satisfy the boson commutation relation: $[\hat{a}_k, \hat{a}_k^\dagger] = 1$. Questions such as how to define the cavity, and what normal modes to use, cannot be answered once and for all, but depend on the particular physical setup in the laboratory (see quote by Willis Lamb at the beginning). Guided by this operational philosophy, we revisit the wave-particle debate on the nature of light in the guise of the following questions.

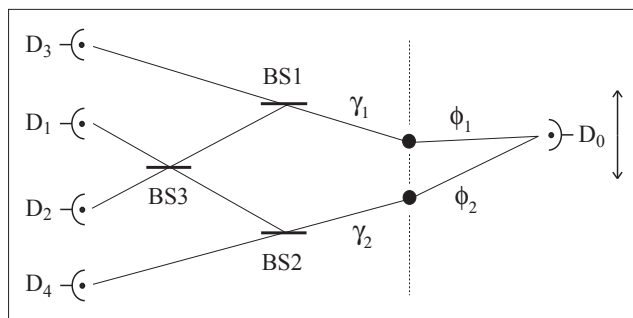


Fig. 6. *Quantum eraser* – One of two atoms (solid circles) emits two photons ϕ_i and γ_i . Interference is observed in ϕ by scanning detector D_0 . Beamsplitters BS1-BS3 direct γ to four detectors. A click in detectors D_3 or D_4 provides which-path information on γ , preventing interference in ϕ . A click in detectors D_1 or D_2 erases which-path information and restores interference in ϕ . Figure adapted from Ref. [29].

What is a photon, and where is it?

In other words, in what manner (and to what extent) can we regard the photon as a true ‘particle’ that is localized in space? When first introduced, the photon was conceived of as a particulate carrier of discrete light energy, $E = \hbar\nu$, a conception guided by considerations of the interaction between radiation and matter. From semiclassical arguments, we saw how this discreteness was related to finite energy spacings in the atom. Here, we pursue this line of reasoning further to inquire whether a fully quantized theory of matter-radiation interaction can lend a characteristic of *spatial discreteness* to the photon when it interacts with a finite-sized atom. This line of thinking derives from the quantum theory of photodetection³⁶ (which, incidentally, also relies on the photoelectric effect).

Closely related to the issue of photon localization is the (much debated) question of the existence of a photon wave function $\psi(\mathbf{r}, t)$,^{2,7,8} analogous to that of an electron or neutrino (cf. Figure 8). The connection is that if such a wave function exists, then we can interpret $|\psi|^2 dV$ as the probability of finding the photon in an infinitesimal volume element

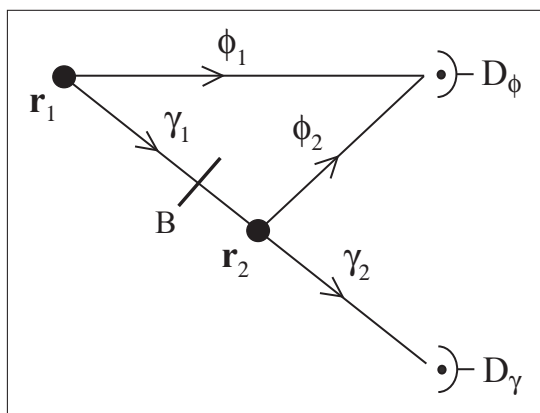


Fig. 7. Two-site downconversion interferometry – Two atoms are located at \mathbf{r}_1 and \mathbf{r}_2 , one of which emits two photons, labeled ϕ_i and γ_i . Detectors D_ϕ and D_γ measure the respective photons. Inserting the beamstop B in the path of γ_1 allows us to infer (potentially, by checking D_γ) which atom emitted the ϕ photon. This potential which-path information is sufficient to prevent the interference of ϕ_1 and ϕ_2 possibilities at D_ϕ . Setup models the experiment of Ref. [34].

dV in space, and pursue the localization of the entire photon to an arbitrarily small volume constrained only by the uncertainty principle. Moreover, a ‘first-quantized theory’ of the electromagnetic field would be interesting from the point of view of discussing various quantum effects that result from wave interference and entanglement. It would also allow us to treat the mechanics of the photon on par with that of massive particles, such as electrons and atoms, and enable a unified treatment of matter-radiation interaction that supersedes the semiclassical theory in rigor, but still avoids the language of field quantization.

Concerning the issue of ‘where’ the photon is, one is reminded of an often asked question in introductory quantum mechanics: “How can a single particle go through both slits in a Young-type experiment?”

Richard Feynman answers this by saying “nobody knows, and its best if you try not to think about it.” This is good advice if you have a picture of a single photon as a particle. On the other hand if you think of the photon as nothing more nor less than a single quantum excitation of the appropriate normal mode, then things are not so mysterious, and in some sense intuitively obvious.

What we have in mind (referring to Figure 1) is to consider a large box having simple normal modes and to put two holes in the box associated with the Young slits. If light is incident on the slits, we will have on the far wall of the box an interference pattern characteristic of classical wave interference, which we can describe as a superposition of normal modes. Now we quantize these normal modes and find that a photodetector on the far wall will indeed respond to the single quantum excitation of a set of normal modes which are localized at the peaks of the interference pattern, and will not respond when placed at the nodes. In this sense, the issue is

a non sequitur. The photon is common to the box and has no independent identity in going through one hole or the other.

But to continue this discussion, let us ask what it is that the photodetector responds to. As we will clarify below, this is essentially what has come to be called the photon wave function.² Historical arguments have tended to disfavor the existence of such a quantity. For example, in his book on quantum mechanics,³⁷ Hendrik Kramers asks whether “it is possible to consider the Maxwell equations to be a kind of Schrödinger equation for light particles.” His bias against this view is based on the disparity in mathematical form of the two types of equations (specifically, the number of time derivatives in each). The former admits *real* solutions ($\sin vt$ and $\cos vt$) for the electric and magnetic waves, while the latter is restricted to *complex* wave functions (e^{ivt} or e^{-ivt} , but not both). Another argument is mentioned by David Bohm in his quantum theory book,³⁸ where he argues that there is no quantity for light equivalent to the electron probability density $P(x) = |\psi(x)|^2$:

There is, strictly speaking, no function that represents the probability of finding a light quantum at a given point. If we choose a region large compared with a wavelength, we obtain approximately

$$P(x) \cong \frac{\mathcal{E}^2(x) + \mathcal{H}^2(x)}{8\pi h \nu(x)},$$

but if this region is defined too well, $\nu(x)$ has no meaning.

Bohm goes on to argue that the continuity equation, which relates the probability density and current density of an electron, cannot be written for light. That is, a precise statement of the conservation of probability cannot be made for the photon. In what follows, we will see that we can partially overcome the objections raised by Kramers and Bohm.

Let us develop the analogy with the electron a bit further. Recall that the wave function of an electron in the coordinate representation is given by $\psi(\mathbf{r}, t) = \langle \mathbf{r} | \psi \rangle$, where $|\mathbf{r}\rangle$ is the position state corresponding to the exact localization of the electron at the point \mathbf{r} in space. Now the question is, can we write something like this for the photon? The answer is, strictly speaking, “no,” because there is no $|\mathbf{r}\rangle$ state for the photon, or more accurately, there is no particle creation operator that creates a photon at an exact point in space. Loosely speaking, even if there were, $\langle \mathbf{r}' | \mathbf{r} \rangle \neq \delta(\mathbf{r} - \mathbf{r}')$ on the scale of a photon wavelength. Nevertheless, we can still define the *detection* of a photon to a precision limited only by the size of the atom (or detector) absorbing it, which can in principle be much smaller than the wavelength. This gives precise, operational meaning to the notion of “localizing” a photon in space.

If we detect the photon by an absorption process, then the interaction coupling the field and the detector is described by the annihilation operator $\hat{E}^+(\mathbf{r}, t)$, defined in Eq. (6). According to Fermi’s Golden Rule, the matrix element of this operator between the initial and final states of the field determines

	Photon	Neutrino
Eikonal physics	Ray optics (Fermat): $\delta \int n ds = 0$	Classical mechanics (Hamilton): $\delta \int L dt = 0$
“Wave” mechanics	Maxwell equations: $\dot{\Psi} = -\frac{i}{\hbar} \begin{bmatrix} 0 & -c\mathbf{s} \cdot \mathbf{p} \\ c\mathbf{s} \cdot \mathbf{p} & 0 \end{bmatrix} \Psi$	Dirac equations: $\dot{\Phi} = -\frac{i}{\hbar} \begin{bmatrix} 0 & -c\boldsymbol{\sigma} \cdot \mathbf{p} \\ c\boldsymbol{\sigma} \cdot \mathbf{p} & 0 \end{bmatrix} \Phi$
Quantum field theory	$\hat{E}^+(\mathbf{r}, t) = \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}}(t) \mathcal{E}_{\mathbf{k}}(\mathbf{r})$ $ \dot{\psi}\rangle = -\frac{i}{\hbar} \mathcal{H}_{\gamma} \psi\rangle$	$\hat{\phi}(\mathbf{r}, t) = \sum_{\mathbf{p}} \hat{c}_{\mathbf{p}}(t) \phi_{\mathbf{p}}(\mathbf{r})$ $ \dot{\phi}\rangle = -\frac{i}{\hbar} \mathcal{H}_{\nu} \phi\rangle$

Fig. 8. Comparison of physical theories of a photon and a neutrino. Eikonal physics describes both in particle terms, showing the parallel between Fermat’s principle in optics and Hamilton’s principle in classical mechanics (L is the Lagrangian). The Maxwell equations can be formulated in terms of photon wave functions, in the same form that the Dirac equations describe the relativistic wave mechanics of the neutrino. Here, Ψ is a six-vector representing the wave functions associated with the electric and magnetic fields, $\mathbf{p} = (\hbar/i)\nabla$ as usual, and $\mathbf{s} = (s_x, s_y, s_z)$ are a set of 3×3 matrices that take the place of the Pauli matrices σ_x, σ_y and σ_z . See Ref [2] for details. Finally, quantum field theory gives a unified description of both the photon and the neutrino in terms of quantized field operators.

the transition probability. If there is only one photon initially in the state $|\psi\rangle$, then the relevant final state is the vacuum state $|0\rangle$. The probability density of detecting this photon at position \mathbf{r} and time t is thus proportional to²

$$G_{\psi}^{(1)} = |\langle 0 | \hat{E}^+(\mathbf{r}, t) | \psi \rangle|^2 = \kappa |\psi_{\mathcal{E}}(\mathbf{r}, t)|^2. \quad (7)$$

Here, κ is a dimensional constant such that $|\psi_{\mathcal{E}}|^2$ has units of inverse volume. The quantity $\psi_{\mathcal{E}}(\mathbf{r}, t)$ may thus be regarded as a kind of ‘electric-field wave function’ for the photon, with $\{\langle 0 | \hat{E}^+(\mathbf{r}, t) \rangle^\dagger = \hat{E}^-(\mathbf{r}, t) | 0 \rangle$ playing the role of the position state $|\mathbf{r}\rangle$. That is, by summing over infinitely many wave vectors in Eq. (6), and appealing to Fourier’s theorem, $\hat{E}^-(\mathbf{r}, t)$ can be interpreted as an operator that *creates* the photon at the position \mathbf{r} out of the vacuum. Of course, we have to be careful not to take this interpretation too precisely.

It is interesting to calculate $\psi_{\mathcal{E}}(\mathbf{r}, t)$ for the photon spontaneously emitted by an atom when it decays. Consider a two-level atom located at \mathbf{r}_0 , initially excited in level a and decaying at a rate Γ to level b , as shown in Figure 2. The emitted field state $|\psi\rangle$ is a superposition of one-photon states $|1_k\rangle$, summed over all modes k , written as

$$|\psi\rangle = \sum_k \frac{g_{ab,k} e^{-i\mathbf{k} \cdot \mathbf{r}_0}}{(v_k - \omega) + i\Gamma/2} |1_k\rangle, \quad (8)$$

where ω is the atomic frequency, and $g_{ab,k}$ is a coupling constant that depends on the dipole moment between levels a and b . The spectrum of the emitted field is approximately Lorentzian, which corresponds in the time domain to an exponential decay of the excited atom. Calculating $\psi_{\mathcal{E}}(\mathbf{r}, t)$ for this state, we obtain

$$\psi_{\mathcal{E}}(\mathbf{r}, t) = K \frac{\sin \eta}{r} \theta(t - r/c) \exp[-i(\omega + i\Gamma/2)(t - r/c)], \quad (9)$$

where K is a normalization constant, $r = |\mathbf{r} - \mathbf{r}_0|$ is the radial distance from the atom, and η is the azimuthal angle with respect to the atomic dipole moment. The step function $\theta(t - r/c)$ is an indication that nothing will be detected until the light from the atom reaches the detector, travelling at the speed c . Once the detector starts seeing the pulse, the probability of detection $|\psi_{\mathcal{E}}|^2$ decays exponentially in time at the rate Γ . The spatial profile of the pulse mimics the radiation pattern of a classical dipole.

To what extent can we interpret Eq. (9) as a kind of wave function for the emitted photon? It certainly has close parallels with the Maxwell theory, since it agrees with what we would write down for the electric field in the far zone of a damped, radiating dipole. We can go even further, and introduce *vector* wave functions $\Psi_{\mathcal{E}}$ and $\Psi_{\mathcal{H}}$ corresponding to the electric and magnetic field vectors \mathbf{E} and \mathbf{H} respectively, and show that these satisfy the Maxwell equations (see Figure 8). This formalism provides the so-called “missing link” between classical Maxwell electrodynamics and quantum field theory.⁷ But we have to be careful in how far we carry the analogy with mechanics. For example, there is no real position operator $\hat{\mathbf{r}}$ for the photon in the wave-mechanical limit, as there is for a first-quantized electron. Nevertheless, the wave function $\psi_{\mathcal{E}}(\mathbf{r}, t)$ does overcome the main objection of Kramers (since it is complex) and partially overcomes that of Bohm (photodetection events are indeed localized to distances smaller than a wavelength).

The real payoff of introducing a photon wave function comes when we generalize this quantity to two or more pho-

tons. A ‘two-photon wave function’ $\Psi_{\mathcal{E}}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)$ may be introduced along similar lines as above, and used to treat problems in second-order interferometry (see Ref [2], chap. 21). Entanglement between the two photons results in an inseparability of the wave function: $\Psi_{\mathcal{E}}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) \neq \phi_{\mathcal{E}}(\mathbf{r}_1, t_1) \gamma_{\mathcal{E}}(\mathbf{r}_2, t_2)$, as in the example of the two-photon state in Eq. (4). The novel interference effects associated with such states may be explained in terms of this formalism.

Thus, the photon wave function concept is useful in comparing the interference of classical and quantum light, and allows us to home in on the key distinction between the two paradigms. In particular, through association with photodetection amplitudes, multi-photon wave functions incorporate the phenomenology of quantum-correlated measurement, which is key to explaining the physics of entangled light.

Conclusion

What is a photon? In this article, we have strived to address this concept in unambiguous terms, while remaining true to its wonderfully multi-faceted nature. The story of our quest to understand the character of light is a long one indeed, and parallels much of the progress of physical theory. Dual conceptions of light, as wave and particle, have co-existed since antiquity. Quantum mechanics officially sanctions this duality, and puts both concepts on an equal footing (to wit, the quantum eraser). The quantum theory of light introduces vacuum fluctuations into the radiation field, and endows field states with quantum, many-particle correlations. Each of these developments provides us with fresh insight on the photon question, and allows us to hone our perspective on the wave-particle debate.

The particulate nature of the photon is evident in its tendency to be absorbed and emitted by matter in discrete units, leading to quantization of light energy. In the spatial domain, the localization of photons by a photodetector makes it possible to define a ‘wave function’ for the photon, which affords a ‘first-quantized’ view of the electromagnetic field by analogy to the quantum mechanics of material particles. Quantum interference and entanglement are exemplified by one-photon and two-photon wave functions, which facilitate comparisons to (and clarify departures from) classical wave optics. Moreover, this interpretive formalism provides a bridge between the two ancient, antithetical conceptions of light – its locality as a particle, and its functionality as a wave.

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