Use of Helmholtz Coils for Magnetic Measurements

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***Abstract*—Helmholtz coils can be used for the measurement of open-circuit magnetization of most permanent magnet materials. This paper describes the physics of the measurement, lists the materials that can be measured, derives the coil constant, and derives a correction factor for the measurement of arc magnets.**

Introduction

THE WIDESPREAD use of hard ferrite, samarium cobalt, and neodymium iron boron permanent magnets has created a need to measure the magnetic properties of samples that are short in the magnetic direction. Two common techniques already exist to measure hysteresis loops, or at least the second quadrant of the loop; they are the vibrating sample magnetometer [1] and the hysteresisgraph. While these tests give the most complete information, samples must be cut to a specific size for the measurement, making the tests destructive to the sample. Because of the sample preparation time, neither technique is applicable for testing large groups of magnets.

Open-circuit magnetic measurements are also used for testing permanent magnets. The two common methods are the slip coil/fluxmeter and the Hall probe/gaussmeter. Only one point on the demagnetizing curve can be determined by an open-circuit measurement, making interpretation of the results more difficult. The principal advantages of open-circuit tests are that they are easy and nondestructive measurements.

High H_c materials, such as hard ferrite, samarium cobalt, and neodymium iron boron, are used in different shapes than the traditional Alnico magnets. The magnetic length of a high H_c material is almost always the shortest dimension, to take full advantage of its nearly linear B versus H behavior. Also, very small (less than 13 mm on a side) samarium cobalt and neodymium iron boron magnets are common in many applications. The use of a search coil or Hall probe is undesirable for these materials in such cases. To test a short magnet with a search coil, it is difficult to fabricate a coil and fixture that can be placed around the sample and give repeatable readings. Hall probes become increasingly unreliable as the magnet dimensions shrink, because the distance between the magnet surface and the Hall probe element becomes a significant, yet poorly controlled, parameter from probe to probe. The exact placement of the probe on the surface of the magnet becomes more critical. Helmholtz coils overcome these difficulties.

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Description of Helmholtz Coils and the Measurement

Helmholtz coils are a pair of thin, parallel, and identical coils separated by a distance equal to their radius. These coils are commonly used to generate small, but highly uniform magnetic fields in the space between the coils [2]. However, this concept can be reversed to use the coils as flux sensing coils, instead of flux generating coils, to accurately measure open-circuit magnetization.

Fig. 1 shows a typical configuration of Helmholtz coils. The coils are connected in series, so that their signals add, and are fed directly into an integrating voltmeter or fluxmeter.

A measurement is made by placing the magnet at the center of the coils, as shown in Fig. 1, and zeroing the integrating voltmeter or fluxmeter. The magnet is then removed from the coil, parallel to the coil axis, to a distance such that the sample has no influence on the reading, typically 75 to 100 cm. The open-circuit magnetization of the sample is related to the time-integrated voltage by

$$4\pi M_0 = \frac{C}{V} \int E \, dt \qquad (1)$$

as first suggested by Martin and Benz [3], where V is the volume of the sample and C is a constant for the coil pair. A special correction is required for arc magnets and is described in Appendix I. The coil constant, C, can be estimated by

$$C = \frac{1.398 \times 10^8 \, r}{N} \quad \text{cm}^3 \cdot \text{G/V} \cdot \text{s} \qquad (2)$$

where r is the coil radius in centimeters and N is the number of turns per coil. (A derivation of (2) is given in Appendix II.) While (2) accurately estimates the coil constant, it is better to determine the coil constant by using several well characterized magnet samples, and to use (2) as a check. Also, (1) and (2) can be used to help design Helmholtz coils, by estimating the signal generated by a given magnet sample in a coil pair with a certain N and r.

Table I lists the coil constant calculated by (2) and determined by calibration with known magnet samples. The data show excellent agreement between the two approaches.

An alternate technique for the measurement is to rotate the magnet sample by 180° rather than extracting the sample. In this case, the generated voltage would be doubled and the coil constant would be halved.

If the open-circuit load line of the sample is known, the

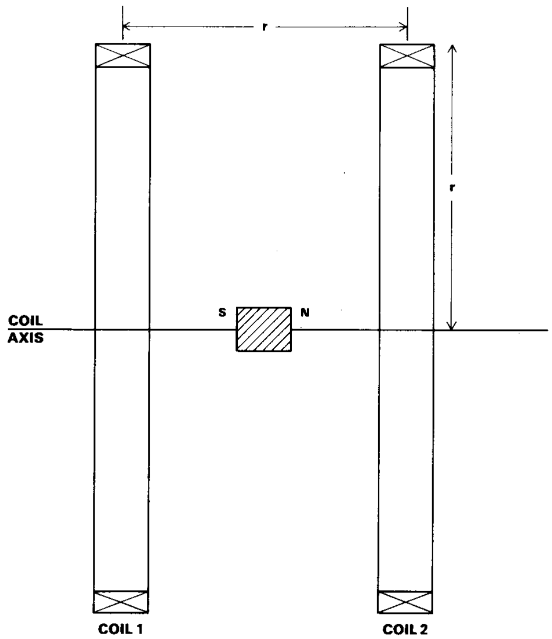

Fig. 1. Cross-sectional view of Helmholtz coils.

TABLE I
COMPARISON OF CALCULATED AND EMPIRICALLY DERIVED COIL CONSTANTS

Coil Size		Coil Constant ($cm^3 \cdot G/V \cdot s$)		
(cm)	N (turns)	Calculated	Derived	% Diff.
5.08	300	2.367	2.359	0.35
7.62	300	3.551	3.549	0.14
10.16	250	5.681	5.667	0.25
12.70	200	8.877	8.865	0.14

open-circuit induction (B_0) can be calculated by

$$B_0 = \frac{B/H}{(B/H + 1)} 4\pi M_0 \tag{3}$$

where B/H is the load line, taken as a positive number.

Construction of Helmholtz Coils

The coil forms and support system should be nonmetallic. The coil radius should be chosen so that it is at least 2.5 times the size of the largest dimension of the largest magnet to be tested. This constraint keeps all samples in the region of uniform sensitivity in the coils. One hundred or more turns per coil are commonly used, but (1) and (2) should be used to assure adequate sensitivity for small magnets, and to avoid overloading the instrumentation for large magnets. The wire diameter should be chosen so that the total coil resistance is consistent with the input impedance of the instrumentation, AWG 22 to 30 are typically used. Measurements should be made in an area away from stray magnetic field sources, e.g., other magnets, transformers, or fluorescent lights.

Typically, the repeatability of the measurement is 0.5 percent or less. The overall accuracy is usually in the 1- to 2-percent range.

Conclusion

Helmholtz coils are a versatile means of making rapid and accurate magnetic measurements on high H_c permanent magnet materials. The coils are particularly useful for small magnets, unusual shapes, such as arcs, or if a wide variety of shapes and sizes are to be measured. In many cases, a single coil pair can measure all the various magnets tested at a given facility.

Appendix I
Using Helmholtz Coils to Measure Arc Magnets

Helmholtz coils are sensitive only to magnetization that is parallel to the coil axis. In cases where the magnetization is not completely parallel to the coil axis, such as an arc magnet, the Helmholtz coils reading is lower than the true magnetization of the sample. A correction is given for an arc magnet with true radial orientation and for an intermediate case, called pseudo-radial orientation.

Fig. 2 shows the situation for a simple arc magnet with radial orientation placed in the center of the coils. By definition of radial orientation, the magnetization is perpendicular to the surface of the arc S. The component of magnetization parallel to the coil axis, at any point on the surface S is

$$M_{\parallel} = M \cos \theta \tag{4}$$

where θ is the angle between the coil axis and the point in question. The net magnetization parallel to the coil axis is determined by integrating over the entire arc

$$M_{\parallel} = \frac{\int_{-\alpha/2}^{\alpha/2} M \cos \theta \, d\theta}{\int_{-\alpha/2}^{\alpha/2} d\theta} = \frac{M2 \sin (\alpha/2)}{\alpha} \tag{5}$$

where the angles are in radians.

An intermediate condition between parallel orientation and radial orientation is shown in Fig. 3 and is termed pseudo-radial orientation. In this case, the magnetization is assumed to be half way between the parallel and radial cases. The component of magnetization parallel to the coil axis is

$$M_{\parallel} = M \cos (\theta/2) \tag{6}$$

where θ is the angle defined in (4). Again, the resultant magnetization is found by integration over the entire arc

$$M_{\parallel} = \frac{\int_{-\alpha/2}^{\alpha/2} M \cos \left(\frac{\theta}{2}\right) d\theta}{\int_{-\alpha/2}^{\alpha/2} d\theta} = \frac{M4 \sin (\alpha/4)}{\alpha} \tag{7}$$

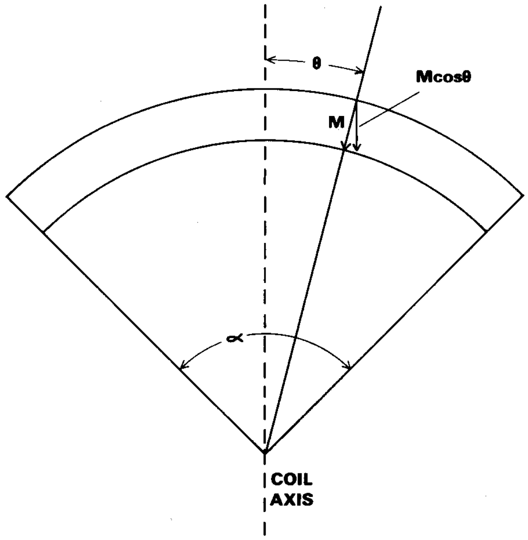

Fig. 2. Arc magnet with radial orientation.

TABLE II
HELMHOLTZ COILS CORRECTIONS FOR ARC MAGNETS

Arc Angle (degrees)	True Radial M(Helmholtz)/M(true)	Pseudo-Radial M(Helmholtz)/M(true)
10	1.00	1.00
20	0.99	1.00
30	0.99	1.00
40	0.98	0.99
50	0.97	0.99
60	0.95	0.99
70	0.94	0.98
80	0.92	0.98
90	0.9	0.97
100	0.88	0.97
110	0.85	0.96
120	0.83	0.95
130	0.8	0.95
140	0.77	0.94
150	0.74	0.93
160	0.71	0.92
170	0.67	0.91
180	0.64	0.90

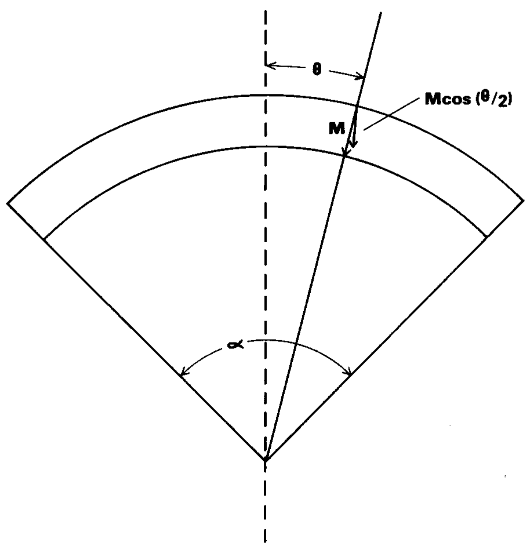

Fig. 3. Arc magnet with pseudo-radial orientation.

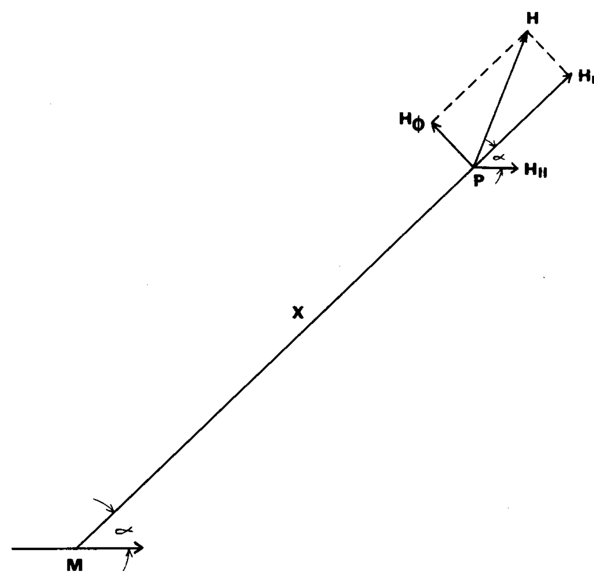

Fig. 4. Field of a dipole magnet.

where the angles are in radians. Correction factors, based on (5) and (7), are listed in Table II for various arc angles.

As one would expect, the correction is small for small arc angles, for either type of orientation. In general, hard ferrite arc magnets tend to agree with the radial model and are usually made with relatively wide arc angles (100° to 140°). Samarium cobalt and neodymium iron boron magnets tend to agree with the pseudo-radial model [4].

APPENDIX II
DERIVATION OF EQUATION (2)

Because the Helmholtz coils have a region of uniform sensitivity near the center, a permanent magnet that is small compared to the diameter of the coils can be approximated as a magnetic dipole. The magnetic field generated by such a dipole at a point P, as shown in Fig. 4, can be written as

$$H_r = \frac{2m \cos \alpha}{x^3} \tag{8}$$

$$H_\phi = \frac{m \sin \alpha}{x^3} \tag{9}$$

where m is the magnetic moment of the dipole [5]. To find the flux that interacts with the coil, the component of field parallel to the dipole, at a point P, $H_\parallel$ can be written as

$$\begin{aligned} H_\parallel &= H_r \cos \alpha - H_\phi \sin \alpha \\ &= \frac{m}{x^3}(3\cos^2 \alpha - 1). \end{aligned} \tag{10}$$

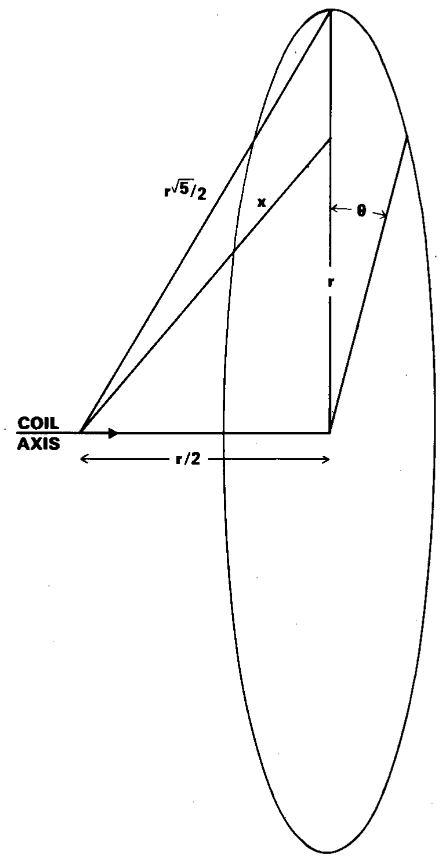

Fig. 5. Interaction of a dipole magnet with a single turn in the Helmholtz coils.

The amount of flux that interacts with a single turn of the Helmholtz coils is found by integrating $H_{\|}$ over the area of a turn, as shown in Fig. 5. This flux can be expressed as

$$\phi = \int_A H_{\|} \, dA = \int_0^{2\pi} \int_{(r/2)}^{r(\sqrt{5}/2)} H_{\|} \, x \, dx \, d \qquad (11)$$

where r is the coil radius. The integral over ϕ reduces to 2π, since $H_{\|}$ and x are independent of ϕ. Also x is related to α by

$$x = r/(2 \cos \alpha). \qquad (12)$$

The integral can be reduced to

$$\phi = 2\pi m \int_{(r/2)}^{r(\sqrt{5}/2)} \frac{(3r^2 - 1)}{4x^2} \frac{dx}{x^2}. \qquad (13)$$

Solving for ϕ we find

$$\phi = \frac{16\pi m}{r 5 \sqrt{5}}. \qquad (14)$$

Magnetic moment is related to the magnetization by

$$m = V(4\pi M)/4\pi \qquad (15)$$

where V is the volume of the sample. From Faraday's law, we have

$$\int E \, dt = 10^{-8} N\phi \qquad (16)$$

where N is the number of turns in the coil. Substituting (15) into (14) and then (14) into (16), and by comparing the combined results to (1), the coil constant is given by

$$C = \frac{1.398 \times 10^8 r}{N} \text{ cm}^3 \cdot \text{G} \cdot \text{turn/V} \cdot \text{s} \qquad (17)$$

where N is the number of turns per coil.

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