# Why do bosons tend to occupy the same state?

I understand that the symmetry of the wavefunction allows many bosons to be in the same one-particle state, but I can't see why they should prefer to do that rather than occupying different states.

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Suppose you have two distinguishable coins that can either come up heads or tails. Then there are four equally likely possibilities,

$$HH,\ HT,\ TH,\ TT$$

There is a 50% chance for the two coins to have the same result.

If the coins were fermions and "heads/tails" were quantum modes, $|HH\rangle$ and $|TT\rangle$ states wouldn't be allowed, so there is a 0% chance for the two coins to have the same result.

If the coins were bosons, then all states are allowed. But there's a twist: bosons are identical particles. The states $|HT\rangle$ and $|TH\rangle$ are precisely the same state (or more precisely, in the $3$D symmetrised Hilbert space, the state $|HT\rangle + |TH\rangle$), namely the one with one particle in each of the two modes. So there are three possibilities :

$$|HH\rangle,\ |TT\rangle,\ |HT\rangle + |TH\rangle$$

and hence *in the microcanonical ensemble* (where each distinct quantum state is equally probable) there is a $2/3$ chance of double occupancy, not $1/2$. That's what people mean when they say bosons "clump up", though it's not really a consequence of bosonic statistics, just a consequence of the *particles being identical*. Whenever a system of bosonic particles is in thermal equilibrium, there exist fewer states with the bosons apart than you would naively expect, if you treated them as distinguishable particles, so you are more likely to see them together.

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- Thank you very much for your answer. However, there's a thing that troubles me: your argument works for two bosons that can choose between two states, but let's say that our two bosons can choose among $k$ states. Then there will be $k$ cases in which they occupy the same state, and $k(k-1)/2$ cases in which they don't. If $k > 3$, these last cases are more than the others.

- Yes, but compare this to the naive non-identical particle reasoning: in that case there are $k$ states where they are the same, but $k(k-1)$ when they aren't. For the bosons, this latter number is cut in half. The statement is not that any two bosons are more than 50% likely to be in the same state, it's that you get **more clumping with bosons than with distinguishable particles** when you average over all states.

- Now I think I understand. So, two bosons actually don't need to have a high probability of being in the same state. They just have more probability of being so (roughly the double in our example) than they would if they were distinguishable.

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